# Bell Nonlocality and Entanglement in $e^+e^- \to Y\bar{Y}$ at BESIII


**Sihao Wu(吴思浩)**

**University of Science and Technology of China**


**Based on [Phys. Rev. D 110, 054012]**

**In collaboration with:**

**Chen Qian (Baqis), Qun Wang (USTC), Xiao-Rong Zhou(USTC).**

# Outline

- **Motivation & Introduction.**
- **Bell Nonlocality and Entanglement for $Y\bar{Y}$ System.**
- **Quantum State Tomography & Experiment.**
- **Loopholes in HEP Experiment.**

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# When QIS Meets HEP

➢ **In history, researchers investigate the quantum information in atomic, optical and condensed matter physics.**


- **"for experiments with *entangled photons*, establishing the violation of *Bell inequalities* and pioneering quantum information science."**

➢ **In recent years, the development of theory and experiment makes it possible to investigate the quantum information in scale of particle physics at high-energy colliders (e.g. top quarks).**

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**Observation of quantum entanglement with top quarks at the ATLAS detector**

The ATLAS Collaboration

*Nature* **633**, 542–547 (2024) | Cite this article

- **"Quantum *entanglement* at the highest energy scale."**

# QIS in Top Quarks (2020 ~ Now)

## ➢ Theoretical Articles

- Y. Afik, J. R. M. de Nova, *Entanglement* and quantum tomography with *top quarks* at the LHC, Eur. Phys. J. Plus 136, 907 (2021).
- Y. Afik and J. R. M. de Nova, Quantum information with *top quarks* in QCD, Quantum 6, 820 (2022).
- Y. Afik and J. R. M. de Nova, Quantum Discord and Steering in *Top Quarks* at the LHC, Phys. Rev. Lett. 130, 221801 (2023).
- ……

## ➢ Experimental Articles

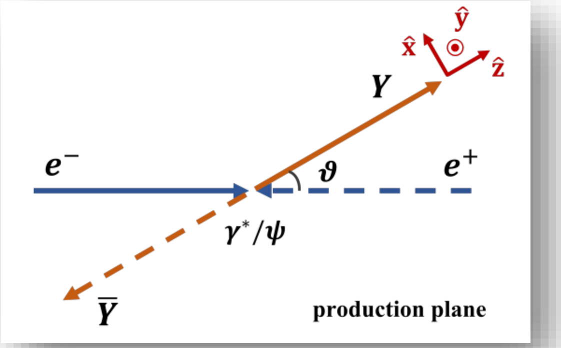
- The ATLAS Collaboration, Observation of quantum *entanglement* with *top quarks* at the ATLAS detector, Nature volume 633, pages542–547 (2024).
- The CMS Collaboration, Observation of quantum *entanglement* in *top quark* pair production in proton–proton collisions at $\sqrt{s} = 13$ TeV, Rep. Prog. Phys. 87 (2024).
- The CMS Collaboration, Measurements of polarization and spin correlation and observation of *entanglement* in *top quark* pairs using lepton + jets events from proton-proton collisions at $\sqrt{s} = 13$ TeV, Phys. Rev. D 110, 112016 (2024).

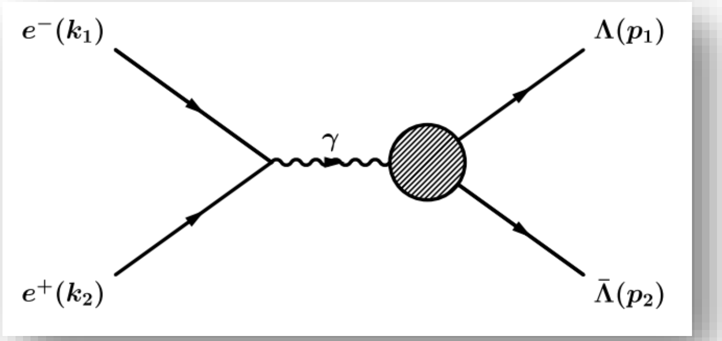
# Quantum correlations in Λ Hyperons (1980s ~ Now)

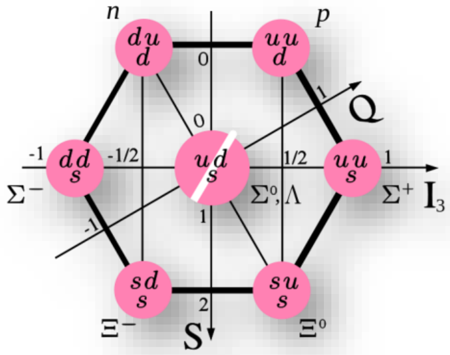
- In 1981, Törnqvist gave an "Suggestion for Einstein-Podolsky-Rosen experiments using reactions like $e^+e^- \to \Lambda\bar{\Lambda} \to p\pi^-\bar{p}\pi^+$" [N. A. Törnqvist, Found. Phys. 11, 171 (1981)].

  - N. A. Törnqvist, Phys. Lett. A 117, 1 (1986).
  - S. Baranov, J. Phys. G 35, 075002 (2008).
  - S. Chen, Y. Nakaguchi, and S. Komamiya, Prog. Theor. Exp. Phys. 2013, 063A01 (2013).
  - Y. Shi and J.-C. Yang, Eur. Phys. J. C 80, 116 (2020).
  - C. Qian, J.-L. Li, A. S. Khan, and C.-F. Qiao, Phys. Rev. D 101, 116004 (2020).
  - ……

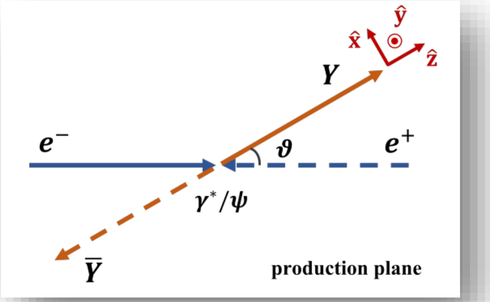
- Inspired by the studies of top quarks and the proposal given by Törnqvist, we try to study the "Bell nonlocality and entanglement in $e^+e^- \to Y\bar{Y}$ at BESIII"



# Spin Density Operator for $Y\bar{Y}$




- The two-qubit spin density operator for hyperon-antihyperon system reads [G. Fäldt and A. Kupsc, 2017; E. Perotti et al., 2019).] : $\rho_{Y\bar{Y}} = (1/4)\Theta_{\mu\nu}\sigma_\mu \otimes \sigma_\nu$

