Rethinking the $N\phi$ scattering length with unphyscial kaon mass in one-loop

严茂俊

西南大学

合作者：张旭，安春生，邓成荣

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Motivation

Theor. predictions

- The non-relativistic Yukawa potential $V_{(Q\bar{Q})A} = -\alpha e^{-\mu r}/r$ is matched to the Pomeron interaction. S. J. Brodsky et al,PRL64(1990); H. Gao et al, PRC63(2001).
- $\{N\phi,\ \Lambda K^*\}$ coupled channel scattering, F. Huang et al,PRC73(2006); J.J Xie et al PLB774(2017);C.S. An et al, PRC98(2018); J. He et al, PRD98(2018).
- Analysis in correlation function： attractive $N\phi$ scattering E. Chizzali et al, PLB848(2024); L. M. Abreu et al, PLB860(2025).

Latt. prediction

- $J = 3/2$ $N\phi$ scattering length $(-1.7,\ -0.9)\,\mathrm{fm}$ fitted to two-pion exchange with $m_\pi = 146.4\,\mathrm{MeV}$ by HAL QCD, Yan Lyu et al, PRD106(2022)

Exp. prediction

- Re. $N\phi$ scattering length around $-1\,\mathrm{fm}$ w/o $\Lambda K^*, \Sigma K^*$ coupled channel scattering by Alice, S. Acharyaet al, PRL127(2021)

$N\phi$ interaction to one loop

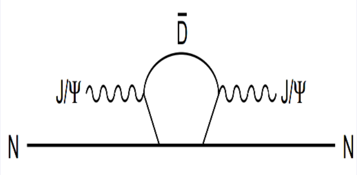
OBE

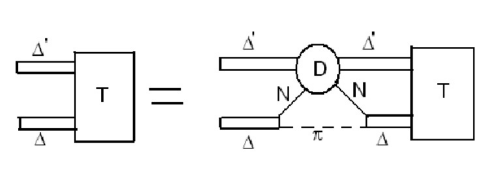
- η-exchange is perturbative.
- σ coupling to strange quark is not clear.
- $\omega - \phi$ mixing is tiny

Two-pion exchange in bubbles

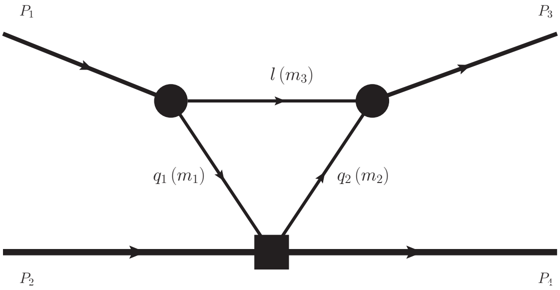
- Two-pion exchange in bubbles is negligible with a vanished WT in $\phi\pi(K)$ scattering.

Dynamics in triangle diagrams S.J. Brodsky et al, PLB 412 (1997); A. Gal et al, PRL111(2013).





Dynamics in the triangle diagram



- $\mathcal{L}_{\phi_{K\bar{K}}} = ig\phi^{\mu}\left(\overline{K}\partial_{\mu}\cdot K - K\partial_{\mu}\cdot\overline{K}\right)$
- $L_1 = \langle \bar{B} i\gamma^{\mu}\frac{1}{4F_{\pi}^2}\left[\left(\Phi\partial_{\mu}\Phi - \partial_{\mu}\Phi\Phi\right)B - B\left(\Phi\partial_{\mu}\Phi - \partial_{\mu}\Phi\Phi\right)\right]\rangle$
- Divergent part driven by a WT in $NK(\bar{K})$ sacattering
- Convergent part driven by a resummation WT in $NK(\bar{K})$ sacattering

Dynamics in the triangle diagram

The S-wave projected loop integral w/o couplings

$$I(l) = \int \frac{d^4 l}{(2\pi)^4} \frac{\left[4l^2 + \vec{P}_1 \cdot \vec{P}_3\right] u(P_2)\gamma^0 \bar{u}(P_1)}{(l^2 - m_1^2)\left[(P_1 - l)^2 - m_2^2\right]\left[(P_3 - l)^2 - m_3^2\right]},$$ con. with large valued l.

