Parity and Environmental Effects on Quantum Entanglement and Bell Tests

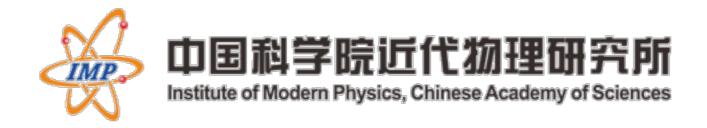
Yong Du (杜勇)

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Workshop on Quantum Entanglement at the Energy Frontier PKU, Apr 26, 2025

Based on

2409.15418, with Xiao-Gang He, Chia-Wei Liu, Jian-Ping Ma



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nature > nature physics > editorials > article
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Editorial Published: 08 September 2022

Survey the foundations

Nature Physics 18, 961 (2022) Cite this article

3939 Accesses | 1 Citations | 7 Altmetric | Metrics

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It is easy to dismiss research into the foundations of quantum mechanics as irrelevant to physicists in other areas. Adopting this attitude misses opportunities to appreciate the richness of quantum mechanics. $w/a = \frac{1}{2} \left(\frac{1}{2} \right)^{1/2} \left(\frac{1}{2} \right)^{1/2}$

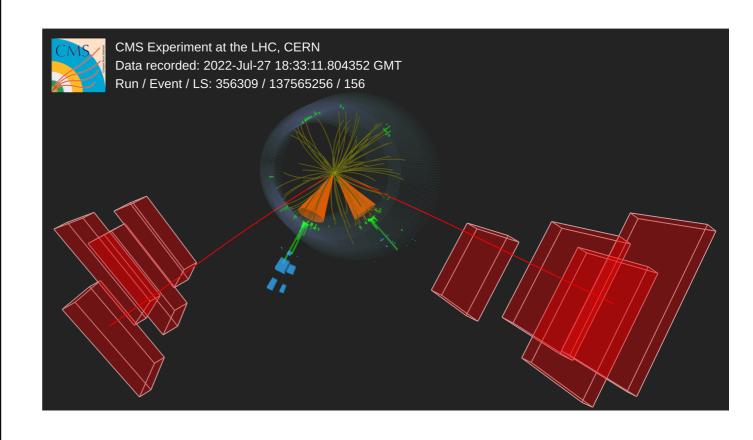
Qubits: low- vs high-energy production examples

Cascade photons

2275 Å H2 ARC * **FILTER** 4227 Å 5513 Å **FILTER** FILTER LENS PHOTO MULTIPLIER MULTIPLIER #2 LINEAR POLARIZER LINEAR POLARIZER Ca BEAM OVEN (4.5 nsec)

Kocher & Commins, PRL, 1967

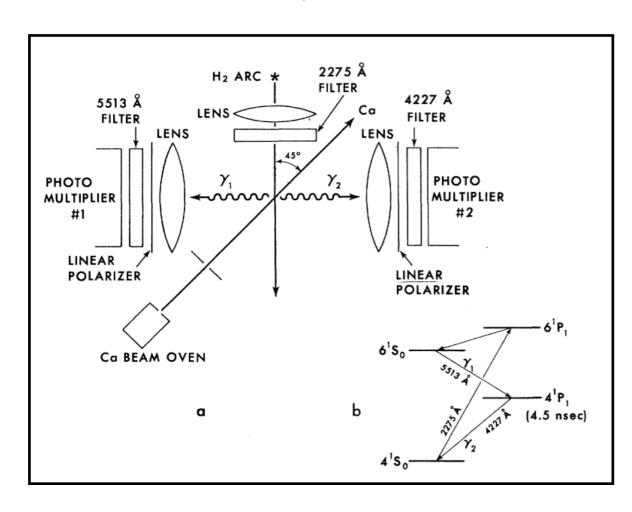
$t\bar{t}$ at the LHC



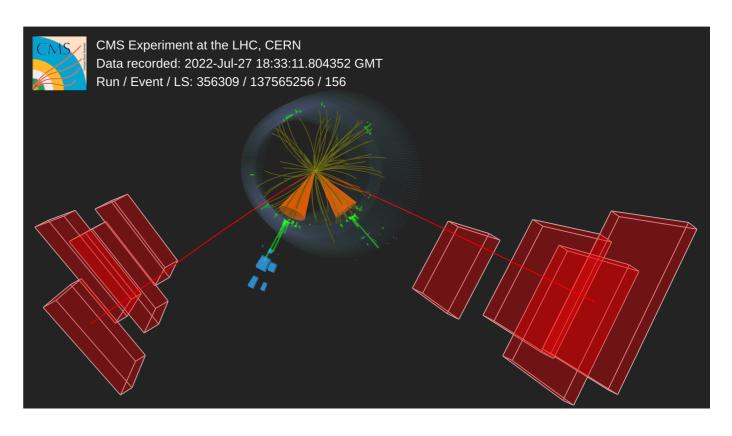
CMS-PHO-EVENTS-2022-033

Qubits: low- vs high-energy production examples

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CMS-PHO-EVENTS-2022-033

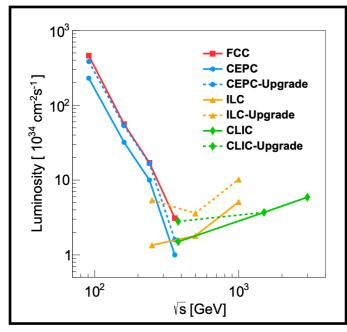
The challenges lie in performing a loophole free test, and now there are well-established methods for that. Luminosity will probably be the key ingredient as we'll see later.

The high-luminosity of BESIII/STCF/SuperKEKB/CEPC/FCC-ee makes it ideal for this kind of studies

STCF CDR, 2303.15790



CME (GeV)	Lumi (ab ⁻¹)	Samples	$\sigma(nb)$	No. of Events	Remarks
3.097	1	J/ψ	3400	3.4×10^{12}	
3.670	1	$\tau^+\tau^-$	2.4	2.4×10^{9}	
3.686	1	$\psi(3686)$	640	6.4×10^{11}	
		$\tau^+\tau^-$	2.5	2.5×10^{9}	
		$\psi(3686) \rightarrow \tau^+\tau^-$		2.0×10^{9}	
3.770	1	$D^0ar{D}^0$	3.6	3.6×10^{9}	
		$D^+ ar{D}^-$	2.8	2.8×10^{9}	
		$D^0ar{D}^0$		7.9×10^{8}	Single tag
		$D^+ar{D}^-$		5.5×10^{8}	Single tag
		$\tau^+\tau^-$	2.9	2.9×10^{9}	
4.009	1	$D^{*0}\bar{D}^{0} + c.c$	4.0	1.4×10^9	$CP_{D^0\bar{D}^0} = +$
		$D^{*0}\bar{D}^{0} + c.c$	4.0	2.6×10^{9}	$CP_{D^0\bar{D}^0} = -$
		$D_s^+D_s^-$	0.20	2.0×10^{8}	
		$\tau^+\tau^-$	3.5	3.5×10^{9}	
4.180	1	$D_s^{+*}D_s^{-}$ +c.c.	0.90	9.0×10^{8}	
		$D_s^{+*}D_s^{-}$ +c.c.		1.3×10^{8}	Single tag
		$\tau^+\tau^-$	3.6	3.6×10^{9}	
4.230	1	$J/\psi \pi^+\pi^-$	0.085	8.5×10^{7}	
		$\tau^+\tau^-$	3.6	3.6×10^{9}	
		$\gamma X(3872)$			
4.360	1	$\psi(3686)\pi^{+}\pi^{-}$	0.058	5.8×10^{7}	
		τ+τ-	3.5	3.5×10^{9}	
4.420	1	$\psi(3686)\pi^{+}\pi^{-}$	0.040	4.0×10^{7}	
		$\tau^+\tau^-$	3.5	3.5×10^{9}	
4.630	1	$\psi(3686)\pi^{+}\pi^{-}$	0.033	3.3×10^{7}	
		$\Lambda_c \bar{\Lambda}_c$	0.56	5.6×10^{8}	
		$\Lambda_c \bar{\Lambda}_c$		6.4×10^{7}	Single tag
		$\tau^+\tau^-$	3.4	3.4×10^{9}	
4.0-7.0	3	300-point scan with 10 MeV steps, 1 fb ⁻¹ /point			
> 5	2–7	Several ab ⁻¹ of high-energy data, details dependent on scan resul			