- cross section of $\Lambda\bar{\Lambda}$
- spin polarization of $\bar{\Lambda}$

$$\boxed{\Theta_{\mu\nu} = \frac{1}{1+\alpha_\psi \cos^2\vartheta}\left[\begin{array}{c|ccc} 1+\alpha_\psi\cos^2\vartheta & 0 & \beta_\psi \sin\vartheta\cos\vartheta & 0 \\ \hline 0 & \sin^2\vartheta & 0 & \gamma_\psi \sin\vartheta\cos\vartheta \\ \beta_\psi \sin\vartheta\cos\vartheta & 0 & -\alpha_\psi \sin^2\vartheta & 0 \\ 0 & \gamma_\psi \sin\vartheta\cos\vartheta & 0 & \alpha_\psi + \cos^2\vartheta \end{array}\right].}$$

- spin polarization of $\Lambda$
- spin-spin correlation

- Parameters

| | $\mathcal{B}(\times 10^{-4})$ | $\alpha_\psi$ | $\Delta\Phi/\mathrm{rad}$ | References |
|---|---|---|---|---|
| $\Lambda\bar{\Lambda}$ | 19.43(33) | 0.475(4) | 0.752(8) | [31,42] |
| $\Sigma^+\bar{\Sigma}^-$ | 15.0(24) | −0.508(7) | −0.270(15) | [43,44] |
| $\Xi^-\bar{\Xi}^+$ | 9.7(8) | 0.586(16) | 1.213(49) | [32,45] |
| $\Xi^0\bar{\Xi}^0$ | 11.65(4) | 0.514(16) | 1.168(26) | [46,47] |

$$\beta_\psi = \sqrt{1-\alpha_\psi^2}\sin(\Delta\Phi), \qquad \gamma_\psi = \sqrt{1-\alpha_\psi^2}\cos(\Delta\Phi).$$

- $\Delta\Phi$ is the relative phase between EMFFs for hyperons.

# Spin Density Operator for $Y\bar{Y}$

➢ **Two-qubit density operator for $Y\bar{Y}$**

$$\rho_{Y\bar{Y}} = \frac{1}{4}\left(\mathbb{1}\otimes\mathbb{1} + \mathbf{B}^{+}\cdot\boldsymbol{\sigma}\otimes\mathbb{1} + \mathbb{1}\otimes\mathbf{B}^{-}\cdot\boldsymbol{\sigma} + \sum_{i,j} C_{ij}\sigma_i\otimes\sigma_j\right),$$

$$\begin{aligned}\mathbf{B}^{+} &\equiv \mathrm{Tr}\left[\rho_{Y\bar{Y}}(\boldsymbol{\sigma}\otimes\mathbb{1})\right],\\ \mathbf{B}^{-} &\equiv \mathrm{Tr}\left[\rho_{Y\bar{Y}}(\mathbb{1}\otimes\boldsymbol{\sigma})\right],\\ C_{ij} &\equiv \mathrm{Tr}\left[\rho_{Y\bar{Y}}(\sigma_i\otimes\sigma_j)\right].\end{aligned}$$

➢ **$\Lambda(\bar{\Lambda})$ spin polarization & $\Lambda\bar{\Lambda}$ spin correlation**

$$B_y^{+} = B_y^{-} = \frac{\sqrt{1-\alpha_\psi^2}\,\sin(\Delta\Phi)\sin\vartheta\cos\vartheta}{1+\alpha_\psi\cos^2\vartheta},$$

- **spin polarization of $\Lambda$ and $\bar{\Lambda}$**

$$C_{xx} = \frac{\sin^2\vartheta}{1+\alpha_\psi\cos^2\vartheta},\qquad C_{yy} = \frac{-\alpha_\psi\sin^2\vartheta}{1+\alpha_\psi\cos^2\vartheta},\qquad C_{zz} = \frac{\alpha_\psi+\cos^2\vartheta}{1+\alpha_\psi\cos^2\vartheta},$$

$$C_{xz} = C_{zx} = \frac{\sqrt{1-\alpha_\psi^2}\,\cos(\Delta\Phi)\sin\vartheta\cos\vartheta}{1+\alpha_\psi\cos^2\vartheta}.$$

- **spin correlation of $\Lambda$ and $\bar{\Lambda}$**

# Local Unitary Equivalence & $X$ States

- Before our investigation, it is convenient to transform the two-qubit state to the ***X state*** by swapping the y and z axes and then diagonalizing $C_{ij}$ in $Y$ and $\bar{Y}$'s rest frame.

$$\rho^X_{Y\bar{Y}} = \underline{(U_Y \otimes U_{\bar{Y}}) \rho_{Y\bar{Y}} (U_Y \otimes U_{\bar{Y}})^\dagger} = \frac{1}{4}\left(\mathbb{1} \otimes \mathbb{1} + a\sigma_z \otimes \mathbb{1} + \mathbb{1} \otimes a\sigma_z + \sum_i t_i \sigma_i \otimes \sigma_i\right),$$

**Corresponding to**

$$\Theta^X_{\mu\nu} = \left[\begin{array}{c|ccc} 1 & 0 & 0 & a \\ \hline 0 & t_1 & 0 & 0 \\ 0 & 0 & t_2 & 0 \\ a & 0 & 0 & t_3 \end{array}\right],$$

$$a = \frac{\beta_\psi \sin\vartheta \cos\vartheta}{1 + \alpha_\psi \cos^2\vartheta},$$

$$t_{1,2} = \frac{1 + \alpha_\psi \pm \sqrt{(1 + \alpha_\psi \cos 2\vartheta)^2 - (\beta_\psi \sin 2\vartheta)^2}}{2(1 + \alpha_\psi \cos^2\vartheta)},$$

$$t_3 = \frac{-\alpha_\psi \sin^2\vartheta}{1 + \alpha_\psi \cos^2\vartheta}.$$

- The states described by $\rho_{Y\bar{Y}}$ and $\rho^X_{Y\bar{Y}}$ are said to be ***local unitary equivalent*** in the sense that they have same quantum correlation properties such as Bell nonlocality and entanglement.

