Doubly heavy tetraquark spectrum correlated to heavy baryons with heavy antiquark-diquark symmetry

Speaker: Liu-Yu Zhang
Collaborate with: Tian-Wei Wu, Yong-Liang Ma
zhangly279@mail2.sysu.edu.cn

School of Science
Sun Yat-sen University

March 24, 2025 @Lanzhou University

Outline

1. Background

2. Theory and Framework

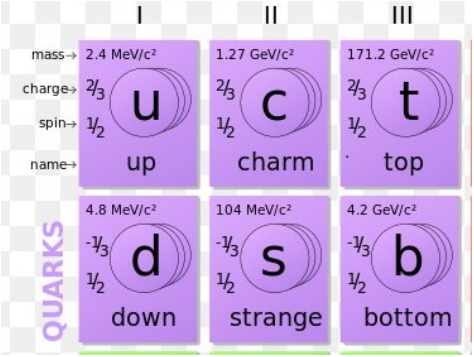
3. Calculation Procedure

4. Calculation Results and Figures

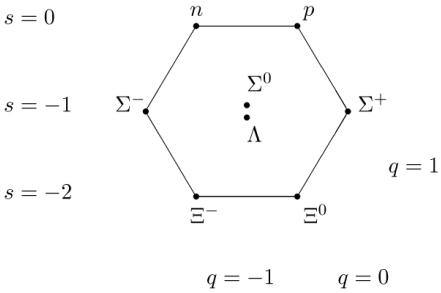
5. Summary

Quark model

What are particles composed of ? how are they classified ?

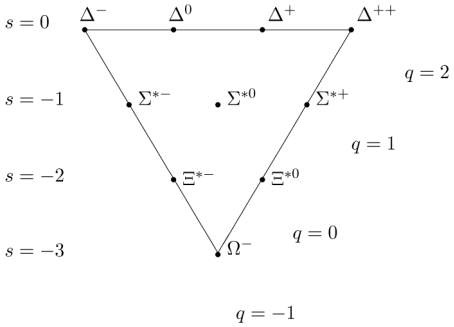


quarks

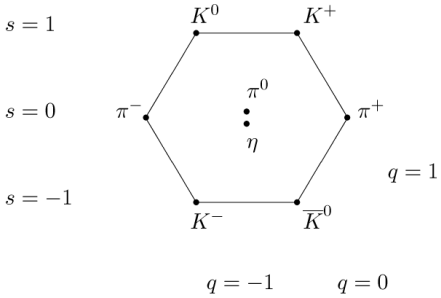


baryon octet

Quark model



baryon decuplet



meson octet

Conventional hadron

Conventional quark model's core ideal: the quark model assumes that mesons are bound states of quark - antiquark pairs, and baryons are bound states of three quarks

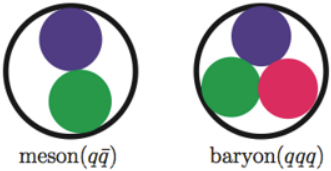


Figure: meson and baryon

Are there other types of hadronic states ?

Exotic hadronic states

multiquark states: composed of more than three quarks
glueballs: composed of several gluons
hybrids: composed of both quarks and gluons

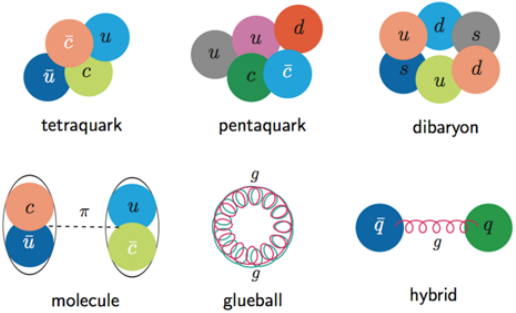


Figure: Exotic hadronic states

$T_{cc}(3875)$

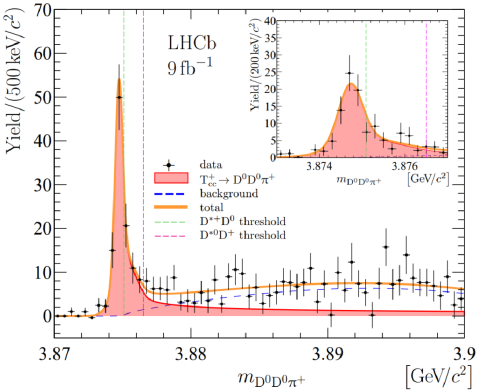


Figure 1: Distribution of $D^0D^0\pi^+$ mass. Distribution of $D^0D^0\pi^+$ mass where the contribution of the non-D^0 background has been statistically subtracted. The result of the fit described in the text is overlaid.

In 2021, $T_{cc}(3875)$ was discovered[1].

Hadronic molecule ?

[2] M.-L. Du, V. Baru, X.-K. Dong, A. Filin, F.-K. Guo, C. Hanhart, A. Nefediev, J. Nieves, and Q. Wang, Physical Review D 105 (2022)
[3] T. Kinugawa and T. Hyodo, Physical Review C 109 (2024)
[4] M. Albaladejo, Physics Letters B 829, 137052 (2022)
[5] G.-J. Wang, Z. Yang, J.-J. Wu, M. Oka, and S.-L. Zhu, Science Bulletin 69, 3036–3041 (2024)

Compact tetraquark ?

