

Investigation on the Ω(2012) from QCD sum rules

Niu Su

Southeast University

Collaborators: Hua-Xing Chen, Philipp Gubler, Atsushi Hosaka

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Contents

- Background
- P-wave Ω baryon currents
- QCD sum rule analyses
- Numerical analyses
- Summary and outlook

New baryon candidates

• Significant progress has been achieved in baryon spectroscopy, with an increasing number of new baryon candidates

Heavy baryons
Light baryons

 $E_c(2923)$ $\Lambda_b(5912)$ $E_c(2939)$ $\Lambda_b(5920)$ $E_c(2965)$ $E_b(6087)$ $E_b(6100)$ Ξ(1620) Ξ(1690) Ξ(1820) Ω(2012)

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Discovery of the $\Omega(2012)$

• In 2018, the excited Ω baryon, $\Omega(2012)$, was discovered for the first time by the Belle experiment in $\Xi^0 K^-$ and $\Xi^- K_s^0$ invariant mass spectrum in $\Upsilon(1S)$, $\Upsilon(2S)$, $\Upsilon(3S)$ decays



Belle Collaboration, Phys. Rev. Lett. 121 (2018) 052003

$$M = 2012.5 \pm 0.7 \pm 0.5 \text{ MeV},$$

$$\Gamma = 6.4^{+2.5}_{-2.0} \text{ MeV}.$$



Interpretation of the $\Omega(2012)$

• In 2021, the experimental evidence has been further strengthened by $\Omega_c \rightarrow \pi^+ \Omega(2012) \rightarrow \pi^+ (\overline{K}\Xi)^-$ decay

Belle Collaboration, Phys. Rev. D 104 (5) (2021) 052005

 In the past six months, the BESIII and ALICE experiments have confirmed its existence successively
 Arxiv: 2411.11648

Arxiv: 2502.18063

5

The mass of Ω(2012) is about 340 MeV higher than the ground state Ω(1672) and is just 16.6 MeV be pw the K⁻ and Ξ*(1530) mass threshold

the first P-wave excited **Ω** baryon a hadronic molecular

P-wave excited Ω baryon state

• The conventional quark model may naively explain the $\Omega(2012)$ to be the first P-wave excited Ω baryon

before after K. T. Chao, N. Isgur and G. Karl, Phys. Rev. D 23, 155 (1981)
S. Capstick, N. Isgur, Phys. Rev. D 34 (1986) 9, 2809-2835
L. Y. Xiao and X. H. Zhong, Phys. Rev. D 98, 034004 (2018)
Z.Y. Wang, L.C. Gui, Q.F. Lu, L.Y. Xiao, X.-H. Zhong, Phys. Rev. D98 (11) (2018) 114023
A. J. Arifi, D. Suenaga, A. Hosaka, Y. Oh, Phys. Rev. D 105 (9) (2022) 094006

• One important feature of this picture is that there should be spin-orbit partners of both $J^P = 1/2^-$ and $J^P = 3/2^-$

P-wave excited Ω baryon state

- The spin $3/2^-$ state may decay to $\overline{K}\Xi$ via D-wave, or decay to $\overline{K}\Xi(1530)$ via S-wave, but with a small phase space factor (small decay width)
- The spin $1/2^-$ state will be easier to decay to $\overline{K}\Xi$ via S-wave with no phase space suppression, or decay to $\overline{K}\Xi(1530)$ via D-wave (large decay width)
- Considering $\Omega(2012)$ has a narrow width, its spin-parity is more likely to be $3/2^-$

Molecule state

• A molecular picture has been proposed and extensively discussed via various methods, the spin $1/2^-$ state is not easily generated in the this picture and it lead to a large contribution to $\Omega(2012) \rightarrow \overline{K}\Xi(1530) \rightarrow \overline{K}\Xi\pi$

Phys. Rev. D 98, 054009 (2018), Phys. Rev. D 98,056013 (2018), Phys. Rev. D 98,076012 (2018), Eur. Phys. J. C 78, 857 (2018), Phys. Rev. D 102, 074025 (2020), Phys. Rev. C 103 (2) (2021) 025202, Phys. Rev. D 106, 054028 (2022), Chin. Phys. Lett. 41 (2024) 8, 081402

