



南京大學  
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# Impact of Quark Compositions in Sigma Mesons on Nuclear Matter and Neutron Star

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Lanzhou University, Lanzhou, Mar. 22nd





# Outline

- Motivation (**Phenomenons** and **theories for dense nuclear matter**)
- Theoretical framework and phenomenological analysis (**Nuclear matter** properties and **neutron star** structures with an **extended linear sigma model** (mixing picture between 2- and 4-quark configurations of scalar mesons))
- Summary and outlook



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# Motivation

**Rich** phenomena of dense environments

**Weak** parameterizations in past studies on nuclear matter





# Nuclei structures (low/intermediate densities)

- Hadron interactions around saturation density  $n_0 = 0.16 \text{fm}^{-3}$  are crucial to nuclei structures, e.g.  $^{24}\text{Mg}$ ,  $^{90}\text{Zr}$ ,  $^{116}\text{Sn}$  and  $^{208}\text{Pb}$

$$E(n, \alpha) = E_0(n) + E_{\text{sym}}(n)\alpha^2 + O(\alpha^4)$$

B. A. Li and C. M. Ko, Nucl. Phys. A 618, 498 (1997).  
 D. H. Youngblood, H. L. Clark, and Y. W. Lui, Phys. Rev. Lett. 82, 691 (1999).  
 A. W. Steiner and B. A. Li, Phys. Rev. C 72, 041601 (2005).  
 L. W. Chen, C. M. Ko, and B. A. Li, Phys. Rev. C 72, 064309 (2005).  
 A. E. L. Dieperink, Y. Dewulf, D. Van Neck, M. Waroquier, and V. Rodin, Phys. Rev. C 68, 064307 (2003).  
 S. Karataglidis, K. Amos, B. A. Brown, and P. K. Deb, Phys. Rev. C 65, 044306 (2002).  
 R. J. Furnstahl, Nucl. Phys. A 706, 85 (2002).  
 B. A. Brown, Phys. Rev. Lett. 85, 5296 (2000).

$$\alpha = (n_n - n_p) / (n_n + n_p)$$

$$E_0(n) = E_0(n_0) + \frac{K_0}{2!} \chi^2 + \frac{J_0}{3!} \chi^3 + O(\chi^4)$$

$$n = n_n + n_p$$

A. Sedrakian, J. J. Li, and F. Weber, Prog. Part. Nucl. Phys. 131, 104041 (2023)  
 M. Dutra, et al, Phys. Rev. C 85, 035201 (2012).  
 J. M. Lattimer and Y. Lim, Astrophys. J. 771, 51 (2013).  
 M. Farine, J. M. Pearson, and F. Tondeur, Nucl. Phys. A 615, 135 (1997).

$$E_{\text{sym}}(n) = E_{\text{sym}}(n_r) + L(n_r) \chi_r + O(\chi_r^2)$$

$$\chi \equiv (n - n_0) / 3n_0$$

$n_0 \rightarrow 0.155 \pm 0.050 \text{ (fm}^{-3}\text{)}$   
 $E_0(n_0) \rightarrow -15.0 \pm 1.0 \text{ (MeV)}$   
 $E_{\text{sym}}(n_0) \rightarrow 30.9 \pm 1.9 \text{ (MeV)}$   
 $K_0 \rightarrow 230 \pm 30 \text{ (MeV)}$   
 $L_0 \rightarrow 52.5 \pm 17.5 \text{ (MeV)}$   
 $J_0 \rightarrow -700 \pm 500 \text{ (MeV)}$

$$\mathcal{L}_I = \bar{\psi} \left[ i\gamma_\mu \partial^\mu - M - g_\sigma \sigma - g_\omega \gamma_\mu \omega^\mu - g_\rho \gamma_\mu \tau_a \rho^{a\mu} \right] \psi$$



# Neutron star (high densities)

- The density in the cores of NSs always reach nearly  $8n_0$
- M-R relations/ tidal deformations are sensitive to the EOS behavior throughout the whole density regions

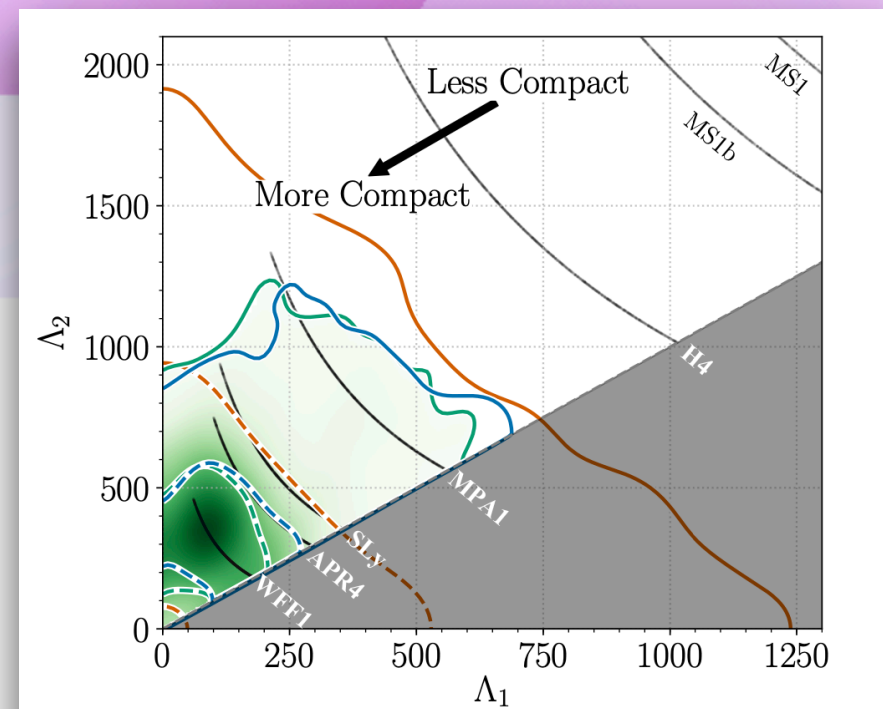


FIG. 1. Marginalized posterior for the tidal deformabilities of the two binary components of GW170817. The green shading shows the posterior obtained using the  $\Lambda_a(\Lambda_s, q)$  EOS-insensitive relation to impose a common EOS for the two bodies, while the green, blue, and orange lines denote 50% (dashed) and 90% (solid) credible levels for the posteriors obtained using EOS-insensitive relations, a parametrized EOS without a maximum mass requirement, and independent EOSs (taken from [52]), respectively. The gray shading corresponds to the unphysical region  $\Lambda_2 < \Lambda_1$  while the seven black scatter regions give the tidal parameters predicted by characteristic EOS models for this event [113, 115, 121–125].

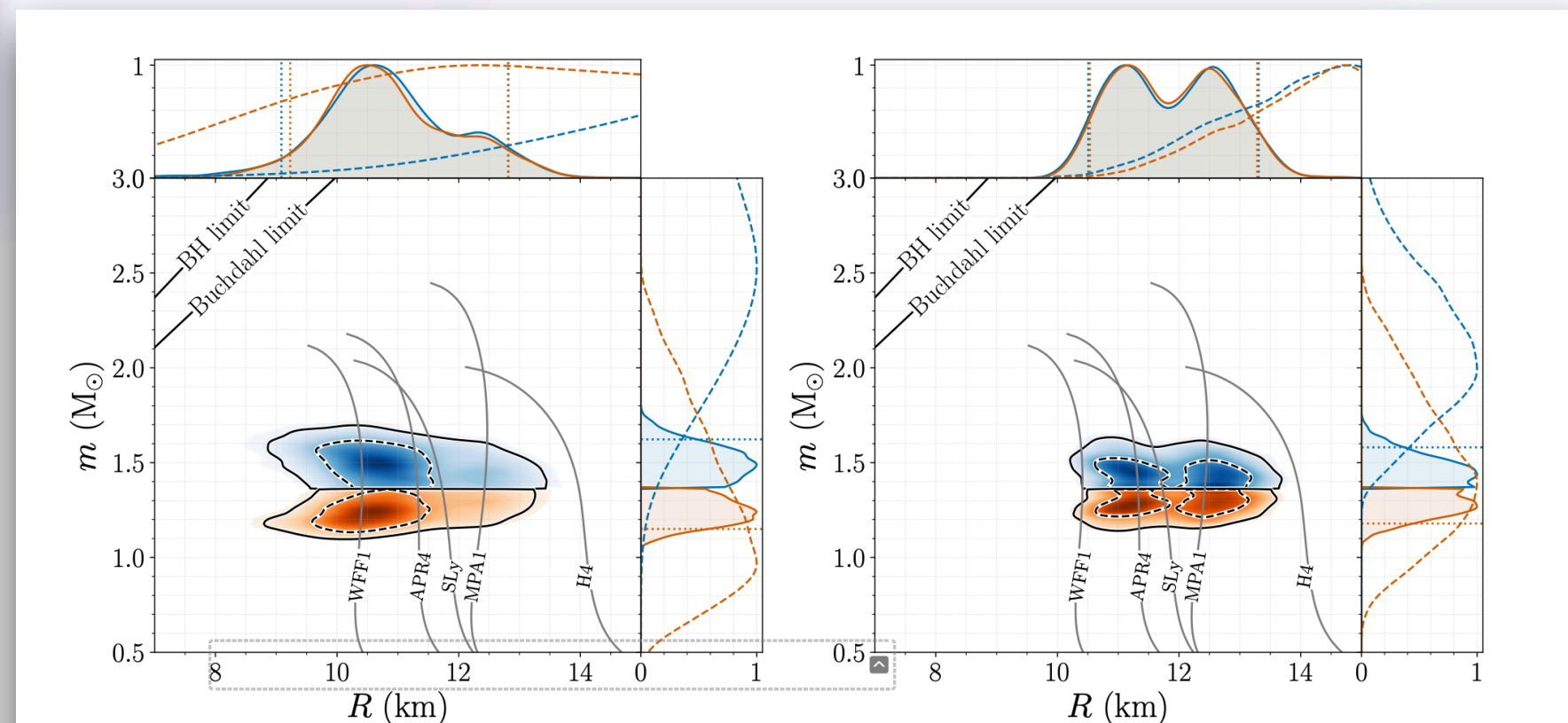


FIG. 3. Marginalized posterior for the mass  $m$  and areal radius  $R$  of each binary component using EOS-insensitive relations (left panel) and a parametrized EOS where we impose a lower limit on the maximum mass of  $1.97 M_\odot$  (right panel). The top blue (bottom orange) posterior corresponds to the heavier (lighter) NS. Example mass-radius curves for selected EOSs are overplotted in gray. The lines in the top left denote the Schwarzschild BH ( $R = 2m$ ) and Buchdahl ( $R = 9m/4$ ) limits. In the one-dimensional plots, solid lines are used for the posteriors, while dashed lines are used for the corresponding parameter priors. Dotted vertical lines are used for the bounds of the 90% credible intervals.

## TOV equation

TABLE I. Source properties for GW170817: we give ranges encompassing the 90% credible intervals for different assumptions of the waveform model to bound systematic uncertainty. The mass values are quoted in the frame of the source, accounting for uncertainty in the source redshift.