[6] T.-W. Wu and Y.-L. Ma, Phys. Rev. D 107, L071501 (2023)
[7] X.-Z. Weng, W.-Z. Deng, and S.-L. Zhu, Chinese Physics C 46, 013102 (2022)
[8] Y.-X. Song and D.-J. Jia, Communications in Theoretical Physics 75, 055201 (2023)
[9] S.-Y. Li, Y.-R. Liu, Z.-L. Man, Z.-G. Si, and J. Wu, Physical Review D 110 (2024)

[1] R. Aaij et al. (LHCb), Nature Phys. 18, 751 (2022)

Outline

1. Background

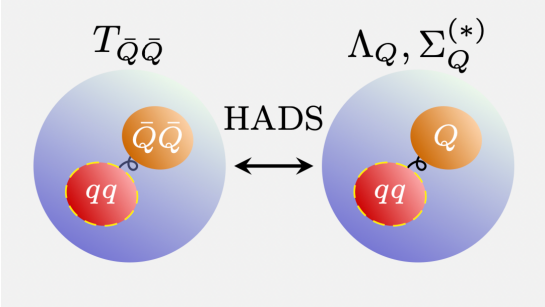
2. Theory and Framework

3. Calculation Procedure

4. Calculation Results and Figures

5. Summary

Heavy antiquark-diquark symmetry (HADS)



In the doubly heavy tetraquark, the heavy antidiquark $[\bar{Q}\bar{Q}]$ can be regarded as a heavy quark Q.

Heavy antiquark-diquark symmetry (HADS)

- In a heavy hadron, the heavy quark's mass is far heavier then the light quark. The heavy quark's mass is equivalent to infinity compared to the light quark.
- In the limit of heavy quark, the heavy di-quark's color structure in the teraquark is same as one heavy quark's color structure in the baryons.

Constituent Quark Model

$$M = \sum_i m_i + \sum_{i<j} (F_i \cdot F_j)[B_{ij} + (\sigma_i \cdot \sigma_j)\alpha_{ij}/m_i m_j]$$

F_i: the color operators
σ: the spin operators
B_{ij}: the binding between a heavy quark (c and b) and a flavored quark (s, c, and b) for describing the heavy baryons
m_i: the ith quark mass
α: the quark-quark spin hyperfine interaction coupling
Where $i, j = 1, 2, 3$.

Constituent Quark Model

In Refs. [10, 11], the model is developed to describe the heavy baryons and successfully predict the Ξ_{cc} mass.
And in Refs [6], the model is developed to successfully calculate the baryons mass.

[10] M. Karliner, B. Keren-Zur, H. J. Lipkin, and J. L. Rosner, Annals Phys. 324, 2 (2009)
[11] M. Karliner and J. L. Rosner, Phys. Rev. D 90, 094007 (2014)
[6] T.-W. Wu and Y.-L. Ma, Phys. Rev. D 107, L071501 (2023)

Mass formula of baryon

Table: Calculate the baryon masses with Mass formula of Constituent Quark Model[6]

State	$I(J^+)$	$\sum_i m_i + \sum_{i<j}(F_i \cdot F_j)[B_{ij} + (\sigma_i \cdot \sigma_j)\alpha_{ij}/m_i m_j]$	This model	PDG2022
N	$1/2(1/2^+)$	$3m_q^b + 2\alpha/(m_q^b)^2$	939.3	938.9
Δ	$1/2(3/2^+)$	$3m_q^b - 2\alpha/(m_q^b)^2$	1246.5	1232
Λ	$0(1/2^+)$	$2m_q^b + m_s^b + 2\alpha/(m_q^b)^2$	1111.2	1115.68
Σ	$1(1/2^+)$	$2m_q^b + m_s^b - \frac{2}{3}[\alpha/(m_q^b)^2 - 4\alpha/(m_q^b m_s^b)]$	1176.9	1193.1
Σ^*	$1(3/2^+)$	$2m_q^b + m_s^b - \frac{2}{3}[\alpha/(m_q^b)^2 + 2\alpha/(m_q^b m_s^b)]$	1385.6	1384.6
Ξ	$1/2(1/2^+)$	$2m_s^b + m_q^b - \frac{2}{3}[\alpha/(m_s^b)^2 - 4\alpha/(m_q^b m_s^b)]$	1321.2	1318.3
Ξ^*	$1/2(3/2^+)$	$2m_s^b + m_q^b - \frac{2}{3}[\alpha/(m_s^b)^2 + 2\alpha/(m_q^b m_s^b)]$	1529.9	1533.4
Ω	$0(3/2^+)$	$3m_s - 2\alpha/(m_s^2)$	1679.5	1672.45
Λ_c	$0(1/2^+)$	$2m_q^b + m_c^b + 2\alpha/(m_q^b)^2$	2290.9	2286.46
Σ_c	$1(1/2^+)$	$2m_q^b + m_c^b - \frac{2}{3}[\alpha/(m_q^b)^2 - 4\alpha/(m_q^b m_c^b)]$	2452.2	2453.54
Σ_c^*	$1(3/2^+)$	$2m_q^b + m_c^b - \frac{2}{3}[\alpha/(m_q^b)^2 + 2\alpha/(m_q^b m_c^b)]$	2517.4	2518.13
Ξ_c	$1/2(1/2^+)$	$m_c^b + m_q^b + m_s^b - \frac{2}{3}[B_{cs} - 3\alpha/(m_s^b m_q^b)]$	2476.4	2469.1
Ξ_c'	$1/2(1/2^+)$	$m_c^b + m_q^b + m_s^b - \frac{2}{3}[B_{cs} + \alpha/(m_s^b m_q^b) - 2\alpha/(m_c^b m_s^b) - 2\alpha/(m_c^b m_q^b)]$	2579.1	2578.5
Ξ_c^*	$1/2(3/2^+)$	$m_c^b + m_q^b + m_s^b - \frac{2}{3}[B_{cs} + \alpha/(m_s^b m_q^b) + \alpha/(m_c^b m_s^b) + \alpha/(m_c^b m_q^b)]$	2633.8	2645.63
Ω_c	$0(1/2^+)$	$m_c^b + 2m_s^b - \frac{2}{3}[2B_{cs} + \alpha/(m_s^b)^2 - 4\alpha/(m_c^b m_s^b)]$	2711.2	2695.2
Ω_c^*	$0(3/2^+)$	$m_c^b + 2m_s^b - \frac{2}{3}[2B_{cs} + \alpha/(m_s^b)^2 + 2\alpha/(m_c^b m_s^b)]$	2755.5	2765.9
Λ_b	$0(1/2^+)$	$2m_q^b + m_b^b + 2\alpha/(m_b^b)^2$	5622.3	5619.6
Σ_b	$1(1/2^+)$	$2m_q^b + m_b^b - \frac{2}{3}[\alpha/(m_q^b)^2 - 4\alpha/(m_q^b m_b^b)]$	5812.3	5813.1
Σ_b^*	$1(3/2^+)$	$2m_q^b + m_b^b - \frac{2}{3}[\alpha/(m_q^b)^2 + 2\alpha/(m_q^b m_b^b)]$	5834.5	5832.53
Ξ_b	$1/2(1/2^+)$	$m_b^b + m_q^b + m_s^b - \frac{2}{3}[B_{bs} - 3\alpha/(m_s^b m_q^b)]$	5801.7	5794.5
Ξ_b'	$1/2(1/2^+)$	$m_b^b + m_q^b + m_s^b - \frac{2}{3}[B_{bs} + \alpha/(m_s^b m_q^b) - 2\alpha/(m_b^b m_s^b) - 2\alpha/(m_b^b m_q^b)]$	5928.4	5935.02
Ξ_b^*	$1/2(3/2^+)$	$m_b^b + m_q^b + m_s^b - \frac{2}{3}[B_{bs} + \alpha/(m_s^b m_q^b) + \alpha/(m_b^b m_s^b) + \alpha/(m_b^b m_q^b)]$	5947.1	5955.32
Ω_b	$0(1/2^+)$	$m_b^b + 2m_s^b - \frac{2}{3}[2B_{bs} + \alpha/(m_s^b)^2 - 4\alpha/(m_b^b m_s^b)]$	6049.8	6046.1