 In experiments, it was first reported that such a three-body decay was not observed in 2019, but in 2022 the measurement was revisited and the possibility of the three-body decay was discussed
 S. Jia, et al, Phys. Rev. D 100 (3) (2019) 032006 Belle collaboration, Phys. Lett. B 860 (2025) 139224

$$\mathcal{R}_{\Xi\bar{K}}^{\Xi\pi\bar{K}} \equiv \frac{\mathcal{B}[\Omega(2012) \to \Xi(1530)\bar{K} \to \Xi\pi\bar{K}]}{\mathcal{B}[\Omega(2012) \to \Xi\bar{K}]} < 11.9\% \longrightarrow 0.99$$

Mixing of two picture

 ● A hybrid picture of three-quark and molecular states was proposed in a coupled-channel approach. It found that both the three-quark core and KE(1530) channel are essential

Q.-F. Lu, H. Nagahiro, A. Hosaka, Phys.Rev. D 107 (1) (2023) 014025

 It would be fair to say that the structure of Ω(2012) is not yet well understood, and this has motivated us to study its further properties based on QCD sum rules

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P-wave ss diquark

- We construct the P-wave Ω baryon currents with a covariant derivative, assuming that the parity of $\Omega(2012)$ is negative
- Ω baryon has three identical strange quarks, we can construct the currents either the ρ mode or λ mode orbital excitation which can be related by Fierz transformation. It is much easier to construct the currents of the ρ mode, which contains the P-wave ss diquark field



P-wave ss diquark

• For the P-wave ss diquark within a derivative, three types of diquarks remain non-vanishing:

$$\epsilon^{abc} s_a^T C \gamma_5 \overset{\leftrightarrow}{D}_{\mu} s_b \qquad \epsilon^{abc} s_a^T C \overset{\leftrightarrow}{D}_{\mu} s_b \qquad \epsilon^{abc} s_a^T C \gamma_{\mu} \gamma_5 \overset{\leftrightarrow}{D}_{\mu} s_b$$

• This diquark has a suitable internal P-wave structure. Before applying a derivative, $s_a^T C \gamma_5 s_b$ has quantum number $J^P = 0^+$ which is S-wave. When a derivative is applied, $s_a^T C \gamma_5 \vec{D} s_b$ becomes a pure P-wave diquark $J^P = 1^-$.

P-wave ss diquark

 For the P-wave ss diquark within a derivative, three types of diquarks remain non-vanishing:

$$\epsilon^{abc} s_a^T C \gamma_5 \overleftrightarrow{D}_{\mu} s_b \quad \epsilon^{abc} s_a^T C \overleftrightarrow{D}_{\mu} s_b \quad \epsilon^{abc} s_a^T C \gamma_{\mu} \gamma_5 \overleftrightarrow{D}_{\mu} s_b$$

• In contrast, the other two diquarks, before applying a derivative, $s_a^T C s_b$ and $s_a^T C \gamma_{\mu} \gamma_5 s_b$ themselves have already P -wave nature $(J^P = 0^- \text{ and } J^P = 1^-)$. When a derivative is applied, they obtain complicated structure unlike the P-wave field

P-wave Ω baryon currents

• Combining the diquark with the third quark field of spin 1/2, we can write the currents for the P-wave Ω baryon with the total angular momentum $J_{tol} = 1/2, 3/2$

$$J = s_1 \otimes s_2 \otimes s_3 \otimes l_{\rho} \otimes l_{\lambda}$$

$$\rightarrow [s_1 \otimes s_2 \otimes l_{\rho}] \otimes s_3 \qquad s_1 / s_2 / s_3 = 1/2$$