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# History of EPR Paradox

- **In 1935, Einstein, Podolsky and Rosen published a famous paper titled "Can Quantum-Mechanical Description of Physical Reality be Considered Complete?". In this paper, Einstein claimed that the QM is an incomplete theory since it did not obey the local realism. (The birth of EPR paradox)**

- **In 1951, David Bohm came up a thought experiment based on spin degree of freedom using electron-positron pair.**

- **In 1964, John Bell introduced the local hidden variable (LHV) theory into the QM and derived a mathematical expression for LHV theory: Bell's inequality [J. S. Bell, Phys. Phys. Fiz. 1, 195 (1964)].**

$$|\mathcal{P}(\mathbf{a},\mathbf{b})-\mathcal{P}(\mathbf{a},\mathbf{c})|\leq 1+\mathcal{P}(\mathbf{b},\mathbf{c}).$$

- **In 1969, Clauser, Horne, Shimony and Holt modified the original Bell's inequality and gave the CHSH inequality [Phys. Rev. Lett. 23, 880 (1969). ]. The CHSH inequality is experiment-friendly and is the most widely used Bell-type inequality till now.**

$$|E(\mathbf{a}_1,\mathbf{b}_1)+E(\mathbf{a}_1,\mathbf{b}_2)+E(\mathbf{a}_2,\mathbf{b}_1)-E(\mathbf{a}_2,\mathbf{b}_2)|\leq 2.$$

# CHSH Inequality

- The nonlocal property in a quantum entangled system can be tested by the violation of Bell inequality. The most widely used Bell-type inequality is the CHSH inequality.

$$\boxed{|\langle A_1 \otimes B_1\rangle + \langle A_1 \otimes B_2\rangle + \langle A_2 \otimes B_1\rangle - \langle A_2 \otimes B_2\rangle| \leq 2.}$$

where $A_{1,2} = \mathbf{a}_{1,2} \cdot \boldsymbol{\sigma}$ and $B_{1,2} = \mathbf{b}_{1,2} \cdot \boldsymbol{\sigma}$ are spin observables for Alice and Bob respectively. By noticing the fact that $\langle A \otimes B\rangle \equiv \mathrm{Tr}\left[\rho_{AB}(\mathbf{a}\cdot\boldsymbol{\sigma} \otimes \mathbf{b}\cdot\boldsymbol{\sigma})\right] = \mathbf{a}^{\mathrm{T}} C \mathbf{b}$ , we can find a compact form of CHSH inequality.

$$\boxed{\left|\mathbf{a}_1^{\mathrm{T}} C\left(\mathbf{b}_1+\mathbf{b}_2\right)+\mathbf{a}_2^{\mathrm{T}} C\left(\mathbf{b}_1-\mathbf{b}_2\right)\right| \leq 2.}$$

with $\boldsymbol{C}$ being the correlation matrix $\boldsymbol{C_{ij}}$ in density operator.

- The maximum of the left-hand side can be obtained by tuning 4 unit vectors, $\mathbf{a_1}, \mathbf{a_2}$ $\mathbf{b_1}, \mathbf{b_2}$

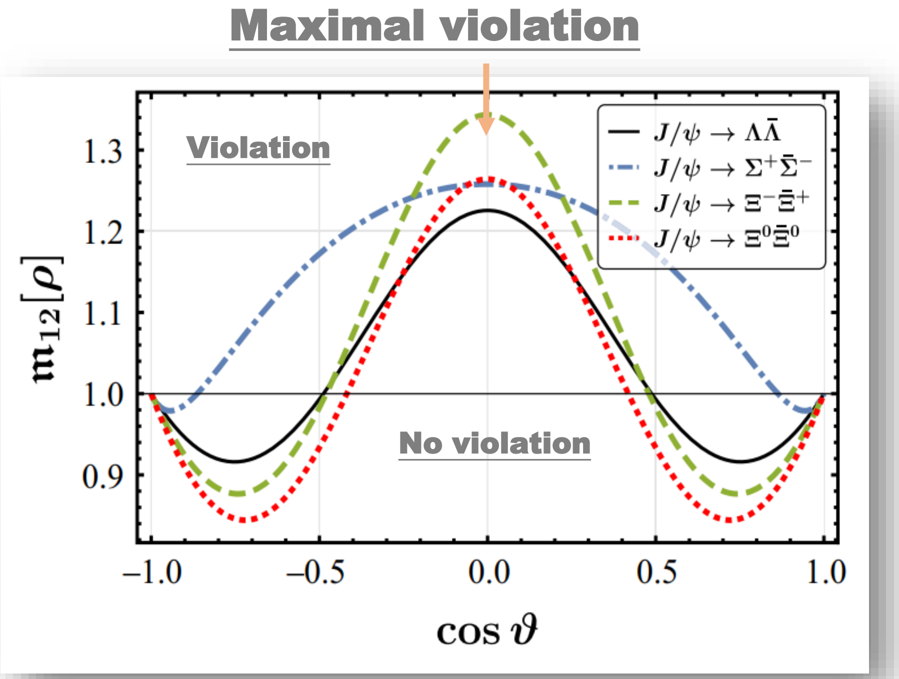
$$\begin{aligned}\mathcal{B}[\rho] &\equiv \max_{\mathbf{a}_1,\mathbf{a}_2,\mathbf{b}_1,\mathbf{b}_2}\left|\mathbf{a}_1^{\mathrm{T}} C\left(\mathbf{b}_1+\mathbf{b}_2\right)+\mathbf{a}_2^{\mathrm{T}} C\left(\mathbf{b}_1-\mathbf{b}_2\right)\right| \\ &= 2\sqrt{m_1+m_2}.\end{aligned}$$

$\boldsymbol{m_1}$ and $\boldsymbol{m_2}$ are the two largest eigenvalues of $\boldsymbol{C^T C}$.

# Bell Nonlocality: Bell Inequality Violation

- Therefore, the CHSH inequality can be violated iff (if and only if) $m_1 + m_2 > 1$ and the maximum possible violation of the CHSH inequality is the upper bound value $2\sqrt{2}$. We define $\mathfrak{m}_{12}[\rho] = m_1 + m_2$ to be a measure of the Bell nonlocality.

- According to the X-type density operator, we can calculate the Bell nonlocality for $Y\bar{Y}$ in $e^+e^-$ annihilation.




$$\mathfrak{m}_{12}\left[\rho^X_{Y\bar{Y}}\right] = \begin{cases} t_1^2 + t_2^2, & \alpha_\psi \geq 0 \\ \max\left\{t_1^2 + t_2^2, t_1^2 + t_3^2\right\}, & \alpha_\psi < 0 \end{cases}$$

| | $\Lambda\bar{\Lambda}$ | $\Sigma^+\bar{\Sigma}^-$ | $\Xi^-\bar{\Xi}^+$ | $\Xi^0\bar{\Xi}^0$ |
|---|---|---|---|---|
| $\mathcal{B}_{\max}$ | 2.214 | 2.243 | 2.318 | 2.249 |
| $\vartheta^*$ | 60.81° | 30.29° | 61.37° | 65.27° |

- Bell inequality is violated in near the transverse scattering;
- The max violation is obtained at $\vartheta = 90°$.
- The nonlocal region in given by $[\vartheta^*, 180° - \vartheta^*]$.

