北京航空航天大學
BEIHANG UNIVERSITY

Towards High-Precision Calculations in Heavy Flavor Physics

Determining Heavy Meson LCDAs from Lattice QCD

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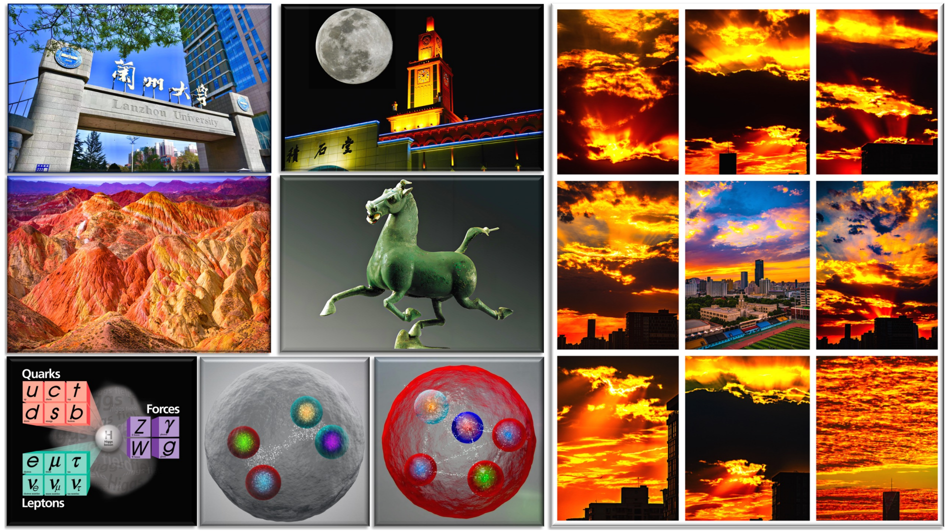
Mar. 24, 2025 @ Lanzhou University

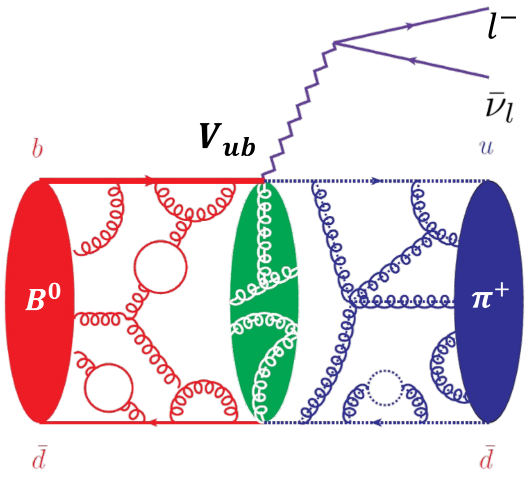


Outline

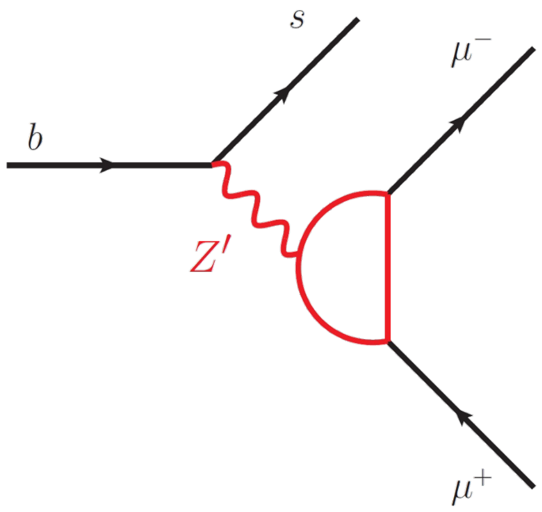
- Motivation
- Sequential Effective Theory to obtain Heavy Meson LCDAs
- Lattice QCD implementation of Sequential Effective Theory
- Phenomenological Discussions
- Summary and Prospect

Motivation

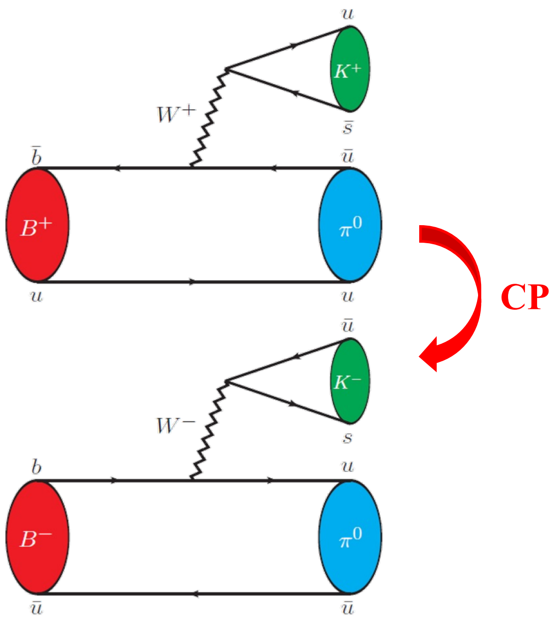
Heavy flavor physics is one of the frontier topics in particle physics:



