$D_0^*(2300)$ and $\omega(782)$ from lattice QCD

燕浩波

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Based on:

- 1. Yan et al., PRD 111 (2025) 014503
- 2. Yan et al., PRL 133 (2024) 211906, Editors' Suggestion
- 3. Yan et al., arXiv 2504.xxxx on OpTion
- 4. Yan et al., arXiv 250x.xxxx on coupled channel $D_0^*(2300)$



Introduction

- Most new particles discovered are hadronic resonances in the non-perturbative regime
- LHCb discovered a tetraquark candidate $T_{cc}(3875) \rightarrow DD^* \rightarrow DD\pi$ in 2022¹



- Previous lattice studies lifted the pion mass and study the two-body DD^* scattering² and found the corresponding pole
- The real three-body scattering problem relies on the $D\pi$ scattering amplitudes as an input
- We need to know how to solve the two-body and three-body problems, along the way we collect the fruits

¹LHCb collaboration, NP 18 (2022) 751

²Padmanath *et al.*, PRL 129 (2022) 032002; Chen *et al.*, PLB 833 (2022) 137391; Lyu *et al.*, PRL 131 (2023) 161901; Collins *et al.*, PRD 109 (2024) 094509

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 $D_0^*(2300) / \omega(782)$

Introduction

• A broad resonance $D_0^* \to D\pi$ was found by Belle collboration³ in 2004



- The mass of $D_0^*(2300)$ is almost identical to $D_{s0}^*(2317)$, which is **not** consistent with the traditional quark model predictions⁴. This can be explained by the strong coupling to DK^5
- UChPT: The possible two-pole structure, and $D_0^*(2100)$ should be the lightest charmed scalar meson⁶

 ³Satpathy *et al.*, PRB 159 (2003) 553.
 ⁴Du *et al.*, PRD 98 (2018) 094018.
 ⁵Chen *et al.*, Rep. Prog. Phys. 80 (2017) 076201
 ⁶Albaladejo *et al.*, PLB 767 (2017) 465.

Lattice setup

- Turn off the weak and electromagnetic interactions
- Since the invention of lattice field theory⁷, the calculation of hadron spectroscopy in the non-perturbative regime has been pursued to understand the structure of particles from the first principle

configuration	volume	a/fm	$m_\pi/{ m MeV}$	$N_{\rm cfgs}$
C48P14	$48^3 \times 96$	0.10530(18)	135.5(1.6)	132
F32P21	$32^3 \times 64$	0.07746(18)	210.9(2.2)	459
F48P21	$48^3 \times 96$	0.07746(18)	207.2(1.1)	222
F32P30	$32^3 \times 96$	0.07746(18)	303.2(1.3)	567
F48P30	$48^3 \times 96$	0.07746(18)	303.4(9)	201
H48P32	$48^3 \times 144$	0.05187(26)	317.2(0.9)	274

- 4 different pion masses to track the chiral behavior
- 3 lattice spacings to estimate the discretization error

⁷Wilson, PRD 10 (1974) 2445

Operator construction

- Symmetry broken $O(3) \rightarrow O_h$ or even smaller group
- A Mathematica package **OpTion**⁸ (**Op**erator construc**Tion**) is developed to construct general *N*-hadron operators

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🐢 wittscien Possible to print also JLS with partial wave method 86dd411-last month 🕥 12 Commits			OpTion (Operator construction) is a Mathematica package for building
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OpTion			Packages
Ecores MIT Mathematica 30.x 11.x 12.x 13.x			Publish your first package
OpTion (Operator construction) is a Mathematica package for operator construction in lattice QCD.			Languages

• The package has been widely used in CLQCD collaboration

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<sup>8</sup>https://github.com/wittscien/OpTion
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Finite-volume spectra

 $m_{\pi} \approx 303 \text{ MeV}$



- $\bullet\,$ The emergence of D^*
- \bullet Strong attraction in $\mathit{S}\text{-wave}$ and small δ_1

Pole positions of $D_0^*(2300)$ as a function of m_π



- Two-pole structure is to be found in the coupled $D\pi D\eta D_s \bar{K}$ scattering
- $1/a_0$ at the physical point: not consistent with the experiment⁹
- An clear trend for the motion of the $D_0^*(2300)$ pole is identified

bound state \rightarrow virtual state \rightarrow resonance

⁹ALICE (2024)

Move on to the three-body problem

Now we have the two-body input – how do we approach the $DD\pi$ channel?



Gravitational three-body problem

- Goal: space-time trajectories unsolvable
- Birth of mathematical chaos

Quantum mechanical three-body problem

- Goal: rigorous scattering theory solvable
- Spectra from lattice QCD
- Tests of the fundamental theory

Three-body problem



For ω -meson

- Three-body problem with resonances in two-body problem
 - $\blacktriangleright \ \pi\pi\pi \to \omega$
 - $\blacktriangleright \ \pi\pi \to \rho$
- Challenging isospin in the $\pi\pi\pi$ channel
- crucial for T_{cc} ^a, Roper, ...