# Quantum Entanglement

- For a bipartite quantum system, a state is said to be ***separable*** iff the following decomposition holds

$$\boxed{\rho_{AB} = \sum_k p_k \rho_A^k \otimes \rho_B^k,}$$

otherwise is called ***entangled*** (Entangled state can not write as a mixture of product states).

- The ***concurrence*** is an entanglement monotone, so it can be regarded as an entanglement measure [Wootters, PRL (1998)].

$$\mathcal{C}[\rho] = \max\{0, \mu_1 - \mu_2 - \mu_3 - \mu_4\}.$$

- $\mathcal{C}[\rho] = 0$, separable (no entanglement)
- $\mathcal{C}[\rho] > 0$, entangled
- $\mathcal{C}[\rho] = 1$, maximal entangled

where $\mu_i$ $(i = 1, 2, 3, 4)$ are the eigenvalues of the Hermitian matrix $\sqrt{\sqrt{\rho}\tilde{\rho}\sqrt{\rho}}$ with decreasing order and $\tilde{\rho} = \sigma_y \otimes \sigma_y \rho^* \sigma_y \otimes \sigma_y$ .

# Quantum Entanglement

$$\sigma_x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \ \sigma_y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}, \ \sigma_z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}.$$

- **We expand the Pauli matrices in the X-type spin density operator for the hyperon-antihyperon** $\rho^X_{Y\bar{Y}} = \frac{\mathbb{1}\otimes\mathbb{1} + a\sigma_z\otimes\mathbb{1} + \mathbb{1}\otimes a\sigma_z + \sum_i t_i\sigma_i\otimes\sigma_i}{4}.$ **We can obtain a 4 by 4 matrix resembling to the letter "X".**

$$\rho^X_{Y\bar{Y}} = \frac{1}{4}\begin{bmatrix} 1+2a+t_3 & 0 & 0 & t_1-t_2 \\ 0 & 1-t_3 & t_1+t_2 & 0 \\ 0 & t_1+t_2 & 1-t_3 & 0 \\ t_1-t_2 & 0 & 0 & 1-2a+t_3 \end{bmatrix},$$

- **The properties of X states are extensively studied in QIS and he *concurrence* for $X$ states has a closed formula [Quesada *et al*., Journal of Modern Optics (2012) ].**

$$\mathcal{C}[\rho^X] = 2\max\{0, |\rho_{14}| - \sqrt{\rho_{22}\rho_{33}}, |\rho_{23}| - \sqrt{\rho_{11}\rho_{44}}\},$$

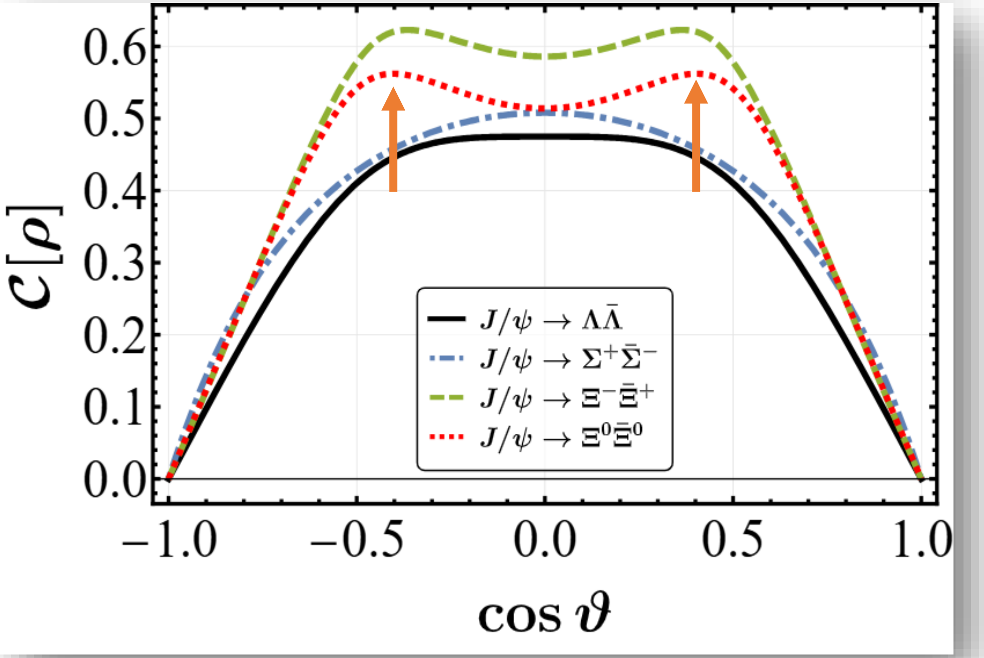
**Where $\rho_{ij}$ are the entries in the X-type density matrix above.**

# Quantum Entanglement

➤ **Concurrence as an entanglement measure in $Y\bar{Y}$ *system.***

$$\mathcal{C}[\rho^X_{Y\bar{Y}}] = |t_2| = \frac{\left|1+\alpha_\psi - \sqrt{(1+\alpha_\psi \cos 2\vartheta)^2 - (\beta_\psi \sin 2\vartheta)^2}\right|}{2(1+\alpha_\psi \cos^2\vartheta)}$$

➤ **Concurrence as a function of $\cos\vartheta$ for different hyperons, $\Lambda$, $\Sigma^+$ and $\Xi^{-,0}$.**

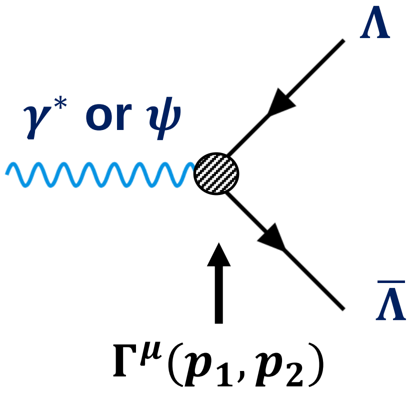



| | $\Lambda\bar{\Lambda}$ | $\Sigma^+\bar{\Sigma}^-$ | $\Xi^-\bar{\Xi}^+$ | $\Xi^0\bar{\Xi}^0$ |
|---|---|---|---|---|
| $\mathcal{C}_{\max}$ | 0.475 | 0.508 | 0.623 | 0.562 |
| $\vartheta_{\max}$ | 90° | 90° | 68.60°, 111.40° | 66.26°, 113.74° |