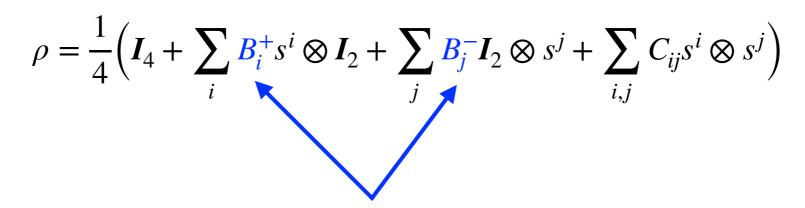
CEPC Snowmass 2021, 2205.08553

So we are good!

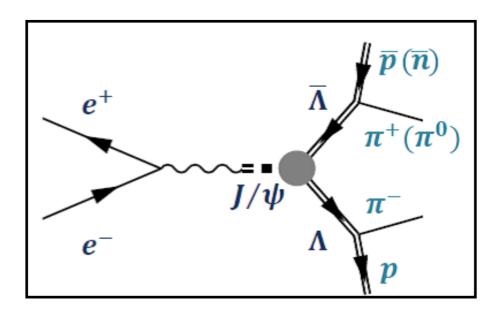
Formalism and Exp Extraction

Formalism and Exp Extraction

The entangled spin-half bipartite system can be properly described by the spin-density matrix expanded in the $SU(2) \otimes SU(2)$ Hilbert space:



Polarization of the subsystem

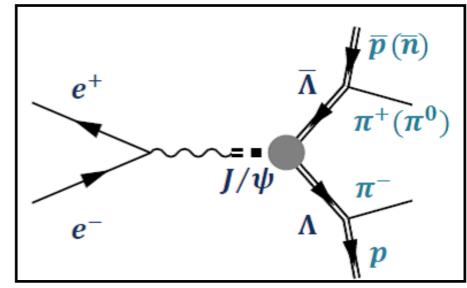


$$\frac{d\Gamma_{\Lambda}}{d\Omega_p} \left(\vec{s}_1, \hat{l}_p \right) \propto 1 + \alpha \vec{s}_1 \cdot \hat{l}_p$$

Formalism and Exp Extraction

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$$\rho = \frac{1}{4} \left(\mathbf{I}_4 + \sum_i \mathbf{B}_i^+ s^i \otimes \mathbf{I}_2 + \sum_j \mathbf{B}_j^- \mathbf{I}_2 \otimes s^j + \sum_{i,j} \mathbf{C}_{ij} s^i \otimes s^j \right)$$



Spin correlation

$$\frac{d\sigma}{d\Omega_k d\Omega_p d\Omega_{\bar{p}}} \propto \text{Tr} \left[\rho \left(1 + \alpha s_1 \cdot \hat{\boldsymbol{l}}_p \right) \left(1 - \bar{\alpha} s_2 \cdot \hat{\boldsymbol{l}}_{\hat{p}} \right) \right]$$

Equally applies to LHC, Belle II, CEPC, FCC-ee $(e^+e^-/pp \to \tau^+\tau^-/t\bar{t}$ for instance)!

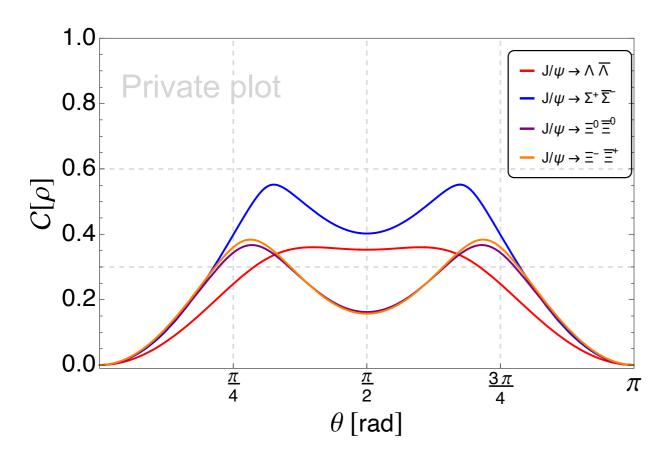
The underlying assumption is that the fermion pair is entangled, which is not guaranteed to be present!

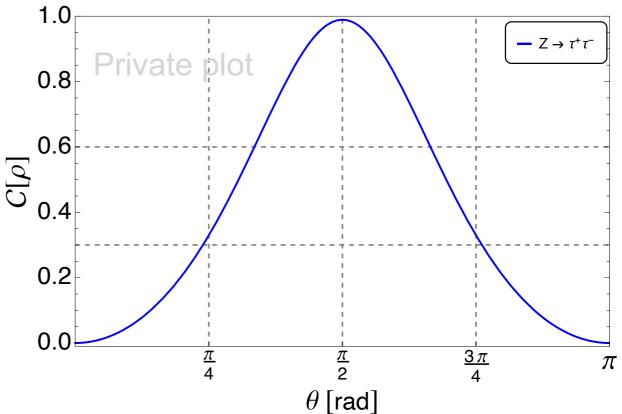
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Concurrence quantifies the entanglement of the fermion pair

$$C(\rho) = \max[0, 2\lambda_{\max} - \operatorname{Tr} R]$$

$$R = \sqrt{\sqrt{\rho}(\sigma_{y} \otimes \sigma_{y})\rho^{*}(\sigma_{y} \otimes \sigma_{y})\sqrt{\rho}}$$