$$= s_{12} \otimes l_{\rho} \otimes s_3 \qquad s_{12} = 0, j_{12} = 1$$

$$= j_{12} \otimes s_3 \qquad l_{\rho} = 1$$

$$= 1/2 \oplus 3/2.$$

• Their corresponding P-wave Ω baryon currents are

$$J = -2\epsilon^{abc} \left[(D^{\mu}s_{a}^{T})C\gamma_{5}s_{b} \right] \gamma_{\mu}s_{c}, \qquad (1)$$
$$J_{\mu} = -2\epsilon^{abc} \left[(D^{\nu}s_{a}^{T})C\gamma_{5}s_{b} \right] (g_{\mu\nu} - \frac{1}{4}\gamma_{\mu}\gamma_{\nu})s_{c} \qquad (2)$$

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• The current matrix element $J_{\mu} = -2\epsilon^{abc} \left[(D^{\nu}s_a^T)C\gamma_5 s_b \right] (g_{\mu\nu} - \frac{1}{4}\gamma_{\mu}\gamma_{\nu})s_c$

$$\langle 0|J_{\mu}|\Omega; 3/2^{-}\rangle = f_{-}u_{\mu}(q) \tag{3}$$

$$\langle 0|J_{\mu}|\Omega; 3/2^{+}\rangle = f_{+}\gamma_{5}u_{\mu}(q) \tag{4}$$

• We study the correlation function with the following Lorentz structure

$$\Pi_{\mu\nu}(q^2) = i \int d^4x e^{iqx} \langle 0 | \mathbf{T}[J_{\mu}(x)J_{\nu}^{\dagger}(0)] | 0 \rangle$$

$$= \left(\frac{q_{\mu}q_{\nu}}{q^2} - g_{\mu\nu}\right) \Pi(q^2) + \cdots .$$
(5)

• $\Pi(q^2)$ can be expressed as a dispersion relation

$$\Pi(q^2) = \int_{s_{<}}^{\infty} \frac{\rho(s)}{s - q^2 - i\varepsilon} ds \qquad \begin{array}{l} \rho(s) \equiv \operatorname{Im}\Pi(s)/\pi & \text{(6)} \\ s_{<} = 9m_s^2 \end{array}$$

• At the hadron level, we obtain the spectral density by inserting the complete set of intermediate hadronic states

$$\rho^{\text{phen}}(s) \equiv \sum_{n} \delta(s - M_{n}^{2}) \langle 0|J_{\mu}|n \rangle \langle n|J_{\nu}^{\dagger}|0 \rangle \stackrel{\rho}{=} \left\{ f_{-}^{2}(\not q + M_{-})\delta(s - M_{-}^{2}) + f_{+}^{2}(\not q - M_{+})\delta(s - M_{+}^{2}) + \theta(s - s_{0})\rho^{\text{cont}}(s), \right\}$$

$$(7)$$

$$M_{-} M_{+} s_{0}$$

• At the hadron level, the correlation function can be given as

$$\Pi^{\text{phen}}(q^2) = f_-^2 \frac{\not q + M_-}{M_-^2 - q^2 - i\epsilon} + f_+^2 \frac{\not q - M_+}{M_+^2 - q^2 - i\epsilon}$$

$$= \Pi^{\text{phen}}_{-}(q^2) \not q + \Pi^{\text{phen}}_{-}(q^2)$$
(8)

$$\rho_{1}^{\text{phen}}(s) = f_{-}^{2}\delta(s - M_{-}^{2}) + f_{+}^{2}\delta(s - M_{+}^{2}), \qquad (9)$$

$$\rho_0^{\text{phen}}(s) = f_-^2 M_- \delta(s - M_-^2) - f_+^2 M_+ \delta(s - M_+^2) \tag{10}$$

• Finally we can get the spectral densities for negative and positive parity states as

$$\rho_{\mp}^{\text{phen}}(s) = \sqrt{s}\rho_1^{\text{phen}}(s) \pm \rho_0^{\text{phen}}(s) \tag{11}$$

