

Existence of  $0^{--} \bar{D}_s D K$  on nature of  $D_{s0}^*(2317)$

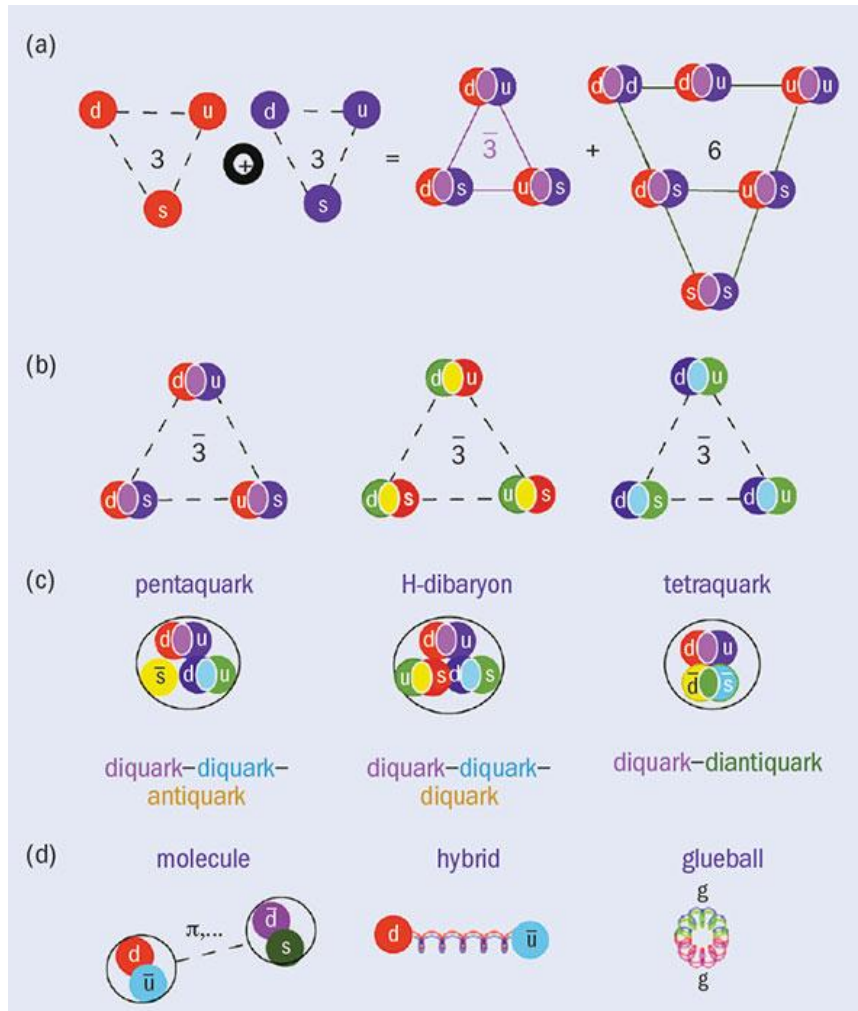
arxiv:2501.11358

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# Exotic hadronic states



**Normal hadron :**

**meson:** quark-antiquark pair

**baryon:** 3 quarks

**Exotic hadron state:**

## 1. Glueball

Composed of gluons

## 2. Hybrid

Composed of quarks and gluons

## 3. Multiquark state

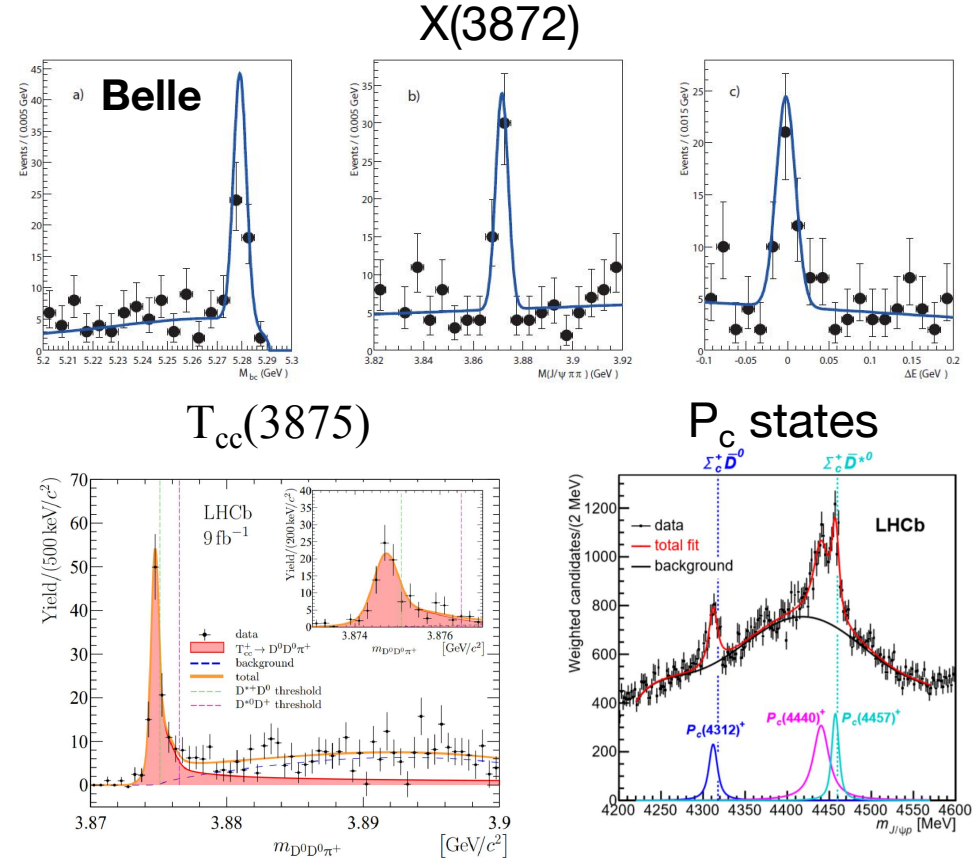