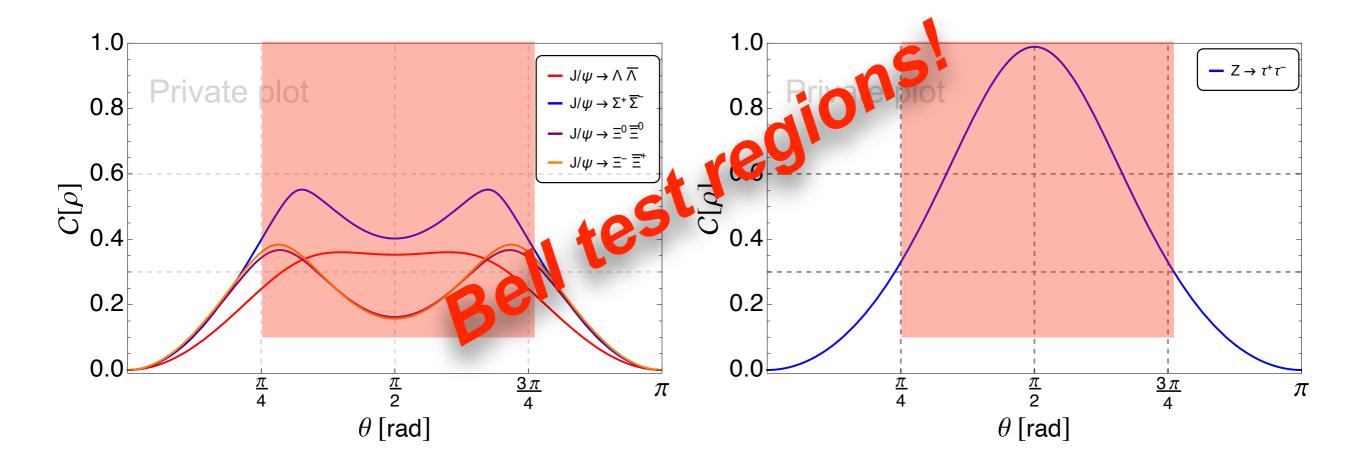


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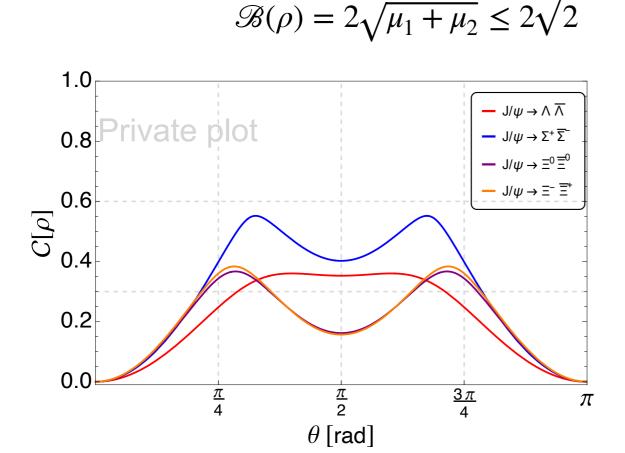
The original Bell inequality requires simultaneously adjusting two directions with a spacelike separation randomly, thus practically very challenging.

The CHSH inequality instead avoids this simultaneity:

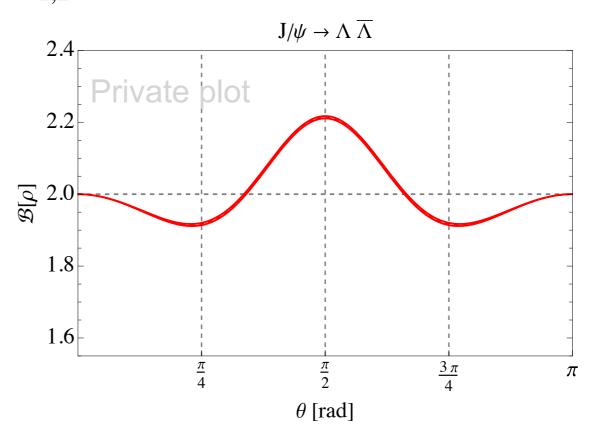
$$\mathscr{B}(\rho) = 2\sqrt{\mu_1 + \mu_2} \le 2\sqrt{2}$$
 $\mu_{1,2}$ the largest two eigenvalues of C^TC

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 $\mu_{1,2}$ the largest two eigenvalues of C^TC



See also Wu et al, 2406.16298

Quantum entanglement

Bell inequality violation

Yong Du (IMP-CAS)

Current studies on Bell tests focused on parity-conserving interactions: QED conserves it, $\Lambda_{\rm OCD}^2 G_F$ suppression for octet baryons from J/ψ decay. But not generically true!

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Q1: Isolating P V, impact on Bell tests?

Q2: What are our targets?



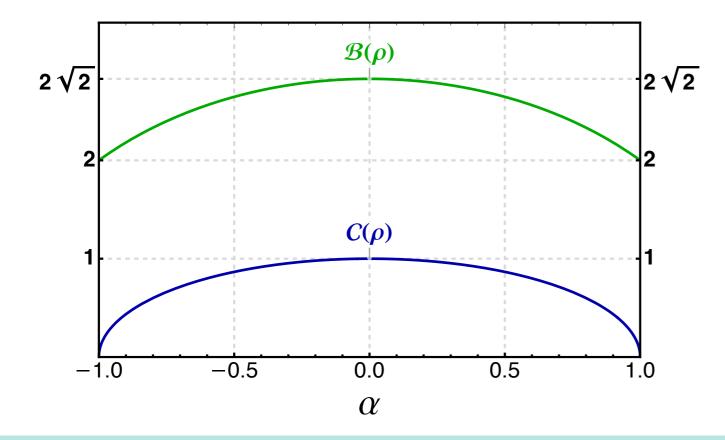
The simplest case is the spin-1/2 bipartite system resulting from spin-0 and spin-1 particle decays

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The silly simple spin-0 case ($h \to f_1 \bar{f}_2$ or crossed channels): $\mathcal{L} = h \bar{f}_1 \left(g_S - g_P \gamma_5 \right) f_2$

$$|\Psi\rangle = \frac{S+P}{\sqrt{2(|S|^2+|P|^2)}} |\uparrow\downarrow\rangle + \frac{S-P}{\sqrt{2(|S|^2+|P|^2)}} |\downarrow\uparrow\rangle$$



$$\alpha = \frac{2\text{Re}(S^*P)}{|S|^2 + |P|^2}$$

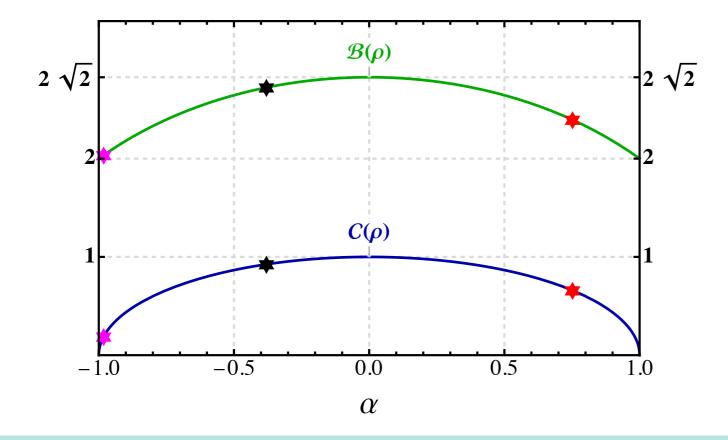
$$S = \sqrt{m_i^2 - (m_1 + m_2)^2} g_S$$