Precisely testing standard model



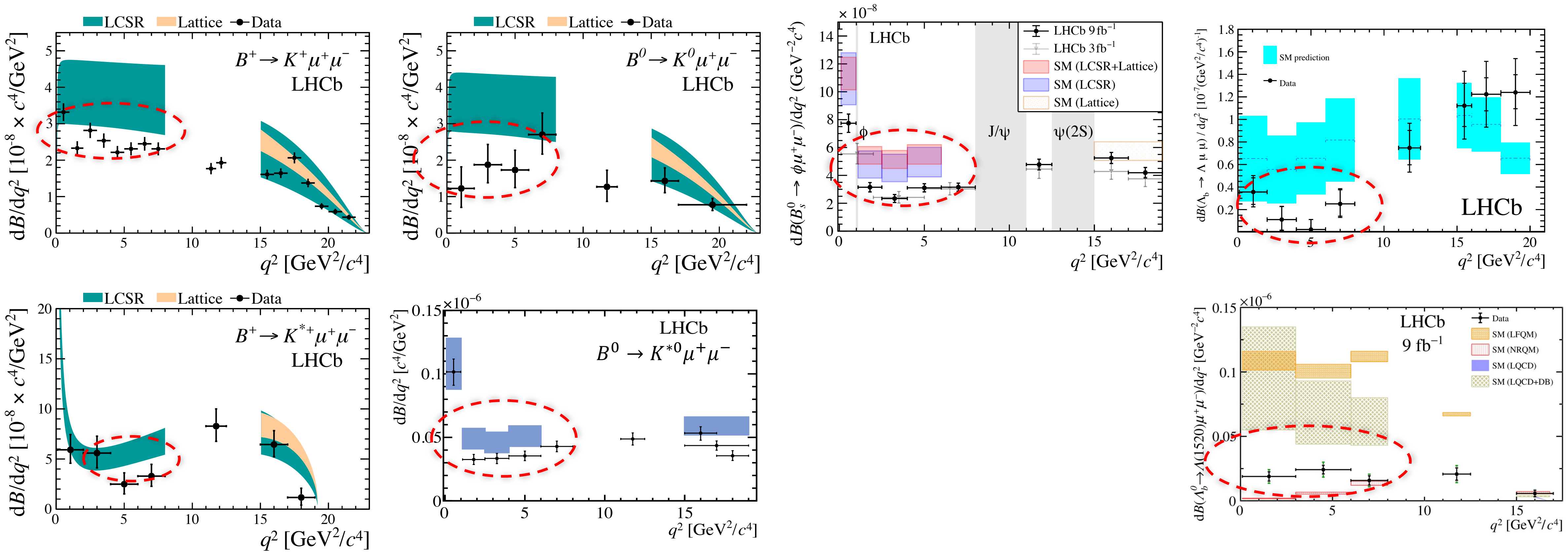
Indirect search for new physics



Study on CP violation

Motivation

Current experimental results show deviations from theoretical prediction…



Motivation

Systematic errors in theoretical calculations primarily from the **nonperturbative inputs**.

Light meson LCDAs

- **Asymptotic form**
  Chernyak, Zhitnitsky, 1977; Lepage, Brodsky, 1979; ……
- **QCD Sum rules**
  Chernyak, Zhitnitsky, 1982; Braun, Filyanov, 1989; ……
- **Dyson-Schwinger Equation**
  Chang, Cloet, et.al, 2013; Gao, Chang, et.al, 2014; ……
- **Global Fits**
  Cheng, et.al, 2020; Hua, Li, Lu, Wang, Xing, 2021; ……
- **Models**
  Arriola, Broniowski, 2002; Zhong, Zhu, et.al, 2021; ……
- **Lattice with OPE**
  Braun, Bruns, et al., 2016; RQCD collaboration, 2019, 2020; ……
- **Lattice with LaMET**
  LP3, 2019; LPC, 2021, 2022; ……

Heavy meson LCDAs

- **Models**
  Grozin, Neubert, 1997; Braun, Ivanov, Korchemsky, 2004; Beneke, Braun, Ji, Wei, 2018; ……

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Heavy meson LCDAs

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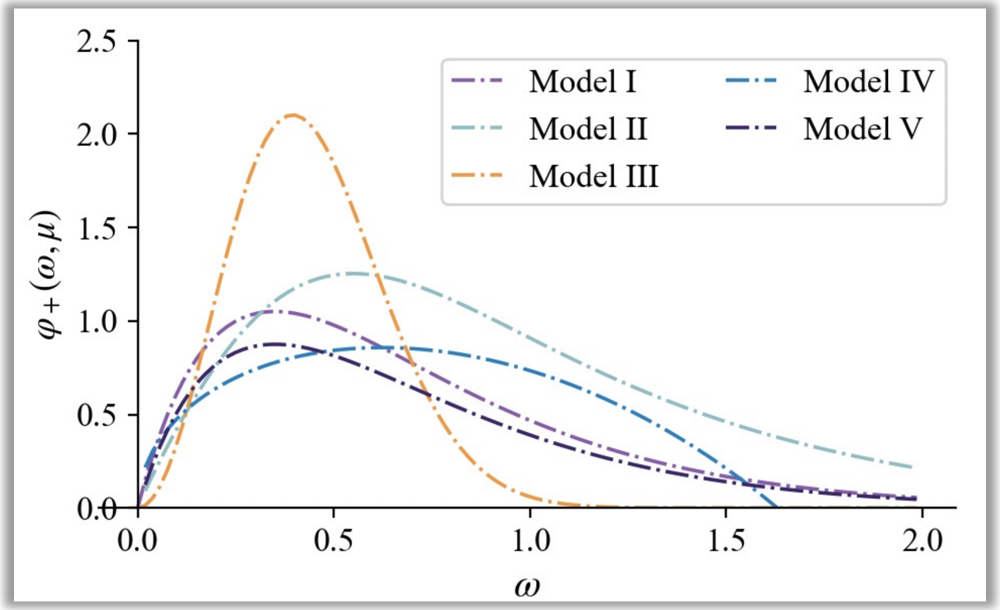


Motivation

Uncertainties from heavy meson LCDAs **dominate** the errors in theoretical calculation.

- For example: $B \to \pi,\ K^*$ form factors from LCSRs:

Gao, Lu, Shen, Wang, Wei, PRD 101 (2020) 074035

Cui, Huang, Shen, Wang, JHEP 03 (2023) 140

$$\mathcal{V}_{B\to K^*}(0) = 0.359^{+0.141}_{-0.085}\Big|_{\lambda_B}\,{}^{+0.019}_{-0.019}\Big|_{\sigma_1}\,{}^{+0.001}_{-0.062}\Big|_{\mu}\,{}^{+0.010}_{-0.004}\Big|_{M^2}\,{}^{+0.016}_{-0.017}\Big|_{s_0}\,{}^{+0.153}_{-0.079}\Big|_{\varphi_\pm(\omega)},$$

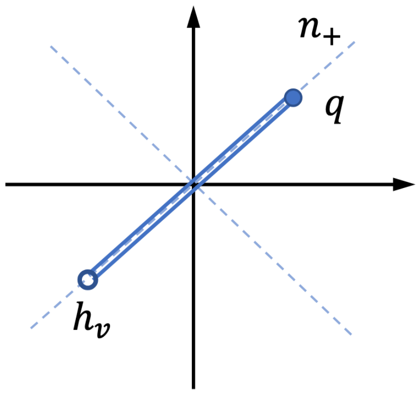
$$f^+_{B\to\pi}(0) = 0.122 \times \Big[1 \pm 0.07\Big|_{S_0^\pi} \pm 0.11\Big|_{\Lambda_q} \pm 0.02\Big|_{\lambda_E^2/\lambda_H^2}\,{}^{+0.05}_{-0.06}\Big|_{M^2} \pm 0.05\Big|_{2\lambda_E^2+\lambda_H^2}\,{}^{+0.06}_{-0.10}\Big|_{\mu_h} \pm 0.04\Big|_{\mu}\,{}^{+1.36}_{-0.56}\Big|_{\lambda_B}\,{}^{+0.25}_{-0.43}\Big|_{\sigma_1,\sigma_2}\Big].$$

Without reliable B LCDA, it is impossible to discuss precision calculation!