- **$Y\bar{Y}$ system is entangled in the whole scattering angle (except at 0 and 180°);**
- **Strong entangled near $\vartheta = 90°$;**
- **The max entanglement are not necessary obtained at $\vartheta = 90°$, e.g. $\Xi^{-,0}$. (because of $\Delta\Phi$)**

# Electromagnetic Form Factors (EMFFs)

- From the $\psi$-hyperon vertex, one can define the Electromagnetic form factors (EMFFs).




$$\Gamma^\mu(p_1, p_2) = F_1(P^2)\gamma^\mu + i\frac{\sigma^{\mu\nu}P_\mu}{2M}F_2(P^2)$$

$$G_E(s) = F_1 + \frac{s}{4M^2}F_2, \qquad G_M(s) = F_1 + F_2.$$

- In the time-like region, $G_E$ and $G_M$ are complex quantities.

$$\alpha_\psi = \frac{s - 4M^2\,|G_E/G_M|^2}{s + 4M^2\,|G_E/G_M|^2} \in [-1, 1],$$

$$\Delta\Phi = \arg\{G_E/G_M\} \in (-\pi, \pi].$$

- Polarization comes from the relative phase between $G_E$ and $G_M$.

$$B_\Lambda = B_{\bar{\Lambda}} = \frac{\sqrt{1-\alpha_\psi^2}\sin\vartheta\cos\vartheta}{1+\alpha_\psi\cos^2\vartheta}\sin(\Delta\Phi),$$

- The non-zero $\Delta\Phi$ will also affect the spin quantum correlations for hyperon systems.

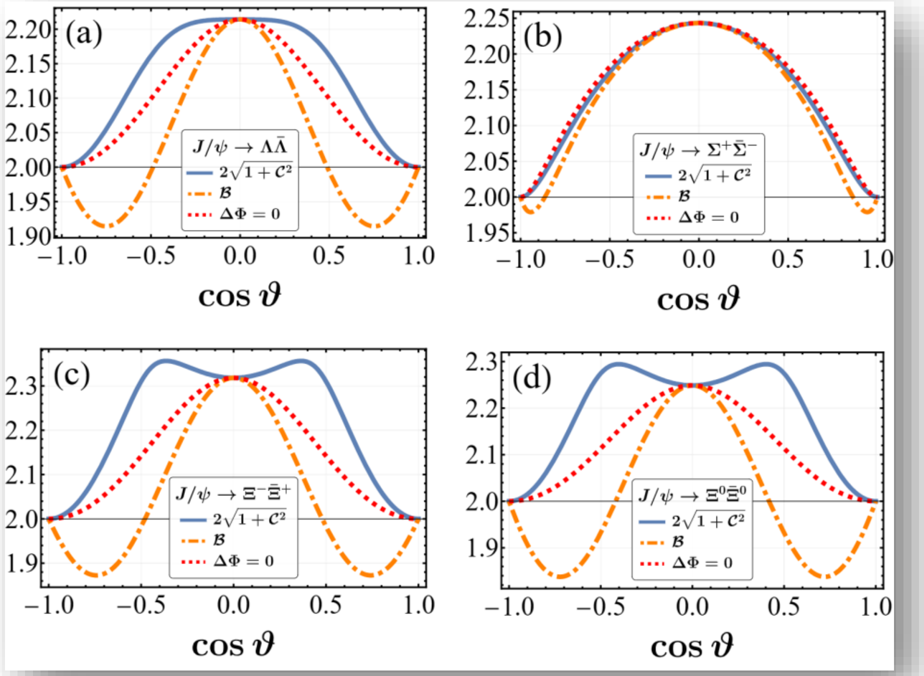
# Bell Nonlocality v.s. Entanglement

- BIV is an subset of quantum entanglement [ Werner, PRL (1989) ].

$$\boxed{\text{Bell Nonlocality} \subseteq \text{Entanglement}}$$

- For a two-qubit density operator $\rho$ with Wootters' concurrence $\mathcal{C}[\rho]$, the maximum violation of the CHSH inequality $\mathcal{B}[\rho]$ has an upper bound [ Verstraete and Wolf, PRL (2002) ]

$$\boxed{\mathcal{B}[\rho] \leq 2\sqrt{1+\mathcal{C}^2[\rho]}.}$$




- Blue lines: Entanglement;
- Orange lines: Bell Inequality Violation
- Red lines: $\Delta\Phi = 0$, ***Bell Nonlocality*** $=$ ***Entanglement***

(Can be found in $e^+e^- \to \tau^+\tau^-$ [ Ehatäht *et al.*, PRD (2024) ].)

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- **Quantum Measurement & Quantum State Tomography.**
- Loopholes in HEP Experiment.

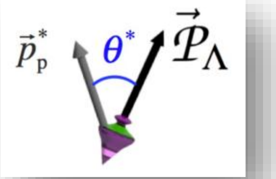
# Generalized Quantum Measurement

[S. Wu, C Qian *et al.*, Chin. Phys. Lett. 41, 110301 (2024)]

- The weak decay of $\Lambda$ hyperon can be treated as ***generalized quantum measurement.***

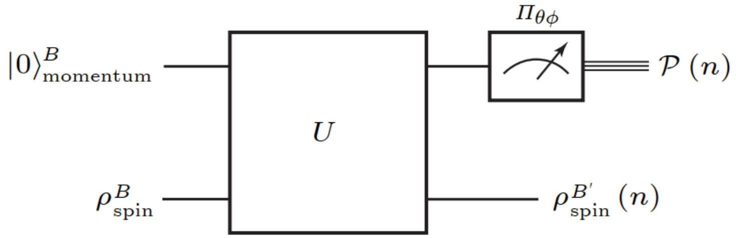
  - **Decay $\Leftrightarrow$ Measurement**

$$\boxed{\frac{1}{\Gamma}\frac{d\Gamma_{\Lambda\to p\pi^-}}{d\Omega} = \mathcal{P}(\mathbf{n}) = \mathrm{Tr}\left(M_{\mathbf{n}}\rho_{\Lambda}M_{\mathbf{n}}^{\dagger}\right),}$$




  - $\mathcal{P}(\mathbf{n})$ is the probability for finding proton in the $\mathbf{n}$ direction.
  - $\{M_n\}$ are measurement operators, $\int d\Omega\, M_n^{\dagger}M_n = \mathbb{I}$, ($M_n^{\dagger}M_n$ POVM)

$$M_{\mathbf{n}} \equiv \frac{1}{\sqrt{4\pi\left(|S|^2+|P|^2\right)}}\left(S + P\boldsymbol{\sigma}\cdot\mathbf{n}\right).$$




