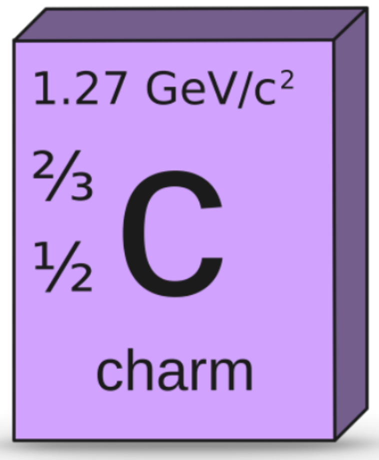

Inclusive charm decays

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第四届强子与重味物理理论与实验联合研讨会

兰州大学，2025年03月21-24日

Semi-inclusive charm decays

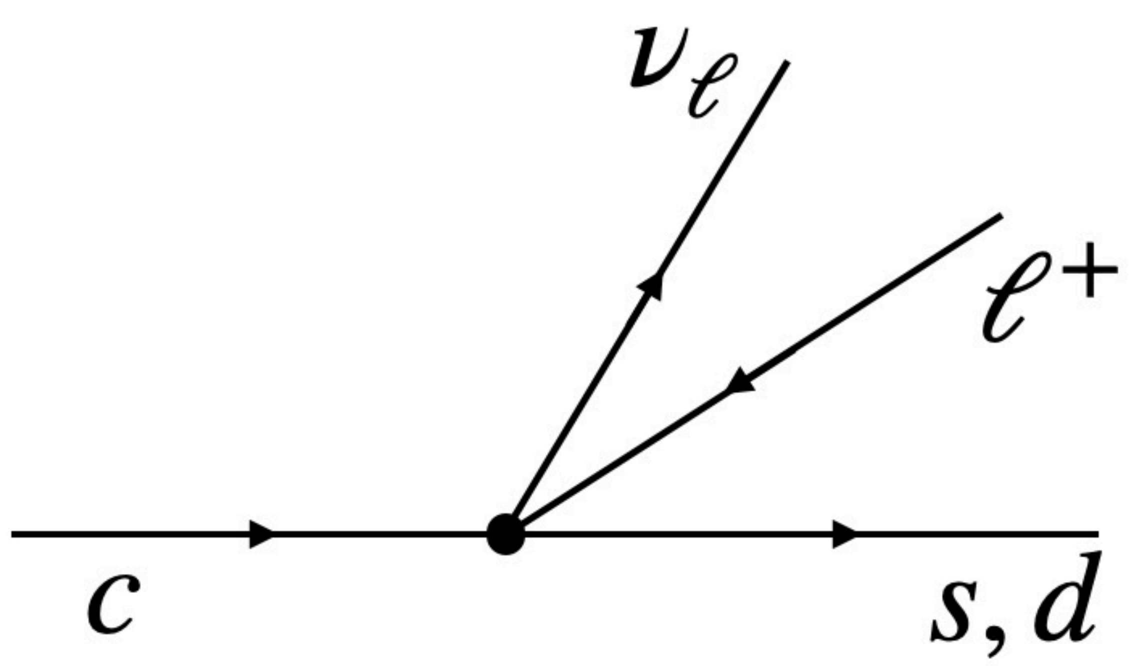
- **Experimental detection of partial final state particles**

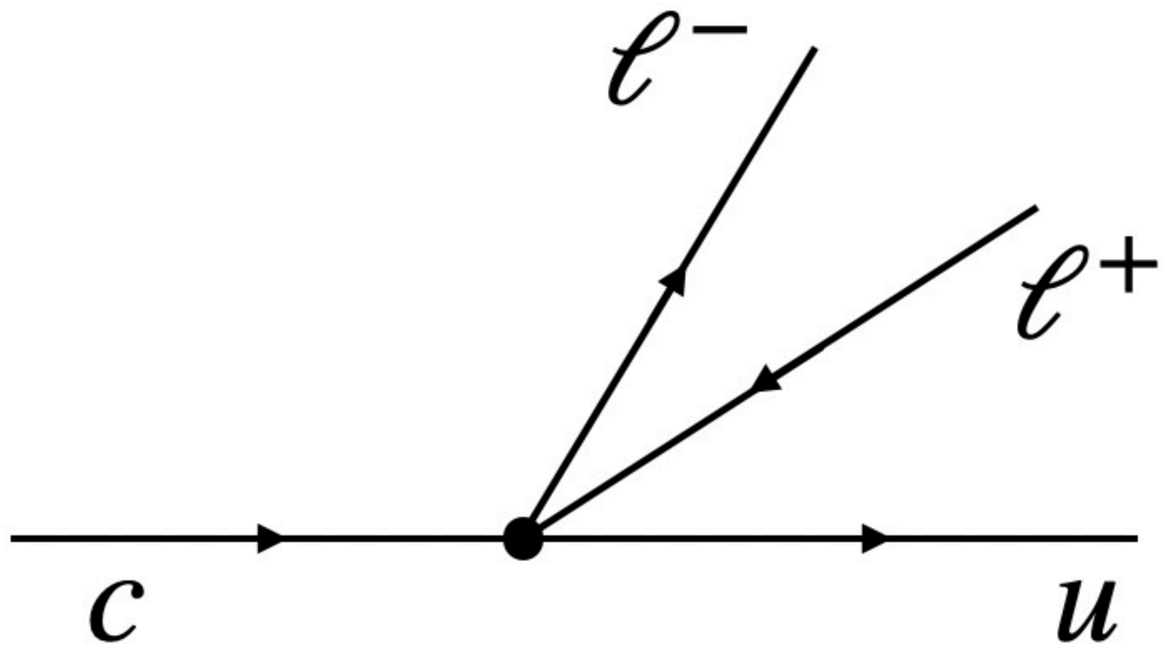
  ➡ $D \to e^+ X$ ($D \to e^+ \nu_e X$, only e^+ is detected)

- **Sum of a group of exclusive channels**

  ➡ $D^0 \to e^+ X_s = D \to e^+ \nu_e K^-, e^+ \nu_e K^- \pi^0, e^+ \nu_e \bar{K}^0 \pi^-, \ldots$

  ➡ $D^0 \to e^+ X_d = D \to e^+ \nu_e \pi^-, e^+ \nu_e \pi^- \pi^0, e^+ \nu_e \pi^- \pi^+ \pi^-, \ldots$





Why inclusive charm decays?

- **As weak decays of heavy hadrons**

  ➡ Probe new physics

  ➡ Understand QCD

- **Compared to exclusive decays**

  ➡ **Better theoretical control**

  ★ More important with stronger experiment

- **Compared to beauty decays**

  ➡ **Special to new dynamics attached with up-type quarks**

  ➡ **More sensitive to power corrections**

  ★ Determination by charm, application in beauty.

Why inclusive charm decays?

- **Resolve (or at least give hints to) current flavor puzzles/anomalies**

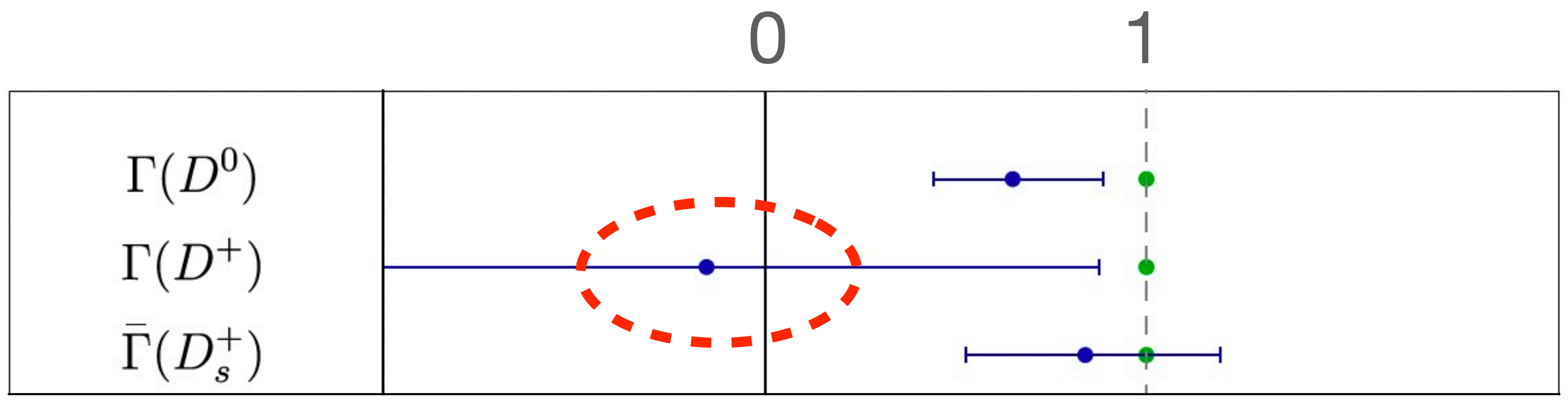
  ➡ Puzzles in charmed hadron lifetimes: theory vs experiment

