Quantum Entanglement in Particle Physics: From Physics to Quantum Information

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Workshop on Quantum Entanglement at the Energy Frontier, Peking University, 2025.4.26.

Quantum Entanglement in High Energy Physics

Yu Shi (施郁)

2024.7.9.

28^{th} LHC MiniWorkshop

Towards high energy quantum information: quantum teleportation using neutral kaons

Yu Shi
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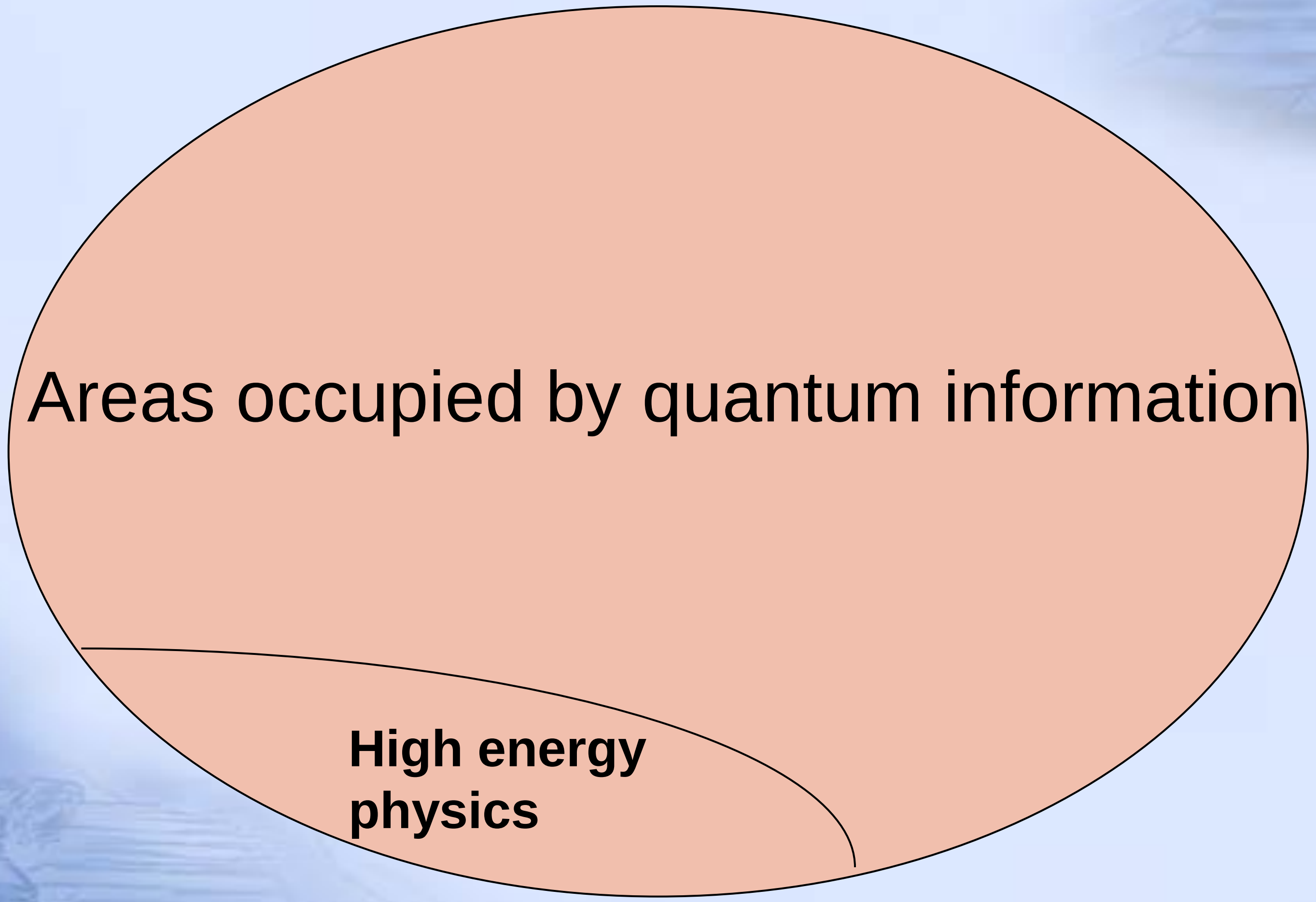
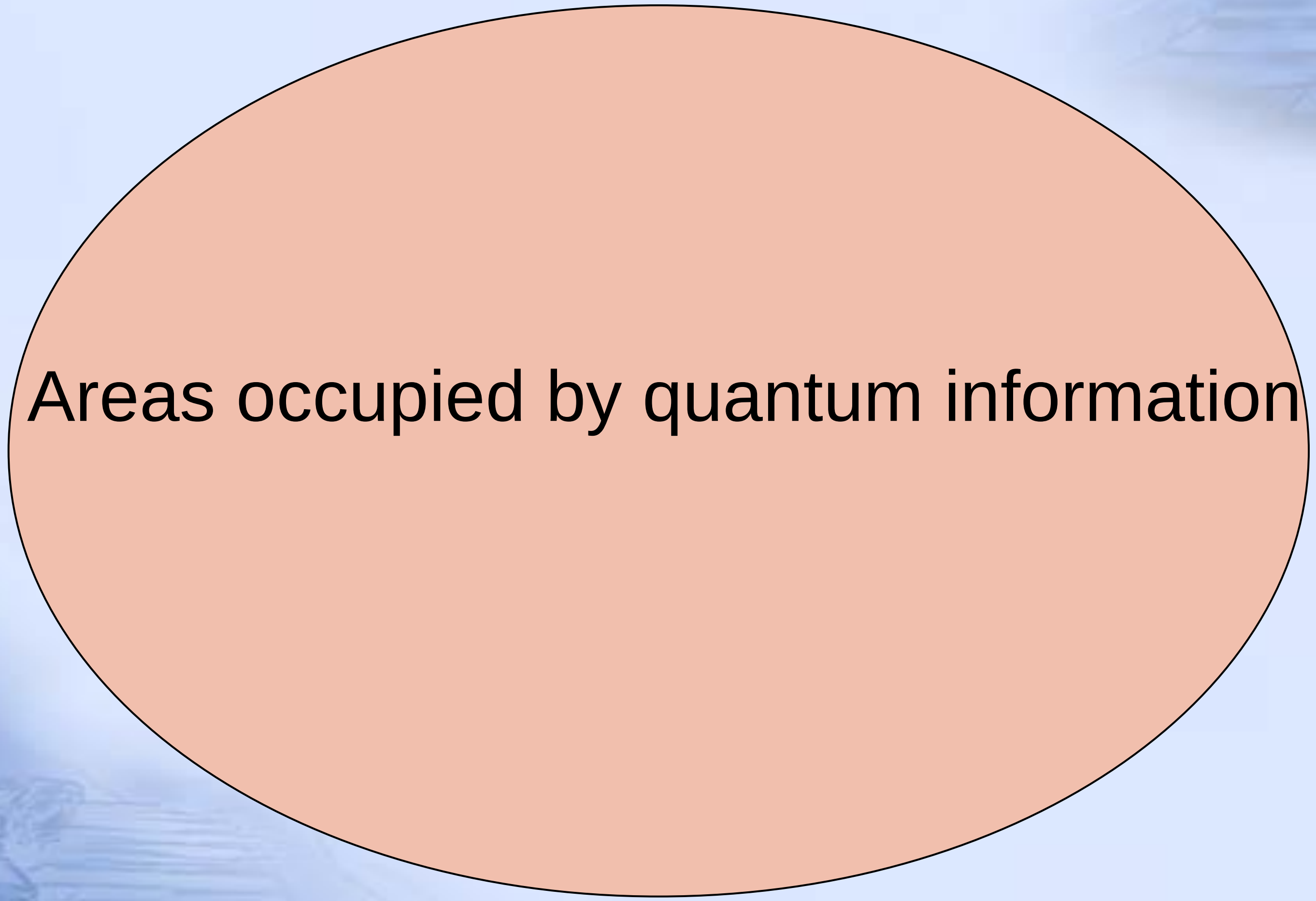
Ref: quant-ph/0605070; appearing in Phys. Lett. B

