Studying time-like proton form factors using vortex state $p\bar{p}$ annihilation

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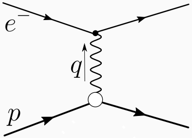
Quantum effects in atomic and particle physics, Zhuhai

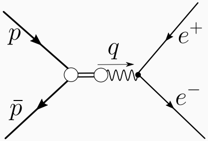
Proton electromagnetic form factors

Form factors describe non-point-like interaction: $\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{pl} |F(q)|^2$

Space-like ($q^2 < 0$)



Time-like ($q^2 > 0$)



- Real valued function
- Has intuitive interpretation: Fourier transform of electric and magnetization distributions

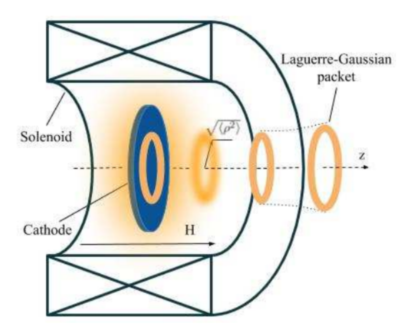
$$\rho(\vec{x}) = \frac{Ze}{(2\pi)^2} \int d^3q F(q) e^{-i\vec{q}\cdot\vec{x}}$$

- No intuitive meaning
- Complex valued: sources of imaginary part — intermediate particles and excitations
- Relative phase is unknown. Can be measured in experiments with polarized beams (not done).

Why vortex protons?

Until now: produced vortex states have small energy (photons, electrons, neutrons and atoms)
Why consider p and even $\bar{p}$?

- high proton mass $\rightarrow$ access to many hadronic reactions
- a milestone on the road to use vortex states in DIS to probe of nucleon structure (spin crisis, OAM of quarks and gluons)
- proposals to generate vortex ions (Karlovets, New J.Phys.23, 033048)



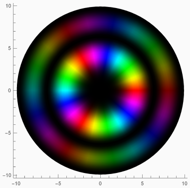
Simple scalar Bessel state

Coordinate space

In cylindrical coordinates (ρ, ϕ, z) for a scalar particle propagating along z direction:

$$\Psi(r,t) = \frac{N}{\sqrt{2E}}\psi_{l,\varkappa}(\mathbf{r})e^{-iEt+ik_z z},$$

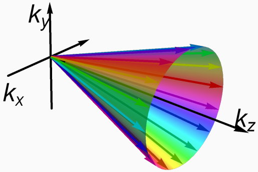
$$\psi_{l,\varkappa}(\mathbf{r}) = \sqrt{\frac{\varkappa}{2\pi}}J_l(\rho\varkappa)e^{il\phi_r},$$



Momentum space

$$\psi_{l,\varkappa}(\mathbf{r}) = \int \frac{d^2\mathbf{k}}{(2\pi)^2}a_{l,\varkappa}(\mathbf{k})e^{i\mathbf{k}\cdot\mathbf{r}},$$

$$a_{l,\varkappa}(\mathbf{k}) = (-i)^l e^{il\phi_k}\sqrt{\frac{2\pi}{\varkappa}}\delta(|\mathbf{k}|-\varkappa).$$



Vortex fermion

- Vortex Bessel fermion can be constructed using PW basis.
  PW basis: A plane wave fermion with $k^\mu = (E,\, \mathbf{k},\, k_z)$, where $\mathbf{k} = |\mathbf{k}|(\cos\phi_k,\, \sin\phi_k)$, $k_z = |\vec{k}|\cos\theta$, and helicity $\lambda = \pm 1/2$:

$$\Psi_{k\lambda}(r) = \frac{N_{\rm PW}}{\sqrt{2E}}\, u_{k\lambda}\, e^{-i\vec{k}\vec{r}}\,.$$

$$u_{k\lambda} = \begin{pmatrix} \sqrt{E+m}\, w^{(\lambda)} \\ 2\lambda\sqrt{E-m}\, w^{(\lambda)} \end{pmatrix},\, v_{k\lambda} = \begin{pmatrix} -\sqrt{E-m}\, w^{(-\lambda)} \\ 2\lambda\sqrt{E+m}\, w^{(-\lambda)} \end{pmatrix},$$