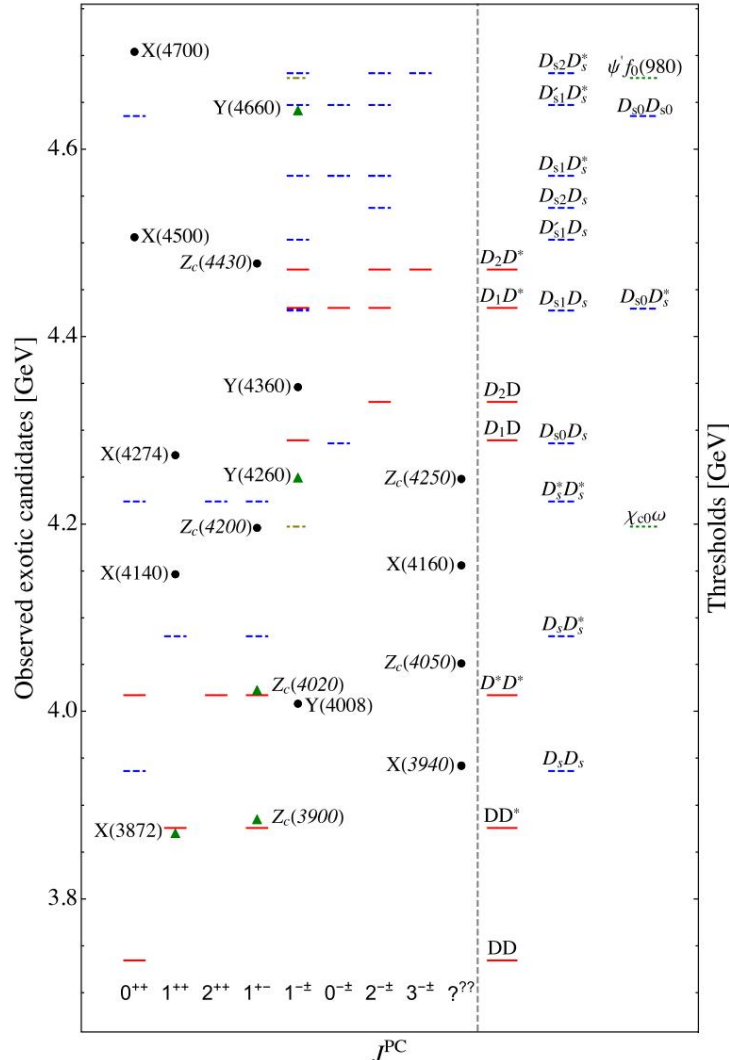
Composed of multi quarks(>3)

## 4. Hadronic molecule

Composed of 2 or more hadrons

# Hadronic molecules

F.K. Guo, et al. Rev.Mod.Phys.90.015004

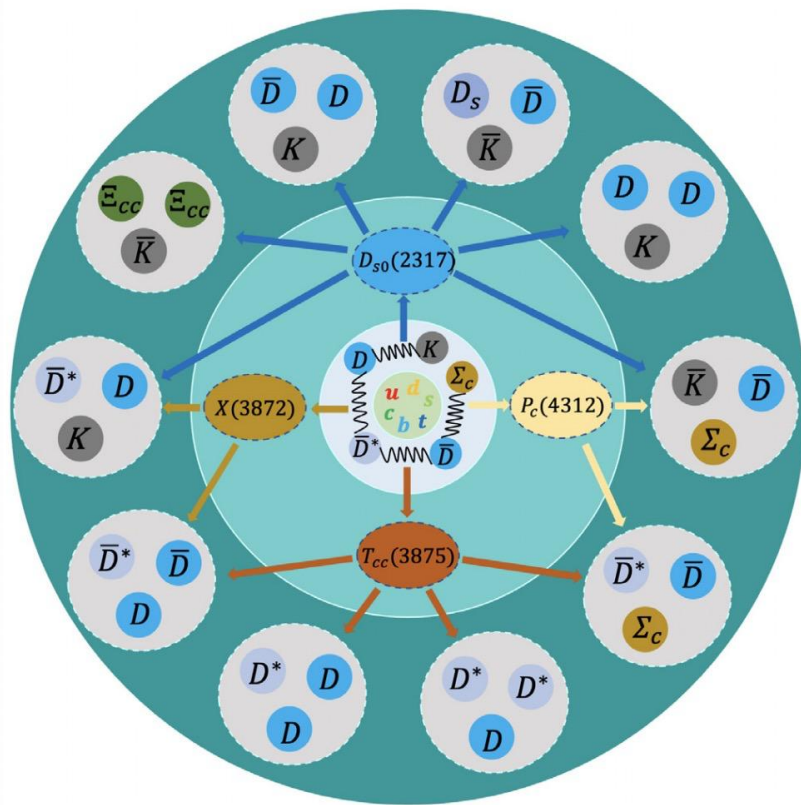


**X(3872)、T<sub>cc</sub>(3875)、P<sub>c</sub> pentaquarks、D<sub>s0</sub><sup>\*</sup>(2317) are interpreted as hadronic molecules.**

Many XYZ particles are close to two hadron mass thresholds.