International Conference on Quantum Foundation and Technology: Frontier and Future, Hangzhou, 27/8/2006

Map of Quantum Physics



New Map of Quantum Physics



EPR 1935

MAY 15, 1935 PHYSICAL REVIEW VOLUME 47

Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?

A. Einstein, B. Podolsky and N. Rosen, *Institute for Advanced Study, Princeton, New Jersey*

(Received March 25, 1935)

In a complete theory there is an element corresponding to each element of reality. A sufficient condition for the reality of a physical quantity is the possibility of predicting it with certainty, without disturbing the system. In quantum mechanics in the case of two physical quantities described by non-commuting operators, the knowledge of one precludes the knowledge of the other. Then either (1) the description of reality given by the wave function in quantum mechanics is not complete or (2) these two quantities cannot have simultaneous reality. Consideration of the problem of making predictions concerning a system on the basis of measurements made on another system that had previously interacted with it leads to the result that if (1) is false then (2) is also false. One is thus led to conclude that the description of reality as given by a wave function is not complete.

OCTOBER 15, 1935 PHYSICAL REVIEW VOLUME 48

Can Quantum-Mechanical Description of Physical Reality be Considered Complete?

N. Bohr, *Institute for Theoretical Physics, University, Copenhagen*

(Received July 13, 1935)

It is shown that a certain "criterion of physical reality" formulated in a recent article with the above title by A. Einstein, B. Podolsky and N. Rosen contains an essential ambiguity when it is applied to quantum phenomena. In this connection a viewpoint termed "complementarity" is explained from which quantum-mechanical description of physical phenomena would seem to fulfill, within its scope, all rational demands of completeness.

Schrödinger 1935

- Another way of expressing the peculiar situation is: the best possible knowledge of the a whole does not necessarily include the best possible knowledge of its parts…I would not call that one but rather the characteristic trait of quantum mechanics, the one that enforces its entire departure from classical lines of thought . By the interaction the two representatives become entangled.

W. Furry 1936

- Schrödinger (and EPR) thought entanglement unreasonable, speculated it might become nonentangled when particles are separated.
- W. Furry thought such a speculation unreasonable.

MARCH 1, 1936 PHYSICAL REVIEW VOLUME 49

Note on the Quantum-Mechanical Theory of Measurement

W. H. FURRY, *Department of Physics, Harvard University*

(Received November 12, 1935)

In recent notes by Einstein, Podolsky and Rosen and by Bohr, attention has been called to the fact that certain results of quantum mechanics are not to be reconciled with the assumption that a system has independently real properties as soon as it is free from mechanical interference. We here investigate in general, and in abstract terms, the extent of this disagreement. When suitably formulated, such an assumption gives to certain types of questions the same answers as does quantum mechanics; this is true of the formulas usually given in discussions of the theory of measurement. There exists, however, a general class of cases in which contradictions occur. That such contradictions are not restricted to the abstract mathematical theory, but can be realized in the commonest physical terms, is shown by the working out of an example.