- For the pole z_A : $l^0 = \omega_1 - i\epsilon$

$$2\pi i \,\mathrm{Res}\, I(z_A) = \int \frac{d^3 l}{(2\pi)^3} \frac{1}{8\pi m_1 m_2 m_3} \frac{8 m_K}{P_1^0 + P_3^2 - 4\omega_1}$$

$N\phi$ potential w/ $2m_K - m_\phi = \delta$ in HAL QCD

- $V^s_{LO} = -\frac{1}{4\pi^2} \frac{m_K}{m_1 m_2 m_3} \int \frac{g_{eff} |\vec{l}|^2 dl}{m_\phi - 2w_1}$, with the product of couplings g_{eff}.
- $\int \frac{l^2}{\delta - \frac{l^2}{m_K}} dl = m_K \left[-\Lambda - \frac{m_K \delta}{\sqrt{-m_K \delta}} \mathrm{ArcT}\left(\frac{\Lambda}{\sqrt{-m_K \delta}}\right)\right] = \tilde{I}^{div.}(\Lambda) + \tilde{I}^{con.}(\delta)$

Dynamics in the triangle diagram

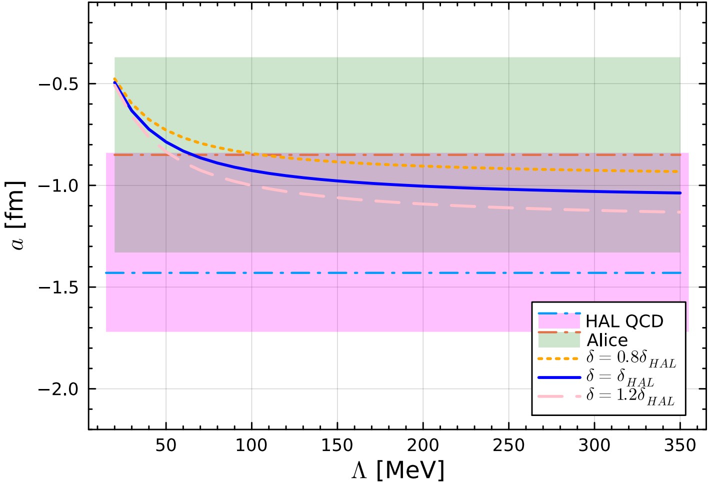
$N\phi$ potential

- $V^{div.} = -\frac{1}{4\pi^2}\frac{m_K^2}{m_1 m_2 m_3}\int V^{WT}(l)\, dl \simeq -\frac{1}{4\pi^2}\frac{m_K^2}{m_1 m_2 m_3} V^{WT}(0)\, l_{max}$.
- $l_{N\bar{K}}^{max} = 245\,\mathrm{MeV}$ cut at $\Lambda(1520)$, $l_{NK}^{max} = 553\,\mathrm{MeV}$ cut at NK^* th.
- $V_{I=0,\,N\bar{K}}^{WT} : V_{I=1,\,N\bar{K}}^{WT} : V_{I=0,\,NK}^{WT} : V_{I=1,\,NK}^{WT} = -3 : -1 : 0 : 2.$
- $V^{div} \simeq \Delta(\Lambda)$
- $\frac{1}{V^{div.} + V^{con.}} \simeq \frac{1}{V^{con.}}\left(1 - \frac{V^{div.}}{V^{con.}}\right).$

