GPDs and GTMDs at nonzero skewness in the proton

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第四届强子与重味物理理论与实验联合研讨会

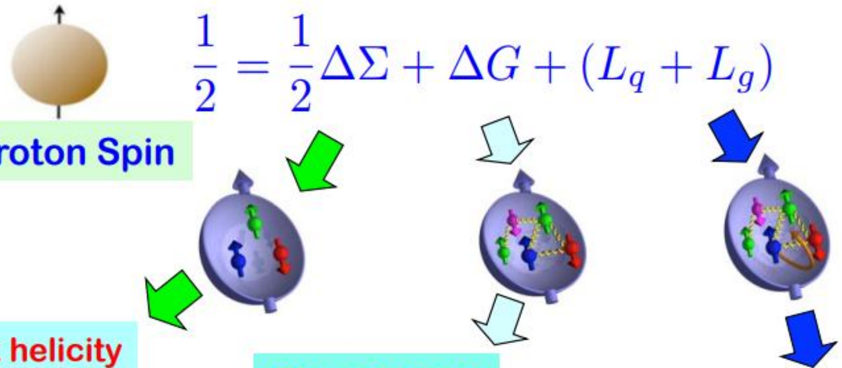
outline

- Overview on the proton spin structure in terms of GPDs and GTMDs
- Model calculations——Gluon GTMDs and impact parameter dependent parton distributions (IPDs)
- Model calculations——Chiral-even twist-3 quark GPDs
- Summary

proton spin structure

$$\frac{1}{2}=\frac{1}{2}\Delta\Sigma+\Delta G+(L_q+L_g)$$

Jaffe-Manohar, Ji, 96, ...



Proton Spin

Quark helicity
Best known

$$\frac{1}{2}\int dx\left(\Delta u+\Delta\bar{u}+\Delta d+\Delta\bar{d}+\Delta s+\Delta\bar{s}\right)$$
$$\sim 30\%$$

Gluon helicity
Start to know

$$\Delta G=\int dx\Delta g(x)$$
$$\sim 40\%(\text{with RHIC data})$$

Orbital Angular Momentum of quarks and gluons
Little known

Net effect of partons'

spin decomposition

Phys. Rev. Lett. 113, 012001 (2014)

$$l_z=-\int dxd^2\boldsymbol{k}_\perp\frac{\boldsymbol{k}_\perp^2}{M^2}F_{1,4}(x,0,0,\boldsymbol{k}_\perp^2,0;\mathcal{W}_{\rm straight}/\mathcal{W}_{\rm LF})$$

kinetic orbit angular momentum (OAM)

canonical OAM

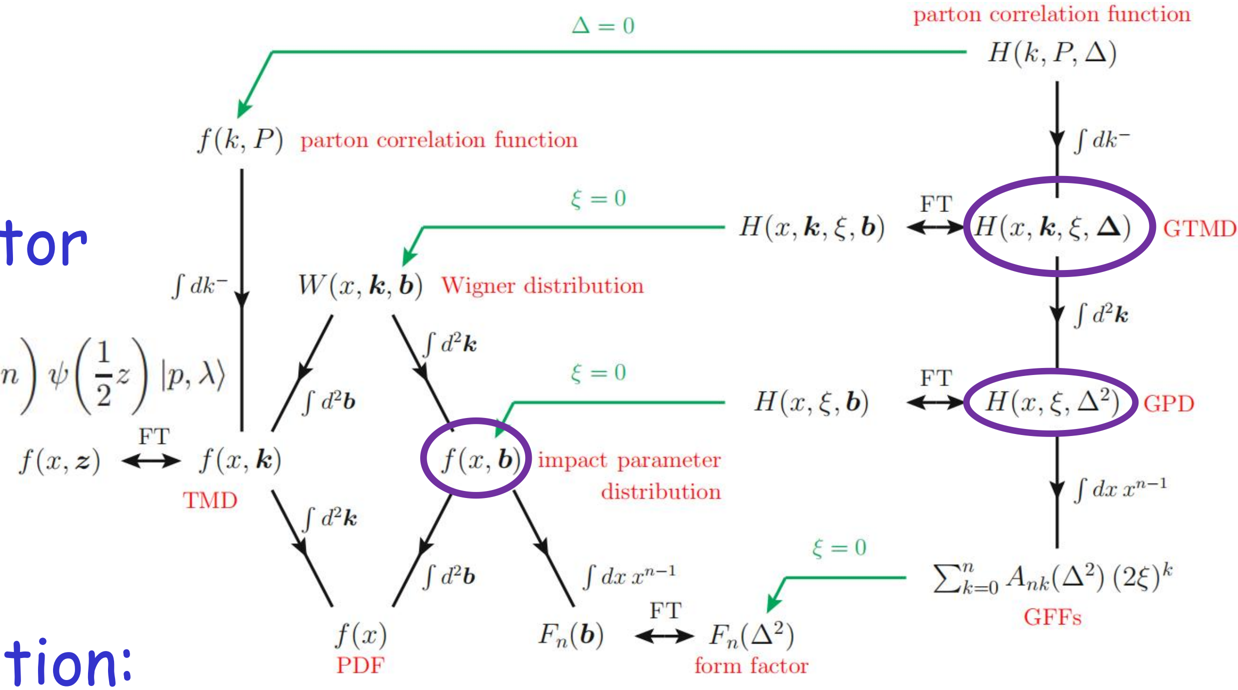
classification

$\vec{J}^q_{\rm Bel}$		$\vec{J}^g_{\rm Bel}$	
$\vec{S}^q_{\rm Ji}$	$\vec{L}^q_{\rm Ji}$	$\vec{J}^g_{\rm Ji}$	
$\vec{S}^q_{\rm gik}$	$\vec{L}^q_{\rm gik}$	$\vec{L}^g_{\rm gik}$	$\vec{S}^g_{\rm gik}$
$\vec{S}^q_{\rm gic}$	$\vec{L}^q_{\rm gic}$	$\vec{L}^g_{\rm gic}$	$\vec{S}^g_{\rm gic}$