  ➡ V_{cb}, V_{ub} puzzles: inclusive vs exclusive

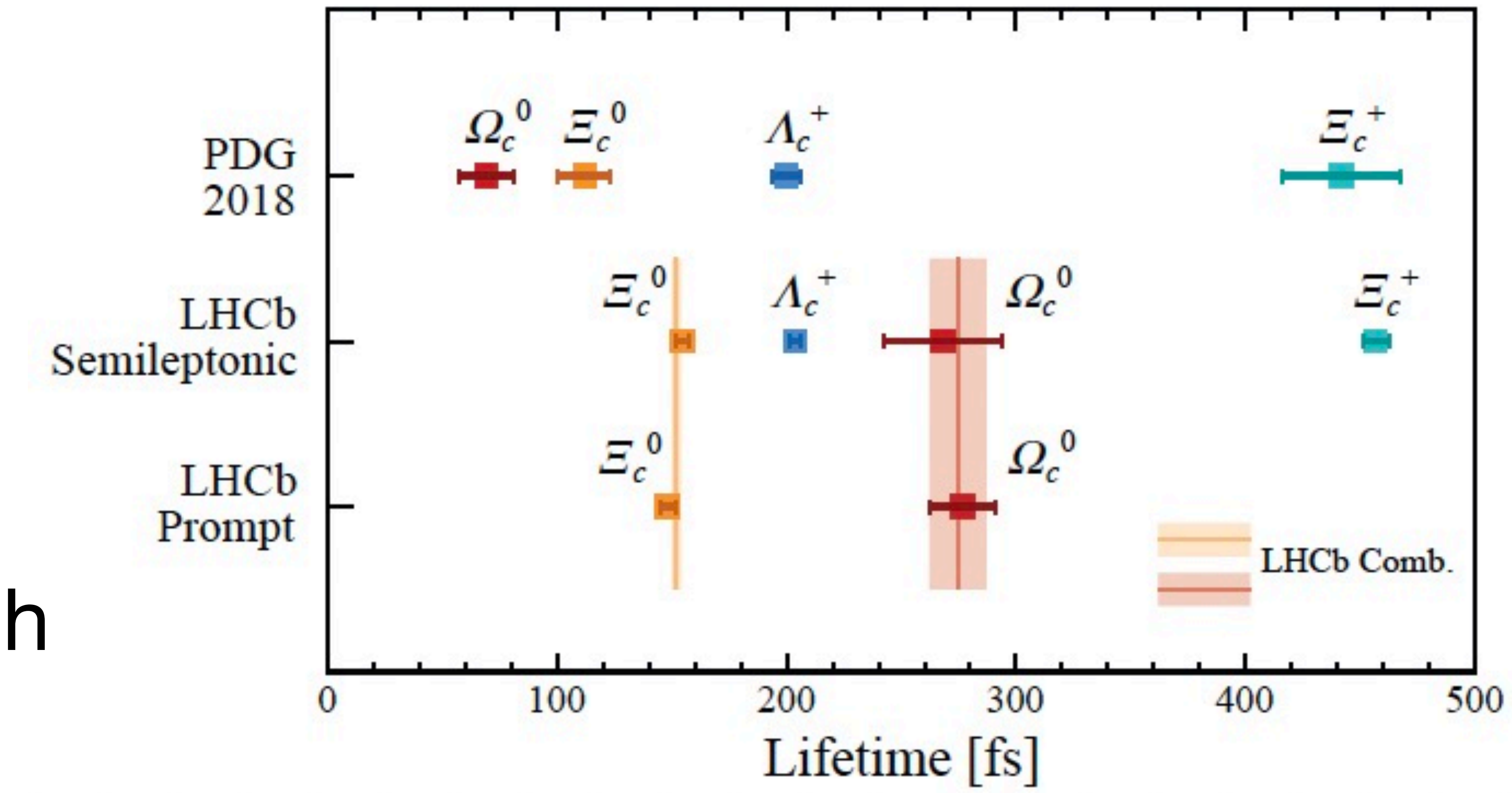
  ➡ $b \to s$ anomalies: P_5' in $B \to K^* \ell\ell$

Why inclusive charm decays?

- **Flavor puzzle 1.** Charmed hadron lifetimes: theory vs experiment



[Lenz et al, ’22]



- **Key issue:** Nonperturbative power corrections with large/unknown uncertainties

- **Solution**: Extraction in the inclusive decay spectrum and application to lifetime

$$\mathcal{O}(1/m_c^3) \Rightarrow \tau(\Xi_c^+) > \tau(\Lambda_c^+) > \tau(\Xi_c^0) > \tau(\boldsymbol{\Omega_c^0}),$$
$$\mathcal{O}(1/m_c^4) \Rightarrow \tau(\boldsymbol{\Omega_c^0}) > \tau(\Xi_c^+) > \tau(\Lambda_c^+) > \tau(\Xi_c^0),$$
$$\mathcal{O}(1/m_c^4) \text{ with } \alpha \Rightarrow \tau(\Xi_c^+) > \tau(\boldsymbol{\Omega_c^0}) > \tau(\Lambda_c^+) > \tau(\Xi_c^0).$$

➡ Dependence on identical hadronic HQET parameters, $\langle H_c | O_i | H_c \rangle$

[Cheng, ’21]

Again a more precise experimental determination of μ_π^2 from fits to semileptonic D^+, D^0 and D_s^+ meson decays – as it was done for the B^+ and B^0 decays – would be very desirable.

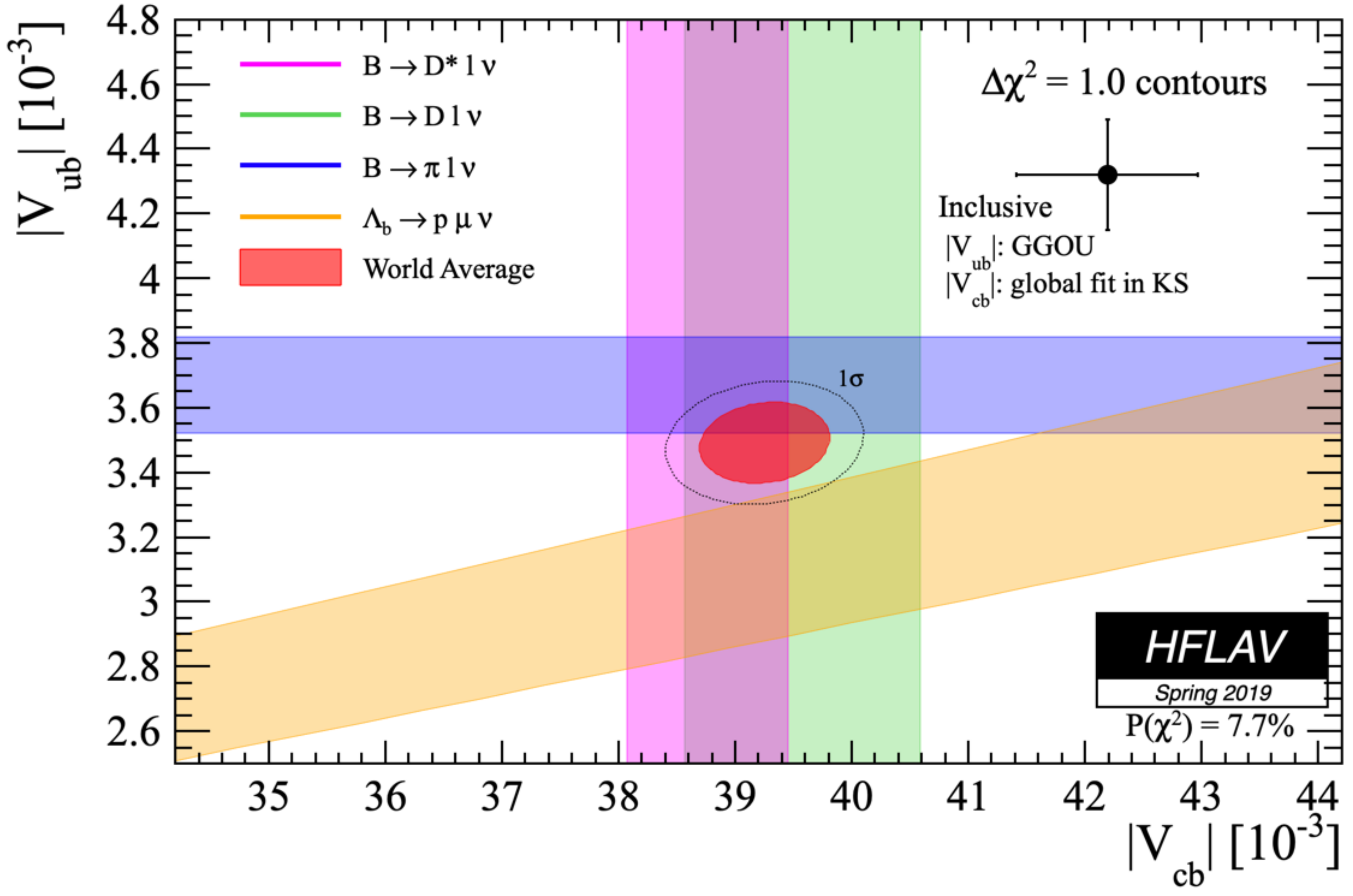
[Lenz et al, ’22]

Why inclusive charm decays?

- **Flavor puzzle 2.** V_{cb}, V_{ub}: inclusive vs exclusive
- **Key issue:** Systematic uncertainties from theoretical inclusive and exclusive frameworks
- **Give hints:**

➡ Test V_{cd}, V_{cs}: inclusive vs exclusive

➡ Await the first inclusive value



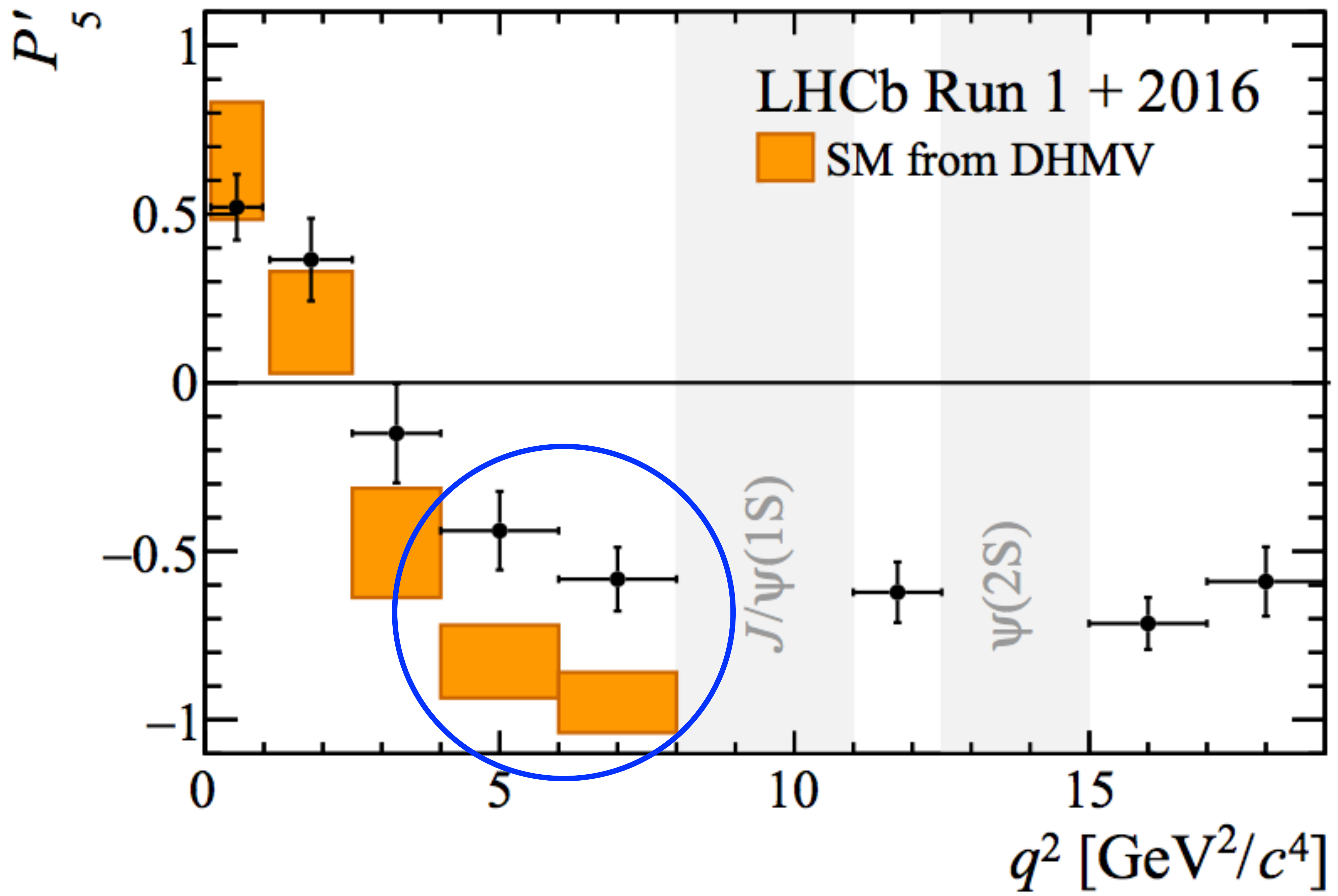
Why inclusive charm decays?