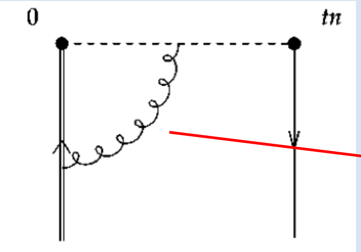
Challenges in first principle calculation

The definition of leading twist heavy meson LCDA:

$$i\tilde{f}_H(\mu)m_H\varphi^+(\omega,\mu) = \int_{-\infty}^{+\infty}\frac{dt}{2\pi}e^{i\omega n_+\cdot vt} \times \langle 0|\bar{q}\,(tn_+)\,\not{n}_+\gamma_5 W_c\,(tn_+,0)\,h_v(0)|H(v)\rangle$$



Challenge 1: **Cusp divergence**



$$O_v^{\rm ren}(t,\mu) = \frac{4}{\hat{\epsilon}}\ln(it\mu)O_v^{\rm bare}(t) + \cdots$$

Braun, Ivanov, Korchemsky, PRD69, 034014 (2004)

- No local limit
- Lattice with OPE, QCD sum rules, …, **FAILED**

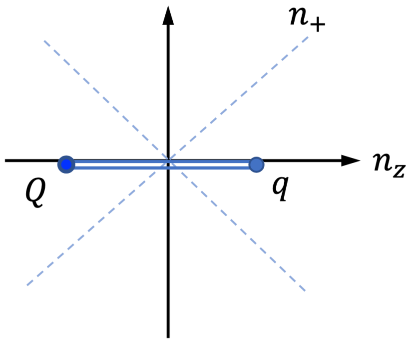
Challenge 2: **StN Problem**

- Simulating the boosted h_v on the lattice will encounter significant signal-to-noise problem.

Mandula, Ogilvie, PRD 45, 2183-2187 (1992), NPB 34, 480-482 (1994)

- Lattice with LaMET, **FAILED**

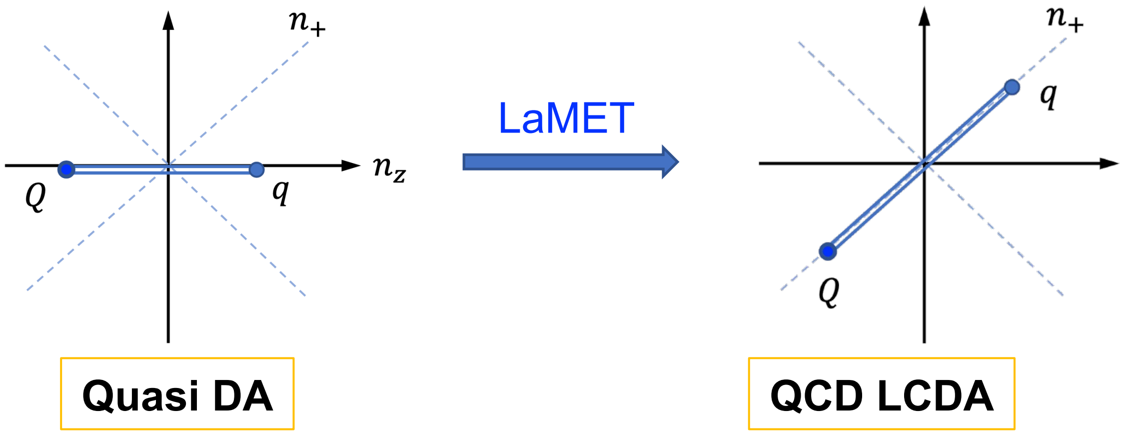
Sequential effective theory to obtain heavy meson LCDAs



Quasi DA

- Start from the equal-time correlators with QCD fields:
  - Equal-time correlations ensure that it is directly calculable from lattice QCD;
  - Without h_v, the issues of cusp divergence and StN problem are both resolved.

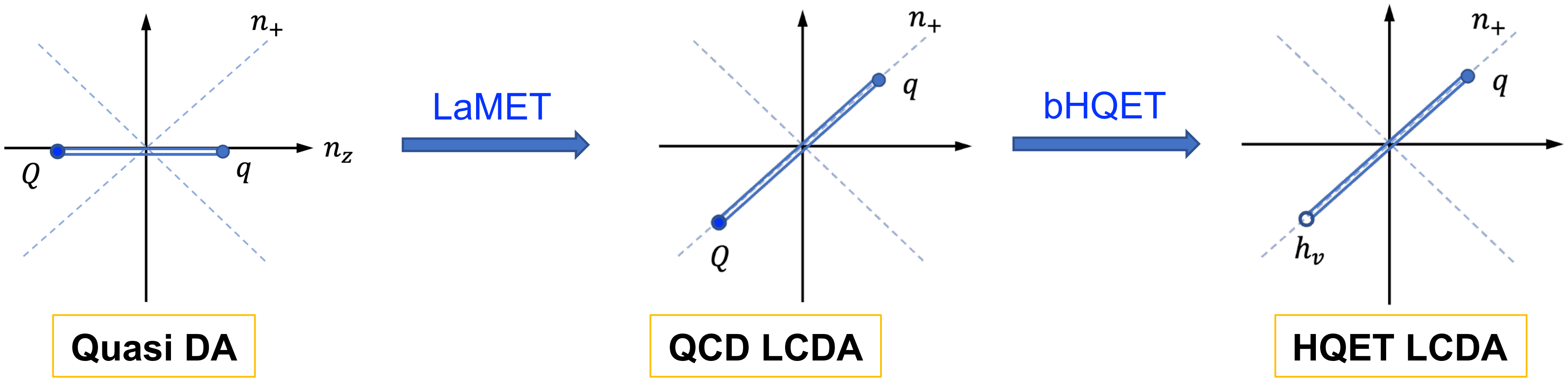
Sequential effective theory to obtain heavy meson LCDAs



- Large-momentum effective theory (LaMET) provides a connection between equal-time correlators and light-cone observables.

