Exploring the inverse problem from the correlation function for system with two open bottom quarks

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Outline

1 previous works

1.1 propose and deveop formalism $\Rightarrow$ molecular probability

LRD, Song, Oset, arXiv: 2306.01607 PLB $T_{cc}(3875)$

Song,LRD, Oset, arXiv: 2307.02382 PRD $X(3872)$

1.2 model-independent fitting $\Rightarrow$ molecular probability

LRD, Abreu, Feijoo, Oset, arXiv:2304.01870 EPJC $T_{cc}(3875)$ resampling method

2 correlation function

2.1 why we choose T_{bb}

Feijoo, Liang, Oset, arXiv: 2108.02730 PRD consistent with LHCb data for $T_{cc}(3875)$

LRD, Oset, Feijoo, et al. arXiv:2201.04840 PRD our prediction for T_{bb}

2.2 model-independent fitting for correlation function $\Rightarrow$ molecular probability

Feijoo, LRD, Abreu, Oset, arXiv:2309.00444 PRD T_{bb} resampling method

3 summary

1 previous works

molecular probability

1.1 propose and deveop formalism arXiv: 2306.01607 PLB $T_{cc}(3875)$

The dilemma between molecular states and compact quark states is the subject of a continuous debate in hadron physics.

The condition that a pole appears at s_0 (the square of the mass of the physical state) below the threshold

$$s_0 - s_R - \tilde{g}^2 G_{DD^*}(s_0) = 0 \qquad \text{with} \quad \sqrt{s_R} = \sqrt{s_{\rm th}} + \Delta\sqrt{s_R}$$

we can obtain the molecular probability

$$P = -\frac{\tilde{g}^2 \frac{\partial G}{\partial s}}{1 - \tilde{g}^2 \frac{\partial G}{\partial s}}\Big|_{s=s_0}$$

at $s_0 = \sqrt{s_{\rm th}} - 0.36$ MeV for $T_{cc}(3875)$ [Nature Physics 18(2022)751]

we discuss $\Delta\sqrt{s_R} = 102$ MeV [PRL119(2017)202002] and different "scale" with $\Delta\sqrt{s_R} = 10$ MeV and $\Delta\sqrt{s_R} = 1$ MeV $\Rightarrow$ obtain different P at s_0.

$\Rightarrow$ **The binding energy by itself cannot give a proof of the nature of the state.**

For scattering length & effective range

The unitarity of the t_{DD^*,DD^*} amplitude

$$\text{Im}\, t^{-1} = \text{Im}\left(\frac{s - s_R}{\tilde{g}^2} - G_{DD^*}(s)\right) = -\text{Im}\, G_{DD^*}(s) = \frac{k}{8\pi\sqrt{s}}$$

with k the meson-meson on shell momentum.
The relationship with the f^{QM} [Quantum Mechanics]

$$t = -8\pi\sqrt{s}\, f^{\text{QM}} \simeq -8\pi\sqrt{s}\, \frac{1}{-\frac{1}{a} + \frac{1}{2}\, r_0\, k^2 - ik}$$

It is easy to induce

$$\begin{aligned} -\frac{1}{a} &= \frac{s_{\text{th}} - s_R}{\tilde{g}^2} - \text{Re}\, G_{DD^*}(s_{\text{th}}) \\ r_0 &= 2\frac{\sqrt{s}}{\mu}\frac{\partial}{\partial s}\left\{(-8\pi\sqrt{s})\left(\frac{s - s_R}{\tilde{g}^2} - \text{Re}\, G_{DD^*}(s)\right)\right\}\Big|_{s=s_{\text{th}}} \end{aligned}$$

scattering and effective range for $q_{\text{max}} = 450$ MeV at $s_0 = \sqrt{s_{\text{th}}} - 0.36$ MeV ($\beta = 0$)

$\Delta\sqrt{s_R}$ [MeV]	a [fm]	r_0 [fm]
0.1	0.87	-114.07
0.3	1.19	-79.33
1	2.10	-38.20
5	4.62	-9.26
10	5.74	-4.51
50	7.25	-0.47
70	7.39	-0.17
102	7.51	0.06

It can be seen that as $\Delta\sqrt{s_R}$ becomes smaller (decreasing the P), a becomes smaller and smaller and r_0 grows indefinitely.

