郑州大学 ZHENGZHOU UNIVERSITY

第四届强子与重味物理理论与实验联合研讨会

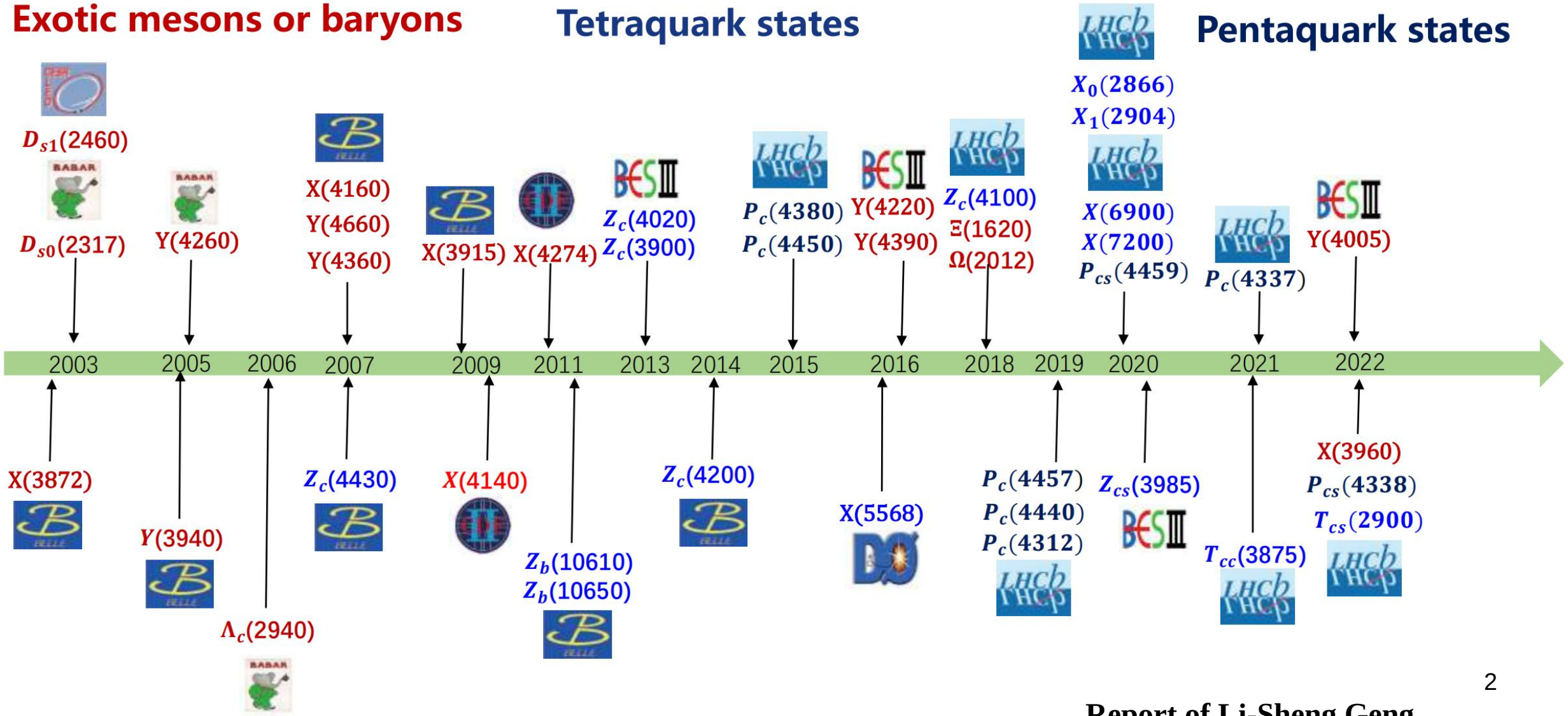
Theoretical study of the open-flavored tetraquark $T_{c\bar{s}}(2900)$ and its spin partner

吕文韬 郑州大学

主要合作者：段漫玉，王恩，陈殿勇，王冠颖，肖楮文，李德民，Eulogio Oset

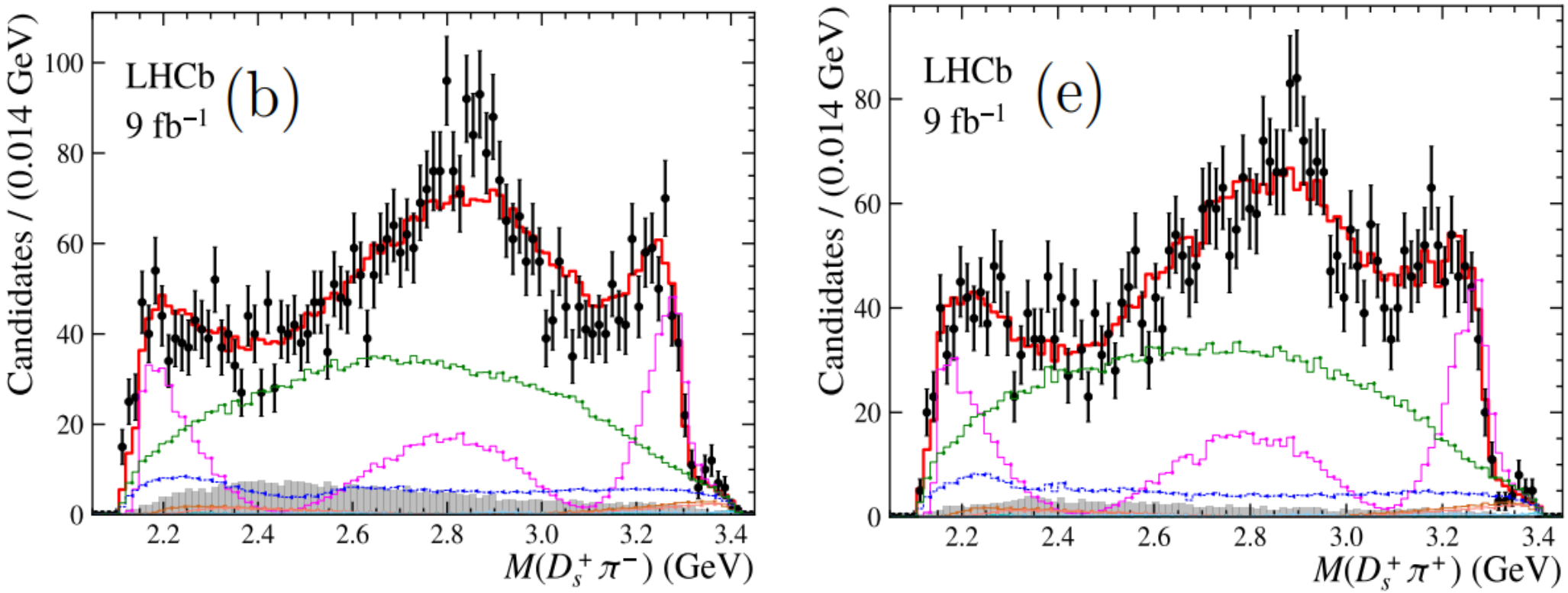
2025年3月24日

Exotic states



$T_{c\bar{s}0}(2900)$ at LHCb

- In 2022， LHCb Collaboration have reported two new open-flavored tetraquark states $T_{c\bar{s}0}(2900)^0[c\bar{s}\bar{u}d]$ and $T_{c\bar{s}0}(2900)^{++}[c\bar{s}u\bar{d}]$



[PhysRevLett. 131 (2023) 4, 041902]

$T_{c\bar{s}0}(2900)$ at LHCb

- $\mathbf{T_{c\bar{s}0}(2900)^0[c\bar{s}\bar{u}d]}$ **and** $\mathbf{T_{c\bar{s}0}(2900)^{++}[c\bar{s}u\bar{d}]}$

$$\mathbf{J^P = 0^+}$$

$$\mathbf{T_{c\bar{s}0}(2900)^0}: \quad M = 2.892 \pm 0.014 \pm 0.015 \text{ GeV},$$

$$\Gamma = 0.119 \pm 0.026 \pm 0.013 \text{ GeV},$$

$$\mathbf{T_{c\bar{s}0}(2900)^{++}}: \quad M = 2.921 \pm 0.017 \pm 0.020 \text{ GeV},$$

$$\Gamma = 0.137 \pm 0.032 \pm 0.017 \text{ GeV}.$$

[PhysRevLett. 131 (2023) 4, 041902]

Interpretation of $T_{c\bar{s}0}(2900)$

ZHENGZHOU UNIVERSITY 郑州大学

- **Compact tetraquark:**

  PRD107,096020(2023) : Constituent quark model

  PRD106,096023(2022) : Multiquark color flux-tube model

  EPJC84,1(2024) : QCD sum rule……

- **D^*K^* molecular state:**

  JPG50,055002 (2023) : QCD Sum rule

  PRD 107, 034018 (2023) : The effective Lagrangian theory

  PRD 107, 094019 (2023) : QCD two-point sum rule……

- **Virtual state or cusp :**

  PRD107,056015(2023) : The interaction of the $D^*K^* - D_s^*\rho$ channel

  EPJC82,955(2022) : Triangle singularity……

- **$T_{c\bar{s}0}(2900)^0$ main decay modes:**

  [PhysRevD.107.034018(2023)]