	Low-spin priors ( $ \chi  \leq 0.05$ )	High-spin priors ( $ \chi  \leq 0.89$ )
Primary mass $m_1$	$1.36\text{--}1.60 M_\odot$	$1.36\text{--}2.26 M_\odot$
Secondary mass $m_2$	$1.17\text{--}1.36 M_\odot$	$0.86\text{--}1.36 M_\odot$
Chirp mass $\mathcal{M}$	$1.188^{+0.004}_{-0.002} M_\odot$	$1.188^{+0.004}_{-0.002} M_\odot$
Mass ratio $m_2/m_1$	$0.7\text{--}1.0$	$0.4\text{--}1.0$
Total mass $m_{\text{tot}}$	$2.74^{+0.04}_{-0.01} M_\odot$	$2.82^{+0.47}_{-0.09} M_\odot$
Radiated energy $E_{\text{rad}}$	$> 0.025 M_\odot c^2$	$> 0.025 M_\odot c^2$
Luminosity distance $D_L$	$40^{+8}_{-14}$ Mpc	$40^{+8}_{-14}$ Mpc
Viewing angle $\Theta$	$\leq 55^\circ$	$\leq 56^\circ$
Using NGC 4993 location	$\leq 28^\circ$	$\leq 28^\circ$
Combined dimensionless tidal deformability $\tilde{\Lambda}$	$\leq 800$	$\leq 700$
Dimensionless tidal deformability $\Lambda(1.4M_\odot)$	$\leq 800$	$\leq 1400$

B. P. Abbott et al. (LIGO Scientific, Virgo), Phys. Rev. Lett. 119, 161101 (2017)  
 B. P. Abbott et al. (LIGO Scientific, Virgo), Phys. Rev. Lett. 121, 161101 (2018)

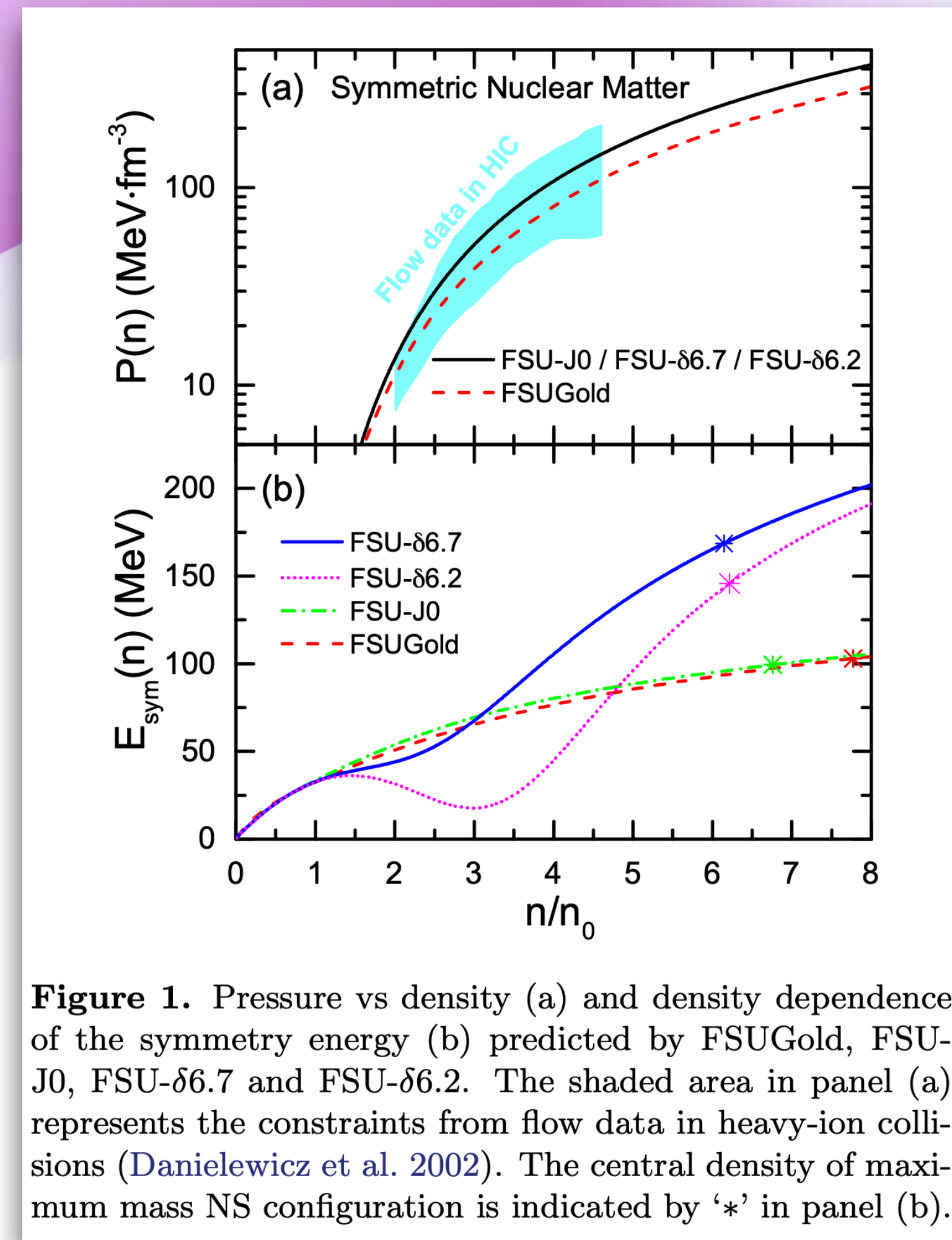
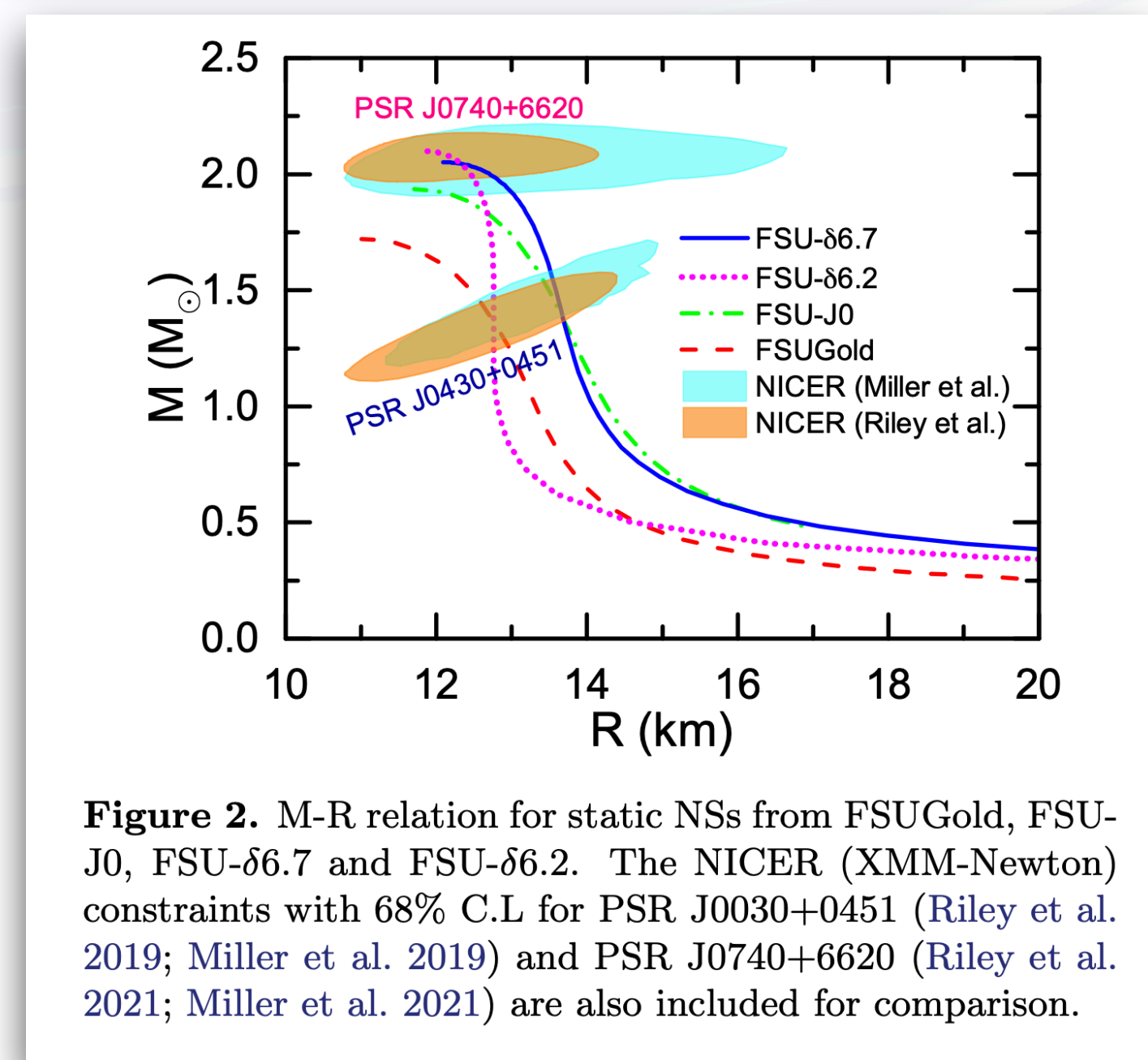
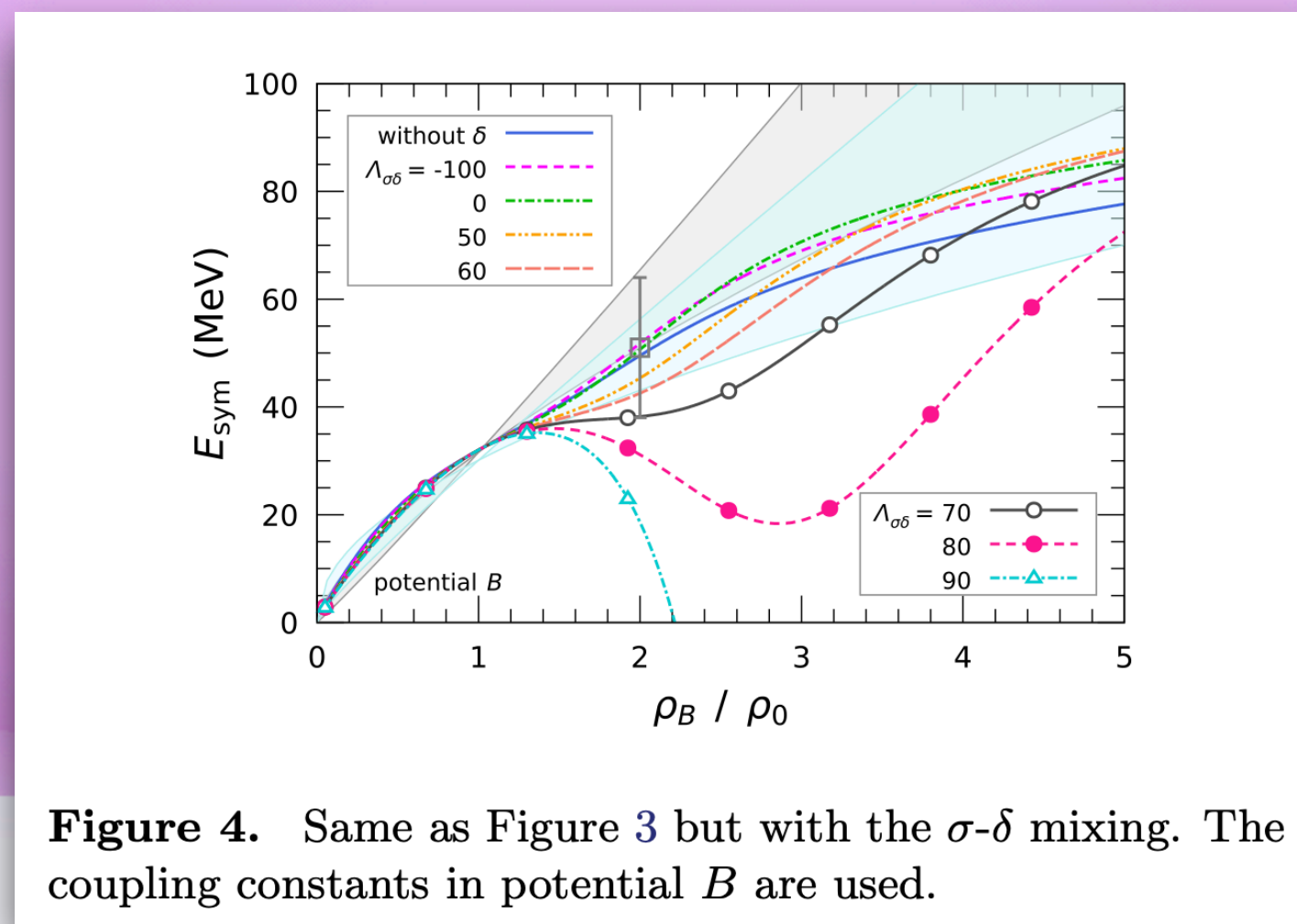
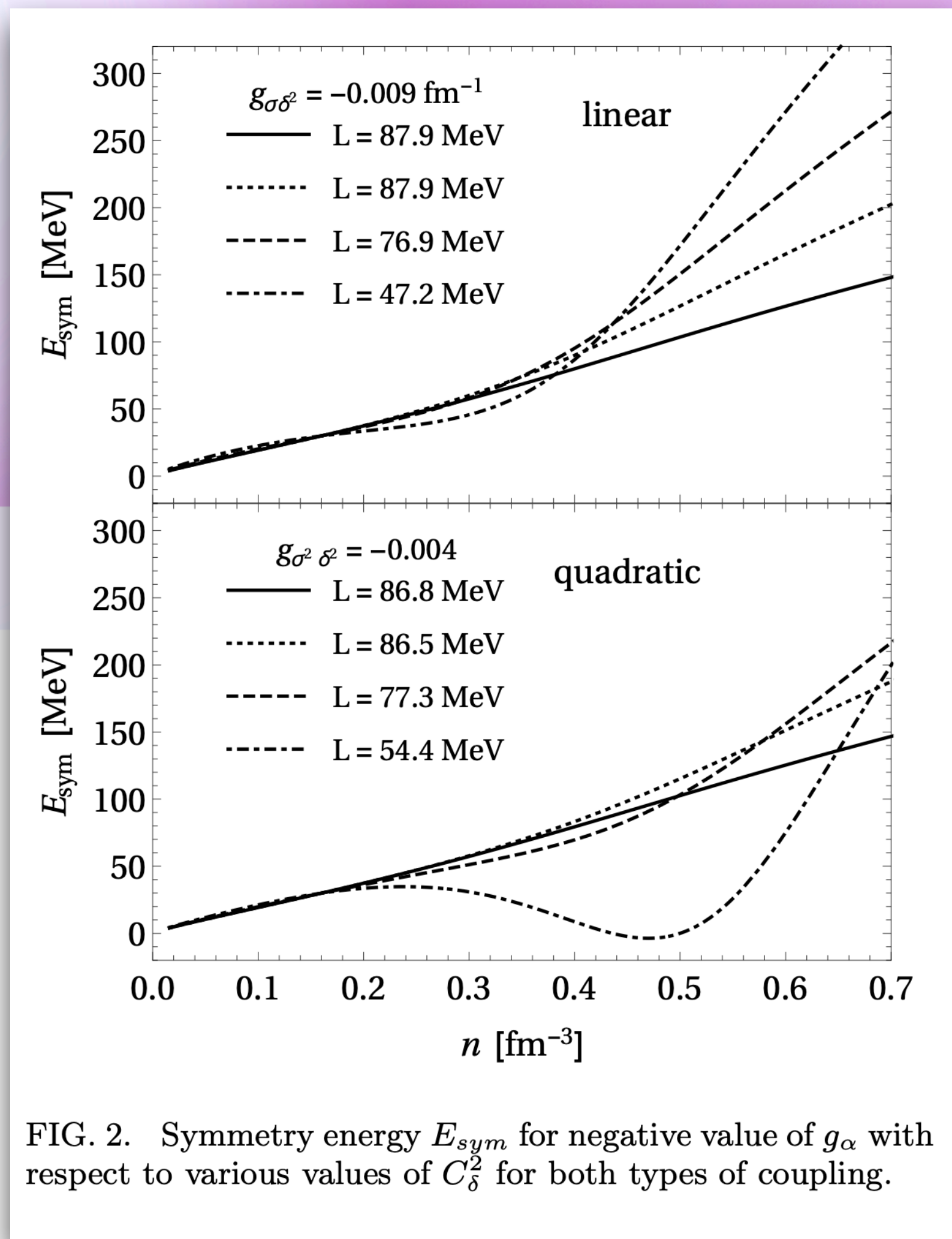
Polytropic process + estimations of strong interaction + TOV equation