The lesson we draw is the a and r_0 are very useful to determine the molecular probability of the state.

arXiv: 2307.02382 PRD $X(3872)$ extension to coupled-channel case

The binding energy by itself does not give us the molecular probability.

$$|D^*\bar{D},\ I=0\rangle \;=\; \frac{1}{\sqrt{2}}(D^{*0}\bar{D}^0 + D^{*+}D^-)$$

$$t_{D^*\bar{D}}(I=0) \;=\; \frac{\tilde{g}^2}{s-s_R}$$

$$\tilde{g}^2 = \left.\frac{s-s_R}{\frac{1}{2}G_1 + \frac{1}{2}G_2}\right|_{s_0}.$$

The loop functions G_i of $i=1$ for $\bar{D}^0 D^{*0}$ and $i=2$ for $D^- D^{*+}$.

$$P_1 = -\left.\frac{\frac{1}{2}\tilde{g}^2 \frac{\partial G_1}{\partial s}}{1-\frac{1}{2}\tilde{g}^2\frac{\partial}{\partial s}(G_1+G_2)}\right|_{s_0}$$

$$P_2 = -\left.\frac{\frac{1}{2}\tilde{g}^2 \frac{\partial G_2}{\partial s}}{1-\frac{1}{2}\tilde{g}^2\frac{\partial}{\partial s}(G_1+G_2)}\right|_{s_0}$$

scattering and effective range for $q_{max} = 450$ MeV $(\beta = 0)$

$\Delta\sqrt{s_R}$	a_1[fm]	$r_{0,1}$[fm]	a_2[fm]	$r_{0,2}$[fm]
0.1	1.42	-663.61	$0.0073 - i\ 0.00003$	$-664.79 - i\ 1.56$
0.3	3.16	-273.51	$0.0176 - i\ 0.00020$	$-273.04 - i\ 1.56$
1	7.48	-89.71	$0.0530 - i\ 0.00180$	$-88.46 - i\ 1.56$
10	18.45	-9.68	$0.3957 - i\ 0.10756$	$-8.10 - i\ 1.56$
50	21.35	-2.29	$0.7558 - i\ 0.58190$	$-0.68 - i\ 1.56$
100	21.78	-1.37	$0.7818 - i\ 0.78157$	$0.25 - i\ 1.56$

- $r_{0,1} = -5.34$ fm LHCb data [PRD102(2020)092005]
  -2.78 fm $< r_{0,1} < 1$ fm, $a_1 \approx 28$ fm [PLB833(2022)137290]
- $\Delta\sqrt{s_R} = 0.1$ MeV, a_1, a_2 become small, and most important, $r_{0,1}, r_{0,2}$ become extremely large, where we had a negligible molecular component $\Rightarrow$ enough to discard this scenario.
- $\Delta\sqrt{s_R} = 100$ MeV, would be basically acceptable, but $P \to 1$.

arXiv: 2306.01607 PLB and arXiv: 2307.02382 PRD

We develop the general formalisms in single-channel and coupled-channel calculations.

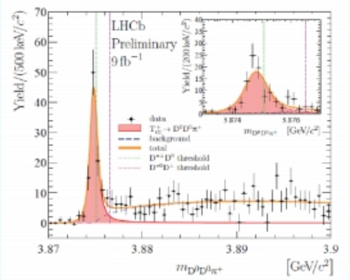
as an application we make the comparison of molecular and compact states for the $T_{cc}(3875)$ and $X(3872)$ in three different scenarios.

- $T_{cc}(3875)$ in the $D^{*+}D^0$ single-channel
- $X(3872)$ in the $D^0\bar{D}^{*0}$ and D^+D^{*-} coupled-channel

the conclusion **binding energy itself does not determine the compositeness of a state,but the additional information of the scattering length and effective range can provide an answer.**

1.2 model-independent fitting arXiv:2304.01870 EPJC

The discovery of the $T_{cc}(3875)$ by LHCb experiment: Nature Physics 18 (2022) 751; Nature Communication 13 (2022) 3351 was a turning point in hadron physics, showing the first evidence of a meson state clearly exotic with two open charm quarks.



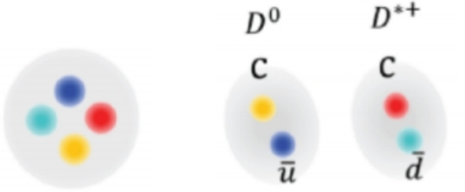
Its mass: $M_{T_{cc}} = M_{D^{*+}D^0} + \delta m_{exp}$

Its width: $\Gamma = 48 \pm 2^{+0}_{-14}$ keV

with $M_{D^{*+}D^0} = 3875.09$ MeV

$\delta m_{exp} = -360 \pm 40^{+4}_{-0}$ keV

$T_{cc}^+(3875)$ $(cc\bar{u}\bar{d})$



Compact (genuine) states? or molecular states? or mixture of both?

we can see the debate for various models for its origin and nature of $T_{cc}(3875)$.