in a most generalized picture

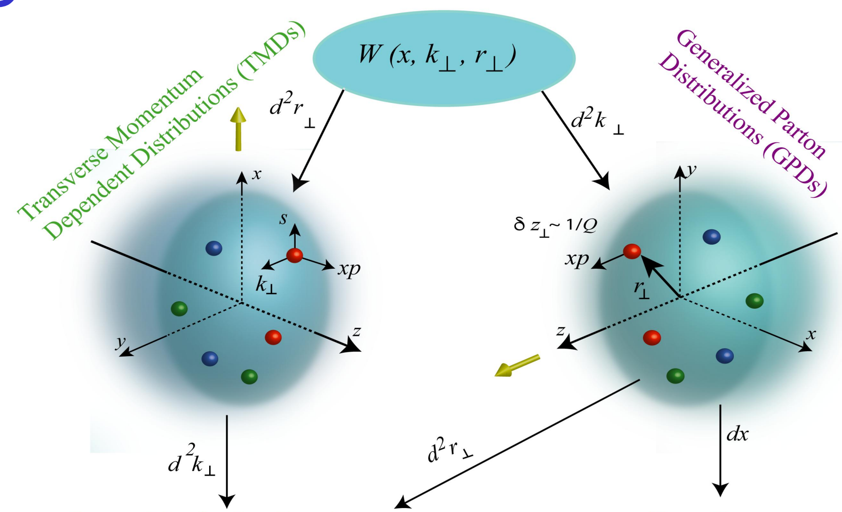
fully-unintegrated two-parton correlator

$$\frac{1}{2}\int\frac{d^4z}{(2\pi)^4}\,e^{ik\cdot z}\,\langle p',\lambda'|\,\bar{\psi}\Big(-\frac{1}{2}z\Big)\,\Gamma\,\mathcal{W}\Big(-\frac{1}{2}z,\frac{1}{2}z\,|\,n\Big)\,\psi\Big(\frac{1}{2}z\Big)\,|p,\lambda\rangle$$



Wigner distribution:
F.T. of GTMD

semi-inclusive:
SIDIS
Drell-Yan
e^+e^- annihilation

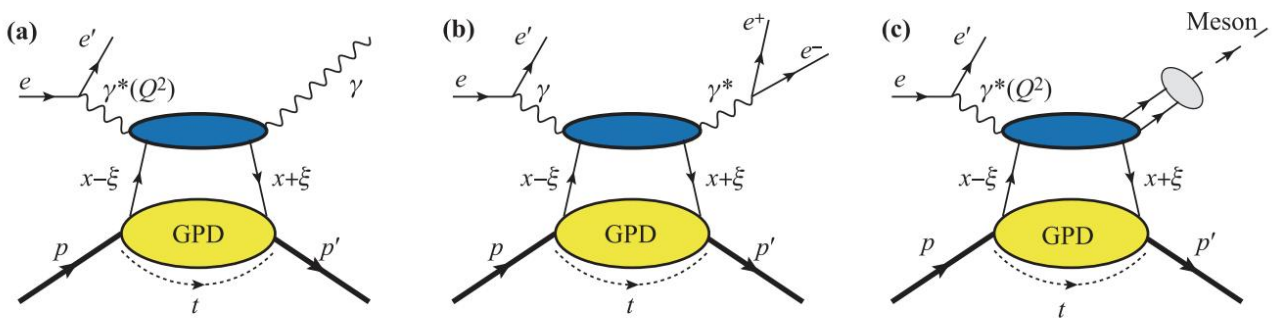


exclusive:
DVCS
HEMP

hard exclusive processes

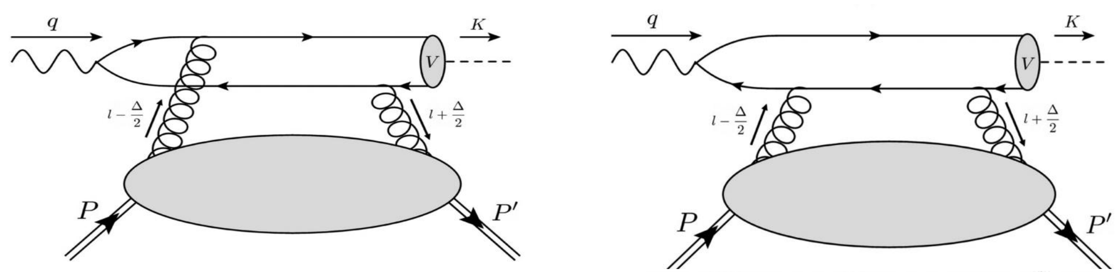
for quark GPDs

Front. Phys. 16(6), 64701 (2021)

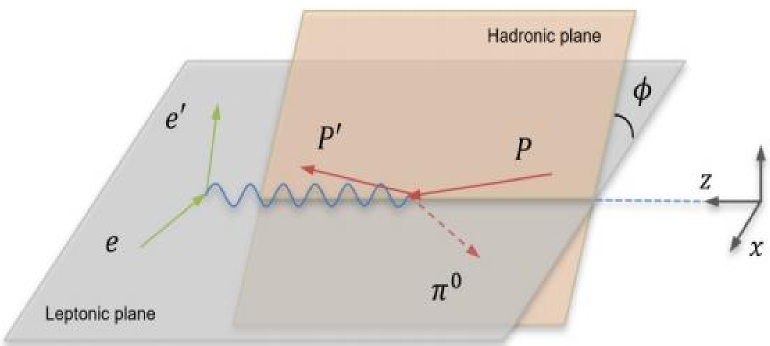


for gluon GPDs

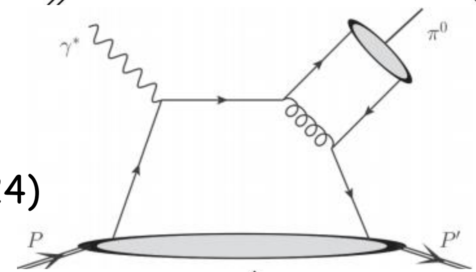
PHYS. REV. D 103, 096010 (2021)



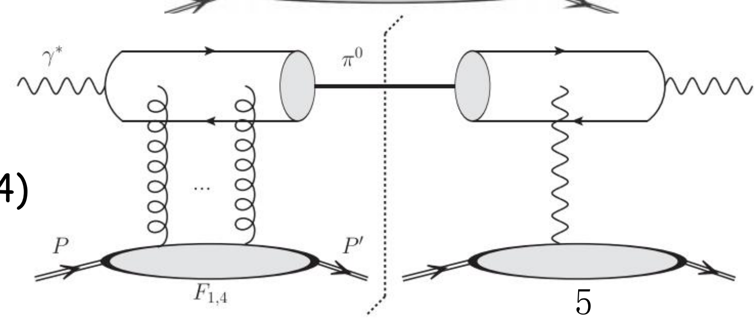
for quark and gluon GTMDs $F_{1,4}$



PHYSICAL REVIEW LETTERS 133, 051901 (2024)



PHYSICAL REVIEW D 109, 096029 (2024)



HERMES, COMPASS, JLab12, EIC, EicC...