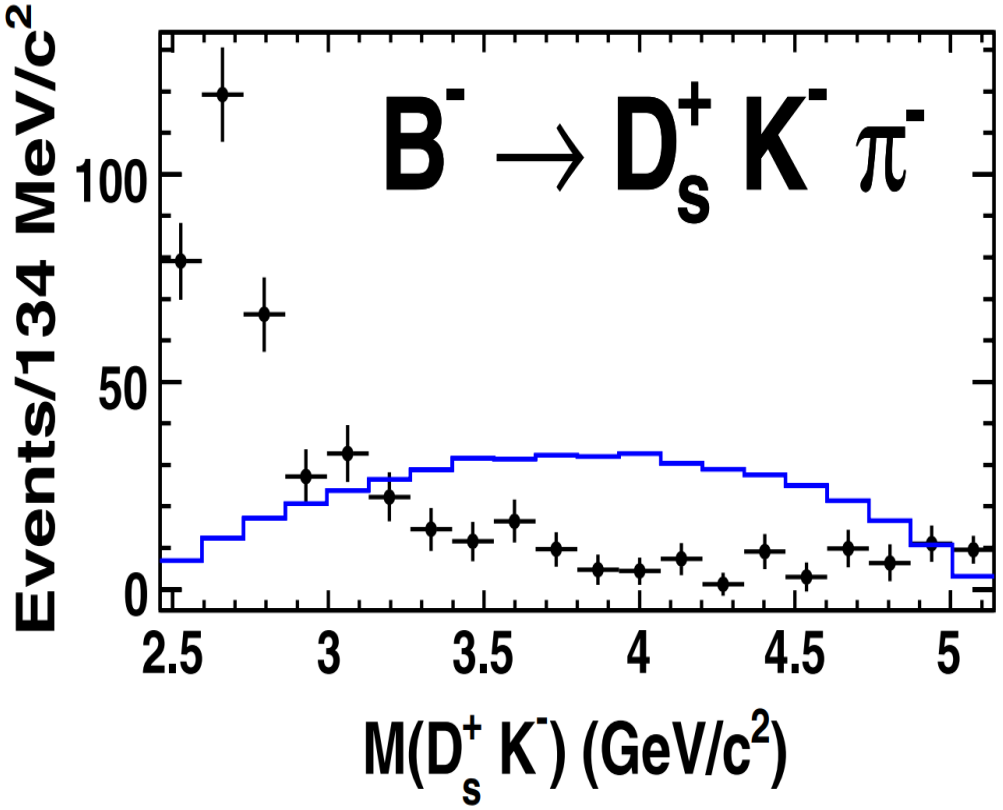
  - $T_{c\bar{s}0}(2900)^0 \to D_s^+\pi^-$
  - $T_{c\bar{s}0}(2900)^0 \to D^0K^0$
  - $T_{c\bar{s}0}(2900)^0 \to D_s^{*+}\rho^-$
  - $T_{c\bar{s}0}(2900)^0 \to D_{s1}^+\pi^-$
  - $T_{c\bar{s}0}(2900)^0 \to D^{*0}(K\pi)^0$

$B^- \to D_s^+ K^- \pi^-$ PRD 109 (2024) 1, 014008

郑州大学 ZHENGZHOU UNIVERSITY

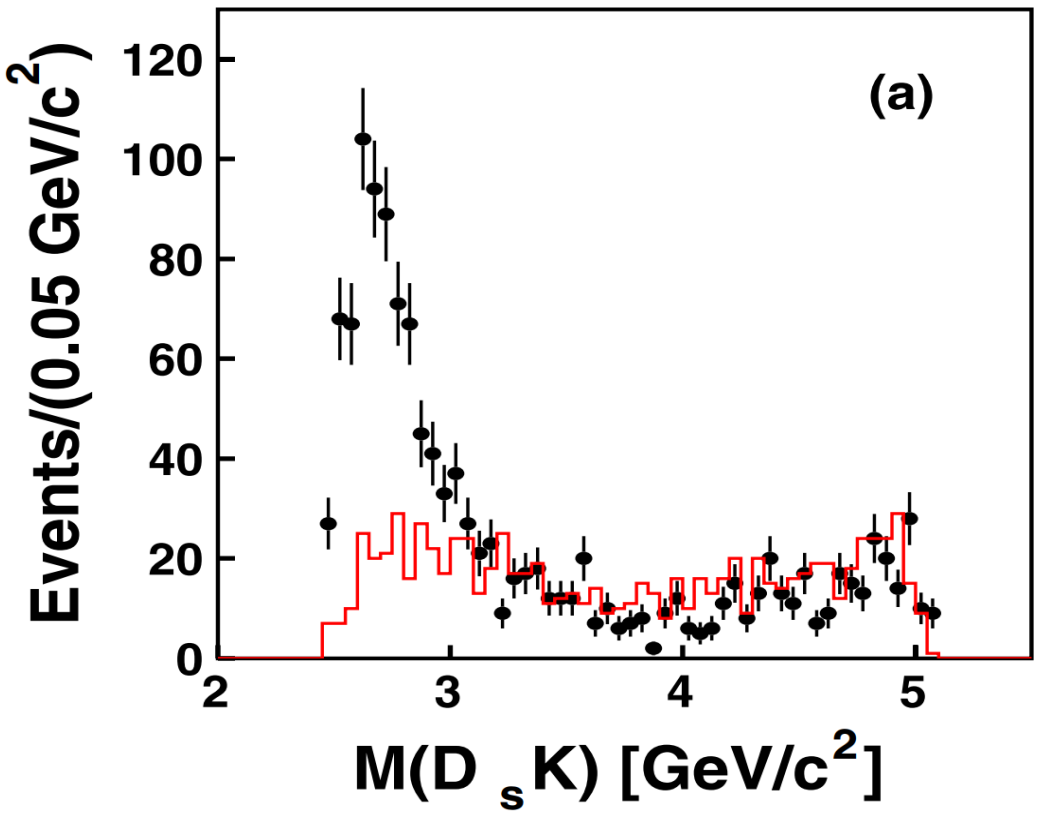
☐ *BABAR* and Belle measurements

[PhysRevLett 100 (2008) 171803]



BABAR data

[PhysRevD 80 (2009) 052005]



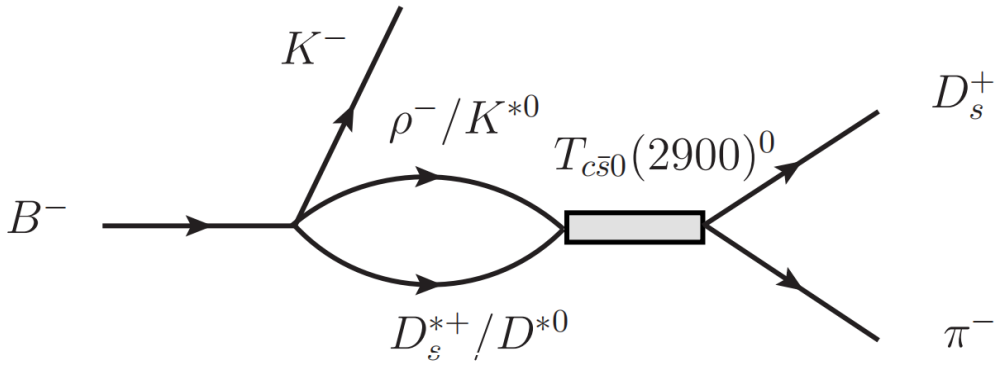
Belle data

$B^- \to D_s^+ K^- \pi^-$

PRD 109 (2024) 1, 014008

郑州大学 ZHENGZHOU UNIVERSITY

The mechanisms from the intermediate $T_{c\bar{s}0}(2900)$



The transition amplitudes

$$t_{\rho^- D_s^{*+} \to D_s^+ \pi^-} = \frac{g_{T_{c\bar{s}},\rho^- D_s^{*+}} g_{T_{c\bar{s}},D_s^+\pi^-}}{M^2_{D_s^+\pi^-} - m^2_{T_{c\bar{s}}} + i m_{T_{c\bar{s}}} \Gamma_{T_{c\bar{s}}}}$$

$$t_{D^{*0}K^{*0} \to D_s^+ \pi^-} = \frac{g_{T_{c\bar{s}},D^{*0}K^{*0}} g_{T_{c\bar{s}},D_s^+\pi^-}}{M^2_{D_s^+\pi^-} - m^2_{T_{c\bar{s}}} + i m_{T_{c\bar{s}}} \Gamma_{T_{c\bar{s}}}}$$

The decay amplitude

$$\mathcal{T}^{T_{c\bar{s}}} = Q\epsilon(V_1)\cdot\epsilon(V_2) \times (C+1) \times [G_{\rho^- D_s^{*+}} t_{\rho^- D_s^{*+} \to D_s^+ \pi^-} + G_{D^{*0}K^{*0}} t_{D^{*0}K^{*0} \to D_s^+ \pi^-}]$$

Loop function

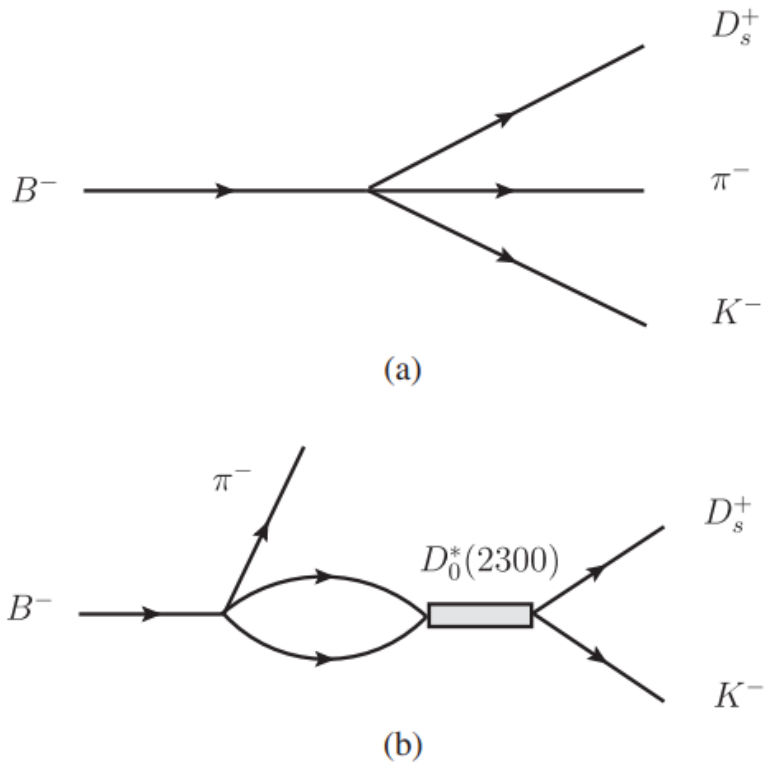
$$G = i\int \frac{d^4 q}{(2\pi)^4} \frac{1}{q^2 - m_1^2 + i\epsilon} \frac{1}{(P-q)^2 - m_2^2 + i\epsilon}$$

