Properties of vortex muon decay: from rotational symmetric states to asymmetric spin-orbit states

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Cataloge

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2. Vortex muon decay
   - Rotational symmetric case
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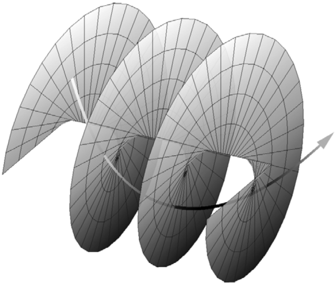
Vortex states are non plane wave solutions (solved in coordinate space) of equation of motion with intrinsic orbital angular momentum.

- Phase factor $e^{i\,\ell\varphi}$ with topological charge $\ell \propto$ OAM;
- Helical phase front;
- Phase singularities on a line.

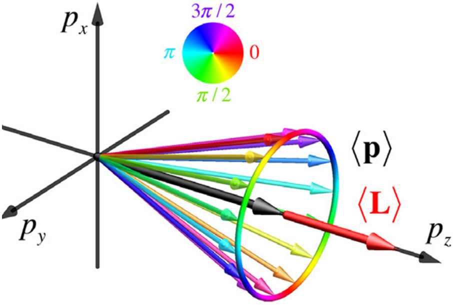


Experimental realizations

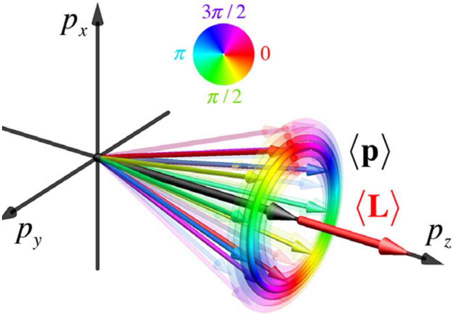
Vortex photon, electron, neutron, atom and so on.

Rotational symmetric momentum space distribution

Bessel type:



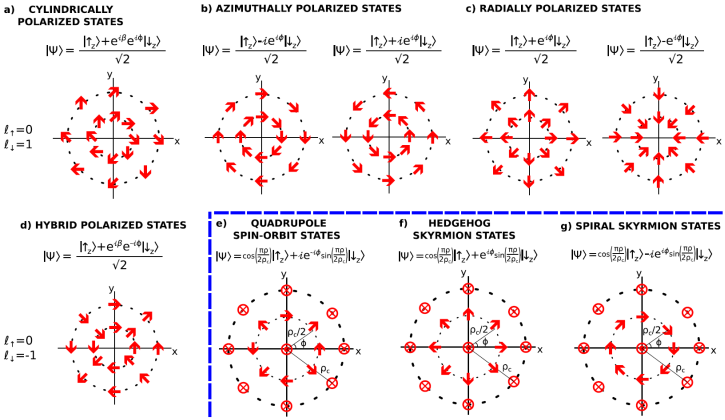
LG type:



Ref: Bliokh K Y, et al. Physics Reports 690 (2017): 1-70.

Important kinetic properties of vortex states

Spin-orbit states: Various polarization possibilities.



Ref: Sarenac D, et al. New Journal of Physics 20 (2018): 103012.

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Vortex muon decay in rotational symmetric case

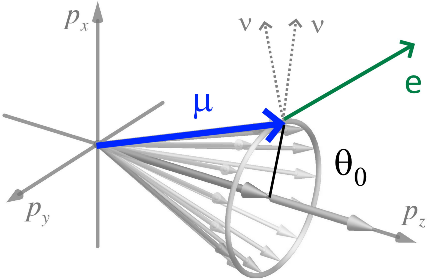
The rotational symmetric states we use

TAM eigenstates with rotational symmetric variables:

- The amplitude in coordinate space;
- The momentum distribution;
- The polarization distribution.

Specially for Bessel vortex muon:

$$d\Gamma_V = \int \frac{d\varphi_p}{2\pi} d\Gamma_{PW}(\vec{p}).$$



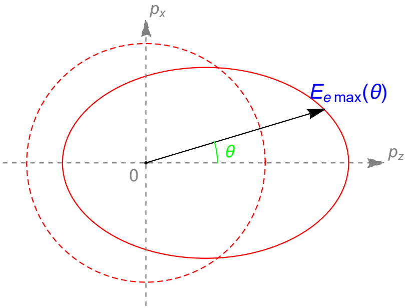
Ref: Ivanov I P. Physical Review D 83 (2011): 093001.