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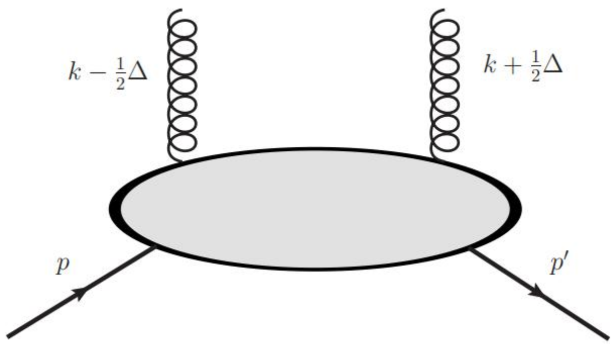
F-type and G-type Gluon GTMDs

off-forward gluon-gluon correlator at $z^+=0$

$$W^{g[ij]}_{\lambda',\lambda}(x,P,\Delta,\boldsymbol{k}_\perp) = \int \frac{dz^- d^2\boldsymbol{z}_\perp}{(2\pi)^3 P^+} e^{ik\cdot z} \left\langle p',\lambda' \left| F_a^{+i}(-\frac{z}{2}) \mathcal{W}_{ab}(-\frac{z}{2},\frac{z}{2}) F_b^{+j}(\frac{z}{2}) \right| p,\lambda \right\rangle \Big|_{z^+=0}$$



kinematics for Gluon GTMDs

parameterization

$$W^g_{\lambda',\lambda} = \delta^{ij}_\perp W^{g[ij]}_{\lambda',\lambda}$$

$$= \frac{1}{2M}\bar{u}(p',\lambda')\left[F^g_{1,1} + \frac{i\sigma^{i+}k^i_\perp}{P^+}F^g_{1,2} + \frac{i\sigma^{i+}\Delta^i_\perp}{P^+}F^g_{1,3} + \frac{i\sigma^{ij}k^i_\perp\Delta^j_\perp}{M^2}F^g_{1,4}\right]u(p,\lambda)$$

$$\widetilde{W}^g_{\lambda',\lambda} = -i\epsilon^{ij}_\perp W^{g[ij]}_{\lambda',\lambda}$$

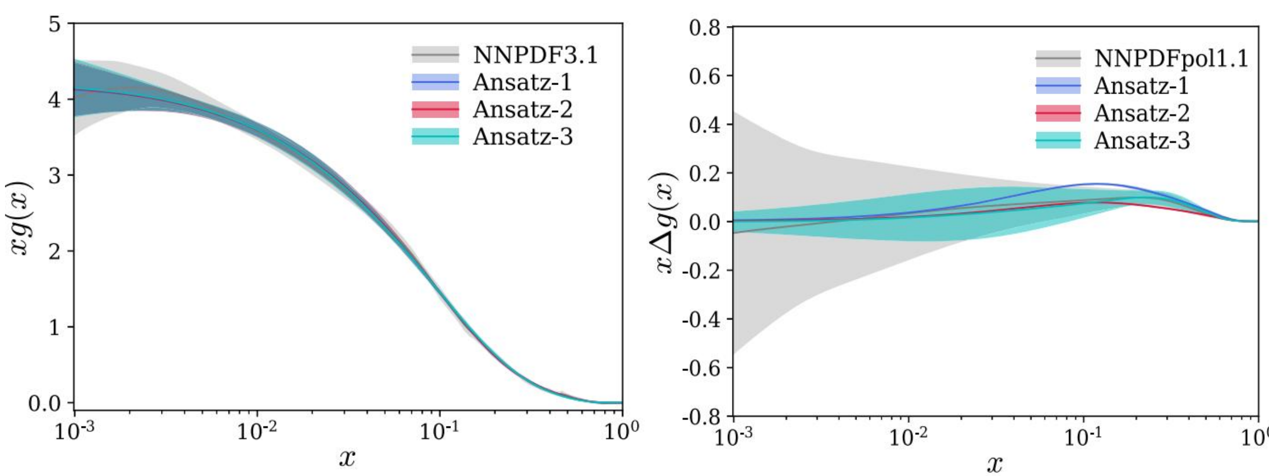
$$= \frac{1}{2M}\bar{u}(p',\lambda')\left[-\frac{i\epsilon^{ij}_\perp k^i_\perp \Delta^j_\perp}{M^2}G^g_{1,1} + \frac{i\sigma^{i+}\gamma_5 k^i_\perp}{P^+}G^g_{1,2} + \frac{i\sigma^{i+}\gamma_5\Delta^i_\perp}{P^+}G^g_{1,3} + i\sigma^{+-}\gamma_5 G^g_{1,4}\right]u(p,\lambda)$$

soft-wall AdS/QCD

$$|p;S\rangle \to |g_{s_g} X_{s_X}(qqq)\rangle$$ minimum Fock state

Fock-state expansion

$$|\Psi^{+}_{\text{two particle}}(p^+, \boldsymbol{p}_\perp = \boldsymbol{0}_\perp)\rangle = \int \frac{d^2\boldsymbol{k}_\perp dx}{16\pi^3\sqrt{x(1-x)}} \times \Big[\psi^{+}_{+1+\frac{1}{2}}(x,\boldsymbol{k}_\perp)\Big|+1,+\frac{1}{2};xp^+,\boldsymbol{k}_\perp\Big\rangle + \psi^{+}_{+1-\frac{1}{2}}(x,\boldsymbol{k}_\perp)\Big|+1,-\frac{1}{2};xp^+,\boldsymbol{k}_\perp\Big\rangle + \psi^{+}_{-1+\frac{1}{2}}(x,\boldsymbol{k}_\perp)\Big|-1,+\frac{1}{2};xp^+,\boldsymbol{k}_\perp\Big\rangle + \psi^{+}_{-1-\frac{1}{2}}(x,\boldsymbol{k}_\perp)\Big|-1,-\frac{1}{2};xp^+,\boldsymbol{k}_\perp\Big\rangle\Big]$$