$B^- \to D_s^+ K^- \pi^-$

PRD 109 (2024) 1, 014008

郑州大学 ZHENGZHOU UNIVERSITY

- The mechanisms from the intermediate $D_0^*(2300)$



- Hadronization

$$H = \pi^-\left(\frac{1}{\sqrt{2}}D^0\pi^0 + \frac{1}{\sqrt{3}}D^0\eta + D^+\pi^- + D_s^+K^-\right)$$

- The decay amplitude

$$\mathcal{T}^{D_0^*(2300)} = Q'(C+1)\left[h_{D_s\bar{K}} + \sum_i h_i\, G_i t_{i\to D_s\bar{K}}\right]$$

$$= \mathcal{T}^{tree} + \mathcal{T}^{S}$$

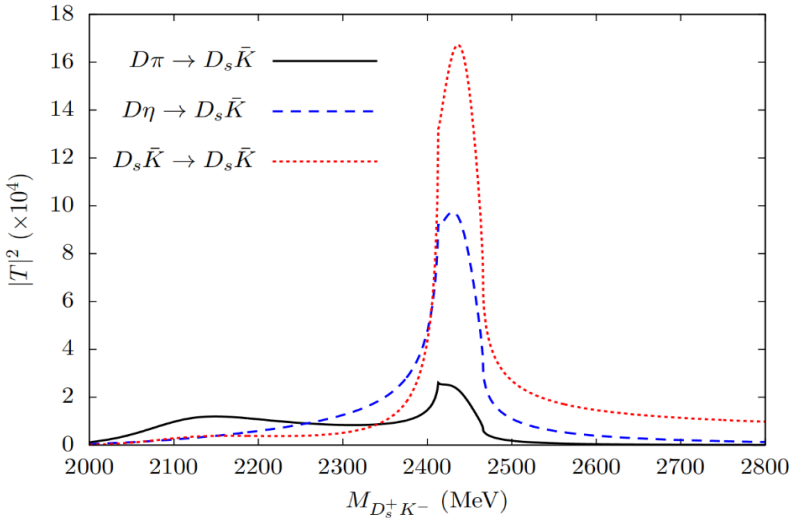
- B-S function

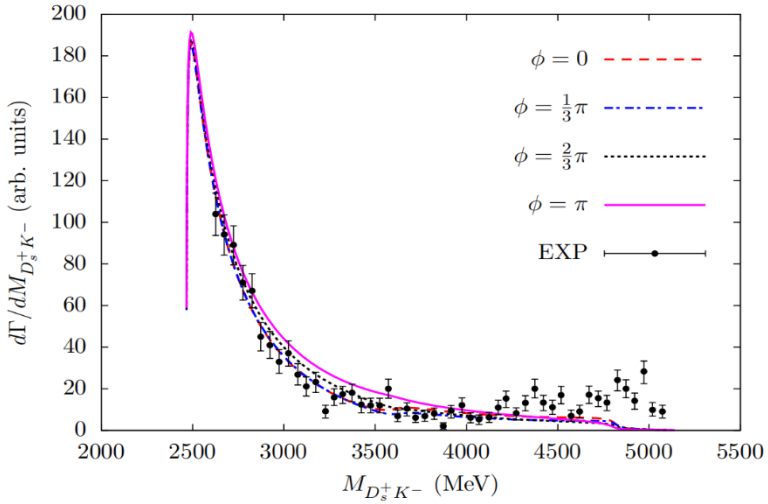
$$T = [1 - VG]^{-1}V$$

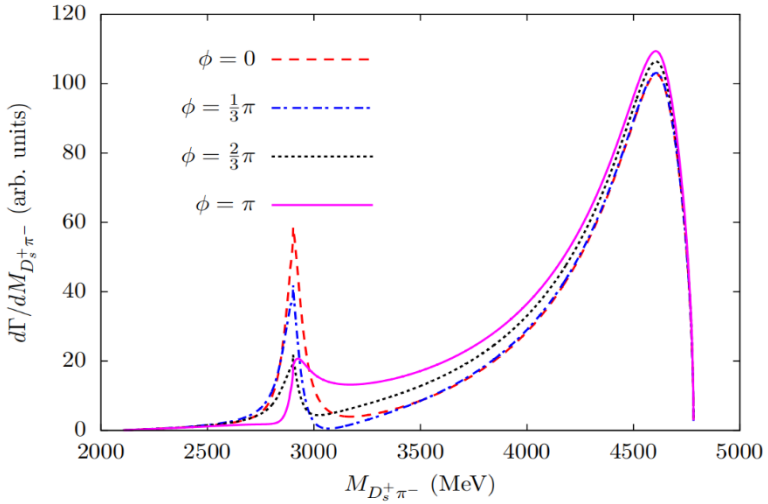
[PhysRevD 87 (2013) 1, 014508]
[PhysRevD 102 (2020) 9, 096020]

$B^- \to D_s^+ K^- \pi^-$

PRD 109 (2024) 1, 014008



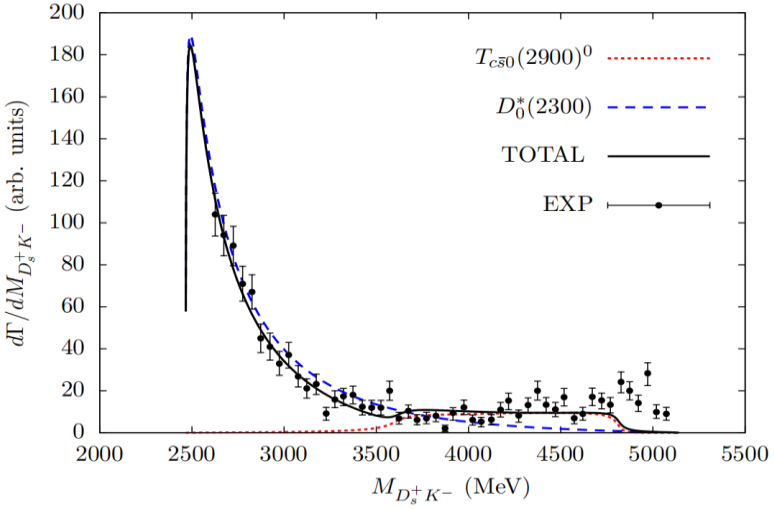


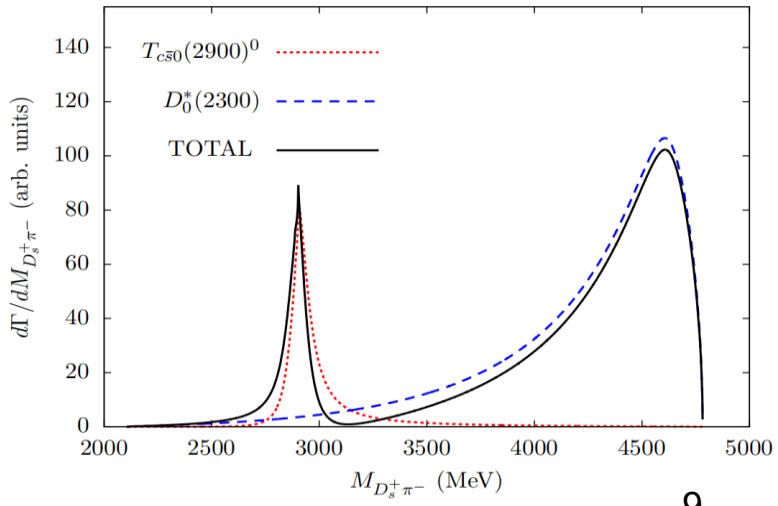


$$\left|\mathcal{T}^{Total}\right|^2 = \left|\mathcal{T}^{T_{c\bar{s}0}} e^{i\phi} + \mathcal{T}^{D_0^*(2300)}\right|^2$$