$$P = \sqrt{m_i^2 - (m_1 - m_2)^2} g_P$$

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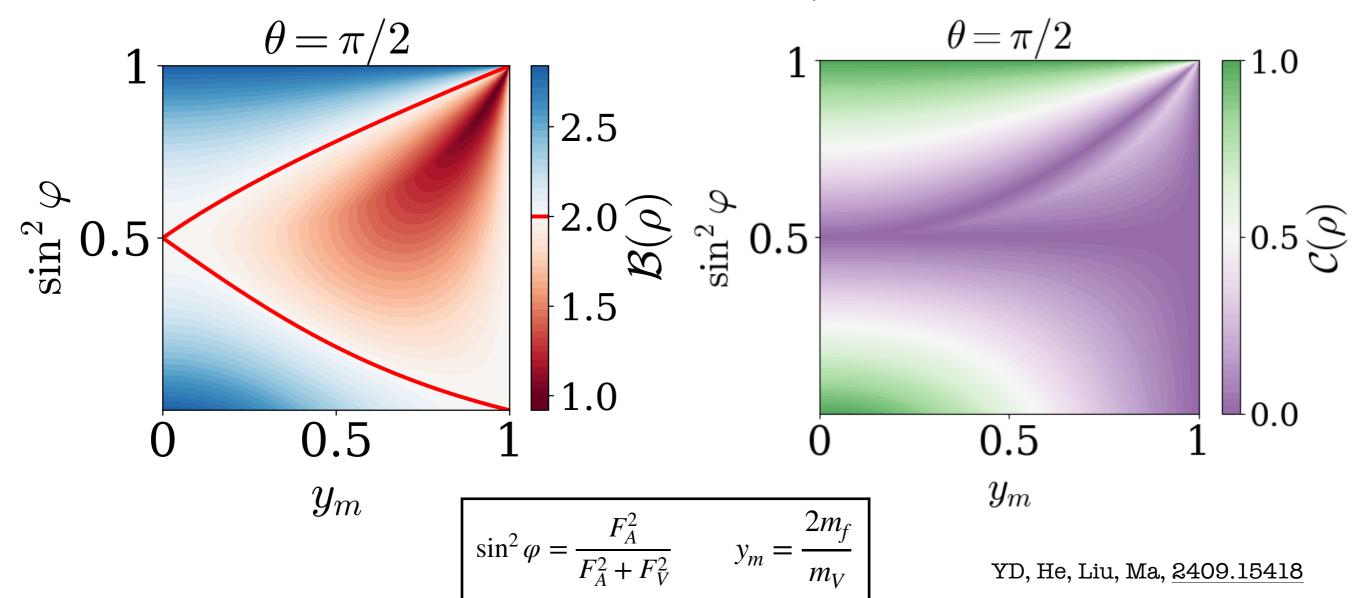
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$$lpha_{\Lambda} = 0.7519 \pm 0.0043$$
 $_{\rm BESIII}, 2204.11058$
 $lpha_{\Xi^0} = -0.3750 \pm 0.0038$
 $_{\rm BESIII}, 2305.09218$
 $lpha_{\Xi^-} = -0.376 \pm 0.008$
 $_{\rm BESIII}, 2105.11155$
 $lpha_{\Sigma^+} = -0.982 \pm 0.14$
 $_{\rm PDG} 2022$

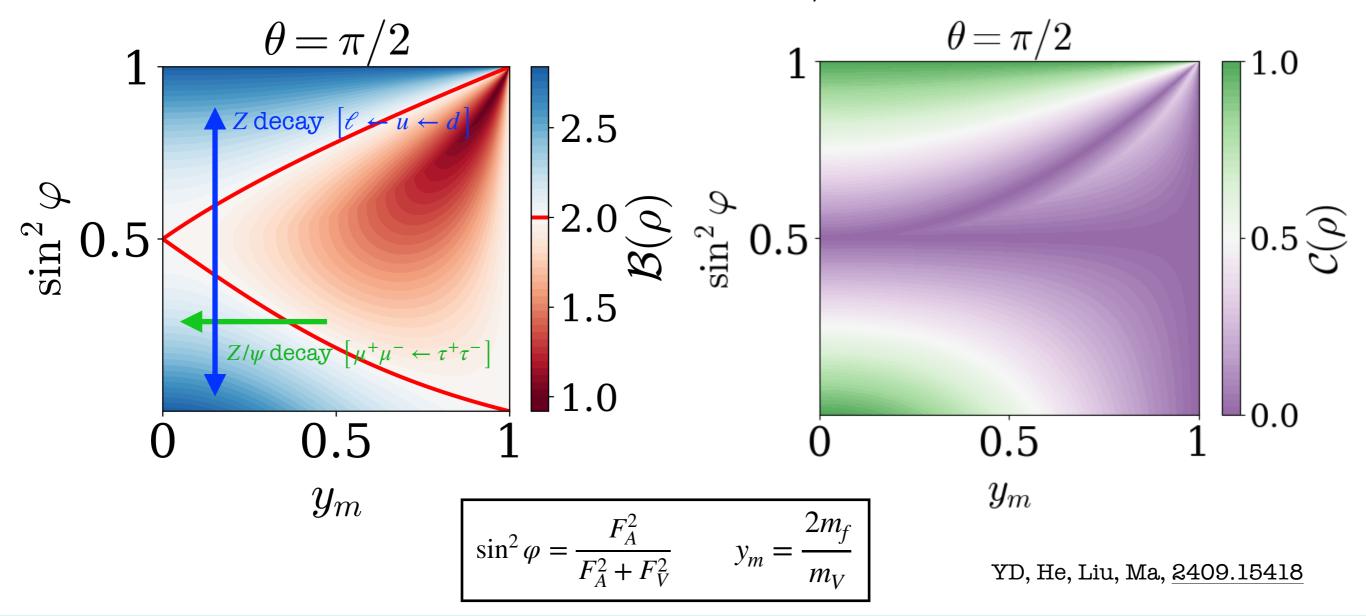
The simplest case is the spin-1/2 bipartite system resulting from spin-0 and spin-1 particle decays

The not that simple spin-1 case $(V \to f\bar{f})$: $\mathscr{L} = V^{\mu}\bar{f}\gamma_{\mu}\left(F_V + F_A\gamma_5\right)f$



