PEKING UNIVERSITY 北京大學

University of Pittsburgh

Quantum State Tomography at Colliders

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2025/04/26 @ Peking University

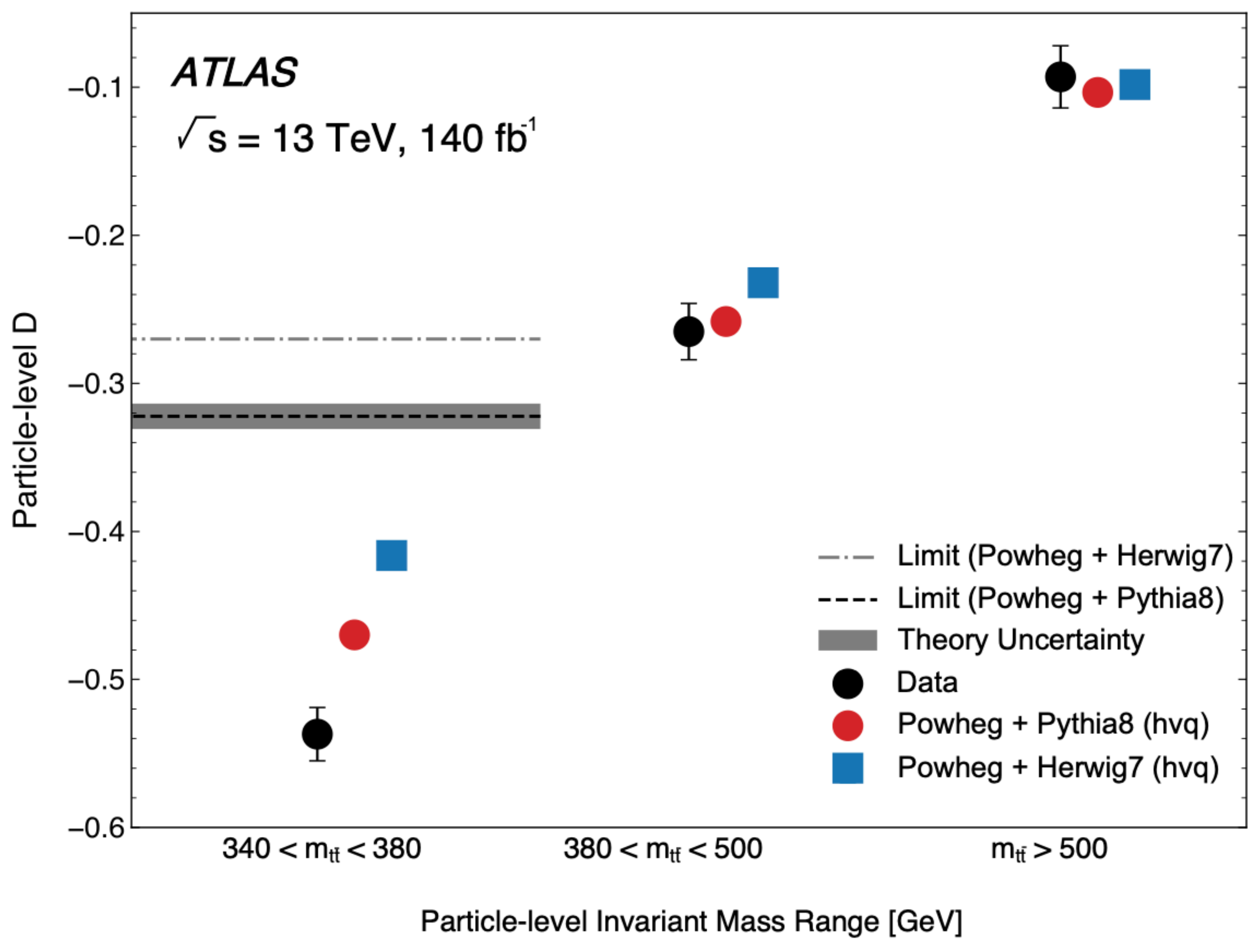
Quantum information at colliders

- Fundamental interests (new environments, new systems)

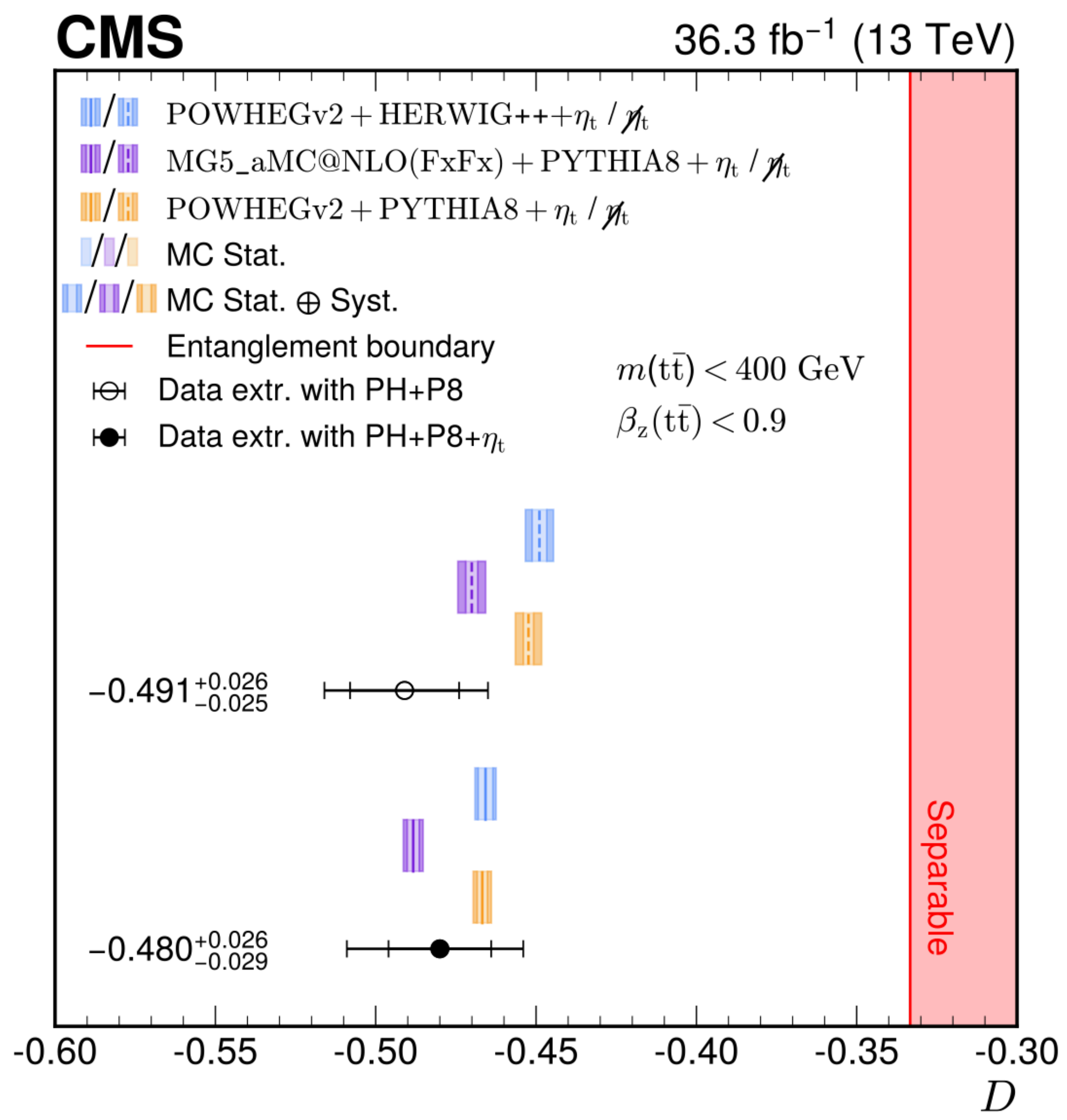
- HEP for QI (rich information, vast number of events…)

see the talk by Yu Shi

- QI for HEP (new methods, techniques, model comparison…)



[ATLAS, Nature 633 (2024) 542]



[CMS, RPP 87 (2024) 117801]

η_t, a psedo scalar that couples to gg and $t\bar{t}$

$$\sigma(\eta_t) = 8.8^{+1.2}_{-1.4}\,\text{pb}$$

[CMS, arXiv:2503.22382]

Quantum Tomography

— a procedure to reconstruct the complete quantum state

- Requirement: a **complementary** set of measurements
- One qubit: 3 operators

$$\rho = \frac{\mathbb{1}_2 + R_i \sigma_i}{2}, \quad R_i = \langle \sigma_i \rangle$$