$$\phi = 0.35\pi$$

$$\Gamma_{T_{c\bar{s}} \to D_s\pi} = 10.45 \text{ MeV}$$



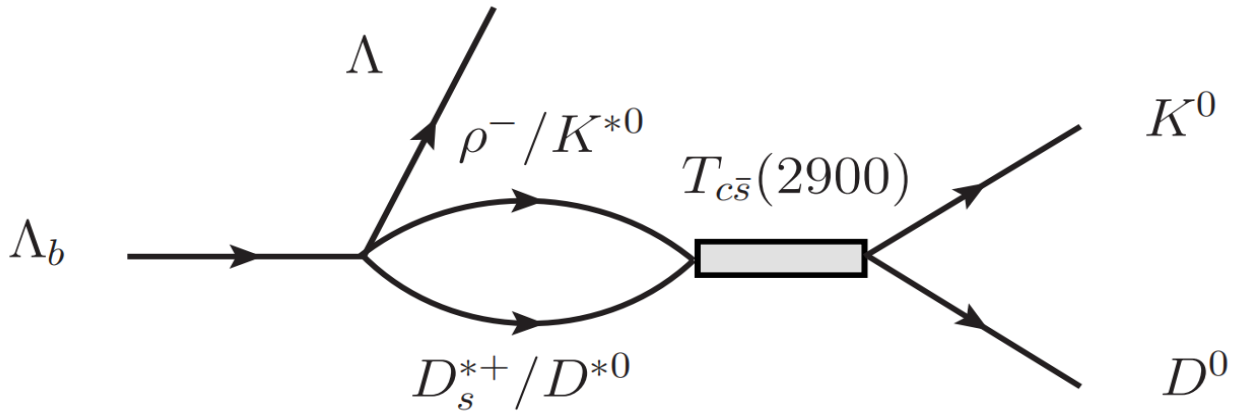


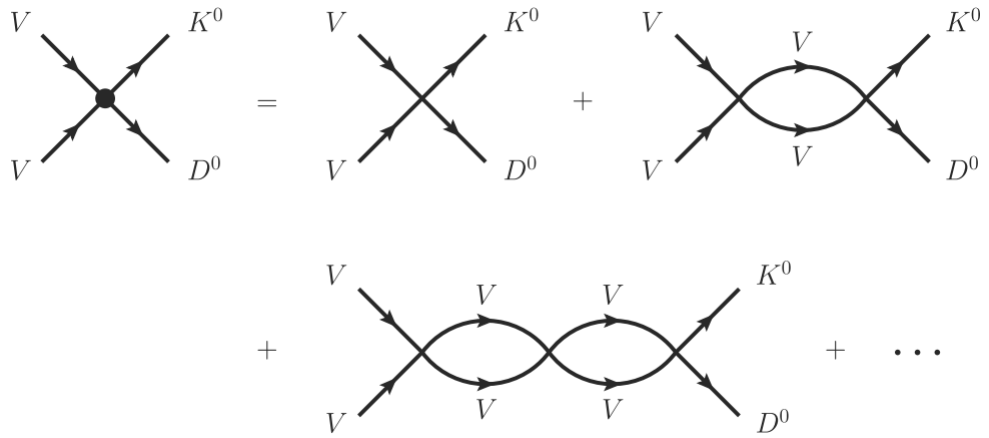
$\Lambda_b \to D^0 K^0 \Lambda$

EPJC 85 (2025) 2, 123

ZHENGZHOU UNIVERSITY 郑州大学

☐ The mechanisms from the intermediate $T_{c\bar{s}0}(2900)$





$$\mathcal{T}^{T_{c\bar{s}}} = -\sqrt{\frac{2}{3}} V_p \Big[C \times \tilde{G}_{D_s^{*+}\rho^-} \tilde{t}_{D_s^{*+}\rho^- \to D_s^{*+}\rho^-} V'_{D_s^{*+}\rho^- \to D^0 K^0}$$
$$+ C \times \tilde{G}_{D_s^{*+}\rho^-} \tilde{t}_{D_s^{*+}\rho^- \to D^{*0}K^{*0}} V'_{D^{*0}K^{*0} \to D^0 K^0}$$
$$+ \tilde{G}_{D^{*0}K^{*0}} \tilde{t}_{D^{*0}K^{*0} \to D_s^{*+}\rho^-} V'_{D_s^{*+}\rho^- \to D^0 K^0}$$
$$+ \tilde{G}_{D^{*0}K^{*0}} \tilde{t}_{D^{*0}K^{*0} \to D^{*0}K^{*0}} V'_{D^{*0}K^{*0} \to D^0 K^0} \Big],$$

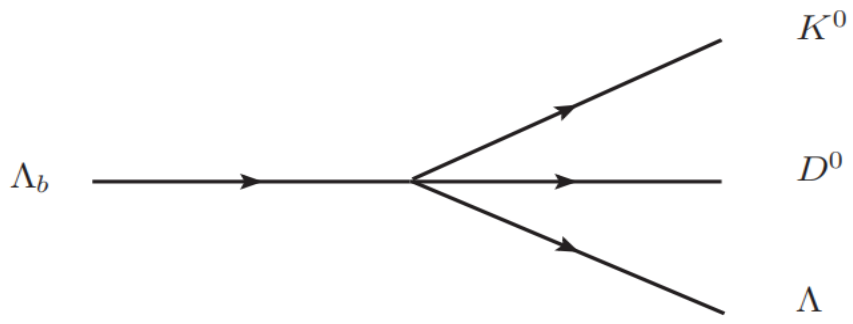
$$\tilde{G}_i V'_{i \to D^0 K^0} + \tilde{G}_i V_{i \to j} \tilde{G}_j V'_{j \to D^0 K^0} + \cdots$$
$$= \tilde{G}[1 - V\tilde{G}]^{-1} V',$$

$\Lambda_b \to D^0 K^0 \Lambda$

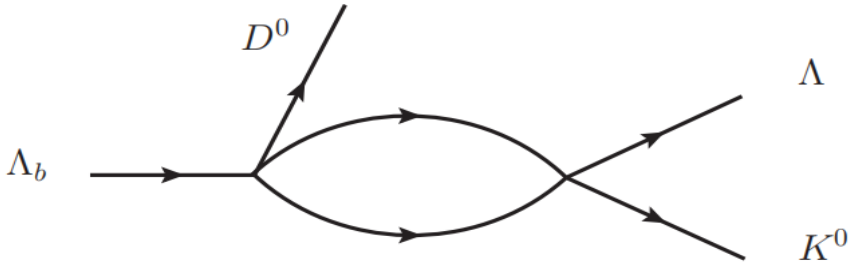
EPJC 85 (2025) 2, 123

郑州大学 ZHENGZHOU UNIVERSITY

☐ The mechanisms from the intermediate $N(1535)$

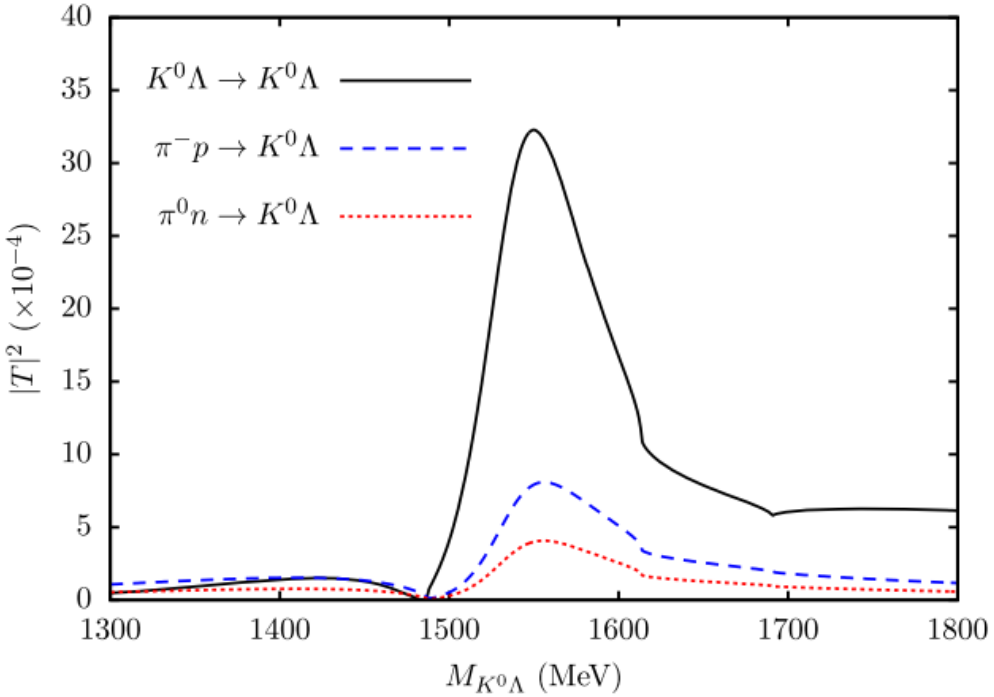


(a)



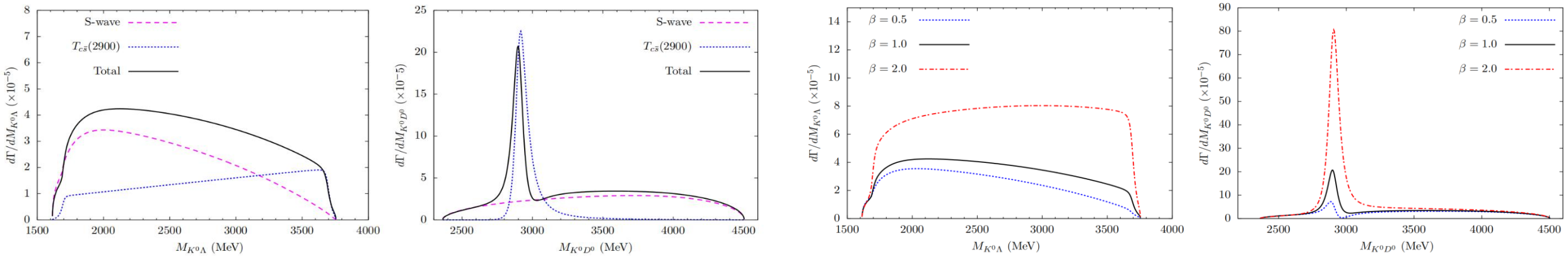
(b)

$$\mathcal{T}^{S-wave} = V'_p \left(h_{K^0\Lambda} + \sum_i h_i G'_i t_{i\to K^0\Lambda} \right)$$

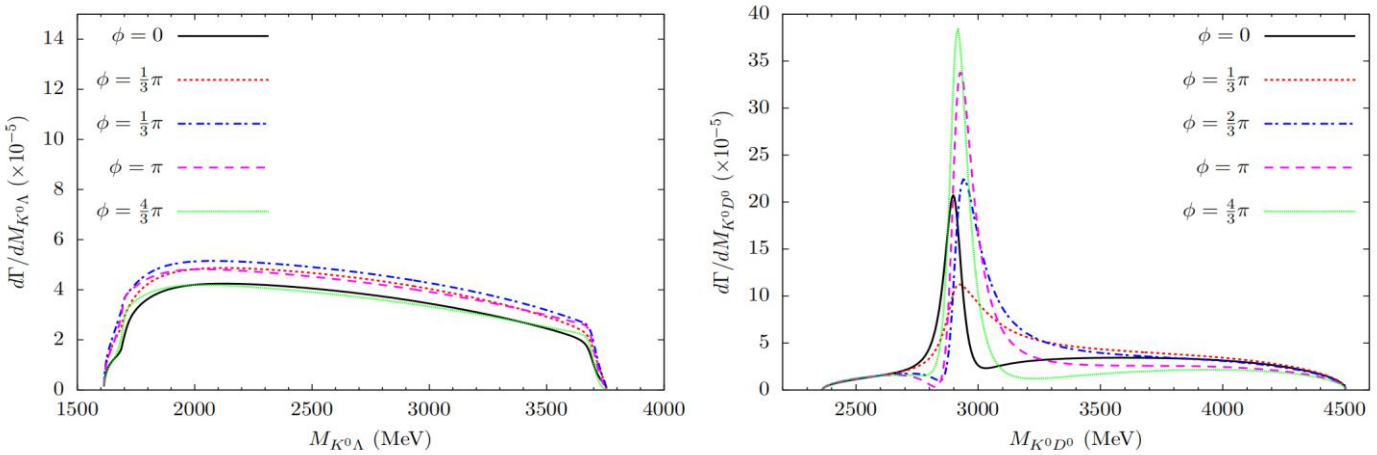


$\Lambda_b \to D^0 K^0 \Lambda$

EPJC 85 (2025) 2, 123



$$\left|\mathcal{T}^{Total}\right|^2 = \left|\mathcal{T}^{T_{c\bar{s}}0} e^{i\phi} + \mathcal{T}^{S-wave}\right|^2$$



