## |V<sub>ub</sub>| "疑难" 和 B 介子四体半轻衰变

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### I: |Vub| 疑难问题

### II: B 介子四体半轻衰变的初步探索

III: 总结和展望

# |V<sub>ub</sub>| 疑难问题

### |Vub| 疑难问题



- $VV^{\dagger} = V^{\dagger}V = I_3$  in the Standard Model
- $VV^{\dagger} \neq V^{\dagger}V \neq I_3$  New Physics
- $|V_{ub}|/|V_{cb}|$  contributes to CPV measurement in B decays
- CKM matrix elements are mainly measured via the charged current processes, i.e, b → ul<sup>-</sup>ν, b → cl<sup>-</sup>ν, c → sl<sup>+</sup>ν
- Flavor changing neutral current precesses are sensitive to new physical contributions, i.e,  $b \rightarrow sl^+l^-, b \rightarrow dl^+l^-$



### $|V_{\mu\nu}|$ 疑难问题

- $|V_{ub}|$  tension  $|V_{ub}| = (3.82 \pm 0.20) \times 10^{-3}$  [PDG 2024]
- $\pm \sim 2.5\sigma$  tension between  $(4.13 \pm 0.25) \times 10^{-3}$  and  $(3.70 \pm 0.16) \times 10^{-3}$ measured via the  $B \to X_{\mu} l^- \bar{\nu}$  and  $B \to \pi l^- \bar{\nu}$  processes, respectively.
- $|V_{cb}|$  tension  $|V_{cb}| = (41.1 \pm 1.2) \times 10^{-3}$  [PDG 2024]
- $\pm \sim 2.5\sigma$  tension between  $(42.2 \pm 0.5) \times 10^{-3}$  and  $(39.8 \pm 0.6) \times 10^{-3}$ measured via the  $B \to X_c l^- \bar{\nu}$  and  $B \to D^{(*)} l^- \bar{\nu}$  processes, respectively.



p-value

0.9

0.8

0.7

0.6

05

0.4

0.3

0.2

0.1

0.0

V<sub>a</sub> (dashed

### B 介子半轻衰变的反常现象

- LFU in  $b \to cl^- \bar{\nu}$  processes  $R_{D^{(*)}} = \mathcal{B}(B \to D^{(*)}\tau^- \bar{\nu})/\mathcal{B}(B \to D^{(*)}\mu^- \bar{\nu})$
- $\ddagger R_D = 0.407 \pm 0.046, R_{D^*} = 0.306 \pm 0.015$  Average with [Belle PRL124, 161803 (2020)]
- $\ddagger 2.1\sigma, 3.0\sigma$  derivations from the SM predictions of  $R_D = 0.298 \pm 0.004, R_{D^*} = 0.254 \pm 0.005$  [HFLAV]
- $\ddagger R_D = 0.441 \pm 0.089, R_{D^*} = 0.281 \pm 0.030$  [LHCb PRL131,111802 (2023)]
- ‡ would make the CKM measurements more complicated if confirmed
- Anomalies in FCNC processes  $B \rightarrow K^* \mu^+ \mu^-$
- $\ddagger~3.6\sigma$  derivation from SM of  $d{\cal B}(B\to K^*\mu^+\mu^-)/dq^2$  in  $q^2\in[1,6]~{\rm GeV}^2$
- $\ddagger~1.9\sigma$  derivation from SM of  $\textit{p}_5'=\textit{S}_5/\sqrt{\textit{F}_L(1-\textit{F}_L)}$  in  $\textit{q}^2\in[4,8]~\text{GeV}^2$



[Heavy Flavour Physics and CP Violation at LHCb: a Ten-Year Review, Front. Phys.18,44601(2023)]

### 解决方案 在传统过程继续奋斗

• 更精确的测量和格点计算

 $|V_{cs}|$  issue  $|V_{cs}| = 0.975 \pm 0.006$  [PDG 2022, 24]

\*  $0.972 \pm 0.007$  and  $0.984 \pm 0.012$  measured via the  $D \rightarrow K l \nu$ and  $D_s \rightarrow \mu^+ \nu_\mu$  processes  $\sim 3\sigma \rightarrow \sim 1.5\sigma$ 理论与实验联合研讨的成功典范