  - Integrating out large-momentum scale, one can obtain the heavy meson LCDA in QCD.

Sequential effective theory to obtain heavy meson LCDAs



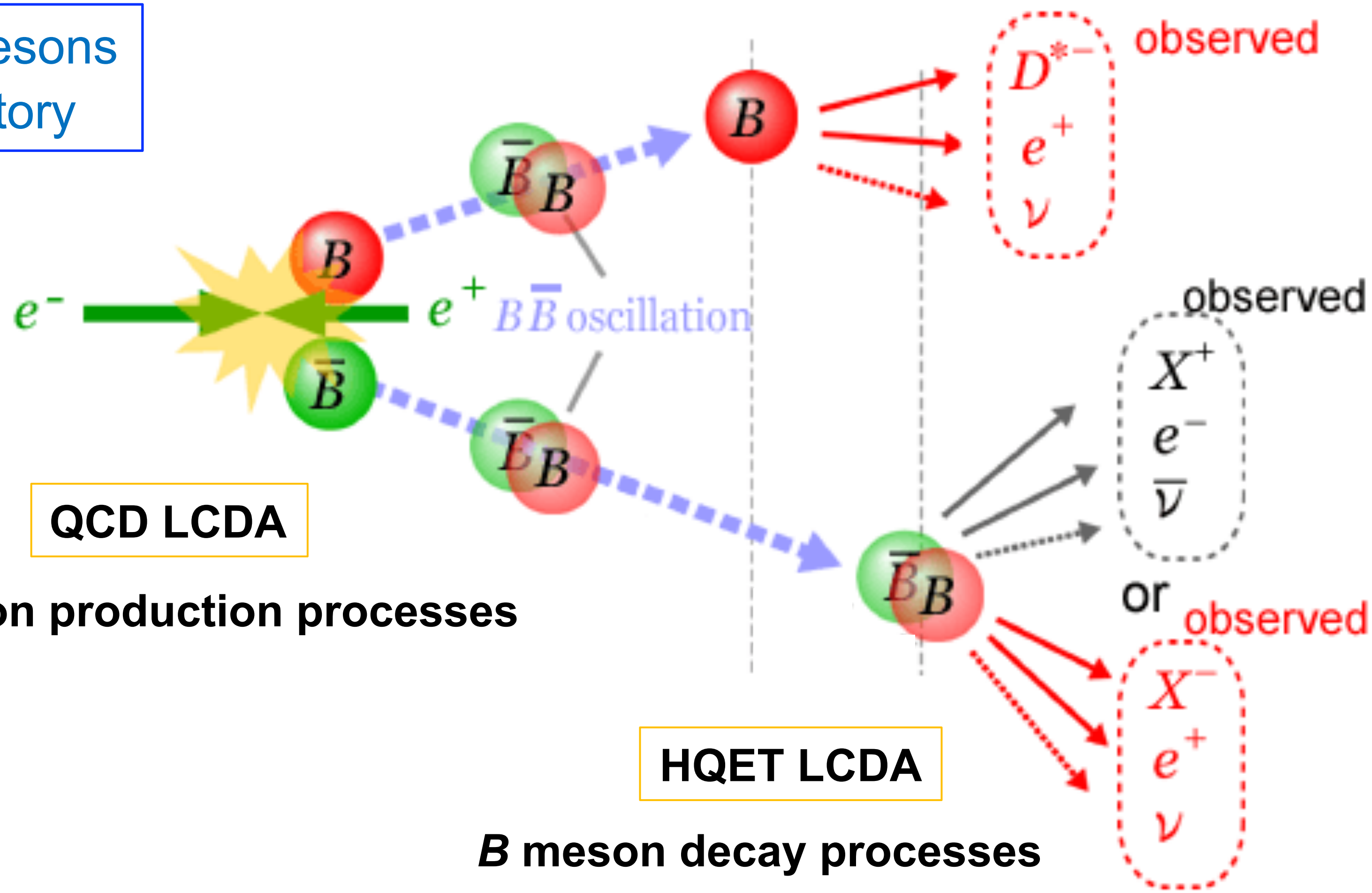
- In the Isgur-Wise limit, QCD LCDAs are related to the HQET LCDAs:
  - The boosted HQET provides the matching between them.

Ishaq, Jia, Xiong, Yang, PRL125(2020)132001; Beneke, Finauri, Vos, Wei, JHEP 09, 066 (2023)

QCD and HQET LCDAs

- Both QCD and HQET LCDAs are key nonperturbative parameters in the study of heavy meson:

The lifetime of B mesons at the KEK B-factory



QCD LCDA

***B* meson production processes**

HQET LCDA

***B* meson decay processes**

Lattice QCD implementation of sequential effective theory

- ✓ A numerical simulation on the finest CLQCD ensemble ($a = 0.05187$ fm);

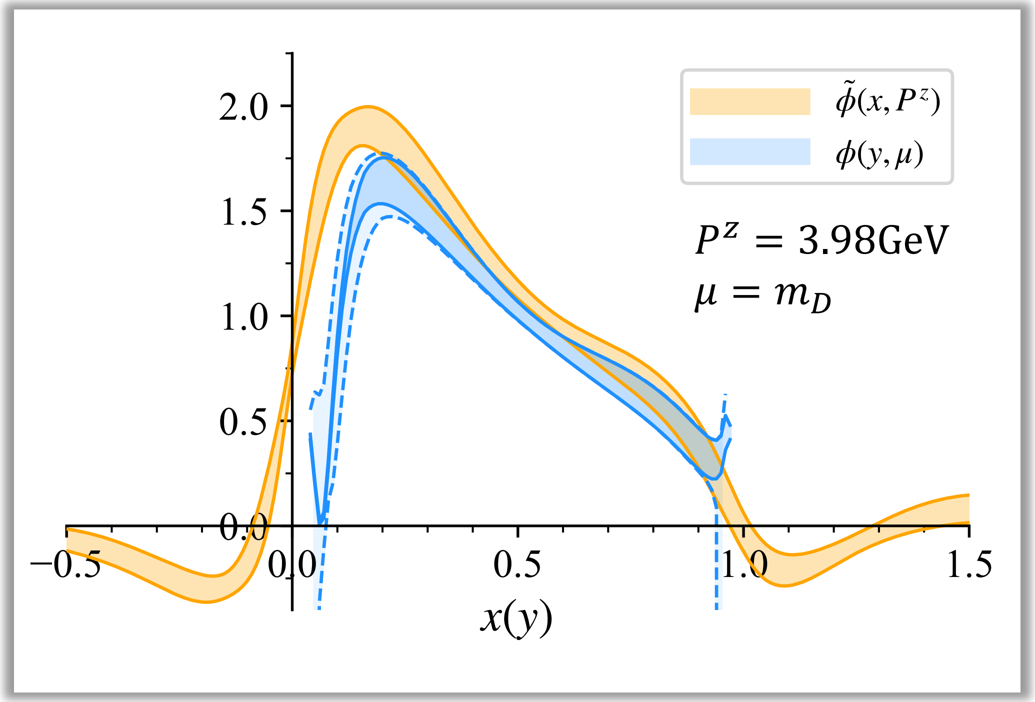
  CLQCD Collaboration, PRD 109, 054507 (2024)