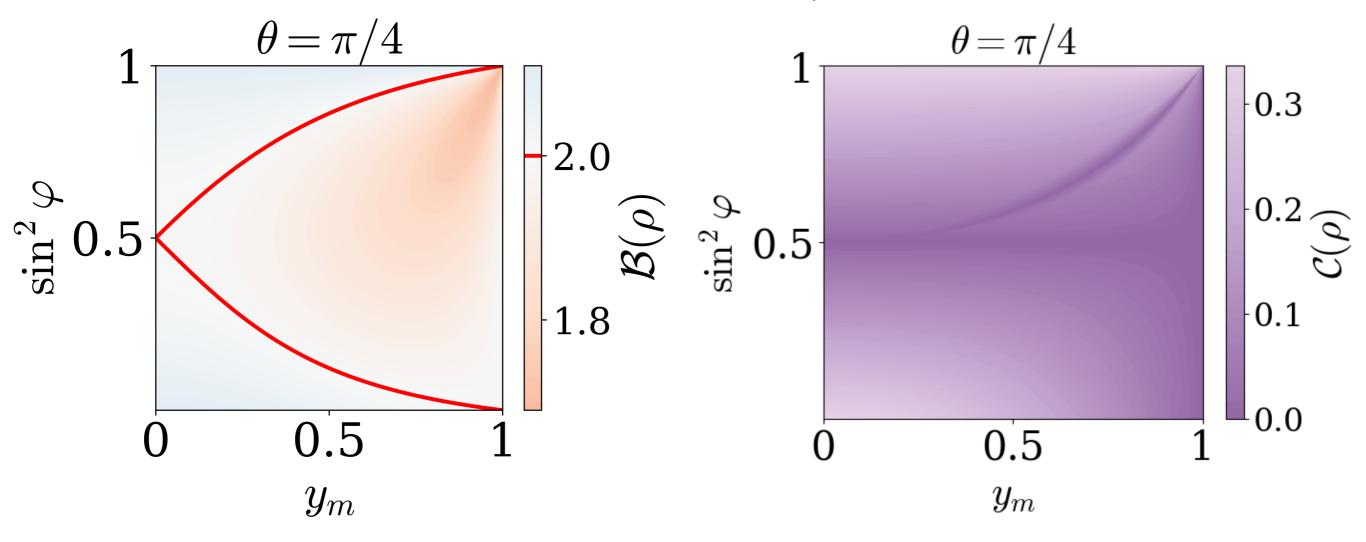
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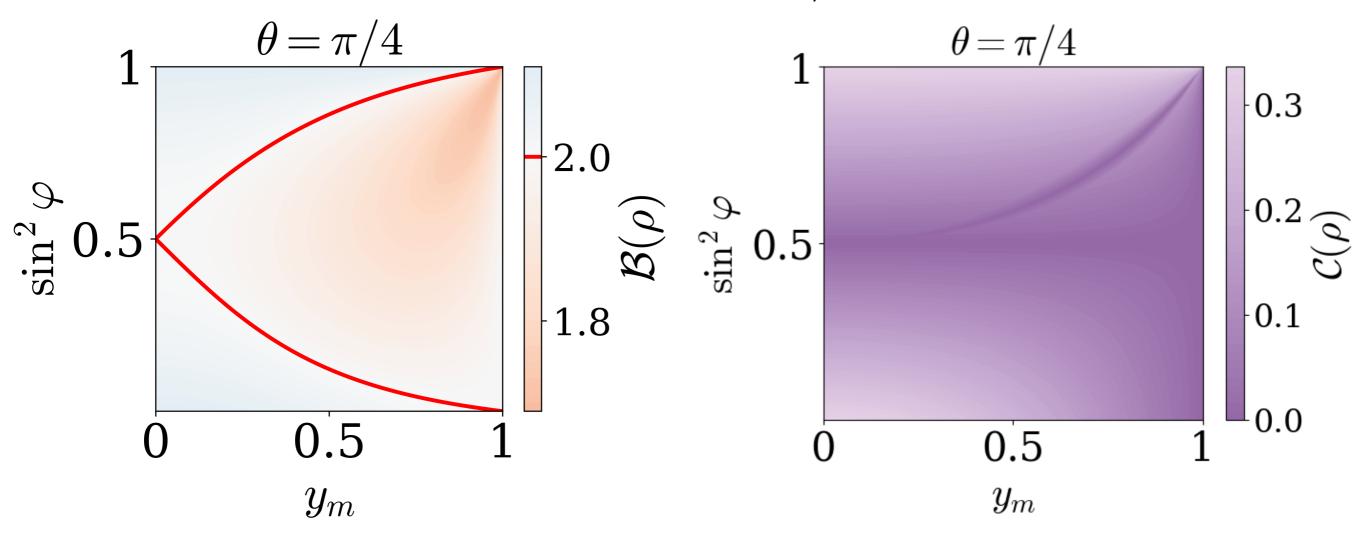
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YD, He, Liu, Ma, 2409.15418

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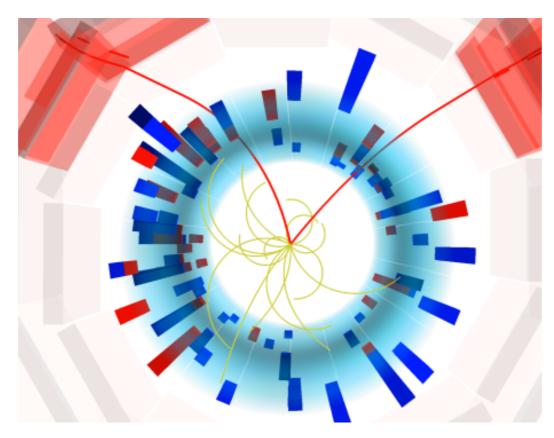
Stat. improvement!

YD, He, Liu, Ma, 2409.15418

What is missing? Interactions with the surroundings (as has always been).

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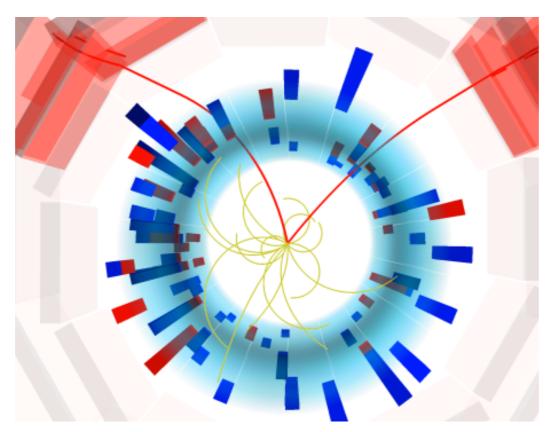
Figure credit: CMS collaboration



This environmental effect is largely overlooked in literature. As I will show soon, this ignorance may lead to misunderstandings in interpreting the physical results.

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Figure credit: CMS collaboration



This environmental effect is largely overlooked in literature. As I will show soon, this ignorance may lead to misunderstandings in interpreting the physical results.

To isolate the effects from the magnetic field and to make our point, we focus on particles decay before hitting the detector. Good examples are τ , Λ_c^+ and Ξ^- for LEP/CEPC/FCC-ee and BESIII/STCF, respectively.