- **Flavor puzzle 3.** $b \to s$ anomalies: P_5' in $B \to K^* \ell\ell$

- **Key issue:** $B \to K^*$ form factor receive large long-distance quark loop contributions, whose first-principle calculation is still missing

- **Give hints:**

  ➡ Test the $c \to u$ transition, by angular distribution in inclusive $D \to X_u \ell\ell$



Theoretical framework

- Optical theorem

$$\sum \langle D|H|X\rangle\langle X|H|D\rangle \propto \mathrm{Im}\int \mathrm{d^4x}\langle \mathrm{D}|\mathrm{T}\{\mathrm{H(x)H(0)}\}|\mathrm{D}\rangle$$

- Operator product expansion (OPE)

➡ Short distance $x \sim 1/m_c$

➡ Dynamical fluctuation in D meson $\sim \Lambda_{\mathrm{QCD}}$

$$T\{H(x)H(0)\} = \sum_n C_n(x)O_n(0) \to 1 + \frac{\Lambda_{\mathrm{QCD}}}{m_c} + \frac{\Lambda_{\mathrm{QCD}}^2}{m_c^2} + \ldots$$

Systematic OPE in HQET.

Theoretical framework

- **Heavy quark effective theory**

$$h_v(x) \equiv e^{-im_c v\cdot x}\frac{1+\gamma\cdot v}{2}c(x) \qquad v=(1,0,0,0)$$

Subtract the big intrinsic momentum,
Leave only ~Λ_{QCD} degrees of freedom.

$$L \ni \bar{h}_v iv\cdot D h_v$$

$$-\bar{h}_v\frac{D_\perp^2}{2m_c}h_v - a(\mu)g\bar{h}_v\frac{\sigma\cdot G}{4m_c}h_v+\ldots$$

Similar to $\dfrac{m}{\sqrt{1-v^2}} = m+\dfrac{1}{2}mv^2+\ldots$

Theoretical framework

Question:

convergent expansion of $\alpha_s(m_c)$ and $\Lambda_{\rm QCD}/m_c$?

- **OPE**

$$T\{H(x)H(0)\} = \sum_n C_n(x)O_n(0)$$

$C_n(x)$

- ☆ LO: $\alpha_s^0(m_c)$
- ☆ NLO: $\alpha_s(m_c)$
- ☆ …

$O_n(0)$

- ☆ Dim-3: $\bar{h}_v h_v$ $(\bar{c}\gamma^\mu c) \rightarrow$ **partonic decay rate.**
- ☆ Dim-5: $\bar{h}_v D_\perp^2 h_v$, $g\bar{h}_v \sigma \cdot G h_v$.
- ☆ Dim-6: $\bar{h}_v D_\mu (v \cdot D) D^\mu h_v$, $(\bar{h}_v \Gamma_1 q)(\bar{q}\Gamma_2 h_v), \ldots$
- ☆ …

- Contribute to inclusive decay rate and also lifetime

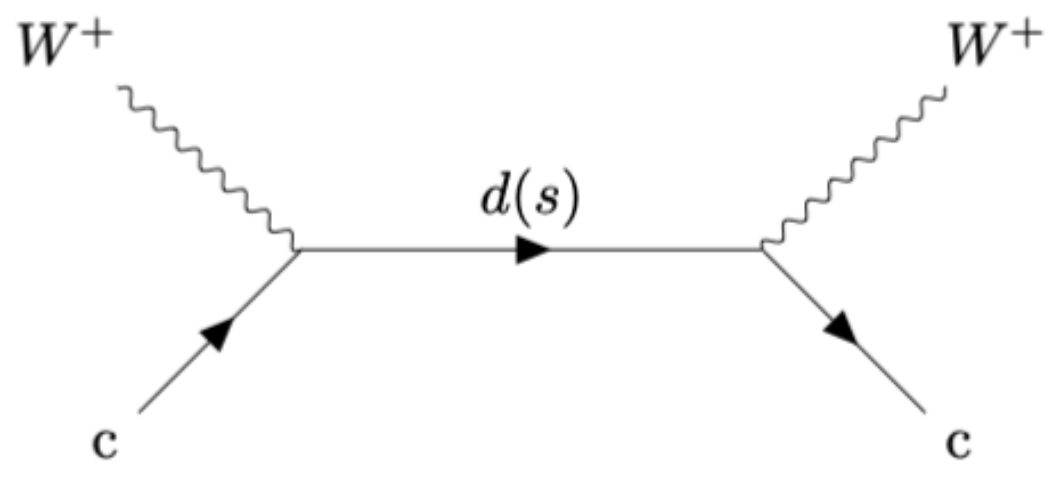
➡ Matrix elements of the same operators

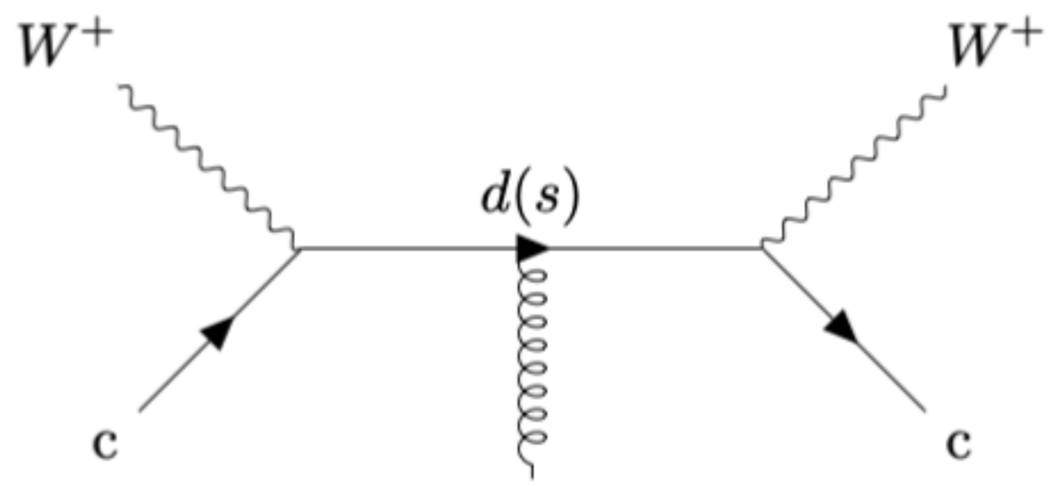
➡ Only different short-distance coefficients

$$\lambda_1 \equiv \frac{1}{4m_D}\langle D|\bar{h}_v(iD)^2 h_v|D\rangle = -\mu_\pi^2$$

$$\lambda_2 \equiv \frac{1}{16(s_c \cdot s_q)}\frac{1}{2m_D}\langle D|\bar{h}_v g\sigma \cdot G h_v|D\rangle = \frac{\mu_G^2}{3}$$

Theoretical results





- Analytical differential decay rate

$$\frac{1}{\Gamma_0}\frac{d\Gamma}{dy} = 12(1-y)y^2\theta(1-y)$$
$$+\frac{2\mu_\pi^2}{m_c^2}\Big[-10y^3\theta(1-y)+2\delta(1-y)\Big]$$
$$-\frac{2\mu_G^2}{3m_c^2}\Big[6y^2(6-5y)\theta(1-y)\Big]+\mathcal{O}(\alpha_s,\frac{\Lambda^3}{m_c^3})$$

$$y \equiv 2E_e/m_c$$

- Up to finite power, the obtained differential decay rate is **NOT** the experimental spectrum

➡ Observables require integration over final states

➡ $\Gamma = \int \frac{d\Gamma}{dy} dy \,, \ \langle E_\ell^n \rangle = \frac{1}{\Gamma}\int \frac{d\Gamma}{dy} E_\ell^n \, dy$ (n=1,2,3,4)

Theoretical results

- Analytical results for total decay rate and energy moments (NNLO & $\Lambda_{\rm QCD}^3/m_c^3$)

NLO analytical integration

NNLO numerical results provided by Long Chen

$$\Gamma_{D_i} = \sum_{q=d,s} \hat{\Gamma}_0 \left|V_{cq}\right|^2 m_c^5 \Big\{ 1 + \frac{\alpha_s}{\pi}\frac{2}{3}\left(\frac{25}{4}-\pi^2\right) + \frac{\alpha_s^2}{\pi^2}\left[\frac{\beta_0}{4}\frac{2}{3}\left(\frac{25}{4}-\pi^2\right)\log\left(\frac{\mu^2}{m_c^2}\right) + 2.14690 n_l - 29.88311\right]$$
$$- 8\rho\delta_{sq} - \frac{1}{2}\frac{\mu_\pi^2(D_i)}{m_c^2} - \frac{3}{2}\frac{\mu_G^2(D_i)}{m_c^2} + 6\frac{\rho_D^3(D_i)}{m_c^3} + \ldots \Big\},$$

[Chen,Chen,Guan,Ma,'23]