^aHansen et al., (2025)

Three-body problem on the lattice – History

Two-body problem: many to investigate but looks good :)

- *ππ*: PRD 100, 114514 (2019)...
- T_{cc}: PRL 129, 032002 (2022)...
- $\Lambda(1405)$: 2307.10413, for review see Eur.Phys.J.ST 230 (2021) 6

Three-body problem: all repulsive except one study :(

- 2019: $\pi\pi\pi$ at maximal isospin, PRL 122, 062503
- 2020: $\pi\pi\pi$ at maximal isospin, PRL 124, 032001
- 2020: $\pi\pi\pi$ at maximal isospin, PRD 101, 114507
- 2020: *KKK* at maximal isospin, $I = \frac{3}{2}$ *KKK*
- 2021: $\pi\pi\pi$ at maximal isospin, PRL 126, 012001
- 2021: $\pi\pi\pi$ at maximal isospin, EPJC 81, 436
- 2021: $\pi\pi\pi \to a_1(1260)$, PRL 127, 222001
- 2023: $\pi\pi K$ and $KK\pi$ at maximal isospin, JHEP 05, 137

ω : the first neutral vector meson (1961)¹



Phenomenologically¹¹,

- ω is the lightest hadron decaying into three particles: $\omega \to 3\pi$
- ullet ω dominates the isoscalar response within the VMD picture of the photon-nucleon interactions
- ω generates the observed repulsion at $< 1\,{
 m fm}$ in the one-boson-exchange picture of the N-N interaction
- $\bullet~\omega$ mixes with the ρ and leads to marked effects in the pion vector form factor
- $\omega
 ho$ mass splitting is phenomenologically interesting, for instance muon g-2 and dark matter

¹⁰Maglic et al. (1961).

¹¹Sakurai (1960); Erkelenz (1974); Brown and Jackson (1976); Barkov et al., 1985; Connell et al. (1997); Bazavov et al. (2021).

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The methodology



- Operator construction from OpTion
- $I = 1 \ \pi\pi$ and $I = 0 \ \pi\pi\pi$ spectra
- Develop the formalism to map finite to infinite volumes
- Solve the integral equations and search the poles

Disclaimers

- $\omega \to \pi^+\pi^-$ is forbidden due to G-parity but only 2%
- $\bullet\,$ Mixing with the ϕ or $\omega(1420)$ are ignored since they are too high to play a role in our analysis
- No continuum extrapolations yet

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Contraction topologies

- Insanely many diagrams (202 for only $\pi\pi\pi \to \pi\pi\pi$, only 9 for the two-body problem)
- The topologies for $\pi\pi\pi \to \pi\pi\pi$



- Distillations² for the vast number of annihilation diagrams
- Collect all operators with different momentum configurations and do GEVP
- The spectra are stable against more non-local operators / thermal pollution / N_v

²Peardon et al., 2009

Finite-volume spectra



- Strong **attraction** in both the $\pi\pi$ and $\pi\pi\pi$ channels
- In the $\pi\pi\pi$ channel, the ground levels indicate **bound** ω at $M_{\pi} \approx 305 \,\mathrm{MeV}$ and **resonating** ω at $M_{\pi} \approx 208 \,\mathrm{MeV}$
- Restricted to be below the $\omega(1420)$ region

Quantization condition

• Using FVU (Finite-Volume Unitarity) of all state-of-art formalisms

$$\begin{cases} \tilde{K}^{-1}(\sigma) - \Sigma^{FV}(\sigma) = 0\\ \det[(\tilde{K}^{-1}(s) - \Sigma^{FV}(s))E_L - (\tilde{B}(s) + \tilde{C}(s))] = 0 \end{cases}$$

- Two-body quantization condition is equivalent to Lüscher's equation
- Combined fit for $\pi\pi$ and $\pi\pi\pi$ spectra for all M_{π} 's (EFT4)³

$$\begin{cases} \searrow & \sim \frac{\sigma - M_{\rho}^2}{g^2} \\ & \searrow & \sim \frac{s(M_{\rho}^2 - \sigma_q + 6g^2 f_{\pi}^2)(M_{\rho}^2 - \sigma_p + 6g^2 f_{\pi}^2)}{g^2 f_{\pi}^6 (s - M_{\omega}^2)} \end{cases}$$

• Parameters: $[g, \delta, M_V, a]$: $(M_\rho = M_V + a M_\pi^2, M_\omega = M_\rho + \delta)$

³For a review of the EFT form, see Meißner (1988).

Finite-volume spectra revisited



- $\chi^2_{\rm dof}({\rm EFT4}) = 2.3$
- Continuous spectra from FVU
- High-lying energies above the cutoff are also well-predicted

Finite-volume spectra revisited



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Pole positions of $\omega(782)$ as a function of m_{π}



• Solve the integral equation $^4~T=B+~C+\int \frac{d^3l}{(2\pi)^3}\frac{B+C}{2E_l(\tilde{K}^{-1}-\Sigma^{IV})}~T$

- ω is indeed a bound state at $M_{\pi} \approx 305 \,\mathrm{MeV}$ and a resonance at $M_{\pi} \approx 208 \,\mathrm{MeV}$
- Extrapolate to the physical pion mass, the poles agree well with the PDG values⁵

⁴Mai and Döring, 2017

⁵See related discussions in Hoferichter (2023), Hoferichter (2019), Hoid (2020), Colangelo (2022), Colangelo (2018).

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Summary

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bound state \rightarrow virtual state \rightarrow resonance

- The package OpTion is developed and widely used For $\omega(782)$,
 - First-ever determination of the ω -meson pole from lattice QCD
 - Development of the FVU, matching EFT and FVU
 - Paved the way to study heavier three-hadron resonances
 - $\bullet\,$ The ρ and ω pole positions at the physical point

$$\sqrt{s_{\rho}} = (748.9(10.0) - i63.5(1.8)) \text{ MeV}$$
$$\sqrt{s_{\omega}} = (778.0(11.2) - i3.0(5)) \text{ MeV}$$

For the future,

- Coupled $D\pi D\eta D_s \bar{K}$ analysis is expected soon
- Extension of the quantization condition is on the way
- Heavier three-body resonances ($\phi(1020)$, Roper, $T_{cc..}$ are on the way)

Thank you!

@Maxim Mai and Marco Garofalo

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$D_0^* \text{ and } \omega$ also thank you! $(\geqq \omega \leqq)$

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