For the momenta, the magnetic field simply induces a rotation along the \hat{z} direction due to the Lorentz force:

$$R_p = e^{-i\boldsymbol{J}\cdot\left(\frac{q\boldsymbol{H}}{m\gamma}\right)t}$$

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For the spins, induction instead of spin precession as described by the Bargmann-Michel-Telegdi equation: Bargmann, Michel, Telegdi, PRL 1959

$$\frac{d\mathbf{S}(t)}{dt} = \frac{ge}{2m}\mathbf{S}(t) \times \left[\gamma \mathbf{H} + (1 - \gamma)\frac{\mathbf{H} \cdot \mathbf{v}}{v^2}\mathbf{v}\right] \equiv \mathbf{S}(t) \times \tilde{\mathbf{H}}$$

First-order Magnus expansion is sufficient $\left| \tilde{\boldsymbol{H}} \right| \approx \frac{ge\tau_f \left| \boldsymbol{H} \right| \gamma}{m} \sim \mathcal{O}(10^{-2} \sim 10^{-4})$

$$R_{s} = e^{-i\boldsymbol{J}\cdot\Omega_{1}(t)} \qquad \Omega_{1}(t) = \int_{0}^{t} dt' \frac{ge}{2m} \left[\gamma \boldsymbol{H} + (1-\gamma) \frac{\boldsymbol{H}\cdot\boldsymbol{v}(t')}{v^{2}} \boldsymbol{v}(t') \right]$$

Due to the magnetic effect

YD, He, Liu, Ma, 2409.15418

$$\rho(0,0) = \frac{1}{4} \left(\mathbf{I}_4 + \sum_i B_i^+ s^i \otimes \mathbf{I}_2 + \sum_j B_j^- \mathbf{I}_2 \otimes s^j + \sum_{i,j} C_{ij} s^i \otimes s^j \right)$$

$$\rho(t_1, t_2) = \frac{1}{4} \left(\mathbf{I}_4 + \sum_i B_i^+ s^i \otimes \mathbf{I}_2 + \sum_j B_j^- \mathbf{I}_2 \otimes s^j + \sum_{i,j} C_{ij} s^i \otimes s^j \right)$$

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Statistical average is taken over the decay time through a Gaussian PDF:

$$p(t_1, t_2) = \frac{1}{2\pi\sigma_{\text{TOF}}^2} e^{-\frac{(t_1 - \tau)^2 + (t_2 - \tau)^2}{2\sigma_{\text{TOF}}^2}}$$

$$\sigma_{\text{TOF}}^{\text{BESIII}} = 33 \text{ ps}$$

$$\sigma_{\text{TOF}}^{\text{LEP}} = 150 \text{ ps}$$

Decay time correlation is ignored as we lack this info (also crucial for a loophole free test!), thus high luminosity would possibly be urgently needed for a loophole free test at colliders.

^{*} a Poisson PDF instead barely affects our conclusion

Q: How large is this environmental effect?

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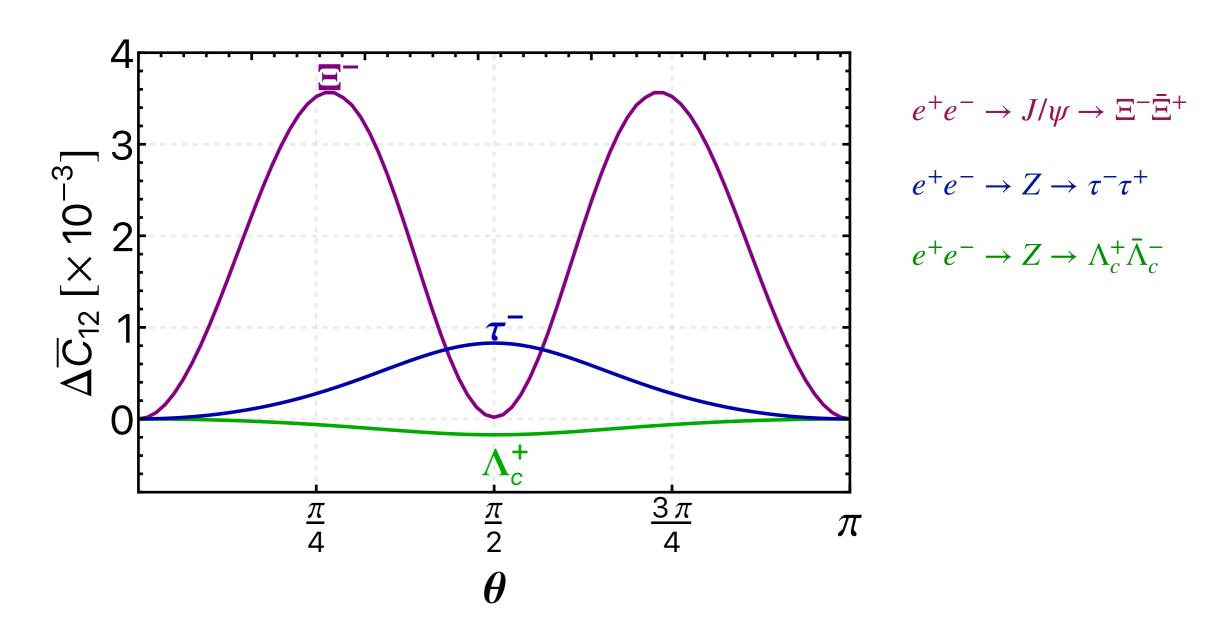
$$\rho = \frac{1}{4} \left(\mathbf{I}_4 + \sum_i B_i^+ s^i \otimes \mathbf{I}_2 + \sum_j B_j^- \mathbf{I}_2 \otimes s^j + \sum_{i,j} C_{ij} s^i \otimes s^j \right)$$

Rotational invariance puts constraints on the generic form of R (backup slide), and P and CP invariance will lead to, for instance, $C_{12} = C_{21}$ under P or CP invariance

$$\Delta \bar{C}_{12} \equiv \bar{C}_{12} - \bar{C}_{21}$$

 $\Delta \bar{C}_{12} \neq 0$ would correspond to a spurious P and/or CP violation due to interaction with the environmental magnetic field.