PHYS. REV. D 103, 036007 (2021)

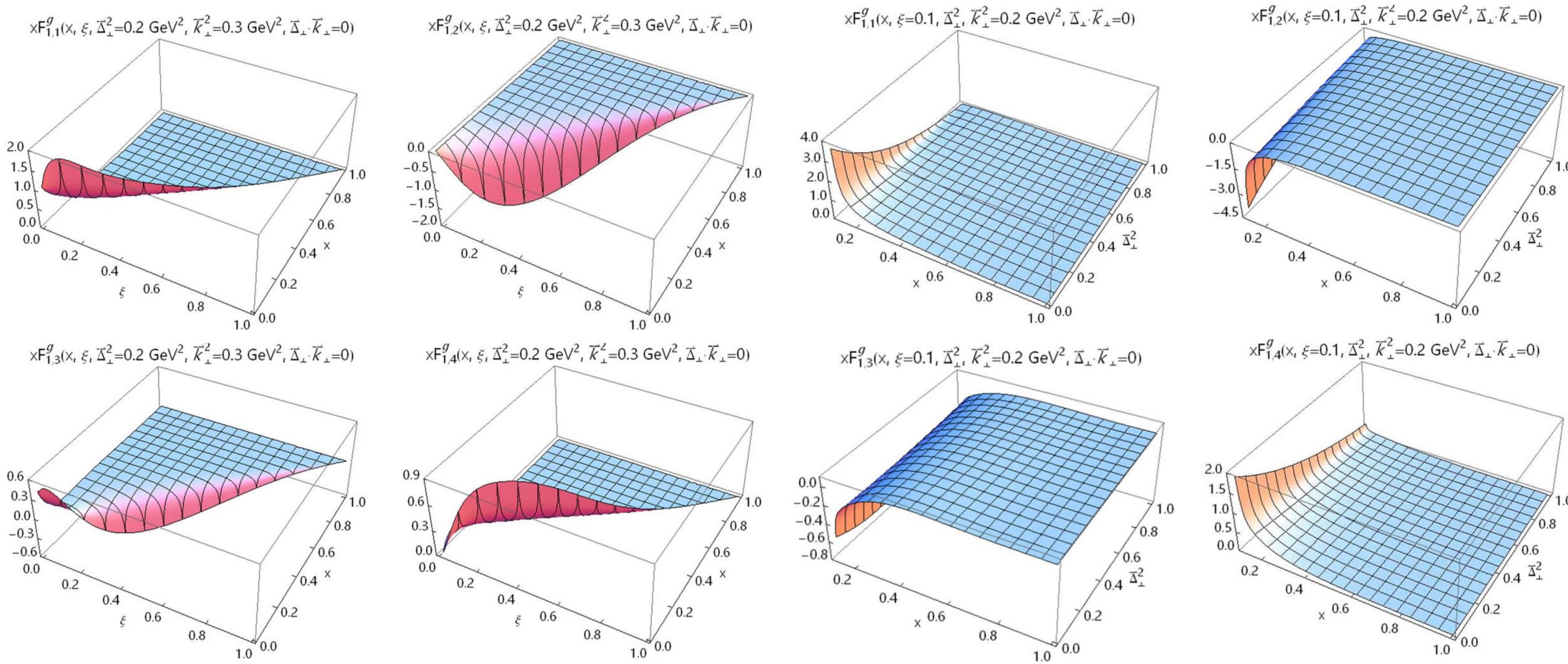
ansatz-3

$$xG^+(x) = x^\alpha[A(1-x)^{4+\beta} + B(1-x)^{5+\beta}](1+\gamma\sqrt{x}+\delta x),$$
$$xG^-(x) = x^\alpha[A(1-x)^{6+\beta'} + B(1-x)^{7+\beta'}](1+\gamma'\sqrt{x}+\delta' x),$$

3-D image of unpolarized gluon GTMDs

as functions of x and ξ at fixed $\boldsymbol{\Delta}_\perp{}^2$ and $\mathbf{k}_\perp{}^2$

as functions of x and $\boldsymbol{\Delta}_\perp{}^2$ at fixed ξ and $\mathbf{k}_\perp{}^2$



canonical OAM

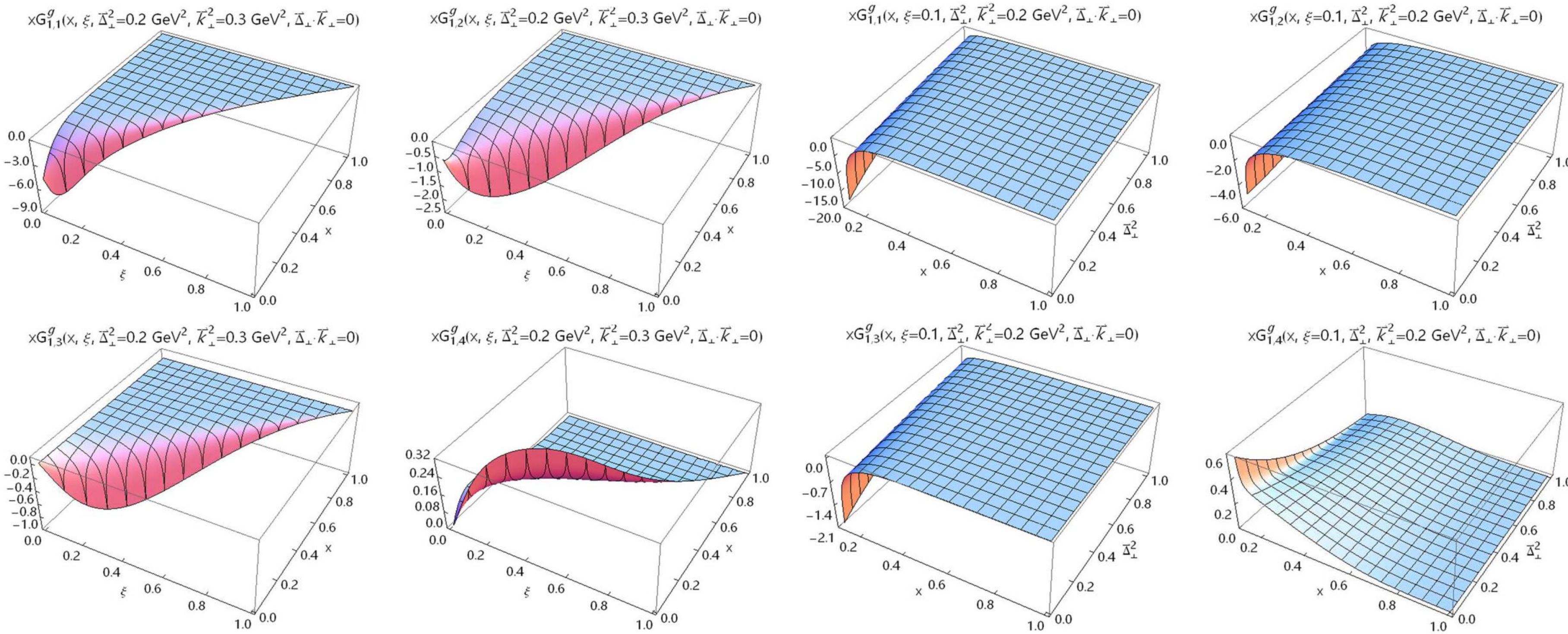
$$l_z^g = \int dx d^2\boldsymbol{k}_\perp d^2\boldsymbol{b}_\perp (\boldsymbol{b}_\perp \times \boldsymbol{k}_\perp)_z W_{LU}(x, \boldsymbol{k}_\perp, \boldsymbol{b}_\perp)$$