# $\delta$ meson effects

T. Miyatsu, M.-K. Cheoun, and K. Saito, *Astrophys. J.* 929, 82 (2022).

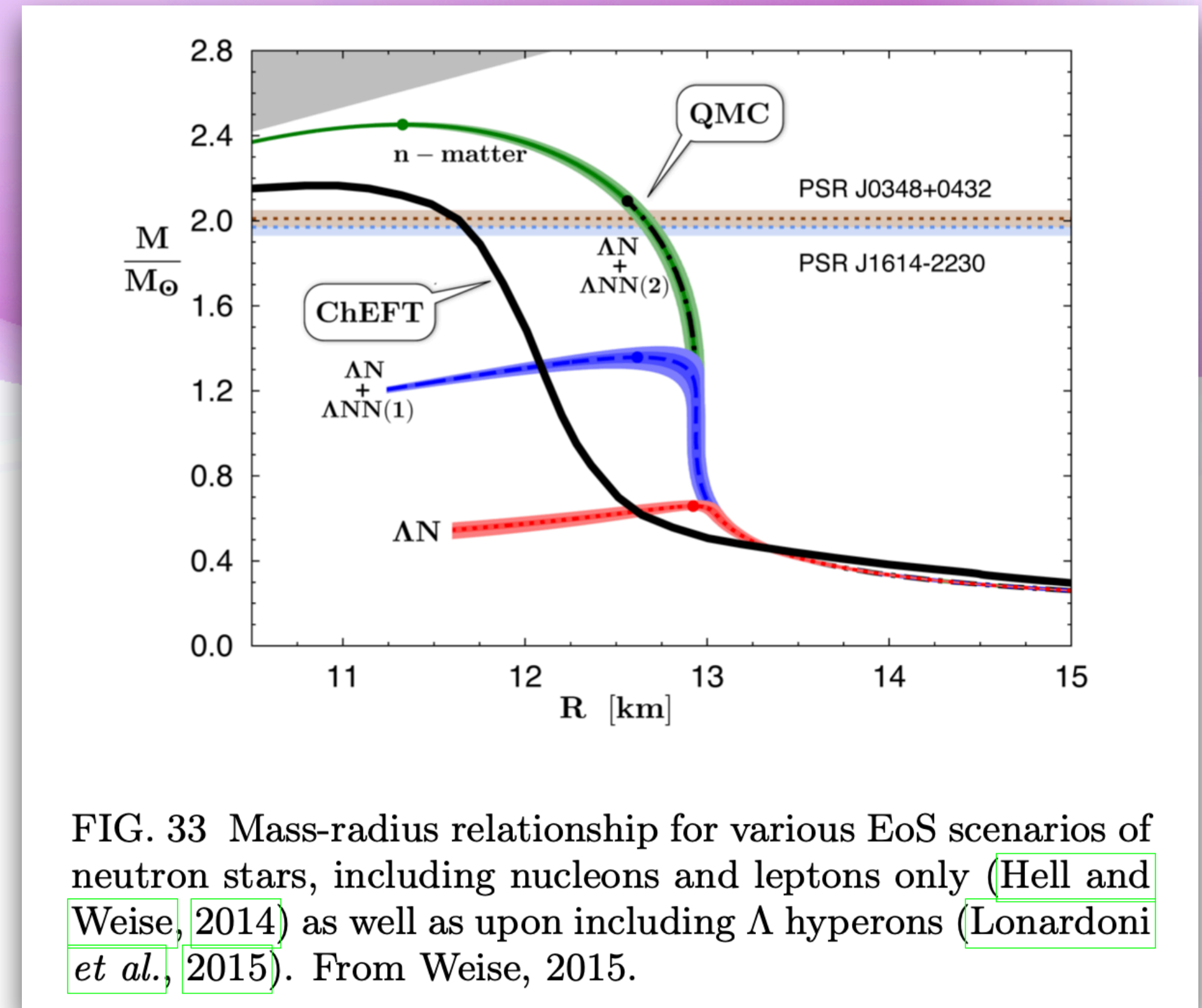
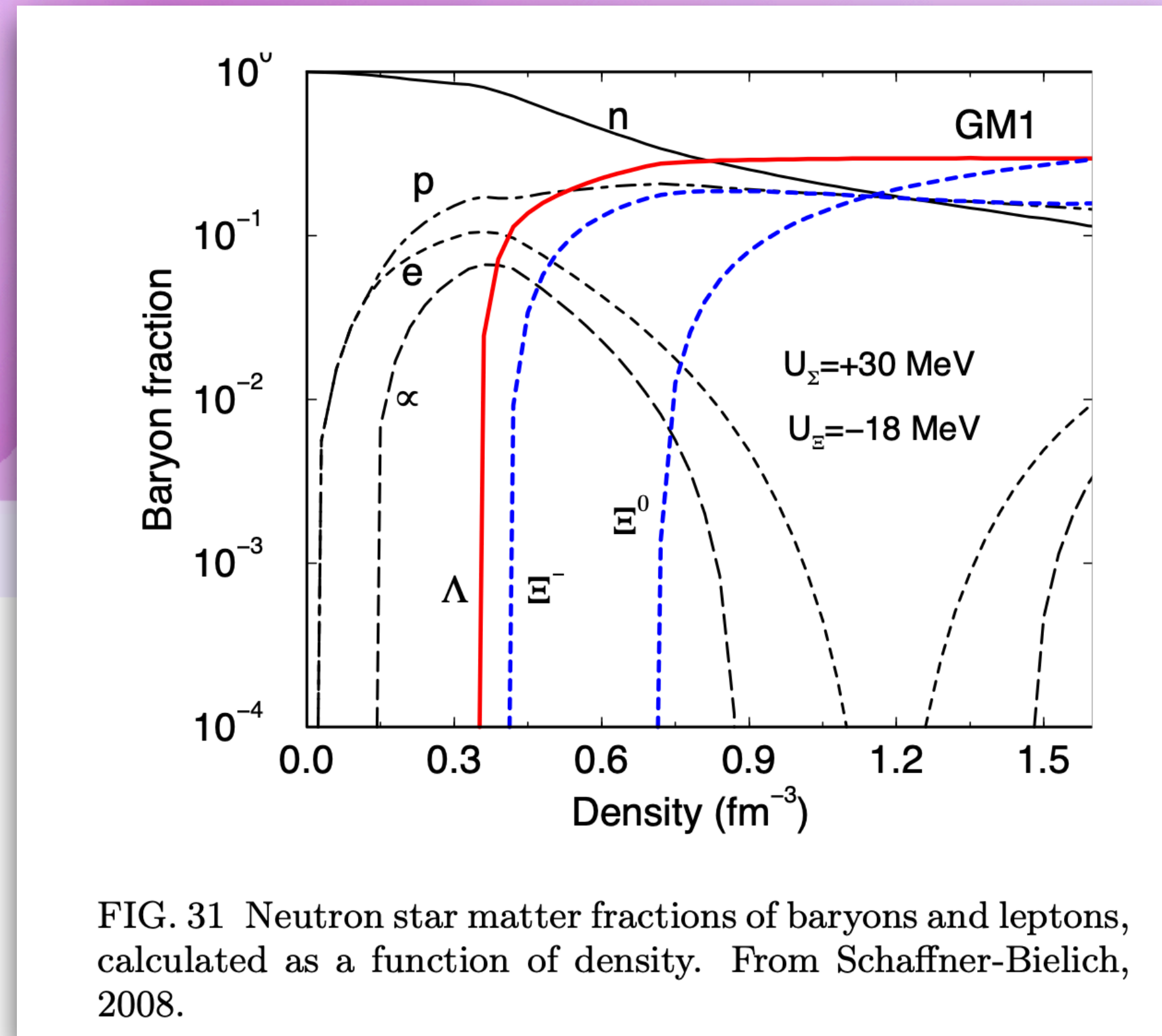
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# Hyperon effects (Strangeness)

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A. Gal, E. V. Hungerford, and D. J. Millener, *Rev. Mod. Phys.* 88, 035004 (2016).