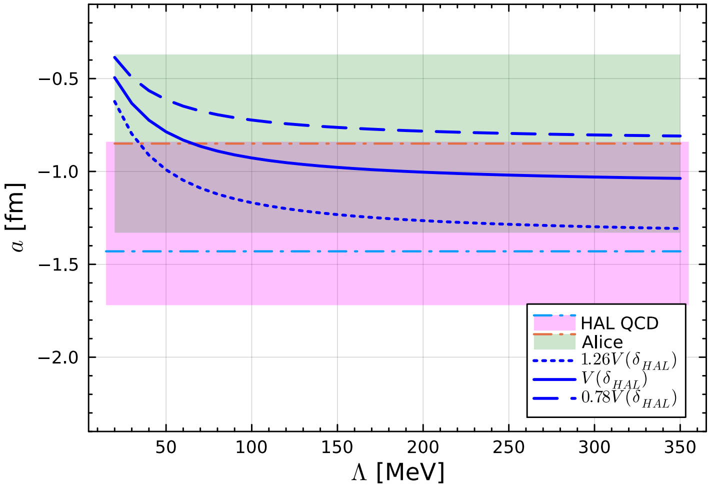
$N\phi$ potential and ERE

- $\frac{8\pi\, th.}{T} = V^{-1} - G = -\frac{1}{a} - ik,\ -\frac{1}{a_{N\phi}} = \frac{1}{V^{con.}}.$
- $V_{LO}^{s,\,con.} = \frac{3g^2 R_{pole}}{4F_\pi^2}\sqrt{m_K\tilde{\delta}}\,\mathrm{ArcT}\left(\Lambda/\sqrt{m_K\tilde{\delta}}\right),$
- $R_{pole} = \frac{g_i^2/(th - s_{pole})}{V^{WT}} \simeq 5.65$ D. Jido et al, NPA725(0223); Z.H. Guo and J. A. Oller, PRC87(2013)
- $Err = (1 \pm r_q)^4 = 1.00^{+0.26}_{-0.22}$ w/ $r_q = \left(m_K^{phy.}/m_K\right).$

$N\phi$ scattering length with unphysical Kaon mass



$N\phi$ scattering length with unphysical Kaon mass



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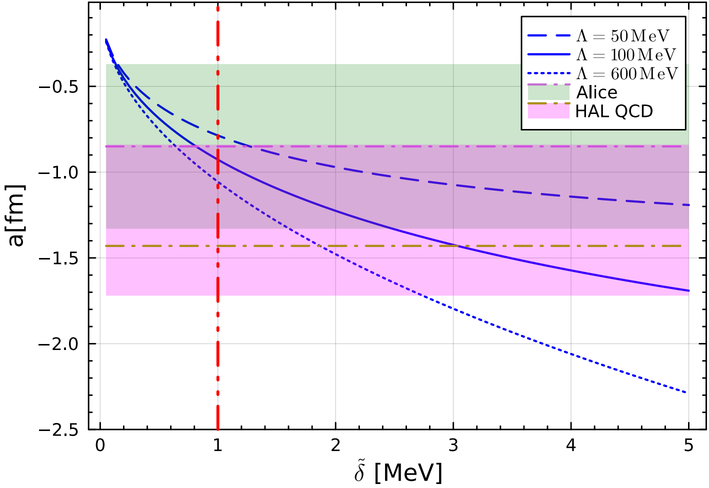
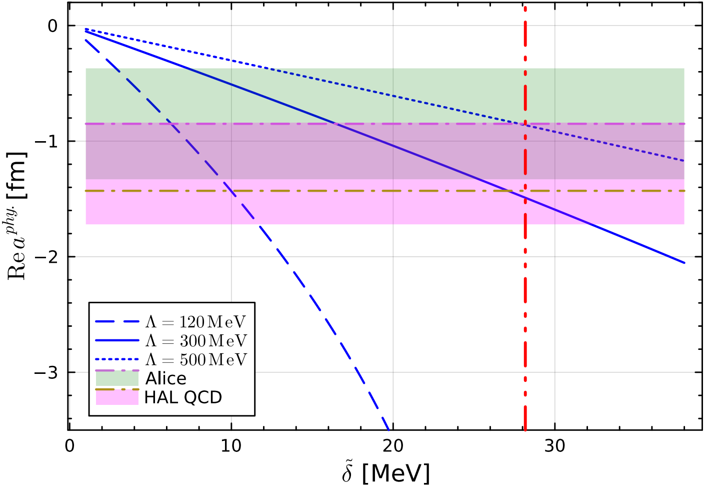


$N\phi$ scattering length with physical Kaon mass



The momentum carried by the outgoning K is $118.17\,\mathrm{MeV}$ in $\phi \to K\bar{K}$.

Power law and Phills-Line

Power law

- V_{LO}^{s} is a distribution of $\tilde{\delta}^{n}$ in the power law, where $n = 0$ and $1/2$ correspond to Λ is very small and large, respectively, which differs from the Van der Waals force proportional to $\delta^{3\,(7/2)}$ ($p_{N\phi}^{6\,(7)}$)
- Differs from the tail of two-pion exchange in HAL QCD.