- At the quark-gluon level, we insert explicit forms of currents into the correlation function, contract out pairs of quark field which are quark propagator of QCD
- We get the master formula expressed in terms of quark propagator

$$\Pi_{\mu\nu}(q) = i \int d^4x e^{iq \cdot x} \langle 0 | \mathcal{T}\{J_{\mu}(x)J^{\dagger}{}_{\nu}(0)\} | 0 \rangle$$

$$\Pi_{\mu\nu}^{\text{OPE}}(q) = \epsilon_{abc} \epsilon_{a'b'c'} \int d^4 x e^{iqx} \langle 0| \left\{ S_s^{ca'}(x) \gamma_{\nu} \widetilde{S}_s^{ab'}(x) \gamma_{\mu} S_s^{bc'}(x) - S_s^{ca'}(x) \gamma_{\nu} \widetilde{S}_s^{bb'}(x) \gamma_{\mu} S_s^{ac'}(x) - S_s^{cb'}(x) \gamma_{\nu} \widetilde{S}_s^{aa'}(x) \gamma_{\mu} S_s^{bc'}(x) + S_s^{cb'}(x) \gamma_{\nu} \widetilde{S}_s^{ba'}(x) \gamma_{\mu} S_s^{ac'}(x) - S_s^{cc'}(x) Tr \left[S_s^{ba'}(x) \gamma_{\nu} \widetilde{S}_s^{ab'}(x) \gamma_{\mu} \right] + S_s^{cc'}(x) Tr \left[S_s^{bb'}(x) \gamma_{\nu} \widetilde{S}_s^{aa'}(x) \gamma_{\mu} \right] \right\} |0\rangle,$$
(12)

 Insert propagator to master formula, we can get the spectral densities in the form of the operator product expansion(OPE)



• Feynman diagrams in the present study



 By equating the spectral densities at the hadron and quarkgluon levels, and by performing the Borel transformation, we derive the sum rule equation as

$$\Pi_{\mp}(s_0, M_B) = 2M_{\mp} f_{\mp}^2 e^{-M_{\mp}^2/M_B^2}$$

= $\int_{s_{<}}^{s_0} (\sqrt{s}\rho_1^{\text{OPE}}(s) \pm \rho_0^{\text{OPE}}(s)) e^{-s/M_B^2} ds.$ (14)

Π_ is the equation for the negative parity state, Π₊ is the equation for the positive parity state

• The masses and decay constants are obtained as

$$M_{\mp}^{2}(s_{0}, M_{B})$$

$$= \frac{\int_{s_{<}}^{s_{0}} (\sqrt{s}\rho_{1}^{\text{OPE}}(s) \pm \rho_{0}^{\text{OPE}}(s))se^{-s/M_{B}^{2}}ds}{\int_{s_{<}}^{s_{0}} (\sqrt{s}\rho_{1}^{\text{OPE}}(s) \pm \rho_{0}^{\text{OPE}}(s))e^{-s/M_{B}^{2}}ds}$$

$$f_{\mp}^{2}(s_{0}, M_{B})$$

$$= \frac{\int_{s_{<}}^{s_{0}} (\sqrt{s}\rho_{1}^{\text{OPE}}(s) \pm \rho_{0}^{\text{OPE}}(s))e^{-s/M_{B}^{2}}ds \times e^{M_{\mp}^{2}/M_{B}^{2}}}{2M_{\mp}}$$
(15)

23

(16)

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• Take $J^p = 3/2^-$ state as a concrete example

$$= \frac{M_{\mp}^{2}(s_{0}, M_{B})}{\int_{s_{<}}^{s_{0}}(\sqrt{s}\rho_{1}^{\text{OPE}}(s) \pm \rho_{0}^{\text{OPE}}(s))se^{-s/M_{B}^{2}}ds}{\int_{s_{<}}^{s_{0}}(\sqrt{s}\rho_{1}^{\text{OPE}}(s) \pm \rho_{0}^{\text{OPE}}(s))e^{-s/M_{B}^{2}}ds}$$

• Two parameters: s_0, M_B

Mass formula

Criteria:

 The convergence of OPE
 The pole contribution
 The mass dependence on these two parameters