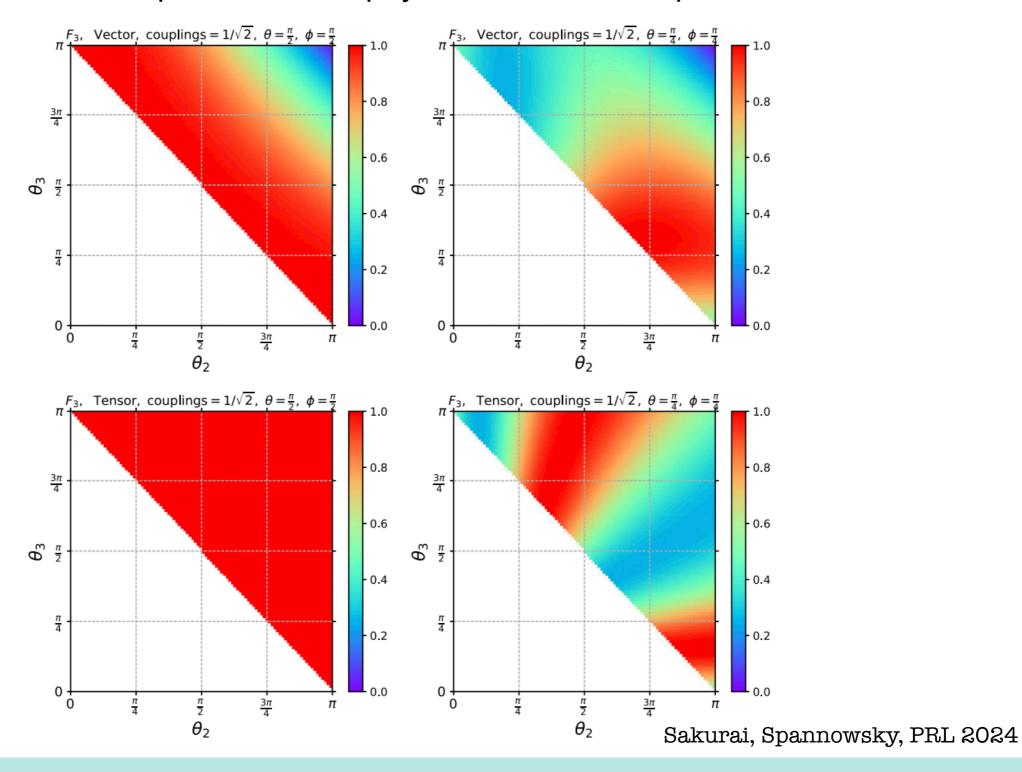
Spurious P and/or CP violation can be of $\mathcal{O}(10^{-3})$ for $|\mathbf{H}| = 1$ tesla.



Non-negligible and may become observable at a future high-lumi electron collider!

Yong Du (IMP-CAS)

The spin correlation can be easily modified by the presence of new physics, e.g., U(1) gauge boson or 4-fermion operators: New physics in the heatmap



Up to now, the discussion, though free of referring to any specific local hidden variable theory, however does rely on the knowledge of a quantum one.

Q: Do we have to?

Challenging the validity of general QFT can be achieved from the simple spin-0 h decay:

$$\rho = \frac{1}{4} \left(\mathbf{I}_4 + \sum_i B_i^+ s^i \otimes \mathbf{I}_2 + \sum_j B_j^- \mathbf{I}_2 \otimes s^j + \sum_{i,j} C_{ij} s^i \otimes s^j \right)$$

Assuming no special direction for nature, *i.e.*, SO(3) invariance alone (in the rest frame of h)

$$\vec{B}^{+} = b_{1k}\hat{k}$$
, $\vec{B}^{-} = b_{2k}\hat{k}$ $C_{ij} = c_0 \delta_{ij} + c_2 \epsilon_{ijl} \hat{k}_l + c_5 \left(\hat{k}_i \hat{k}_j - \frac{\delta_{ij}}{3}\right)$

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Angular momentum conservation immediately leads to (3 independent parameters):

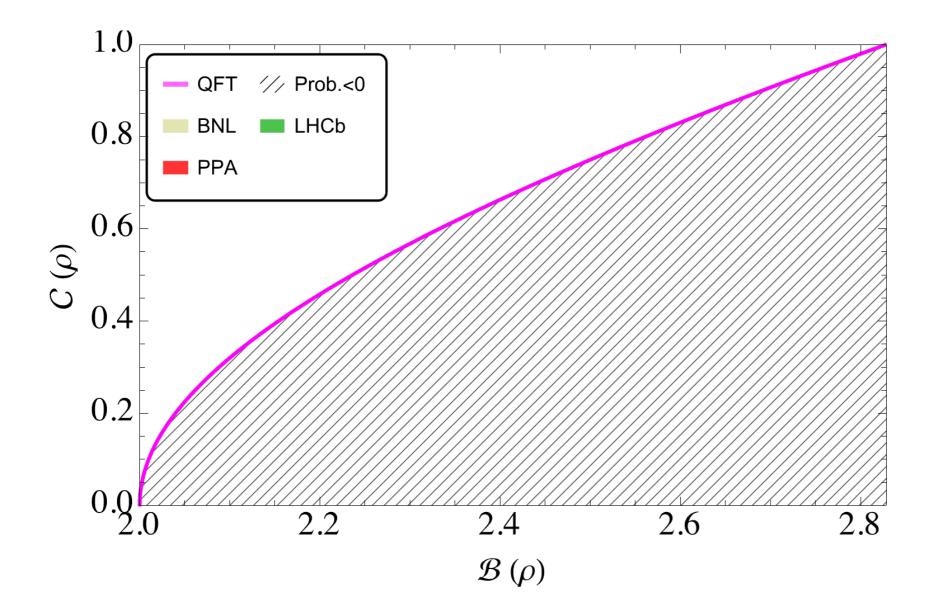
$$b_{1k}=-b_{2k} \qquad c_0=-1-\frac{2}{3}c_5$$

$$\mathcal{B}(\rho)=2\sqrt{2-b_{1k}^2-\epsilon} \qquad \mathrm{C}(\rho)=\frac{1}{2}\left[(\mathcal{B}(\rho)^2-4)(\mathcal{B}(\rho)^2-4+4\epsilon)\right]^{\frac{1}{4}}$$

$$\epsilon=1-b_{1k}^2-c_2^2-(1+c_5)^2. \text{ In } \textit{any } \mathrm{QFT}, \ \epsilon=0 \text{ is guaranteed and } \textit{unprovenly } \mathrm{utilized}$$

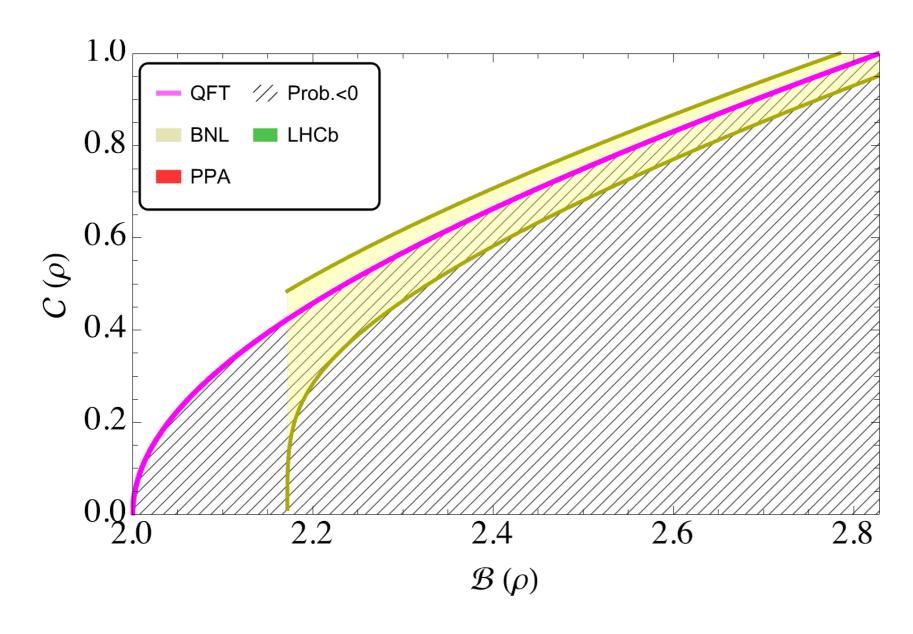
for fitting.