[6] T.-W. Wu and Y.-L. Ma, Phys. Rev. D 107, L071501 (2023)

Outline

1. Background

2. Theory and Framework

3. Calculation Procedure

4. Calculation Results and Figures

5. Summary

Configuration

non-strange quarks:

1. $Q_1 = Q_2$

(a) I=0

$$[[\bar{q}\bar{q}]^3_0[QQ]^{\bar{3}}_1]^0_1$$

(b) I=1

$$[[\bar{q}\bar{q}]^3_1[QQ]^3_1]^1_0$$

$$[[\bar{q}\bar{q}]^3_1[QQ]^{\bar{3}}_1]^1_1$$

$$[[\bar{q}\bar{q}]^3_1[QQ]^{\bar{3}}_1]^1_2$$

2. $Q_1 \neq Q_2$

(a) I=0

$$[[\bar{q}\bar{q}]^3_0[bc]^{\bar{3}}_0]^0_0$$

$$[[\bar{q}\bar{q}]^3_0[bc]^3_1]^0_1$$

(b) I=1

$$[[\bar{q}\bar{q}]^3_1[bc]^{\bar{3}}_0]^1_1$$

$$[[\bar{q}\bar{q}]^3_1[bc]^3_1]^1_0$$

$$[[\bar{q}\bar{q}]^3_1[bc]^{\bar{3}}_1]^1_1$$

$$[[\bar{q}\bar{q}]^3_1[bc]^{\bar{3}}_1]^1_2$$

single-strange quarks:

1. $Q_1 = Q_2$

(a) The spins of q quark and s quark can be 0 or 1

$$[[\bar{q}\bar{s}]^3_0[QQ]^{\bar{3}}_1]^0_1$$

$$[[\bar{q}\bar{s}]^3_1[QQ]^{\bar{3}}_1]^0_0$$

$$[[\bar{q}\bar{s}]^3_1[QQ]^{\bar{3}}_1]^0_1$$

$$[[\bar{q}\bar{s}]^3_1[QQ]^{\bar{3}}_1]^0_2$$

2. $Q_1 \neq Q_2$

(a) The spins of q quark and s quark can be 0 or 1

$$[[\bar{q}\bar{s}]^3_0[bc]^{\bar{3}}_0]^0_0$$

$$[[\bar{q}\bar{s}]^3_0[bc]^{\bar{3}}_1]^0_1$$

$$[[\bar{q}\bar{s}]^3_1[bc]^{\bar{3}}_0]^0_1$$

$$[[\bar{q}\bar{s}]^3_1[bc]^{\bar{3}}_1]^0_0$$

$$[[\bar{q}\bar{s}]^3_1[bc]^{\bar{3}}_1]^0_1$$

$$[[\bar{q}\bar{s}]^3_1[bc]^{\bar{3}}_1]^0_2$$

double-strange quarks:

1. $Q_1 = Q_2$

(a) The spins of them can only be 1

$$[[\bar{s}\bar{s}]^3_1[cc]^{\bar{3}}_1]^0_0$$

$$[[\bar{s}\bar{s}]^3_1[cc]^{\bar{3}}_1]^0_1$$

$$[[\bar{s}\bar{s}]^3_1[cc]^{\bar{3}}_1]^0_2$$

$$[[\bar{s}\bar{s}]^3_1[bb]^{\bar{3}}_1]^0_0$$

$$[[\bar{s}\bar{s}]^3_1[bb]^{\bar{3}}_1]^0_1$$

$$[[\bar{s}\bar{s}]^3_1[bb]^{\bar{3}}_1]^0_2$$

2. $Q_1 \neq Q_2$

(a) The spins of them can only be 1

$$[[\bar{s}\bar{s}]^3_1[bc]^{\bar{3}}_0]^0_1$$

$$[[\bar{s}\bar{s}]^3_1[bc]^{\bar{3}}_1]^0_0$$

$$[[\bar{s}\bar{s}]^3_1[bc]^{\bar{3}}_1]^0_1$$

$$[[\bar{s}\bar{s}]^3_1[bc]^{\bar{3}}_1]^0_2$$

Mass formula of the heavy di-quark

the heavy di-quark mass:

$$M_{[Q_1Q_2]_S^C} = m_{Q_1} + m_{Q_2} + (F_1 \cdot F_2)[B_{Q_1Q_2} + (\sigma_1 \cdot \sigma_2)\alpha_{Q_1Q_2}/m_{Q_1}m_{Q_2}], \tag{1}$$