• The convergence of OPE



• The pole contribution

$$PC \equiv \left| \frac{\Pi_{-}(s_{0}, M_{B}^{2})}{\Pi_{-}(\infty, M_{B}^{2})} \right| \ge 40\%.$$

$$1.54 \text{ GeV}^{2} \le M_{B}^{2} \le 1.76 \Longrightarrow s_{0} = 6.0 \text{ GeV}^{2}$$

$$s_{0}^{\min} = 5.3 \text{ GeV}^{2}$$

$$26$$

• The mass of $3/2^-$ state as a function of these two parameters



$$M_{3/2^{-}} = 2.05^{+0.09}_{-0.10} \text{ GeV},$$

$$f_{3/2^{-}} = 0.037^{+0.007}_{-0.007} \text{ GeV}^{3}$$

• The masses and the decay constants for the $J = \frac{1}{2} / \frac{3}{2}$ state using the current with a derivative

$$J = -2\epsilon^{abc} \left[(D^{\mu}s_a^T)C\gamma_5 s_b \right] \gamma_{\mu}s_c$$
$$J_{\mu} = -2\epsilon^{abc} \left[(D^{\nu}s_a^T)C\gamma_5 s_b \right] (g_{\mu\nu} - \frac{1}{4}\gamma_{\mu}\gamma_{\nu})s_c$$

TABLE I: Masses and coupling constants extracted from the currents J in Eq. (5), J_{μ} in Eq. (6), and J'_{μ} in Eq. (30).

Current	state	$e^{\min} \left[C_{e} V^{2} \right]$	Working Regions		Pole [%]	Mass [GeV]	Couple constant $[GeV^3]$
	50400	30 [UCV]	$M_B^2 \; [{\rm GeV}^2]$	$s_0 \; [\text{GeV}^2]$			
J	$ \Omega;1/2^+\rangle$	9.7	1.91 - 2.40	11.0	40-55	$3.05\substack{+0.21\\-0.15}$	$0.168\substack{+0.045\\-0.040}$
	$ \Omega;1/2^-\rangle$	5.5	1.58 - 1.73	6.0	40-47	$2.07\substack{+0.07\\-0.07}$	$0.079\substack{+0.011\\-0.011}$
J_{μ}	$ \Omega;3/2^+\rangle$	10.5	2.09 - 2.30	11.0	40-46	$3.13_{-0.18}^{+0.27}$	$0.074\substack{+0.015\\-0.009}$
	$ \Omega;3/2^-\rangle$	5.3	1.54 - 1.76	6.0	40-51	$2.05\substack{+0.09 \\ -0.10}$	$0.037\substack{+0.007\\-0.007}$
J'_{μ}	$ \Omega';3/2^+\rangle$	3.3	1.48-1.77	4.0	40-52	$1.59^{+0.10}_{-0.12}$	$0.033^{+0.006}_{-0.006}$
	$ \Omega';3/2^- angle$	11.5	3.30-3.93	13.0	40-51	$3.15_{-0.17}^{+0.16}$	$0.092^{+0.018}_{-0.018}$

• BESIII collaboration report a new excited Ω baryon $\Omega(2109)$ along with evidence for $\Omega(2012)$



Arxiv: 2411.11648

its width is 18.3 MeV
 larger than Ω(2012)

• The masses and the decay constants for the $J = \frac{1}{2} / \frac{3}{2}$ state using the current with a derivative

$$J = -2\epsilon^{abc} \left[(D^{\mu}s_a^T)C\gamma_5 s_b \right] \gamma_{\mu}s_c$$
$$J_{\mu} = -2\epsilon^{abc} \left[(D^{\nu}s_a^T)C\gamma_5 s_b \right] (g_{\mu\nu} - \frac{1}{4}\gamma_{\mu}\gamma_{\nu})s_c$$

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J'_{μ}	$ \Omega';3/2^+\rangle$	3.3	1.48 - 1.77	4.0	40-52	$1.59\substack{+0.10 \\ -0.12}$	$0.033\substack{+0.006\\-0.006}$
	$ \Omega';3/2^- angle$	11.5	3.30-3.93	13.0	40-51	$3.15_{-0.17}^{+0.16}$	$0.092^{+0.018}_{-0.018}$