# Three-body molecules

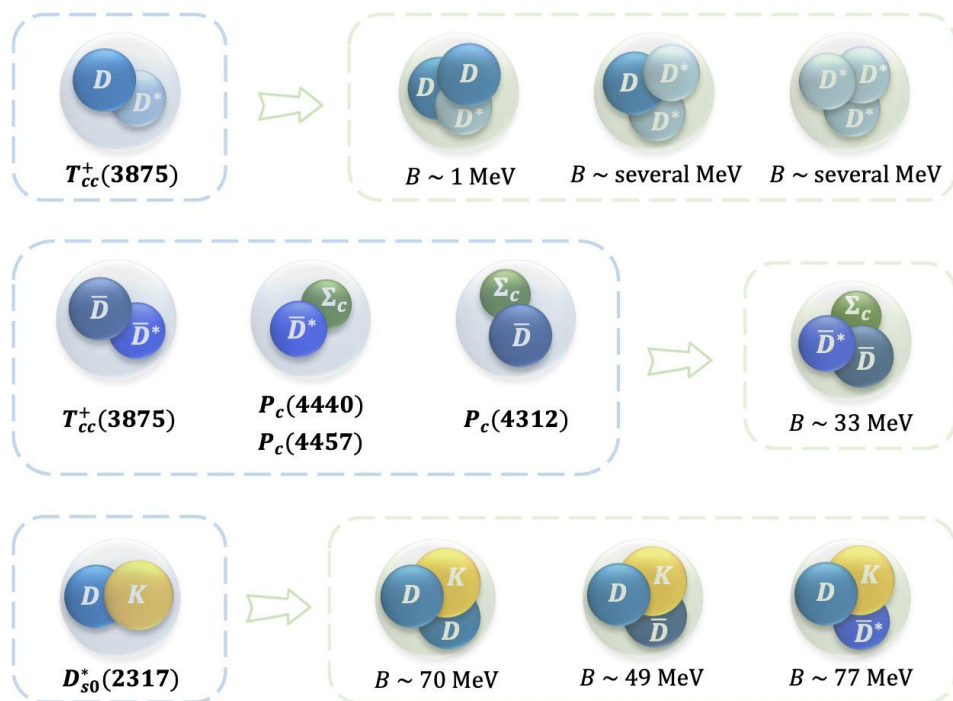
T.W. Wu, et al. Science Bulletin 67 (2022) 1735-1738



M.Z. Liu, et al. Phys.Rept. 1108 (2025) 1-108

Observed States

Three – body Molecules



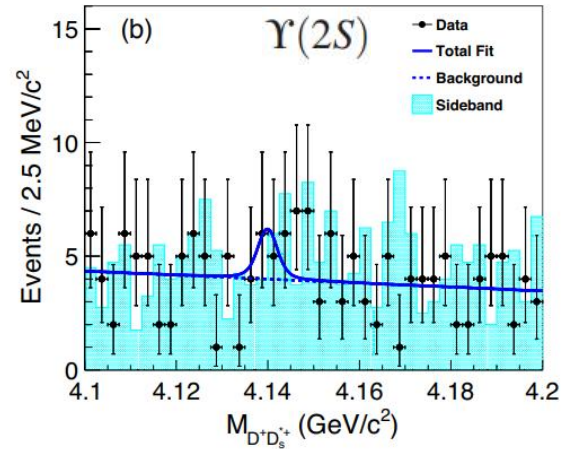
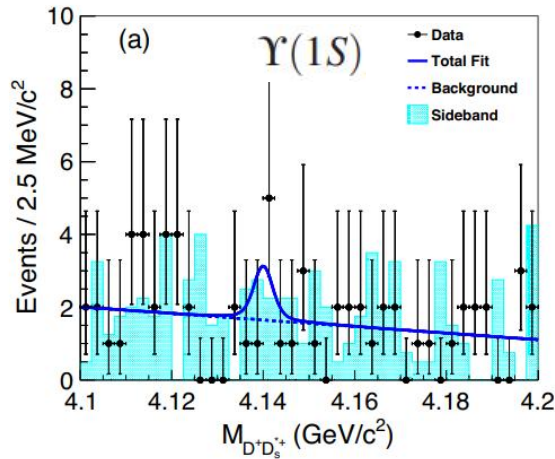
Three-body molecules based on two-body candidates



# Searching for DDK

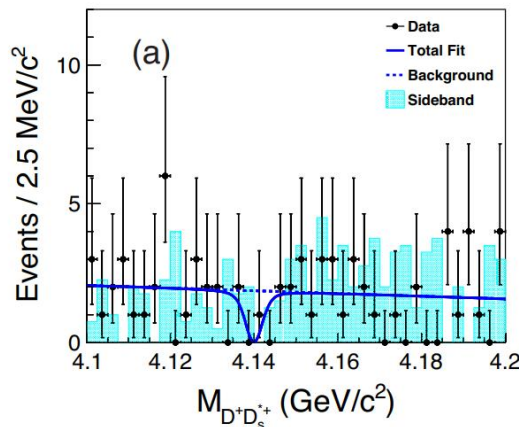
Phys.Rev. D102 (2020) 112001@Belle

$$\Upsilon(1S)/\Upsilon(2S) \rightarrow R^{++} + \text{anything}, R^{++} \rightarrow D^+ D_s^{*+}$$