Different context: Testing QED

- Wheeler: asymmetry in coincident measurement of photons produced from the e+e- annihilation. Got the quantitative result wrong.
- Price-Ward, Snyder-Pasternack-Hornbostel got the correct result.

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物　理　学　进　展
PROGRESS IN PHYSICS

粒子物理中量子纠缠的历史起源

施　郁[1,2]

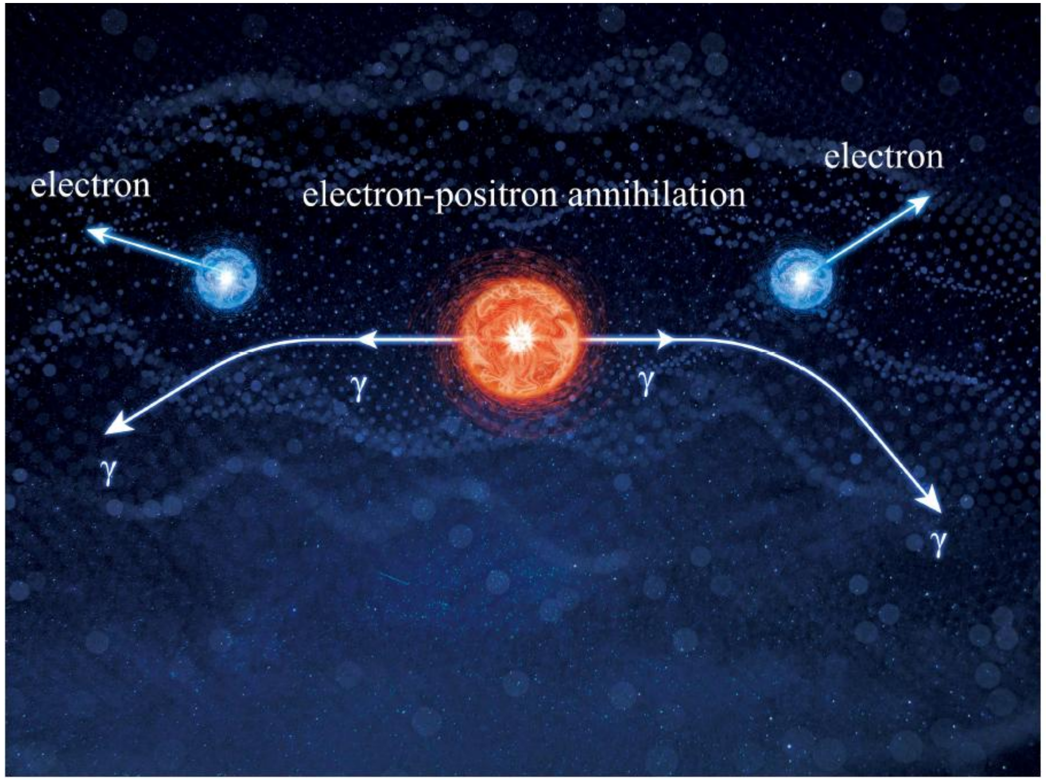


图 1. 正负电子湮没后产生相背运动的两个光子，分别被电子散射。

Electron-positron annihilation and quantum entanglement

Yu Shi, Scientific spirit of C. S. Wu: from quantum entanglement to parity nonconservation

Wu, Shaknov, 1950

The Angular Correlation of Scattered Annihilation Radiation*

C. S. WU AND I. SHAKNOV
Pupin Physics Laboratories, Columbia University, New York, New York

November 21, 1949

- Test QED, Wheeler proposed to study electron-positron annihilation. Further calculations by Ward-Price, and by Snyder-Pasternack-Hornbostel.
- Two photons created in electron-positron annihilation, polarizations are always perpendicular, scattered by electrons respectively.
- For different scattering angles, measure the asymmetries between the two situations of moving directions being perpendicular and parallel.
- Maximum=2.85, when the scattering angle=82°
- The polarization determines the scattering distribution.
- Previous experiments not satisfactory.
- Sensitivity of the γ detector by Wu and Shaknov was 10 times that of previous ones.
- Positron source Cu64 was activated by deuterium bombardment in Columbia cyclotron.
- Result: 2.04∓0.08. (Theory: 2.00)