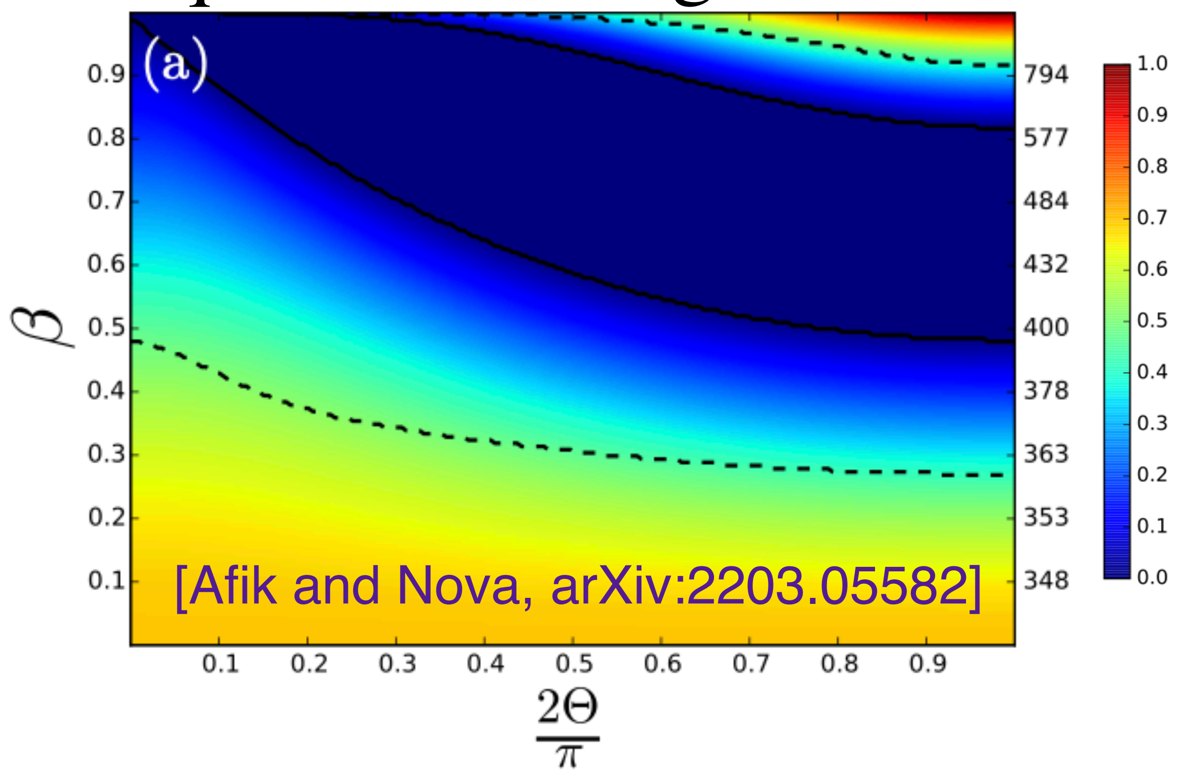
- Two qubit: 3+3+9 operators

$$\rho = \frac{\mathbb{1}_4 + R_i^{\mathcal{A}} \sigma_i \otimes \mathbb{1}_2 + R_i^{\mathcal{B}} \mathbb{1}_2 \otimes \sigma_i + C_{ij} \sigma_i \otimes \sigma_j}{4}$$

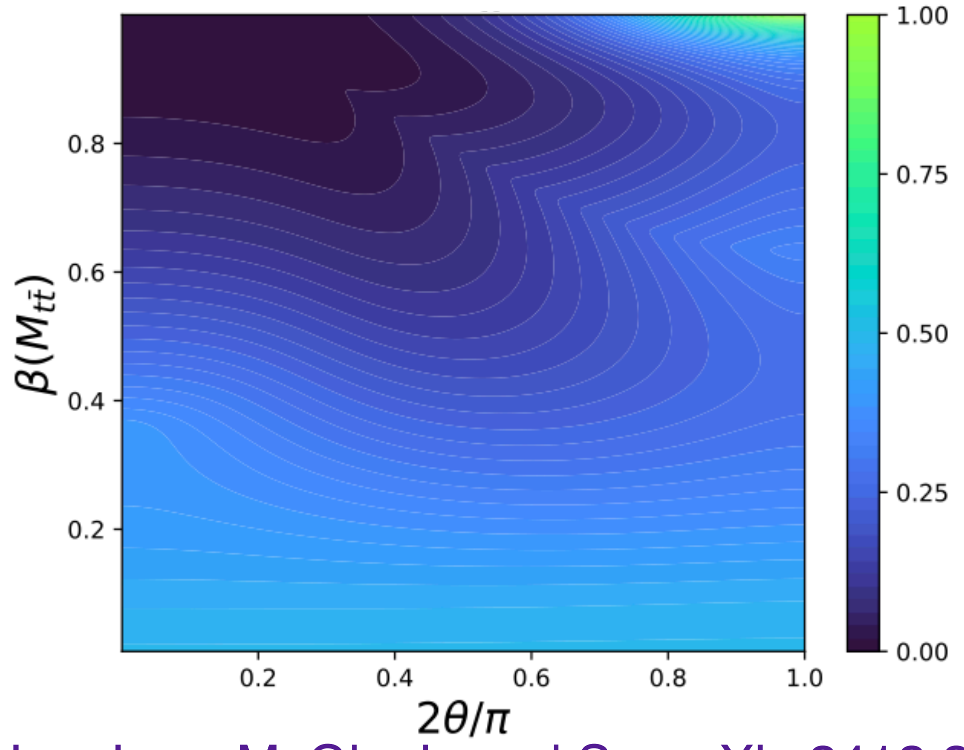
$$R_i^{\mathcal{A}} = \langle \sigma_i \otimes \mathbb{1}_2 \rangle$$
$$R_i^{\mathcal{B}} = \langle \mathbb{1}_2 \otimes \sigma_i \rangle$$
$$C_{ij} = \langle \sigma_i \otimes \sigma_j \rangle$$

- Complete density matrix $\Longrightarrow$ complete quantum information

quantum entanglement

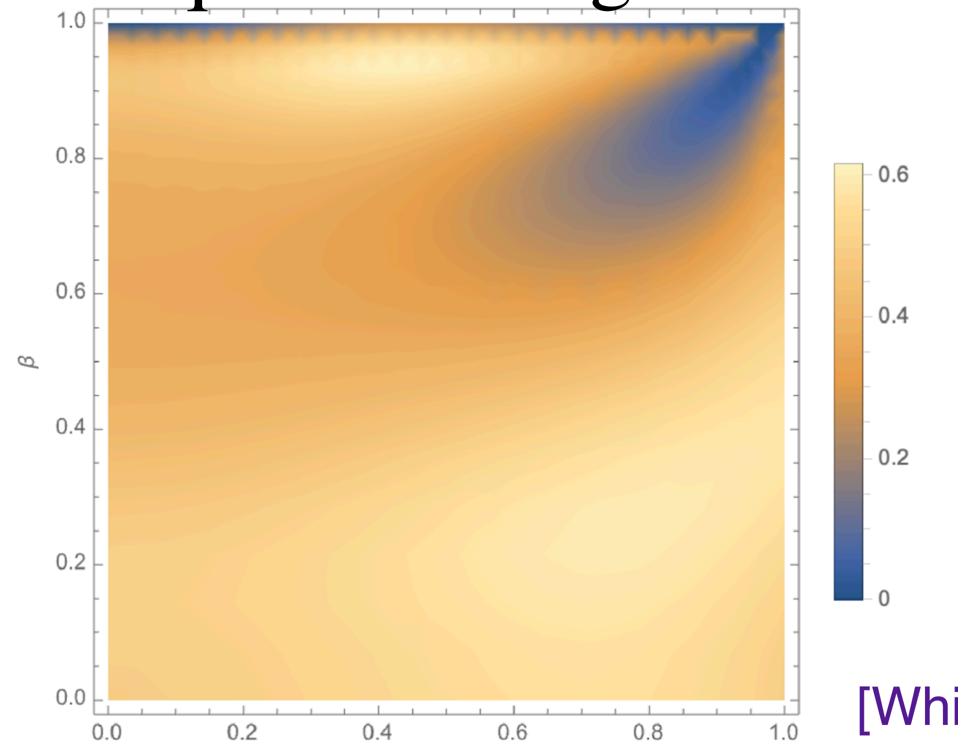


quantum discord



[Han, Low, McGinnis and Su, arXiv:2412.21158]

quantum magic



[White and White, arXiv:2406.07321]

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- Complete density matrix $\Longrightarrow$ complete quantum information

▸ ρ is entangled if $\mathscr{C}(\rho) = \max(0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4) > 0$

λ_i are eigenvalues of $\sqrt{\sqrt{\rho}\tilde{\rho}\sqrt{\rho}}$, $\tilde{\rho} = (\sigma_2 \otimes \sigma_2)\rho^*(\sigma_2 \otimes \sigma_2)$

$$\mathscr{C} = \begin{cases} \frac{-\mathrm{Tr}(C) - 1}{2}, & \text{(spin singlet)} \\ \max_i \left[\frac{\mathrm{Tr}(C) - 2C_{ii} - 1}{2}\right], & \text{(spin triplet)} \end{cases}$$

▸ Bell's inequality is violated if $\left|\vec{a}_1 \cdot C \cdot (\vec{b}_1 - \vec{b}_2) + \vec{a}_2 \cdot C \cdot (\vec{b}_1 + \vec{b}_2)\right| > 2$ for any four directions $\vec{a}_i, \vec{b}_i$

e.g. $\sqrt{2}|C_{xx} \pm C_{yy}| > 2$

Outline

Spin states

$t\bar{t}, \tau^+\tau^- W^+W^- \cdots$
Measure decay distribution

$q\bar{q}$
Measure hadron distribution

Measure $2 \to 2$ kinematics

Some theory inputs

$$\rho = \frac{\mathbb{1}_4 + R_i^{\mathcal{A}} \sigma_i \otimes \mathbb{1}_2 + R_i^{\mathcal{B}} \mathbb{1}_2 \otimes \sigma_i + C_{ij} \sigma_i \otimes \sigma_j}{4}$$

No direct measurements $\sigma_x, \sigma_y, \sigma_z$ in collider environment

$B^0\bar{B}^0, K^0\bar{K}^0$
Measure oscillation pattern

Flavor states

Reconstructing the spin states

Spin-1/2: $t\bar{t}$, $\tau\tau$, $q\bar{q}$

Decay Products as Spin Analyzer

— no SG experiments in collider environments

- Reconstructed $\langle \sigma_i \rangle$ from decay distribution:
- E.g. t quark with spin vector $\vec{s}$:

$$\rho^t = \frac{1}{2}(I_2 + s_i \sigma_i)$$

  - $t \to \ell^+ \nu b$, ℓ^+ tends to fly along top spin

$$\frac{1}{\Gamma}\frac{d\Gamma}{d\Omega} = \frac{1}{2}(1 + \ell_i^+ s_i)$$

$\ell^+ = \hat{p}_{\ell^+}$
Lorentz structure $\vec{p} \cdot \vec{s}$ violates Parity