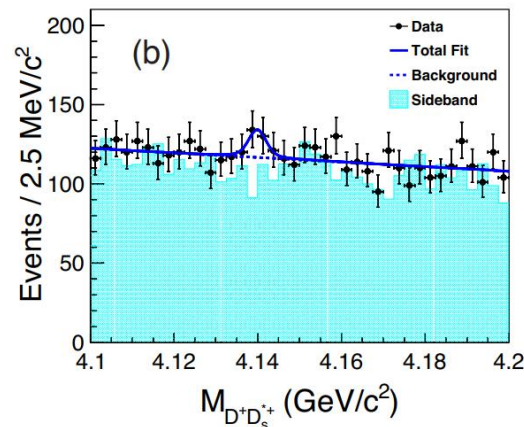


$$e^+ e^- \rightarrow R^{++} + \text{anything at } \sqrt{s} = 10.520/10.580/10.867 \text{ GeV}, R^{++} \rightarrow D^+ D_s^{*+}$$

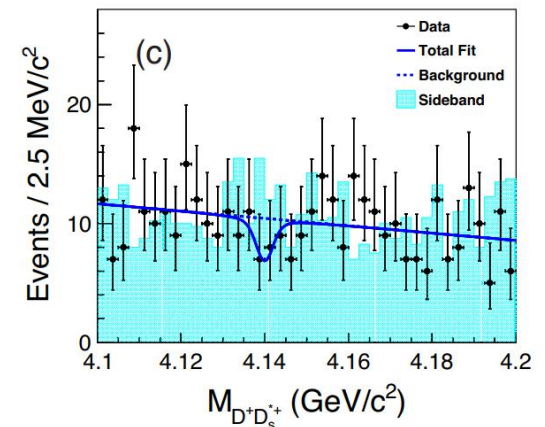
$\sqrt{s} = 10.520 \text{ GeV}$



$\sqrt{s} = 10.580 \text{ GeV}$



$\sqrt{s} = 10.867 \text{ GeV}$



# Features of $\bar{D}_s DK$ system

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## How to find and verify three-body molecules?

✓ **No mixture of conventional hadrons**

$0^{--}$  exotic quantum numbers

✓ **Two-body system can not bind**

Suppressed by the OBE model or OZI rule

✓ **Produced in both  $e^+e^-$  and pp collisions**

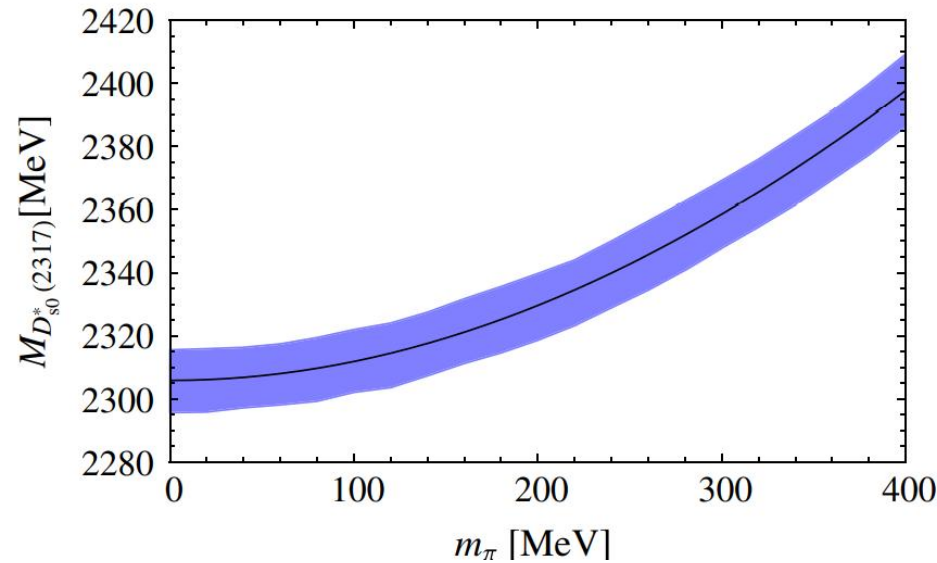
Hidden charm/strange state easier to observe



# Molecular explanation of $D_{s0}^*(2317)$

M. Altenbuchinger et al. Phys.Rev.D 89, 014026

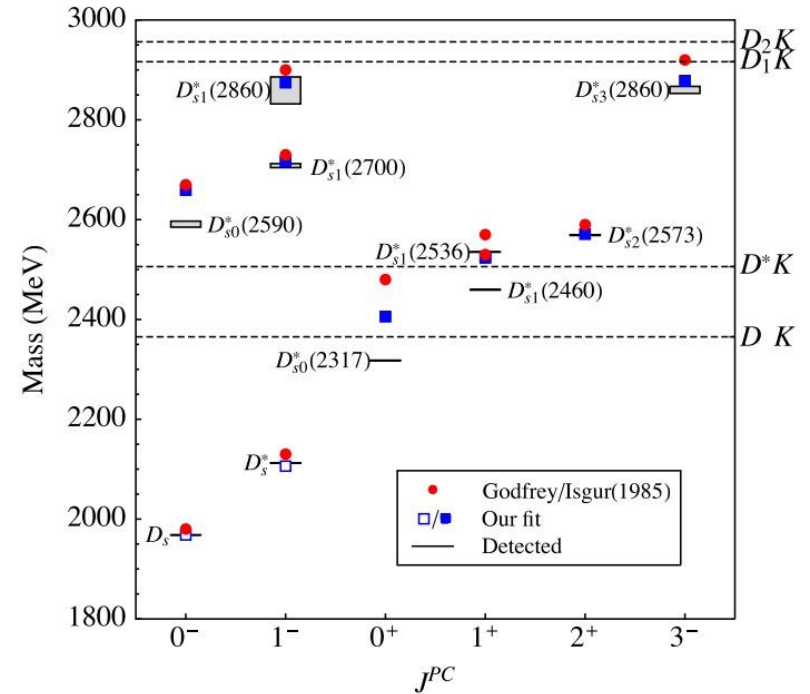
Z. Yang et al. PhysRevLett.128.112001



DK pole position@ChPT

TABLE V. Pole positions  $\sqrt{s} = M - i\frac{\Gamma}{2}$  (in units of MeV) of charm mesons dynamically generated in the HQS UChPT.