Chemical potential equilibrium, e.g.  $\mu_{\Lambda} = \mu_p + \mu_e$



# Method to handle dense system



## 1. Relativistic mean field approximation

J. D. Walecka, Ann. Phys. 83, 491 (1974).

$$\mathcal{L} = \mathcal{L}_N + \mathcal{L}_\sigma + \mathcal{L}_\omega + \mathcal{L}_I$$

$$\mathcal{L}_N = \bar{\psi} \left( i\gamma_\mu \partial^\mu - m_N \right) \psi$$

$$\mathcal{L}_\sigma = \frac{1}{2} \left( \partial_\mu \phi \partial^\mu \phi - m_\sigma^2 \phi^2 \right)$$

$$\mathcal{L}_\omega = -\frac{1}{4} W_{\mu\nu} W^{\mu\nu} + \frac{1}{2} m_\omega^2 \omega_\mu \omega^\mu$$

$$\mathcal{L}_I = \mathcal{L}_{\sigma N} + \mathcal{L}_{\omega N} = g_\sigma \phi \bar{\psi} \psi - g_\omega \omega^\mu \bar{\psi} \gamma_\mu \psi$$

$$\left( i\gamma_\mu \partial^\mu - g_\omega \gamma_0 \omega^0 - m_N \right) \psi = 0, \quad m_N^* = m_N - g_\sigma \phi,$$

$$\phi = \frac{g_\sigma}{m_\sigma^2} \rho_s, \quad \rho_s = \langle \bar{\psi} \psi \rangle,$$

$$\omega^0 = \frac{g_\omega}{m_\omega^2} \rho_B, \quad \rho_B = \langle \psi^\dagger \psi \rangle,$$

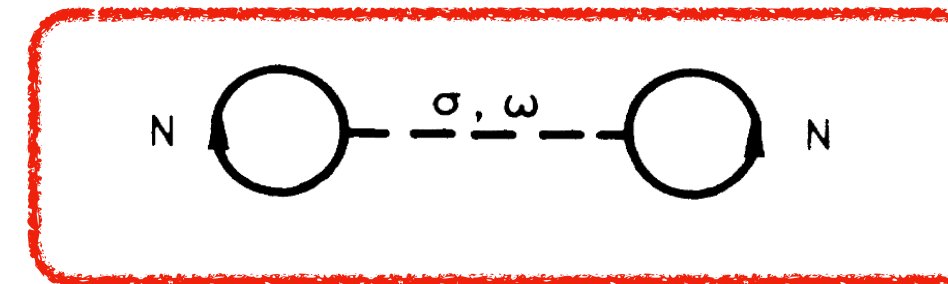
## 2. Relativistic Hartree-Fock method

$$\mathcal{L}_I = -g_\sigma \bar{\psi} \psi - g_\omega \bar{\psi} \gamma_\mu \omega^\mu \psi + \frac{f_\omega}{2M} \bar{\psi} \sigma_{\mu\nu} \partial^\nu \omega^\mu \psi$$

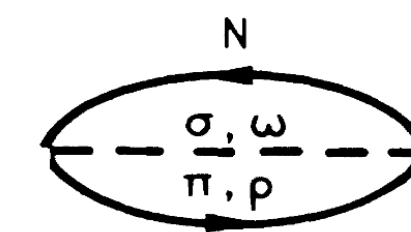
$$-g_\rho \bar{\psi} \gamma_\mu \rho^\mu \cdot \tau \psi + \frac{f_\rho}{2M} \bar{\psi} \sigma_{\mu\nu} \partial^\nu \rho^\mu \cdot \tau \psi$$

$$-e \bar{\psi} \gamma_\mu \frac{1}{2} (1 + \tau_3) A^\mu \psi + \mathcal{L}_{\pi NN}$$

$$\Sigma(\mathbf{p}) = \Sigma_S(p) + \gamma_0 \Sigma_0(p) + \boldsymbol{\gamma} \cdot \hat{\mathbf{p}} \Sigma_V(p)$$



(a)



(b)

A. Bouyssy, J.-F. Mathiot, N. V. Giai, and S. Marcos, Phys Rev C 36, 380 (1987).





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Can we extend an EFT/model taking care of QCD symmetry patterns into dense nucleon systems?

# An extended linear sigma model in nuclear matter

Phys. Rev. D 109 (2024) 7, 7, YM and Y. L. Ma

P-wave problems in light scalar meson sectors below 1 GeV

F. E. Close and N. A. Tornqvist, J. Phys. G 28, R249 (2002)

A tetra-quark picture to include  $\delta$  ( $a_0(980)$ ) meson and hyperon





# Freedoms to be considered

- Highlights of the parametrization:
  - I. Include meson exchanges, e.g.  $f_0(500)(\sigma)$  and  $a_0(980)(\delta)$
  - II. Include baryon freedoms, e.g. nucleon and hyperon

$$\begin{aligned}
 R^\mu = V^\mu - A^\mu &= \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\omega_N^\mu + \rho^{\mu 0}}{\sqrt{2}} - \frac{f_{1N}^\mu + a_1^{\mu 0}}{\sqrt{2}} & \rho^{\mu+} - a_1^{\mu+} & K^{*\mu+} - K_1^{\mu+} \\ \rho^{\mu-} - a_1^{\mu-} & \frac{\omega_N^\mu - \rho^{\mu 0}}{\sqrt{2}} - \frac{f_{1N}^\mu - a_1^{\mu 0}}{\sqrt{2}} & K^{*\mu 0} - K_1^{\mu 0} \\ K^{*\mu-} - K_1^{\mu-} & \bar{K}^{*\mu 0} - \bar{K}_1^{\mu 0} & \omega_S^\mu - f_{1S}^\mu \end{pmatrix} \\
 L^\mu = V^\mu + A^\mu &= \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\omega_N^\mu + \rho^{\mu 0}}{\sqrt{2}} + \frac{f_{1N}^\mu + a_1^{\mu 0}}{\sqrt{2}} & \rho^{\mu+} + a_1^{\mu+} & K^{*\mu+} + K_1^{\mu+} \\ \rho^{\mu-} + a_1^{\mu-} & \frac{\omega_N^\mu - \rho^{\mu 0}}{\sqrt{2}} + \frac{f_{1N}^\mu - a_1^{\mu 0}}{\sqrt{2}} & K^{*\mu 0} + K_1^{\mu 0} \\ K^{*\mu-} + K_1^{\mu-} & \bar{K}^{*\mu 0} + \bar{K}_1^{\mu 0} & \omega_S^\mu + f_{1S}^\mu \end{pmatrix}
 \end{aligned}$$

$$\Phi = S + iP = \begin{pmatrix} \frac{(\sigma_N + a_0^0) + i(\eta_N + \pi^0)}{\sqrt{2}} & a_0^+ + i\pi^+ & K_S^+ + iK^+ \\ a_0^- + i\pi^- & \frac{(\sigma_N - a_0^0) + i(\eta_N - \pi^0)}{\sqrt{2}} & K_S^0 + iK^0 \\ K_S^- + iK^- & \bar{K}_S^0 + i\bar{K}^0 & \sigma_S + i\eta_S \end{pmatrix}$$

$a_0$  mesons may be crucial to NS tidal deformations and neutron skin of nucleus

$$B_N \equiv \begin{pmatrix} \frac{\Lambda}{\sqrt{6}} + \frac{\Sigma^0}{\sqrt{2}} & \Sigma^+ & p \\ \Sigma^- & \frac{\Lambda}{\sqrt{6}} - \frac{\Sigma^0}{\sqrt{2}} & n \\ \Xi^- & \Xi^0 & -\frac{2\Lambda}{\sqrt{6}} \end{pmatrix}$$

High densities may lead to hyperon cores of NSs

D. Adhikari et al. (PREX), Phys. Rev. Lett. 126, 172502 (2021)  
 B. T. Reed, F. J. Fattoyev, C. J. Horowitz, and J. Piekarewicz, Phys. Rev. Lett. 126, 172503 (2021)  
 F. Li, B. J. Cai, Y. Zhou, W. Z. Jiang, and L. W. Chen, Astrophys. J. 929, 183 (2022)

N. K. Glendenning, Astrophys. J. 293, 470 (1985)  
 N. K. Glendenning and S. A. Moszkowski, Phys. Rev. Lett. 67, 2414 (1991).  
 S. Weissenborn, D. Chatterjee, and J. Schaffner-Bielich, Phys. Rev. C 85, 065802 (2012), [Erratum: Phys. Rev. C 90, 019904 (2014)]



# The chiral transformations



$$SU(3)_R \otimes SU(3)_L$$

$$\Phi' \rightarrow g_L \Phi' g_R^\dagger, \quad \hat{\Phi}' \rightarrow g_L \hat{\Phi}' g_R^\dagger$$

$$L_\mu \rightarrow g_L L_\mu g_L^\dagger, \quad R_\mu \rightarrow g_R R_\mu g_R^\dagger$$

$$N_R^{(RR)} \rightarrow g_R N_R^{(RR)} g_R^\dagger$$

$$N_R^{(LL)} \rightarrow g_R N_R^{(LL)} g_L^\dagger$$

$$2\text{-quark } U(1)_A \text{ 4-quark}$$

$$\Phi' \rightarrow e^{2iv} \Phi', \quad \hat{\Phi}' \rightarrow e^{-4iv} \hat{\Phi}'$$

$$L_\mu \rightarrow L_\mu, \quad R_\mu \rightarrow R_\mu$$

$$N_R^{(RR)} \rightarrow e^{-3iv} N_R^{(RR)}, \quad N_L^{(RR)} \rightarrow e^{-iv} N_L^{(RR)}$$

$$N_R^{(LL)} \rightarrow e^{iv} N_R^{(LL)}, \quad N_L^{(LL)} \rightarrow e^{3iv} N_L^{(LL)}$$

di-quark approximation



# Power counting rules



- ① The operators are limited within dimension-4, for the higher dimensional operators are suppressed by the cutoff scale;
- ② The quark number of an operator is limited within 8 and the number of flavor space traces is limited to only 1, for its suppression by  $N_c$ ;
- ③ The explicit symmetry breaking caused by quark mass is treated as perturbation, and it's ignored in current work.

A. H. Fariborz, R. Jora, and J. Schechter, Phys. Rev. D 77, 034006 (2008).  
A. H. Fariborz, R. Jora, and J. Schechter, Phys. Rev. D 72, 034001 (2005)  
A. H. Fariborz, R. Jora, and J. Schechter, Phys. Rev. D 79, 074014 (2009)  
D. Parganlija, F. Giacosa, and D. H. Rischke, Phys. Rev. D 82, 054024 (2010)



# The lowest order Lagrangians



$$\mathcal{L}_M = \frac{1}{2} \text{Tr} \left( \partial_\mu \Phi' \partial^\mu \Phi'^\dagger \right) + \frac{1}{2} \text{Tr} \left( \partial_\mu \hat{\Phi}' \partial^\mu \hat{\Phi}'^\dagger \right) + c_2 \text{Tr} \left( \Phi' \Phi'^\dagger \right) - c_4 \text{Tr} \left( \Phi' \Phi'^\dagger \Phi' \Phi'^\dagger \right) - d_2 \text{Tr} \left( \hat{\Phi}' \hat{\Phi}'^\dagger \right) - e_3 \left( \epsilon_{abc} \epsilon^{def} \Phi'_d{}^a \Phi'_e{}^b \hat{\Phi}'_f{}^c + \text{h.c.} \right) - c_3 \left[ \gamma_1 \ln \left( \frac{\det \Phi'}{\det \hat{\Phi}'^\dagger} \right) + (1 - \gamma_1) \ln \left( \frac{\text{Tr} \left( \Phi' \hat{\Phi}'^\dagger \right)}{\text{Tr} \left( \hat{\Phi}' \Phi'^\dagger \right)} \right) \right]^2$$