- ✓ Simulate the D meson quasi DA with $m_D \simeq 1.92\mathrm{GeV}$, up to $P^z \simeq 3.98\mathrm{GeV}$;

- ✓ Then match to the QCD LCDA.

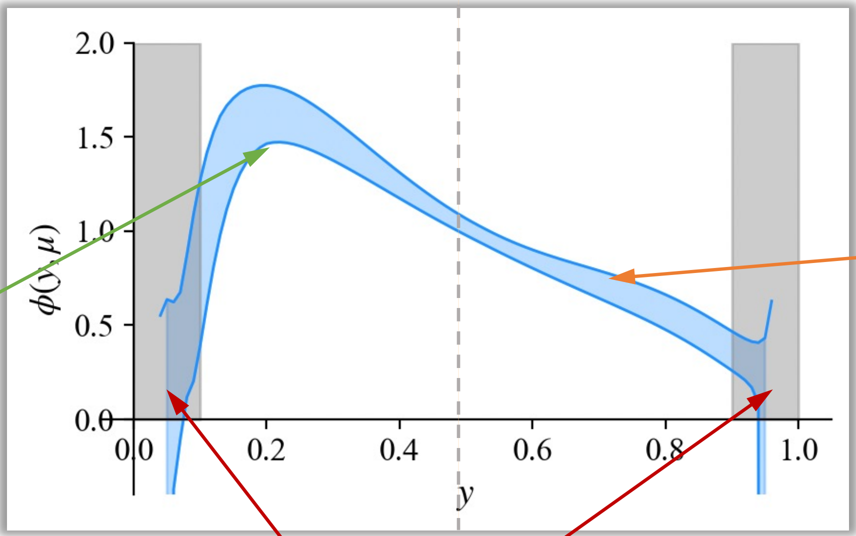
$$\tilde{\phi}(x, P^z) = \int_0^1 C\left(x, y, \frac{\mu}{P^z}\right) \phi(y, \mu) + \mathcal{O}\left(\frac{m_H^2}{(P^z)^2}, \frac{\Lambda_{\mathrm{QCD}}^2}{(xP^z, \bar{x}P^z)^2}\right)$$

Liu, Wang, Xu, QAZ, Zhao, PRD 99, 094036 (2019)
Han, Hua, Ji, Lu, Wang, Xu, QAZ, Zhao, 2410.18654



Lattice QCD implementation of sequential effective theory

QCD LCDA of D meson



Peak region: $y \sim \frac{\Lambda_{QCD}}{m_H}$

- Light quark carries small momentum fraction;
- Related to the HQET LCDA.

Ishaq, Jia, Xiong, Yang, PRL125(2020)132001
Beneke, Finauri, Vos, Wei, JHEP 09, 066 (2023)

End-point region:

- LaMET matching kernel suffer large power corrections.
- Lattice QCD predictions fail

Tail region: $y \sim 1$

- Contain only hard-collinear physics, perturbative calculable;
- Suppressed in LCDA.

Lattice QCD implementation of sequential effective theory

Peak region: $y \sim \frac{\Lambda_{\text{QCD}}}{m_H}$

- Light quark carries small momentum fraction;
- Related to the HQET LCDA.

➢ A multiplicative factorization from QCD LCDA to HQET LCDA in the peak region:

$$\varphi^+_{\text{peak}}(\omega, \mu) = \frac{f_H}{\widetilde{f}_H} \frac{1}{\mathcal{J}_{\text{peak}}} \phi(y, \mu; m_H) + \mathcal{O}\left(\frac{\Lambda_{\text{QCD}}}{m_H}\right)$$

Beneke, Finauri, Vos, Wei, JHEP 09, 066 (2023)

- Nonperturbative, determined from lattice QCD.

Lattice QCD implementation of sequential effective theory

Peak region: $y \sim \frac{\Lambda_{\rm QCD}}{m_H}$

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Beneke, Finauri, Vos, Wei, JHEP 09, 066 (2023)

- Nonperturbative, determined from lattice QCD.

➢ Tail region: perturbative result at 1-loop order

$$\varphi^+_{\rm tail}(\omega,\mu) = \frac{\alpha_s C_F}{\pi\omega}\left[\left(\frac{1}{2} - \ln\frac{\omega}{\mu}\right) + \frac{4\bar{\Lambda}}{3\omega}\left(2 - \ln\frac{\omega}{\mu}\right)\right]$$

Lee, Neubert, PRD 72, 094028 (2005)

Lattice QCD implementation of sequential effective theory

Peak region: $y \sim \frac{\Lambda_{\rm QCD}}{m_H}$

- Light quark carries small momentum fraction;
- Related to the HQET LCDA.