$$= -\int dx d^2\boldsymbol{k}_\perp \frac{\boldsymbol{k}_\perp^2}{M^2} F_{1,4}^g(x, 0, \boldsymbol{k}_\perp^2, 0, 0)$$

3-D image of longitudinally polarized gluon GTMDs

as functions of x and ξ at fixed $\boldsymbol{\Delta}_\perp{}^2$ and $\mathbf{k}_\perp{}^2$

as functions of x and $\boldsymbol{\Delta}_\perp{}^2$ at fixed ξ and $\mathbf{k}_\perp{}^2$



canonical spin-orbit correlation

$$C_z^g = \int dx d^2\boldsymbol{k}_\perp d^2\boldsymbol{b}_\perp (\boldsymbol{b}_\perp \times \boldsymbol{k}_\perp)_z W_{UL}(x, \boldsymbol{k}_\perp, \boldsymbol{b}_\perp)$$

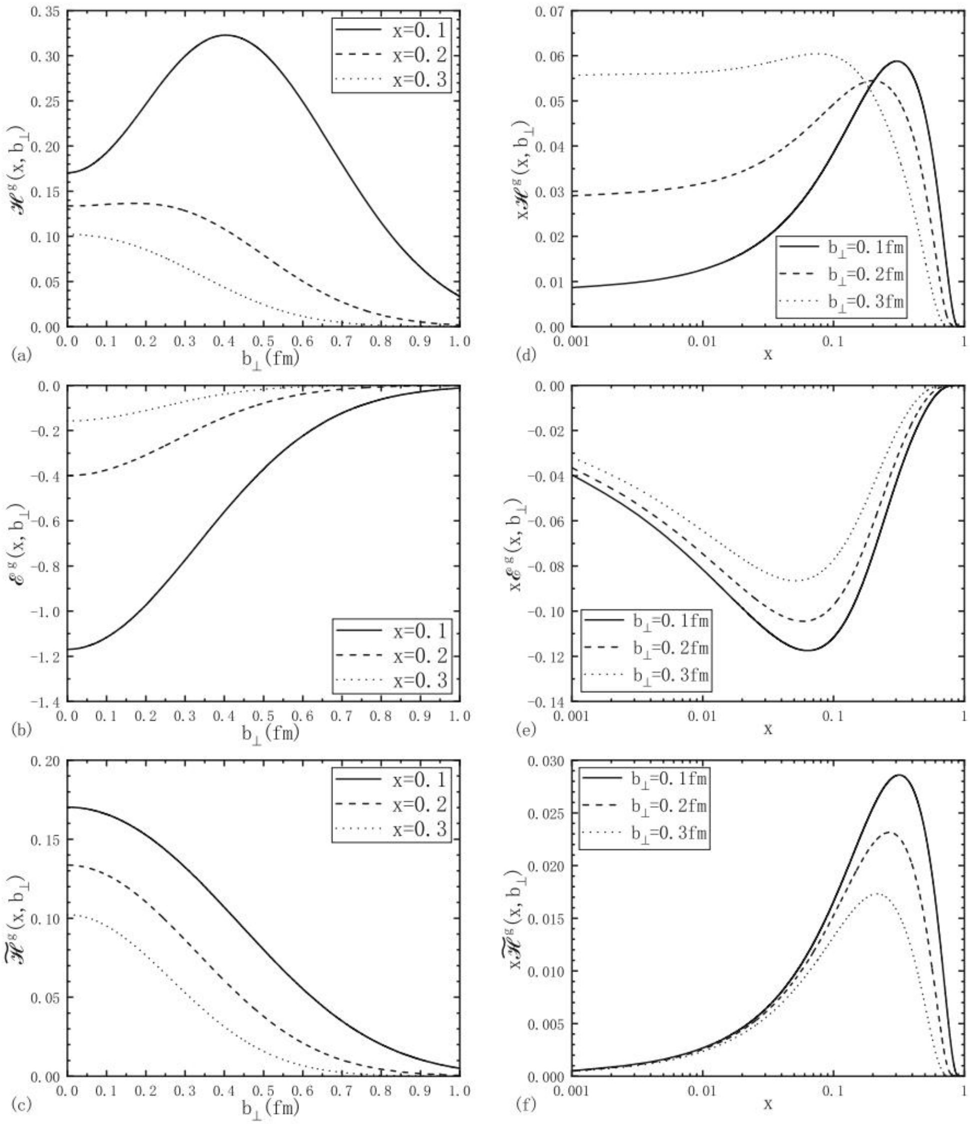
$$= \int dx d^2\boldsymbol{k}_\perp \frac{\boldsymbol{k}_\perp^2}{M^2} G^g_{1,1}(x, 0, \boldsymbol{k}_\perp^2, 0, 0)$$

