Quantum Entanglement Autodistillation in Baryon Pair Decays

Hai-Long Feng

Huazhong University of Science and Technology

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Collaborators: Hao Tang, Wu-zhong Guo, Qin Qin

Background

Inspired by: *Entanglement Autodistillation from Particle Decays* (arXiv:2401.06854)

$$t\bar{t} \to bW^+\bar{t} \xrightarrow{\text{partial trace on } W^+} b\bar{t}$$

Entanglement between b and $\bar{t}$ can increase with some probability. This probabilistic enhancement is known as **autodistillation**.

The process we consider:

$$e^+e^- \to J/\psi, \psi' \to \mathcal{B}\bar{\mathcal{B}}$$

Each baryon then undergoes a weak decay:

- $\mathcal{B} \to \mathscr{b} + M_0$ (baryon decay)
- $\bar{\mathcal{B}} \to \bar{\mathscr{b}} + \bar{M}_0$ (antibaryon decay)

Our goal:

- Compare the entanglement of the initial state $\rho_{\mathcal{B}\bar{\mathcal{B}}}$ and the final state $\rho_{\mathscr{b}\bar{\mathscr{b}}}$

Entanglement Measures

Two commonly used entanglement measures for two-qubit systems:

1. Negativity

$$\mathcal{N}(\rho) = \frac{\sum_i |\lambda_i| - 1}{2}$$

Where:

- ρ^{pt} is the partial transpose of ρ with respect to one subsystem.
- λ_i are the eigenvalues of ρ^{pt}.

2. Concurrence

$$\mathscr{C}(\rho) = \max\left(r_1 - \sum_{i>1} r_i,\, 0\right)$$

Where:

- $R = \rho\,(\sigma_y \otimes \sigma_y)\,\rho^*\,(\sigma_y \otimes \sigma_y)$
- r_i are the square roots of the eigenvalues of R in decreasing order

Specific Process

We consider the cascade decay:

$$J/\psi \to \Xi^- + \bar{\Xi}^+ \quad \text{then} \quad \begin{cases} \Xi^- \to \Lambda + \pi^- \\ \bar{\Xi}^+ \to \bar{\Lambda} + \pi^+ \end{cases} \qquad \begin{cases} \Lambda \to p + \pi^- \\ \bar{\Lambda} \to \bar{p} + \pi^+ \end{cases}$$

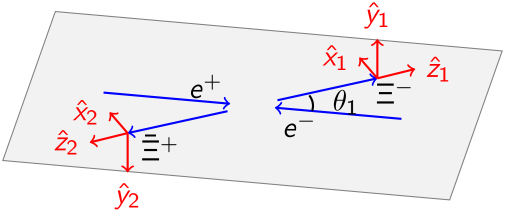


Figure: Orientation of helicity frames in the decays of Ξ^- and $\bar{\Xi}^+$

Spin Configuration

- The spin density matrix of the $\Xi^-\bar{\Xi}^+$ system is given by(Perotti et al. 2019):

$$\rho_{\Xi^-,\bar{\Xi}^+} = \frac{1}{4}\sum_{\mu,\nu=0}^{3} C_{\mu\nu}(\theta_1;\alpha_\psi,\Delta\Phi)\,\sigma_\mu^{\Xi^-}\otimes\sigma_\nu^{\bar{\Xi}^+},$$

- Under weak decays, the spin operators transform as:

$$\sigma_\mu^{\Xi^-}\to\sum_{\nu=0}^{3} a_{\mu\nu}^{\Xi^-}\sigma_\nu^{\Lambda},\quad \sigma_\mu^{\bar{\Xi}^+}\to\sum_{\nu=0}^{3} a_{\mu\nu}^{\bar{\Xi}^+}\sigma_\nu^{\bar{\Lambda}},$$

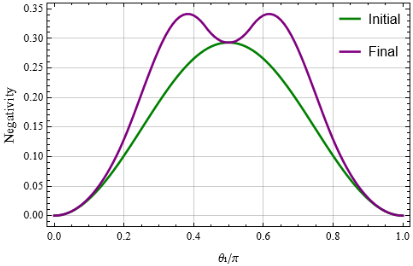
where

$$a_{\mu\nu}^{\Xi^-} = a_{\mu\nu}^{\Xi^-}(\theta_\Lambda,\phi_\Lambda;\alpha_D^{\Xi^-},\phi_D^{\Xi^-}),\quad a_{\mu\nu}^{\bar{\Xi}^+} = a_{\mu\nu}^{\bar{\Xi}^+}(\theta_{\bar{\Lambda}},\phi_{\bar{\Lambda}};\alpha_D^{\bar{\Xi}^+},\phi_D^{\bar{\Xi}^+}).$$