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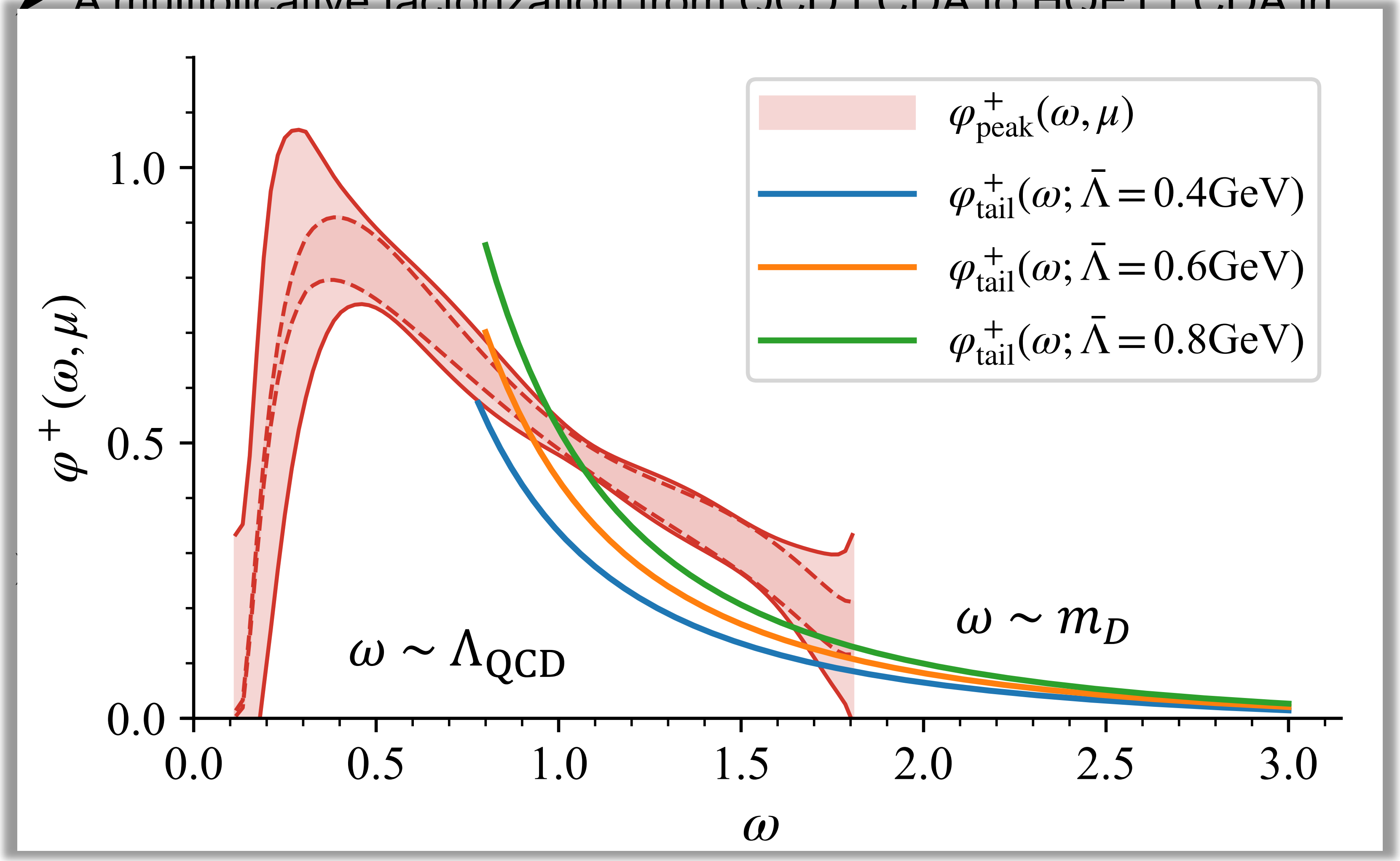
➢ A multiplicative factorization from QCD LCDA to HQET LCDA in



Lattice QCD implementation of sequential effective theory

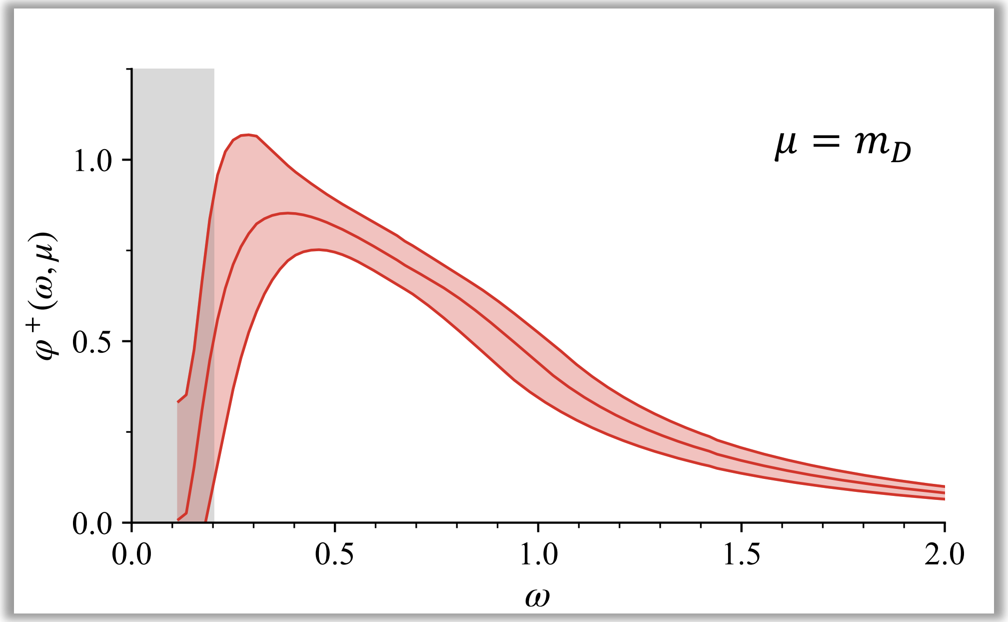
Finally, we obtain the final result of HQET LCDA.

- A verification of the two-step factorization method, the numerical result is still preliminary.
- Considered the systematic errors in lattice analysis:

  From extrapolation, scale uncertainty, ……
- Some key systematic errors are still absent:

  Only one lattice spacing,

  Power corrections within two matchings are still significant, ……

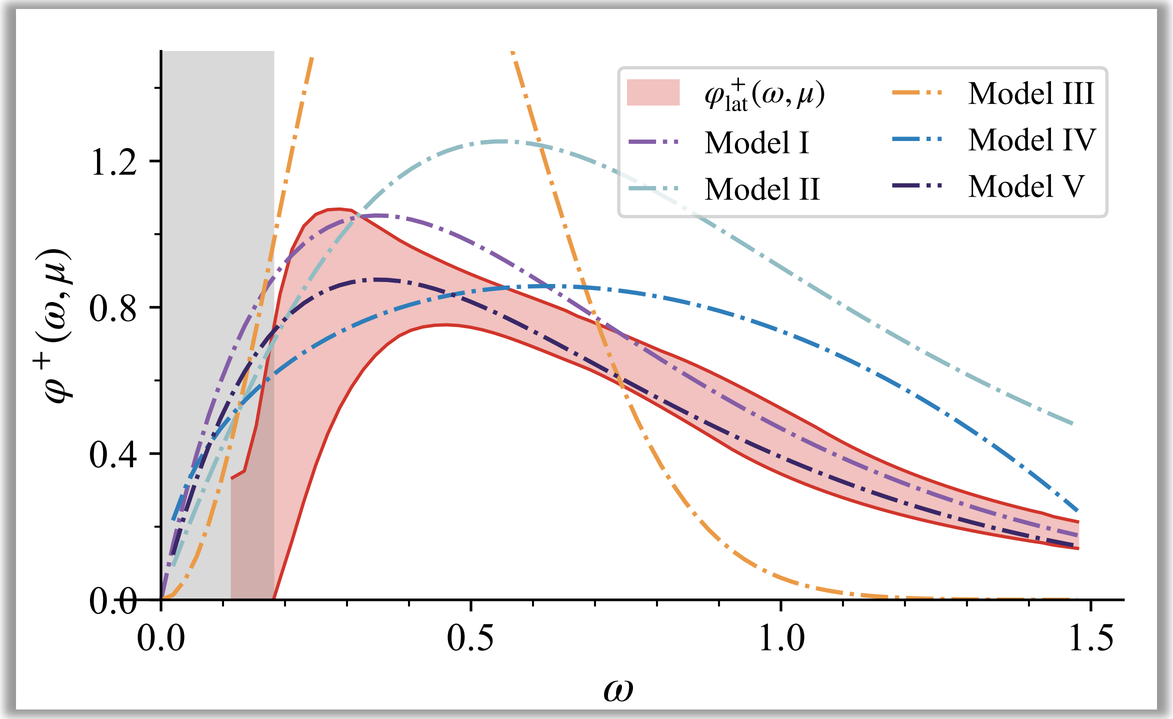


A more systematic study is underway…

Discussion I: Comparison with Models

- Our result is basically consistent with most of the model estimates, and will also provide a first-principle constrains on the existing models.

- For theoretical calculations, result from first-principles will help to REMOVE the primary uncertainties arising from the model parametrizations.



$$\mathcal{V}_{B\to K^*}(0) = 0.359^{+0.141}_{-0.085}\Big|_{\lambda_B}{}^{+0.019}_{-0.019}\Big|_{\sigma_1}{}^{+0.001}_{-0.062}\Big|_{\mu}{}^{+0.010}_{-0.004}\Big|_{M^2}{}^{+0.016}_{-0.017}\Big|_{s_0}{}^{+0.153}_{-0.079}\Big|_{\varphi_\pm(\omega)}$$

Discussions II: Inverse moment and Model-independent Parameterizations

Significant uncertainties from λ_B and σ_1:

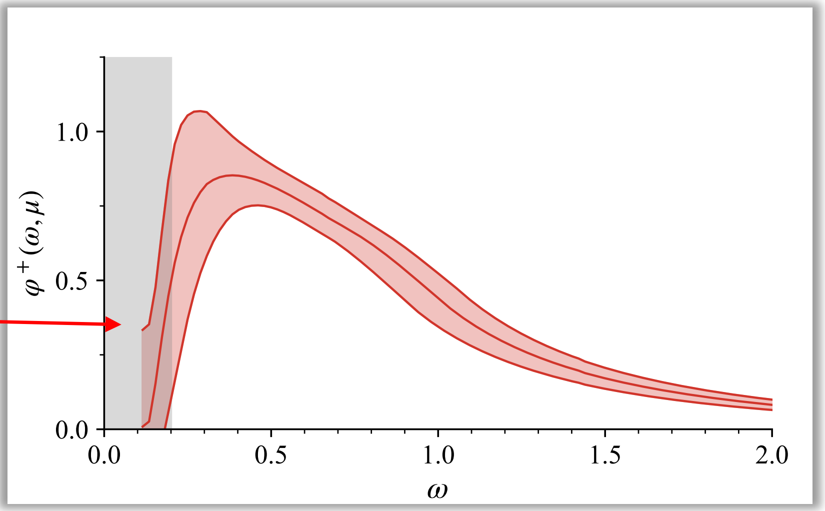
Gao, Lu, Shen, Wang, Wei, PRD 101 (2020) 074035

$$\mathcal{V}_{B\to K^*}(0) = 0.359^{+0.141}_{-0.085}\Big|_{\lambda_B}{}^{+0.019}_{-0.019}\Big|_{\sigma_1}{}^{+0.001}_{-0.062}\Big|_{\mu}{}^{+0.010}_{-0.004}\Big|_{M^2}{}^{+0.016}_{-0.017}\Big|_{s_0}{}^{+0.153}_{-0.079}\Big|_{\varphi_\pm(\omega)},$$

Definition of Inverse and inverse-logarithmic moments:

$$\lambda_B^{-1}(\mu) = \int_0^\infty \frac{d\omega}{\omega}\varphi^+(\omega,\mu),$$

$$\sigma_B^{(n)}(\mu) = \lambda_B(\mu)\int_0^\infty \frac{d\omega}{\omega}\ln\left(\frac{\mu}{\omega}\right)^{(n)}\varphi^+(\omega,\mu).$$



The power corrections at small ω makes the integral non-computable.

- Model independent parameterization forms to reconstruct the full distribution of HQET LCDA

Discussions II: Inverse moment and Model-independent Parameterizations

Strategy I: expansion in small-ω region

$$\varphi^{+}(\omega,\mu)=\sum_{n=1}^{N}c_n\frac{\omega^n}{\omega_0^{n+1}}e^{-\omega/\omega_0},$$

Strategy II: expansion in generalized Laguerre polynomials

Feldmann, Lughausen, Dyk, JHEP10 (2020) 162

$$\varphi^{+}(\omega,\mu)=\frac{\omega e^{-\omega/\omega_0}}{\omega_0^2}\sum_{k=0}^{K}\frac{a_k(\mu)}{1+k}L_k^{(1)}(2\omega/\omega_0),$$