Check uncertainties of the weight parameters

Considering different the phase interference

$\bar{B}_s^0 \to K^0 D^0 \pi^0$

EPJC 85 (2025) 1, 90

郑州大学 ZHENGZHOU UNIVERSITY

☐ The mechanisms from the intermediate K^*

$$\mathcal{T}^{K^*(892)} = \frac{V^{K^*(892)} |\mathbf{p}^*_{\pi^0}||\mathbf{p}^*_{D^0}|\cos\theta}{M^2_{K^0\pi^0} - M^2_{K^*(892)} + i M_{K^*(892)}\Gamma_{K^*(892)}},$$

$$\mathcal{T}^{K_0^*(1430)} = \frac{V^{K_0^*(1430)} M_{K_0^*(1430)}\Gamma_{K_0^*(1430)}}{M^2_{K^0\pi^0} - M^2_{K_0^*(1430)} + i M_{K_0^*(1430)}\Gamma_{K_0^*(1430)}},$$

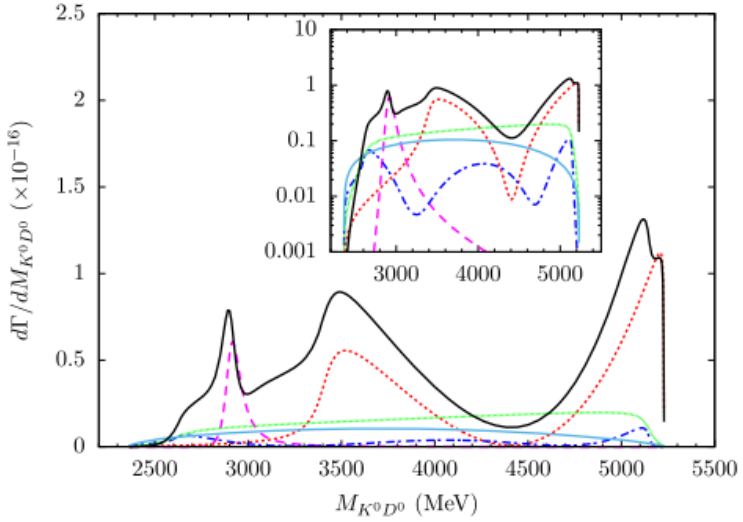
$$\mathcal{T}^{K_2^*(1430)} = \frac{V^{K_2^*(1430)} |\mathbf{p}^*_{\pi^0}|^2 (3\cos^2\theta - 1)}{M^2_{K^0\pi^0} - M^2_{K_2^*(1430)} + i M_{K_2^*(1430)}\Gamma_{K_2^*(1430)}},$$

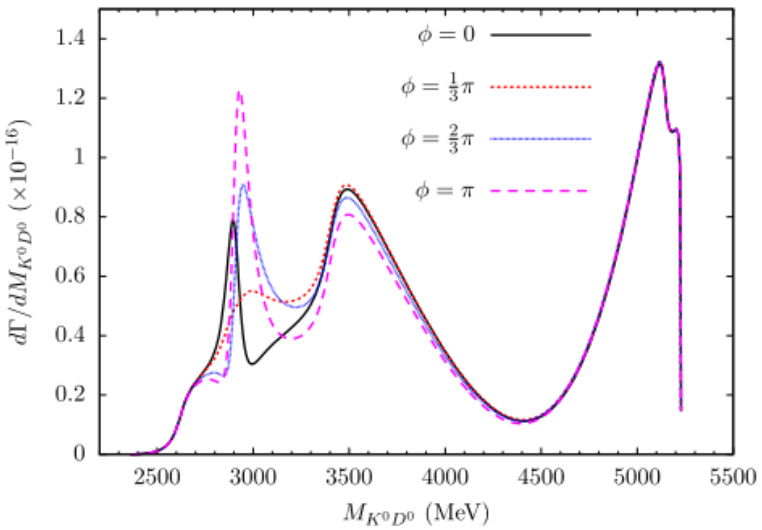
States	J^P	Mass	Width	$\mathcal{B}(K^* \to K\pi)$	$\mathcal{B}(B_s \to \bar{D}^0 \bar{K}^{0*})$
$K^*(892)$	1^-	895.55	47.3	100%	4.4×10^{-4}
$K^*(1410)$	1^-	1414	232	6.6%	3.9×10^{-4}
$K_0^*(1430)$	0^+	1425	270	93%	3.0×10^{-4}
$K_2^*(1430)$	2^+	1427.3	100	49.9%	1.1×10^{-4}

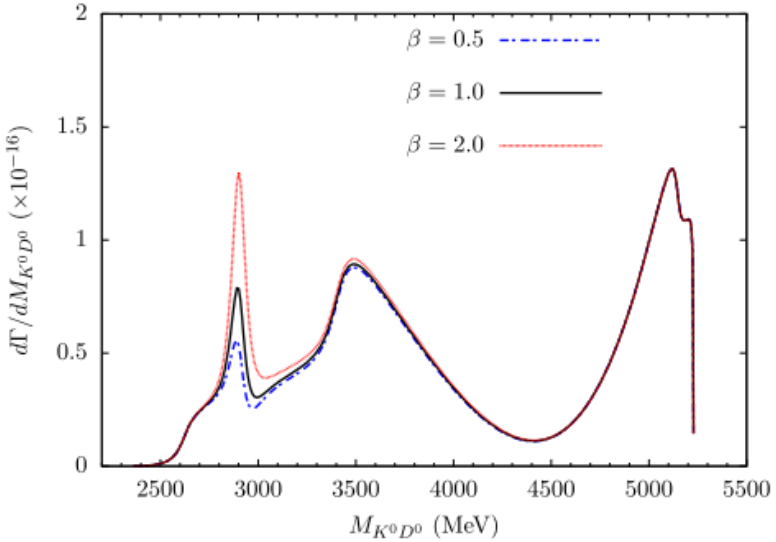
$$|\mathcal{T}^{\text{Total}}|^2 = |\mathcal{T}^{BG} + \mathcal{T}^{T_{c\bar{s}}} + \mathcal{T}^{K_0^*(1430)} + \mathcal{T}^{K^*(892)} + \mathcal{T}^{K_2^*(1430)}|^2.$$