A general potential for $i=1$ for $D^0 D^{*+}$ and $i=2$ for $D^+ D^{*0}$ channels

$$V = \begin{pmatrix} V_{11} & V_{12} \\ V_{12} & V_{22} \end{pmatrix},$$

from where the scattering matrix is

$$T = [1 - VG]^{-1} V,$$

where G_i are the loop functions regularized in the cutoff method, with

$$G = \int_{|\boldsymbol{q}|<q_{\rm max}} \frac{d^3 q}{(2\pi)^3} \frac{\omega_1 + \omega_2}{2\,\omega_1 \omega_2} \frac{1}{s - (\omega_1 + \omega_2)^2 + i\epsilon}$$

where $\omega_i = \sqrt{\boldsymbol{q}^2 + m_i^2}$, m_1 is the mass of the D and m_2 that of D^*. The value of $q_{\rm max}$ reflects the range of the interaction in momentum space and will be obtained from the fits to the data.

From the effective range expansion and scattering matrix, we can obtain scattering lengths and effective ranges

$i = 1$: at threshold of $D^0 D^{*+}$

$$
\begin{aligned}
-\frac{1}{a_1} &= -8\pi\sqrt{s}\,T_{11}^{-1} = -8\pi\sqrt{s}\left[\frac{1 - V_{11} G_2}{V_{11} + (V_{12}^2 - V_{11}^2)\,G_2} - Re\,G_1\right]\Big|_{s=s_1} \\
r_{0,1} &= -\frac{\sqrt{s_1}}{\mu_1}\frac{\partial}{\partial s}\left\{16\pi\sqrt{s}\left[\frac{1 - V_{11} G_2}{V_{11} + (V_{12}^2 - V_{11}^2)\,G_2} - Re\,G_1\right]\right\}\Big|_{s=s_1}
\end{aligned}
$$

$i = 2$: at threshold of $D^+ D^{*0}$

$$
\begin{aligned}
-\frac{1}{a_2} &= -8\pi\sqrt{s}\,T_{22}^{-1} = -8\pi\sqrt{s}\left[\frac{1 - V_{11} G_1}{V_{11} + (V_{12}^2 - V_{11}^2)\,G_1} - Re\,G_2\right]\Big|_{s=s_2} \\
r_{0,2} &= -\frac{\sqrt{s_2}}{\mu_2}\frac{\partial}{\partial s}\left\{16\pi\sqrt{s}\left[\frac{1 - V_{11} G_1}{V_{11} + (V_{12}^2 - V_{11}^2)\,G_1} - Re\,G_2\right]\right\}\Big|_{s=s_2}
\end{aligned}
$$

From the residues of the T matrix at binding of T_{cc}, wa can derive the couplings and the probabilities of the channels

$$
\begin{aligned}
g_1^2 &= \lim_{s\to s_0}(s-s_0)\,T_{11} = \frac{V_{11}+(V_{12}^2-V_{11}^2)\,G_2}{\frac{\partial}{\partial s}\,\mathrm{DET}}\Big|_{s=s_0}, \\
g_2^2 &= \lim_{s\to s_0}(s-s_0)\,T_{22} = \frac{V_{11}+(V_{12}^2-V_{11}^2)\,G_1}{\frac{\partial}{\partial s}\,\mathrm{DET}}\Big|_{s=s_0}, \\
g_1 g_2 &= \lim_{s\to s_0}(s-s_0)\,T_{12} = \frac{V_{12}}{\frac{\partial}{\partial s}\,\mathrm{DET}}\Big|_{s=s_0}
\end{aligned}
$$

we have the probabilities for the $D^0 D^{*+}$ and $D^+ D^{*0}$ channels, respectively, as

$$
P_1 = -g_1^2\,\frac{\partial G_1}{\partial s}\Big|_{s=s_0}, \qquad P_2 = -g_2^2\,\frac{\partial G_2}{\partial s}\Big|_{s=s_0}
$$

and the nonmolecular component

$$
Z = 1-(P_1+P_2)
$$

Conditions and parameters

isospin basis

$$|D^*D, I=0\rangle = -\frac{1}{\sqrt{2}}(D^{*+}D^0 - D^{*0}D^+)$$

$$|D^*D, I=1\rangle = -\frac{1}{\sqrt{2}}(D^{*+}D^0 + D^{*0}D^+)$$

a) due to isospin symmetry
$V_{11} = V_{22}$

b) from the experimental analysis
$V_{12} > 0\,; \quad V_{12} > |V_{11}|$

c) demanding a pole at s_0
reduce a parameter

We have a energy dependent potential

$$V_{11} = V'_{11} + \frac{\alpha}{m_V^2}(s - s_0)$$

$$V_{12} = V'_{12} + \frac{\beta}{m_V^2}(s - s_0)$$

where α, β are dimensionless free parameters, $m_V = 800$ MeV, s_0 is the mass squared of the T_{cc}.

We have five free parameters q_{max}, V'_{11}, V'_{12} and α, β (dimensionless) in our fits.