Dim-5, $\Lambda_{\rm QCD}^2/m_c^2$

Dim-6, $\Lambda_{\rm QCD}^3/m_c^3$

Theoretical results

- Analytical results for total decay rate and energy moments (NNLO & $\Lambda^3_{\rm QCD}/m_c^3$)

$$\langle E_e\rangle_{D_i} = \frac{\hat\Gamma_0}{\Gamma_{D_i}} \sum_{q=d,s} |V_{cq}|^2\, m_c^6 \left[\frac{3}{10} + \frac{\alpha_s}{\pi} a_1^{(1)} + \frac{\alpha_s^2}{\pi^2} a_1^{(2)} - 3\rho\delta_{sq} - \frac{1}{2}\frac{\mu_G^2(D_i)}{m_c^2} + \frac{139}{30}\frac{\rho_D^3(D_i)}{m_c^3} + \frac{3}{10}\frac{\rho_{LS}^3(D_i)}{m_c^3} + \ldots\right],$$

$$\langle E_e^2\rangle_{D_i} = \frac{\hat\Gamma_0}{\Gamma_{D_i}} \sum_{q=d,s} |V_{cq}|^2\, m_c^7 \left[\frac{1}{10} + \frac{\alpha_s}{\pi} a_2^{(1)} + \frac{\alpha_s^2}{\pi^2} a_2^{(2)} - \frac{6}{5}\rho\delta_{sq} + \frac{1}{12}\frac{\mu_\pi^2(D_i)}{m_c^2} - \frac{11}{60}\frac{\mu_G^2(D_i)}{m_c^2} + \frac{17}{6}\frac{\rho_D^3(D_i)}{m_c^3} \right.$$
$$\left. + \frac{7}{30}\frac{\rho_{LS}^3(D_i)}{m_c^3} + \ldots\right],$$

$$\langle E_e^3\rangle_{D_i} = \frac{\hat\Gamma_0}{\Gamma_{D_i}} \sum_{q=d,s} |V_{cq}|^2\, m_c^8 \left[\frac{1}{28} + \frac{\alpha_s}{\pi} a_3^{(1)} + \frac{\alpha_s^2}{\pi^2} a_3^{(2)} - \frac{1}{2}\rho\delta_{sq} + \frac{1}{14}\frac{\mu_\pi^2(D_i)}{m_c^2} - \frac{1}{14}\frac{\mu_G^2(D_i)}{m_c^2} + \frac{223}{140}\frac{\rho_D^3(D_i)}{m_c^3} \right.$$
$$\left. + \frac{1}{7}\frac{\rho_{LS}^3(D_i)}{m_c^3} + \ldots\right],$$

$$\langle E_e^4\rangle_{D_i} = \frac{\hat\Gamma_0}{\Gamma_{D_i}} \sum_{q=d,s} |V_{cq}|^2\, m_c^9 \left[\frac{3}{224} + \frac{\alpha_s}{\pi} a_4^{(1)} + \frac{\alpha_s^2}{\pi^2} a_4^{(2)} - \frac{3}{14}\rho\delta_{sq} + \frac{3}{64}\frac{\mu_\pi^2(D_i)}{m_c^2} - \frac{13}{448}\frac{\mu_G^2(D_i)}{m_c^2} + \frac{481}{560}\frac{\rho_D^3(D_i)}{m_c^3} \right.$$
$$\left. + \frac{9}{112}\frac{\rho_{LS}^3(D_i)}{m_c^3} + \ldots\right],$$

Mass scheme

- Pole mass scheme

$$\Gamma/\Gamma_{\rm LO} = 1 - 0.768104\alpha_s - 2.37521\alpha_s^2 \approx 1 - 30\% - 36\%$$

Become negative at NNNLO!

- $\overline{\rm MS}$ mass scheme

$$\Gamma/\Gamma_{\rm LO} = 1 + 1.35\alpha_s + 3.02\alpha_s^2 \approx 1 + 52\% + 46\%$$

Very slow convergence!

$$\Gamma = m_c^5(\Gamma^{(0)} + \alpha_s\Gamma^{(1)} + \alpha_s^2\Gamma^{(2)}) = \left(\overline{m}_c(1 + \alpha_s m^{(1)} + \alpha_s^2 m^{(2)})\right)^5 (\Gamma^{(0)} + \alpha_s\Gamma^{(1)} + \alpha_s^2\Gamma^{(2)})$$

- 1S mass scheme (half of J/ψ mass)

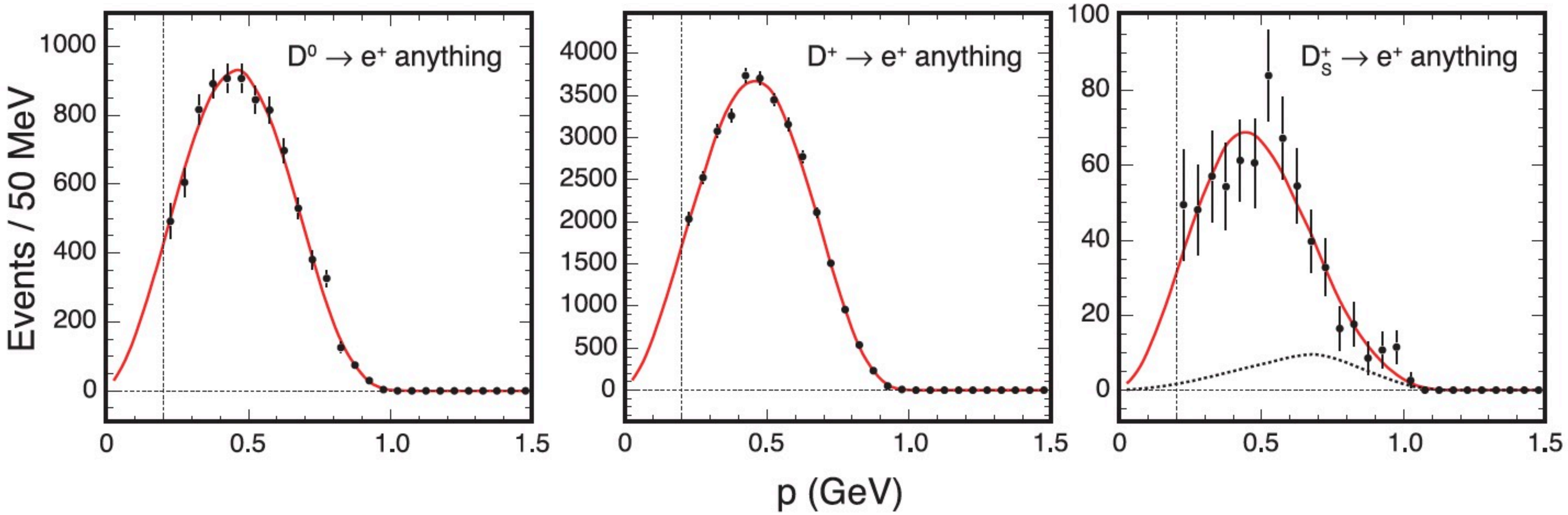
$$\Gamma/\Gamma_{\rm LO} \approx 1 - 13\% - 5\%$$

Answer: convergent expansion of $\alpha_s(m_c)$!

Experimental status

CLEO measurements

$D^0 \to e^+X$ $\quad$ $D^+ \to e^+X$ $\quad$ $D_s^+ \to e^+X$

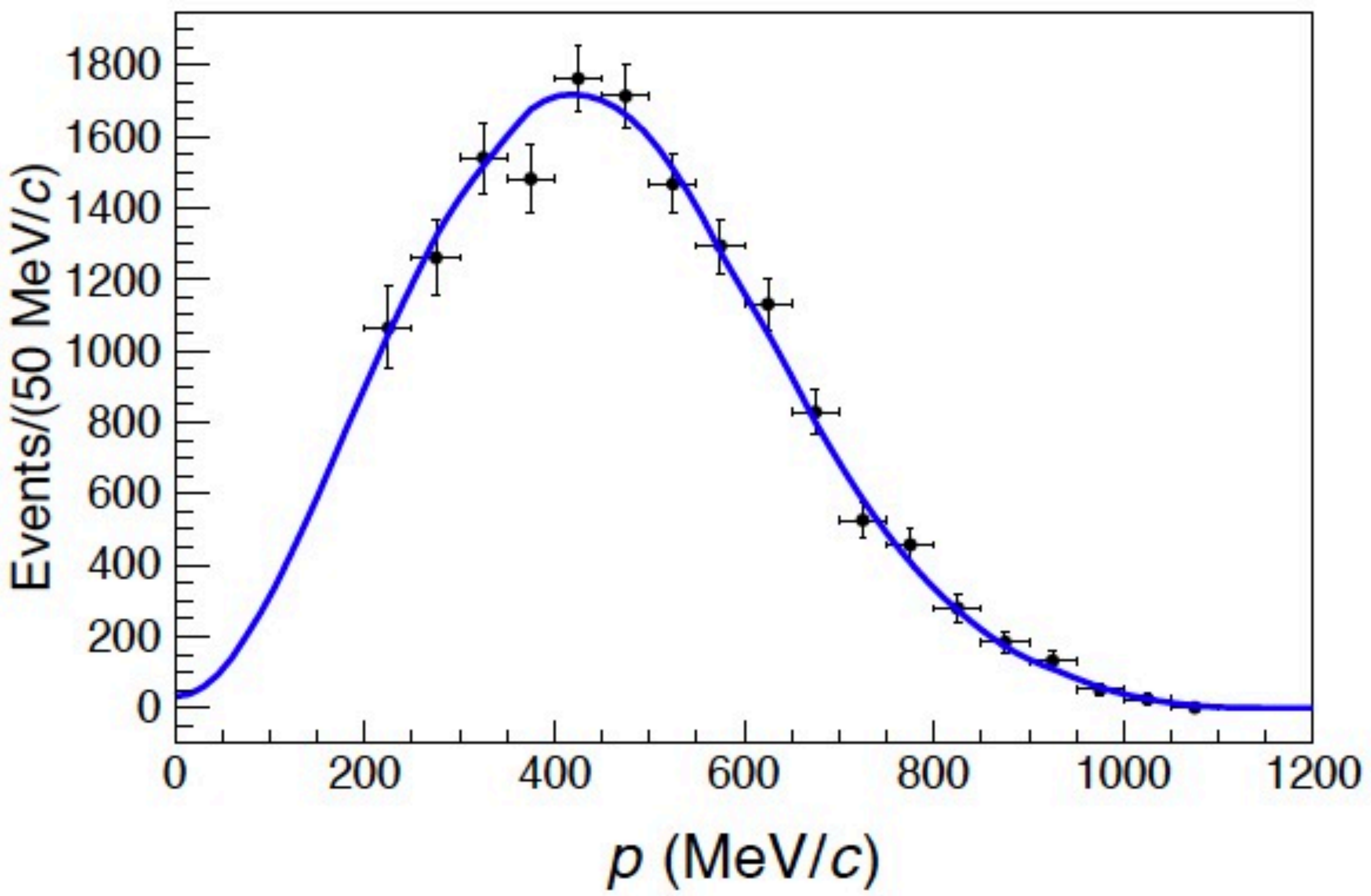


$$\mathcal{B}(D^0 \to X e^+ \nu_e) = (6.46 \pm 0.09 \pm 0.11)\%,$$
$$\mathcal{B}(D^+ \to X e^+ \nu_e) = (16.13 \pm 0.10 \pm 0.29)\%,$$
$$\mathcal{B}(D_s^+ \to X e^+ \nu_e) = (6.52 \pm 0.39 \pm 0.15)\%,$$