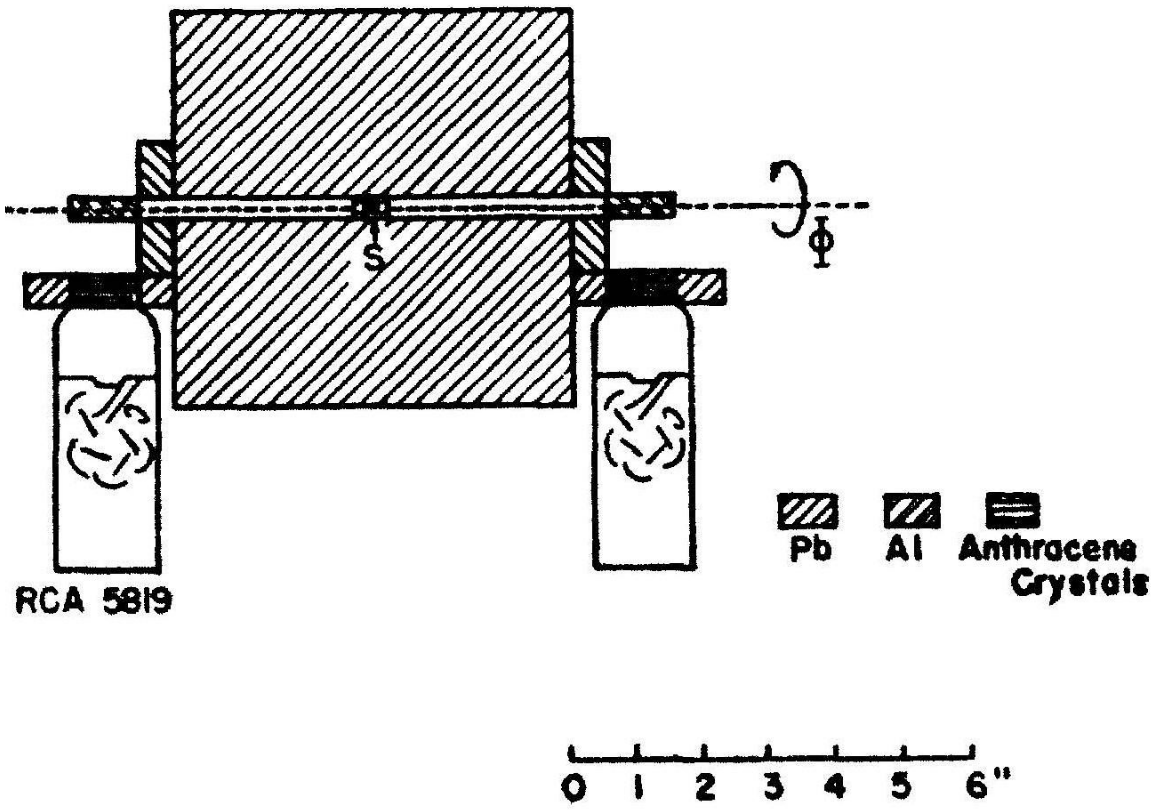
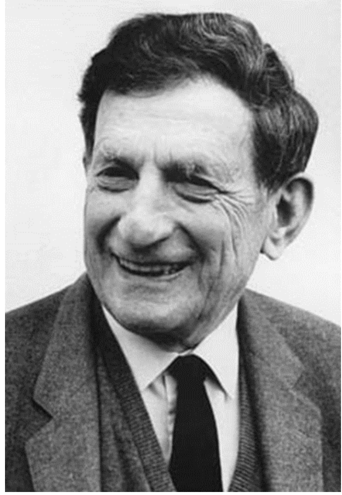


FIG. 1. Schematic diagram of experiment.

Quantum Entanglement

- In 1935, Einstein-Podolsky-Rosen demonstrated the conflict between local realism and quantum mechanics using quantum entangled states. They claimed that quantum mechanics is incomplete.
- In the same year, Schrödinger coined quantum entanglement, noting that it is a characteristic of quantum mechanics.
- In 1957, Bohm-Aharonov noted that Wu-Shaknov achieved photon polarization correlation (Bohm-Aharonov did not use the word "entanglement"). They proved that the non-entangled state could not give the experimental results of Wu-Shaknov.
- For the first time, Wu-Shaknov experimentally achieved a clearly spatially separated quantum entangled state.





1975, Wu's group attempted to test Bell's inequality

- In 1964, Bell proposed the inequality satisfied by local realism (hidden variables).
- Quantum entangled states violate Bell's inequality, but polarization needs to be measured in a direction that is neither parallel nor perpendicular.
- In 1950 Wu-Shaknov experiment, the azimuth angles of the two photons detected were always perpendicular.
- In 1975, Chien-Shiung Wu and his students Kasday and Ullman measured the coincidence probability of two photons in a wide range of polar and azimuthal angles.
- ANGULAR-CORRELATION OF COMPTON-SCATTERED ANNIHILATION PHOTONS AND HIDDEN VARIABLES, KASDAY, LR; ULLMAN, JD, d WU, CS, 1975 | NUOVO CIMENTO B 25 (2) , pp.633-661.
- The results are consistent with the entangled state and contradict the non-entangled state.
- However, it cannot be used to prove that Bell's inequality is violated, because local realism can also calculate Compton scattering, and it cannot explicitly lock the direction of photon detection and polarization in a certain direction.
- Here are high-energy photons, which are not easy for direct polarization measurements (can destroy polarizers). The polarization of low-energy photons can be measured directly with polarizers.
- Contemporary quantum information science is booming by using the entangled states of low-energy photons and other agents.

Scientific Spirit of Chien-Shiung Wu: From Quantum Entanglement to Parity Nonconservation

Yu Shi

(Fudan University)

International Symposium Commemorating the 110th Birth Anniversary of Chien-Shiung Wu

Southeast University

2022.5.31.

Chien-Shiung Wu: A great experimental physicist

- Intelligent, quiet, diligent, hardworking, focused, undaunted by difficulties, dedicated to science. (at her childhood, her father said: "Don't be afraid of difficulties, work hard, and move forward")
- Accurate experiments. Elements of Chinese girls' specialties, and embodiment of the scientific integrity.
- Research areas and achievements are closely related to years of experiences and expertise.
- Attach importance to the development of experimental techniques and instruments, but also to physical significance, master important theories, and verify theories with experiments.
- Her experimental work has great theoretical significance, e.g. the verification of Fermi theory, the prediction of quantum electrodynamics on photon coincidence and quantum entanglement, parity nonconservation, vector current conservation, and double β decay.
- Whether it's overthrowing or verification, do it.
- I think the most valuable is: prioritizing scientific rigor over competition and honor, even when priority and merit are at stake (indeed there was some damage, although publication is respected, may be one of the factors in missing the Nobel Prize). Another manifestation of her scientific spirit.