$$S = \begin{cases} 0 & (1 - \frac{1}{2} - \frac{1}{2} \quad \text{or} \quad 0 + \frac{1}{2} - \frac{1}{2}) \\ 1 & (1 + \frac{1}{2} - \frac{1}{2} \quad \text{or} \quad 0 + \frac{1}{2} + \frac{1}{2}) \\ 2 & (1 + \frac{1}{2} + \frac{1}{2}) \end{cases}$$

$$S = 0 : \sum_{i<j} \sigma_i \cdot \sigma_j = 4 \sum_{i<j} s_i \cdot s_j = 2[0 \times (0+1) - \frac{3}{4} \times 4] = -6$$

$$S = 1 : \sum_{i<j} \sigma_i \cdot \sigma_j = 4 \sum_{i<j} s_i \cdot s_j = 2[1 \times (1+1) - \frac{3}{4} \times 3] = -2$$

$$S = 2 : \sum_{i<j} \sigma_i \cdot \sigma_j = 4 \sum_{i<j} s_i \cdot s_j = 2[2 \times (2+1) - \frac{3}{4} \times 3] = 6$$

Parameters

Table: Masses, bindings, and hyperfine couplings of quarks (in unit of MeV).

Parameters	m_q^b	m_s^b	m_c^b	m_b^b	B_{cs}	B_{bs}	$\alpha/(m_q^b)^2$
values	364.3	536.2	1715.9	5047.3	53.4	62.6	-76.8

These values of the parameters is determined by minimizing the mass difference between the predicted and experimental values of all 23 ground baryons [6].

[6] T.-W. Wu and Y.-L. Ma, Phys. Rev. D 107, L071501 (2023)

Parameters

Table: Masses, bindings, and hyperfine couplings of quarks in doubly heavy tetraquark ground states(in unit of MeV).

Parameters	$m_{[cc]_1^{\bar{3}}}$	$m_{[bb]_1^{\bar{3}}}$	$m_{[cb]_0^{\bar{3}}}$	$m_{[cb]_1^{\bar{3}}}$	$B_{[cc]_1^{\bar{3}}s}$	$B_{[bb]_1^{\bar{3}}s}$	$B_{[cb]_1^{\bar{3}}s}$
values	3300.8	9821.0	6567.0	6600.9	106.8	125.2	116.0

$m_{[Q_1Q_2]_S^C}$:
we determine the values by using the masses of corresponding mesons with constituent quark model.

For example
$m_{[Q_1Q_2]_S^C} = m_{[cc]_1^{\bar{3}}}$:
$m_{[cc]_1^{\bar{3}}} = 2m_c^b + (-2/3)(B_{cc} + \alpha_{cc}/(m_c^b)^2) = 3300.8$ MeV.

$B_{[Q_1Q_2]_S^C s}$:
the binding is spin and color independent, we assume that $B_{[Q_1Q_2]_S^C s} = B_{Q_1 s} + B_{Q_2 s}$.

Parameters

Table: Bindings, and coupling of quarks to calculate the value of $m_{[Q_1Q_2]^C_S}$ (in unit of MeV).

Parameters	B_{cc}	B_{bb}	B_{cb}	$\alpha_{cc}/(m_c^b)^2$	$\alpha_{bb}/(m_b^b)^2$	$\alpha_{cb}/(m_c^b \cdot m_b^b)$
values	217.7	422.0	256.2	-21.2	-11.6	-12.7

cc:

$$m_{J/\Psi} = 2m_c^m + (-4/3)[B_{cc} + \alpha_{cc}/(m_c^m)^2]$$
$$m_{\eta_c} = 2m_c^m + (-4/3)[B_{cc} - 3\alpha_{cc}/(m_c^m)^2]$$

$(i)\alpha_{cc}/(m_c^b)^2$:

$$\alpha_{cc}/(m_c^b)^2 = \alpha_{cc}/(m_c^m)^2 = -3/16(m_{J/\Psi} - m_{\eta_c}) = -21.2\text{MeV}$$

Parameters

$(ii) B_{cc}:$

$\because m_{\Xi_{cc}} = 2m_c^b + m_q^b - 2/3[B_{cc} + \alpha_{cc}/(m_c^b)^2 - 4\alpha/(m_q^b m_c^b)] = 3621.4\text{MeV}$

$\therefore B_{cc} = 217.7\text{MeV}$

(we neglecting the small difference between m_c^b with m_c^m)

bb:

$B_{bb} = -3/16[(3m_{\Upsilon_b} + m_{\eta_b}) - 8m_b^m] = 422.0\text{MeV}$

(there is no experimental data of a doubly bottomed baryon)

Parameters

cb:

we need B_{cb} and $\alpha_{cb}/(m_c^b \cdot m_b^b)$

but we only have $B_c(s=0)$, $B_c^*(s=1)$ hasn't been discovered

we use the theoretically predicted values[11] as $m_{B_c^*} - m_{B_c} = 68.0$ MeV.

$\alpha_{cb}/(m_c^b \cdot m_b^b) = -3/16(m_{B_c^*} - m_{B_c}) = -12.7$ MeV.

$B_{cb} = -3/16[(3m_{B_c^*} + m_{B_c}) - 4m_c^m - 4m_b^m] = 256.2$ MeV.