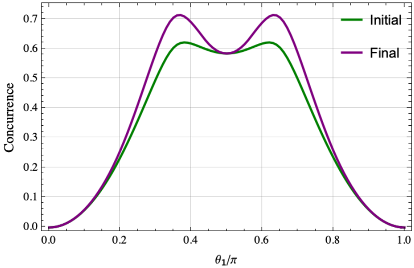
- Then, the spin density matrix of the $\Lambda\bar{\Lambda}$ system becomes:

$$\rho_{\Lambda,\bar{\Lambda}} = \frac{1}{4}\sum_{\mu,\nu=0}^{3}\sum_{\alpha,\beta=0}^{3} C_{\mu\nu}\,a_{\mu\alpha}^{\Xi^-}\,a_{\nu\beta}^{\bar{\Xi}^+}\,\sigma_\alpha^{\Lambda}\otimes\sigma_\beta^{\bar{\Lambda}}.$$

- We compare the entanglement of the initial state $\rho_{\Xi^-,\bar{\Xi}^+}$ with that of the final state $\rho_{\Lambda,\bar{\Lambda}}$, using two standard measures:



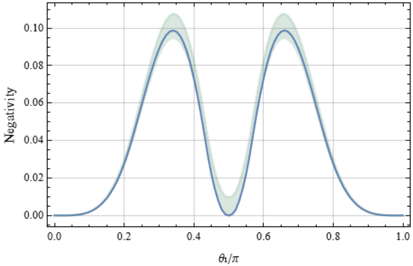
(a) Negativity of $\Xi^-\bar{\Xi}^+$ vs. $\Lambda\bar{\Lambda}$



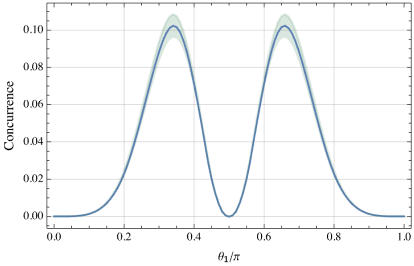
(b) Concurrence of $\Xi^-\bar{\Xi}^+$ vs. $\Lambda\bar{\Lambda}$

Enhancement of Entanglement

- We also examine the *difference* in entanglement before and after the decay.
- The following plots show the enhancement of negativity and concurrence:



(a) Increase in Negativity



(b) Increase in Concurrence

Reconstruction of Spin Configuration

We reconstruct the spin configuration of the mother particles using the angular distributions of their decay products.

Angular decay coefficients(Perotti et al. 2019):

$$
\begin{aligned}
a_{00} &= 1, \\
a_{10} &= \alpha_D^{\Xi^-} \cos\phi \sin\theta, \\
a_{20} &= \alpha_D^{\Xi^-} \sin\phi \sin\theta, \\
a_{30} &= \alpha_D^{\Xi^-} \cos\theta.
\end{aligned}
$$

Step 1: From $\Lambda\bar{\Lambda}$ distribution to $C_{\mu\nu}$:

$$
f_{\Lambda\bar{\Lambda}}(\theta_\Lambda, \phi_\Lambda; \theta_{\bar{\Lambda}}, \phi_{\bar{\Lambda}}) \propto \mathrm{Tr}(\rho_{\Lambda,\bar{\Lambda}}) = \sum_{\mu,\nu=0}^{3} C_{\mu\nu} a_{\mu 0}^{\Xi^-}(\theta_\Lambda, \phi_\Lambda) a_{\nu 0}^{\Xi^+}(\theta_{\bar{\Lambda}}, \phi_{\bar{\Lambda}}).
$$

$$
C_{\mu\nu} \propto \int f_{\Lambda\bar{\Lambda}}(\theta_\Lambda, \phi_\Lambda; \theta_{\bar{\Lambda}}, \phi_{\bar{\Lambda}})\, a_{\mu 0}^{\Xi^-}\, a_{\nu 0}^{\Xi^+}\, \mathrm{d}\theta_\Lambda\, \mathrm{d}\phi_\Lambda\, \mathrm{d}\theta_{\bar{\Lambda}}\, \mathrm{d}\phi_{\bar{\Lambda}}.
$$

Reconstruction of Spin Configuration

Step 2: Transform to $p\bar{p}$ distribution:

$$f_{p\bar{p}}(\theta_p, \phi_p; \theta_{\bar{p}}, \phi_{\bar{p}}) \propto \mathrm{Tr}(\rho_{p\bar{p}}) = \sum_{\mu,\nu=0}^{3} \sum_{\alpha,\beta=0}^{3} C_{\mu\nu}\, a_{\mu\alpha}^{\Xi^-} a_{\nu\beta}^{\bar{\Xi}^+} a_{\alpha 0}^{\Lambda} a_{\beta 0}^{\bar{\Lambda}}.$$

$$C_{\alpha\beta}^{\Lambda\bar{\Lambda}} = \sum_{\mu,\nu=0}^{3} C_{\mu\nu}\, a_{\mu\alpha}^{\Xi^-}\, a_{\nu\beta}^{\bar{\Xi}^+} \propto \int f_{p\bar{p}}(\theta_p, \phi_p; \theta_{\bar{p}}, \phi_{\bar{p}})\, a_{\alpha 0}^{\Lambda}\, a_{\beta 0}^{\bar{\Lambda}}\, \mathrm{d}\theta_p\, \mathrm{d}\phi_p\, \mathrm{d}\theta_{\bar{p}}\, \mathrm{d}\phi_{\bar{p}}.$$