• 更全面更系统的物理分析方法

 $|V_{ub}|$  result from Belle collaboration with Simultaneous Determination in excl. and incl. processes [Belle PRL131, 211801 (2023)]

- \*  $(3.78 \pm 0.23 \pm 0.16 \pm 0.14) \times 10^{-3}$  and  $(3.88 \pm 0.20 \pm 0.31 \pm 0.09) \times 10^{-3}$
- 精细结构、丰富的 QCD 效应 high order QCD corrections, more structures [AK, TM, YMW, JHEP 02 (2013) 010] [AK, TM, AAP, YMW, JHEP 09 (2010) 089]

L			
ETM	PRD96,054514	0.765:0.031	2010-22021
HPQCI	PRD104,034505	0.7380::0.0044	2.4→0.6%
Belle	PRL97,061804, D <sup>8</sup> -+KTV	0.995::0.007::0.022	
BaBar	PRD76,052005, D <sup>8</sup> -+K'e*v	0.727±0.007±0.009	
CLEO	PRD80,032005, DRe*v	0.739±0.007±0.005	
BESH	PRD92,112008, D*→K <sup>0</sup> <sub>1</sub> e*v	0.748:0.007:0.012	-
BESH	$PRD96,\!012002,D^*\!\!\rightarrow\!\!K^0_g e^*\!\!\vee$	0.7246±0.0041±0.0115 -	
BESH	$PRL122,011804, D^0{\rightarrow}K'\mu^{\rm tr}$	0.7327±0.0039±0.0030	
BESH	PRD92,072012, D <sup>8</sup> -+K'e*+	0.7368±0.0026±0.0036	0.7%
DESH	Expected (20fb <sup>-1</sup> ), D <sup>6</sup> -+K <sup>*</sup> e <sup>+</sup> v	0.7368±0.0009±0.0036	
0.2	0.3 0.4 0 f_*	.5 0.6 0.7 <sup>K</sup> (0)	0.8



 $= \mathscr{B}(B \to X_u \ell \nu)$ 



### 解决方案 寻找新的增长极





#### $\downarrow |V_{ub}|f_B$ in pure leptonic decay

- \*  $0.72 \pm 0.09$  MeV from Belle,  $1.01 \pm 0.14$  MeV from BABAR,
  - $0.77\pm0.12$  MeV average [FLAG2021]

#### $\downarrow |V_{ub}|$ in baryon decay

[LHCb Nature Physics 11, 743-747 (2015)]

$$\frac{|V_{ub}|^2}{|V_{cb}|^2} = \frac{\mathcal{B}(\Lambda_b^0 \to p\mu^-\bar{\nu})}{\mathcal{B}(\Lambda_b^0 \to \Lambda_c^+\mu^-\bar{\nu})} R_{\rm FF} = 0.68 \pm 0.07 \Downarrow$$
  
$$\frac{|V_{ub}|}{|V_{cb}|} = 0.083 \pm 0.06 \xrightarrow{|V_{cb}|} |V_{ub}| = (3.72 \pm 0.23) \times 10^{-3}$$

\* consistent with determinations in exclusive  $B \rightarrow \pi l \bar{\nu}$  decay confirms the existing incompatibility with the inclusive sample

$$\ddagger |V_{ub}|/|V_{cb}|$$
 in  $\mathcal{B}(B_s o K^- \mu^+ 
u)/\mathcal{B}(B_s o D_s^- \mu^+ 
u)$  [LHCb prl126, 081804 (2021)]

$$\ddagger |V_{cb}| \text{ in } B_s \to D_s \mu^+ \nu, \quad \frac{d\mathcal{B}(B \to K^* \mu^+ \mu^-)}{dq^2} \text{ and } p'_5 \text{ in } \Lambda^0_b \to \Lambda \mu^+ \mu^-$$

### 解决方案

- 以上过程都只涉及到基态粒子
- 含有激发态粒子的过程也可以提供独立的测量
- $|V_{ub}|$  in the B o
  ho l
  u channel the same b o ul
  u transition as in the golden channel
- simultaneous measurements of the  $\frac{dB}{dq^2}$  for  $B \to \pi^- l^+ \nu_l$  and  $B \to \rho^0 l^+ \nu_l$ [Belle-II arXiv:2407.17403[hep-ex]] [HFLAV 2023] [P. Bharucha and et.al., JHEP 1608. 098]

$$\begin{aligned} |V_{ub}|_{B \to \pi l\nu} &= (3.73 \pm 0.10 \pm 0.16 |_{\rm LQCD+LCSRs}) \times 10^{-3}, \\ |V_{ub}|_{B \to \rho l\nu} &= (3.19 \pm 0.22 \pm 0.26 |_{\rm LCSRs}) \times 10^{-3}. \end{aligned}$$