- The $\Lambda$ spin density operator $\rho_{\Lambda} = \frac{1}{2}(\mathbb{I} + \boldsymbol{\sigma}\cdot\mathbf{B}_{\Lambda})$, the resulting angular distribution of daughter proton reads:

$$\mathcal{P}(\mathbf{n}) = \frac{1}{4\pi}\left(1 + \alpha_{\Lambda}\mathbf{B}_{\Lambda}\cdot\mathbf{n}\right),$$

| $Y$ | $\mathcal{B}(\%)$ | $\alpha_Y$ | References |
|---|---|---|---|
| $\Lambda \to p\pi^-$ | 064 | 0.755(3) | [32,69] |
| $\Sigma^+ \to p\pi^0$ | 052 | −0.994(4) | [44] |
| $\Xi^- \to \Lambda\pi^-$ | 100 | −0.379(4) | [32,45] |
| $\Xi^0 \to \Lambda\pi^0$ | 96 | −0.375(3) | [45,47] |

  - The $\Lambda$ self-analysis property has been used as a spin probe in HEP.

# Generalized Quantum Measurement

- The spin of daughter particle can be obtained through the ***quantum measurement postulate***: (using $\Lambda \to p\pi^-$ as an example)

  - **Post-measurement state:** $$\boxed{\rho_{\text{proton}}(\mathbf{n}) = \frac{M_{\mathbf{n}}\rho_\Lambda M_{\mathbf{n}}^\dagger}{\mathrm{Tr}(M_{\mathbf{n}}\rho_\Lambda M_{\mathbf{n}}^\dagger)} = \frac{1}{2}\left(\mathbb{1} + \boldsymbol{\sigma}\cdot\mathbf{B}_{\text{proton}}\right),}$$

  $\rho(\mathbf{n})$ : spin density operator for daughter proton in the $\mathbf{n}$ direction.

  - **Spin transfer:** [Lee and Yang, Phys. Rev. (1957)]

$$\mathbf{B}_{\text{proton}} = \frac{(\alpha + \mathbf{B}_\Lambda\cdot\mathbf{n})\mathbf{n} + \beta(\mathbf{B}_\Lambda\times\mathbf{n}) + \gamma\mathbf{n}\times(\mathbf{B}_\Lambda\times\mathbf{n})}{1+\alpha\mathbf{B}_\Lambda\cdot\mathbf{n}},$$

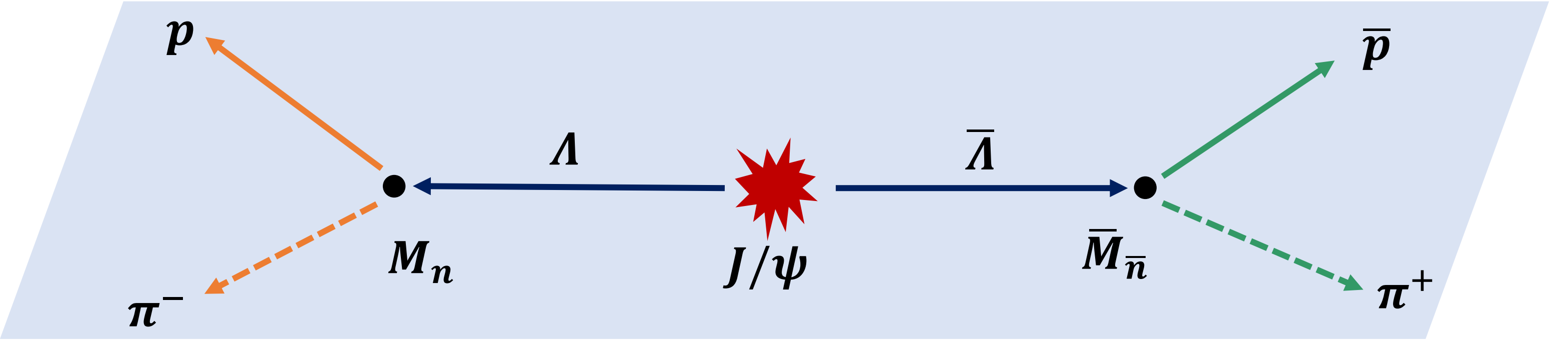
- The ***averaged*** spin density matrix of proton can be expressed by a ***quantum channel***:

$$\mathcal{E}(\rho_\Lambda) = \int \mathrm{d}\Omega\, M_{\mathbf{n}}\rho_\Lambda M_{\mathbf{n}}^\dagger = \frac{1}{2}\left(\mathbb{1} + \frac{1+2\gamma}{3}\boldsymbol{\sigma}\cdot\mathbf{B}_\Lambda\right).$$

  - $\frac{1+2\gamma}{3}\mathbf{B}_\Lambda$ : averaged proton polarization (global polarization).
  - $M_n$ act like Kraus operator in ***operator-sum representation***.

# Generalized Quantum Measurement

- **Adopting the generalized quantum measurement formalism, one can obtain the angular distribution for joint decay of $\Lambda\bar{\Lambda}$, e.g. ($J/\psi \to \Lambda\bar{\Lambda} \to p\pi^-\bar{p}\pi^+$)**




- **The two-qubit density operator for $\Lambda\bar{\Lambda}$ pair:** $\rho_{\Lambda\bar{\Lambda}} = \frac{\mathbb{1}\otimes\mathbb{1}+\mathbf{B}^+\cdot\boldsymbol{\sigma}\otimes\mathbb{1}+\mathbb{1}\otimes\mathbf{B}^-\cdot\boldsymbol{\sigma}+\sum_{i,j}C_{ij}\sigma_i\otimes\sigma_j}{4},$

$$\mathcal{P}(\mathbf{n}) = \mathrm{Tr}\left(M_{\mathbf{n}}\rho_\Lambda M_{\mathbf{n}}^\dagger\right) \xrightarrow{\text{Extend to 2-qubits}} \boxed{\mathcal{P}(\mathbf{n},\bar{\mathbf{n}}) = \mathrm{Tr}\left[\left(M_{\mathbf{n}}\otimes\bar{M}_{\bar{\mathbf{n}}}\right)\rho_{\Lambda\bar{\Lambda}}\left(M_{\mathbf{n}}^\dagger\otimes\bar{M}_{\bar{\mathbf{n}}}^\dagger\right)\right]}$$

**$\mathbf{n}$ and $\bar{\mathbf{n}}$ are directions of $p$ and $\bar{p}$ respectively, and the resulting the joint angular distribution reads:**