Two different strategies to fit the experimental LHCb data [Nature Commun. 13 (2022) 3351]

Two different strategies

by fitting the scattering lengths and effective ranges

$\Downarrow$ fit (a)

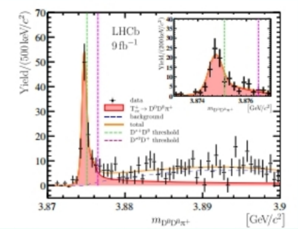
The experimental LHCb data
[Nature Commun. 13 (2022) 3351;
arXiv: 2203.04622 neglecting the D^* width

$$a_1 = 6.134 \pm 0.51 \text{ fm},$$
$$r_{0,1} = -3.516 \pm 0.50 \text{ fm}$$
$$a_2 = (1.707 \pm 0.30) - i\,(1.07 \pm 0.30) \text{ fm},$$
$$r_{0,2} = (0.259 \pm 0.30) - i\,(3.769 \pm 0.30) \text{ fm}$$

by fitting the $D^0D^0\pi^+$ mass distribution

$\Downarrow$ fit (b)

Nature Commun. 13 (2022) 3351



corrected by the experimental resolution and parametrized in terms of a unitary amplitude (consider D^* width explicitly)

in the evaluation

a) correlation between $V'_{11} - V'_{12}$ and q_{max}
There is a tradeoff

b) correlation of α and β
what matters is the $\alpha - \beta$ combination

resampling (bootstrap) method
which is particularly suited for the case that there are strong correlations between the parameters.

fit (a) use six data: $a_1, r_{0,1}$ (real), and $a_2, r_{0,2}$ (complex)
fit (b): direct fit to the $D^0 D^0 \pi^+$ mass distribution

$\Downarrow$ resampling method

to evaluate the average value of these magnitudes and dispersion of each observable

$$\overline{P}_1 = \frac{1}{N} \sum_i P_{1,i}\,, \qquad (\Delta P_1)^2 = \frac{1}{N} \sum_i (P_{1,i} - \overline{P}_1)^2$$

fit (a) The obtained scattering lengths and effective ranges

a_1 [fm]	$r_{0,1}$ [fm]	a_2 [fm]	$r_{0,2}$ [fm]
6.04 ± 0.11	-3.55 ± 0.37	$(1.72 \pm 0.12) - i\,(1.07 \pm 0.03)$	$(0.29 \pm 0.08) - i\,(3.76 \pm 0.14)$

The experimental LHCb data
[Nature Commun. 13 (2022) 3351; arXiv: 2203.04622 neglecting the D^* width

$$a_1 = 6.134 \pm 0.51\ \mathrm{fm}\,, \quad r_{0,1} = -3.516 \pm 0.50\ \mathrm{fm}$$

$$a_2 = (1.707 \pm 0.30) - i\,(1.07 \pm 0.30)\ \mathrm{fm}\,, \quad r_{0,2} = (0.259 \pm 0.30) - i\,(3.769 \pm 0.30)\ \mathrm{fm}$$

It is seen that the agreement with the experiment is remarkable.

fit (a) The obtained coupling constants and probabilities

g_1 [MeV]	g_2 [MeV]	P_1	P_2	Z
3727 ± 54	-3752 ± 164	0.67 ± 0.02	0.26 ± 0.02	0.07 ± 0.03

The obtained P_1, P_2 of the order of 69%, 28% with uncertainties of the order of 2%, and $Z = 0.07 \pm 0.03$ $\Rightarrow$ T_{cc} as a clear molecular state made of the $D^0 D^{*+}$ and $D^+ D^{*0}$ components.

The obtained couplings g_1 and g_2 are very close and with opposite sign
$\Rightarrow$ indicating that we have basically a state of isospin I=0.

fit (b) Direct fit to the $D^0D^0\pi^+$ mass distribution

[Nature Commun. 13 (2022) 3351

we are using now the G functions accounting for the width of the D^*

$$G = \int_{|q|<q_{max}} \frac{d^3q}{(2\pi)^3} \frac{\omega_1+\omega_2}{2\,\omega_1\omega_2} \frac{1}{\sqrt{s}+\omega_1+\omega_2} \frac{1}{\sqrt{s}-\omega_1-\omega_2+i\,\frac{\sqrt{s'}}{2\,m_{D^*}}\Gamma_{D^*}(s')}$$

where $s' = (\sqrt{s}-\omega_D)^2 - q^2$ and $\Gamma_{D^*}(s')$ from PRD104 (2021) 114015

We perform a best fit by taking 44 points to fit the $D^0D^0\pi^+$ mass distribution.