•  $B \rightarrow V \text{ FFs}$  updated via *B*-meson LCSRs [Gao, et.al., PRD 101. 074035(2020)]

$$\begin{split} |V_{ub}|_{B \to \rho l\nu} &= \left(3.05^{+1.34}_{-1.30}|_{\text{theo}} \, {}^{+0.19}_{-0.20}|_{\text{data}}\right) \times 10^{-3}, \\ |V_{ub}|_{B \to \omega l\nu} &= \left(2.54^{+1.09}_{-1.05}|_{\text{theo}} \, {}^{+0.18}_{-0.19}|_{\text{data}}\right) \times 10^{-3} \end{split}$$

- a notably smaller value is obtained in  $B \rightarrow \rho$  transition
- $|V_{ub}|_{B_s \to Kl\nu} = 3.58(9) \times 10^{-3}$  is consistent with the "golden" channel [PRD104. 114041 (2021)]
- the uniformity of |V<sub>ub</sub>| determinations across different exclusive channels ?

## B 介子四体半轻衰变的初步探索

### B 介子四体半轻衰变的探索

- $\rho$  is an unstable resonance that decays into  $\pi\pi$  via the strong interaction
- \* the signal channel in a  $B \rightarrow \rho l \nu$ -type decay is  $B \rightarrow \pi \pi l \nu (B_{l_4})$
- \* the  $\pi\pi$  spectra in [0.554, 0.996] GeV serves as the candidate region for  $\rho$ [X.-W. Kang, B. Kubis, C. Hanhart, and U.-G. Meißner, PRD 89. 053015 (2014)] [S. Faller, T. Feldmann, A. Khodjamirian, T. Mannel and D. van. Dyk PRD 89. 014015 (2014)]
- \* the QCD studies usually treat  $\rho$  as a stable single particle
- mismatch between experimental measurements and theoretical calculations, particularly in accounting for the finite width and nonresonant backgrounds
- how to address the finite width of the  $\rho$  mesons, nonresonant QCD backgrounds, and the effects of different partial waves in the calculation of  $B \rightarrow \pi\pi$  form factor?
- $|V_{cb}|$  in the  $B \to D^* l \nu$  processes, B anomalies in  $B \to K^* l^+ l^-$  processes

$$\begin{split} i\langle \pi^{+}(k_{1})\pi^{-}(k_{2})|\bar{u}\gamma_{\nu}(1-\gamma_{5})b|\bar{B}^{0}(p)\rangle &= F_{\perp}(q^{2},k^{2},\zeta) \frac{2}{\sqrt{k^{2}}\sqrt{\lambda_{B}}} i\epsilon_{\nu\alpha\beta\gamma} q^{\alpha} k^{\beta} \bar{k}^{\gamma} \\ &+ F_{t}(q^{2},k^{2},\zeta) \frac{q_{\nu}}{\sqrt{q^{2}}} + F_{0}(q^{2},k^{2},\zeta) \frac{2\sqrt{q^{2}}}{\sqrt{\lambda_{B}}} \left(k_{\nu} - \frac{k \cdot q}{q^{2}}q_{\nu}\right) \\ &+ F_{\parallel}(q^{2},k^{2},\zeta) \frac{1}{\sqrt{k^{2}}} \left(\bar{k}_{\nu} - \frac{4(q \cdot k)(q \cdot \bar{k})}{\lambda_{B}} k_{\nu} + \frac{4k^{2}(q \cdot \bar{k})}{\lambda_{B}} q_{\nu}\right) \end{split}$$

† 
$$\lambda = \lambda(m_B^2, k^2, q^2)$$
 is the Källén function  
†  $q \cdot k = (m_B^2 - q^2 - k^2)/2$  and  $q \cdot \bar{k} = \sqrt{\lambda}\beta_{\pi}(k^2)\cos\theta_{\pi}/2 = \sqrt{\lambda}(2\zeta - 1)$   
†  $\beta_{\pi}(k^2) = \sqrt{1 - 4m_{\pi}^2/k^2}$ ,  $\theta_{\pi}$  is the angle between the 3-momenta of the neutral pion and the B-meson in the dipion rest frame