Angular decay coefficients:

$$\begin{aligned} a_{00} &= 1, \\ a_{10} &= \alpha_D^{\Lambda} \cos\phi \sin\theta, \\ a_{20} &= \alpha_D^{\Lambda} \sin\phi \sin\theta, \\ a_{30} &= \alpha_D^{\Lambda} \cos\theta. \end{aligned}$$

Mechanism Behind Entanglement Increase

The polarization vector of the daughter baryon can be derived from that of the mother particle via the following transformation (Navas et al. 2024):

$$\mathbf{P}_D = \frac{(\alpha_D + \mathbf{P}_M \cdot \hat{\mathbf{n}})\hat{\mathbf{n}} + \beta_D(\mathbf{P}_M \times \hat{\mathbf{n}}) + \gamma_D\, \hat{\mathbf{n}} \times (\mathbf{P}_M \times \hat{\mathbf{n}})}{1 + \alpha_D\, (\mathbf{P}_M \cdot \hat{\mathbf{n}})},$$

where:

$$\beta_D = \sqrt{1-\alpha_D^2}\,\sin\phi_D, \quad \gamma_D = \sqrt{1-\alpha_D^2}\,\cos\phi_D.$$

Examples:

- For a spin-up mother particle $\mathbf{P}_M = \hat{\mathbf{z}}$, the daughter polarization is:

$$\mathbf{P}_D = (\beta_D,\, \gamma_D,\, \alpha_D + \hat{\mathbf{z}} \cdot \hat{\mathbf{n}})$$

- For a spin-down mother particle $\mathbf{P}_M = -\hat{\mathbf{z}}$, we have:

$$\mathbf{P}_D = (-\beta_D,\, -\gamma_D,\, \alpha_D - \hat{\mathbf{z}} \cdot \hat{\mathbf{n}})$$

Mechanism Behind Entanglement Increase

The decay process can also be understood as a local quantum operation:

$$\rho_{\Lambda,\bar{\Lambda}} = \left(D^{\Xi^-} \otimes D^{\bar{\Xi}^+}\right)^\dagger \rho_{\Xi^-,\bar{\Xi}^+} \left(D^{\Xi^-} \otimes D^{\bar{\Xi}^+}\right)$$

- If $\alpha_D = 0$:
  $\Rightarrow$There is **no parity violation**
  $\Rightarrow$ D is a local unitary (LU) operator
  $\Rightarrow$ Entanglement remains **unchanged**
- If $\alpha_D \neq 0$:
  $\Rightarrow$The decay process **exhibits parity violation**
  $\Rightarrow$ D is a local non-unitary operator
  $\Rightarrow$ Entanglement can be **modified or enhanced**

Irrelevance of ϕ_D to Entanglement

The spinor states correspond to the polarization vectors as follows:

$$(\beta_D, \gamma_D, \alpha_D + \hat{z} \cdot \hat{n}) \to \left(\cos \frac{\theta_1}{2},\ e^{i\phi_1} \sin \frac{\theta_1}{2}\right)$$

$$(-\beta_D, -\gamma_D, \alpha_D - \hat{z} \cdot \hat{n}) \to \left(\cos \frac{\theta_2}{2},\ e^{i\phi_2} \sin \frac{\theta_2}{2}\right)$$

We observe that $\phi_1 = \phi_2 = \phi = \arctan(\cot(\phi_D))$, the change of the decay parameter ϕ_D introduces **a local unitary transformation**:

$$U = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\delta\phi} \end{pmatrix}.$$

So we conclude that ϕ_D does not affect the entanglement.

Conclusion

Key Findings

- We observed the phenomenon of **entanglement autodistillation** in the baryon decay process.
- This phenomenon correlates with the decay parameter α_D, which is itself connected to parity violation.

Thank You

Questions?