IPDs: F.T. of H_g, E_g and $\tilde{H}_g$

IPD: F.T. of GPD

$$\mathcal{X}(x, \boldsymbol{b}_\perp) = \int \frac{d^2\boldsymbol{\Delta}_\perp}{(2\pi)^2} e^{-i\boldsymbol{\Delta}_\perp \cdot \boldsymbol{b}_\perp} X(x, 0, \boldsymbol{\Delta}_\perp^2)$$

IPDs vs $b_\perp$



IPDs vs x

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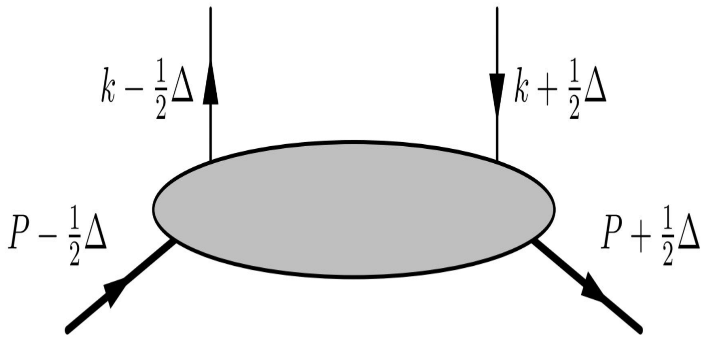
Chiral-even twist-3 quark GPDs

off-forward quark-quark correlator at $z^+=0$ and $\boldsymbol{z}_T=0$

$$F^{[\Gamma]}(x,\Delta;\lambda,\lambda') = \frac{1}{2}\int \frac{dz^-}{2\pi} e^{ik\cdot z} \langle p';\lambda'|\bar{\psi}\left(-\frac{1}{2}z\right)\Gamma \mathcal{W}\left(-\frac{1}{2}z,\frac{1}{2}z\right)\psi\left(\frac{1}{2}z\right)|p;\lambda\rangle\Big|_{z^+=0,\boldsymbol{z}_T=\boldsymbol{0}_T}$$



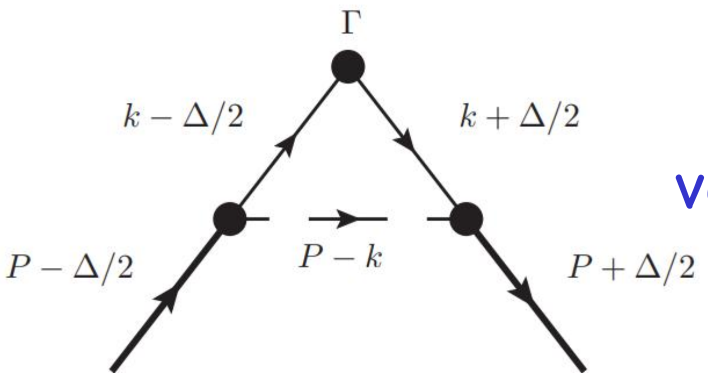
kinematics for quark GPDs

parameterization

$$F^\mu = \bar{u}(p')\left[P^\mu \frac{\gamma^+}{P^+}H + P^\mu \frac{i\sigma^{+\nu}\Delta_\nu}{2MP^+}E + \Delta_T^\mu \frac{1}{2M}G_1 + \gamma_T^\mu (H+E+G_2) + \Delta_T^\mu \frac{\gamma^+}{P^+}G_3 + i\epsilon_T^{\mu\nu}\Delta_\nu \frac{\gamma^+\gamma_5}{P^+}G_4\right]u(p)$$

$$\tilde{F}^\mu = \bar{u}(p')\left[P^\mu \frac{\gamma^+\gamma_5}{P^+}\tilde{H} + P^\mu \frac{\Delta^+\gamma_5}{2MP^+}\tilde{E} + \Delta_T^\mu \frac{\gamma_5}{2M}(\tilde{E}+\tilde{G}_1) + \gamma_T^\mu\gamma_5(\tilde{H}+\tilde{G}_2) + \Delta_T^\mu \frac{\gamma^+\gamma_5}{P^+}\tilde{G}_3 + i\epsilon_T^{\mu\nu}\Delta_\nu \frac{\gamma^+}{P^+}\tilde{G}_4\right]u(p)$$

spectator diquark model



vertex → nucleon-quark-scalar-diquark $ig_s^{\pm} I$

vertex → nucleon-quark-axial-vector-diquark $i\frac{g_a^{\pm}}{\sqrt{2}}\gamma^{\mu}\gamma_5$

exponential form factors

$$g_X^{\pm} \equiv g_X(\boldsymbol{k}_T \pm \frac{\boldsymbol{\Delta}_T}{2}) = g_X \exp\left\{ -\frac{1}{8\Lambda_X^2}\left[\frac{m^2 + (\boldsymbol{k}_T \pm \frac{\boldsymbol{\Delta}_T}{2})^2}{|x|} + \frac{M_X^2 + (\boldsymbol{k}_T \pm \frac{\boldsymbol{\Delta}_T}{2})^2}{1-|x|}\right]\right\}$$