- LQCD (Lattice QCD) in the ρ resonance region with a simple BW model
   [L. Leskovec, S. Meinel, M. Petschlies, J. Negele, S. Paul and A. Pochinsky, arXiv:2501.00903 [hep-lat]]
- **HChPT** (Heavy-meson Chiral Perturbative Theory) in the large  $q^2$  by taking dispersive methods in terms of Omnés functions
  - · [X.-W. Kang, B. Kubis, C. Hanhart, and U.-G. Meißner, PRD 89. 053015 (2014)]
  - in the full phase-space by a novel parameterization with unitarity
  - [F. Herren, B. Kubis and R. van Tonder, arXiv:2502.20960 [hep-ph]]
- QCDF (QCD factorization) in the large dipion mass
  - · [P. Böer, T. Feldmann and D. van Dyk, JHEP02, 133(2017)]

 $T_I \propto F_{Bto\pi} \otimes \phi_{\pi}, T_{II} \propto \phi_B \otimes \phi_{\pi} \otimes \phi_{\pi}$ 

- LCSRs (Light-cone sum rules) in the small and intermediate  $q^2$ 
  - [S. Cheng, A. Khodjamirian and J. Virto, JHEP 05, 157(2017)] B-meson LCSRs
  - [C. Hambrock and A. Khodjamirian, NPB 905(2016)379-390]  $2\pi$ DAs LCSRS of  $F_{\parallel,\perp}$
  - [S. Cheng, A. Khodjamirian and J. Virto, PRD(R) 96 (2017)051901] timelike-helicity FF  $F_t$  and  $F_0$
  - [S. Cheng, PRD 99 (2019) 053005]  $2\pi$ DAs updates and  $B \rightarrow [\pi\pi]_{S,P}$  FFs
  - [S. Cheng and J.M Shen, EPJC(2020)6:554, S. Cheng and S.L Zhang, EPJC (2024)84:379] Pheno
  - [S. Cheng, arXiv: 2502.07333[hep-ph]] first study of twist-three  $2\pi$ DAs and  $|V_{ub}|$  extraction

•  $B 
ightarrow \pi \pi$  form factors from the *B*-meson LCSRs

$$F_{\mu\nu}(k,q) = i \int d^4 x \, e^{ik \cdot x} \langle 0 | \mathrm{T}\{j_{\mu}(x), j_{\nu}^{V-A}(0)\} | \overline{B}^0(q+k) \rangle$$
  
$$\equiv \varepsilon_{\mu\nu\rho\sigma} q^{\rho} k^{\sigma} F_{(\varepsilon)}(k^2, q^2) + ig_{\mu\nu} F_{(g)}(k^2, q^2) + iq_{\mu} k_{\nu} F_{(qk)}(k^2, q^2) + \cdots$$

•  $B \rightarrow \pi \pi$  form factors from the  $2\pi \text{DAs LCSRs}$ 

$$F_{\mu}(k_{1}, k_{2}, q) = i \int d^{4}x e^{iq \cdot x} \langle \pi^{+}(k_{1})\pi^{-}(k_{2}) | \mathrm{T}\{j_{\mu}^{V-A}(x), j_{5}(0)\} | 0 \rangle$$
  
$$\equiv \varepsilon_{\mu\nu\rho\sigma} q^{\nu} k_{1}^{\rho} k_{1}^{\sigma} F^{V} + q_{\mu} F^{(A,q)} + k_{\mu} F^{(A,k)} + \bar{k}_{\mu} F^{(A,\bar{k})}$$

- Advantages of the  $2\pi$ DAs LCSRs
  - $\ast~$  the QCD calculation does not rely on any resonant model, however, encounter a inverse problem in B-meson LCSRs

$$2 \operatorname{Im} F_{\mu\nu}(k,q) = \int d\tau_{2\pi} \langle 0|\bar{d}\gamma_{\mu}u|\pi^{+}\pi^{-}\rangle \langle \pi^{+}\pi^{-}|\bar{u}\gamma_{\nu}(1-\gamma_{5})b|\bar{B}^{0}\rangle + \cdots$$