$$w^{(+1/2)} = \begin{pmatrix} c_i\, e^{-i\phi_i/2} \\ s_i\, e^{i\phi_i/2} \end{pmatrix},\, w^{(-1/2)} = \begin{pmatrix} -s_i\, e^{-i\phi_i/2} \\ c_i\, e^{i\phi_i/2} \end{pmatrix},$$

  where $c_i \equiv \cos(\theta_i/2)$, $s_i \equiv \sin(\theta_i/2)$.
- Construction of Bessel vortex state using PW basis:

$$\Psi_{\varkappa m k_z \lambda}(r) = \frac{N_{\rm Bes}}{\sqrt{2E}} \int \frac{d^2\mathbf{k}}{(2\pi)^2}\, a_{\varkappa m}(\mathbf{k})\, u_{k\lambda}\, e^{-i\vec{k}\cdot\vec{r}}, \qquad a_{\varkappa m}(\mathbf{k}) = (-i)^m\, e^{im\phi_k} \sqrt{\frac{2\pi}{\varkappa}}\, \delta(|\mathbf{k}| - \varkappa)\,.$$

Scattering of two scalar Bessel states

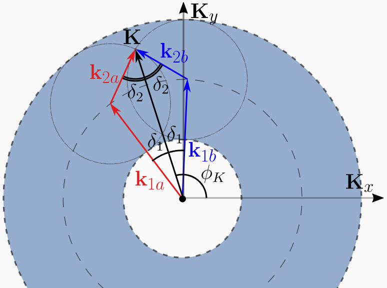
1st particle: $k_1 = (E_1, \mathbf{k}_1, k_{1z})$ with OAM m_1; 2nd particle: $k_2 = (E_2, \mathbf{k}_2, k_{2z})$ with OAM m_2

The S-matrix element:

$$S = \frac{N_{Bes}^2}{N_{PW}^2} \int \frac{d^2\mathbf{k}_1}{(2\pi)^2} \frac{d^2\mathbf{k}_2}{(2\pi)^2} a_{\varkappa_1, m_1}(\mathbf{k}_1) a_{\varkappa_2, -m_2}(\mathbf{k}_2) S_{PW} = \frac{i(2\pi)^4 \delta(E)\delta(k_z)}{\sqrt{16 E_1 E_2 E_3 E_4}} N_{Bes}^2 N_{PW}^2 \frac{(-i)^{m_1 - m_2}}{(2\pi)^3 \sqrt{\varkappa_1 \varkappa_2}} \mathcal{J}$$

Vortex scattering amplitude:

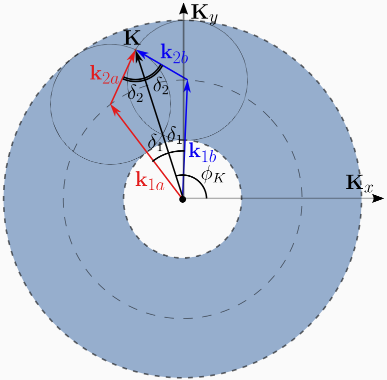
$$\mathcal{J} = \int d^2\mathbf{k}_1 d^2\mathbf{k}_2 e^{i(m_1\phi_1 - m_2\phi_2)} \delta(|\mathbf{k}_1| - \varkappa_1)\delta(|\mathbf{k}_2| - \varkappa_2)\delta^{(2)}(\mathbf{k}_1 + \mathbf{k}_2 - \mathbf{K})\mathcal{M}(k_1, k_2, k_3, k_4)$$



The dynamics is determined by $\mathcal{J}$. The integral in $\mathcal{J}$ is nonzero only when:

$$|\kappa_1 - \kappa_2| \leq |\mathbf{K}| < \kappa_1 + \kappa_2$$

Scattering of 2 Bessel states



configuration a: $\phi_1 \to \phi_K + \delta_1 \quad \phi_2 \to \phi_K - \delta_2$

configuration b: $\phi_1 \to \phi_K - \delta_1 \quad \phi_2 \to \phi_K + \delta_2$