[CLEO, '09]

BESIII measurements

$D_s^+ \to e^+X$



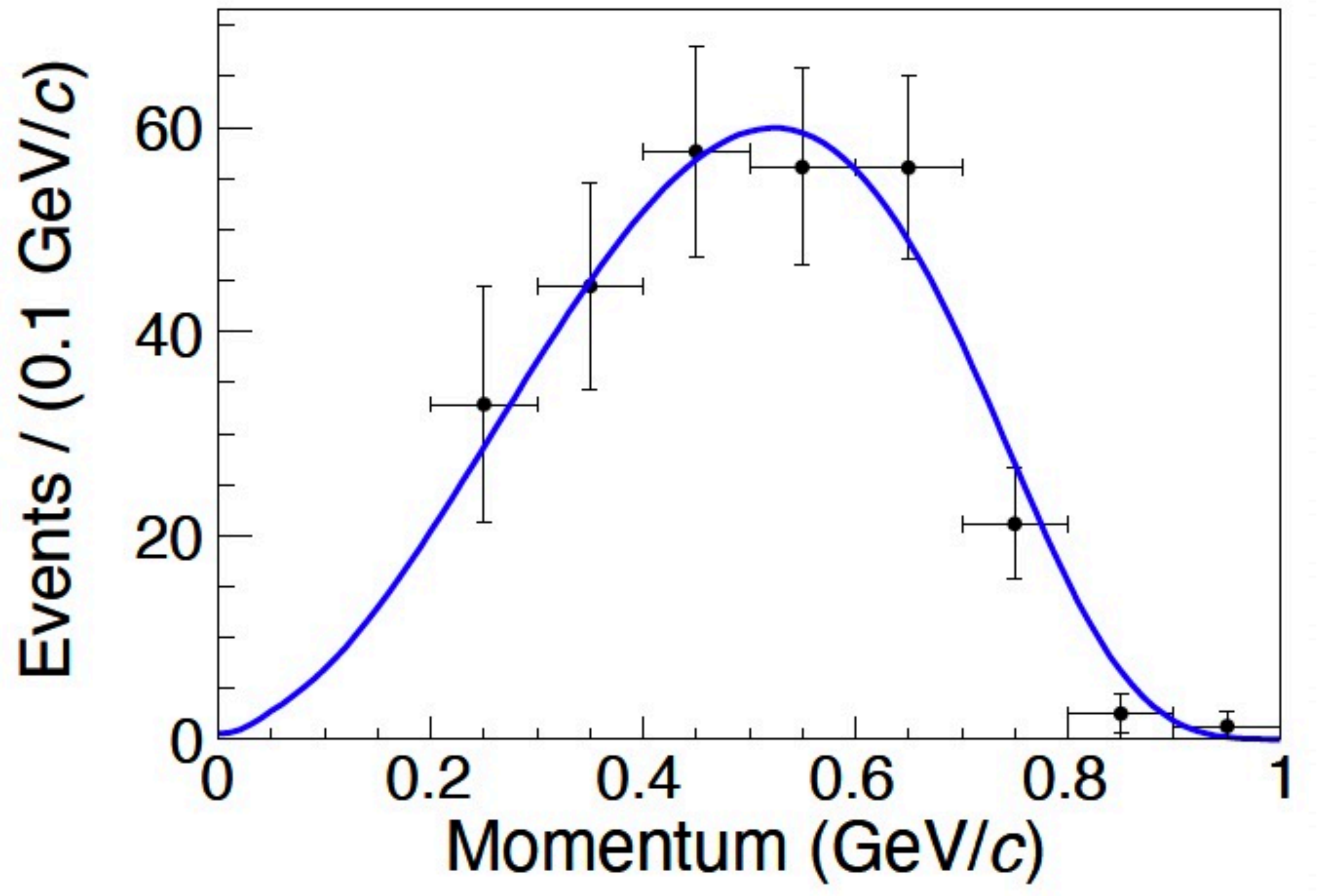
$$B(D_s^+ \to X e^+ \nu_e) = (6.30 \pm 0.13 \pm 0.10)\,\%$$

[BESIII, '21]

Experimental status

BESIII measurements

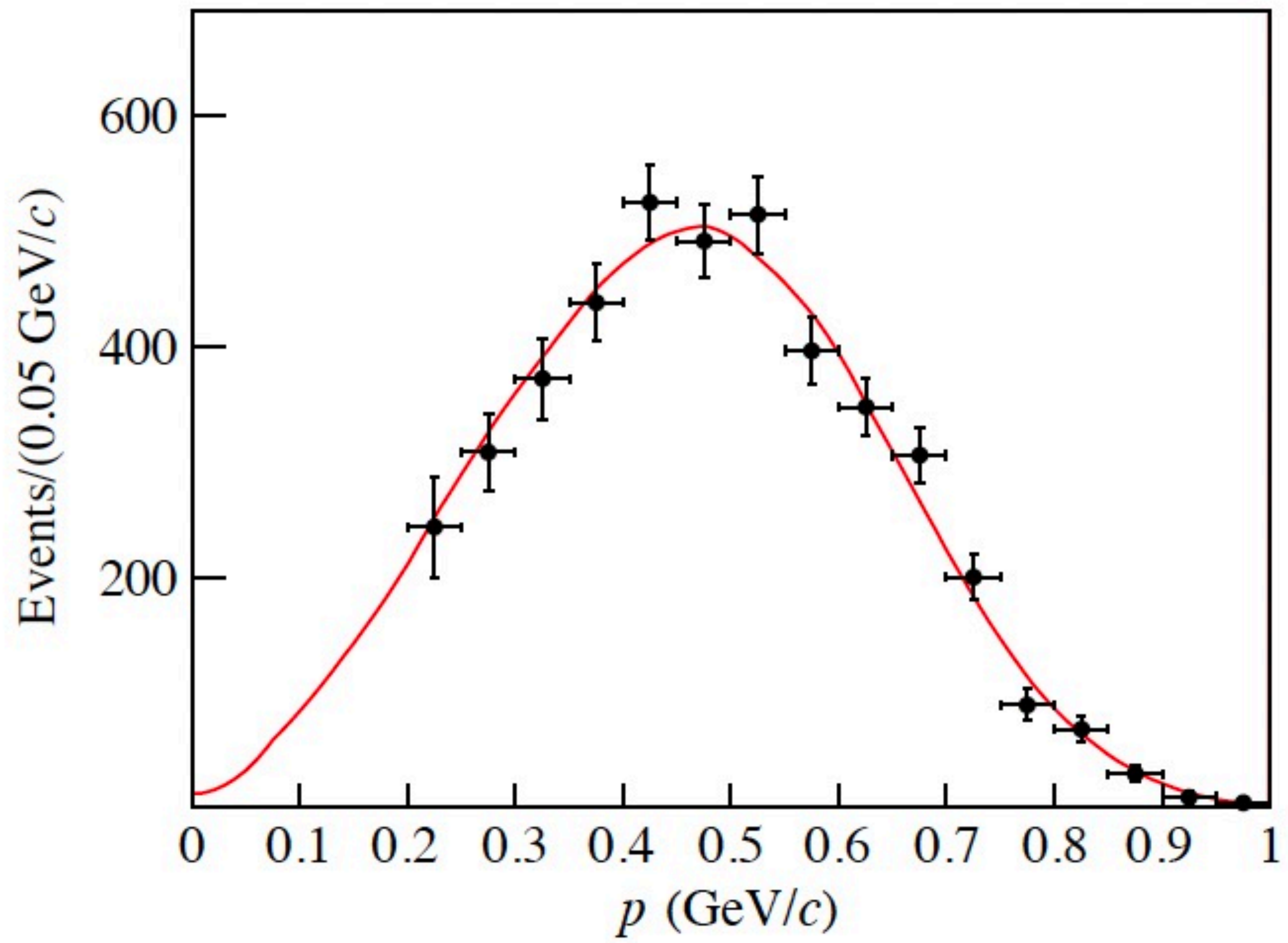
$\Lambda_c \to e^+X$



$$B(\Lambda_c^+ \to Xe^+\nu_e) = (3.95 \pm 0.34 \pm 0.09)\,\%$$

[BESIII (567 pb^{-1}), ’18]

$\Lambda_c \to e^+X$



$$\mathcal{B}(\Lambda_c^+ \to Xe^+\nu_e) = (4.06 \pm 0.10_{\text{stat.}} \pm 0.09_{\text{syst}})\%$$