- \* provides a unifies framework to predict contributions from different partial-waves
- \* theoretical uncertainties are better controlled, higher-power corrections are suppressed by  ${\cal O}(1/m_b)$  when  $k^2$  is not too large

### $2\pi \mathsf{DAs}$

- Chiral-even LC expansion with gauge factor [x, 0] [Polyakov 1999, Diehl 1998]  $\langle \pi^{a}(k_{1})\pi^{b}(k_{2})|\overline{q}_{f}(zn)\gamma_{\mu}\tau q_{f}(0)|0\rangle = \kappa_{ab} k_{\mu} \int dx e^{iuz(k\cdot n)} \Phi_{\parallel}^{ab,ff'}(u,\zeta,k^{2})$
- ‡ Three independent kinematic variables
- 2 $\pi$ DAs is decomposed in terms of  $C_n^{3/2}(2z-1)$  and  $C_\ell^{1/2}(2\zeta-1)$

$$\begin{split} \Phi^{l=1}(z,\zeta,k^2,\mu) &= 6z(1-z) \sum_{n=0,\text{even}}^{\infty} \sum_{l=1,\text{odd}}^{n+1} B_{n\ell}^{l=1}(k^2,\mu) C_n^{3/2}(2z-1) C_{\ell}^{1/2}(2\zeta-1) \\ \Phi^{l=0}(z,\zeta,k^2,\mu) &= 6z(1-z) \sum_{n=1,\text{odd}}^{\infty} \sum_{l=0,\text{even}}^{n+1} B_{n\ell}^{l=0}(k^2,\mu) C_n^{3/2}(2z-1) C_{\ell}^{1/2}(2\zeta-1) \end{split}$$

- How to describe the evolution from  $4m_{\pi}^2$  to large invariant mass  $k^2$  ?
- $\ddagger$  Watson theorem of  $\pi\text{-}\pi$  scattering amplitudes

$$B_{n\ell}^{l}(k^{2}) = B_{n\ell}^{l}(0) \exp\left[\sum_{m=1}^{N-1} \frac{k^{2m}}{m!} \frac{d^{m}}{dk^{2m}} \ln B_{n\ell}^{l}(0) + \frac{k^{2N}}{\pi} \int_{4m_{\pi}^{2}}^{\infty} ds \frac{\delta_{\ell}^{l}(s)}{s^{N}(s-k^{2}-i0)}\right]$$

 $\bigtriangleup 2\pi {\sf DAs}$  in a wide range of energies is given by  $\delta_\ell^I$  and a few subtraction constants

### $2\pi \mathsf{DAs}$

• The subtraction constants of  $B_{n\ell}(s)$  at low s (around the threshold)

(nl)	$B_{n\ell}^{\parallel}(0)$	$\mathbf{c}_1^{\parallel,(\mathit{nl})}$	$\frac{d}{dk^2} \ln B^{\parallel}_{n\ell}(0)$	$B_{n\ell}^{\perp}(0)$	$c_1^{\perp,(\mathit{nl})}$	$\frac{d}{dk^2} \ln B_{n\ell}^{\perp}(0)$
(01) (21) (23)	$\begin{vmatrix} 1 \\ -0.113 \rightarrow 0.218 \\ 0.147 \rightarrow -0.038 \end{vmatrix}$	0 -0.340 0	$1.46 \rightarrow 1.80$ 0.481 0.368	$\begin{array}{c} 1 \\ 0.113 \rightarrow 0.185 \\ 0.113 \rightarrow 0.185 \end{array}$	0 -0.538 0	$0.68 \rightarrow 0.60$ -0.153 0.153
(10) (12)	-0.556 0.556	-	0.413 0.413		-	

m riangle firstly studied in the effective low-energy theory based on instanton vacuum [Polyakov 1999]

 $\triangle$  updated with the kinematical constraints and the new  $a_2^{\pi}$  ,  $a_2^{
ho}$  [SC 2019, 2023]

- All the above discussions are at leading twist
- twist three 2πDAs are studied recently [S. Cheng, arXiv:2502.07333[hep-ph]]