[11] M. Karliner and J. L. Rosner, Phys. Rev. D 90, 094007 (2014)

Outline

1. Background

2. Theory and Framework

3. Calculation Procedure

4. Calculation Results and Figures

5. Summary

Results

Table: Doubly heavy tetraquark ground states aroused from heavy anti-baryons $\bar{\Lambda}_c(\bar{\Lambda}_b)$ and $\bar{\Sigma}_c(\bar{\Sigma}_b)$ with HADS

State	Configuration	$I(J^P)$	Mass formula
$T_{cc}(3876)$	$[[\bar{q}\bar{q}]^3_0[cc]^{\bar{3}}_1]^0_1$	$0(1^+)$	$2m_q^b + m_{[cc]^{\bar{3}}_1} + 2\alpha/(m_q^b)^2$
$T_{cc}(4035)$	$[[\bar{q}\bar{q}]^3_1[cc]^{\bar{3}}_1]^1_0$	$1(0^+)$	$2m_q^b + m_{[cc]^{\bar{3}}_1} - \frac{2}{3}[\alpha/(m_q^b)^2 - 8\alpha_{[cc]^{\bar{3}}_1 q}/(m_q^b m_{[cc]^{\bar{3}}_1})]$
$T_{cc}(4058)$	$[[\bar{q}\bar{q}]^3_1[cc]^{\bar{3}}_1]^1_1$	$1(1^+)$	$2m_q^b + m_{[cc]^{\bar{3}}_1} - \frac{2}{3}[\alpha/(m_q^b)^2 - 4\alpha_{[cc]^{\bar{3}}_1 q}/(m_q^b m_{[cc]^{\bar{3}}_1})]$
$T_{cc}(4103)$	$[[\bar{q}\bar{q}]^3_1[cc]^{\bar{3}}_1]^1_2$	$1(2^+)$	$2m_q^b + m_{[cc]^{\bar{3}}_1} - \frac{2}{3}[\alpha/(m_q^b)^2 + 4\alpha_{[cc]^{\bar{3}}_1 q}/(m_q^b m_{[cc]^{\bar{3}}_1})]$
$T_{bb}(10396)$	$[[\bar{q}\bar{q}]^3_0[bb]^{\bar{3}}_1]^0_1$	$0(1^+)$	$2m_q^b + m_{[bb]^{\bar{3}}_1} + 2\alpha/(m_q^b)^2$
$T_{bb}(10586)$	$[[\bar{q}\bar{q}]^3_1[bb]^{\bar{3}}_1]^1_0$	$1(0^+)$	$2m_q^b + m_{[bb]^{\bar{3}}_1} - \frac{2}{3}[\alpha/(m_q^b)^2 - 8\alpha_{[bb]^{\bar{3}}_1 q}/(m_q^b m_{[bb]^{\bar{3}}_1})]$
$T_{bb}(10593)$	$[[\bar{q}\bar{q}]^3_1[bb]^{\bar{3}}_1]^1_1$	$1(1^+)$	$2m_q^b + m_{[bb]^{\bar{3}}_1} - \frac{2}{3}[\alpha/(m_q^b)^2 - 4\alpha_{[bb]^{\bar{3}}_1 q}/(m_q^b m_{[bb]^{\bar{3}}_1})]$
$T_{bb}(10608)$	$[[\bar{q}\bar{q}]^3_1[bb]^{\bar{3}}_1]^1_2$	$1(2^+)$	$2m_q^b + m_{[bb]^{\bar{3}}_1} - \frac{2}{3}[\alpha/(m_q^b)^2 + 4\alpha_{[bb]^{\bar{3}}_1 q}/(m_q^b m_{[bb]^{\bar{3}}_1})]$
$T_{cb}(7142)$	$[[\bar{q}\bar{q}]^3_0[cb]^{\bar{3}}_0]^0_0$	$0(0^+)$	$2m_q^b + m_{[cb]^{\bar{3}}_0} + 2\alpha/(m_q^b)^2$
$T_{cb}(7176)$	$[[\bar{q}\bar{q}]^3_0[cb]^{\bar{3}}_1]^0_1$	$0(1^+)$	$2m_q^b + m_{[cb]^{\bar{3}}_1} + 2\alpha/(m_q^b)^2$
$T_{cb}(7347)$	$[[\bar{q}\bar{q}]^3_1[cb]^{\bar{3}}_0]^1_1$	$1(1^+)$	$2m_q^b + m_{[cb]^{\bar{3}}_0} - (2/3)[\alpha/(m_q^b)^2]$
$T_{cb}(7358)$	$[[\bar{q}\bar{q}]^3_1[cb]^{\bar{3}}_1]^1_0$	$1(0^+)$	$2m_q^b + m_{[cb]^{\bar{3}}_1} - (2/3)[\alpha/(m_q^b)^2 - 8\alpha/(m_q^b m_{[cb]^{\bar{3}}_1})]$
$T_{cb}(7369)$	$[[\bar{q}\bar{q}]^3_1[cb]^{\bar{3}}_1]^1_1$	$1(1^+)$	$2m_q^b + m_{[cb]^{\bar{3}}_1} - (2/3)[\alpha/(m_q^b)^2 - 4\alpha/(m_q^b m_{[cb]^{\bar{3}}_1})]$
$T_{cb}(7392)$	$[[\bar{q}\bar{q}]^3_1[cb]^{\bar{3}}_1]^1_2$	$1(2^+)$	$2m_q^b + m_{[cb]^{\bar{3}}_1} - (2/3)[\alpha/(m_q^b)^2 + 4\alpha/(m_q^b m_{[cb]^{\bar{3}}_1})]$

Result

Table: Doubly heavy tetraquark ground states aroused from heavy anti-baryons $\bar{\Xi}_c(\bar{\Xi}_b)$ with HADS