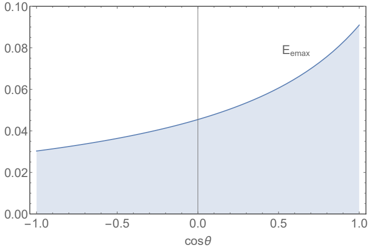
Kinematic limitation: plane wave muon decay

For different outgoing angle θ, there is only one threshold:

$$E_e < E_{e\,\max} = \frac{m^2}{2E(1-\beta\,\cos\theta)}, \quad \beta = \sqrt{E^2 - m^2}/E,$$

where electron mass is ignored.

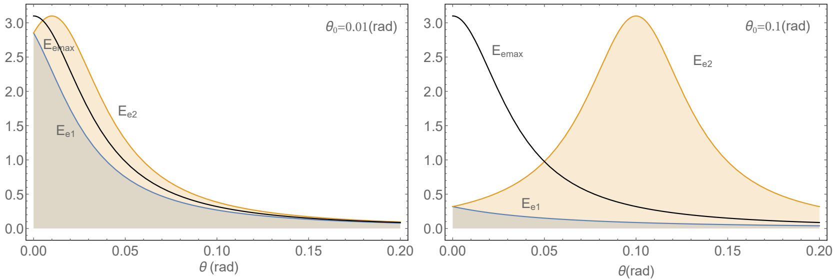




Kinematic limitation: vortex muon decay

For different outgoing angle θ, there are two thresholds:

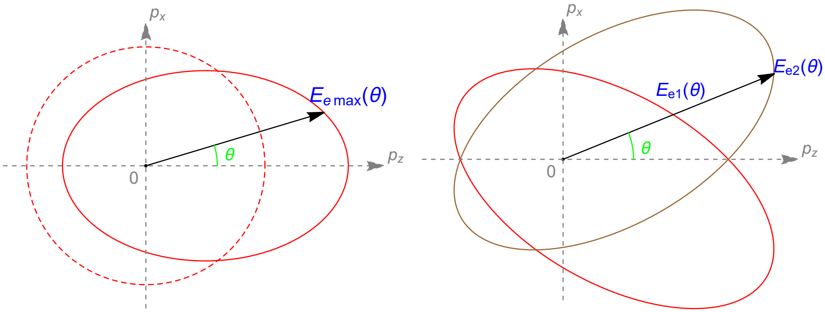
$$E_{e1} = \frac{m^2}{2E(1-\beta\cos(\theta+\theta_0))}, \; E_{e2} = \frac{m^2}{2E(1-\beta\cos(\theta-\theta_0))}.$$



Plane wave case vs Vortex case

Kinematic limitation: vortex muon decay

- $0 < E_e < E_{e1} \rightarrow$ All plane wave components have contribution.
- $E_{e1} < E_e < E_{e2} \rightarrow$ Some plane wave components have contribution.



Plane wave case vs Vortex case

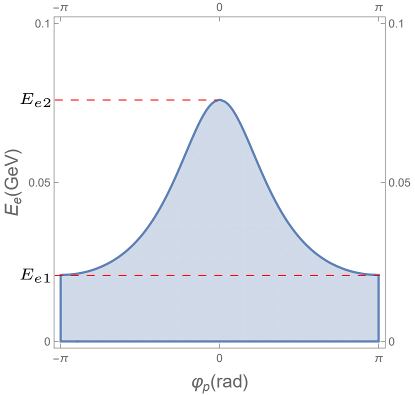
Kinematic limitation: vortex muon decay

At different electron energy and fixed outgoing angle, azimuth angle of plane wave components that have contribution to result satisfies

$$-\pi \leq -\tau \leq \varphi_{\mathbf{p}} \leq \tau \leq \pi,$$

where τ is determined by E_e:

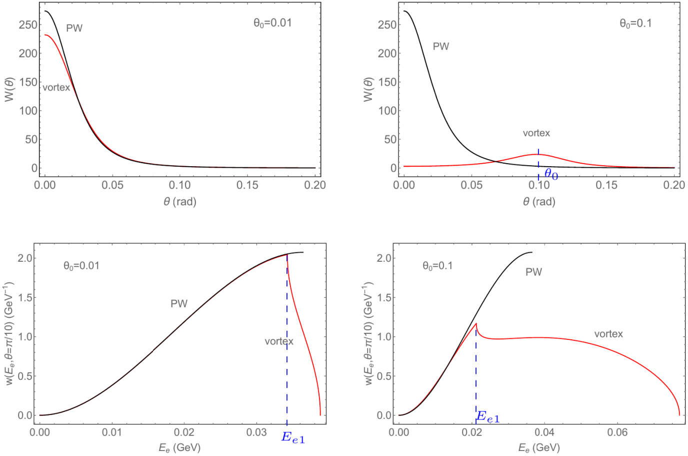
$$\cos\tau = \frac{(E_e - E_{e1})E_{e2} - (E_{e2} - E_e)E_{e1}}{E_e(E_{e2} - E_{e1})}$$



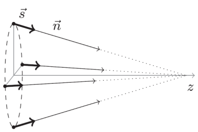
Value range of "$E_e - \varphi_{\mathbf{p}}$" with $E = 3.1$ GeV, $\theta_0 = 0.1$ and $\theta = \pi/10$.

Results for unpolarized case

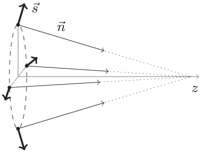
Angular distribution (rotational symmetric) vs Energy spectrum



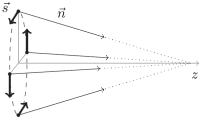
Differential decay width of vortex muon at $E = 3.01$GeV, $\theta_0 = 0.01 \approx 0.6^\circ$ (left) and $\theta_0 = 0.1 \approx 6^\circ$ (right), angular distribution (up) vs energy spectrum (down).



Parallel polarization

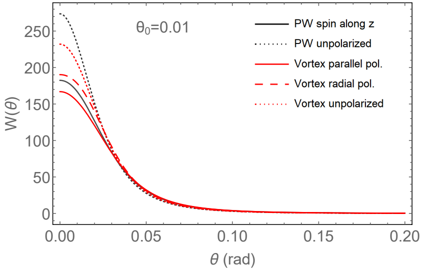


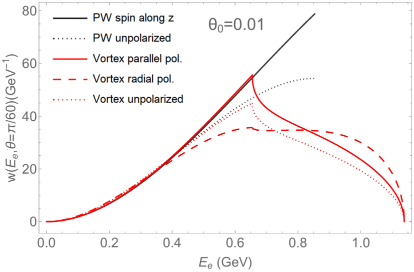
Radial polarization



Azimuth polarization

Angular distribution vs Energy spectrum:





$\theta_0 = 0.01 \approx 0.6^\circ$ (all), $\theta = 3^\circ$ (right).

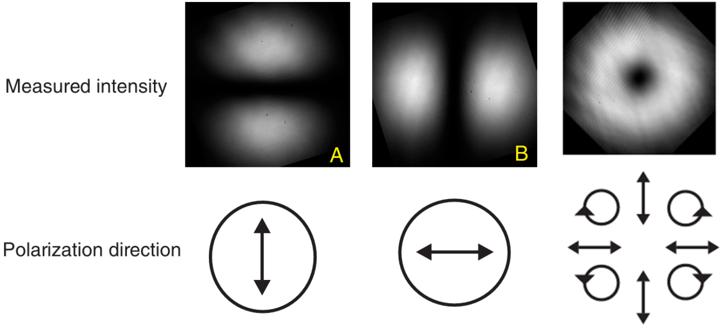
Conclusions on rotational symmetric vortex muon decay

- Rotational symmetric electron distribution.
- Single peak for electron angular distribution near conical angle.
- Longer energy spectrum at certain angles.
- Energy spectrum at relatively high energy region significantly reveals vortex and polarization characteristics.