- A. H. Fariborz, R. Jora, and J. Schechter, Phys. Rev. D 72, 034001 (2005)  
 A. H. Fariborz, R. Jora, and J. Schechter, Phys. Rev. D 77, 034006 (2008)  
 A. H. Fariborz, R. Jora, and J. Schechter, Phys. Rev. D 79, 074014 (2009)





$$\begin{aligned}
\mathcal{L}_V = & -\frac{1}{8} \text{Tr} \left( L_{\mu\nu} L^{\mu\nu} + R_{\mu\nu} R^{\mu\nu} \right) + g_1 \left[ \text{Tr} \left( \partial_\nu R_\mu R^\mu R^\nu \right) + \text{Tr} \left( \partial_\nu L_\mu L^\mu L^\nu \right) \right] + \\
& g_2 \left[ \text{Tr} \left( \partial_\nu R_\mu R^\nu R^\mu \right) + \text{Tr} \left( \partial_\nu L_\mu L^\nu L^\mu \right) \right] + g_3 \left[ \text{Tr} \left( R_\mu R^\mu R_\nu R^\nu \right) + \text{Tr} \left( L_\mu L^\mu L_\nu L^\nu \right) \right] + \\
& g_4 \left[ \text{Tr} \left( R_\mu R^\nu R_\mu R^\nu \right) + \text{Tr} \left( L_\mu L^\nu L_\mu L^\nu \right) \right] + h_1 \left[ \text{Tr} \left( \Phi'^{\dagger} \Phi' R_\mu R^\mu \right) + \text{Tr} \left( \Phi' \Phi'^{\dagger} L_\mu L^\mu \right) \right] + \\
& h_2 \left[ \text{Tr} \left( L^\mu \Phi' R_\mu \Phi'^{\dagger} \right) \right] + h_3 \left[ \text{Tr} \left( \Phi'^{\dagger} \partial_\mu \Phi' R^\mu \right) + \text{Tr} \left( \Phi' \partial_\mu \Phi'^{\dagger} L^\mu \right) \right] + \\
& \frac{a_1}{2} \epsilon_{abc} \epsilon^{def} \left[ \left( R_\mu \right)_{ad} \left( R_\nu \right)_{be} \left( \Phi'^{\dagger} \Phi \right)_{cf} + \left( L_\mu \right)_{ad} \left( L_\nu \right)_{be} \left( \Phi \Phi'^{\dagger} \right)_{cf} \right] + \\
& \frac{a_2}{2} \epsilon_{abc} \epsilon^{def} \left[ \left( R_\mu \right)_{ad} \left( R_\nu \right)_{be} \left( R^\mu R^\nu \right)_{cf} + \left( L_\mu \right)_{ad} \left( L_\nu \right)_{be} \left( L^\mu L^\nu \right)_{cf} \right] + \\
& \frac{a_3}{2} \epsilon_{abc} \epsilon^{def} \left[ \left( R_\mu \right)_{ad} \left( R^\mu \right)_{be} \left( R^\nu R_\nu \right)_{cf} + \left( L_\mu \right)_{ad} \left( L^\mu \right)_{be} \left( L^\nu L_\nu \right)_{cf} \right] + \\
& \frac{a_4}{2} \epsilon_{abc} \epsilon^{def} \left[ \left( R_\mu \right)_{ad} \left( R^\nu \right)_{be} \left( \partial^\mu R_\nu \right)_{cf} + \left( L_\mu \right)_{ad} \left( L^\nu \right)_{be} \left( \partial^\mu L_\nu \right)_{cf} \right]
\end{aligned}$$



# Di-quark approximation

L. Olbrich, Master's thesis, Goethe U., Frankfurt (main) (2015).

L. Olbrich, M. Zétényi, F. Giacosa, and D. H. Rischke, Phys. Rev. D 93, 034021 (2016).



$$N_{R, L}^{(RR)} = \frac{1}{\sqrt{2}} \frac{1 \pm \gamma_5}{2} B, \quad N_{R, L}^{(LL)} = -\frac{1}{\sqrt{2}} \frac{1 \pm \gamma_5}{2} B$$

$$\begin{aligned} \mathcal{L}_B = & \text{Tr}(\bar{B} i \not{\partial} B) + c \text{Tr} \left( \bar{B} \gamma_\mu V^\mu B + \bar{B} \gamma_\mu \gamma_5 A^\mu B \right) + c' \text{Tr} \left( \bar{B} \gamma_\mu \bar{B} V^\mu + \bar{B} \gamma_\mu \gamma_5 \bar{B} A^\mu \right) + \\ & h \epsilon_{abc} \epsilon^{def} \left[ (\bar{B})_{ad} \gamma_\mu (B)_{be} (V^\mu)_{cf} + (\bar{B})_{ad} \gamma_\mu \gamma_5 (B)_{be} (A^\mu)_{cf} \right] - \\ & \frac{g}{2} \text{Tr} \left[ \bar{B} (\Phi' + \Phi'^\dagger) B + \bar{B} \gamma_5 (\Phi' - \Phi'^\dagger) B \right] - \\ & \frac{e}{2} \epsilon_{abc} \epsilon^{def} \left[ (\bar{B})_{ad} (\Phi' + \Phi'^\dagger)_{be} (B)_{cf} + (\bar{B})_{ad} \gamma_5 (\Phi' - \Phi'^\dagger)_{be} (B)_{cf} \right] \end{aligned}$$



# The Lagrangian at the lowest order for RMF

$$\mathcal{V}_M = c_2 \text{Tr} S'^2 - d_2 \text{Tr} \hat{S}'^2 - c_4 \text{Tr} S'^4 - 2e_3 \epsilon_{abc} \epsilon_{def} S'_{ad} S'_{be} \hat{S}'_{cf}$$

$$\mathcal{V}_V = \tilde{h}_2 \text{Tr} (S'^2 V^2) + \tilde{g}_3 \text{Tr} V^4 +$$
$$a_1 \epsilon_{abc} \epsilon_{def} V_{ad} V_{be} (S'^2)_{cf}$$

$$\mathcal{L}_B^{\text{RMF}} = \text{Tr} \left( \bar{B} i \gamma_\mu \partial^\mu B \right) + c \text{Tr} \left( \bar{B} \gamma^0 V B \right) - g \text{Tr} \left( \bar{B} S' B \right)$$
$$+ h \epsilon_{abc} \epsilon_{def} \bar{B}_{ad} \gamma^0 B_{be} V_{cf}$$
$$- e \epsilon_{abc} \epsilon_{def} \bar{B}_{ad} \gamma^0 B_{be} S'_{cf}$$

Phys. Rev. D 109 (2024) 7, 7, YM and Y. L. Ma

Improvement of parameter space





# SSB of chiral symmetry



● Spontaneous symmetry breaking down from  $SU(3)_L \otimes SU(3)_R$  to  $SU(3)_V$

$$\langle \sigma' \rangle = \sqrt{3}\alpha \text{ and } \langle \hat{\sigma}' \rangle = \sqrt{3}\beta$$

● Mixing between 2-quark and 4-quark configurations

$$\sigma = \cos \theta_0 \sigma' + \sin \theta_0 \hat{\sigma}'$$

$$a_0 = \cos \theta_8 a'_0 + \sin \theta_8 \hat{a}'_0$$

$$f_0 = \cos \theta_8 f'_0 + \sin \theta_8 \hat{f}'_0$$



# Phenomenological analysis



## Parameter space choice

	$\alpha(\text{MeV})$	$\beta(\text{MeV})$	$e_3(\text{MeV})$	$c_4$	$h_2$	$\tilde{g}_3$	$c$	$g$	$a_1$	$\tilde{g}$	$e$	$g_{\sigma NN}$	$g_{a NN}$	$g_{f NN}$	$g_{\omega NN}$	$g_{\rho NN}$
el-g30eg	61.4	26.4	-2100	45.6	79.3	0.397	9.51	6.54	4.10	-2.61	8.75	-5.98	-0.671	2.68	6.06	3.45
el-g30e	61.1	24.4	-2050	43.6	80.0	0.542	11.4	0.234	4.17	-0.790	15.1	-6.20	-5.03	3.00	6.09	5.30
el-g30g	61.1	24.7	-2060	44.0	80.1	1.59	-0.792	15.4	4.25	11.5	-0.027	-6.17	5.12	2.95	-6.09	5.30
el-g350eg	61.2	25.6	-2100	44.4	79.9	51.5	10.1	6.35	4.14	-2.65	9	-6.12	-0.852	2.85	6.37	3.71
el-g3100eg	60.8	24.0	-2090	42.4	80.8	100	10.6	7.10	4.19	-2.88	8.34	-6.36	-0.442	3.19	6.73	3.85
el-g3150eg	60.7	24.3	-2110	42.0	81.0	150	11.1	7.15	4.18	-3.05	8.30	-6.38	-0.413	3.20	7.09	4.04



# Bare mass parameters and NM properties

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Physical quantities in unit of MeV

	B.E.	$E_{\text{sym}}$	$K_0$	$L_0$	$J_0$	$m_\rho$	$m_\omega$	$m_\sigma$	$m_{\sigma'}$	$m_{a_0}$	$m_{a_0'}$	$m_N$
<b>Empirical</b>	$-15.0 \pm 1.0$	$30.9 \pm 1.9$	$250 \pm 50$	$52.5 \pm 17.5$	$-700 \pm 500$	$763 \pm 2$	$783 \pm 1$	$475 \pm 75$	$1350 \pm 100$	$995 \pm 25$	$1410 \pm 120$	$939 \pm 1$
el-g30eg	-14.6	30.1	415	92.2	421	763	783	503	1520	977	1510	939
el-g30e	-14.6	31.6	420	85.8	479	763	783	525	1510	991	1480	939
el-g30g	-14.6	30.9	418	83.6	451	763	783	522	1510	989	1480	939
el-g350eg	-15.2	30.9	370	80.7	-392	763	783	498	1520	983	1500	939
el-g3100eg	-15.4	31.4	317	71.7	-1020	763	783	502	1510	994	1470	939
el-g3150eg	-15.6	31.6	253	63.7	-1470	763	783	485	1510	991	1470	939
<b>TM1</b>	-16.3	36.9	280	113	-247	770	783	511	—	—	—	938
FSU - $\delta 6.7$	-16.3	32.7	229	53.5	-322	763	783	492	—	980	—	938

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J. M. Lattimer and Y. Lim, Astrophys. J. 771, 51 (2013).