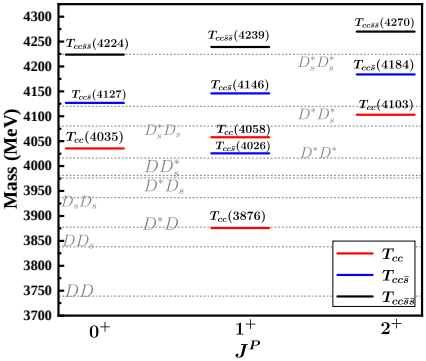
State	Configuration	$I(J^P)$	Mass formula
$T_{cc\bar{s}}(4026)$	$[[\bar{q}\bar{s}]^3_0[cc]^3_1]^{\frac{1}{2}}_1$	$\frac{1}{2}(1^+)$	$m_q^b+m_s^b+m_{[cc]^{\bar{3}}_1}-(2/3)[B_{[cc]^{\bar{3}}_1 s}-3\alpha/(m_q^b m_s^b)]$
$T_{cc\bar{s}}(4127)$	$[[\bar{q}\bar{s}]^3_1[cc]^3_1]^{\frac{1}{2}}_0$	$\frac{1}{2}(0^+)$	$m_q^b+m_s^b+m_{[cc]^{\bar{3}}_1}-(2/3)[B_{[cc]^{\bar{3}}_1 s}+\alpha/(m_q^b m_s^b)-4\alpha/(m_q m_{[cc]^{\bar{3}}_1})-4\alpha/(m_s^b m_{[cc]^{\bar{3}}_1})]$
$T_{cc\bar{s}}(4146)$	$[[\bar{q}\bar{s}]^3_1[cc]^3_1]^{\frac{1}{2}}_1$	$\frac{1}{2}(1^+)$	$m_q^b+m_s^b+m_{[cc]^{\bar{3}}_1}-(2/3)[B_{[cc]^{\bar{3}}_1 s}+\alpha/(m_q^b m_s^b)-2\alpha/(m_q^b m_{[cc]^{\bar{3}}_1})-2\alpha/(m_s^b m_{[cc]^{\bar{3}}_1})]$
$T_{cc\bar{s}}(4184)$	$[[\bar{q}\bar{s}]^3_1[cc]^3_1]^{\frac{1}{2}}_2$	$\frac{1}{2}(2^+)$	$m_q^b+m_s^b+m_{[cc]^{\bar{3}}_1}-(2/3)[B_{[cc]^{\bar{3}}_1 s}+\alpha/(m_q^b m_s^b)+2\alpha/(m_q^b m_{[cc]^{\bar{3}}_1})+2\alpha/(m_s^b m_{[cc]^{\bar{3}}_1})]$
$T_{bb\bar{s}}(10534)$	$[[\bar{q}\bar{s}]^3_0[bb]^3_1]^{\frac{1}{2}}_1$	$\frac{1}{2}(1^+)$	$m_q^b+m_s^b+m_{[bb]^{\bar{3}}_1}-(2/3)[B_{[bb]^{\bar{3}}_1 s}-3\alpha/(m_q^b m_s^b)]$
$T_{bb\bar{s}}(10660)$	$[[\bar{q}\bar{s}]^3_1[bb]^3_1]^{\frac{1}{2}}_0$	$\frac{1}{2}(0^+)$	$m_q^b+m_s^b+m_{[bb]^{\bar{3}}_1}-(2/3)[B_{[bb]^{\bar{3}}_1 s}+\alpha/(m_q^b m_s^b)-4\alpha/(m_q^b m_{[bb]^{\bar{3}}_1})-4\alpha/(m_s^b m_{[bb]^{\bar{3}}_1})]$
$T_{bb\bar{s}}(10666)$	$[[\bar{q}\bar{s}]^3_1[bb]^3_1]^{\frac{1}{2}}_1$	$\frac{1}{2}(1^+)$	$m_q^b+m_s^b+m_{[bb]^{\bar{3}}_1}-(2/3)[B_{[bb]^{\bar{3}}_1 s}+\alpha/(m_q^b m_s^b)-2\alpha/(m_q^b m_{[bb]^{\bar{3}}_1})-2\alpha/(m_s^b m_{[bb]^{\bar{3}}_1})]$
$T_{bb\bar{s}}(10679)$	$[[\bar{q}\bar{s}]^3_1[bb]^3_1]^{\frac{1}{2}}_2$	$\frac{1}{2}(2^+)$	$m_q^b+m_s^b+m_{[bb]^{\bar{3}}_1}-(2/3)[B_{[bb]^{\bar{3}}_1 s}+\alpha/(m_q^b m_s^b)+2\alpha/(m_q^b m_{[bb]^{\bar{3}}_1})+2\alpha/(m_s^b m_{[bb]^{\bar{3}}_1})]$
$T_{cb\bar{s}}(7286)$	$[[\bar{q}\bar{s}]^3_0[cb]^3_0]^{\frac{1}{2}}_0$	$\frac{1}{2}(0^+)$	$m_q^b+m_s^b+m_{[cb]^{\bar{3}}_0}-(2/3)[B_{[cb]^{\bar{3}}_0 s}-3\alpha/(m_q^b m_s^b)]$
$T_{cb\bar{s}}(7320)$	$[[\bar{q}\bar{s}]^3_0[cb]^3_1]^{\frac{1}{2}}_1$	$\frac{1}{2}(1^+)$	$m_q^b+m_s^b+m_{[cb]^{\bar{3}}_1}-(2/3)[B_{[cb]^{\bar{3}}_1 s}-3\alpha/(m_q^b m_s^b)]$
$T_{cb\bar{s}}(7425)$	$[[\bar{q}\bar{s}]^3_1[cb]^3_0]^{\frac{1}{2}}_1$	$\frac{1}{2}(1^+)$	$m_q^b+m_s^b+m_{[cb]^{\bar{3}}_0}-(2/3)[B_{[cb]^{\bar{3}}_0 s}+\alpha/(m_q^b m_s^b)]$
$T_{cb\bar{s}}(7440)$	$[[\bar{q}\bar{s}]^3_1[cb]^3_1]^{\frac{1}{2}}_0$	$\frac{1}{2}(0^+)$	$m_q^b+m_s^b+m_{[cb]^{\bar{3}}_1}-(2/3)[B_{[cb]^{\bar{3}}_1 s}+\alpha/(m_q^b m_s^b)-4\alpha/(m_q^b m_{[cb]^{\bar{3}}_1})-4\alpha/(m_s^b m_{[cb]^{\bar{3}}_1})]$
$T_{cb\bar{s}}(7449)$	$[[\bar{q}\bar{s}]^3_1[cb]^3_1]^{\frac{1}{2}}_1$	$\frac{1}{2}(1^+)$	$m_q^b+m_s^b+m_{[cb]^{\bar{3}}_1}-(2/3)[B_{[cb]^{\bar{3}}_1 s}+\alpha/(m_q^b m_s^b)-2\alpha/(m_q^b m_{[cb]^{\bar{3}}_1})-2\alpha/(m_s^b m_{[cb]^{\bar{3}}_1})]$
$T_{cb\bar{s}}(7468)$	$[[\bar{q}\bar{s}]^3_1[cb]^3_1]^{\frac{1}{2}}_2$	$\frac{1}{2}(2^+)$	$m_q^b+m_s^b+m_{[cb]^{\bar{3}}_1}-(2/3)[B_{[cb]^{\bar{3}}_1 s}+\alpha/(m_q^b m_s^b)+2\alpha/(m_q^b m_{[cb]^{\bar{3}}_1})+2\alpha/(m_s^b m_{[cb]^{\bar{3}}_1})]$