My articles and talks on C. S. Wu's contributions

- Video: https://www.koushare.com/video/details/164531
- Chinese slides:https://mp.weixin.qq.com/s/lilZY3ClxvfP9Q8DSt2j_A
- Chinese transcript: Micius Forum (墨子沙龙), June 2, 2023, https://mp.weixin.qq.com/s/inwKeaTyI9FxT-qEEgsKDA
- English version: arXiv:2504.16978
- YS, Chien-Shiung Wu as the experimental pioneerin quantum entanglement: A 2022 note, Mod. Phys. Lett. A 40, 2530001 (2025), arXiv:2502.06458

粒子物理中量子纠缠的历史起源：吴健雄、杨振宁、李政道以及其他先驱

原创 施郁 墨子沙龙 2023年03月17日 16:06 上海

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粒子物理中量子纠缠的历史起源

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摘要： 本文系统深入地梳理了粒子物理中量子纠缠的历史起源。1957 年，玻姆和阿哈诺罗夫指出，1949 年吴健雄和萨克诺夫的实验实现了爱因斯坦–波多尔斯基–罗森关联。事实上，这是历史上第一次在实验中明确实现空间分离的量子纠缠。惠勒最早建议这个实验，作为对量子电动力学的检验，但是计算有误，正确的理论计算来自沃德和普赖斯，以及斯奈德、帕斯特纳克和奥恩博斯特尔，也符合杨振宁 1949 年的选择定则。1964 年贝尔不等式发表后，人们考虑，它是否可以通过吴–萨克诺夫实验检验。这推动了该领域的发展，吴健雄小组也做了新的实验。1957 年，李政道、厄梅和杨振宁确立了 K 介子的量子力学形式，并发现中性 K 介子是一个双态系统。1958 年，基于与杨振宁 1949 年选择定则类似的方法，戈德哈贝尔、李政道和杨振宁最早写下 K 介子对的纠缠态，其中单个 K 介子可以带电，也可以电中性。这首次给出光子以外的高能粒子的内部自由度纠缠。1960 年，作为没有发表的工作，李政道和杨振宁又讨论了中性 K 介子对的纠缠态。本文也顺便介绍了几位物理学家，特别是沃德。

- Wu-Shaknov: first experimental realization of clearly spatially separated quantum entanglement.
- Early work on quantum entanglement in particle physics has historically promoted entanglement study.

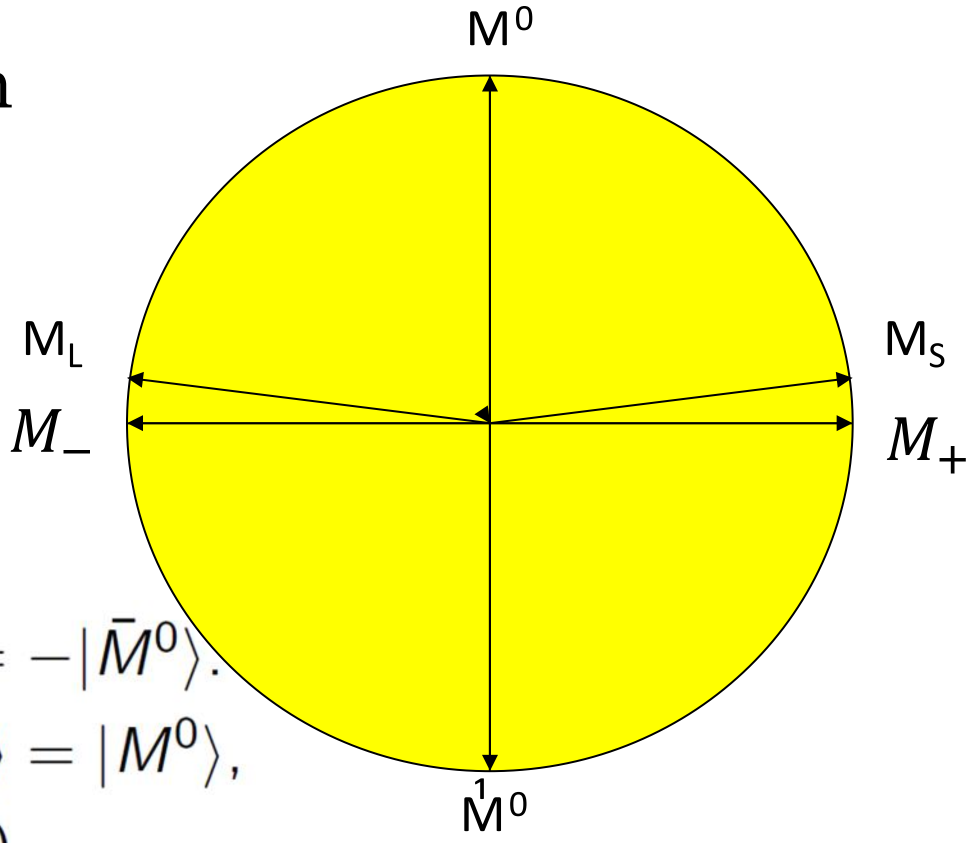
Entanglement between pseudoscalar mesons

- First noted by Lee and Yang in 1960 in an unpublished work.

Two dimensional Hilbert space

- Particle and antiparticle form a 2D Hilbert space.
- similar to qubit.

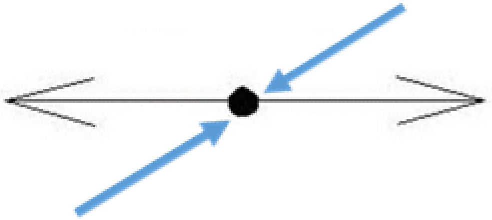
M=K,B,D.