M. Farine, J. M. Pearson, and F. Tondeur, Nucl. Phys. A 615, 135 (1997).

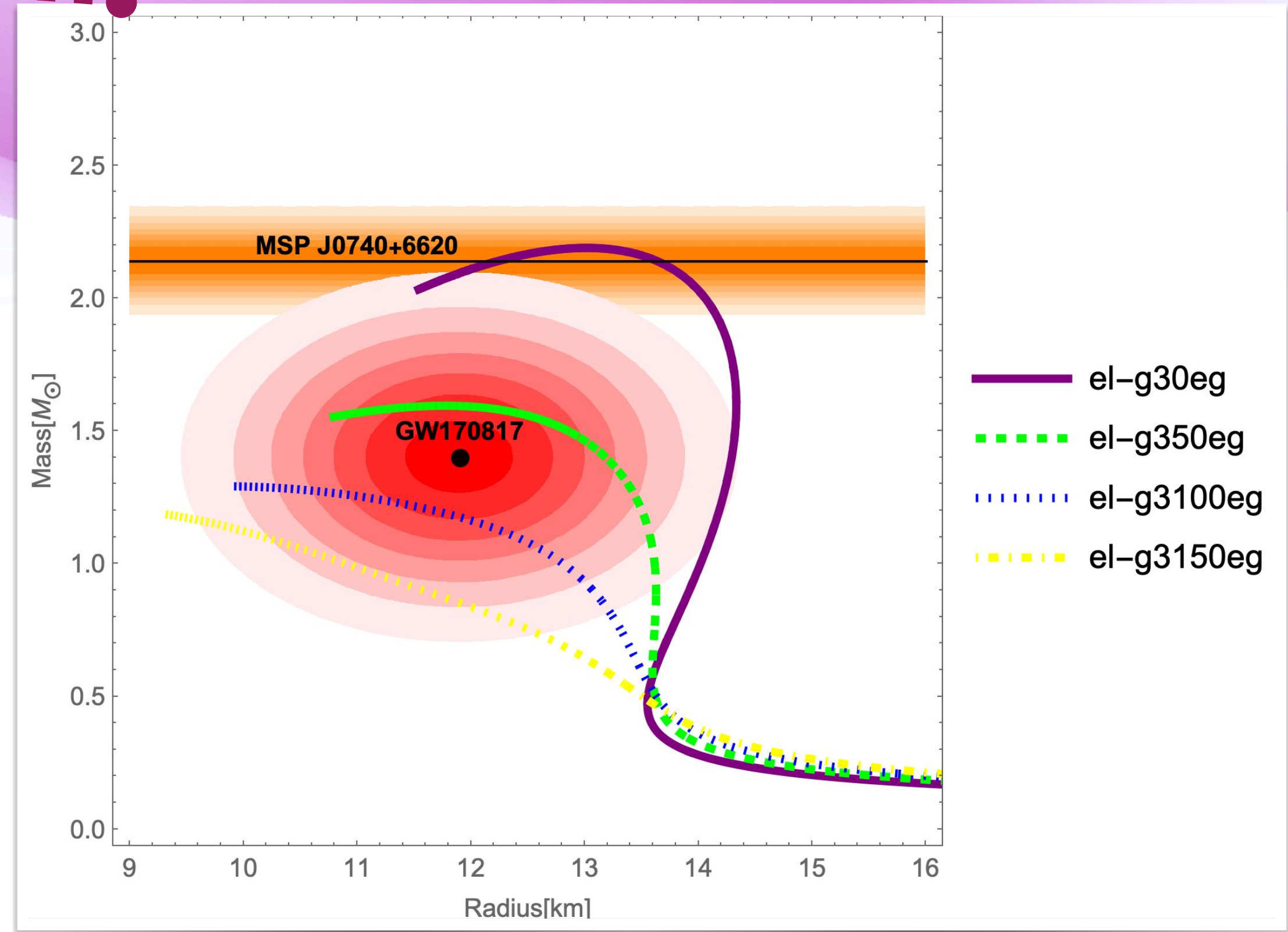
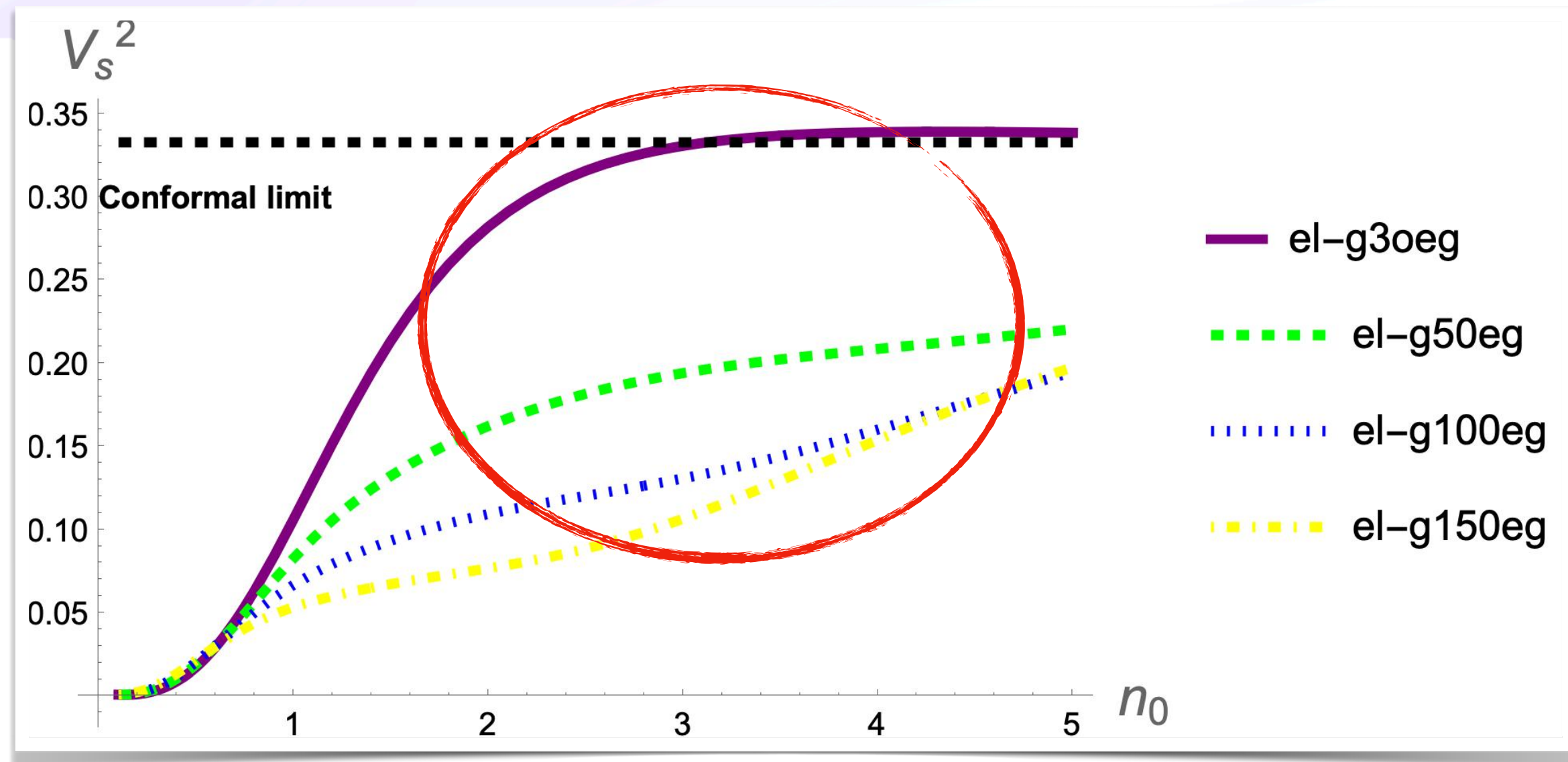
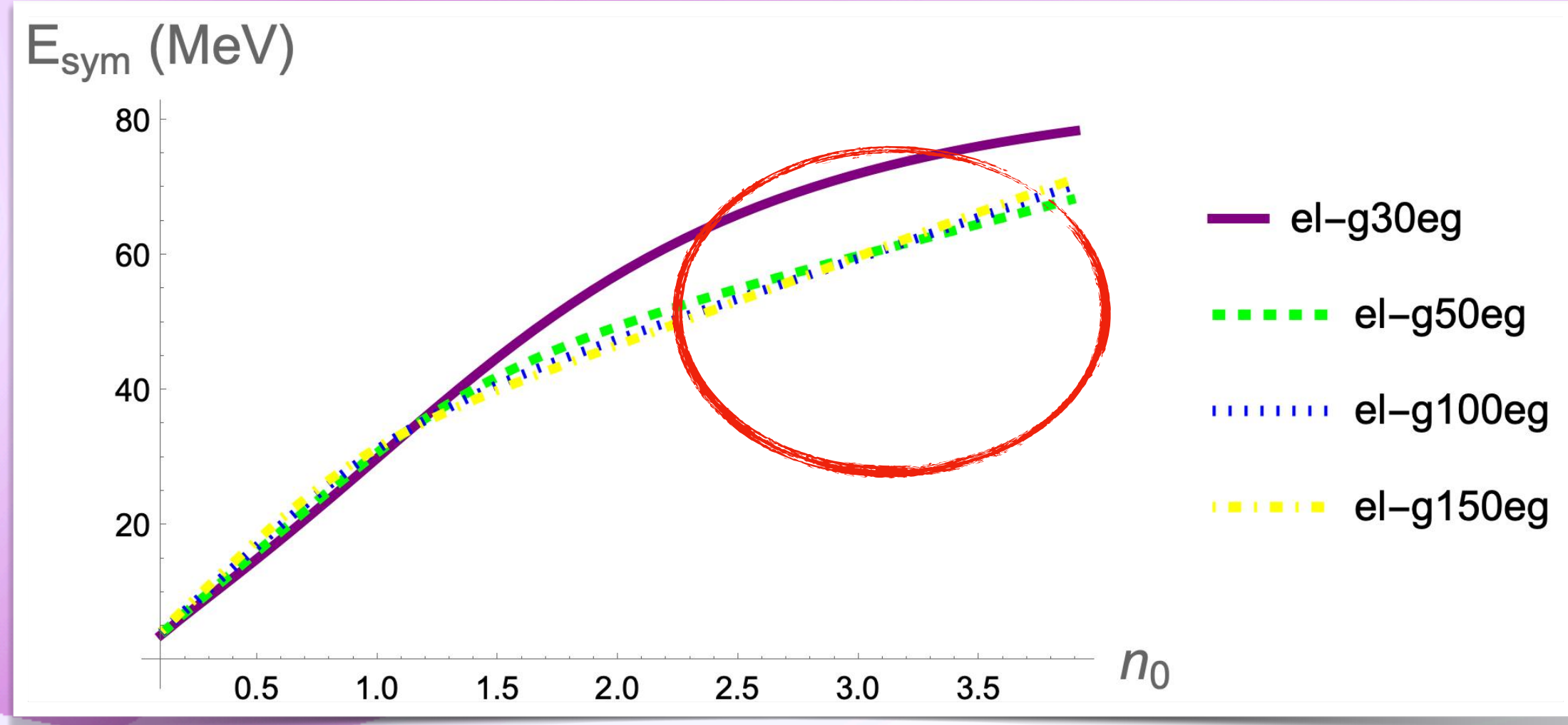
M. Dutra, et al., Phys. Rev. C 85, 035201 (2012).

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# Comparison among different $g_3$ cases