➢ Results @ $\mu=m_D$

Order of Expansion		λ_B (GeV)	$\sigma_B^{(1)}$	$\sigma_B^{(2)}$	Pamameters
	$N=1$	0.424(41)	2.17(12)	6.36(51)	$\omega_0=0.388(46),\ c_1=0.916(81)$
Strategy I:	$N=2$	0.428(42)	2.16(10)	6.29(47)	$\omega_0=0.340(80),\ c_1=0.68(36),\ c_2=0.11(16)$
	$N=3$	0.418(67)	2.18(15)	6.33(80)	$\omega_0=0.32(15),\ c_1=0.63(44),\ c_2=0.08(23), c_3=0.02(11)$
	$K=0$	0.421(33)	2.17(8)	6.35(32)	$\omega_0=0.389(29),\ a_0=0.925(62)$
Strategy II:	$K=1$	0.424(35)	2.18(8)	6.36(32)	$\omega_0=0.446(95),\ a_0=1.05(19),\ a_1=0.15(26)$
	$K=2$	0.396(45)	2.12(12)	6.27(41)	$\omega_0=0.47(10),\ a_0=1.14(21),\ a_1=0.17(24),\ a_2=0.16(18)$

Discussions II: Inverse moment and Model-independent Parameterizations

Numerical results of λ_B and $\sigma_B^{(1)}$ at $\mu = 1\text{GeV}$:

		λ_B (GeV)	$\sigma_B^{(1)}$
Our results		0.376(63)	1.66(13)
Experiment	Belle 2018	> 0.24	
Other theoretical approach	Khodjamirian, Mandal, Mannel, 2020	0.383(153)	
	Gao, Lu, Shen, Wang, Wei, 2020	$0.343^{+0.064}_{-0.079}$	
	Lee, Neubert, 2005	0.48(11)	1.6(2)
	Braun, Ivanov, Korchemsky, 2004	0.46(11)	1.4(4)
	Grozin, Neubert, 1997	0.35(15)	
	Mandal, Nandi, Ray, 2024	0.338(68)	

Summary and Prospect

- ✓ We present a first lattice-implementable method to extract the heavy meson LCDA, and implement it on a CLQCD ensemble.
- ✓ Although the results are preliminary, they can be continually improved.
- ✓ The phenomenological implications demonstrate that our results will significantly advance the theoretical studies towards the frontier of high precision.

More importantly, improving the reliability of our results for the next stage:

- How to properly control the power corrections within two step factorization?
- More systematic lattice QCD calculations: more a, larger P^z, …

A more systematic lattice QCD calculation is on the road……

Summary and Prospect

➢ A more systematic lattice QCD calculation is on the road:

	Current Status	Ongoing work
Continuum limit	None. Single lattice spacing @ $a = 0.05187\text{fm}$	**Present.** Multi lattice spacing @ $a = \{0.07750, 0.05187, 0.0375\}\text{fm}$
Chiral limit	Nonphysical masses @ $m_\pi \simeq 300\text{MeV}$, $m_D \simeq 1.9\text{GeV}$	Nonphysical masses @ $m_\pi \simeq 300\text{MeV}$, $m_D \simeq 1.9\text{GeV}$
Infinite volume limit	----	----
Large momentum limit	None. Only $P^z \simeq 4\text{GeV}$	**Present.** Multi P^z to realize large momentum extrapolation.
Nonperturbative renormalization	Simplified hybrid renormalization scheme.	Self renormalization in the hybrid scheme.

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Thanks for your attention!

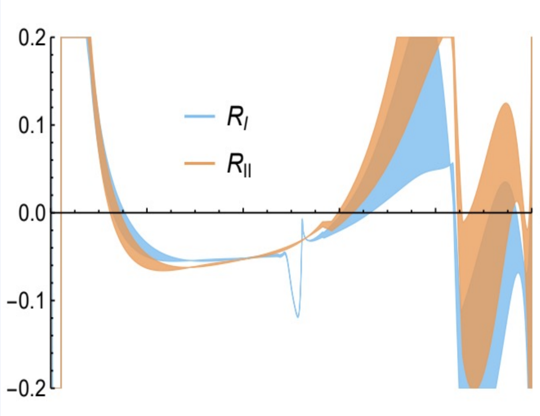
Backup Slides

Power corrections

$(P^z, m_H, \Lambda_{\text{QCD}})$ → **Quasi DA** —**LaMET**→ $(m_H, \Lambda_{\text{QCD}})$ **QCD LCDA** —**bHQET**→ (Λ_{QCD}) **HQET LCDA**

In LaMET: $\Lambda_{\text{QCD}}, m_H \ll P^z \;\Rightarrow\; \mathcal{O}\left(\frac{m_H^2}{(P^z)^2}, \frac{\Lambda_{\text{QCD}}^2}{(xP^z, \bar{x}P^z)^2}\right)$

Mass correction $m_H^2/(P^z)^2$:



- Smaller than 20% in most region;
- Smaller than 10% in the region we perform bHQET matching.

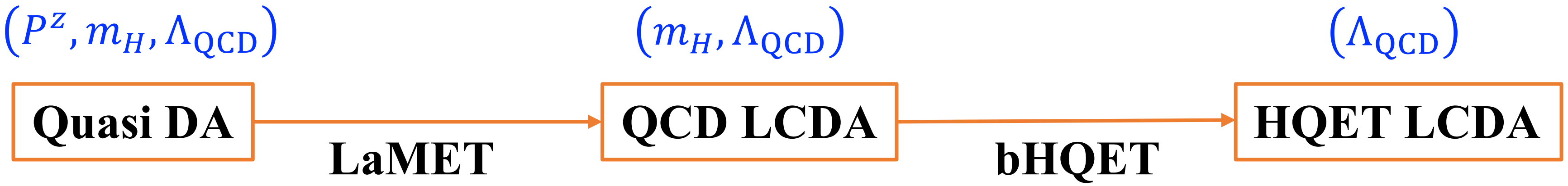
Han, Wang, Zhang, Zhang, *PRD* 110, 094038 (2024)

Power correction $\Lambda_{\text{QCD}}^2/(xP^z)^2$:

- Significant at end-point region ($x \to 0, 1$) of the QCD LCDA;
- Can be improved by considering the renormalon resummation, …

Su, Holligan, Ji, Yao, Zhang, Zhang, NPB 991, 116201 (2023)

Power corrections



In bHQET: $\Lambda_{\rm QCD} \ll m_H \Rightarrow \mathcal{O}\left(\frac{\Lambda_{\rm QCD}}{m_H}\right)$

- A possible solution proposed in [Deng, Wang, Wei, Zeng, 2409.00632]
- HQET LCDA shows degeneracy in the Dirac structures due to heavy quark spin symmetry;
- This power correction can be estimated from pseudoscalar and vector meson HQET LCDA.