- $\mathcal{F}|M^0\rangle = |M^0\rangle$, $\mathcal{F}|\bar{M}^0\rangle = -|\bar{M}^0\rangle$.
- $CP|M^0\rangle = |\bar{M}^0\rangle$, $CP|\bar{M}^0\rangle = |M^0\rangle$,
- $|M_\pm\rangle = \frac{1}{\sqrt{2}}(|M^0\rangle \pm |\bar{M}^0\rangle)$.
- $CP|M_\pm\rangle = \pm|M_\pm\rangle$.

$C=-1$ entangled state

e+e-collision，resonant state decays to entangled pair of partile s and antiparticle。



$$|\Psi_-\rangle = \frac{1}{\sqrt{2}}(|M^0\rangle_a|\bar{M}^0\rangle_b - |\bar{M}^0\rangle_a|M^0\rangle_b),$$

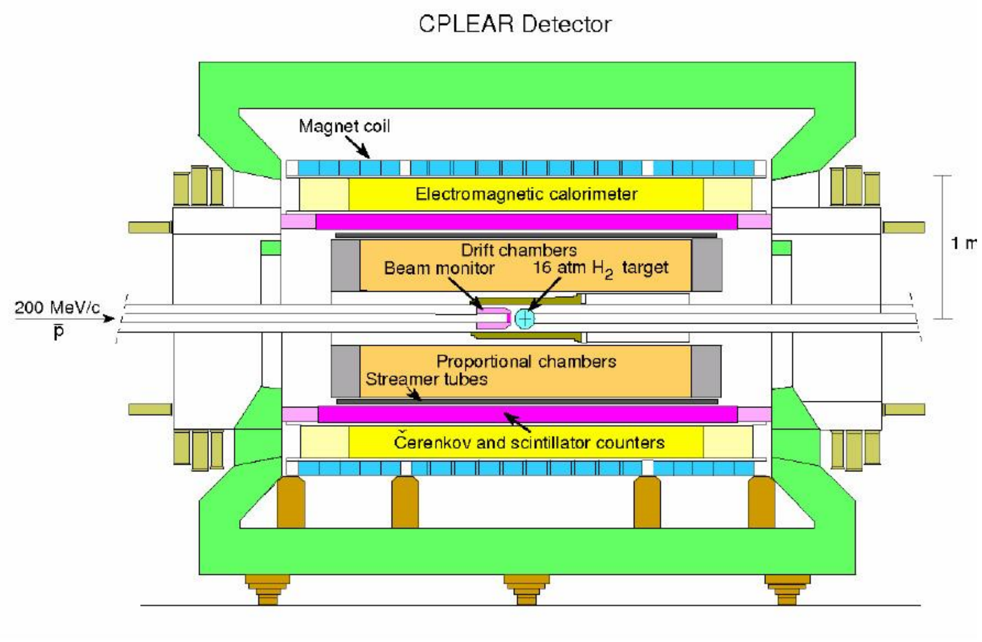
produced from a source of $J^{PC}=1^{--}$, e.g. $K^0\bar{K}^0$ produced at ϕ decay in the KLOE, and $B_d^0\bar{B}_d^0$ produced at $\Upsilon(4s)$ in the BELLE detector.

- $|\Psi_-\rangle$ is also *exactly*

$$|\Psi_-\rangle = \frac{1}{\sqrt{2}}(|M_-\rangle_a|M_+\rangle_b - |M_+\rangle_a|M_-\rangle_b).$$

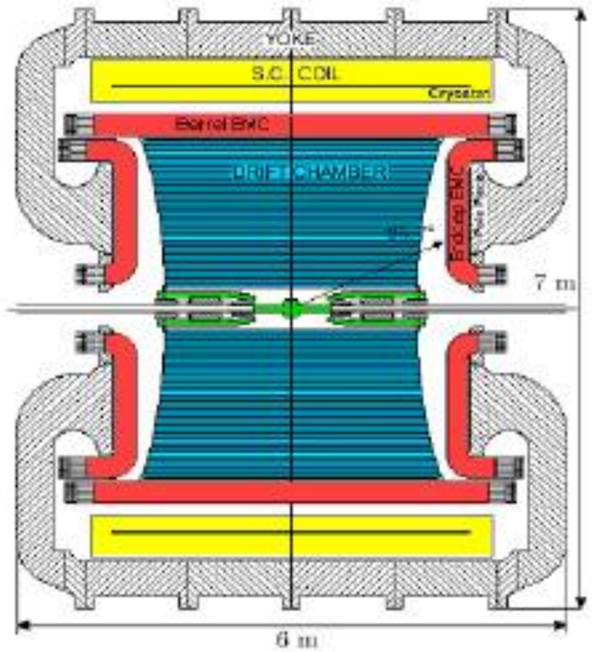
Experimental confirmation (1)

- $K^0\overline{K}^0$ produced in p+p- annihilation in the CPLEAR detector in CERN (1998).