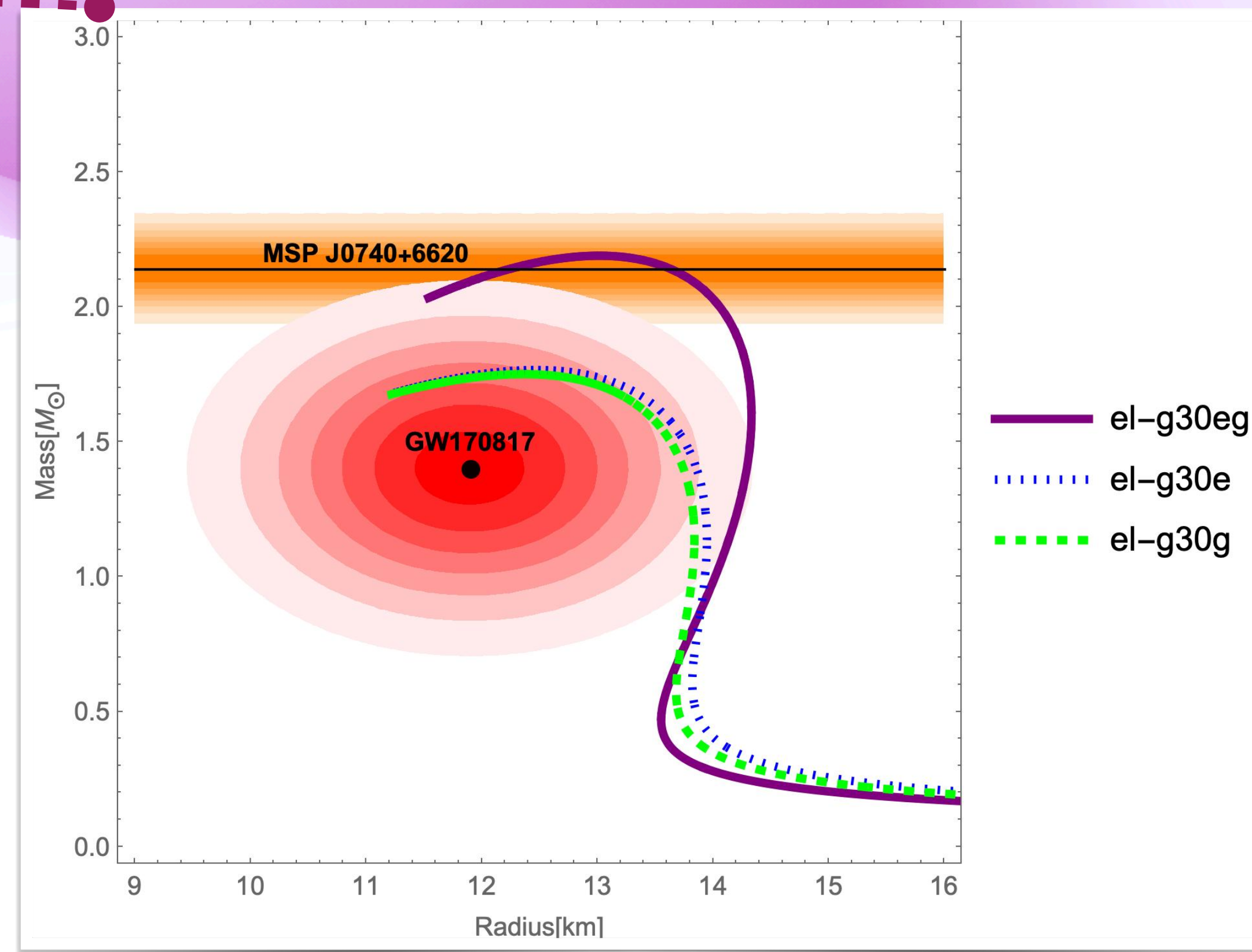
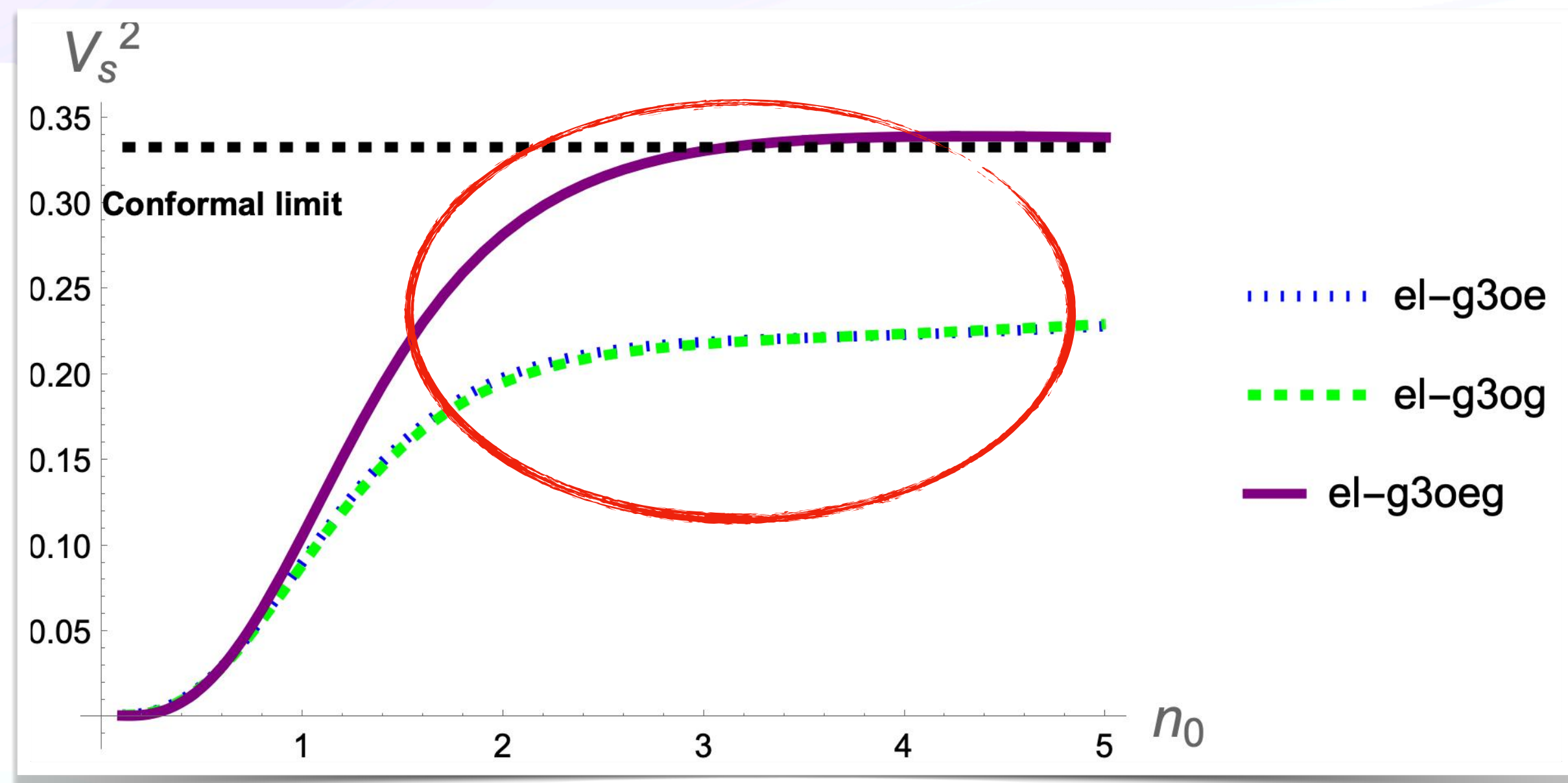
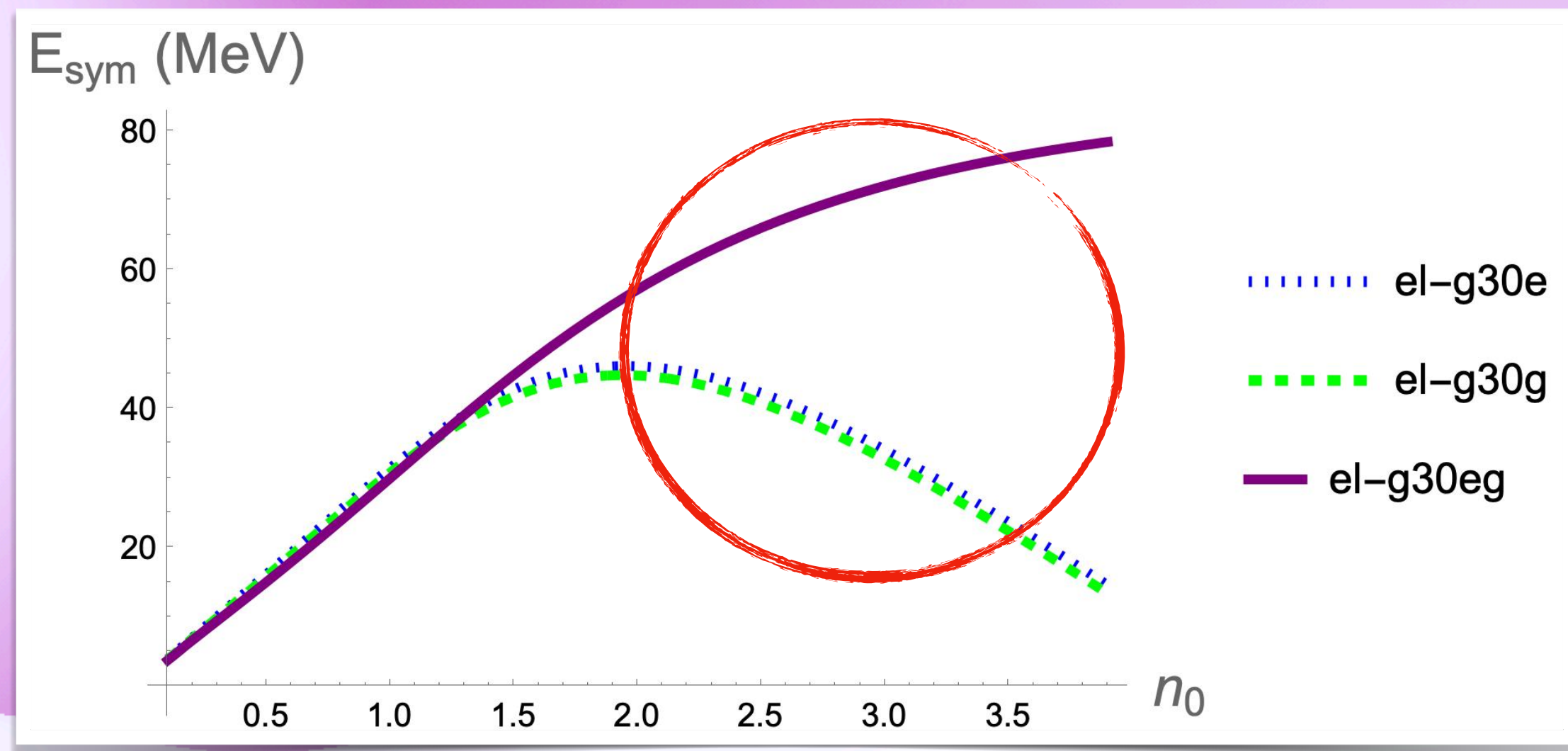
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# Comparison among different $g_{aNN}$ cases

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# Comparison with Walecka-type models