$\Downarrow$ resampling method for estimating the statistical uncertainties

we obtain the average values of the observables and their dispersion in the resampling method

fit (b) The obtained observables of scattering lengths and effective ranges

The obtained average values and their dispersion

$$\begin{aligned}
a_1 &= (7.60 \pm 0.14) - i\,(1.73 \pm 0.09))\ \text{fm} \\
r_{0,1} &= -2.94 \pm 0.04\ \text{fm} \\
a_2 &= (1.99 \pm 0.07) - i\,(1.25 \pm 0.23))\ \text{fm} \\
r_{0,2} &= (0.11 \pm 0.17) - i\,(2.74 \pm 0.22)\ \text{fm}
\end{aligned}$$

Compared with [Nature Commun. 13 (2022) 3351] when the D^* width is explicitly taken into account

$$a_1^{\text{exp}} = [(7.16 \pm 0.51) - i\,(1.85 \pm 0.28)]\ \text{fm}\,, \qquad a_2^{\text{exp}} = (1.76 - i\,1.82)\ \text{fm}$$

It is seen that the agreement is perfect within errors.

fit (b) The obtained binding energy, width, coupling constants and probabilities

The obtained average values and their dispersion

$$
\begin{aligned}
\mathrm{B} &= 360 \pm 2 \ \mathrm{keV}\,, \quad \Gamma = 38 \pm 1 \ \mathrm{keV} \\
\mathrm{g}_1 &= 3875 \pm 51 \ \mathrm{MeV}\,, \quad \mathrm{g}_2 = -4077 \pm 72 \ \mathrm{MeV} \\
\mathrm{P}_1 &= 0.697 \pm 0.017\,, \quad \mathrm{P}_2 = 0.301 \pm 0.009
\end{aligned}
$$

The obtained couplings g_1 and g_2 are very close and with opposite sign and very similar to those with the fit (a) $\Rightarrow$ indicating that we have basically a state of isospin I=0.

The obtained $\mathrm{P}_1 + \mathrm{P}_2 = 0.998 \pm 0.024$ in fit (b), which are also remarkably similar to those with fit (a) $\Rightarrow$ indicating again the molecular nature of the T_{cc} state.

We have performed fits to the data of the LHCb collaboration by using two different strategies.

In a model independent way

1) starting from the scattering length and effective range of the $D^{*+}D^0$, $D^{*0}D^+$ channels;

2) or from the experimental $D^0D^0\pi^+$ mass distribution.

We conclude that the T_{cc} is a molecular state of the $D^{*+}D^0$, $D^{*0}D^+$ components.

using all the available experimental information (not only the binding but also the scattering length and effective range) was essential to reach the present conclusions.

2 correlation function

molecular probability

Experiments on correlation functions

- The study of correlation functions in pairs of particles observed in high energy p-p, p-A and A-A collisions is turning into a very useful tool to determine the basic properties of the pair interaction.
  [ALICE Collaboration, Nature 588(2021)232]
- Experimental work in the strangeness sector is abundant.
  [ALICE Collaboration, Nature 588(2021)232; PRC99(2019)024001; PRL124(2020)092301; PLB833(2022)137272; PLB805(2020)135419; PLB797(2019)134822; PRL123(2019)112002; PRL127(2021)172301; PLB829(2022)137060]
- the ALICE collaboration is starting to explore the charm sector measuring correlation functions in high-multiplicity pp reactions at 13 TeV.
  [ALICE Collaboration, PRD106(2022)052010]
- In the future one will also have access to the bottom sector.

2.1 why we choose T_{bb}

Feijoo, Liang, Oset, arXiv: 2108.02730 PRD consistent with LHCb data for $T_{cc}(3875)$ Chiral Unitary Approach

LRD, Molina, Oset, arXiv:2110.15270 PRD extension

PHYSICAL REVIEW D **104**, 114015 (2021)

$D^0D^0\pi^+$ mass distribution in the production of the T_{cc} exotic state

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We perform a unitary coupled channel study of the interaction of the $D^{*+}D^0$, $D^{*0}D^+$ channels and find a state barely bound, very close to isospin $I = 0$. We take the experimental mass as input and obtain the width of the state and the $D^0D^0\pi^+$ mass distribution. When the mass of the T_{cc} state quoted in the experimental paper from raw data is used, the width obtained is of the order of the 80 keV, small compared to the value given in that work. Yet, when the mass obtained in an analysis of the data considering the experimental resolution is taken, the width obtained is about 43 keV and both the width and the $D^0D^0\pi^+$ mass distribution are in remarkable agreement with the results obtained in that latter analysis.

D^*D system for $T_{cc}(3875)$

PHYSICAL REVIEW D **105**, 016029 (2022)

Prediction of new T_{cc} states of D^*D^* and $D_s^*D^*$ molecular nature

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We extend the theoretical framework used to describe the T_{cc} state as a molecular state of D^*D and make predictions for the D^*D^* and $D_s^*D^*$ systems, finding that they lead to bound states only in the $J^P = 1^+$ channel. Using input needed to describe the T_{cc} state, basically one parameter to regularize the loops of the Bethe-Salpeter equation, we find bound states with bindings of the order of MeV and similar widths for the D^*D^* system, while the $D_s^*D^*$ system develops a strong cusp around the threshold.

we extend to make further predictions for the D^*D^* and $D_s^*D^*$ systems.