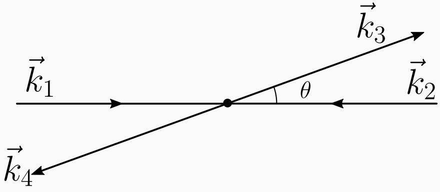
$$\delta_1 = \arccos\left(\frac{\kappa_1^2 - \kappa_2^2 + \mathbf{K}^2}{2\kappa_1|\mathbf{K}|}\right) \quad \delta_2 = \arccos\left(\frac{\kappa_2^2 - \kappa_1^2 + \mathbf{K}^2}{2\kappa_2|\mathbf{K}|}\right)$$

$$\mathcal{J} = \frac{e^{i(m_1 - m_2)\phi_K}}{\sin(\delta_1 + \delta_2)}\left(\mathcal{M}_a e^{i(m_1\delta_1 + m_2\delta_2)} + \mathcal{M}_b e^{-i(m_1\delta_1 + m_2\delta_2)}\right)$$

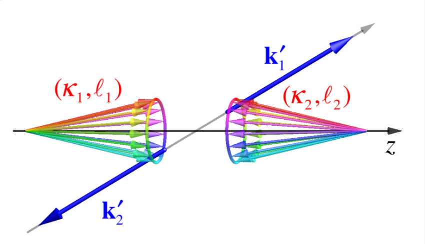
Cross section: $d\sigma \propto |\mathcal{J}|^2 d^2\mathbf{k}_3 d^2\mathbf{k}_4$

Kinematics of final 2 particle state

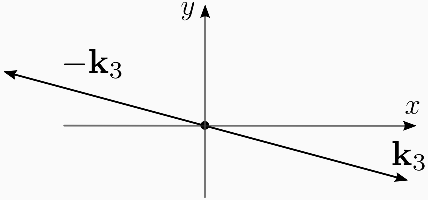
Plane wave scattering



Double vortex scattering



Transverse momenta plane



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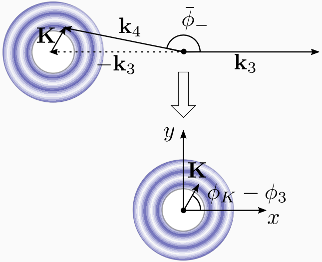

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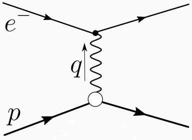

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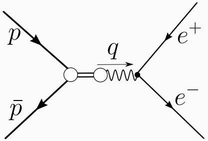
Time-like ($q^2 > 0$)



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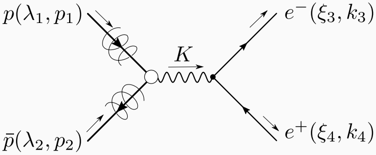
Proton electromagnetic form factors

The plane wave amplitude is product of hadron J^μ and lepton L_μ currents.



$$\mathcal{M} = \frac{e^2}{s} J^\mu L_\mu,$$

Nucleon current J^μ is

$$\bar{v}_{\lambda_2}(k_2)\left[\gamma^\mu F_1(s) + \frac{F_2(s)}{2M}\sigma^{\mu\nu}K_\nu\right]u_{\lambda_1}(k_1) = \bar{v}_{\lambda_2}(k_2)\left[\gamma^\mu G_M(s) + \frac{P^\mu}{2M}\frac{G_M(s) - G_E(s)}{1-\tau}\right]u_{\lambda_1}(k_1),$$

$$G_E = F_1 + \tau F_2, \qquad G_M = F_1 + F_2.$$

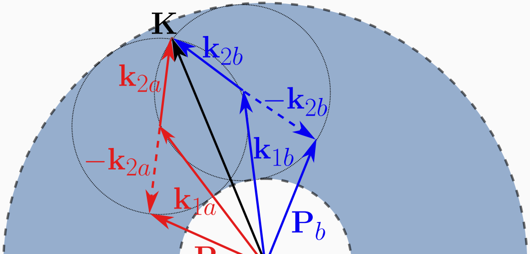
where $\sigma_{\mu\nu} = [\gamma_\mu, \gamma_\nu]/2$, $K = k_1 + k_2$, $s = K^2$, $P = k_2 - k_1$, and $\tau = K^2/4M^2$.