$(S, I)$	$J^P = 0^+$	$J^P = 1^+$
$(1, 0)$	$2317 \pm 10$	$2457 \pm 17$
$(0, 1/2)$	$(2105 \pm 4) - i(103 \pm 7)$	$(2248 \pm 6) - i(106 \pm 13)$

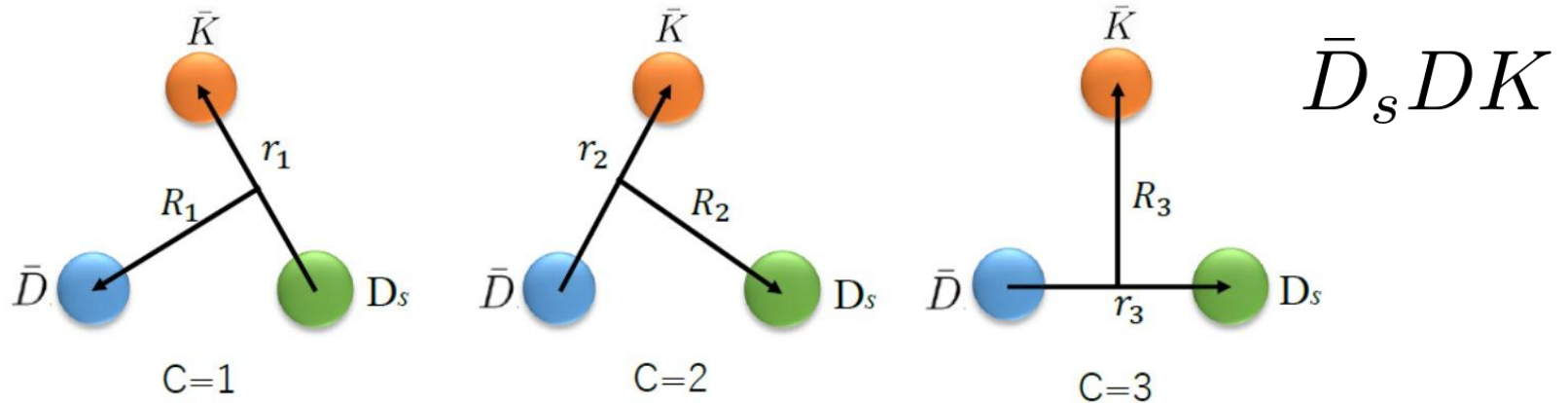


68 % DK+32 %  $c\bar{s}$ @LQCD

**ChPT, Lattice and Exp. all support that the  $D_{s0}(2317)$  is a DK molecule or at least has a large DK component.**



# Wave function of $\bar{D}_s D K$ with C parity



$$\Psi^C = \frac{1}{\sqrt{2}} (\Psi_{\bar{D}_s D K} + C \Psi'_{D_s \bar{D} \bar{K}}), \quad \langle \Psi^C | (H - E) | \Psi^C \rangle = 0$$

$$C = \pm 1$$

$$\Psi'_{D_s \bar{D} \bar{K}} = \hat{C} \Psi_{\bar{D}_s D K} = \sum_{c=1,3} \Phi(r'_c, R'_c).$$

$$H = T + V + V_C$$

Two-body  
interaction

C-parity  
interaction

Solve three-body problem with **GEM**

# Two-body interactions

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**DK** interaction parameterized by a constant  $C_a$

with **Contact-range EFT approach**

**DK** –  $D_s\eta$  coupled channel interaction in matrix form

$$V_{DK-D_s\eta}^{J^P=0^+} = \begin{pmatrix} C_a & -\frac{\sqrt{3}}{2}C_a \\ -\frac{\sqrt{3}}{2}C_a & 0 \end{pmatrix}$$

$V_{DK}$  in a Gaussian form in coordinate space

$$V(r) = C_a \frac{e^{-(r/R_c)^2}}{\pi^{3/2} R_c^3},$$

With SU(3) flavor symmetry

$$C_a^{DK} : C_a^{\bar{D}_s K} : C_a^{\bar{D}_s D} \approx 1 : 0.5 : 0.1$$

# DK interaction fitting $D_{s0}(2317)$

Considering  $D_{s0}(2317)$  as a  $DK - D_s\eta$  molecule +  $c\bar{s}$  state

TABLE IV.  $D_{s0}^*(2317)$  coupling to its constituents (in units of GeV). **Momentum space**