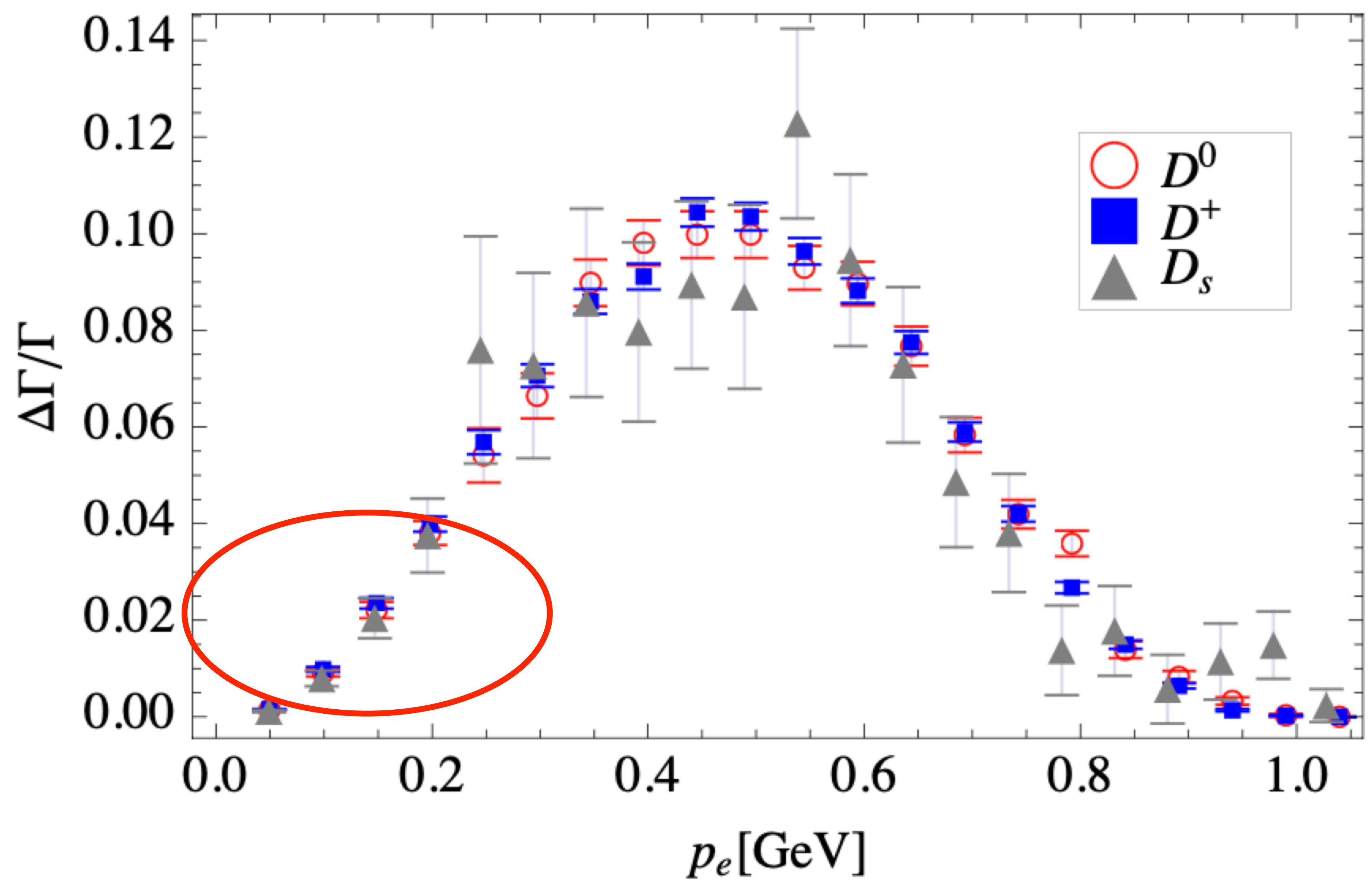
[BESIII (4.5 fb^{-1}), ’23]

Experimental status

Extrapolation

Data

$$\frac{d\Gamma}{dy} = ay^2(1+by)(1-y)$$



Lab frame

Lorentz boost

$$\langle E_e\rangle_{exp}^{D_s} = 0.437(6)\text{GeV}, \quad \langle E_e^2\rangle_{exp}^{D_s} = 0.220(5)\text{GeV}^2 \quad \langle E_e^3\rangle_{exp}^{D_s} = 0.121(4)\text{GeV}^3, \quad \langle E_e^4\rangle_{exp}^{D_s} = 0.072(3)\text{GeV}^4$$

$$\langle E_e\rangle_{exp}^{D^0} = 0.462(5)\text{GeV}, \quad \langle E_e^2\rangle_{exp}^{D^0} = 0.242(5)\text{GeV}^2 \quad \langle E_e^3\rangle_{exp}^{D^0} = 0.138(4)\text{GeV}^3, \quad \langle E_e^4\rangle_{exp}^{D^0} = 0.084(3)\text{GeV}^4$$

$$\langle E_e\rangle_{exp}^{D^+} = 0.455(4)\text{GeV}, \quad \langle E_e^2\rangle_{exp}^{D^+} = 0.236(4)\text{GeV}^2 \quad \langle E_e^3\rangle_{exp}^{D^+} = 0.134(3)\text{GeV}^3, \quad \langle E_e^4\rangle_{exp}^{D^+} = 0.081(3)\text{GeV}^4$$

Uncertainties are obtained assuming independent bins

Rest frame

Global fit

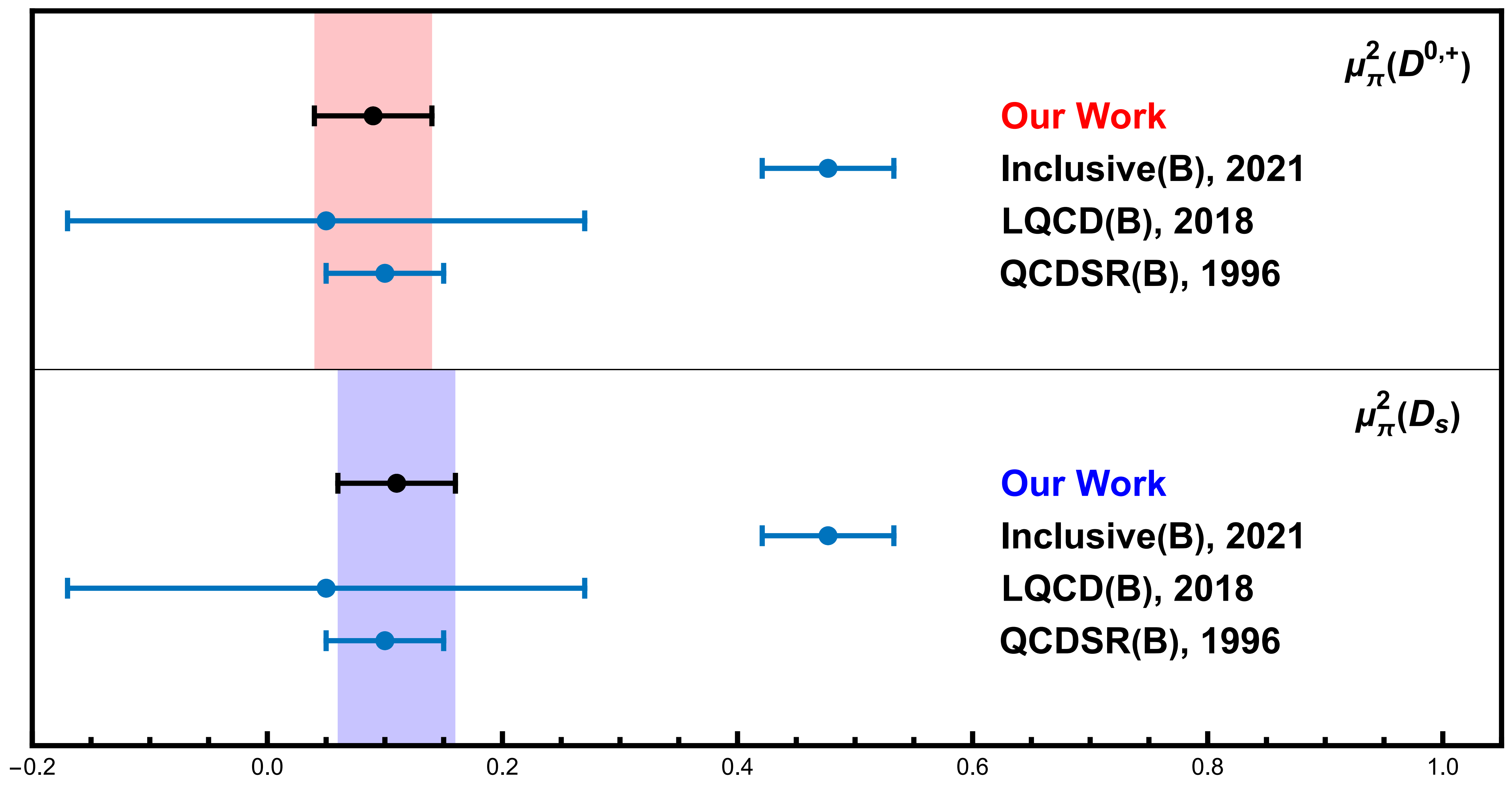
$\overline{\text{MS}}$ scheme	$\chi^2/\text{d.o.f.}$	D_i	μ_π^2/GeV^2	μ_G^2/GeV^2	$\rho_{\text{D}}^3/\text{GeV}^3$	ρ_{LS}^3/GeV^3
Scenario 1	4.5	$D^{0,+}$	0.09 ± 0.01	0.27 ± 0.14	-	-
		D_s	0.09 ± 0.02	0.39 ± 0.12	-	-
Scenario 2	2.1	$D^{0,+}$	0.11 ± 0.02	0.26 ± 0.14	-0.002 ± 0.002	0.003 ± 0.002
		D_s	0.12 ± 0.02	0.38 ± 0.13	-0.003 ± 0.002	0.005 ± 0.002