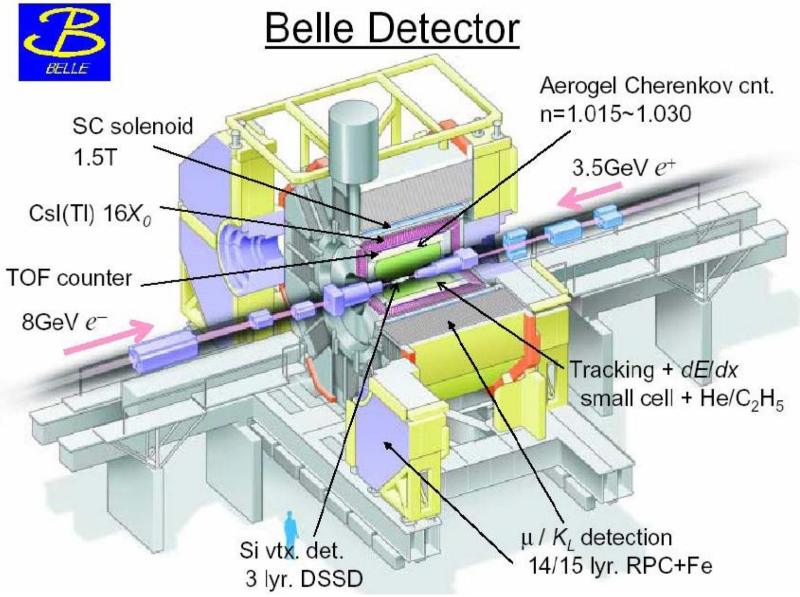
Experimental confirmation (2)

- $K^0\overline{K}^0$ produced in φ decay (1GeV) in the KLOE detector in 2003.



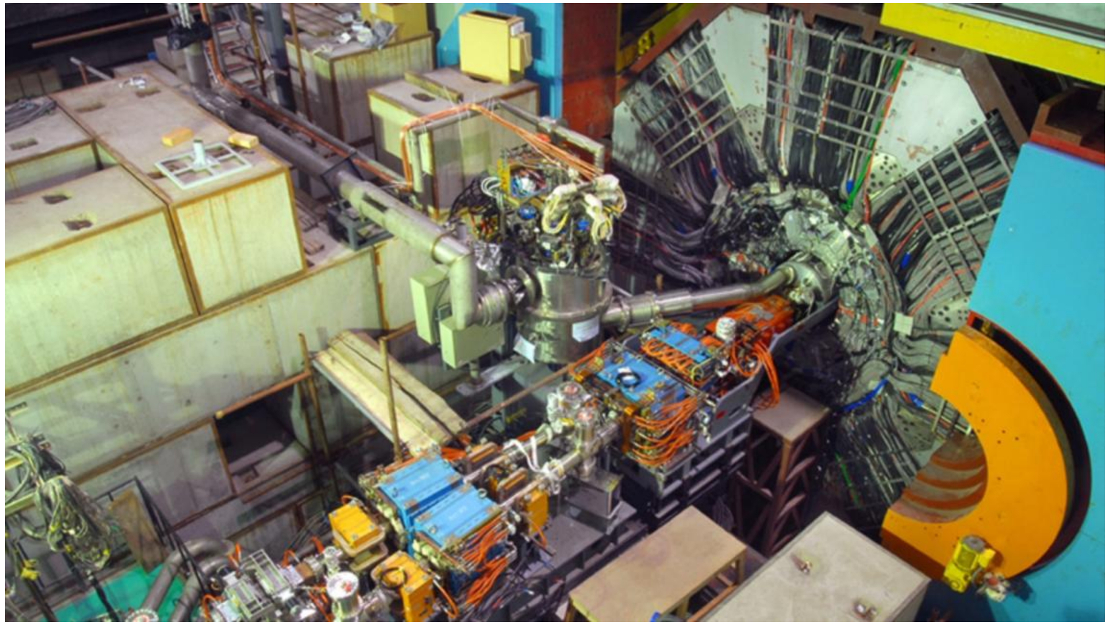
Experimental confirmation (3)

- $B^0\bar{B}^0$ meson entangled states produced in γ decay (10GeV) from e-e+ annihilation in the BELLE detector in KEKB (20



Experimental confirmation (4)

- $D^0\overline{\mathrm{D}}^0$ produced in decay of ψ at 3.773GeV from ee+ annihilation in the BESIII detector (2015).



Our Work

1. 1st Proposal for a High Enegy Quantum Information Process: quantum teleportation in terms of entangled mesons

- YS, Phys. Lett. B 641, 75 (06); 641 (2006) 492.
- YS and Y. L. Wu, EPJC 55, 477 (08).

2. Using Quantum Entanglement to Study CP and CPT Violations

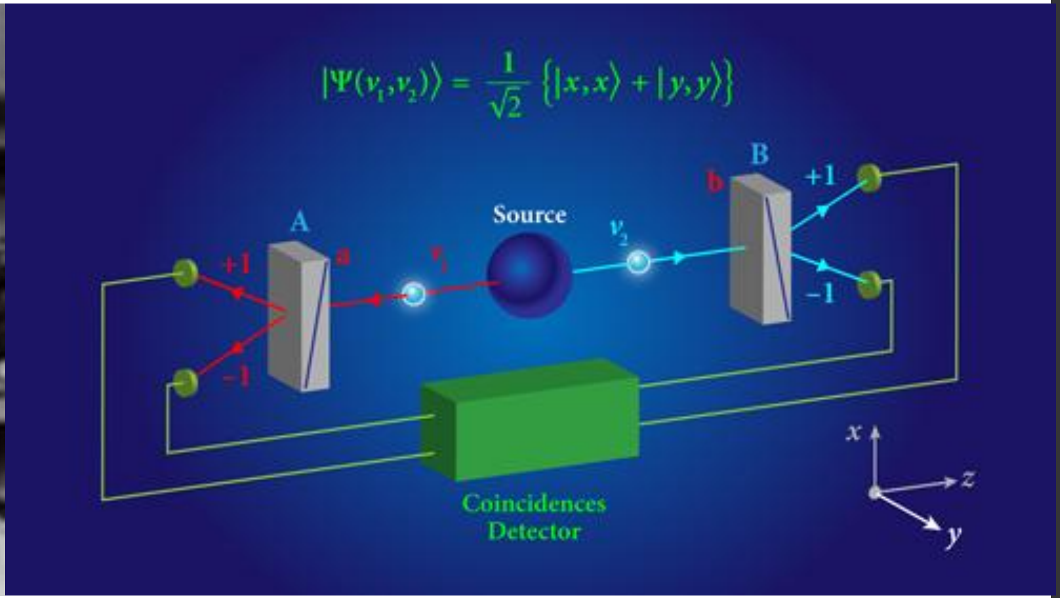
- Z. Huang, YS, Euro. Phys. J. C 72, 1900 (2012).
- YS, Euro. Phys. J. C 73, 2506 (2013.
- Z. Huang, YS, Phys. Rev. D 89, 016018 (2014).