The extension to systems with two open bottom quarks

LRD, Oset, Feijoo, et al. arXiv:2201.04840 PRD Chiral Unitary Approach

An extension of the local hidden gauge approach where the vector mesons are exchanged.

PHYSICAL REVIEW D **105**, 074017 (2022)

Masses and widths of the exotic molecular $B_{(s)}^{(*)}B_{(s)}^{(*)}$ states

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We study the interaction of the doubly bottom systems BB, B^*B, B_sB, B_s^*B, B^*B^*, B^*B_s, $B^*B_s^*$, B_sB_s, $B_sB_s^*$, $B_s^*B_s^*$ by means of vector meson exchange with Lagrangians from an extension of the local hidden gauge approach. The full s-wave scattering matrix is obtained implementing unitarity in coupled channels by means of the Bethe-Salpeter equation. We find poles below the channel thresholds for the attractively interacting channels B^*B in $I=0$, $B_s^*B - B^*B_s$ in $I=\frac{1}{2}$, B^*B^* in $I=0$, and $B_s^*B^*$ in $I=\frac{1}{2}$, all of them with $J^P=1^+$. For these cases the widths are evaluated identifying the dominant source of imaginary part. We find binding energies of the order of 10–20 MeV, and the widths vary much from one system to the other: of the order of 10–100 eV for the B^*B system and $B_s^*B - B^*B_s$, about 6 MeV for the B^*B^* system and of the order of 0.5 MeV for the $B_s^*B^*$ system.

local hidden gauge approach

Bando, Kugo, Yamawaki, Phys. Rep. 164, 217 (1988);
Harada, Yamawaki, Phys. Rep. 381, 1 (2003);
Meissner, Phys. Rep. 161, 213 (1988);
Nagahiro, Roca, Hosaka, Oset, Phys. Rev. D 79, 014015 (2009)

We study the interaction of the BB, B^*B, B_sB, B_s^*B, B^*B^*, B^*B_s, $B^*B_s^*$, B_sB_s, $B_sB_s^*$, $B_s^*B_s^*$ doubly bottom systems.

The full s-wave scattering matrix is obtained implementing unitarity in coupled-channels by means of the Bethe-Salpeter equation.

TABLE V. States of $J^P=1^+$ obtained from different configurations. The binding B is referred to the closest threshold.

States	M (MeV)	B (MeV)	Γ
B^*B $(I=0)$	10583	21	14 eV
$B_s^*B - B^*B_s$ $(I=\frac{1}{2})$	10681	11	45 eV
B^*B^* $(I=0)$	10609	40	33 MeV
$B_s^*B^*$ $(I=\frac{1}{2})$	10727	13	7 MeV

T_{bb} $\Rightarrow$ for $B^{*+}B^0$, $B^{*0}B^+$ coupled-channel calculation, we obtain this state with $I=0$ and a binding of 21 MeV.

arXiv:2201.04840 PRD

IV. RESULTS

A. B^*B states

In the first place we show the results that we obtain for the $I=0, J^P=1^+$ B^*B system. As we pointed out, the only source of imaginary part comes from the $B^* \to B\gamma$ decay, with a very small width, as shown in Eq. (14). We thus should expect bound states with a very narrow width. Indeed, in Fig. 5 we plot the modulus squared of the

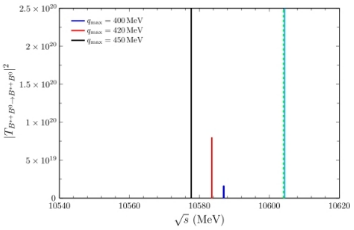


FIG. 5. Squared amplitude $|T_{B^{*+}B^0 \to B^{*+}B^0}|^2$. The vertical lines indicate the $B^{*0}B^+$ and $B^{*+}B^0$ thresholds at 10604.04 MeV and 10604.35 MeV, respectively.

$$g_i g_j = \lim_{s \to s_R} (s - s_R) t_{ij}(s), \tag{26}$$

which is nothing but the residue at the pole.

We find, for $q_{\max} = 420$ MeV,

$$g_1 = 35954 \text{ MeV}, \qquad g_2 = -35798 \text{ MeV}, \tag{27}$$

where g_1, g_2, have opposite sign as we anticipated. According to Eq. (13) this indicates a very neat $I=0$ state, as we anticipated that only the $I=0$ component could lead to a bound state.

The larger distance to the thresholds of the $B^{*+}B^0$, $B^{*0}B^+$ states has as a consequence a smaller isospin breaking than the one found in the T_{cc} state, as can be seen by the proximity of g_2 to $-g_1$.