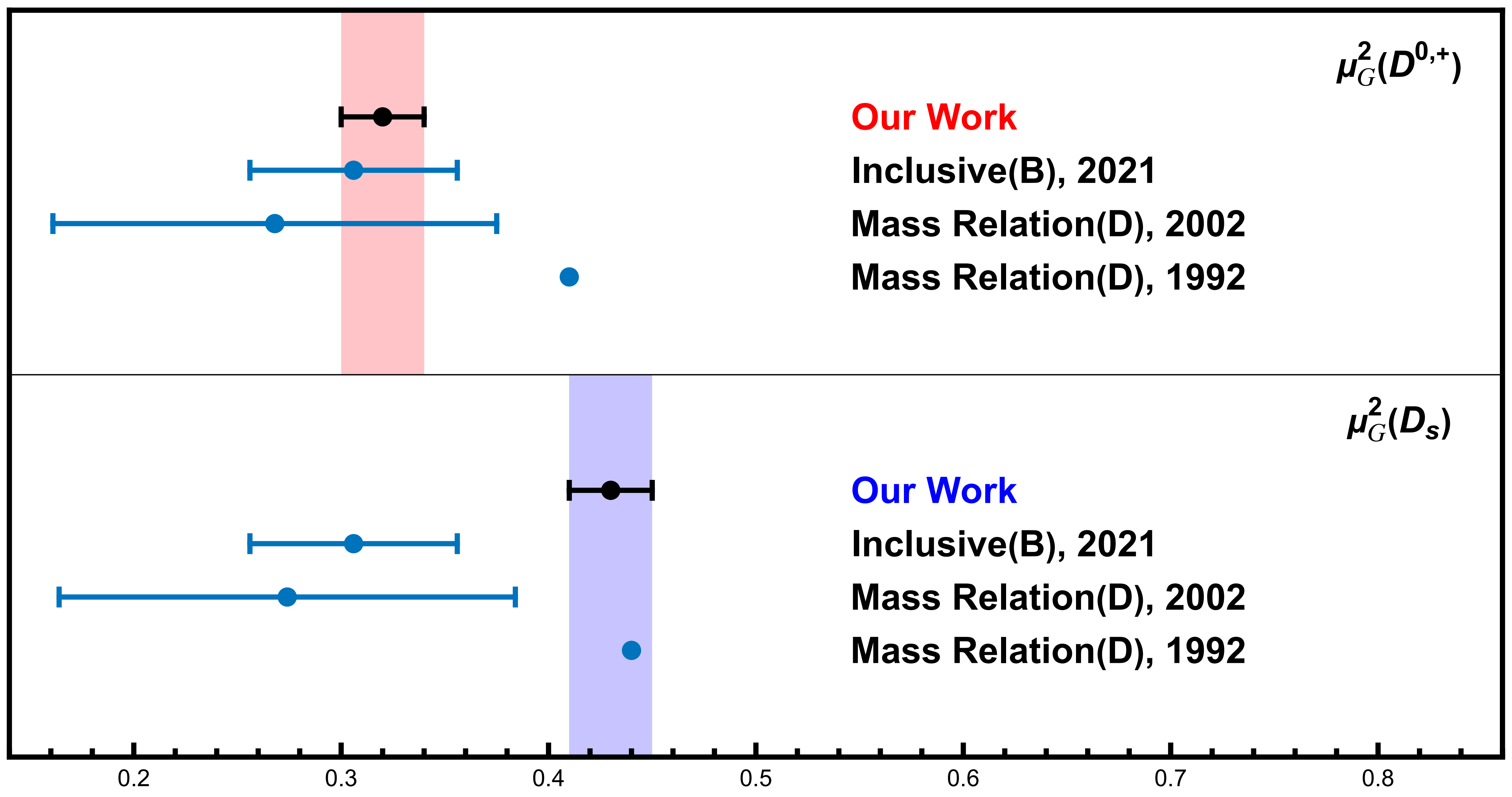
1S scheme	$\chi^2/\text{d.o.f.}$	D_i	μ_π^2/GeV^2	μ_G^2/GeV^2	$\rho_{\text{D}}^3/\text{GeV}^3$	ρ_{LS}^3/GeV^3
Scenario 1	4.9	$D^{0,+}$	0.04 ± 0.01	0.33 ± 0.02	-	-
		D_s	0.06 ± 0.02	0.44 ± 0.02	-	-
Scenario 2	0.33	$D^{0,+}$	0.09 ± 0.02	0.32 ± 0.02	-0.003 ± 0.002	0.004 ± 0.002
		D_s	0.11 ± 0.02	0.43 ± 0.02	-0.004 ± 0.002	0.005 ± 0.002

Difference between Scenario 1 & 2 as systematic uncertainties.

Global fit

$$\mu_\pi^2(D^{0,+}) = (0.09 \pm 0.05)\mathrm{GeV}^2, \qquad \mu_\pi^2(D_s^+) = (0.11 \pm 0.05)\mathrm{GeV}^2,$$
$$\mu_G^2(D^{0,+}) = (0.32 \pm 0.02)\mathrm{GeV}^2, \qquad \mu_G^2(D_s^+) = (0.43 \pm 0.02)\mathrm{GeV}^2,$$
$$\rho_D^3(D^{0,+}) = (-0.003 \pm 0.002)\mathrm{GeV}^3, \qquad \rho_D^3(D_s^+) = (-0.004 \pm 0.002)\mathrm{GeV}^3,$$
$$\rho_{LS}^3(D^{0,+}) = (0.004 \pm 0.002)\mathrm{GeV}^3, \qquad \rho_{LS}^3(D_s^+) = (0.005 \pm 0.002)\mathrm{GeV}^3.$$

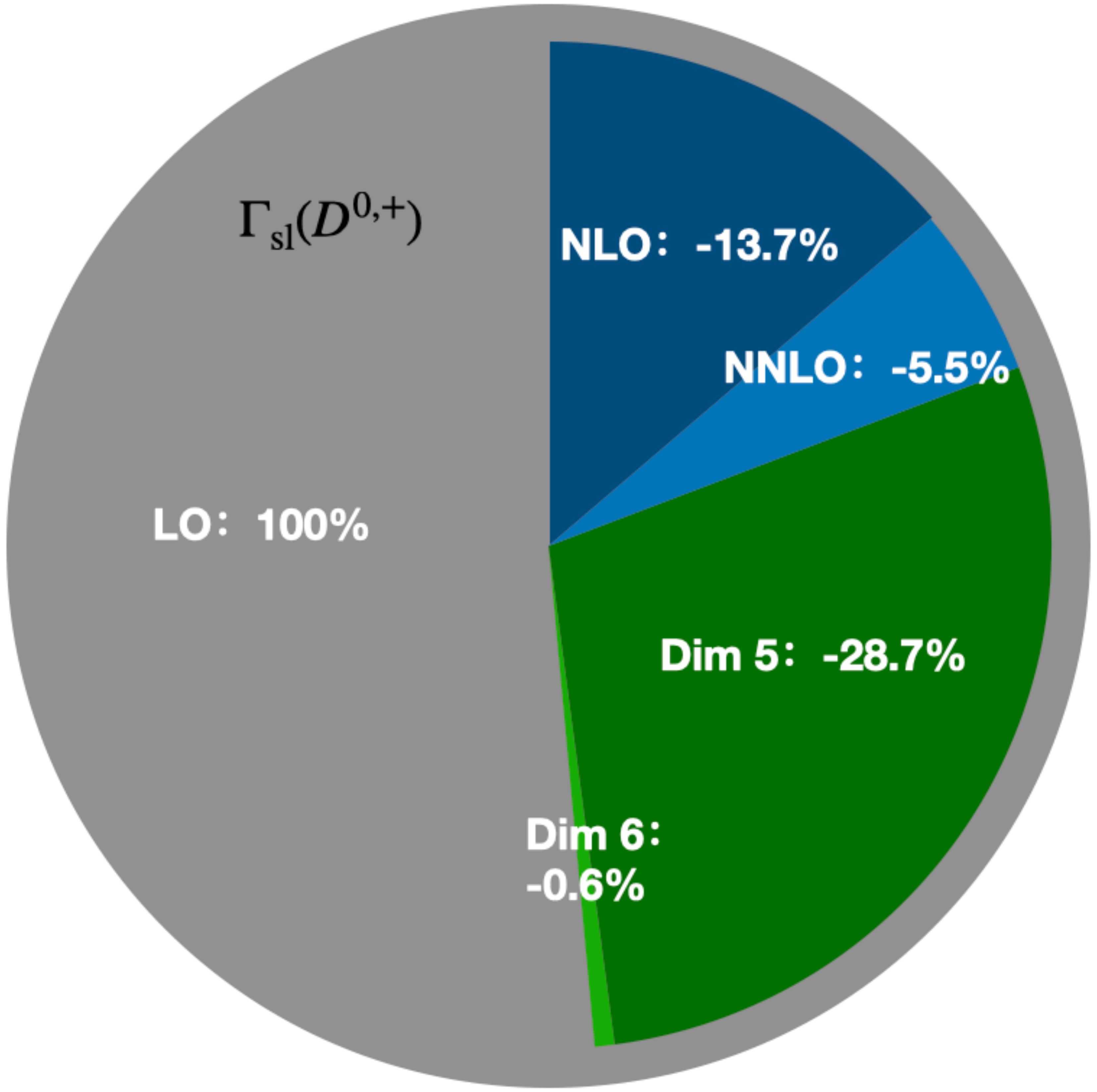


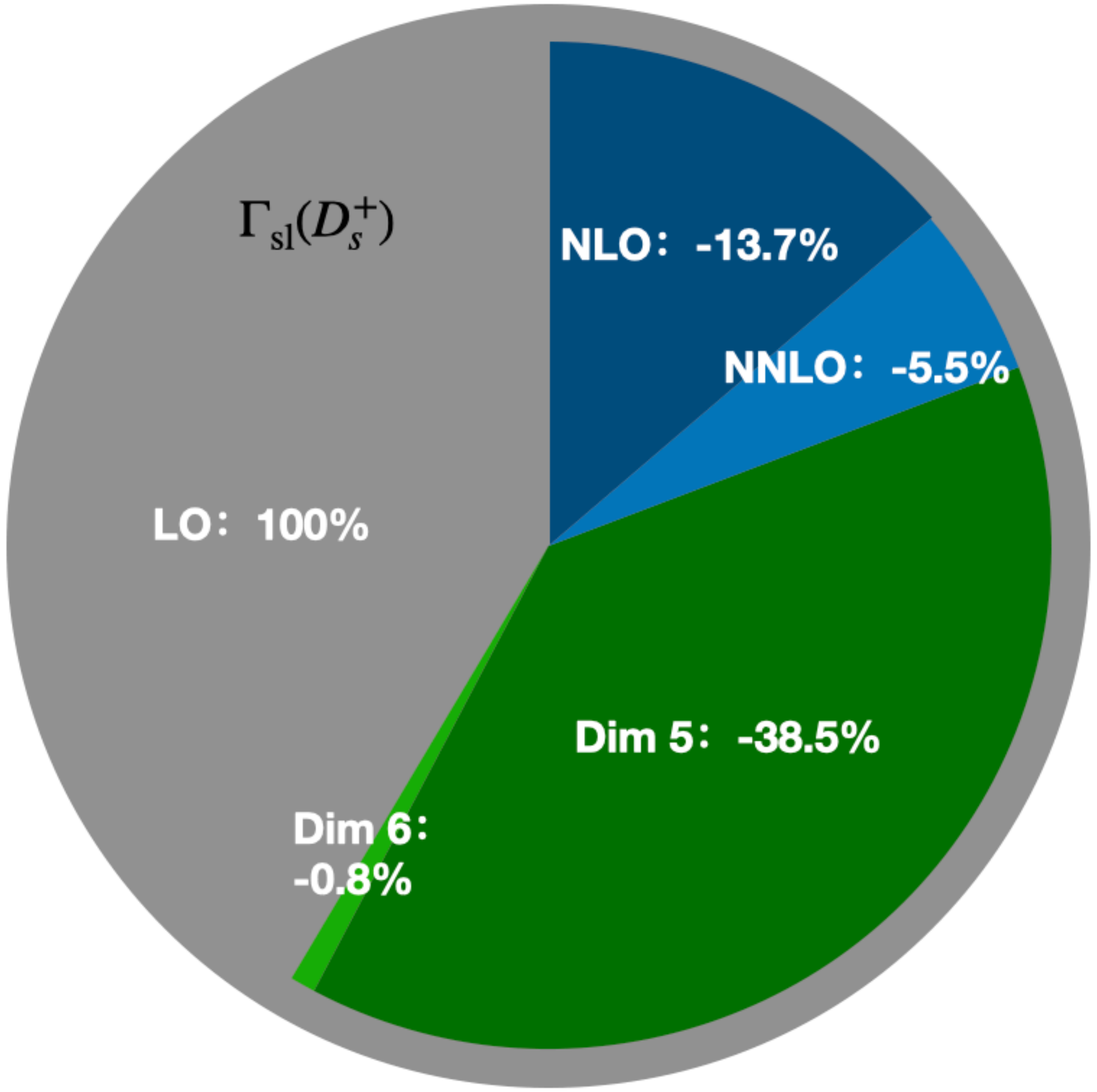


Considerable SU(3) and heavy quark symmetry breaking.