Proton electromagnetic form factors

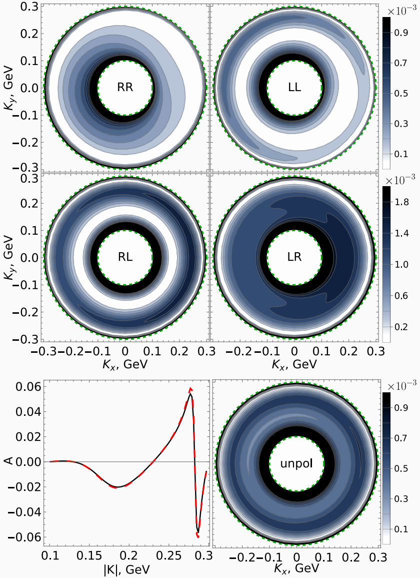
$$\mathcal{J} = \frac{e^{i(m_1 - m_2)\phi_K}}{\sin(\delta_1 + \delta_2)} \left(\mathcal{M}_a e^{i(m_1\delta_1 + m_2\delta_2)} + \mathcal{M}_b e^{-i(m_1\delta_1 + m_2\delta_2)} \right).$$

$$J^\mu = \bar{v}_{\lambda_2}(k_2) \left[\gamma^\mu G_M(s) + \frac{P^\mu}{2M} \frac{G_M(s) - G_E(s)}{1 - \tau} \right] u_{\lambda_1}(k_1),$$

$$P = k_2 - k_1$$



Results



Parameters:

$$m_1 = 7/2; \qquad m_2 = 3/2;$$
$$E_1 = 1.2 \text{ GeV}, \qquad E_2 = \sqrt{\kappa_2^2 + M^2 + p_{1z}^2}$$
$$\kappa_1 = 0.2 \text{ GeV}, \qquad \kappa_2 = 0.1 \text{ GeV};$$
$$K_z = 0, \qquad |\mathbf{k}_3| = 0.8 \text{ GeV}$$

$$A = \frac{\int d\phi_K |\mathcal{J}|^2 \sin(\phi_K - \phi_3)}{\int d\phi_K |\mathcal{J}|^2}.$$

Few observations:

- number of fringes is defined by angular momenta of initial states (m_1, m_2)
- Different Mandelstam s in one setup $s = (E_1 + E_2)^2 - \mathbf{K}^2$

Helicity structure

$$\mathcal{M} = \mathcal{M}_{\parallel} + \mathcal{M}_{\perp}$$

$$|\mathcal{J}|^2 = |\mathcal{J}_{\parallel} + \mathcal{J}_{\perp}|^2 = |\mathcal{J}_{\parallel}|^2 + |\mathcal{J}_{\perp}|^2 + 2\mathrm{Re}[\mathcal{J}_{\parallel}\mathcal{J}_{\perp}^{\dagger}].$$

$$\mathfrak{C}^{m_1}_{m_2} = \cos(m_1\delta_1 + m_2\delta_2), \quad V_{\pm} = \sqrt{E_1^+ E_2^-} \pm \sqrt{E_1^- E_2^+}, \quad W_{\pm} = \sqrt{E_1^+ E_2^+} \pm \sqrt{E_1^- E_2^-}.$$

RR/LL

$$\sum_{\xi} |\mathcal{J}_{\parallel}|^2 \propto 32 E_3 E_4 \sin^2\theta_3 (\mathfrak{C}^{m_1-\lambda}_{m_2-\lambda})^2 |G_M|^2 \times |W_- - \frac{1 - G_E/G_M}{2M(1-\tau)} V_+ P_z|^2.$$