 $2\pi$ DAs widely used in the three-body *B* decays studied from pQCD and QCDF are the asymptotic formula [J. Chai, S. Cheng and A.J Ma PRD 105 (2022) 033003]

normalized to unit as  $\Gamma_{m,M_2}^{d-1}(0) = 1$ . When the invariant mass of dimeson system is small, the higher  $\mathcal{O}(s)$  terms in the expansion of coefficient  $B_{nl}(s,\mu)$  around the resonance pole can be safely neglected due to the large suppression  $\mathcal{O}(s/m_h^2)$  in contrast to the energetic dimeson system in *B* decay, so the relation  $B_{n1}(s,\mu) \to a_n(\mu)\Gamma_{m,M_2}^{d-1}(s)$  can be obtained in the lowest partial wave approximation. This argument induces the basic assumption in PQCD that the energetic dimeson DAs can be deduced from the DAs of resonant meson by replacing the decay constant by the 16/22

• Starting with the correlation functions

$$\Gamma_{\mu} = i \int d^{4}x e^{iq \cdot x} \langle \pi^{+}(k_{1})\pi^{0}(k_{2}) | \mathrm{T}\{j_{\mu}^{V-A}(x), j_{5}(0)\} | 0 \rangle$$

$$\equiv \ \varepsilon_{\mu\nu\rho\sigma} q^{\nu} k_{1}^{\rho} k_{1}^{\sigma} F^{V} + q_{\mu} F^{(A,q)} + k_{\mu} F^{(A,k)} + \bar{k}_{\mu} F^{(A,\bar{k})},$$

$$\Gamma_5 = i \int d^4 x e^{i q \cdot x} \langle \pi^+(k_1) \pi^0(k_2) | \mathrm{T}\{j_5(x), j_5(0)\} | 0 \rangle \propto \mathcal{F}_t(k^2, q^2)$$

• take the perpendicular FF  $\textit{F}_{\perp}$  for example

$$F_{\perp}(q^{2},k^{2},\theta_{\pi}) = \frac{im_{b}^{2}\sqrt{\lambda k^{2}}}{2m_{B}^{2}f_{B}} \Big\{ \int_{u_{0}}^{1} du \Big[ \frac{\Phi_{\perp}}{um_{b}f_{2\pi}^{\perp}\beta_{\pi}\cos\theta_{\pi}} + \frac{\Phi_{\perp}^{(a)}}{2u^{2}M^{2}} \Big] e^{\frac{m_{B}^{2}-s(u,q^{2})}{M^{2}}} + \frac{\Phi_{\perp}^{(a)}(u_{0})}{2u_{0}(s_{0}-q^{2})} e^{\frac{m_{B}^{2}-s_{0}}{M^{2}}} \Big\}$$

 $\downarrow$  partial – wave expansion

$$F_{\perp}^{(l'=1,3,\cdots)}(q^{2},k^{2}) = \frac{im_{b}^{2}\sqrt{\lambda k^{2}}}{2m_{B}^{2}f_{B}}\sqrt{\frac{2l'+1}{2}}\frac{(l'-1)!}{(l'+1)!}\sum_{n=0,\text{even}}^{\infty}\sum_{l=1,\text{odd}}^{n+1} B_{nl}^{\perp}(k^{2},\mu)J_{n}^{\perp}(s_{0},M^{2},q^{2},k^{2})I_{ll'}^{\parallel},$$

$$J_{n}^{\perp} = \int_{u_{0}}^{1} du \frac{6u\bar{v}C_{n}^{3/2}(2u-1)}{um_{b}f_{2\pi}^{\perp}}e^{\frac{m_{B}^{2}-s(u,q^{2})}{M^{2}}}, \ I_{ll'}^{\parallel} = \int_{-1}^{1} d(\cos\theta_{\pi})\frac{\sin\theta_{\pi}}{\beta_{\pi}\cos\theta_{\pi}}P_{l'}^{(1)}(\cos\theta_{\pi})P_{l}^{(0)}(\beta_{\pi}\cos\theta_{\pi}).$$





- twist-three contribution vanish in  $F_{\perp,\parallel}^{(l=1,3,\cdots)}$
- persist in  $F_{t,0}^{(l=0,1,\cdots)}$ , give a significant correction  $\sim 40\%$  to leading twist result
- $\rho$  resonance is dominate in  $F_{\perp,\parallel,t,0}^{(l=1)}(q^2,k^2)$ , *F*-wave contributions are negligible
- S-wave component is dominate in the small  $k^2,\,D$ -wave component is comparable in the large  $k^2$  and small  $q^2$