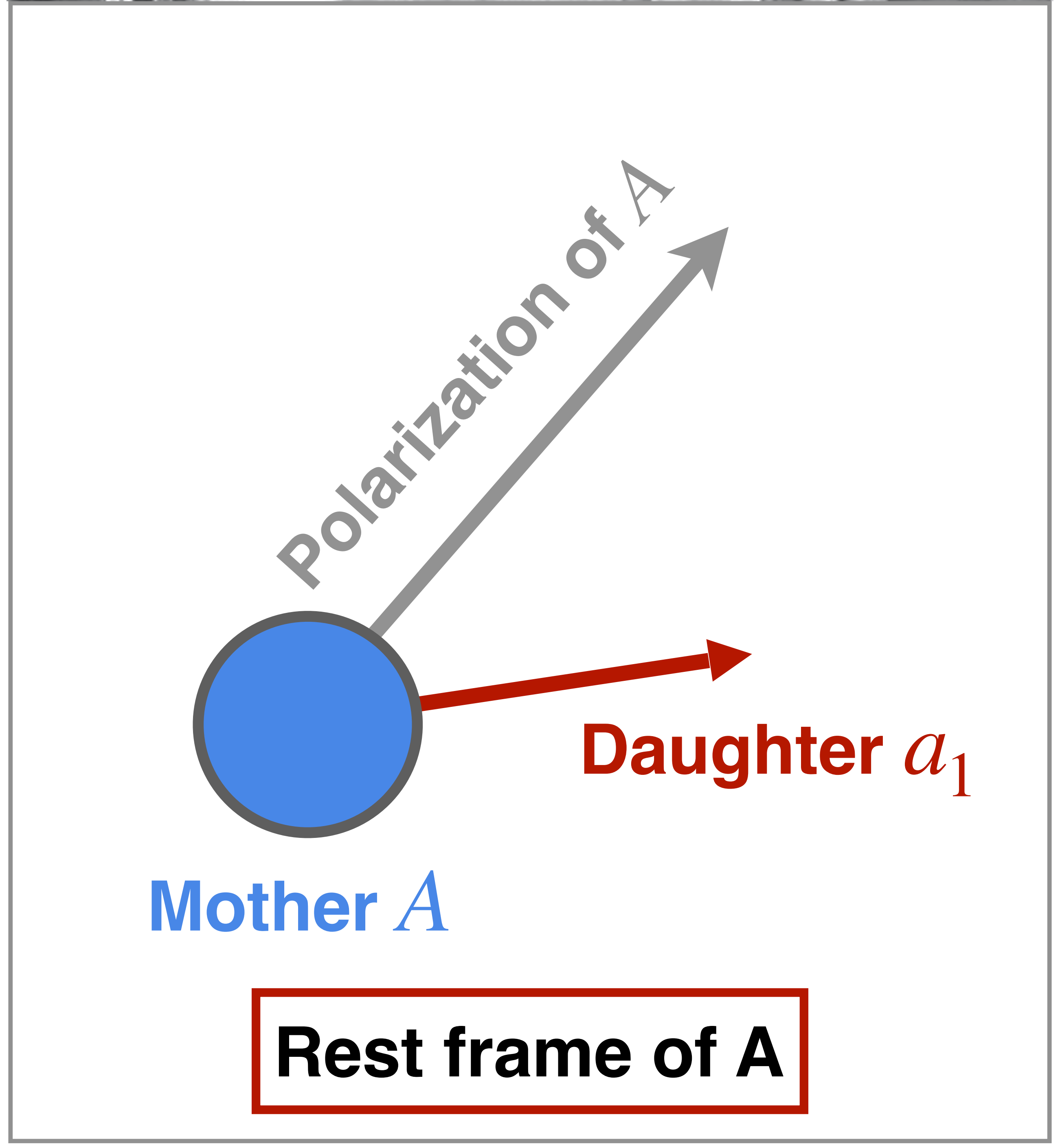
  - Decay distribution $\Longrightarrow$ spin

$$s_i = \langle \sigma_i \rangle = 3\langle \ell_i^+ \rangle$$

- Similarly, for $t\bar{t}$ pair

$$\rho = \frac{I_4 + s_i \sigma_i \otimes I_2 + \bar{s}_i I_2 \otimes \sigma_i + C_{ij} \sigma_i \otimes \sigma_j}{4}$$

$$s_i = 3\langle \ell_i^+ \rangle, \quad \bar{s}_i = -3\langle \ell_i^- \rangle, \quad C_{ij} = -9\langle \ell_i^+ \ell_j^- \rangle$$



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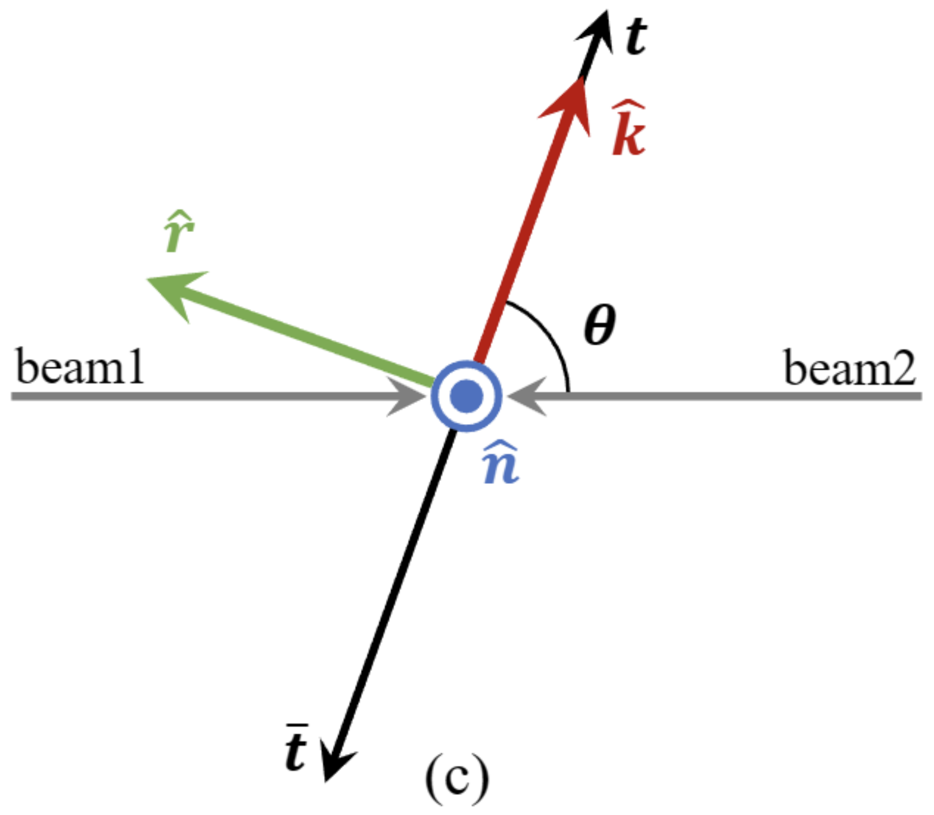
This choice is artificial and there are basis dependences

[**KC**, T. Han and M. Low, arXiv:2311.09166]
[**KC**, T. Han and M. Low, arXiv:2407.01672]



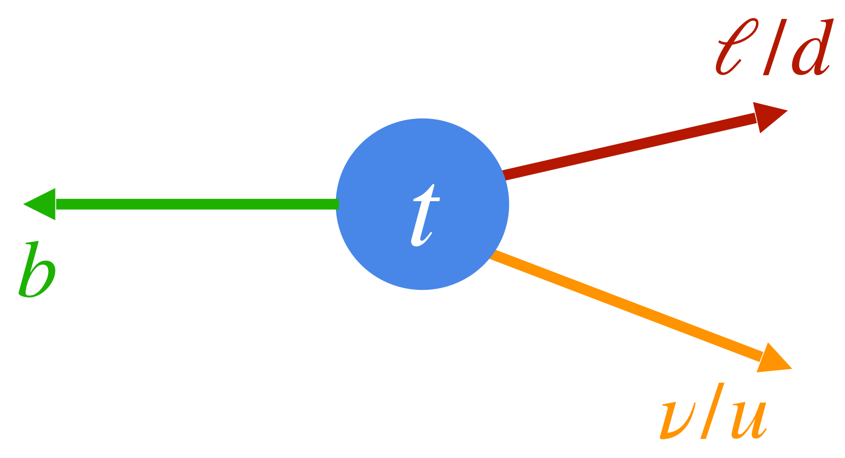
Example basis choices, $i, j \in \{k, n, r\}$

Spin analysing power

- $t \to \ell^+ + X,\ \bar{t} \to \ell^- + X$

$$s_i = 3\langle \ell_i^+ \rangle, \quad \bar{s}_i = -3\langle \ell_i^- \rangle, \quad C_{ij} = -9\langle \ell_i^+ \ell_j^- \rangle$$

- Different decay products have different capabilities of analysing spin



$$\frac{1}{\Gamma}\frac{d\Gamma}{d\Omega} = \frac{1}{2}\left(1 + \alpha_a(\hat{p}_a \cdot \vec{s})\right)$$

[B. Tweedie, 1401.3021]