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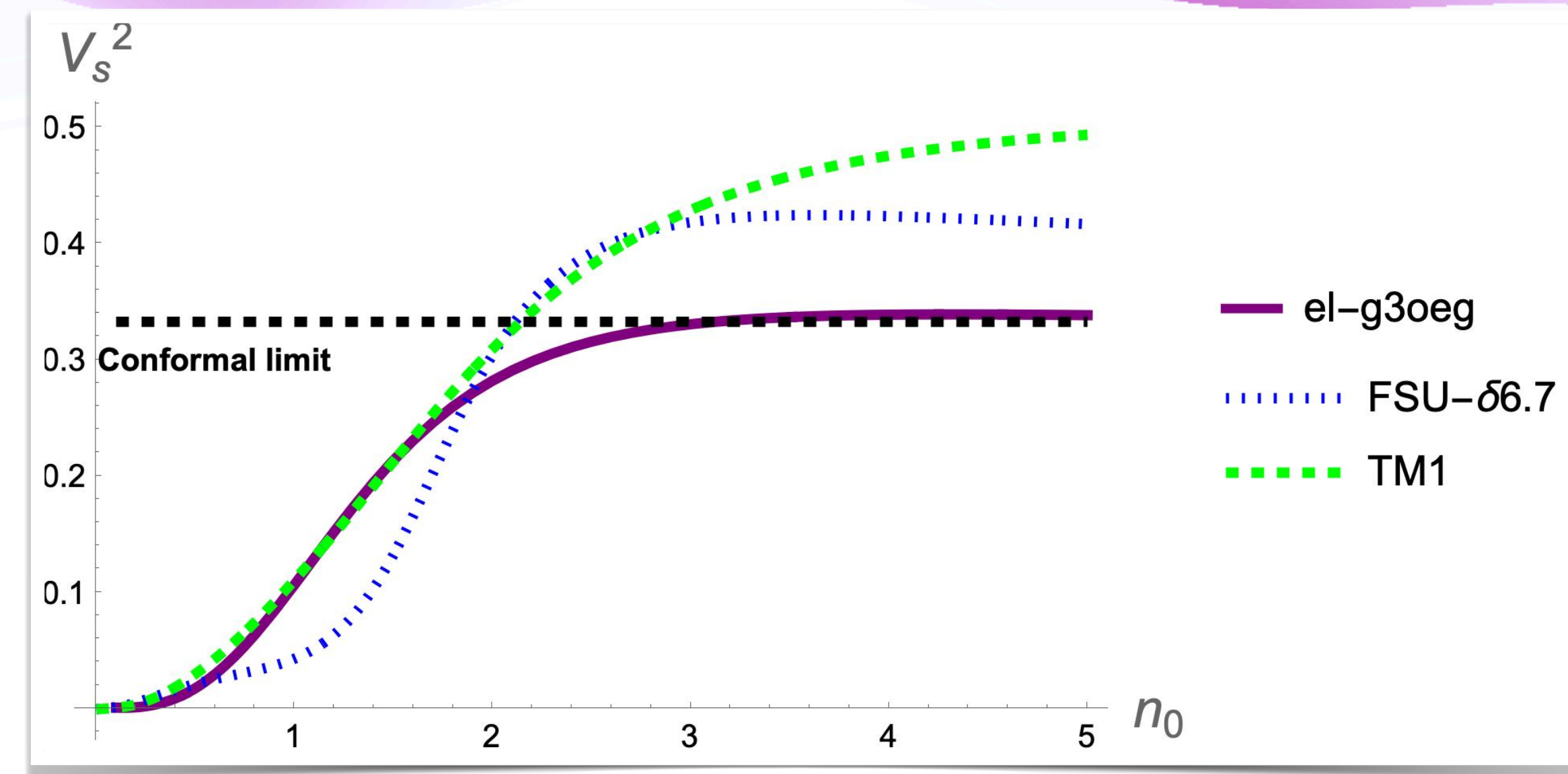
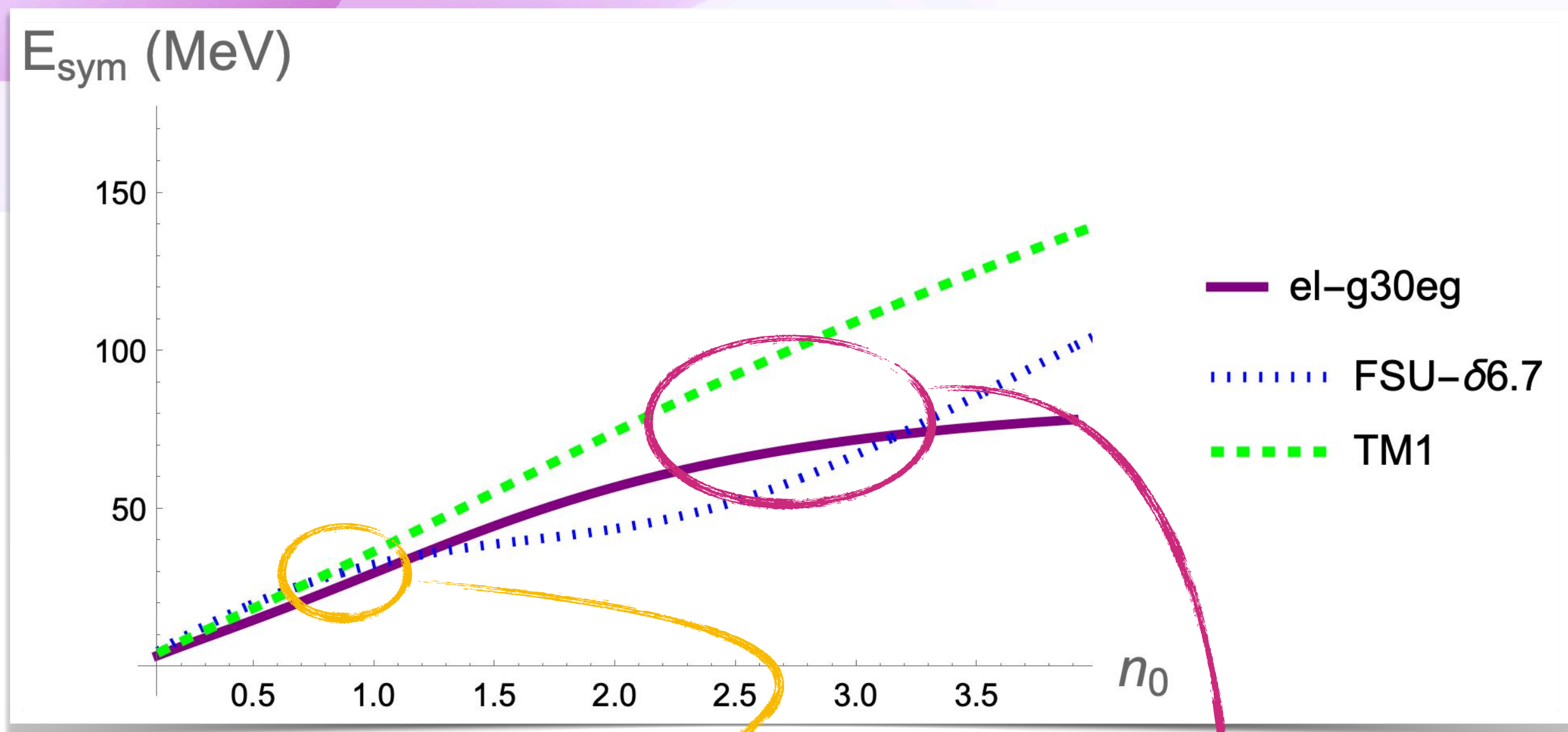
Saturation density

A. Sedrakian, J. J. Li, and F. Weber, Prog. Part. Nucl. Phys. 131, 104041 (2023)

	Empirical	ELSM	TM1	FSU – $\delta 6.7$
$n_0(\text{fm}^{-3})$	$0.155 \pm 0.005$	0.155	0.145	0.148

## Symmetric nuclear matter

## Pure neutron matter



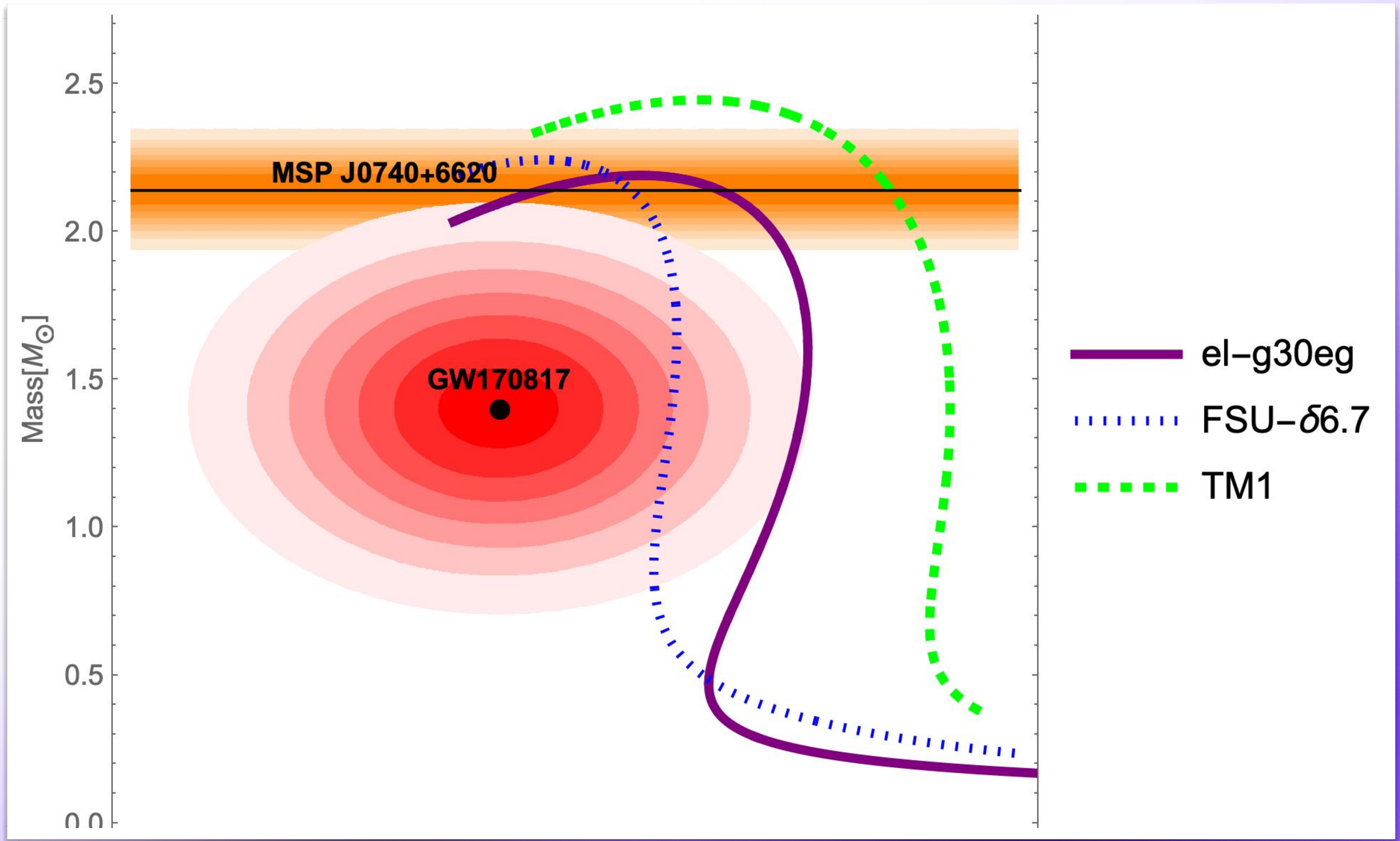
$$L(2/3n_0) \geq 49 \text{ MeV}$$

$$\text{GW170817 } \Lambda_{1.4} \leq 580$$

D. Adhikari et al., PREX, Phys. Rev. Lett. 126, 172502 (2021)  
B. T. Reed, F. J. Fattoyev, C. J. Horowitz, and J. Piekarewicz, Phys. Rev. Lett. 126, 172503 (2021)

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# The percentage of 2/4-quark configuration of scalar mesons



	$\sigma$		$\delta$	
	$p_4$	$p_2$	$p_4$	$p_2$
el-g30g	51.7%	48.3%	77.9%	22.1%
el-g30e	51.2%	48.7%	77.2%	22.8%
el-g30eg	54.2%	45.8%	81.5%	18.5%
el-g350eg	52.4%	47.6%	77.3%	20.7%
el-g3100eg	49.2%	50.8%	74.4%	25.6%
el-g3150eg	48.9%	51.1%	74.3%	25.7%

Slight difference of configurations  
but large difference at macroscopic level



# The scalar meson couplings from different approaches



MeV

	$C_{\sigma\delta^2}$	$C_{\sigma^2\delta^2}$
Zabari-19	$\pm 1.77$	$\pm 0.004$
FSU- $\delta 6.7$	—	2.63
el-g30g	-1860	-9.40
el-g30e	-1940	-9.71
el-g30eg	-1480	-7.70
el-g350eg	-1690	-8.75
el-g3100eg	-2190	-11.0
el-g3150eg	-2160	-11.0

N. Zabari, S. Kubis, and W. Wójcik, Phys. Rev. C 99, 035209 (2019)

F. Li, B.-J. Cai, Y. Zhou, W.-Z. Jiang, and L.-W. Chen, Astrophys. J. 929, 183 (2022).

Quite different parameter space!





# Summary and outlook

- I. The EFTs/models of low energy QCD can reproduce the NM properties and NS structures;
- II. Regarding the well-reproduced vacuum spectra and NM properties at low densities, different parameter space choices significantly affect the neutron star (NS) structure;
- III. These astrophysical objects may serve as a promising test field for strong interaction theories/models, with more detailed analysis forthcoming (the connections between microscopic symmetry and macroscopic observations).



# Strangeness in this framework

In collaboration with Lu-Qi Zhang (张璐琦)



Explicit chiral symmetry breaking due to quark mass

$$\mathcal{L}_{\text{S.B.}} = -b_1 \text{Tr}(\xi S^3) - G \text{Tr}(\xi S) + b_2 \text{Tr}(V^2 \xi S) + b_3 \epsilon^{ijk} \epsilon^{lmn} (V)_{il} (V)_{jm} (\xi S)_{kn} \\ - b_4 \text{Tr}[\bar{B} \xi B] - b_5 \epsilon^{ijk} \epsilon^{lmn} (\bar{B})_{il} (\xi)_{jm} (B)_{kn}$$

Gell-Mann Okubo Formula

$$\xi = \xi_1 \lambda_1 + \xi_3 \lambda_3 + \xi_8 \lambda_8$$

Non-zero  $\langle a_0 \rangle$  and  $\langle f_0 \rangle$

$$m_{\Sigma^+} = \frac{1}{2} (-2m_n + 3m_\Lambda - 2m_{\Xi^-} + 3m_{\Sigma^0})$$

$$m_{\Sigma^-} = \frac{1}{2} (2m_n - 3m_\Lambda + 2m_{\Xi^-} + m_{\Sigma^0})$$

$$m_{\Xi^0} = -m_n - m_p + 3m_\Lambda - m_{\Xi^-} + m_{\Sigma^0}.$$

$$m_{a_0}^2 + m_{f_0}^2 = 2m_\sigma^2$$

Analysis about NS is on the way





*Thank you!*