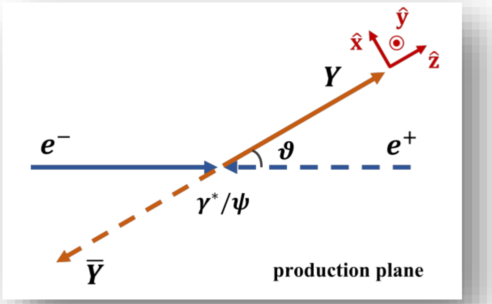
$$\frac{1}{\Gamma}\frac{\mathrm{d}\Gamma(\Lambda\bar{\Lambda}\to p\pi^-\bar{p}\pi^+)}{\mathrm{d}\Omega_{\mathbf{n}}\mathrm{d}\Omega_{\bar{\mathbf{n}}}} = \frac{1}{(4\pi)^2}\left(1+\alpha_\Lambda\mathbf{B}^+\cdot\mathbf{n}+\alpha_{\bar{\Lambda}}\mathbf{B}^-\cdot\bar{\mathbf{n}}+\alpha_\Lambda\alpha_{\bar{\Lambda}}\sum_{i,j}C_{ij}n_i\bar{n}_j\right),$$

# Quantum State Tomography

[See the Talk by Cheng Kun]

- Joint angular distribution of $e^+e^- \to \Lambda\bar{\Lambda} \to p\pi^-\bar{p}\pi^+$.

$$I(\vartheta;\theta,\bar{\theta}) = \frac{1}{(4\pi)^2}\Big[1 + \alpha_Y \sum_i B_i^+(\vartheta)\cos\theta_i + \alpha_{\bar{Y}} \sum_j B_j^-(\vartheta)\cos\bar{\theta}_j + \alpha_Y\alpha_{\bar{Y}} \sum_{i,j} C_{ij}(\vartheta)\cos\theta_i\cos\bar{\theta}_j\Big],$$




- All 15 real parameters can be extracted from the experiment data by *method of moment*：

$$B_i^+(\vartheta) = \frac{3}{\alpha_Y}\langle\cos\theta_i\rangle, \qquad B_j^- = \frac{3}{\alpha_{\bar{Y}}}\langle\cos\bar{\theta}_j\rangle, \qquad C_{ij}(\vartheta) = \frac{9}{\alpha_Y\alpha_{\bar{Y}}}\langle\cos\theta_i\cos\bar{\theta}_j\rangle.$$

- The meaning of quantum state tomography： *angular distribution ⟹ Spin*

$$\left[\begin{array}{c|c} \langle 1\rangle & \frac{3}{\alpha_{\bar{\Lambda}}}\langle\cos\bar{\theta}_j\rangle \\ \hline \frac{3}{\alpha_{\Lambda}}\langle\cos\theta_i\rangle & \frac{9}{\alpha_\Lambda\alpha_{\bar{\Lambda}}}\langle\cos\theta_i\cos\bar{\theta}_j\rangle \end{array}\right] \Rightarrow \left[\begin{array}{c|c} 1 & \mathbf{B}^{-\mathrm{T}} \\ \hline \mathbf{B}^+ & C_{ij} \end{array}\right].$$

# Quantum State Tomography

- For any reaction of $1/2+1/2 \to 1/2+1/2$ (e.g. $e^+e^- \to \Lambda\bar{\Lambda}$, or $p\bar{p} \to \Lambda\bar{\Lambda}$), the spin density operator can be reconstructed through *quantum state tomography* as:

$$\Theta_{\mu\nu}(\vartheta) = \left[\begin{array}{c|ccc} 1 & 0 & B_y & 0 \\ \hline 0 & C_{xx} & 0 & C_{xz} \\ B_y & 0 & C_{yy} & 0 \\ 0 & C_{xz} & 0 & C_{zz} \end{array}\right],$$

- Imposing $\mathcal{P}$ and $\mathcal{CP}$ conservation

$$B_x^\pm = B_z^\pm = 0, B_y^+ = B_y^-$$

$$C_{xz} = C_{zx}, \quad C_{xy} = C_{yx} = C_{yz} = C_{zy} = 0$$

- **Bell nonlocality** $\mathfrak{m}_{12}$ is given by the sum of two largest eigenvalues of $C^T C$,

$$C_{yy}^2, \quad \frac{1}{4}\left[C_{xx} + C_{zz} \pm \sqrt{4C_{xz}^2 + (C_{xx} - C_{zz})^2}\right]^2.$$

- Since the spin density operator is local unitary equivalent to X states, the **concurrence** can be calculated by

$$\mathcal{C} = \frac{1}{2}\max\left\{0, \sqrt{4C_{xz}^2 + (C_{xx} - C_{zz})^2} - |1 - C_{yy}|, |C_{xx} + C_{zz}| - \sqrt{(1 + C_{yy})^2 - 4B_y^2}\right\}.$$

# Outline

- Motivation & Introduction.
- Bell Nonlocality and Entanglement for $Y\bar{Y}$ System.
- Quantum State Tomography & Experiment.
- **Loopholes in HEP Experiment.**

# Loophole-free Experiment

- In history, the Bell test in the lab suffers from the loopholes. Those loopholes may open possibilities for local realism to give the output of the experiment, and thus makes the results unreliable.