Spin Analyzer a	Power α_a
lepton/down-quark	1.00
neutrino/up-quark	-0.34
b-quark or W	$\mp$0.40
soft-quark	0.50

- $t \to a + X,\ \bar{t} \to b + X$

$$s_i = \frac{3\langle \hat{p}_{a,i} \rangle}{\alpha_a}, \quad \bar{s}_i = \frac{3\langle \hat{p}_{b,i} \rangle}{\alpha_b}, \quad C_{ij} = \frac{9\langle \hat{p}_{a,i}\hat{p}_{b,j} \rangle}{\alpha_a \alpha_b}$$

Some other spin-1/2 particles:

$$\alpha_{\tau^+ \to \pi^+ \nu} = 1$$
$$\alpha_{\Lambda_b \to \Lambda_c^+ \pi^-} \approx 1$$
$$\alpha_{\Lambda_c^+ \to \Lambda \pi^+} = -0.79$$

[Du, He, Liu and Ma, arXiv:2409.15418]

- Simililar reconstruction for qutrits ZZ, W^+W^-, ZW, etc . [A. Barr, arXiv:2106.01377] different entries sensitive to different channels

[Q.Bi, Q.-H. Cao, **KC** and H. Zhang, arXiv:2307.14895]

also see the talk by Yong Du and Sihao Wu

Experimental reconstructions

- True distribution —**Fold**→ Detected distribution
  Smearing, missing $\nu\ldots$

$$\frac{d\sigma}{d\Omega_1 d\Omega_2} \propto \sum_{ss'\bar{s}\bar{s}'} \bar{\rho}_{ss',\bar{s}\bar{s}'}\Gamma_{ss'}\Gamma_{\bar{s}\bar{s}'}$$

$$\Gamma_{ss'} = \mathcal{M}_{t_s\to X}\mathcal{M}^*_{t_{s'}\to X}$$

Unfolding,
Templete fitting
Machine Learning
…

$t\bar{t}$ [ATLAS, arXiv:2311.07288]
[CMS, arXiv:2406.03976]
…

$\tau^+\tau^-$ [Zhang, Zhou, Liu, Li, Hsu, Han, Low and Wu, arXiv:2504.01496]
also see the talk by Baihong Zhou

- Sometimes, the concurrence have a simple form

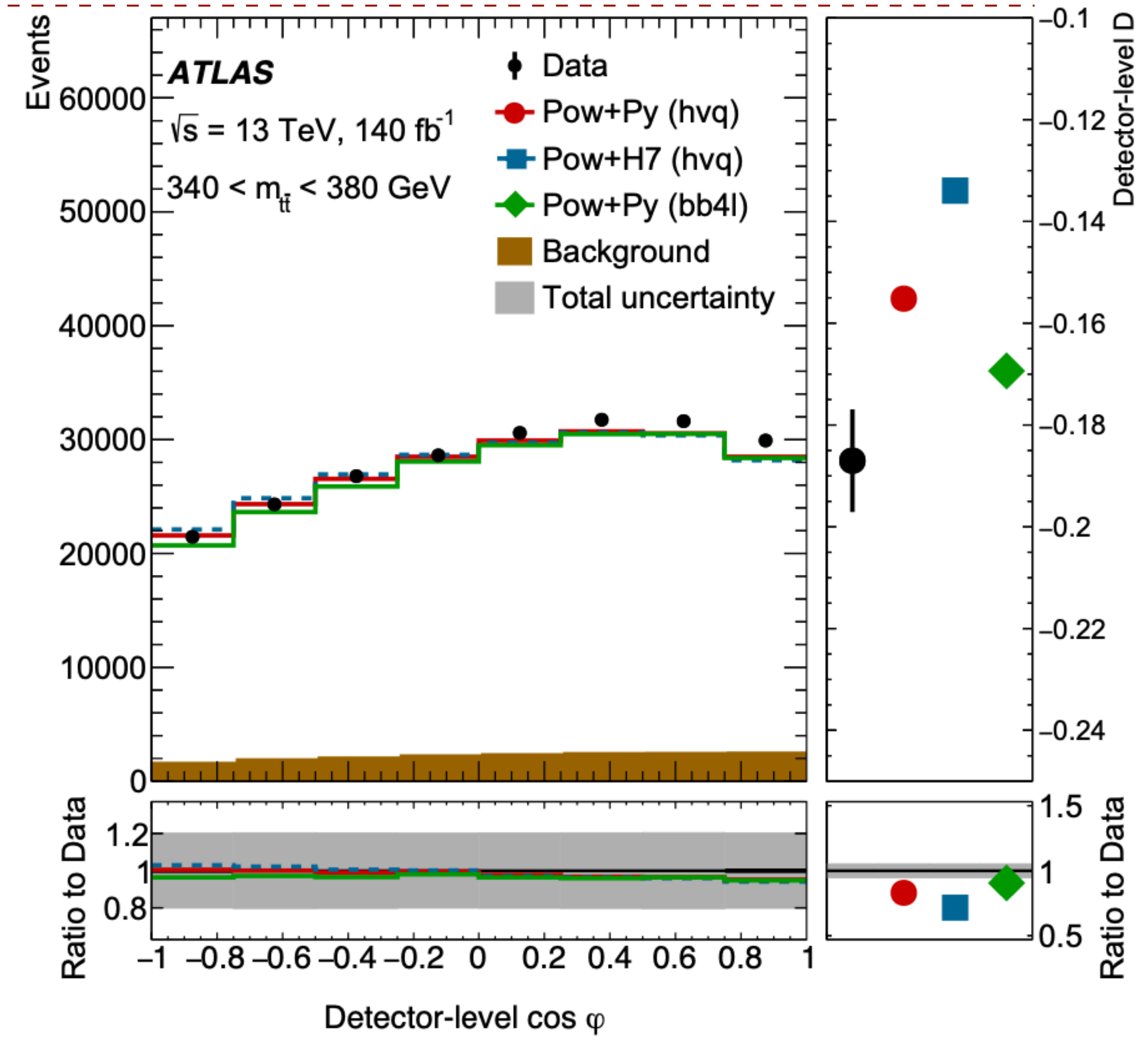
$$\mathscr{C}(\rho) = \frac{-\operatorname{tr}(C) - 1}{2}$$

Can be measured from single distribution

$$\frac{1}{\sigma}\frac{d\sigma}{d\cos\varphi_{\ell^+\ell^-}} = \frac{1}{2}\left(1 - \frac{\operatorname{tr}(C)}{3}\cos\varphi_{\ell^+\ell^-}\right)$$

Fold →

[Afik and de Nova, arXiv:2003.02280]

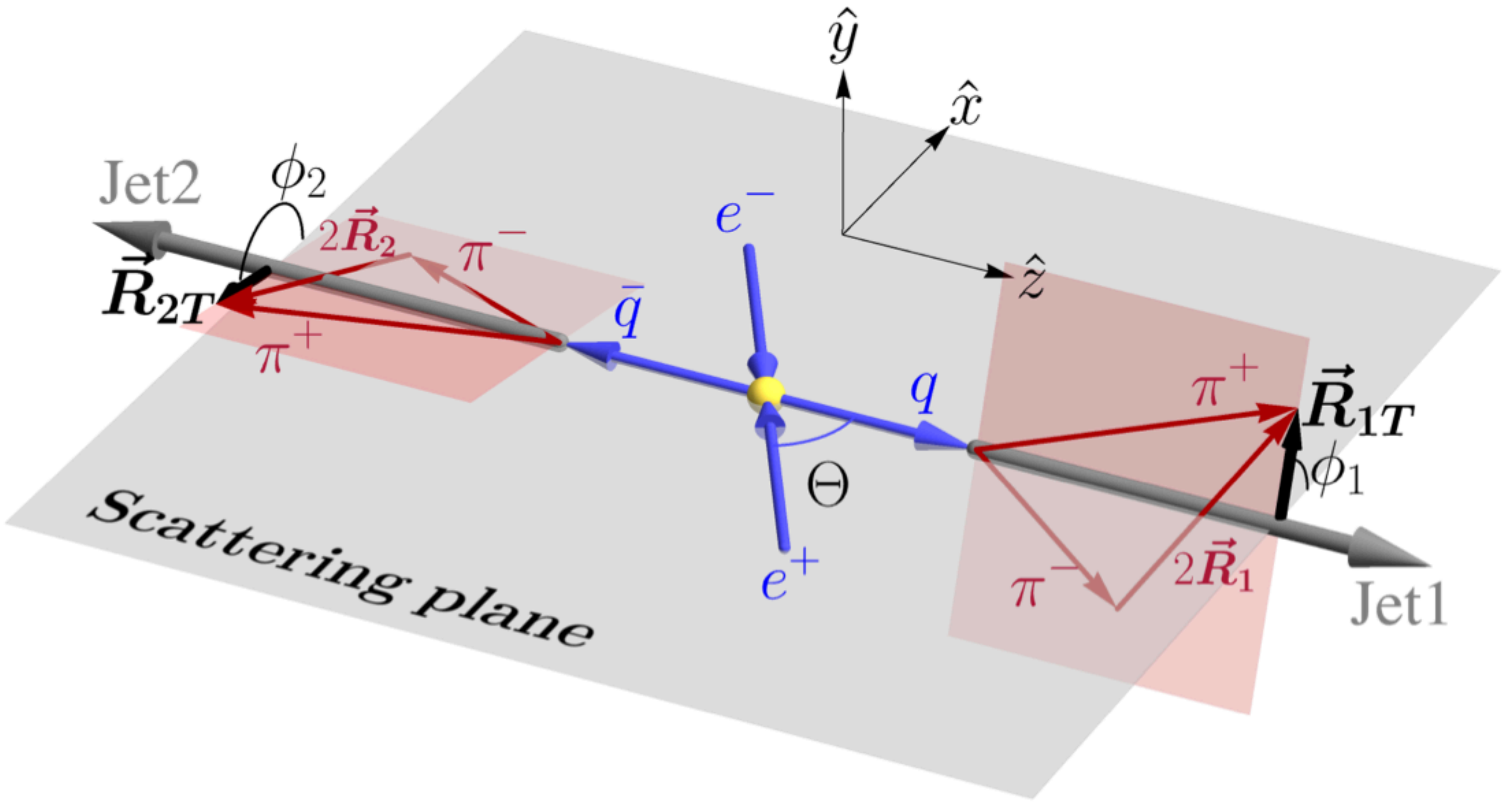


see more in the talk by Veljko Maksimovic

Spin analyser for quarks

— spin dependent fragmentation functions

$t\bar{t}, \tau^+\tau^-, W^+W^-, \text{etc}$	$e^+e^- \to q\bar{q} \to hadrons$
Narrow width approximation $\Downarrow$ production $\otimes$ decay	Collinear factorization $\Downarrow$ production $\otimes$ fragmentation
Parity violation $\Downarrow$ $\vec{p}_{\text{decay}}$ tends align with spin direction $\pm\vec{s}$	Parity conserving $\Downarrow$ $\vec{p}_{\text{hadron}}$ correlated with $\vec{s}$ under other Lorentz structures?