RL/LR

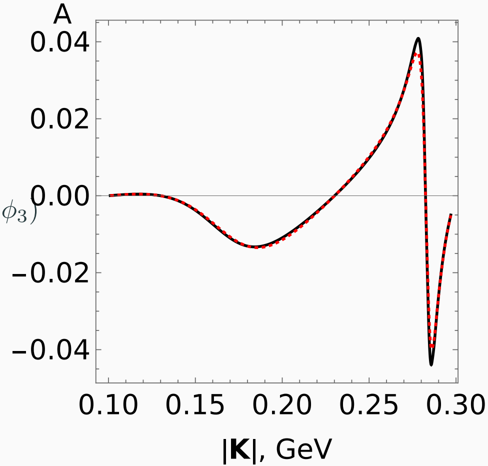
$$\sum_{\xi} |\mathcal{J}_{\gamma,\perp}|^2 = 32 |G_M|^2 W_+^2 E_3 E_4 (\mathfrak{C}^{m_1-\lambda}_{m_2+\lambda})^2 (1 + \cos^2\theta_3).$$

Helicity structure

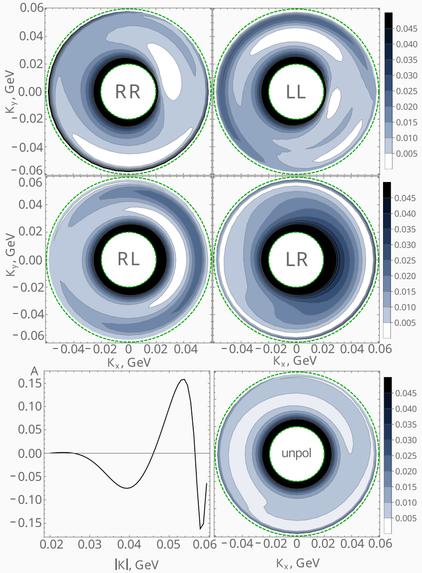
Numerator, RR/LL keeping only terms $\propto \sin(\phi_K - \phi_3)$

$$\sum_{\xi} \mathrm{Re}\{\mathcal{J}_{\parallel}\mathcal{J}_{\perp}^{*}\} \propto 16E_3E_4\sin(2\theta_3)|G_M|^2\sin(\phi_K-\phi_3)$$
$$\times\, 2\lambda|\mathbf{K}|\left(\mathfrak{C}^{m_1-\lambda}_{m_2-\lambda}\right)^2 \mathrm{Im}\{G_E/G_M\}\frac{(|\vec{k}_1|+|\vec{k}_2|)}{1-\tau}.$$

in approximation $\mathbf{K} \ll \kappa_{3,4}$



Non-relativistic regime



Parameters:

$$m_1 = 7/2, \qquad m_2 = 3/2,$$
$$E_1 = 939 \text{ MeV}, \qquad E_2 = \sqrt{\varkappa_2^2 + M^2 + k_{1z}^2},$$
$$\varkappa_1 = 40 \text{ MeV}, \qquad \varkappa_2 = 20 \text{ MeV},$$
$$K_z = 0,$$

The asymmetry is not suppressed by the hadron mass in non-relativistic regime, but defined by how non-paraxial are initial vortex states. Therefore, this method is feasible in low energy experiments.

Form factor vs charge distribution from wave function

Possible misconception if think about form factors as Fourier transform of electric and magnetization distributions

$$\rho(\vec{x}) = \frac{Ze}{(2\pi)^2} \int d^3 q F(q) e^{-i\vec{q}\cdot\vec{x}}$$

Question: how charge distribution of non-point-like particle interplays with distribution encoded by wave function?

Example: electron cloud in atoms - point-like Mott scattering cross section can be reduced by poor overlaping of wave functions, but interaction in every point is with same strength α_{em} (at all energy scales)

Form factor gives interaction strength dependency at every point (effect is independent from wave function overlapping)

Conclusions

- It is possible to extract the relative phase shift of proton electromagnetic form factors in $p\bar{p}$ annihilation
- The asymmetry of differential cross section is proportional to relative form factor phase
- allows measurement at different Mandelstam s in one setup, useful if phase changes rapidly
- The asymmetry is not suppressed in non-relativistic limit, therefore can be measured in experiments with low energy vortex hadrons.