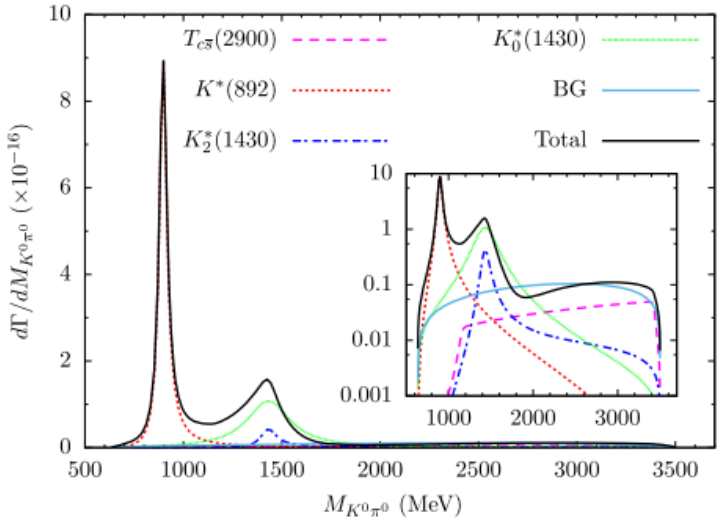
$\bar{B}_s^0 \to K^0 D^0 \pi^0$

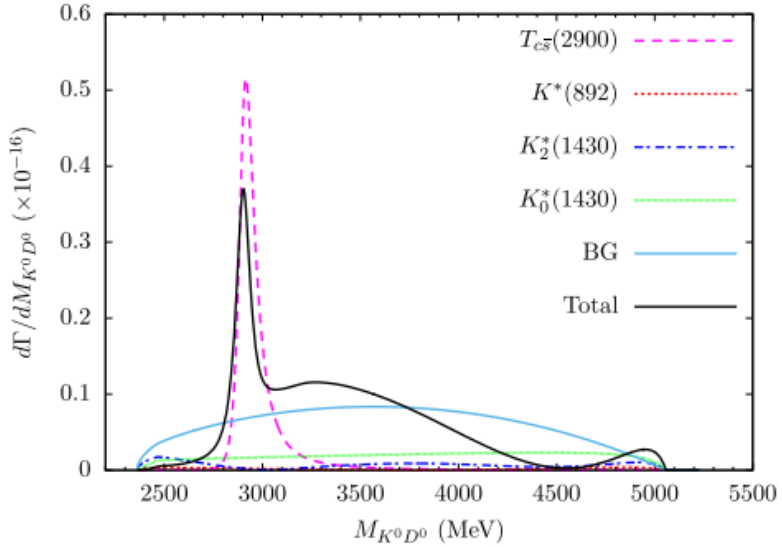
EPJC 85 (2025) 1, 90

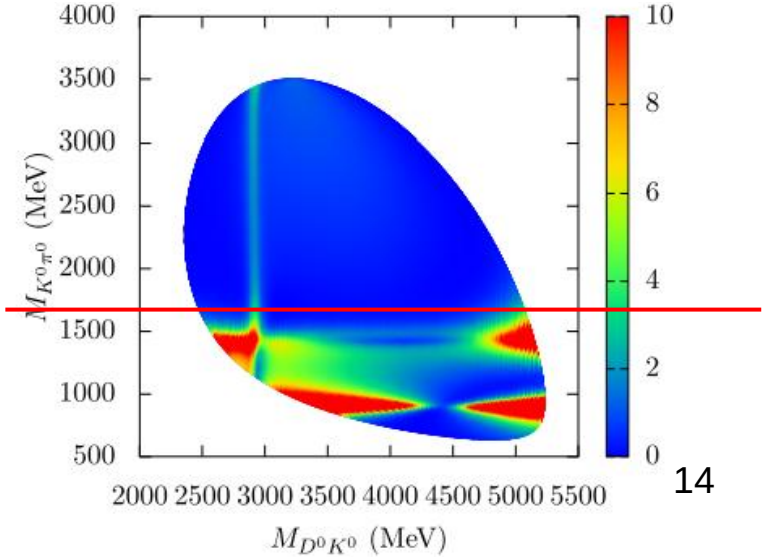






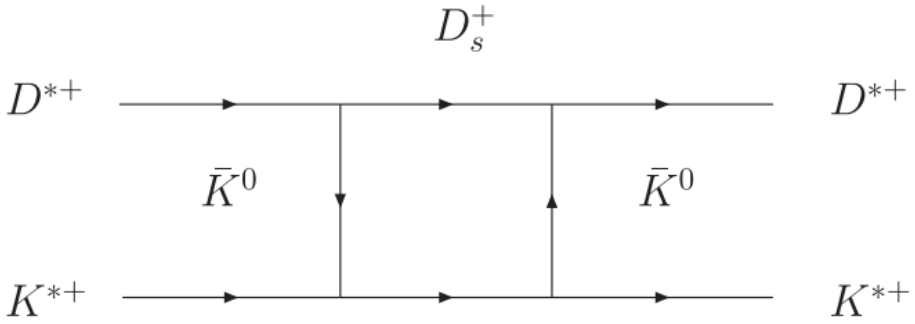






Spin partner $T_{c\bar{s}2}$

Molina-Oset, PRD107(2023)056015



$I[J^P]$	$\sqrt{s_0}$	Γ_0	Experiment
$1[0^+]$	2920 (Cusp)	130	$m = 2908 \pm 11 \pm 20$ $\Gamma = 136 \pm 23 \pm 11$
$1[1^+]$	2923 (Cusp)	145	...
$1[2^+]$	2834	19	...

Duan-Guo-Chen-Wang, PRD108(2023)074006

TABLE II. The pole positions and effective couplings evaluated for $I = 1$, $J = 0$ on different RSs with $\mu = 1500$ MeV. The threshold of $D_s^*\rho$ is 2887 MeV.

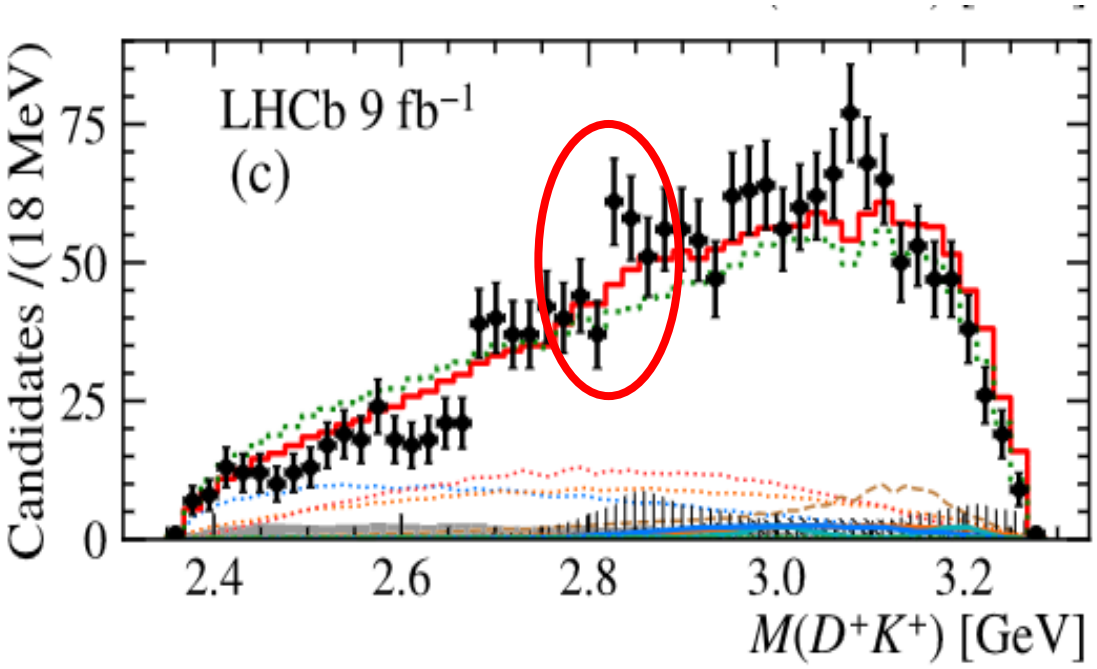
RS	α	$\sqrt{s}_{\rm pole}$ [MeV]	$\lvert g_{D^*K^*}\rvert$ [MeV]	$\lvert g_{D_s^*\rho}\rvert$ [MeV]
$\{1,1\}$	$-1.65 \sim -1.60$	$2885 \sim 2887$	$5531 \sim 2198$	$5379 \sim 2082$
$\{2,1\}$	$-1.60 \sim -1.55$	$2887 \sim 2885$	$1755 \sim 8202$	$1650 \sim 7348$
$\{1,2\}$	$-1.39 \sim -1.35$	$2885 \sim 2887$	$6587 \sim 1625$	$7886 \sim 1865$
$\{2,2\}$	$-1.35 \sim -1.28$	$2887 \sim 2885$	$1415 \sim 4202$	$1613 \sim 4672$

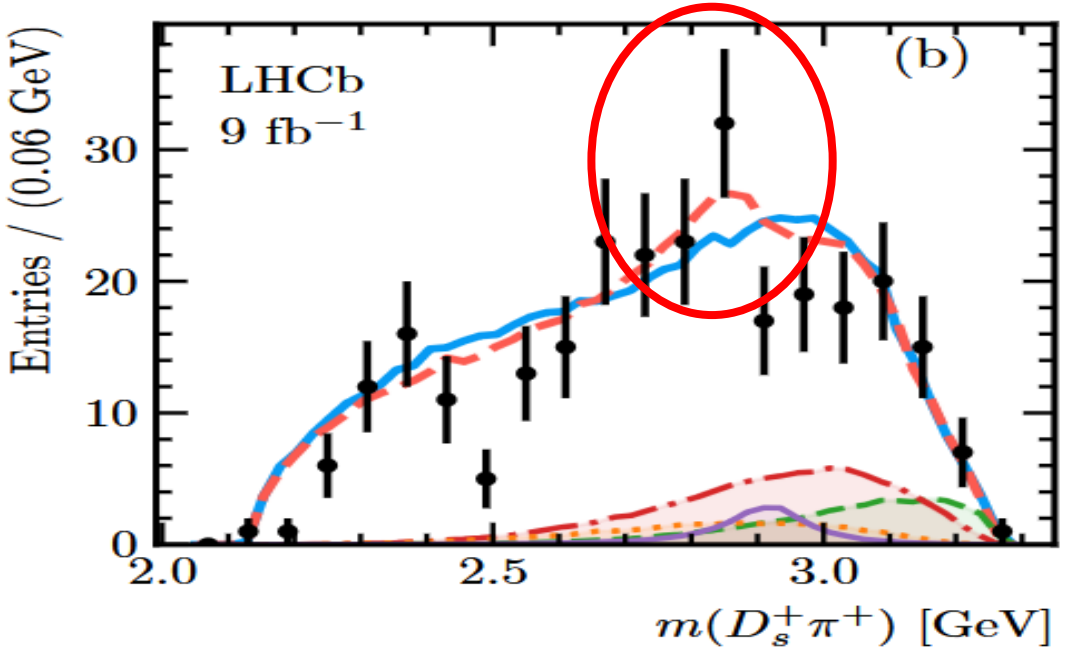
TABLE III. The pole positions evaluated in the sectors of $I = 1, J = 1$ and $I = 1, J = 2$ on the different RSs with $-1.65 < \alpha < -1.55$ and $-1.39 < \alpha < -1.28$. The "—" indicates that no pole is found. In the present work, we only consider the energy region safe from the left-hand cut, i.e., $\sqrt{s} > 2780$ MeV.

	$I = 1, J = 1$		$I = 1, J = 2$	
RS	α	$\sqrt{s}_{\rm pole}$ [MeV]	α	$\sqrt{s}_{\rm pole}$ [MeV]
$\{1,1\}$	$-1.65 \sim -1.61$	$2886 \sim 2887$	$-1.31 \sim -1.28$	$2780 \sim 2806$
$\{2,1\}$	$-1.61 \sim -1.55$	$2887 \sim 2883$	...	...
$\{1,2\}$	$-1.39 \sim -1.36$	$2886 \sim 2887$	...	...
$\{2,2\}$	$-1.36 \sim -1.28$	$2887 \sim 2885$	...	...

Spin partner $T_{c\bar{s}2}$ with mass around 2.8GeV was predicted!