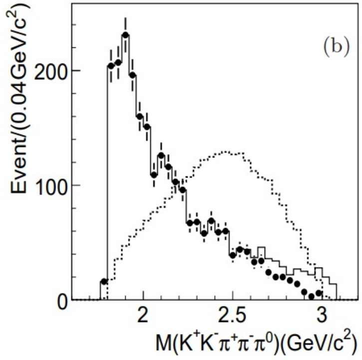
Quark mass dependence

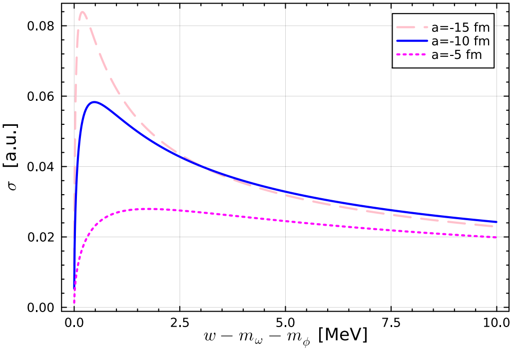
Varying the quark mass and determining the $N\phi$ scattering length.

Phillips line: $n = 1/2$

- The analogy is $B_3 = m_n + 2m_K = 2\tilde{\delta} = 2m_K - m_\phi$, where $K\bar{K}$ scattering in P-wave differs from the nn scatetring in S wave. In the S-wave three-body scattering involving the $K\Lambda(1405)$ two-body scattering, the corresponding spin-parity is $1/2^{+}$ and couples to $1/2^{-}$, $3/2^{-}$ in an additional P-wave transition.

$\omega\phi$ threshold enhancement





M. Ablikim et al. (BESIII), PRD87, 032008 (2013)

A pole from $\omega\phi$ scattering around the threshold in addition to $f_0(1710)$?

Summary

1. The $N\phi$ scattering length evaluated in the triangle diagram with including $\Lambda(1405)$ matches to the one from HAL QCD, where the open channel effect in $\Sigma\pi$, $N\bar{K}$ scattering is ignored.
2. The scattering length is a power law of $K\bar{K}$ binding to ϕ and differs from two-pion exchange and Van der Waals force.
3. When $n = 1/2$, the Phillips line appears in $N\phi$ scattering.
4. When the 3B pole closes to the 2B threshold, the ERE is distorted.
5. This dynamics driven from the triangle diagram is adaptable to $\omega\phi$ threshold enhancement.
6. $P_c(4312)\pi$ couple to $I = 3/2$ sector.
7. $w_{c1}\pi$, $X(3872)\pi$ couple to $Z_c(4020)$.

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Thanks !

Backup

The $\mathrm{SU}(3)$ matrices for the mesons and the baryons are the following

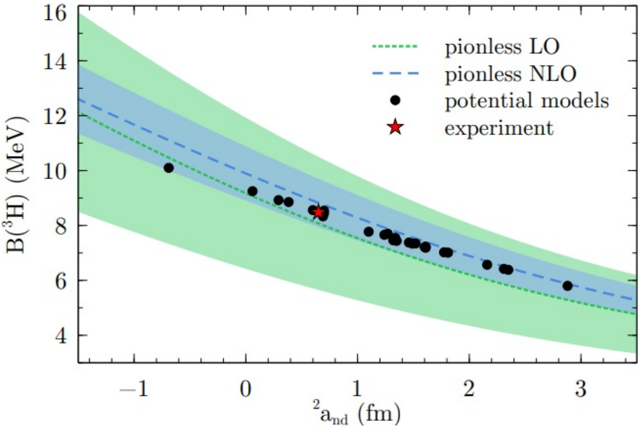
$$\Phi = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta \end{pmatrix},$$

$$B = \begin{pmatrix} \frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & \Sigma^+ & p \\ \Sigma^- & -\frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & n \\ \Xi^- & \Xi^0 & -\frac{2}{\sqrt{6}}\Lambda \end{pmatrix}.$$

Table 1: Inputs of the isospin-averaged hadron masses

Hadron	Lattice [MeV]	Expt. [MeV]
K	524.7	495.6
ϕ	1048.0	1019.5
N	954.0	938.9

Backup: Phillips line



H.W. Hammer et al, Rev.Mod.Phys. 92 (2020)