propagator → scalar diquark $\frac{i}{(P-k)^2 - M_s^2 + i\epsilon}$

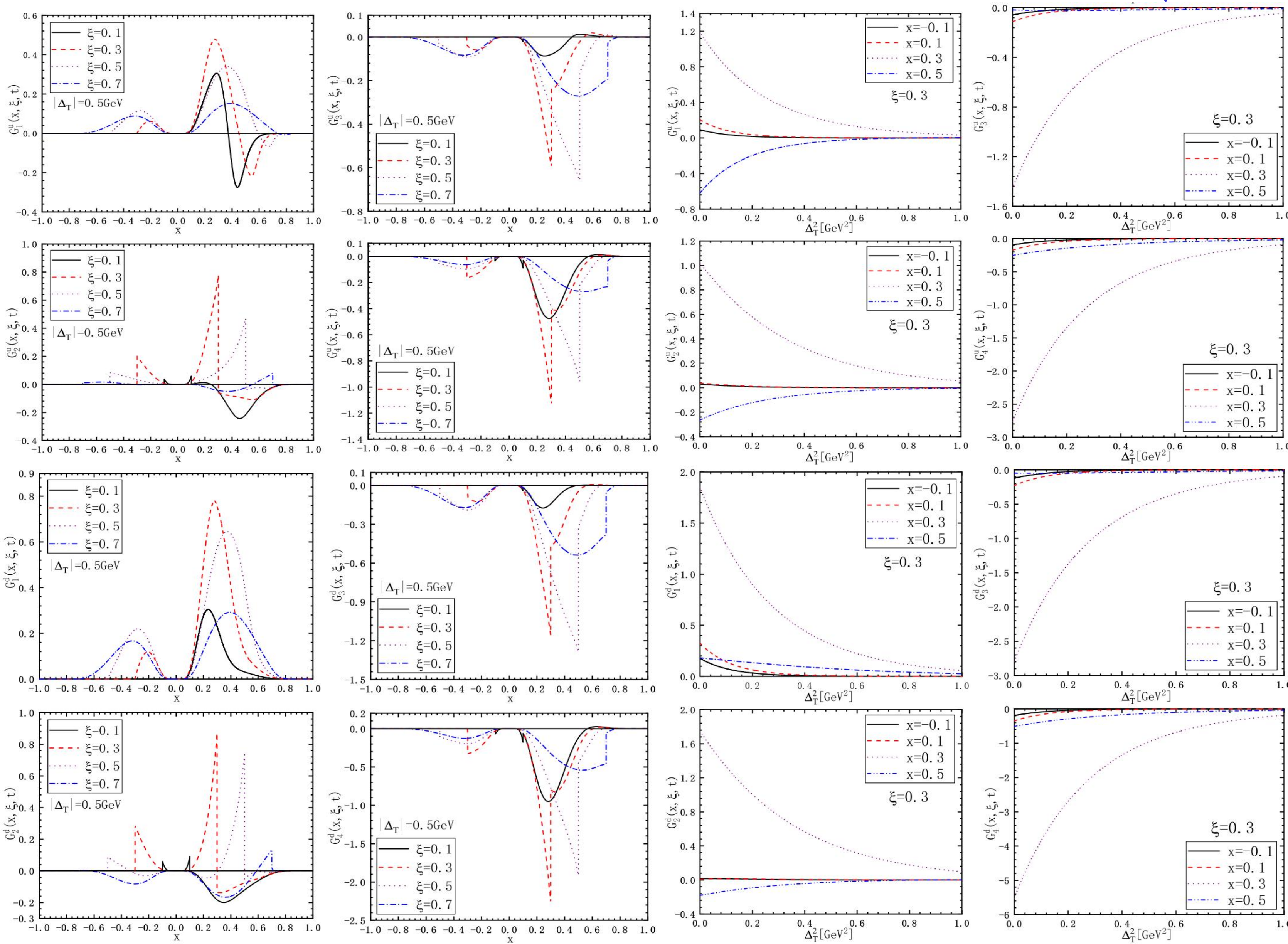
propagator → axial-vector diquark $\frac{i}{(P-k)^2 - M_a^2 + i\epsilon} d^{\mu\nu}(P-k)$

polarization tensor

$$d^{\mu\nu}(P-k) = -g^{\mu\nu} + \frac{(P-k)^{\mu} n_-^{\nu} + (P-k)^{\nu} n_-^{\mu}}{(P-k)\cdot n_-} - \frac{M_a^2}{[(P-k)\cdot n_-]^2} n_-^{\mu} n_-^{\nu}$$