Convergence

Contributions to the inclusive D and Ds decay widths





Answer: convergent expansion of $\alpha_s(m_c)$ and Λ_{QCD}/m_c !

Summary and Prospect

- α_s-expansion and heavy quark expansion are **valid** in inclusive charm decays
- HQE parameters in inclusive charm decays are determined by data model independently for the **first time**
- **Possible improvements**

➡ Include higher order radiative corrections, $\mathcal{O}(\alpha_s^3)$

➡ Include higher power corrections, complete dimension-6 and -7 operator

➡ Extend the study to charmed baryons

➡ ……

Wishlist

- Measurements performed in the **rest frame** of charmed hadrons

- **Direct measurements** of $\langle E_e^n \rangle$, instead of the electron energy spectrum

- Measurements of q^2 **moments**, good for higher-dimensional operators

- Separate X_d, X_s, to give **first** inclusive measurements of V_{cd}, V_{cs}

Thank you!

Backup

Mass scheme transformation

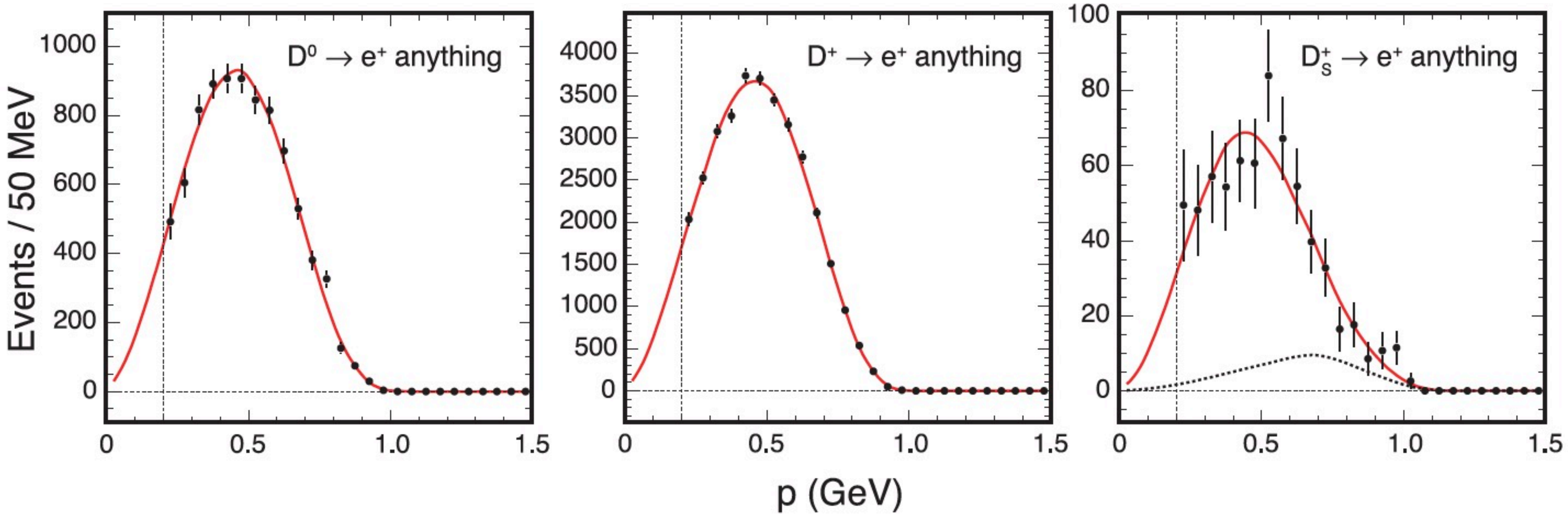
$$m_c = \overline{m}_c(\mu)\left[1 + \frac{\alpha_s(\mu)}{\pi}\left(\frac{4}{3} + \log\left(\frac{\mu^2}{\overline{m}_c^2}\right)\right) + \frac{\alpha_s^2(\mu)}{\pi^2}\frac{1}{288}\Big(112\pi^2 + 2905 + 16\pi^2\log(4) - 48\zeta(3)\right.$$
$$\left. - 12(2n_f - 45)\log^2\left(\frac{\mu^2}{\overline{m}_c^2}\right) - 4(26n_f - 519)\log\left(\frac{\mu^2}{\overline{m}_c^2}\right) - 2\left(71 + 8\pi^2\right)n_f\Big) + \mathcal{O}(\alpha_s^3)\right]$$

$$m_c = m_{c,1\mathrm{S}} + m_{c,1\mathrm{S}}\frac{\alpha_s(\mu)^2 C_F^2}{8}\left\{1 + \frac{\alpha_s}{\pi}\left[\left(-\log\left(\alpha_s(\mu)m_{c,1\mathrm{S}}C_F/\mu\right) + \frac{11}{6}\right)\beta_0 - 4 + \frac{\pi}{8}C_F\alpha_s\right] + \ldots\right\}$$

Experimental status

CLEO measurements

$D^0 \to e^+X$ $D^+ \to e^+X$ $D_s^+ \to e^+X$

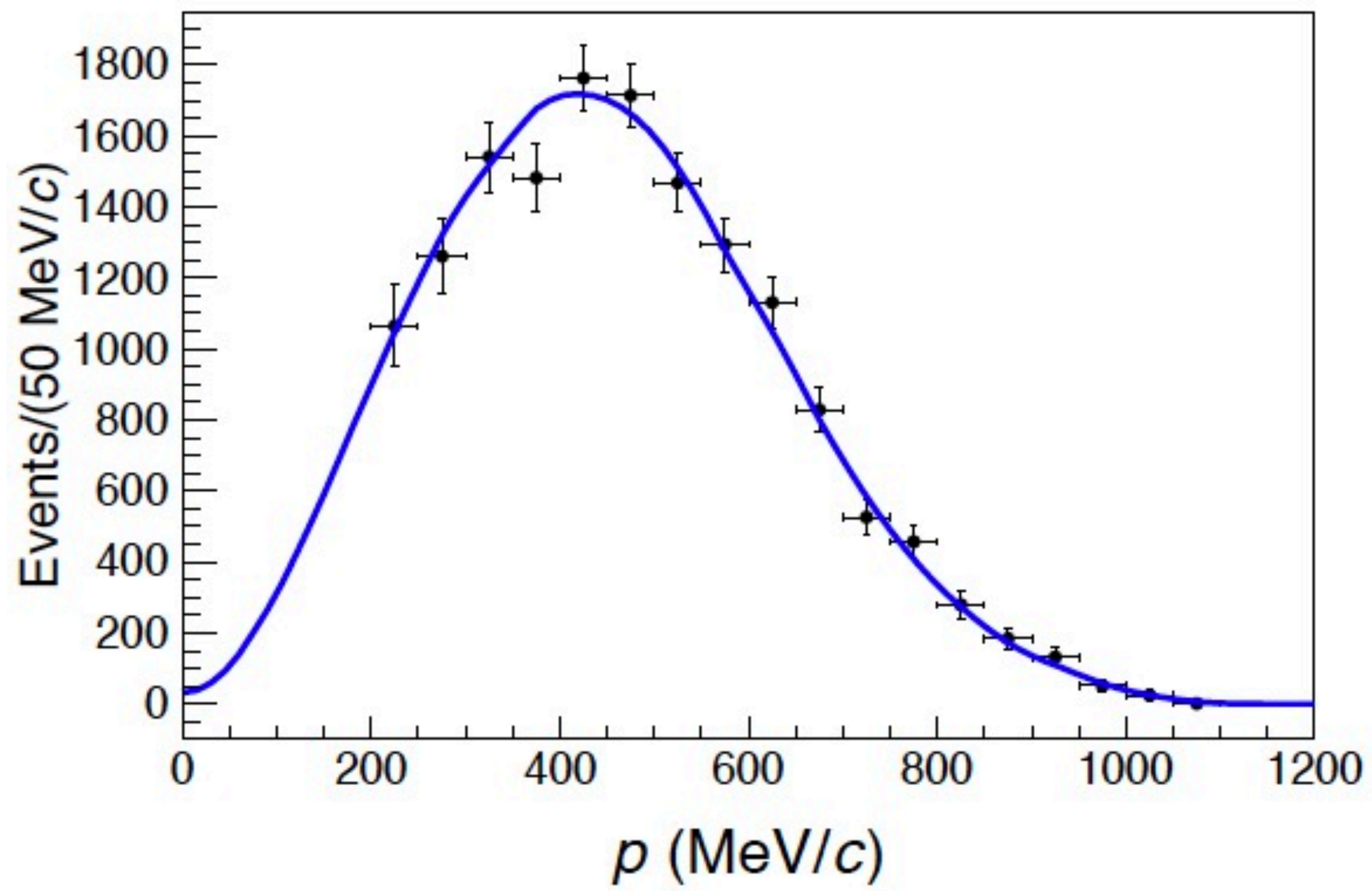


; 3.0×10^6 $D^0\bar{D}^0$ and 2.4×10^6 D^+D^- pairs, and is used to
ays. The latter data set contains 0.6×10^6 $D_s^{*\pm}D_s^{\mp}$ pairs,

[CLEO (818pb^{-1}($D^{0,\pm}$), 602pb^{-1}($D_s^{\pm}$)), '09]

BESIII measurements

$D_s^+ \to e^+X$



$E_{\rm cm}$ (MeV)	$\int\mathcal{L}\,dt$ (pb^{-1})	$N_{D_s}(\times10^6)$
4178	$3189.0\pm0.9\pm31.9$	6.4
4189	$526.7\pm0.1\pm2.2$	1.0
4199	$526.0\pm0.1\pm2.1$	1.0
4209	$517.1\pm0.1\pm1.8$	0.9
4219	$514.6\pm0.1\pm1.8$	0.8
$4225-4230$ [32]	$1047.3\pm0.1\pm10.2$ [33]	1.3

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