The width of the states can be obtained directly from the width of the peak zooming in the plots in Fig. 5 or alternatively using that, at the peak,

$$T_{11} = \frac{g_1^2}{s - s_R + iM_R\Gamma_R} \Rightarrow \Gamma_R = -\frac{g_1^2 \mathrm{Im}\{T_{11}\}}{M_R |T_{11}|^2}. \tag{28}$$

Either way gives the width of the doubly bottom generated state: 25, 14 and 4 eV for $q_{\max} = 400$, 420 and 450 MeV respectively. These quantities are indeed extremely small, in line with the estimated 0.4 keV of the $B^* \to B\gamma$ decay

- We predicted T_{bb} state with a binding of 21 MeV
- $t_{ij} \simeq \frac{g_i g_j}{s - s_R}$, $i = 1, 2$ for $B^{*+}B^0$ and $B^{*0}B^+$ channels finding g_1=35954 MeV and g_2=-35798 MeV with opposite sign for $q_{\max}$=420 MeV (as we anticipated and a very neat I=0 state)

2.2 model-independent fitting for correlation function T_{bb}

Feijoo, LRD, Abreu, Oset, arXiv:2309.00444 PRD resampling method + inverse problem

$$C_{B^0 B^{*+}}(p_{B^0}) = 1 + 4\pi\,\theta(q_{\text{max}} - p_{B^0}) \int_0^{+\infty} dr\, r^2\, S_{12}(r) \Big\{ \Big| j_0(p_{B^0} r) + T_{11}(E)\widetilde{G}^{(1)}(r;E) \Big|^2 + \Big| T_{21}(E)\widetilde{G}^{(2)}(r;E) \Big|^2 - j_0^2(p_{B^0} r) \Big\}$$

$$C_{B^+ B^{*0}}(p_{B^+}) = 1 + 4\pi\,\theta(q_{\text{max}} - p_{B^+}) \int_0^{+\infty} dr\, r^2\, S_{12}(r) \Big\{ \Big| j_0(p_{B^+} r) + T_{22}(E)\widetilde{G}^{(2)}(r;E) \Big|^2 + \Big| T_{12}(E)\widetilde{G}^{(1)}(r;E) \Big|^2 - j_0^2(p_{B^+} r) \Big\}$$

with $E = \sqrt{s}$ and $p_{B^0} = \frac{\lambda^{\frac{1}{2}}(s, m_{B^0}^2, m_{B^{*+}}^2)}{2\sqrt{s}}$, $p_{B^+} = \frac{\lambda^{\frac{1}{2}}(s, m_{B^+}^2, m_{B^{*0}}^2)}{2\sqrt{s}}$.

$\widetilde{G}^{(i)}(r;E)$ and $S_{12}(r)$

where T_{ij} are the scattering matrices and the $\widetilde{G}^{(i)}$ function is given by

$$\widetilde{G}^{(i)}(r;E) = \int_{|\vec{q}\,|<q_{\max}} \frac{d^3\vec{q}}{(2\pi)^3} \frac{\omega_1^{(i)}(q)+\omega_2^{(i)}(q)}{2\omega_1^{(i)}(q)\,\omega_2^{i}(q)} \frac{j_0(qr)}{s-\left[\omega_1^{(i)}(q)+\omega_2^{(i)}(q)\right]^2+i\epsilon}$$

with the source function

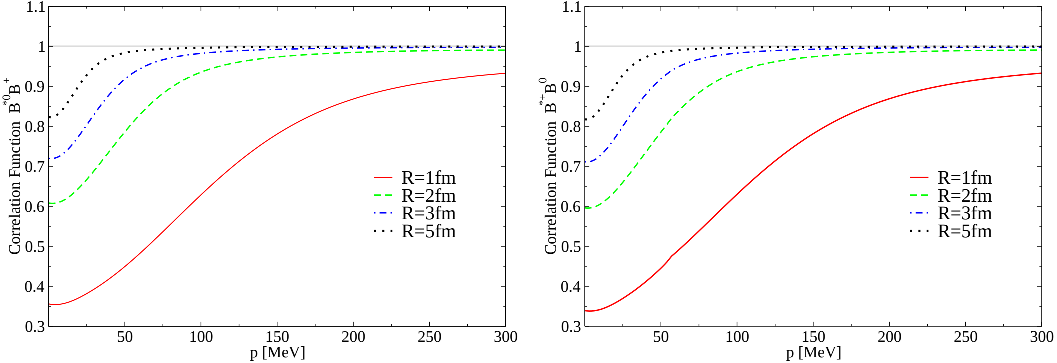
$$S_{12}(r) = \frac{1}{(\sqrt{4\pi})^3 R^3} \exp\left(-\frac{r^2}{4R^2}\right)$$

with R the source size as input.

for inverse problem, we use R^{input}=1 fm and R^{input}=5 fm in the calculation.