### $|V_{ub}|$ extraction from $B_{l4}$ decay

• 2D partial differential decay width

$$\frac{d^2\Gamma}{dq^2dk^2} = G_F^2|V_{ub}|^2 \frac{\beta_{\pi}\sqrt{\lambda}q^2}{3(4\pi)^5 m_B^3} \Big[ \left( |F_0^{(5)}|^2 + |F_0^{(P)}|^2 \right) + \beta_{\pi}^2 \left( |F_{\parallel}^{(P)}|^2 + |F_{\perp}^{(P)}|^2 \right) + \cdots \Big]$$

• Partial branching fractions  $\Delta B^i$  (in unit of  $10^{-5}$ ) in different bins take the PDG average value of  $|V_{ub}| = (3.82 \pm 0.20) \times 10^{-3}$ 

bins	$\sqrt{s}$	$q^2$	$ riangle \mathcal{B}^i$	$\triangle B^i$ [Belle 2021]
1	$[4m_{\pi}^2, 0.6]$	[0, 8]	$0.27 \pm 0.03 \pm 0.06$	$0.84^{+0.39}_{-0.32} \pm 0.18$
2	(0.6, 0.9]	[0,4]	$1.91 \pm 0.21 \pm 0.38$	$2.39^{+0.53}_{-0.47} \pm 0.32$
3	(0.6, 0.9]	(4, 8]	$1.54 \pm 0.17 \pm 0.27$	$2.16^{+0.47}_{-0.42} \pm 0.23$
4	(0.9, 1.2]	[0,4]	$0.65 \pm 0.07 \pm 0.12$	$0.70^{+0.32}_{-0.25} \pm 0.20$
5	(0.9, 1.2]	(4, 8]	$0.41 \pm 0.04 \pm 0.08$	$0.64^{+0.28}_{-0.22} \pm 0.11$
6	(1.2, 1.5]	[0, 4]	$0.57 \pm 0.05 \pm 0.10$	$0.91^{+0.35}_{-0.28} \pm 0.12$
7	(1.2, 1.5]	(4, 8]	$0.16 \pm 0.02 \pm 0.02$	$0.64^{+0.32}_{-0.26} \pm 0.08$

•  $|V_{ub}|$  extraction in the regions of ho and  $f_0$  resonances

 $|V_{ub}|_{B^+ \to [\rho^0 \to] \pi^+ \pi^- l^+ \nu_l} = (4.27 \pm 0.49|_{\text{Data}} \pm 0.55|_{\text{LCSRs}}) \times 10^{-3}$ 

 $|V_{ub}|_{B^+ \to [f_0 \to]\pi^+\pi^- l^+\nu_l} = (3.96 \pm 0.47|_{\text{Data}} \pm 0.52|_{\text{LCSRs}}) \times 10^{-3}$ 

# 总结和展望

### 总结和展望

- |V<sub>ub</sub>| "疑难"是重味物理研究的一个核心问题
- B 介子四体半轻衰变(B<sub>4</sub>)提供了一个新的解决方案
- 两介子系统光锥分布振幅  $(2\pi DAs)$  是  $B_4$  研究的关键部件
- 首次在三扭度水平研究了 2πDAs, 实现了在 B<sub>l4</sub> 过程抽取 |V<sub>ub</sub>|
- 解释了通过  $B \rightarrow \rho l \nu$  过程测量的较小结果
- 当前的理论误差和实验误差都比较大,但是都可以被进一步减小
- ‡ Belle-II 积分亮度未来五年有望达到 3ab<sup>-1</sup>,实验精度将至少提高 3 倍
- ‡ LCSRs 和 LQCD 对形状因子的联合分析,至少可以将理论误差减小一半
- B14 过程将在 |Vub| 测量和味物理反常检验中扮演重要角色
- ‡  $B^0 \to \pi^- \pi^0 l^+ \nu_l$ 和  $B^+ \to \pi^0 \pi^0 l^+ \nu_l$ 过程的测量将进一步帮助理解  $2\pi$ DAs
- B14 过程的研究正在进入高精度时代

Thank you for your patience.