$$\frac{d\sigma}{d\Omega_1 d\Omega_2} = \sigma_{\text{hard}} \sum_{ss'\bar{s}\bar{s}'} \bar{\rho}_{ss',\bar{s}\bar{s}'} \mathcal{D}^{ss'}_{\pi^+\pi^-/q} \mathcal{D}^{\bar{s}\bar{s}'}_{\pi^+\pi^-/\bar{q}}$$

e.g. di-hadron fragmentation functions (diFF)

Total fragmentation rate
$\sigma = \sigma_{\text{hard}} \otimes D_1^q$

$$\frac{1}{2}\text{Tr}(\mathcal{D}_{\pi^+\pi^-/q}) = D_1^q(z_1, M_1),$$ Unpolarized diFF

Dependence on longitudinal spin and transverse spin of the quark

$$\frac{1}{2}\text{Tr}(\sigma_z \mathcal{D}_{\pi^+\pi^-/q}) = 0,$$

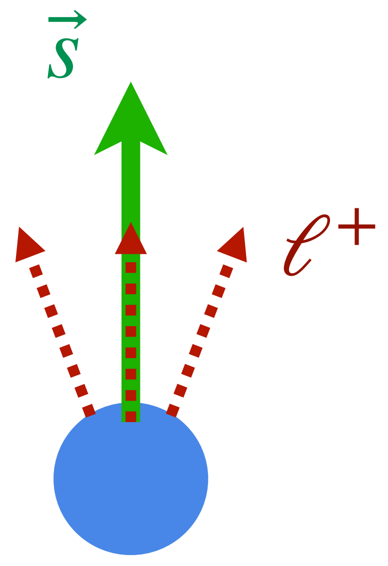
$$\frac{1}{2}\text{Tr}(\sigma_i \mathcal{D}_{\pi^+\pi^-/q}) = -\frac{\varepsilon_T^{ij} R^j_{1,T}}{|\vec{R}_{1,T}|} H_1^{\sphericalangle,q}(z_1, M_1),$$ Interference diFF

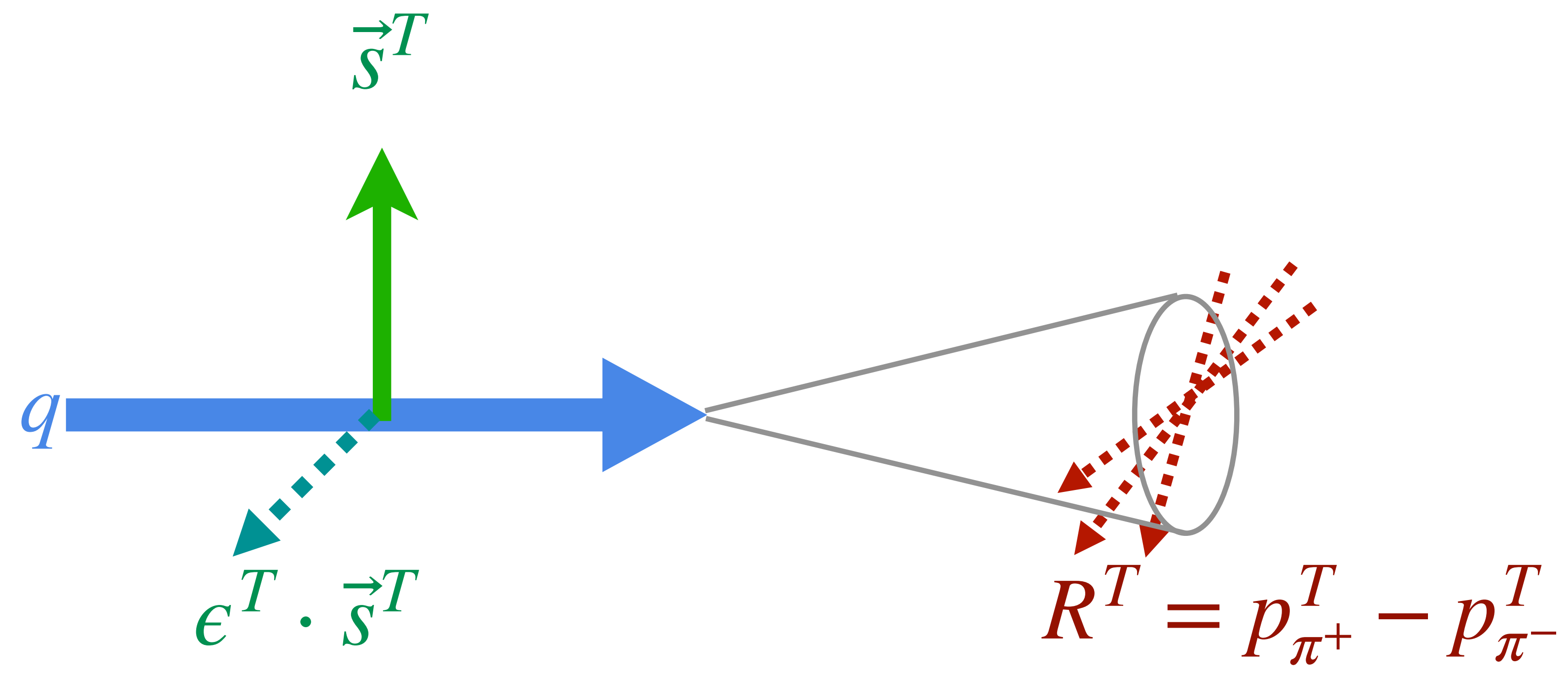
see the talk by Shu-Yi Wei

Spin analyser for quarks

— spin dependent fragmentation functions

- Difference:





- Similarity:

$$\frac{1}{\sigma}\frac{d\sigma}{d\phi_1 d\phi_2} = \frac{1}{4\pi^2} + \frac{\alpha_a\alpha_b}{64}\left[\frac{C_{xx}-C_{yy}}{2}\cos(\phi_1+\phi_2) + \frac{C_{xx}+C_{yy}}{2}\cos(\phi_1-\phi_2)\right]$$

$$\frac{1}{\sigma}\frac{\mathrm{d}\sigma}{\mathrm{d}\phi_1\mathrm{d}\phi_2} = \frac{1}{4\pi^2}\left[1+\frac{\sum_q e_q^2 H_1^{\sphericalangle,q}(z_1,M_1)H_1^{\sphericalangle,\bar{q}}(z_2,M_2)}{\sum_q e_q^2 D_1^q(z_1,M_1)D_1^{\bar{q}}(z_2,M_2)}\left(\frac{C_{xx}-C_{yy}}{2}\cos(\phi_1+\phi_2) - \frac{C_{xx}+C_{yy}}{2}\cos(\phi_1-\phi_2)\right)\right]$$

$$\mathcal{B}_- \equiv C_{xx} - C_{yy} = \frac{2\langle\cos(\phi_1+\phi_2)\rangle}{\alpha^{z_1,z_2}_{M_1,M_2}} = \frac{A_{12}}{\alpha^{z_1,z_2}_{M_1,M_2}}$$

Existing measurement from [Belle, arXiv:1104.2425]

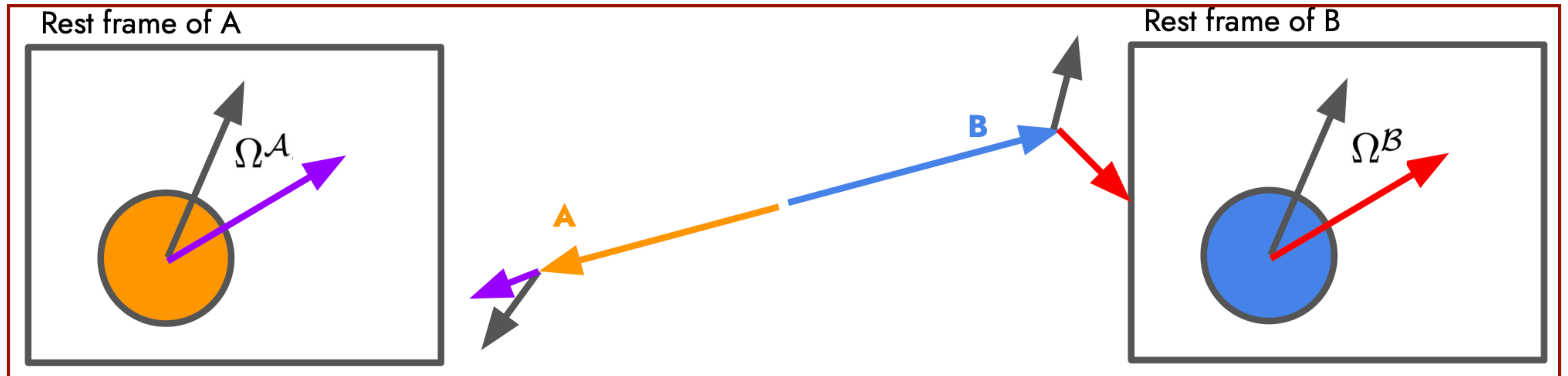
$$\alpha^{z_1,z_2}_{M_1,M_2} = \frac{1}{2}\frac{\sum_q e_q^2 H_1^{\sphericalangle,q}(z_1,M_1)H_1^{\sphericalangle,\bar{q}}(z_2,M_2)}{\sum_q e_q^2 D_1^q(z_1,M_1)D_1^{\bar{q}}(z_2,M_2)}$$