vector GPDs

as functions of x at fixed $|\Delta_T|$ as functions of Δ_T^2 at fixed ξ

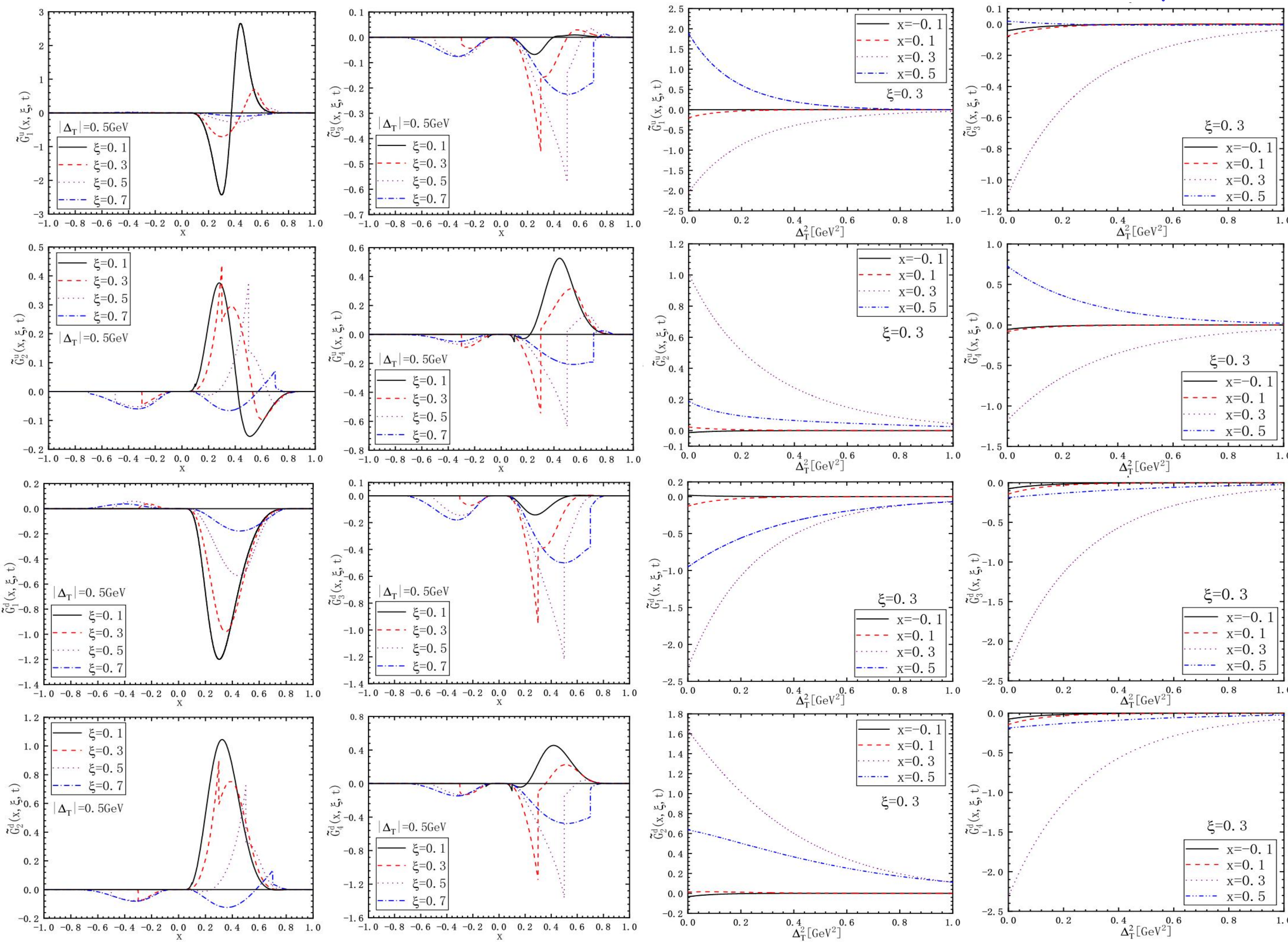


u quark

d quark

axial-vector GPDs

as functions of x at fixed $|\Delta_T|$ as functions of Δ_T^2 at fixed ξ



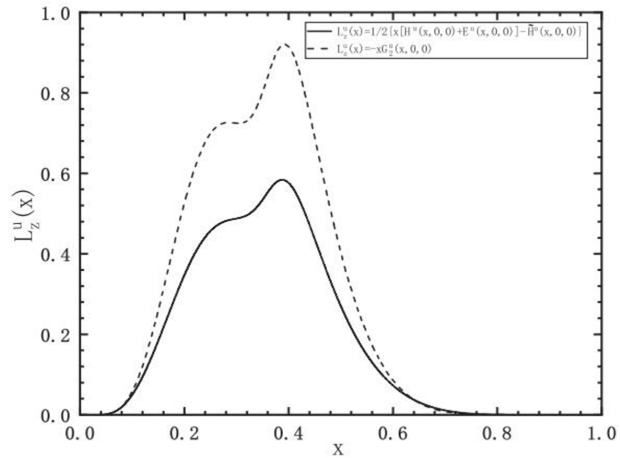
u quark

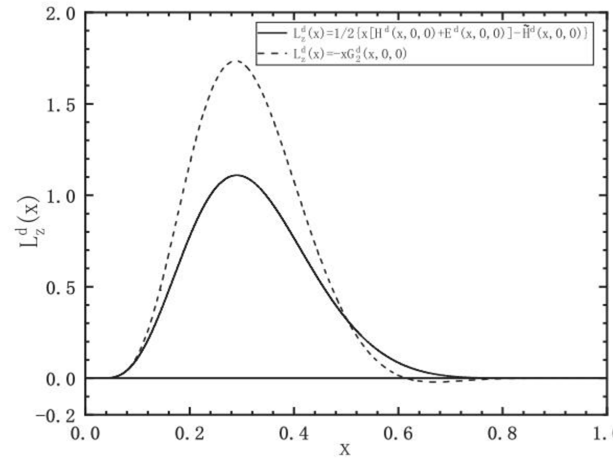
d quark

sum rules

kinetic OAM

$$L_z^q = -\int_{-1}^{1} dx x G_2^q(x,0,0)$$

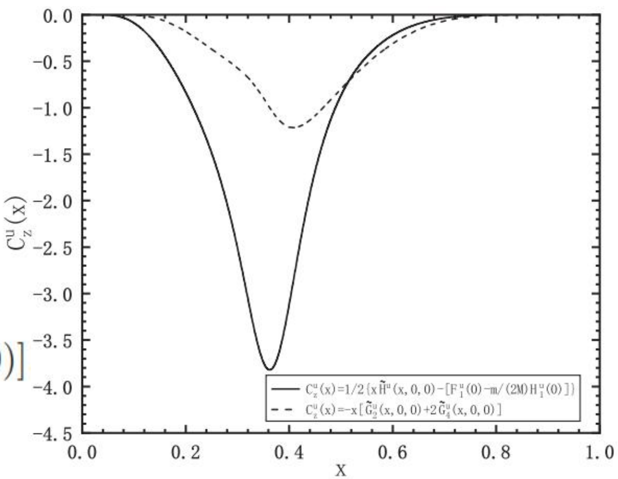


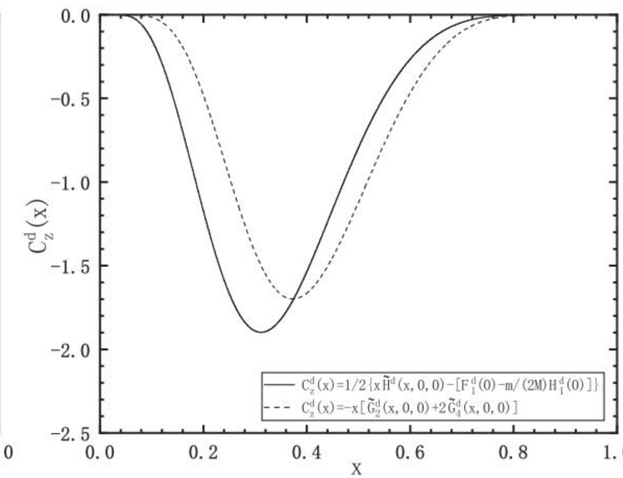


$$\int_{-1}^{1} dx x G_2^q(x,\xi,t) = \frac{1}{2}\int_{-1}^{1} dx\{\tilde{H}^q(x,\xi,t) - x[H^q(x,\xi,t) + E^q(x,\xi,t)]\}$$

kinetic spin-orbit correlation

$$C_z^q = -\int_{-1}^{1} dx x[\tilde{G}_2^q(x,0,0) + 2\tilde{G}_4^q(x,0,0)]$$





$$\frac{1}{2}\int_{-1}^{1} dx\left\{x\tilde{H}^q(x,\xi,t) - \left[F_1^q(t) - \frac{m}{2M}H_1^q(t)\right]\right\} = -\int_{-1}^{1} dx x[\tilde{G}_2^q(x,\xi,t) + 2\tilde{G}_4^q(x,\xi,t)]$$

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summary

- Study on GPDs and GTMDs, particular at nonzero skewness, are extremely important for a complete understanding of hadron structure

- Future measurement on GPD-related and GTMD-related observables at existing and planned facilities combined with phenomenological analysis can provide more precise information on the parton structure inside hadrons

Thank you

谢谢大家！