• The masses and the couple constants for the $J^p = 3/2$ state using the current without a derivative

$$J'_{\mu} = -\sqrt{3}\epsilon^{abc}s_a^T C\gamma_{\mu}s_b s_c$$

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Current	state	$s_0^{\min} [\text{GeV}^2]$	Working Regions		Pole [%]	Mass [CoV]	Couple constant $[CeV^3]$
			$M_B^2 \; [{\rm GeV}^2]$	$s_0 \; [\text{GeV}^2]$			
J	$ \Omega;1/2^+\rangle$	9.7	1.91-2.40	11.0	40-55	$3.05_{-0.15}^{+0.21}$	$0.168^{+0.045}_{-0.040}$
	$ \Omega;1/2^-\rangle$	5.5	1.58 - 1.73	6.0	40-47	$2.07\substack{+0.07 \\ -0.07}$	$0.079^{+0.011}_{-0.011}$
J_{μ}	$ \Omega;3/2^+\rangle$	10.5	2.09 - 2.30	11.0	40-46	$3.13_{-0.18}^{+0.27}$	$0.074\substack{+0.015\\-0.009}$
	$ \Omega:3/2^{-}\rangle$	5.3	1.54-1.76	6.0	40-51	$2.05^{+0.09}_{-0.10}$	$0.037^{+0.007}_{-0.007}$
J'_{μ}	$ \Omega';3/2^+\rangle$	3.3	1.48 - 1.77	4.0	40-52	$1.59^{+0.10}_{-0.12}$	$\Omega(1672).033^{+0.006}_{-0.006}$
	$ \Omega';3/2^-\rangle$	11.5	3.30-3.93	13.0	40-51	$3.15^{+0.16}_{-0.17}$	$0.092^{+0.018}_{-0.018}$

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Summary and outlook

- We have constructed the P-wave Ω baryon currents with a derivative of spin 1/2 and 3/2 by performing the spin projections
- •We have performed the parity projected QCD sum rules to separate the positive parity state and negative parity state
- Our QCD sum rule results predict that Ω(2012) is likely to be a P wave excited Ω baryon with spin 3/2⁻ and Ω(2109) is likely to be a P -wave excited Ω baryon with spin 1/2⁻
- We will study their decay properties to better understand their structure in the future



Thanks for your attention !

Backup slides

Backup

Propagator of the strange quark

$$iS_{s}^{ab}(x-y)$$
(28)

$$= \langle 0|\mathbf{T}[s^{a}(x)\bar{s}^{b}(y)]|0\rangle$$

$$= \frac{i\delta^{ab}}{2\pi^{2}(x-y)^{4}}(\hat{x}-\hat{y}) - \frac{\delta^{ab}}{12}\langle\bar{s}s\rangle$$

$$+ \frac{i}{32\pi^{2}(x-y)^{2}}\frac{\lambda_{ab}^{n}}{2}g_{s}G_{\mu\nu}^{n}\left(\sigma^{\mu\nu}(\hat{x}-\hat{y}) + (\hat{x}-\hat{y})\sigma^{\mu\nu}\right)$$

$$- \frac{1}{4\pi^{2}(x-y)^{4}}\frac{\lambda_{ab}^{n}}{2}g_{s}G_{\mu\nu}^{n}x^{\mu}y^{\nu}(\hat{x}-\hat{y})$$

$$+ \frac{\delta^{ab}(x-y)^{2}}{192}\langle g_{s}\bar{s}\sigma Gs\rangle - \frac{m_{s}\delta^{ab}}{4\pi^{2}(x-y)^{2}}$$

$$+ \frac{i\delta^{ab}}{48}m_{s}\langle\bar{s}s\rangle(\hat{x}-\hat{y}) + \frac{i\delta^{ab}}{8\pi^{2}(x-y)^{2}}m_{s}^{2}(\hat{x}-\hat{y}).$$