Free test of QFT along with the Bell tests



If $\epsilon \neq 0$ were observed, new paradigm beyond the QFT will be needed!

Free test of QFT along with the Bell tests

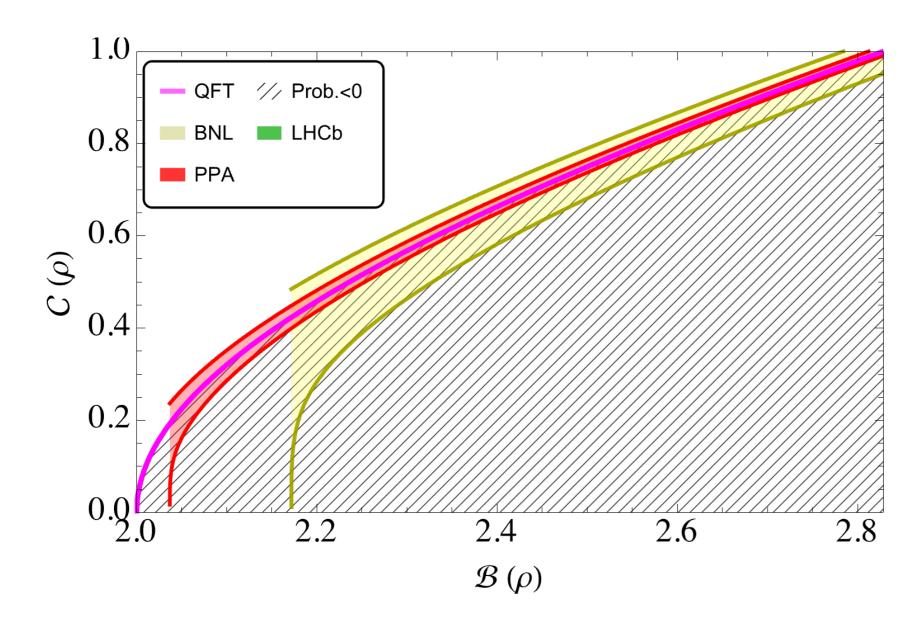


 $\epsilon_{\rm BNL} = -0.025 \pm 0.154$

Cronin & Overseth, PR 1963

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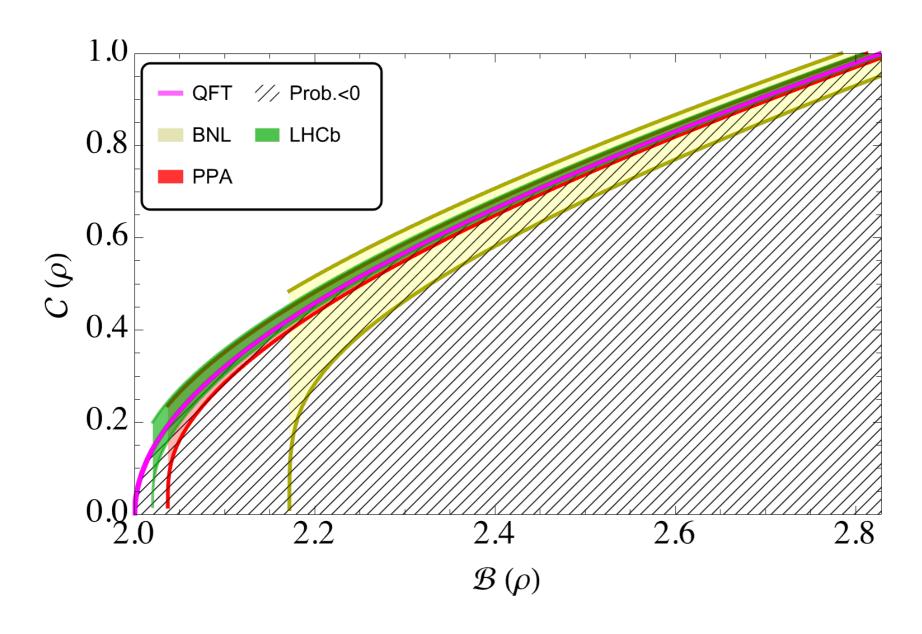
Free test of QFT along with the Bell tests



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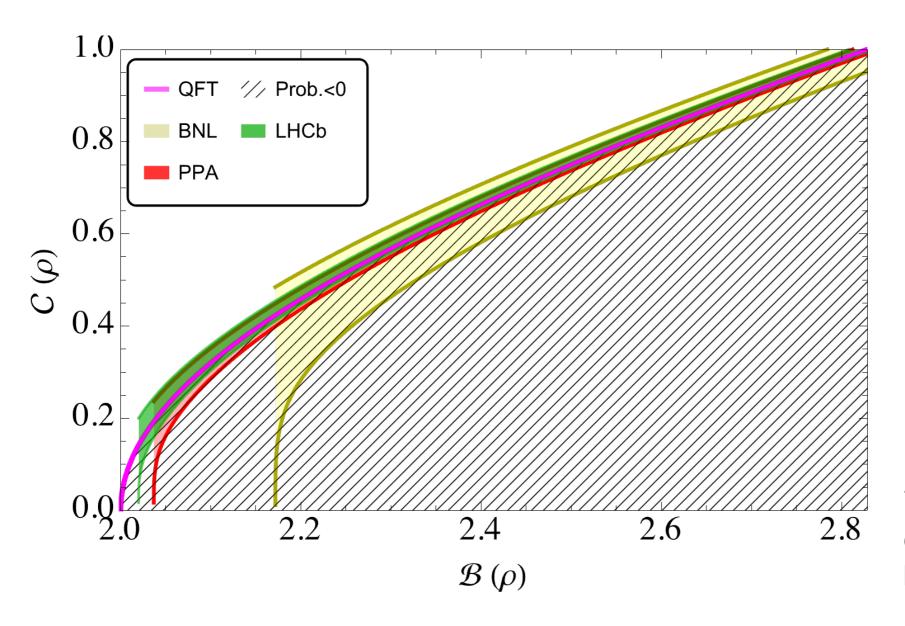
Overseth & Roth, PRL 1967

$$\epsilon_{\rm LHCb} = 0.02 \pm 0.04$$

Private communication w/ Yanxi Zhang (2409.02759)

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* Not mentioning BESIII here as $\epsilon=0$ is taken as the starting point.

If $\epsilon \neq 0$ were observed, new paradigm beyond the QFT will be needed!

Time for data reanalysis is NOW!

Summary

- The entangled fermion pair can be utilized for testing quantum entanglement and Bell nonlocality. We found parity violation could significantly modify the spin correlations of the bipartite system from both spin-0 and spin-1 particle decays.
- * The largely overlooked environmental effect was examined and a spurious P and/or CP violation of $\mathcal{O}(10^{-4} \sim 10^{-3})$ can be induced. This has to be subtracted for a genuine determination of P and CP violation at a future high-lumi lepton collider.
- We also propose a free test of the QFT framework using the simplest spin-0 decay and encourage our experimental colleagues to do such tests NOW

Backup