Vortex muon decay in rotational asymmetric case

The rotational asymmetric spin-orbit states

- The amplitude in coordinate space is symmetric or asymmetric;
- The momentum distribution is symmetric;
- The polarization distribution is asymmetric.

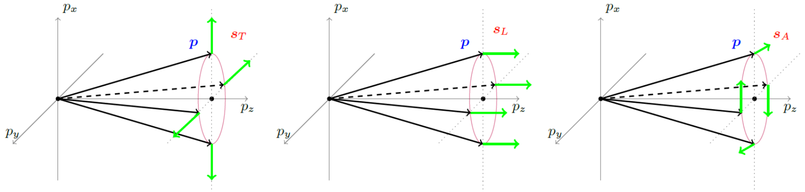


Ref: Christian Maurer, et al. New Journal of Physics 9 (2027): 78.

Constructing vortex muon with Z_n symmetry:

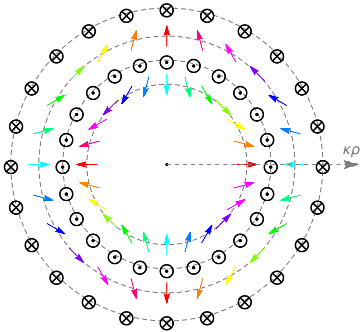
Polarization of plane wave components: $s^\nu = (S^0, \mathbf{S})$, where

$$S^0 = \beta\,(\mathbf{n}\cdot\mathbf{S}),$$
$$\mathbf{S} = a_p(\varphi_p)\,\mathbf{s}_T + b_p(\varphi_p)\,\mathbf{s}_L + c_p(\varphi_p)\,\mathbf{s}_A.$$



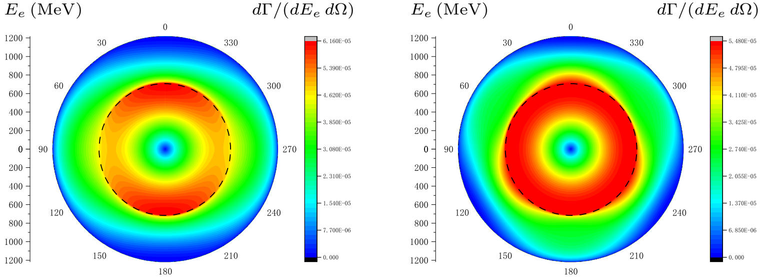
To get the state we want, we can just set $a_p(\varphi_p)$, $b_p(\varphi_p)$, $c_p(\varphi_p)$ as periodic functions of φ_p. Common period $(2\pi/n)$ of them means Z_n symmetry.

Polarization distribution in transverse plane for vortex muon with Z_n symmetry under paraxial approximation:



$(b_p = 0,\ a_p = \cos n\varphi_p,\ c_p = \sin n\varphi_p)$ with $n = 2$ (left) and $n = 3$ (right).

Results: Electron energy spectrum at certain θ



$(b_p = 0,\ a_p = \dfrac{\gamma\ \cos n\varphi_p}{\sqrt{\sin^2\theta_0 + \gamma^2\cos^2\theta_0}},\ c_p = \sin n\varphi_p)$ with $n = 2$ (left) and $n = 3$ (right). $E = 3.1$GeV, $\theta_0 = 0.01$, $\theta = \pi/60$, $\ell = 12$.

Conclusions on rotational asymmetric vortex muon decay

- Particular electron energy spectrum which is the same as symmetric case.
- Rotational asymmetric electron distribution which follows Z_n symmetry of initial muon.
- Significantly different behaviors for electron energy spectrum inside and outside E_{e1}.

Prospects

Prospects

- Thanks to the self-analyzing nature of its decay, vortex muon may serve as a good probe to study evolution of charged vortex particle in electromagnetic fields.
- The kind of vortex fermions with Z_n symmetry we constructed provides a good stage for study Z_n symmetry in decay or collision processes.
- Other processes (for both rotational symmetric and asymmetric case) that share some similar features with the process we have considered may get same results. For example, decay of the vortex neutron (See ref. [Wei Kou, Bing'ang Guo, Xurong Chen, arXiv:2407.06520] and [I. Pavlov, A. Chaikovskaia, D. Karlovets, arXiv:2411.16231]).