References I

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$C_{\mu\nu}$

Regardless of the normalization and the overall phase, the $\{C_{\mu\nu}\}$ are given by(Perotti et al. 2019)

$$
\begin{aligned}
C_{00} &= 2(1 + \alpha_\psi \cos^2\theta_1), \\
C_{02} &= 2\sqrt{1-\alpha_\psi^2}\sin\theta_1\cos\theta_1\sin(\Delta\Phi), \\
C_{11} &= 2\sin^2\theta_1, \\
C_{13} &= 2\sqrt{1-\alpha_\psi^2}\sin\theta_1\cos\theta_1\cos(\Delta\Phi), \\
C_{20} &= -C_{02}, \\
C_{22} &= \alpha_\psi C_{11}, \\
C_{31} &= -C_{13}, \\
C_{33} &= -2(\alpha_\psi + \cos^2\theta_1).
\end{aligned}
$$

α_D and parity violation

$$\alpha_D^{\Xi^-} = -0.390 \pm 0.007 \quad \text{and} \quad \phi_D^{\Xi^-} = -1.2 \pm 1.0^\circ.$$
$$\alpha_D^{\bar{\Xi}^+} = 0.371 \pm 0.007 \quad \text{and} \quad \phi_D^{\bar{\Xi}^+} = -1.2 \pm 1.2^\circ.$$

The decay parameter $\alpha_D, \alpha_D \in [-1, 1]$, can be determined from the angular distribution asymmetry of the b baryon in the B baryon rest frame. The distribution is given as(Batozskaya 2025)

$$\frac{1}{\Gamma}\frac{d\Gamma}{d\Omega} = \frac{1}{4\pi}(1 + \alpha_D \mathbf{P}_B \cdot \hat{\mathbf{n}}),$$

where $\mathbf{P}_B$ is the B baryon polarization vector and $\hat{\mathbf{n}}$ is the direction of the b baryon momentum in the B baryon rest frame.

What is SLOCC?

SLOCC is equivalent to say that there exists arbitrary operators A_i such that(Verstraete, Dehaene, and De Moor 2003)

$$|\psi\rangle = A_1 \otimes \ldots \otimes A_N |\phi\rangle.$$

Here is example of SLOCC equivalence(**Horodecki˙2009**): the state

$$|\psi\rangle = a|00\rangle + b|11\rangle$$

with $a > b > 0$ can be converted (up to irrelevant phase) into $|\phi^+\rangle$ by filter $A \otimes I$, with $A = \begin{bmatrix} \frac{b}{a} & 0 \\ 0 & 1 \end{bmatrix}$ with probability $p = 2|b|^2$; so it is possible to consider representative state for each class.

Why D is unitary?

To see this, consider a general linear operator D acting on the Hilbert space of the mother particle. Suppose the initial polarization states $|\psi_1\rangle$ and $|\psi_2\rangle$ form an orthonormal basis, and are mapped under D to $D|\psi_1\rangle = |\phi_1\rangle$ and $D|\psi_2\rangle = |\phi_2\rangle$, which remain orthonormal when $\alpha_D = 0$. Since the inner products are preserved, i.e.,

$$\langle\phi_i|\phi_j\rangle = \langle D\psi_i|D\psi_j\rangle = \langle\psi_i|D^\dagger D|\psi_j\rangle = \delta_{ij},$$

this implies that $D^\dagger D = I$ on the span of $\{|\psi_1\rangle, |\psi_2\rangle\}$, and hence D is unitary on that subspace.

Decay parameters

The amplitude for a spin-1/2 hyperon decaying into a spin-1/2 baryon and a spin-0 meson may be written in the form(Navas et al. 2024)

$$M = G_F\, m_\pi^2 \cdot \bar{B}_f\, (A - B\gamma_5)\, B_i\,,$$

The parameters α, β, and γ are defined as

$$\alpha = 2\mathrm{Re}(s^* p)/(|s|^2 + |p|^2),$$

$$\beta = 2\mathrm{Im}(s^* p)/(|s|^2 + |p|^2),$$

$$\gamma = (|s|^2 - |p|^2)/(|s|^2 + |p|^2),$$

where $s = A$ and $p = |\mathbf{p}_f| B/(E_f + m_f)$; here E_f and $\mathbf{p}_f$ are the energy and momentum of the final baryon. The parameters α, β, and γ satisfy

$$\alpha^2 + \beta^2 + \gamma^2 = 1$$

α_ψ and $\Delta\Phi$

(Ablikim et al. 2022)

$$\alpha_\psi = 0.586 \pm 0.012|_{\text{stat}} \pm 0.010|_{\text{syst}}$$

$$\Delta\Phi = 1.213 \pm 0.046|_{\text{stat}} \pm 0.016|_{\text{syst}}.$$

The parameters α_ψ and $\Delta\Phi$ are related to two production amplitudes, where α_{vf} parameterises the Ξ^- angular distribution. The $\Delta\Phi$ is the relative phase between the two production amplitudes (in the so-called helicity representation) and governs the polarisation P_y of the produced Ξ^- and $\overline{\Xi}^+$ as well as their spin correlations C_{ij}.