Results

Table: Doubly heavy tetraquark ground states aroused from heavy anti-baryons $\bar{\Omega}_c(\bar{\Omega}_b)$ with HADS

State	Configuration	J^P	Mass formula
$T_{cc\bar{s}\bar{s}}(4224)$	$[[\bar{s}\bar{s}]^3_1[cc]^{\bar{3}}_1]_0$	0^+	$2m_s^b + m_{[cc]^{\bar{3}}_1} - (2/3)[2B_{[cc]^{\bar{3}}_1 s} + \alpha/(m_s^b)^2 - 8\alpha/(m_s^b m_{[cc]^{\bar{3}}_1})]$
$T_{cc\bar{s}\bar{s}}(4239)$	$[[\bar{s}\bar{s}]^3_1[cc]^{\bar{3}}_1]_1$	1^+	$2m_s^b + m_{[cc]^{\bar{3}}_1} - (2/3)[2B_{[cc]^{\bar{3}}_1 s} + \alpha/(m_s^b)^2 - 4\alpha/(m_s^b m_{[cc]^{\bar{3}}_1})]$
$T_{cc\bar{s}\bar{s}}(4270)$	$[[\bar{s}\bar{s}]^3_1[cc]^{\bar{3}}_1]_2$	2^+	$2m_s^b + m_{[cc]^{\bar{3}}_1} - (2/3)[2B_{[cc]^{\bar{3}}_1 s} + \alpha/(m_s^b)^2 + 4\alpha/(m_s^b m_{[cc]^{\bar{3}}_1})]$
$T_{bb\bar{s}\bar{s}}(10740)$	$[[\bar{s}\bar{s}]^3_1[bb]^{\bar{3}}_1]_0$	0^+	$2m_s^b + m_{[bb]^{\bar{3}}_1} - (2/3)[2B_{[bb]^{\bar{3}}_1 s} + \alpha/(m_s^b)^2 - 8\alpha/(m_s^b m_{[bb]^{\bar{3}}_1})]$
$T_{bb\bar{s}\bar{s}}(10745)$	$[[\bar{s}\bar{s}]^3_1[bb]^{\bar{3}}_1]_1$	1^+	$2m_s^b + m_{[bb]^{\bar{3}}_1} - (2/3)[2B_{[bb]^{\bar{3}}_1 s} + \alpha/(m_s^b)^2 - 4\alpha/(m_s^b m_{[bb]^{\bar{3}}_1})]$
$T_{bb\bar{s}\bar{s}}(10755)$	$[[\bar{s}\bar{s}]^3_1[[bb]^{\bar{3}}_1]_2$	2^+	$2m_s^b + m_{[bb]^{\bar{3}}_1} - (2/3)[2B_{[bb]^{\bar{3}}_1 s} + \alpha/(m_s^b)^2 + 4\alpha/(m_s^b m_{[bb]^{\bar{3}}_1})]$
$T_{cb\bar{s}\bar{s}}(7508)$	$[[\bar{s}\bar{s}]^3_1[cb]^{\bar{3}}_0]_1$	1^+	$2m_s^b + m_{[cb]^{\bar{3}}_0} - (2/3)[2B_{[cb]^{\bar{3}}_0 s} + \alpha/(m_s^b)^2]$
$T_{cb\bar{s}\bar{s}}(7527)$	$[[\bar{s}\bar{s}]^3_1[cb]^{\bar{3}}_1]_0$	0^+	$2m_s^b + m_{[cb]^{\bar{3}}_1} - (2/3)[2B_{[cb]^{\bar{3}}_1 s} + \alpha/(m_s^b)^2 - 8\alpha/(m_s^b m_{[cb]^{\bar{3}}_1})]$
$T_{cb\bar{s}\bar{s}}(7535)$	$[[\bar{s}\bar{s}]^3_1[cb]^{\bar{3}}_1]_1$	1^+	$2m_s^b + m_{[cb]^{\bar{3}}_1} - (2/3)[2B_{[cb]^{\bar{3}}_1 s} + \alpha/(m_s^b)^2 - 4\alpha/(m_s^b m_{[cb]^{\bar{3}}_1})]$
$T_{cb\bar{s}\bar{s}}(7550)$	$[[\bar{s}\bar{s}]^3_1[cb]^{\bar{3}}_1]_2$	2^+	$2m_s^b + m_{[cb]^{\bar{3}}_1} - (2/3)[2B_{[cb]^{\bar{3}}_1 s} + \alpha/(m_s^b)^2 + 4\alpha/(m_s^b m_{[cb]^{\bar{3}}_1})]$