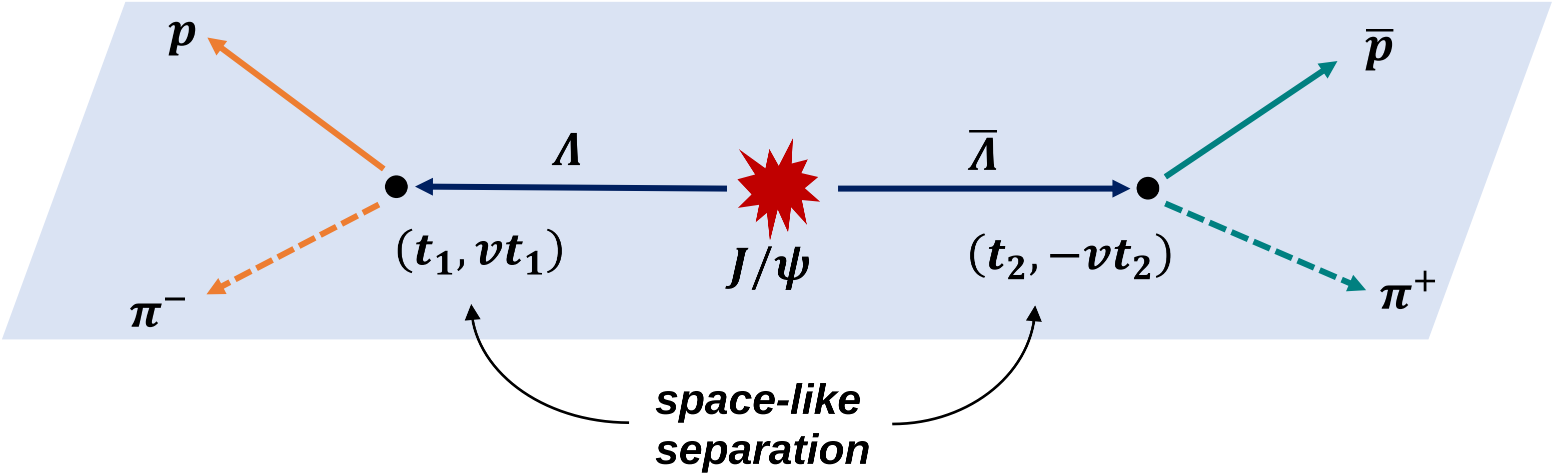
  - Anton Zeilinger *et al.*, Violation of Bell's Inequality under Strict *Einstein Locality Conditions*, PRL (1998).
  - ……
  - B. Hensen *et al.*, *Loophole-free* Bell inequality violation using electron spins separated by 1.3 kilometers, *Nature* (2015)
  - Marissa Giustina *et al.*, Significant-*Loophole-Free* Test of Bell’s Theorem with Entangled Photons, PRL (2015).
  - Lynden K. Shalm *et al.*, Strong *Loophole-Free* Test of Local Realism, PRL (2015).
  - Simon Storz *et al.*, *Loophole-free* Bell inequality violation with superconducting circuits, *Nature* (2023).

- Loopholes at High-energy Colliders

  - M. Fabbrichesi, R. Floreanini, E. Gabrielli, and L. Marzola, Phys. Rev. D 110, 053008 (2024).
  - R. Demina and G. Landi, Phys. Rev. D 111, 012013 (2025).
  - M. Fabbrichesi, R. Floreanini, L. Marzola, arXiv: 2503.18535 (2025).
  - ……

# Loophole-free Experiment

➢ **In high-energy colliders, the most important loophole is the Locality Loophole.**



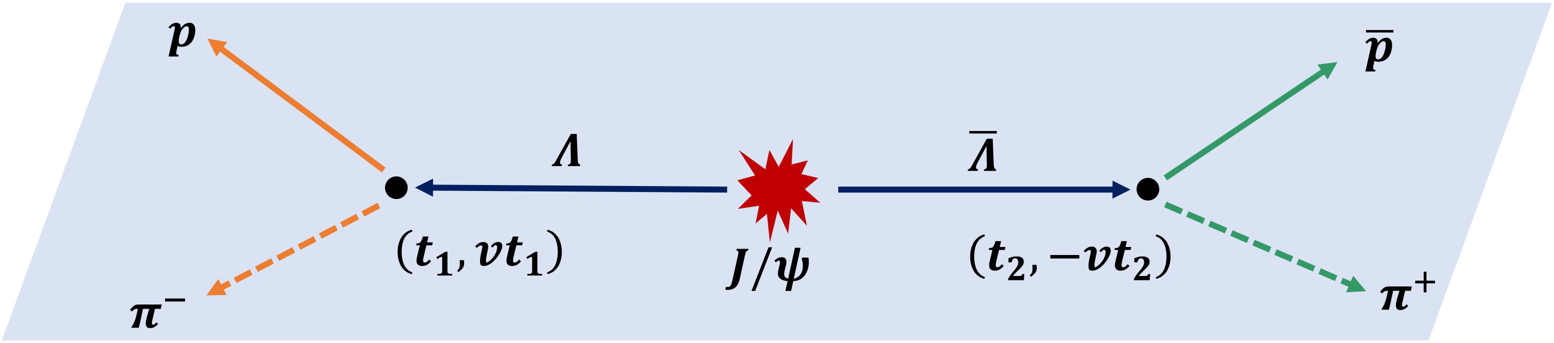

- ***Locality Loophole*: *if two decay events are time-like separate, $\Lambda$ and $\bar{\Lambda}$ may communicate with each other and thus cheat the experiment result.***

➢ ***In order to close the Locality Loophole,* $(t_1, vt_1)$ and $(t_2, -vt_2)$ should be space-like separate:** $\Delta s^2 = c^2(t_1 - t_2)^2 - (vt_1 + vt_2)^2 < 0$ .

- **space-like separation condition:** $$\boxed{\frac{|t_1 - t_2|}{t_1 + t_2} < \frac{v}{c} = \beta_\Lambda}$$

# Loophole-free Experiment

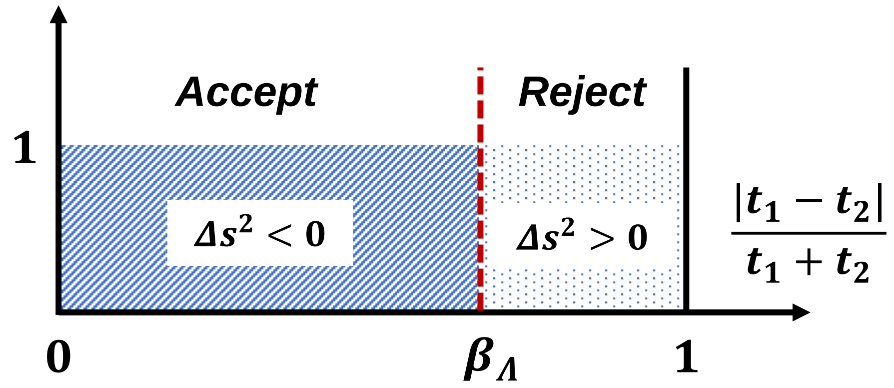
- In high-energy colliders, the most important loophole is the Locality Loophole.




$\boxed{\frac{|t_1 - t_2|}{t_1 + t_2} < \beta_\Lambda}$ $\xrightarrow{t_1, t_2 \sim f(t) = (1/T)e^{-t/T}}$ $\frac{|t_1 - t_2|}{t_1 + t_2}$ is an uniform distribution in $[0, 1]$.

- In $e^+e^- \to J/\psi \to \Lambda\bar{\Lambda}$, the velocity of hyperon is $\beta_\Lambda = v/c \approx 0.69$, which means:

  - $69\%$ events are space-like separate, accept;
  - $31\%$ events are time-like separate, reject



# Thank You.