Some hints of $T_{c\bar{s}2}$





$$B^+ \to D^{*-}D^+K^+$$

[PhysRevLett 133 (2024) 131902]

$$B^+ \to D^{*-}D_s^+\pi^+$$

JHEP 08 (2024) 165

$B^+ \to D^{*-} D^+ K^+$

EPJC84 (2024) 12, 1302

郑州大学 ZHENGZHOU UNIVERSITY

□Formalism

$$\frac{d\Gamma}{dM_{\rm inv}(D^+K^+)d\tilde{\Omega}} = \frac{1}{(2\pi)^4}\frac{1}{8M_B^2} p_{D^{*-}}\,\tilde{k}\sum|t|^2,$$

$$t = \epsilon_\mu(D^{*-})\,P_B^\mu\,(aY_{00} + bY_{20} + cY_{10}),$$

$$\sum|t|^2 = \left(\frac{M_{B^+}}{M_{D^{*-}}}\right)^2 \mathbf{p}^2_{D^{*-}}\Big(|a|^2Y_{00}^2 + |b|^2Y_{20}^2 + |c|^2Y_{10}^2 + 2Re(ab^*)Y_{00}Y_{20} + 2Re(ac^*)Y_{00}Y_{10} + 2Re(bc^*)Y_{20}Y_{10}\Big).$$

$$\int d\Omega Y^*_{l_3m_3}Y_{l_2m_2}Y_{l_1m_1} = \left[\frac{(2l_1+1)(2l_2+1)}{4\pi(2l_3+1)}\right]^{\frac{1}{2}} \mathcal{C}(l_1l_2l_3;m_1m_2m_3) \times \mathcal{C}(l_1l_2l_3;000),$$

$$\frac{d\Gamma_l}{dM_{\rm inv}} = \int d\tilde{\Omega}\frac{d\Gamma}{dM_{\rm inv}d\tilde{\Omega}}Y_{l0},$$

$$\frac{d\,\Gamma_0}{dM_{\rm inv}} = FAC\left[|a|^2 + |b|^2 + |c|^2\right],$$

$$\frac{d\,\Gamma_1}{dM_{\rm inv}} = FAC\left[2\,Re(ac^*) + \frac{2}{\sqrt{5}}\,2\,Re(bc^*)\right],$$

$$\frac{d\,\Gamma_2}{dM_{\rm inv}} = FAC\left[\frac{2}{7}\sqrt{5}\,|b|^2 + \frac{2}{5}\sqrt{5}\,|c|^2 + 2\,Re(ab^*)\right],$$

$$\frac{d\,\Gamma_3}{dM_{\rm inv}} = FAC\sqrt{\frac{15}{7}}\,\frac{3}{5}\,2\,Re(bc^*),$$

$$\frac{d\,\Gamma_4}{dM_{\rm inv}} = FAC\,\frac{6}{7}|b|^2,$$

$$FAC = \frac{1}{\sqrt{4\pi}}\frac{1}{(2\pi)^4}\frac{1}{8M_{B^+}^2}\,\mathbf{p}^2_{D^{*-}}\,p_{D^{*-}}\,\tilde{k}\left(\frac{M_{B^+}}{M_{D^{*-}}}\right)^2$$

$B^+ \to D^{*-} D^+ K^+$

EPJC84 (2024) 12, 1302

郑州大学 ZHENGZHOU UNIVERSITY

$$t = \epsilon_\mu(D^{*-})\, P_B^\mu\, (aY_{00} + bY_{20} + cY_{10}), \qquad a_i \to \tilde{a}_i \frac{\vec{k}}{M_B}$$

$$aY_{00} = \left(a_0 + a_0' \frac{M_B^2}{M_{\rm inv}^2(D^+K^+) - M_R^2 + i M_R \Gamma_R} \right) Y_{00},$$

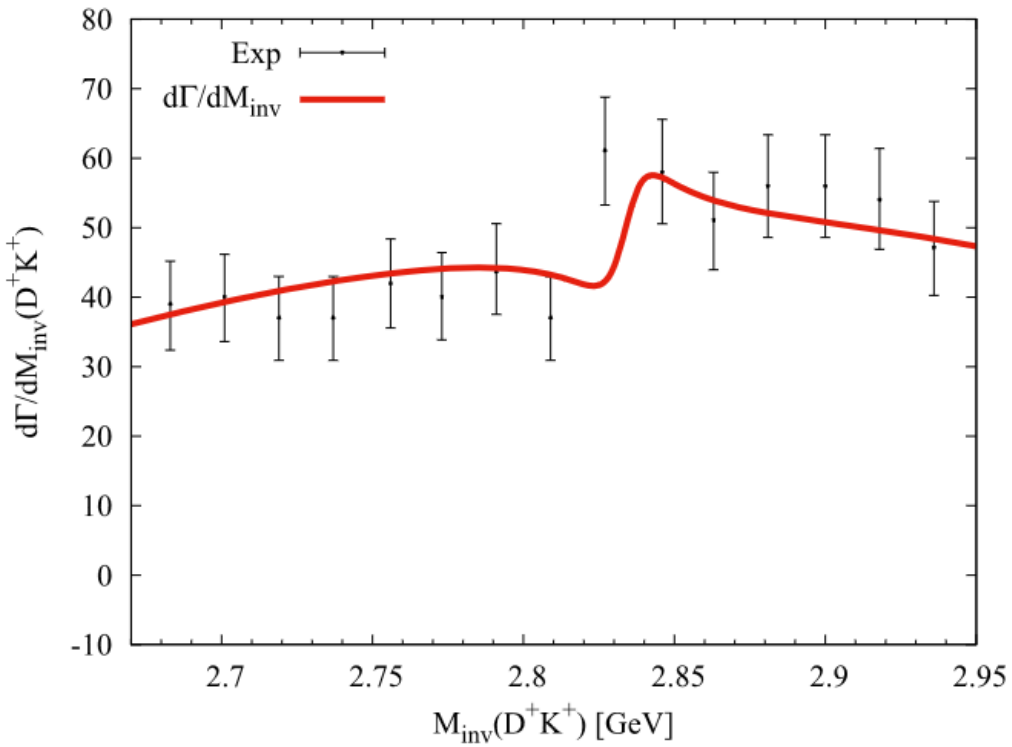
$$\frac{d\,\Gamma_0}{dM_{\rm inv}} = FAC\left[|a|^2 + |b|^2 + |c|^2\right],$$

$$\frac{d\,\Gamma_1}{dM_{\rm inv}} = FAC\left[2\,Re(ac^*) + \frac{2}{\sqrt{5}}\,2\,Re(bc^*)\right],$$

$$\frac{d\,\Gamma_2}{dM_{\rm inv}} = FAC\left[\frac{2}{7}\sqrt{5}\,|b|^2 + \frac{2}{5}\sqrt{5}\,|c|^2 + 2\,Re(ab^*)\right],$$

$$\frac{d\,\Gamma_3}{dM_{\rm inv}} = FAC\sqrt{\frac{15}{7}}\,\frac{3}{5}\,2\,Re(bc^*),$$

$$\frac{d\,\Gamma_4}{dM_{\rm inv}} = FAC\,\frac{6}{7}|b|^2,$$

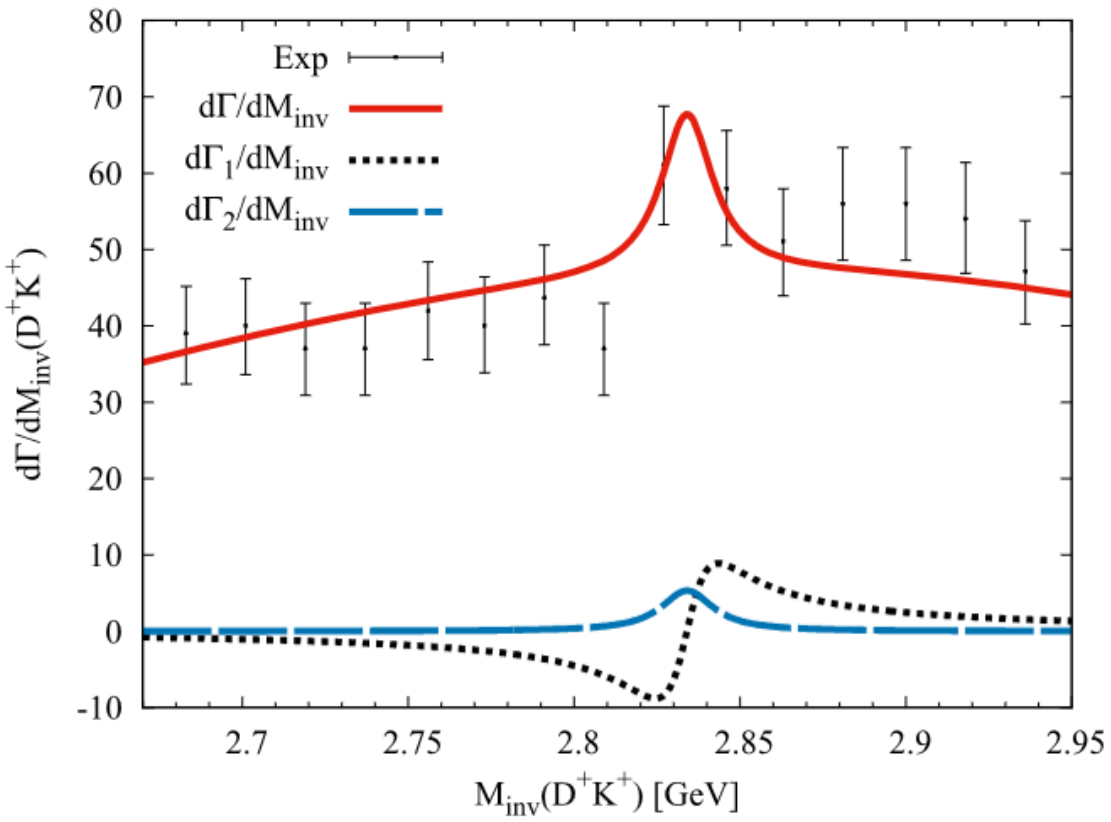


$B^+ \to D^{*-} D^+ K^+$

EPJC84 (2024) 12, 1302

$$aY_{00} + cY_{10}$$
$$= a_1 Y_{00} + c' \frac{M_B \tilde{k}}{M_{\rm inv}^2(D^+K^+) - M_R^2 + i M_R \Gamma_R} Y_{10}$$

$$\frac{d\,\Gamma_0}{dM_{\rm inv}} = FAC\left[|a|^2 + |b|^2 + |c|^2\right],$$
$$\frac{d\,\Gamma_1}{dM_{\rm inv}} = FAC\left[2\,Re(ac^*) + \frac{2}{\sqrt{5}}\,2\,Re(bc^*)\right],$$
$$\frac{d\,\Gamma_2}{dM_{\rm inv}} = FAC\left[\frac{2}{7}\sqrt{5}\,|b|^2 + \frac{2}{5}\sqrt{5}\,|c|^2 + 2\,Re(ab^*)\right],$$
$$\frac{d\,\Gamma_3}{dM_{\rm inv}} = FAC\sqrt{\frac{15}{7}}\,\frac{3}{5}\,2\,Re(bc^*),$$
$$\frac{d\,\Gamma_4}{dM_{\rm inv}} = FAC\,\frac{6}{7}|b|^2,$$