Mass Spectrum



$D_sD_s << T_{cc\bar{s}\bar{s}}(4224) \approx D_s^*D_s^*(4224)$

$D_s^*D_s^* << T_{cc\bar{s}\bar{s}}(4270)$

$DD_s < D^*D_s^* < T_{cc\bar{s}}(4127)$

$DD_s^* < D^*D_s < D^*D_s^* < T_{cc\bar{s}}(4146)$

$D^*D_s^* < T_{cc\bar{s}}(4184)$

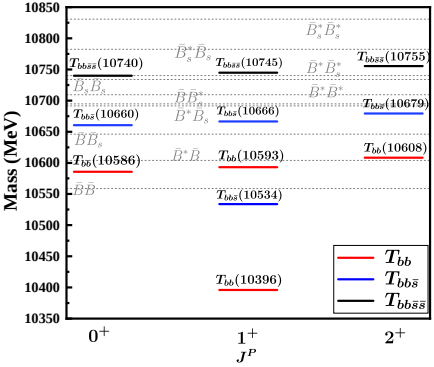
$DD < D^*D^* < T_{cc}(4035)$

$D^*D < D^*D^* < T_{cc}(4058)$

$T_{cc}(3876) \approx D^*D << D^*D^*$

$D^*D^* << T_{cc}(4103)$

Mass Spectrum



$T_{bb\bar{s}\bar{s}}(10745) < \bar{B}_s^*\bar{B}_s << \bar{B}_s^*\bar{B}_s^*$

$T_{bb\bar{s}\bar{s}}(10755) << \bar{B}_s^*\bar{B}_s^*$

$T_{bb\bar{s}}(10666) < \bar{B}^*\bar{B}_s < \bar{B}\bar{B}_s^* < \bar{B}^*\bar{B}_s^*$

$T_{bb\bar{s}}(10534) << \bar{B}^*\bar{B}_s < \bar{B}\bar{B}_s^* < \bar{B}^*\bar{B}_s^*$

$T_{bb\bar{s}}(10679) << \bar{B}^*\bar{B}_s^*$

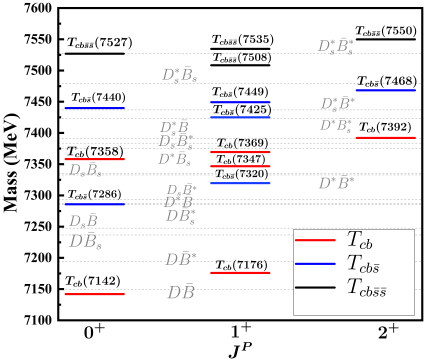
$T_{bb}(10593) < \bar{B}^*\bar{B} << \bar{B}^*\bar{B}^*$

$T_{bb}(10396) << \bar{B}^*\bar{B} < \bar{B}^*\bar{B}^*$

$T_{bb}(10608) << \bar{B}^*\bar{B}^*$

.

Mass Spectrum



$D_s\bar{B}_s << T_{cb\bar{s}\bar{s}}(7527) \approx D_{\bar{s}}^*\bar{B}_{\bar{s}}^*$

$D_s\bar{B}_{\bar{s}}^* << D_s^*\bar{B}_s < D_s^*\bar{B}_s^* < T_{cb\bar{s}\bar{s}}(7535)$

$D_{\bar{s}}^*\bar{B}_s^* < T_{cb\bar{s}\bar{s}}(7550)$

$D\bar{B}_s < D_s\bar{B} << D^*\bar{B}_s^* < D_{\bar{s}}^*\bar{B}^* < T_{cb\bar{s}}(7440)$

$D\bar{B}_{\bar{s}}^* < D^*\bar{B}_s < D_s^*\bar{B} < D^*\bar{B}_s^* < D_s^*\bar{B}^* < T_{cb\bar{s}}(7449)$

$D^*\bar{B}_s^* < D_s^*\bar{B}^* < T_{cb\bar{s}}(7468)$

$D\bar{B} << D^*\bar{B}^* < T_{cb}(7358)$

$T_{cb}(7142) < D\bar{B} << D^*\bar{B}^*$

$D\bar{B}^* < D^*\bar{B} < D^*\bar{B}^* < T_{cb}(7369)$

$D\bar{B}^* < D^*\bar{B} < D^*\bar{B}^* < T_{cb}(7347)$

$T_{cb}(7176) < D\bar{B}^* < D^*\bar{B} < D^*\bar{B}^*$

$D^*\bar{B}^* << T_{cb}(7392)$

Outline

1. Background

2. Theory and Framework

3. Calculation Procedure

4. Calculation Results and Figures

5. Summary

Summary

- We calculate the masses of the teraquarks which have the HADS partners with HADS and constituent quark model.
- Our analysis of the mass spectrum reveals that tetraquark states containing the b quark are more likely to have lower masses compared to their corresponding meson pair.
- The binding of the b quark is stronger than that of the c quark.

Thank you very much!

References

[1] R. Aaij *et al.* (LHCb), Nature Phys. **18**, 751 (2022), arXiv:2109.01038 [hep-ex] .

[2] M.-L. Du, V. Baru, X.-K. Dong, A. Filin, F.-K. Guo, C. Hanhart, A. Nefediev, J. Nieves, and Q. Wang, Physical Review D **105** (2022), 10.1103/physrevd.105.014024.

[3] T. Kinugawa and T. Hyodo, Physical Review C **109** (2024), 10.1103/physrevc.109.045205.

[4] M. Albaladejo, Physics Letters B **829**, 137052 (2022).

[5] G.-J. Wang, Z. Yang, J.-J. Wu, M. Oka, and S.-L. Zhu, Science Bulletin **69**, 3036–3041 (2024).

[6] T.-W. Wu and Y.-L. Ma, Phys. Rev. D **107**, L071501 (2023), arXiv:2211.15094 [hep-ph] .

[7] X.-Z. Weng, W.-Z. Deng, and S.-L. Zhu, Chinese Physics C **46**, 013102 (2022).

[8] Y.-X. Song and D.-J. Jia, Communications in Theoretical Physics **75**, 055201 (2023).

[9] S.-Y. Li, Y.-R. Liu, Z.-L. Man, Z.-G. Si, and J. Wu, Physical Review D **110** (2024), 10.1103/physrevd.110.094044.

[10] M. Karliner, B. Keren-Zur, H. J. Lipkin, and J. L. Rosner, Annals Phys. **324**, 2 (2009), arXiv:0804.1575 [hep-ph] .

[11] M. Karliner and J. L. Rosner, Phys. Rev. D **90**, 094007 (2014), arXiv:1408.5877 [hep-ph] .