All these diFFs are public available in [JAM, arXiv:2308.14857]

[**KC** and B. Yan, arXiv:2501.03321]

see more in Bin Yan's talk

Quantum tomography of spin states at colliders



$$\frac{1}{\sigma}\frac{\mathrm{d}^4\sigma}{\mathrm{d}^2\Omega^{\mathcal{A}}\mathrm{d}^2\Omega^{\mathcal{B}}} = \frac{1}{(4\pi)^2}\left(1+\sum_i\left(\kappa^{\mathcal{A}}\,B_i^{\mathcal{A}}\Omega_i^{\mathcal{A}}+\kappa^{\mathcal{B}}\,B_i^{\mathcal{B}}\Omega_i^{\mathcal{B}}\right)+\sum_{i,j}\kappa^{\mathcal{A}}\kappa^{\mathcal{B}}\,\Omega_i^{\mathcal{A}}C_{ij}\Omega_j^{\mathcal{B}}\right),$$

Theory input is needed to perform quantum tomography at colliders

Theory input for a spin analysor (decay/fragmentation)
Spin is related with decay/fragmentation distribution

$$\rho_{\mathcal{AB}} = \frac{1}{4}\left(\mathbf{I}_4 + B_i^{\mathcal{A}}\sigma_i\otimes\mathbf{I}_2 + B_i^{\mathcal{B}}\mathbf{I}_2\otimes\sigma_i + C_{ij}\sigma_i\otimes\sigma_j\right)$$

→ **Quantun information……**

Theory input on the production
Spin is related with kinematics

Initial states and scattering kinematic
θ, β

[**KC**, T. Han and M. Low, arXiv:2410.08303]
[Gao, Ruzi, Li, Zhou, Chen, Zhang, Sun and Li, arXiv:2411.12518]
……

also see talk by Leyun Gao

Reconstructing the flavor states

Quantum correlations in flavor space

- Particles can be entangled in flavor space, as informative as spin
  - E.g., meson pair decayed from $\Upsilon(J^{PC} = 1^{--})$

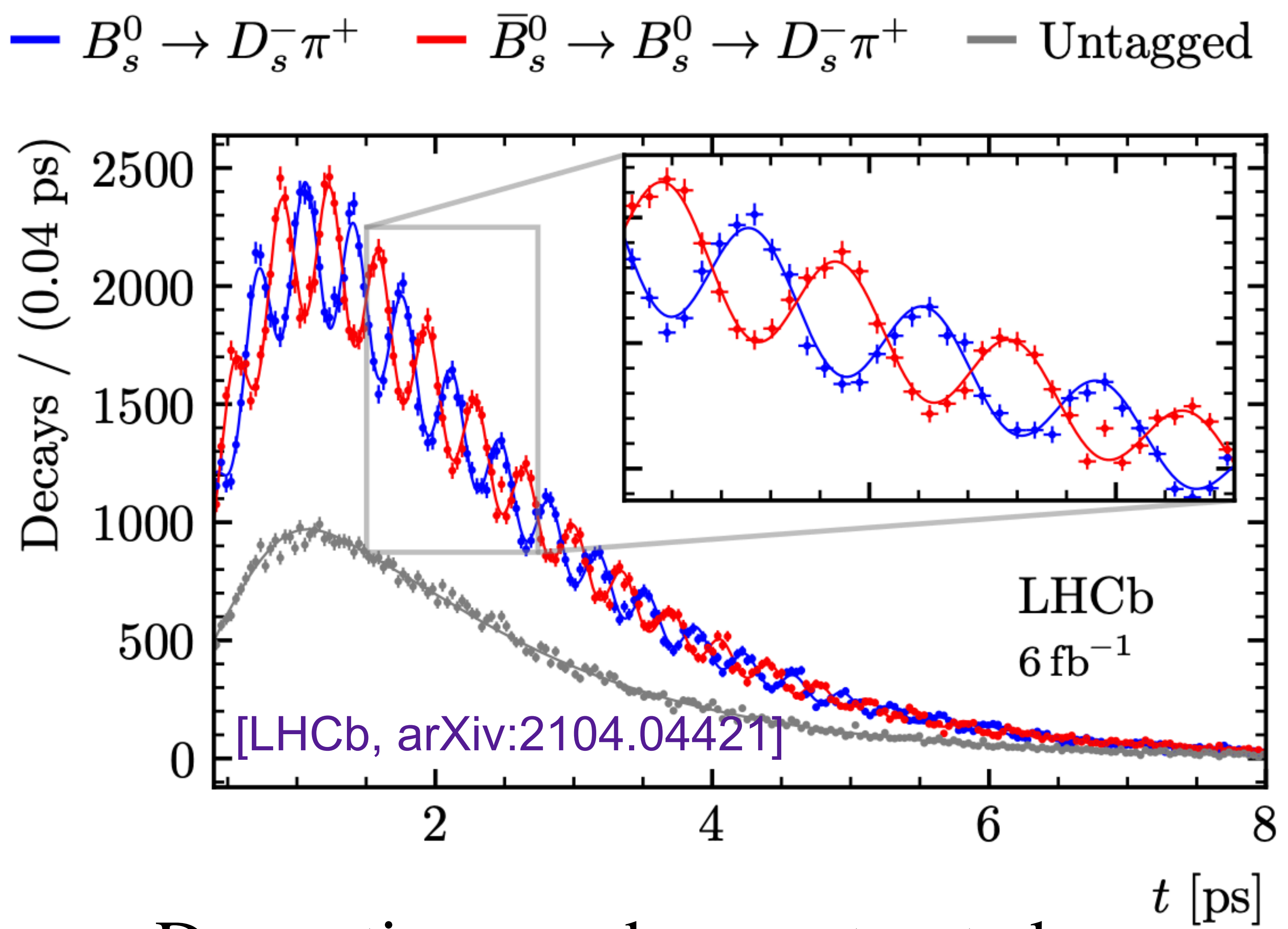
$$\frac{1}{\sqrt{2}}(|B^0\bar{B}^0\rangle - |\bar{B}^0B^0\rangle)$$

  - Vast number of events

$$\text{e.g., } B_s\bar{B}_s\text{: } 7\times 10^6 \text{ at Belle, } 8\times 10^8 \text{ at LHCb}$$

$$K_0\bar{K}_0\text{: } 8\times 10^9 \text{ at KLOE and KLOE-2, etc}$$

  - Precise measurement of flavor oscillation and correlation.



Decay time can be constructed very well!

- Quantum correlation, Bell inequality, quantum decoherence, etc have been studied in BB at Belle, KK at KLOE…

[Belle, hep-ph/0702267; KLOE, hep-ph/0607027,etc]
[Y. Shi, quant-ph/0605070, 1102.2828, etc]

e.g. see the talk by Neetu Raj Singh Chundawat and Hailong Feng

- Reconstruct the complete **flavor density matrix**?

$$\rho_{MM} = \frac{\mathbb{1}_4 + R_i^{\mathcal{A}}\sigma_i \otimes \mathbb{1}_2 + R_i^{\mathcal{B}}\mathbb{1}_2 \otimes \sigma_i + C_{ij}\sigma_i \otimes \sigma_j}{4}$$

Flavor density matrix of MM pair when they are produced, i.e., at $t = 0$

σ_i in spin space v.s. flavor space

— achieving a complementary set of measurements

- One qubit: 3 operators

$$\rho = \frac{\mathbb{1}_2 + R_i \sigma_i}{2}, \quad R_i = \langle \sigma_i \rangle$$

- Spin-1/2 particle as qubit:
  - $\vec{\sigma}$ is embeded in 3d spatial space.
  - Different direction to measure spin → complementary set of observables.
  - Collider envoronment: infer $\langle \sigma_i \rangle$ from decay
- **What about qubit in flavor space**?
  - An obvious challenge: cannot choose“*direction*” of measurement freely?
  - e.g. flavor tagging, only σ_z: $\sigma_z |B^0\rangle = |B^0\rangle, \quad \sigma_z |\bar{B}^0\rangle = -|\bar{B}^0\rangle$

Flavor/Mass eigenstate

- Hamitonian in flavor eigenstate: $|M\rangle, |\bar{M}\rangle$: $H = \mathbf{M} - i\mathbf{\Gamma}/2 = \begin{pmatrix} m - i\frac{\Gamma}{2} & P^2 \\ Q^2 & m - i\frac{\Gamma}{2} \end{pmatrix},$
- Mass eigenstate is

$$|M_1\rangle = p|M\rangle + q\left|\bar{M}\right\rangle, \qquad (m_1, \Gamma_1)$$
$$|M_2\rangle = p|M\rangle - q\left|\bar{M}\right\rangle, \qquad (m_2, \Gamma_2)$$