Correlation functions for $B^{*0}B^{+}$ and $B^{*+}B^{0}$ pairs

$B^{*0}B^{+}$ threshold at 10604.04 MeV $\qquad$ $B^{*+}B^{0}$ threshold at 10604.35 MeV



- given the proximity of the two thresholds, the two curves are remarkably similar.
- correlation functions change appreciably with different source size.

we will use resampling method to produce synthetic data for inverse problem.

$$V_{11} = V'_{11} + \frac{\alpha}{m_V^2}(s - s_0)\,, \quad V_{12} = V'_{12} + \frac{\beta}{m_V^2}(s - s_0)$$

six free parameters in the fitting $q_{\max}, V'_{11}, V'_{12}, \alpha, \beta, R$.

resampling method for inverse problem

a. first we start from the generated correlation functions and produce synthetic data

choose 31 points from each correlation function with a homogeneous error corresponding to the 10% of the minimum value taken by the correlation function.

b. we use resampling method to evaluate the observables and uncertainties for the inverse problem

run 50 best fits with the resampled data and in each of the fits we determine their average and their dispersion

final results

The obtained scattering lengths and effective ranges

R^{input}	a_1 [fm]	$r_{0,1}$ [fm]	a_2 [fm]	$r_{0,2}$ [fm]
1 fm	0.85 ± 0.18	-0.11 ± 0.51	$(0.81 \pm 0.13) - i\,(0.03 \pm 0.03)$	$(0.43 \pm 0.11) - i\,(0.38 \pm 0.29)$
5 fm	0.85 ± 0.19	-0.92 ± 1.78	$(0.77 \pm 0.13) - i\,(0.05 \pm 0.06)$	$(0.26 \pm 0.40) - i\,(0.87 \pm 1.13)$

The scattering lengths are determined with a 20% precision while, for the components of the effective ranges, we get larger uncertainties.

The obtained coupling constants and bindings

R^{input}	g_1 [MeV]	g_2 [MeV]
1 fm	33039 ± 14744	-32031 ± 17367
5 fm	30970 ± 19666	-31181 ± 19718

$$R^{\text{input}} = 1 \text{ fm}: \quad B = -22 \pm 15 \text{ MeV}$$
$$R^{\text{input}} = 5 \text{ fm}: \quad B = -22 \pm 21 \text{ MeV}$$

- The average values provided by the bootstrap method for both cases are in remarkable agreement with the results in PRD 105 (2022) 074017
- interesting to note that g_1 and g_2 are very similar with opposite sign in both analysis, which indicates that they correspond to a state of $I = 0$.

$$|B^* B, I = 0\rangle = -\frac{1}{\sqrt{2}}(B^{*+}B^0 - B^{*0}B^+)$$

The obtained P_1, P_2 and Z

R^{input}	P_1	P_2	Z
1 fm	0.44 ± 0.06	0.43 ± 0.05	0.13 ± 0.11
5 fm	0.41 ± 0.11	0.39 ± 0.11	0.20 ± 0.22

The probabilities of P_1 ($B^0 B^{*+}$) and P_2 ($B^+ B^{*0}$), are very close to 0.5 each, in both cases.

$$R^{\text{input}} = 1 \text{ fm}: \quad P_1 + P_2 = 0.87 \pm 0.11\,, \quad Z = 0.13 \pm 0.11$$
$$R^{\text{input}} = 5 \text{ fm}: \quad P_1 + P_2 = 0.80 \pm 0.20\,, \quad Z = 0.20 \pm 0.22$$

The sum of $P_1 + P_2$ is compatible with 1 within errors, indicating that the nature of the state is mainly molecular, and the Z (nonmolecular component) basically compatible with zero.

recall the source sizes

To conclude, one of the most important results of the present study is that one can obtain values for both sources as

$$R^{\text{input}} = 1 \text{ fm} : R = 0.974 \pm 0.024 \, \text{fm}$$
$$R^{\text{input}} = 5 \text{ fm} : R = 5.052 \pm 0.614 \, \text{fm}$$

with a notable precision comparing them to the starting input of R=1 fm and R=5 fm used to generate the synthetic data.

in the case of R^{input}=1 fm the obtained error of R is of the order of 2.5%, while for R^{input}=5 fm the uncertainty is about the 12%.

Summary

1 previous works

1.1 propose and deveop formalisms $\Rightarrow$ molecular probability

The binding energy by itself cannot give a proof of the nature of the state.
The lesson we draw is the a and r_0 are very useful to determine the molecular probability of the state.

1.2 model-independent fitting $\Rightarrow$ molecular probability

using two different strategies resampling method

$\Rightarrow$ conclude that the T_{cc} is a molecular state of the $D^{*+}D^0$, $D^{*0}D^+$ components

2 correlation function

model-independent fitting $\Rightarrow$ molecular probability

resampling method $+$ inverse problem

The correlation functions and its inverse problem can be used to determine the nature of molecular probability of the state.

These findings can encourage experiments to look for correlation functions and extract valuable information of observables with acceptable precision.

Thank you for your attention