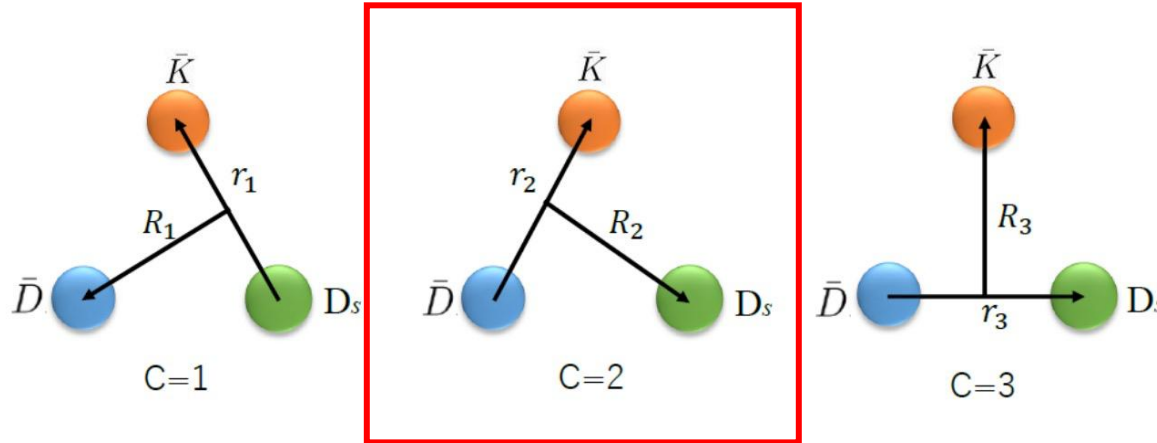
Couplings	$\Lambda = 0.50$	$\Lambda = 1.00$	$\Lambda = 1.50$	$\Lambda = 2.00$	$\Lambda = 0.50$	$\Lambda = 1.00$	$\Lambda = 1.50$	$\Lambda = 2.00$
$g_{D_{s0}^*DK}$	19.37	14.72	13.32	12.66	16.20	12.28	11.16	10.63
$g_{D_{s0}^*D_s\eta}$	13.23	9.54	8.40	7.86	10.42	7.70	6.89	6.50
$C_a(\text{fm}^2)$	-5.78	-1.84	-1.03	-0.71	-6.96	-2.06	-1.12	-0.75
Compositeness	$\Lambda = 0.50$	$\Lambda = 1.00$	$\Lambda = 1.50$	$\Lambda = 1.00$	$\Lambda = 0.50$	$\Lambda = 1.00$	$\Lambda = 1.50$	$\Lambda = 2.00$
$P_{DK}$	0.92	0.90	0.89	0.88	0.65	0.63	0.62	0.62
$P_{D_s\eta}$	0.08	0.10	0.11	0.12	0.05	0.07	0.08	0.08

**Coordinate space**

Components of $D_{s0}^*(2317)$	$M(DK - D_s\eta)$	$M(DK)$	$M(c\bar{s})$ [4]	$P(c\bar{s})$	$P(DK)$	$P(D_s\eta)$
70% molecule+30% $c\bar{s}$	2280	2349	2406	30%	60%	10%
100% molecule	2318	2358	2406	0%	90%	10%
50% molecule+50% $c\bar{s}$	2230	2336	2406	50%	42%	8%

[4] Z. Yang et al. PhysRevLett.128.112001

# C-parity interaction $\bar{D}_s D_{s0}^* - D_s \bar{D}_{s0}^*$



$V_C$  simplifies as

$\eta$  exchange potential

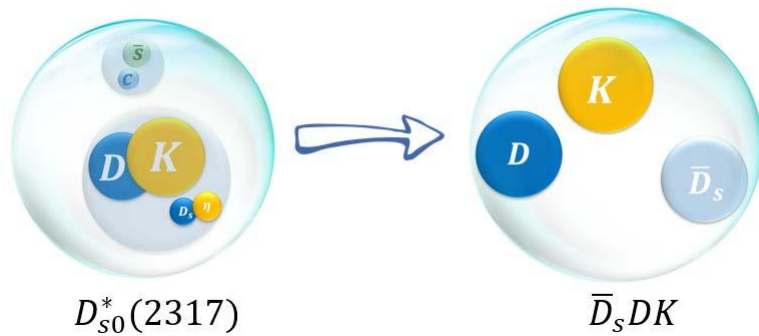
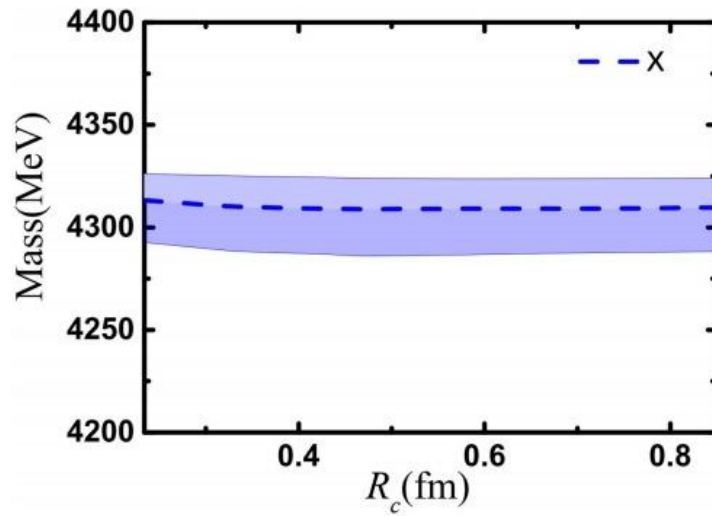
$$V_{\bar{D}_s D_{s0}^* - D_s \bar{D}_{s0}^*}^C = -\frac{2}{3} C \frac{k^2}{f_\pi^2} q_0^2 Y(r, m_{eff}, \Lambda)$$

$$q_0 = m_{D_{s0}^*} - m_{D_s}, k = 0.56, \quad \Lambda = \alpha \Lambda_{QCD} + m_{eff}, m_{eff} = \sqrt{m_\eta^2 - (m_{D_{s0}^*} - m_{D_s})^2}.$$

# Binding and weights of Jacobi channels

$0^{--} \bar{D}_s DK$  molecule:

Sets	B.E.( $0^{--}$ )	$P_{\bar{D}_s K-D}$	$P_{DK-\bar{D}_s}$	$P_{\bar{D}_s D-K}$
$\alpha = 1$	$22_{-14}^{+23}$	$11_{-1}^{+1} \%$	$78_{+2}^{-1} \%$	$11_{-1}^{+0} \%$
$\alpha = 2$	$20_{-13}^{+22}$	$10_{-1}^{+1} \%$	$80_{+2}^{-1} \%$	$10_{-1}^{+0} \%$



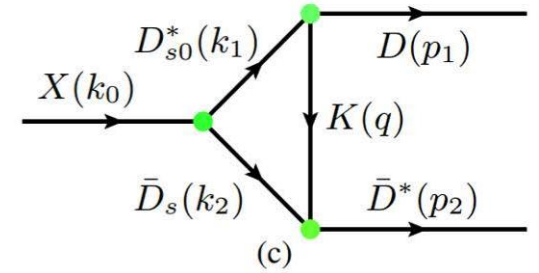
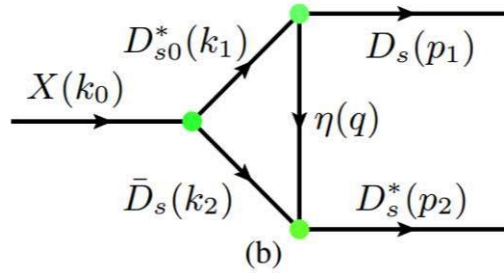
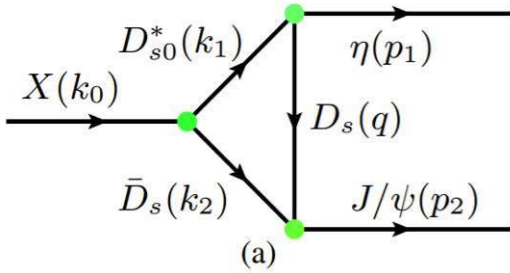
$$0^{--} \bar{D}_s DK : 4310_{-24}^{+14} \text{ MeV}$$

FIG. 5. Mass of  $X$  as a function of the cutoff  $R_c$ .



# Decays of $0^{--} \bar{D}_s D K$ molecule

## Triangle diagrams of the strong decays



$$i\mathcal{M}_a = g_{XD_{s0}^*\bar{D}_s} g_{D_{s0}^*D_s\eta} g_{\psi\bar{D}_sD_s} \int \frac{d^4q}{(2\pi)^4} (k_2^\mu - q^\mu) \frac{1}{k_1^2 - m_{D_{s0}^*}^2} \frac{1}{k_2^2 - m_{\bar{D}_s}^2} \frac{1}{q^2 - m_{D_s}^2} \varepsilon_\mu(p_2) F(q^2),$$

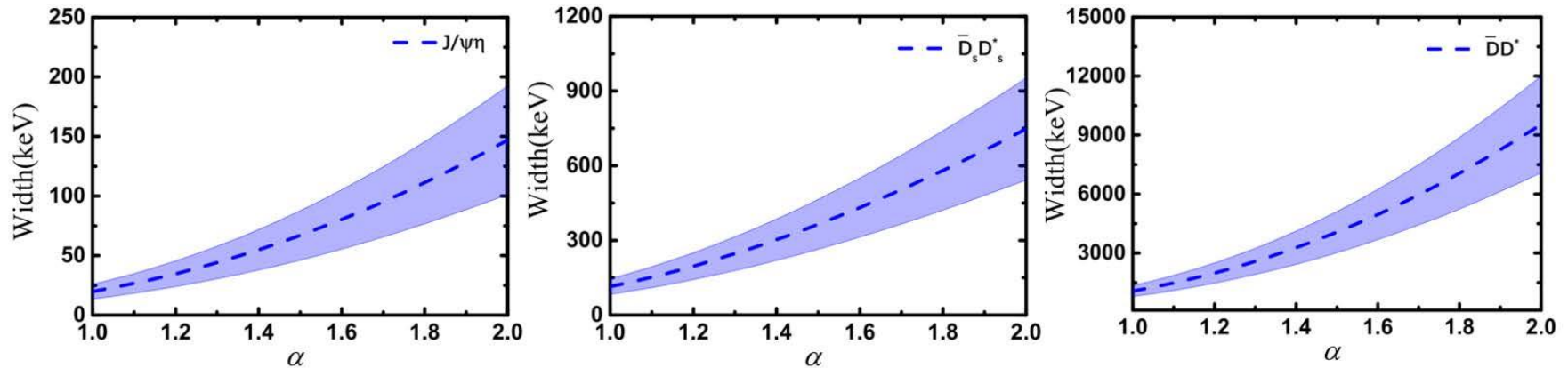
$$i\mathcal{M}_b = g_{XD_{s0}^*\bar{D}_s} g_{D_{s0}^*D_s\eta} g_{\bar{D}_sD_s^*\eta} \int \frac{d^4q}{(2\pi)^4} q^\mu \frac{1}{k_1^2 - m_{D_{s0}^*}^2} \frac{1}{k_2^2 - m_{\bar{D}_s}^2} \frac{1}{q^2 - m_\eta^2} \varepsilon_\mu(p_2) F(q^2),$$

$$i\mathcal{M}_c = g_{XD_{s0}^*\bar{D}_s} g_{D_{s0}^*DK} g_{\bar{D}_sD^*K} \int \frac{d^4q}{(2\pi)^4} q^\mu \frac{1}{k_1^2 - m_{D_{s0}^*}^2} \frac{1}{k_2^2 - m_{\bar{D}_s}^2} \frac{1}{q^2 - m_K^2} \varepsilon_\mu(p_2) F(q^2),$$

$$\Gamma = \frac{1}{2J+1} \frac{1}{8\pi} \frac{|\vec{p}|}{M^2} |\bar{\mathcal{M}}|^2$$

$$F(q, \Lambda, m) = \left( \frac{\Lambda^2 - m_E^2}{\Lambda^2 - q^2} \right)^2,$$