- CP conserving case: $|M_{1/2}\rangle \propto |M\rangle \pm |\bar{M}\rangle$

difference between flavor and mass eigenstate leads to flavor oscillation

- $\rho_M = \dfrac{\mathbb{1}_2 + R_i\sigma_i}{2}$, R_i still have physical meaning; conversions are:
  - z-direction $\Longrightarrow$ flavor eigenstate, (oscillates with y) $|M\rangle, R_z = 1$; $|\bar{M}\rangle, R_z = -1$
  - x-direction $\Longrightarrow$ mass eigenstates (and CP eigenstates) $|M_1\rangle, R_x = 1$; $|M_2\rangle, R_x = -1$

State evolution in Bloch-vector space

$$\rho(t) \propto U(t)\rho U(t)^\dagger \qquad U = \begin{pmatrix} \frac{1}{2}(e^{-\Gamma_1 t/2 - im_1 t} + e^{-\Gamma_2 t/2 - im_2 t}) & \frac{q}{2p}(e^{-\Gamma_1 t/2 - im_1 t} - e^{-\Gamma_2 t/2 - im_2 t}) \\ \frac{p}{2q}(e^{-\Gamma_1 t/2 - im_1 t} - e^{-\Gamma_2 t/2 - im_2 t}) & \frac{1}{2}(e^{-\Gamma_1 t/2 - im_1 t} + e^{-\Gamma_2 t/2 - im_2 t}) \end{pmatrix}$$

- State evolution (Operator evolution)
  - Neglect CPV and decay ($\Gamma \ll \Delta m$):

Schrödinger picture density matrix $\rho = \frac{I_2 + \vec{R}\cdot\vec{\sigma}}{2}$

$$\frac{d\rho(t)}{dt} = -i[H,\rho(t)] = \left(\vec{X} \times \vec{R}(t)\right)\cdot\vec{\sigma},$$
$$\frac{d\vec{R}(t)}{dt} = \vec{X} \times \vec{R}(t).$$

Heisenberg picture operator $A = \vec{a}\cdot\vec{\sigma}$

$$\frac{dA(t)}{dt} = i[H,A(t)] = -\left(\vec{X} \times \vec{a}(t)\right)\cdot\vec{\sigma},$$
$$\frac{d\vec{a}(t)}{dt} = -\vec{X} \times \vec{a}(t).$$

  - Precession around $\vec{X} = (\Delta m, 0, 0)$
- Example operator $\sigma_z |M\rangle = +|M\rangle, \quad \sigma_z |\bar{M}\rangle = -|\bar{M}\rangle$
  - Flavor tagging at different times $\Longrightarrow$ both σ_z and σ_y

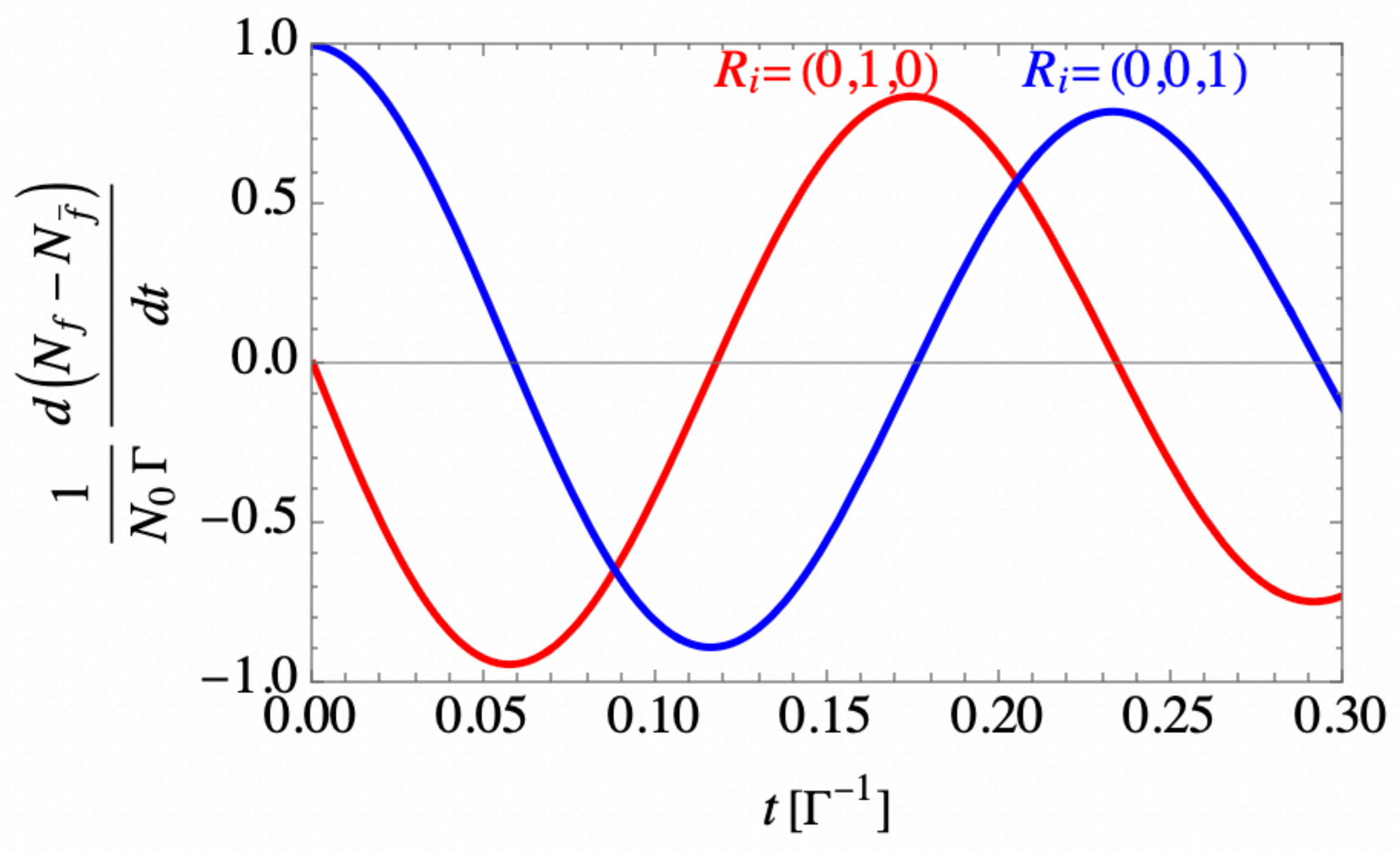
Collapse the superposition

- Decay final state that is NOT a CP eigenstate: (e.g. $B^0 \to \ell^+ \nu X^-$ while $\bar{B}^0 \to \ell^- \bar{\nu} X^+$)
  - $M \to f$ with $CP|f\rangle = |\bar{f}\rangle \neq |f\rangle$: project to $|M\rangle, |\bar{M}\rangle$ with $P_{M/\bar{M}} = \frac{1 \pm R_z}{2}$
- Decay rate asymmetry to $f, \bar{f}$:

$$(N_f - N_{\bar{f}}) \sim \langle \sigma_z \rangle = R_z$$

- Observable:

$$\frac{1}{N_0} \frac{d(N_f - N_{\bar{f}})}{dt} = e^{-\Gamma t} (R_z \cos(\Delta m t) - R_y \sin(\Delta m t)) \Gamma_{M \to f}$$



$\Delta m \approx 27\,\Gamma$,
slightly damped oscillation

- Both R_y and R_z at $t = 0$ are obtained as they oscillated into each other.

Collapse the superposition

$$\rho_M = \frac{\mathbb{1}_2 + R_i \sigma_i}{2}$$

$$|M_{1/2}\rangle \propto |M\rangle \pm |\bar{M}\rangle$$

- Two kinds of decay final states, CP eigenstate or not
  - $M \to f$ with $CP|f\rangle = |\bar{f}\rangle \neq |f\rangle$: project to $|M\rangle, |\bar{M}\rangle$ with $P_{M/\bar{M}} = \frac{1 \pm R_z}{2}$, (oscillates with R_y)
  - $M \to f_\eta$ with $CP|f_\eta\rangle = \eta|f_\eta\rangle$: project to $|M_1\rangle, |M_2\rangle$ with $P_{M_1/M_2} = \frac{1 \pm R_x}{2}$

$\mathrm{Br}(M \to f_{\eta_{CP}})$		$\mathrm{Br}(M \to f)$	
$B_d^0 \to J/\psi K_S$	$(8.91 \pm 0.21) \times 10^{-4}$	$B_d^0 \to \ell^+ \nu_\ell X^-$	$(20.66 \pm 0.56)\%$
$B_s^0 \to J/\psi \eta$	$(3.9 \pm 0.7) \times 10^{-4}$	$B_s^0 \to \ell^+ \nu_\ell X^-$	$(19.2 \pm 1.6)\%$

Combine different channels?