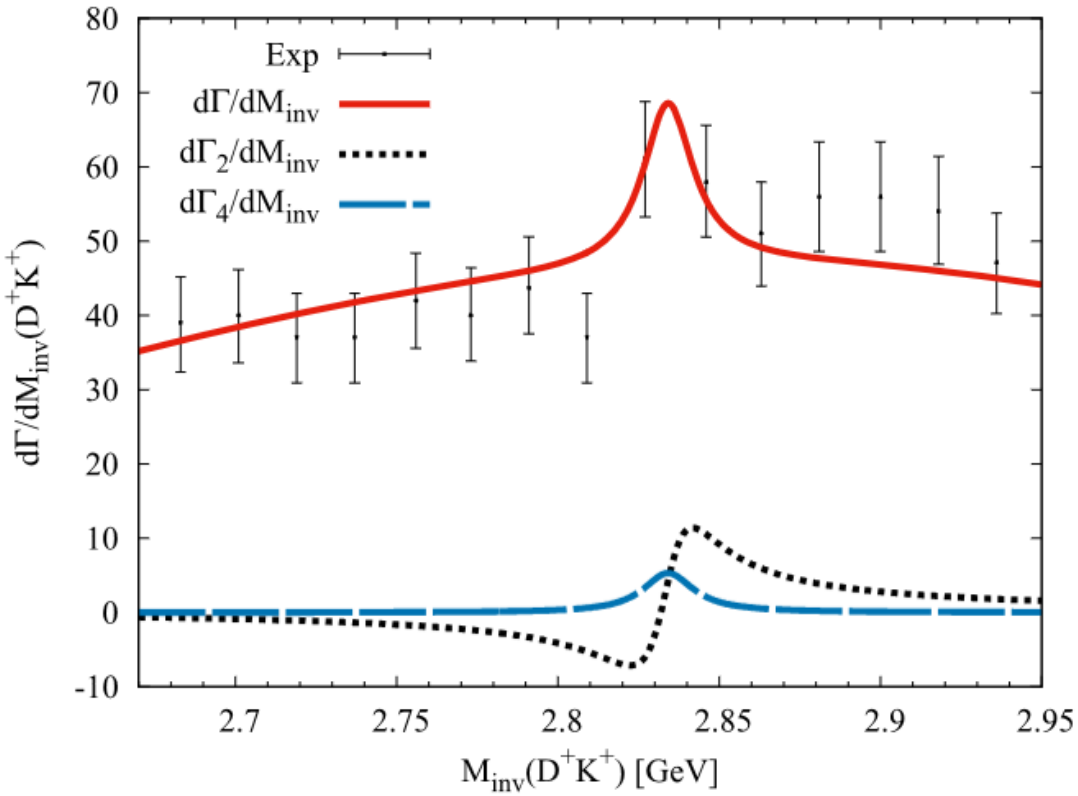


$B^+ \to D^{*-} D^+ K^+$

EPJC84 (2024) 12, 1302

$$aY_{00} + bY_{20}$$
$$= a_2 Y_{00} + b' \frac{\tilde{k}^2}{M_{\rm inv}^2(D^+K^+) - M_R^2 + iM_R\Gamma_R} Y_{20},$$

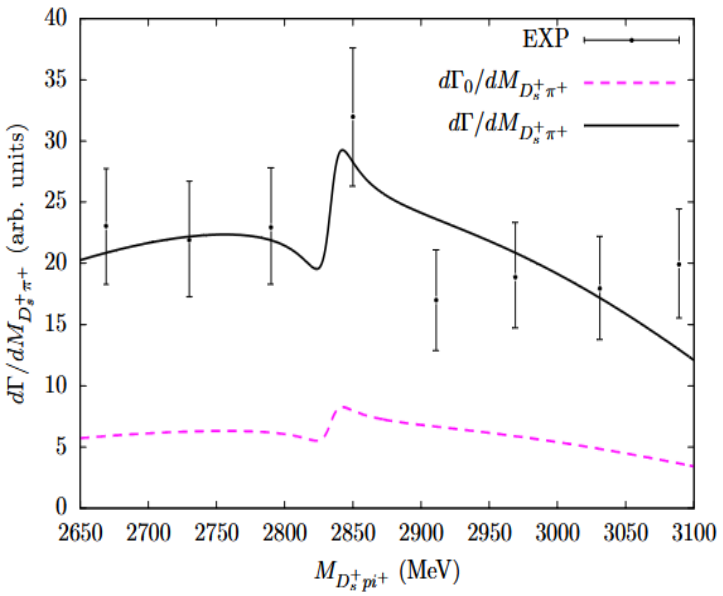
$$\frac{d\,\Gamma_0}{dM_{\rm inv}} = FAC\left[|a|^2 + |b|^2 + |c|^2\right],$$
$$\frac{d\,\Gamma_1}{dM_{\rm inv}} = FAC\left[2\,Re(ac^*) + \frac{2}{\sqrt{5}}\,2\,Re(bc^*)\right],$$
$$\frac{d\,\Gamma_2}{dM_{\rm inv}} = FAC\left[\frac{2}{7}\sqrt{5}\,|b|^2 + \frac{2}{5}\sqrt{5}\,|c|^2 + 2\,Re(ab^*)\right],$$
$$\frac{d\,\Gamma_3}{dM_{\rm inv}} = FAC\sqrt{\frac{15}{7}}\,\frac{3}{5}\,2\,Re(bc^*),$$
$$\frac{d\,\Gamma_4}{dM_{\rm inv}} = FAC\,\frac{6}{7}|b|^2,$$

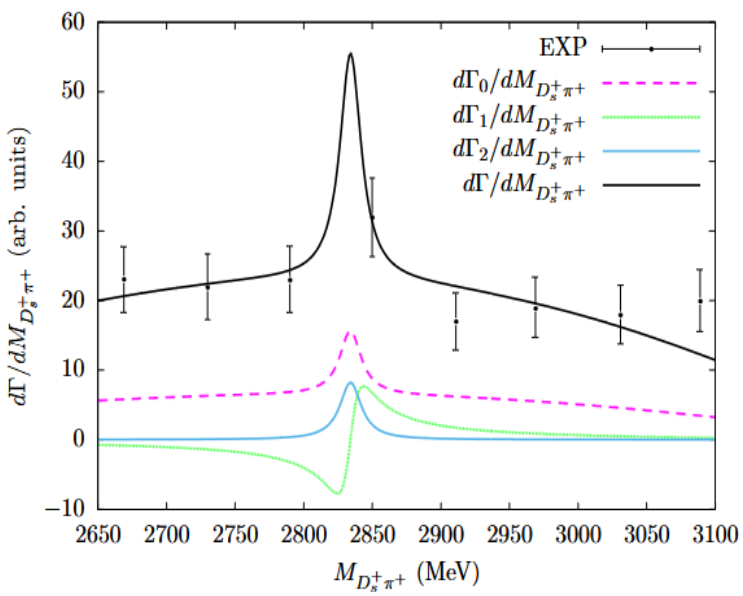


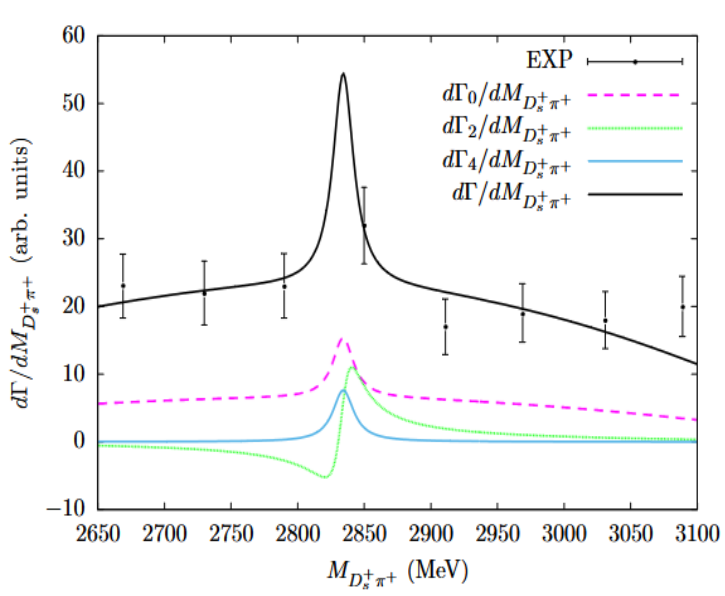
$B^+ \to D^{*-} D_s^+ \pi^+$

arXiv:2501.02839

郑州大学 ZHENGZHOU UNIVERSITY







$$aY_{00} = \left(a_0 + a_0' \frac{M_{B^+}^2}{M_{\rm inv}^2(D_s^+\pi^+) - M_R^2 + iM_R\Gamma_R}\right) Y_{00}$$

$$aY_{00} + cY_{10} = a_1 Y_{00} + c' \frac{M_{B^+}\tilde{k}}{M_{\rm inv}^2(D_s^+\pi^+) - M_R^2 + iM_R\Gamma_R} Y_{10}$$

$$aY_{00} + bY_{20} = a_2 Y_{00} + b' \frac{\tilde{k}^2}{M_{\rm inv}^2(D_s^+\pi^+) - M_R^2 + iM_R\Gamma_R} Y_{20}$$

Summary

- The $D_s^+\pi^-$ and D^0K^0 invariant mass distributions of $B^- \to D_s^+\pi^-K^-$, $\Lambda_b \to D^0K^0\Lambda$, $\bar{B}_s^0 \to K^0D^0\pi^0$ show a significant peak structure of the $T_{c\bar{s}0}(2900)$, respectively.
- The measurements of $B^+ \to D^{*-}D^+K^+$ and $B^+ \to D^{*-}D_s^+\pi^+$ show some hints for the predicted $T_{c\bar{s}2}$.
- The angular mass distributions are very different for each of the spin assumptions $J = 0, 1, 2$.

Thank you very much！