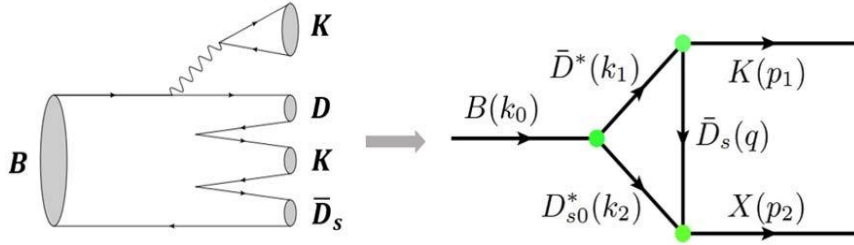
# Partial decay widths



$$X \rightarrow J/\psi \eta \sim 10^1 \quad X \rightarrow \bar{D}_s D_s^* \sim 10^2 \quad X \rightarrow \bar{D}^* D \sim 10^3$$

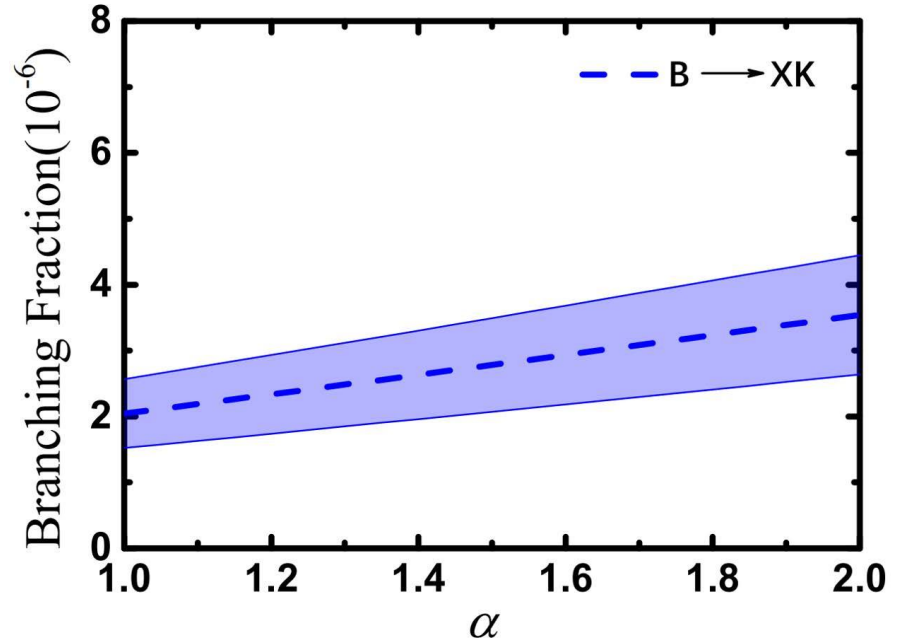
**Dominant decay:**  $X \rightarrow \bar{D}^* D$

# Productions of $B^- \rightarrow \bar{D}_s D K$ molecule



$$\mathcal{M} = g_{\bar{D}^* \bar{D}_s K} g_{D_s^* \bar{D}_s X} \mathcal{A}(B \rightarrow D_s^* \bar{D}^*)^\mu \frac{-g^{\mu\nu} + \frac{k_1^\mu k_1^\nu}{k_1^2}}{(k_1^2 - m_{\bar{D}^*}^2)(k_2^2 - m_{D_s^*}^2)(q^2 - m_{\bar{D}_s}^2)} p_1^\nu F(q^2),$$

$$\mathcal{A}(B \rightarrow D_s^* \bar{D}^*) = \frac{G_F}{\sqrt{2}} V_{cb} V_{cs} a_1 f_{D_s^*} \{ -q_1 \cdot \varepsilon(q_2) (m_{D^*} + m_B) A_1(q_1^2) + (k_0 + q_2) \cdot \varepsilon(q_2) q_1 \cdot (k_0 + q_2) \frac{A_2(q_1^2)}{m_{D^*} + m_B} + (k_0 + q_2) \cdot \varepsilon(q_2) [(m_{D^*} + m_B) A_1(q_1^2) - (m_B - m_{D^*}) A_2(q_1^2) - 2m_{D^*} A_0(q_1^2)] \},$$



**Most promising process:**  $B^+ \rightarrow (X \rightarrow D^{*-} D^+) K^+ \sim 10^{-6}$

LHC integrated luminosity: 50 fb <sup>-1</sup>	Events: 10
350 fb <sup>-1</sup>	100

- 4310 MeV  $\mathbf{0^{--}}$   $\bar{\mathbf{D}}_s \mathbf{D} \mathbf{K}$  exotic molecule is predicted on the nature of  $\mathbf{D}_{s0}^*$  (2317) as a molecule and  $\mathbf{c} \bar{\mathbf{s}}$  mixture
- Main decay  $\mathbf{X} \rightarrow \bar{\mathbf{D}}^* \mathbf{D}$  with several MeV width
- Production process  $\mathbf{B} \rightarrow (\mathbf{X} \rightarrow \bar{\mathbf{D}}^* \mathbf{D}) \mathbf{K}$  with  $10^{-6}$  branching fraction

**Experiment searches are strongly recommended!**