Collapse the superposition

— x direction in the Bloch vector space

- Conside a meson that only decay to flavor eigenstate $|f\rangle$ (such as semileptonic) or CP-even eigenstate $|f_+\rangle$

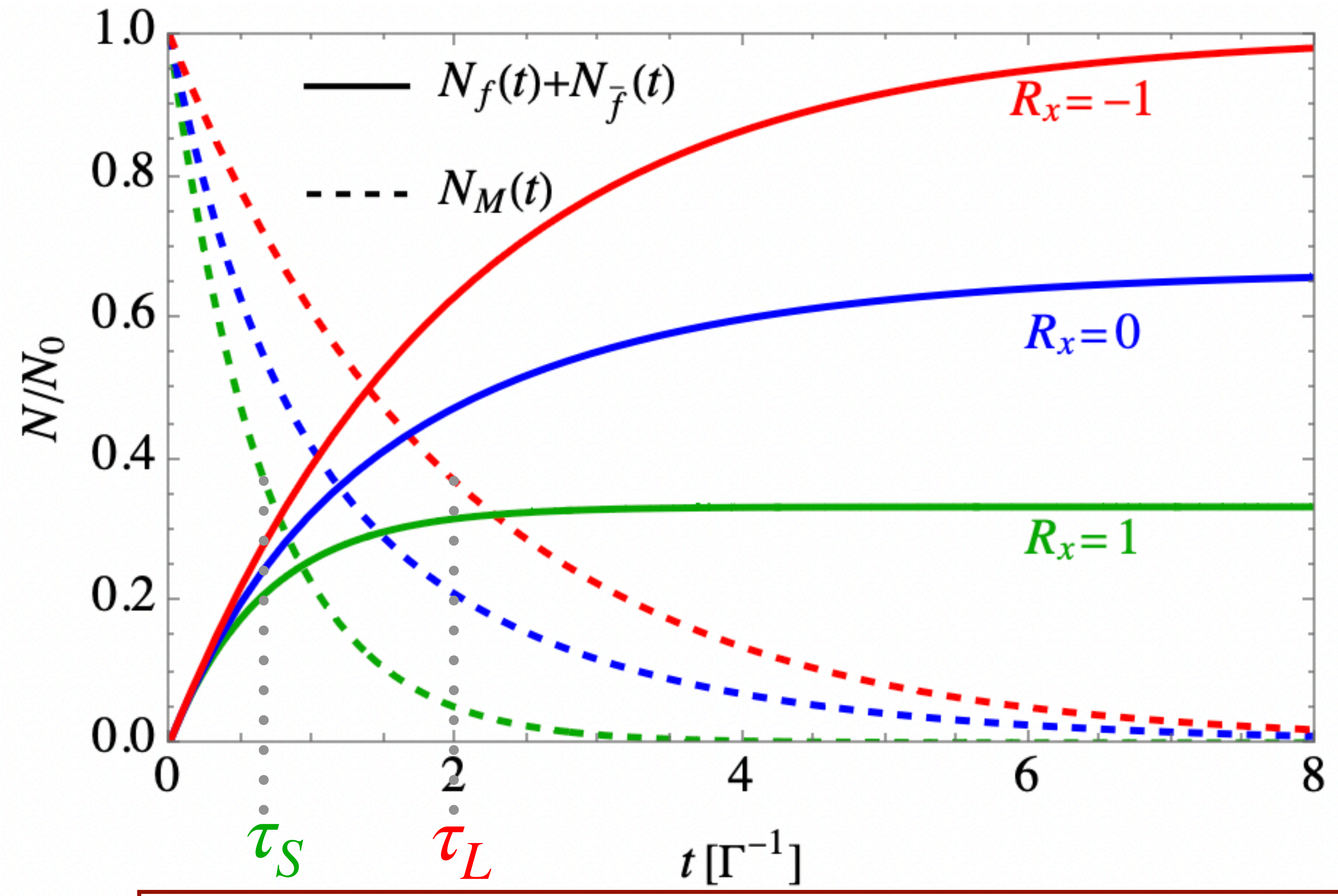
$$\Gamma_{M(t)\to f_+} = \frac{1+R_x(t)}{2}\Gamma_{M_1\to f_+} = (1+R_x(t))\Gamma_{M\to f_+}$$

- Both $M(t) \to f_+$ and $M(t) \to f/\bar{f}$ depend on R_x

affected by branching fraction

  - $R_x = 1$, M_1 can decay to both f_+ and $f/\bar{f}$
  - $R_x = -1$, M_2 can't decay to f_+, only $f/\bar{f}$

- Semi-leptonic channel is enough. e.g. $B_s \to \ell^+\nu_\ell X + h.c.$



difference between each lines $\propto$ $\Delta\Gamma$
i.e., decay to CP eigenstates.

Observables in semileptonic decay channel

- Reconstruct ρ_{MM} at $t = 0$.

$$\rho_{MM} = \frac{\mathbb{1}_4 + R_i^{\mathcal{A}} \sigma_i \otimes \mathbb{1}_2 + R_i^{\mathcal{B}} \mathbb{1}_2 \otimes \sigma_i + C_{ij} \sigma_i \otimes \sigma_j}{4}$$

- One meson:
  - $N_f - N_{\bar{f}} \Longrightarrow R_y, R_z$
  - $N_f + N_{\bar{f}} \Longrightarrow R_x$

$$\frac{1}{N_0} \frac{d(N_f - N_{\bar{f}})}{dt} = e^{-\Gamma t} \left(\boxed{R_z} \cos(\Delta m t) - \boxed{R_y} \sin(\Delta m t) \right) \Gamma_{M \to f}$$

$$\frac{1}{N_0} \frac{d(N_f + N_{\bar{f}})}{dt} = e^{-\Gamma t} \left(\cosh(\Delta \Gamma t/2) - \boxed{R_x} \sinh(\Delta \Gamma t/2) \right) \Gamma_{M \to f}$$

- Meson pair:
  - Four observables from the correlation between the above two

$\mathscr{H}_A \otimes \mathscr{H}_B$ — $N_{f\bar{f}}$: meson A decay to f **and** meson B decay to $\bar{f}$

$I_2 \otimes I_2 \longrightarrow N_{\text{tot}} = N_{ff} + N_{\bar{f}f} + N_{f\bar{f}} + N_{\bar{f}\bar{f}}$

$\sigma_z \otimes \sigma_z \longrightarrow A_{ff} = N_{ff} - N_{\bar{f}f} - N_{f\bar{f}} + N_{\bar{f}\bar{f}} = N_{\text{like}} - N_{\text{unlike}}$

$\sigma_z \otimes I_2 \longrightarrow A_f^{\mathcal{A}} = N_{ff} + N_{f\bar{f}} - N_{\bar{f}f} - N_{\bar{f}\bar{f}}$

$I_2 \otimes \sigma_z \longrightarrow A_f^{\mathcal{B}} = N_{ff} - N_{f\bar{f}} + N_{\bar{f}f} - N_{\bar{f}\bar{f}}$

Sensitivity estimation

- We consider a C-odd flavor state $(|M\bar{M}\rangle - |\bar{M}M\rangle)/\sqrt{2}$
- B_s pair
  - x-direction is hard to measure, y and z are the same
- Feasibility:
  - B_sB_s: $10^5 \sim 10^6$ at LHCb (HL-LHC)
  - More B_s and B_d at BelleII, the reconstruction of individual decay time is developing.
  - KK: 8×10^9 at KLOE and KLOE-2, a fraction of 10^{-5} decay to lepton in the first period we consider.

	$B_s^0\bar{B}_s^0$ fitted	$K^0\bar{K}^0$ fitted	Obs.
$R_x^{\mathcal{A}}$	-0.01 ± 0.06	-0.002 ± 0.006	$N_{\rm tot}$
$R_x^{\mathcal{B}}$	-0.01 ± 0.06	-0.003 ± 0.006	
$R_y^{\mathcal{A}}$	0.000 ± 0.003	0.005 ± 0.006	$A_f^{\mathcal{A}}$
$R_z^{\mathcal{A}}$	0.000 ± 0.003	0.003 ± 0.005	
$R_y^{\mathcal{B}}$	0.000 ± 0.003	0.005 ± 0.006	$A_f^{\mathcal{B}}$
$R_z^{\mathcal{B}}$	0.001 ± 0.003	0.002 ± 0.004	
C_{xx}	-1.2 ± 1.0	-1.005 ± 0.012	$N_{\rm tot}$
C_{yx}	0.00 ± 0.06	0.005 ± 0.008	$A_f^{\mathcal{A}}$
C_{zx}	0.00 ± 0.05	0.006 ± 0.006	
C_{xy}	0.00 ± 0.05	0.006 ± 0.007	$A_f^{\mathcal{B}}$
C_{xz}	0.00 ± 0.05	0.004 ± 0.006	
C_{yy}	-1.001 ± 0.004	-1.003 ± 0.008	A_{ff}
C_{yz}	0.001 ± 0.003	0.000 ± 0.007	
C_{zy}	0.000 ± 0.003	0.000 ± 0.006	
C_{zz}	-1.000 ± 0.003	-1.001 ± 0.003	
Concurrence	1.1 ± 0.5	1.005 ± 0.007	

Tab. Statistical uncertinty with 10^6 events

[**KC**, T. Han, M. Low and T. Wu, in preparation]

Usually no first principle calculation of the meson flavor states, but measuring it is possible!

Summary

Spin states

$t\bar{t}, \tau^+\tau^- W^+W^- \cdots$
Measure decay distribution

$q\bar{q}$
Measure hadron distribution

Measure $2 \to 2$ kinematics

Theory input on spin analysors

Theory input on production mechanism

$$\rho = \frac{\mathbb{1}_4 + R_i^{\mathcal{A}} \sigma_i \otimes \mathbb{1}_2 + R_i^{\mathcal{B}} \mathbb{1}_2 \otimes \sigma_i + C_{ij} \sigma_i \otimes \sigma_j}{4}$$

No direct measurements $\sigma_x, \sigma_y, \sigma_z$ in collider environment

Theory input on the state evolution and decay

$B^0\bar{B}^0, K^0\bar{K}^0$
Measure oscillation pattern

Flavor states

Generally no first principle prediction of the flavor state

- Some other topics beyond quantum tomography: quantum process tomography, QI and model building, etc

e.g. see the talk by Zhewei Yin and Alim Ruzi