3. Genuine T Violation Signal in terms of entangled D mesons

- YS, JC Yang, Time reversal symmetry violation in entangled pseudoscalar neutral charmed mesons, Phys. Rev. D 98, 075079 (2018).

Inequalities from Hidden variables, violated by QM

Bell Inequality



- In 1964, Bell proposed the inequality which the theory of local hidden variables satisfies $A(\mathbf{a},\lambda)=\pm 1$, $B(\mathbf{b},\lambda)=\pm 1$, $P(\mathbf{a},\mathbf{b})=\int d\lambda\rho(\lambda)A(\mathbf{a},\lambda)B(\mathbf{b},\lambda)$
- Perfect anticorrelation: $1+P(\mathbf{a},\mathbf{b})\geq |P(\mathbf{a},\mathbf{b})-P(\mathbf{a},\mathbf{c})|$
- QM： $P(\mathbf{a},\mathbf{b})=\langle \boldsymbol{\sigma}\cdot\mathbf{a}\ \boldsymbol{\sigma}\cdot\mathbf{b}\rangle = -\mathbf{a}\cdot\mathbf{b}$， violates BI。

1969
Clauser, Holt, Shimony, Hone

- More realistic.
- $S=P(\mathbf{a},\mathbf{b})+P(\mathbf{a},\mathbf{b}')$
  $+P(\mathbf{a}',\mathbf{b}) - P(\mathbf{a}',\mathbf{b}')$
- $-2 \le S \le 2$
- Violated by entangled states

Crypto-nonlocal realism

- Which should be abandoned, locality or realism?
- Leggett consider crypto-nonlocal realism and derived Leggett Inequlity (violated by QM) :
  - The physical state is a statistical mixture of sub-ensembles of various polarization directions.
  - Given the hidden variable, the measured quantity also depends on the polarizer axis on the other side (non-local).
  - For each sub-ensemble, the measured quantity obeys Malus Law (which is local) for the average of the hidden parameters.

$$\bar{A}(\mathbf{u}) = \int d\lambda \rho_{\mathbf{u},\mathbf{v}}(\lambda) A(\mathbf{a},\mathbf{b},\lambda) = \mathbf{u}\cdot\mathbf{a}$$

Some special characteristics of using high energy particles to test violations of BI and LI

- Relativistic, massive, involves electroweak and strong interactions.
- Decays are similar to measurements.
- The decay mode, product, and time of each particle may be determined by the hidden varibles of the particle source, so the decays of two particles may also be correlated.
- Hence a measurement setup loophole.

What we did

- For the hidden variables, it is assumed that the measurement settings are also determined by the hidden variables; The decay is determined by the hidden variables at the particle source.
- Generalize crypto non-local realism and Leggett inequality.
- Generalize local realism and Bell's inequality

4. Hyperon spin entanglement is used to generalize Bell's inequality (decay is also determined by hidden variables)

YS and JC Yang, Entangled baryons: violation of inequalities based on local realism assuming dependence of decays on hidden variables, European Physical Journal C 80, 116 (2020).

5. Entangled mesons violate a generalized Leggett Inequality， in which decays also depend on hidden variables

- YS and JC Yang, Particle physics violating crypto-nonlocal realism, European Physical Journal C 80, 861 (2020).

The measurement setting ("axis") is also depend on hidden variables

Average over the distribution of hidden variables

$$\int d\lambda \rho'_{\mathbf{u},\mathbf{v},\mathbf{a},\mathbf{b}}(\lambda) A(\mathbf{u},\mathbf{v},\tilde{\mathbf{a}}(\lambda),\tilde{\mathbf{b}}(\lambda),\lambda) = \bar{A}(\mathbf{u},\mathbf{a}) = \mathbf{u}\cdot\mathbf{a},$$

$$\int d\lambda \rho'_{\mathbf{u},\mathbf{v},\mathbf{a},\mathbf{b}}(\lambda) B(\mathbf{v},\mathbf{u},\tilde{\mathbf{b}}(\lambda),\tilde{\mathbf{a}}(\lambda),\lambda) = \bar{B}(\mathbf{v},\mathbf{b}) = \mathbf{v}\cdot\mathbf{b}.$$

$$\overline{AB}(\mathbf{u},\mathbf{v},\mathbf{a},\mathbf{b}) = \int d\lambda \rho'_{\mathbf{u},\mathbf{v},\mathbf{a},\mathbf{b}}(\lambda) A(\mathbf{u},\mathbf{v},\tilde{\mathbf{a}}(\lambda),\tilde{\mathbf{b}}(\lambda),\lambda) B(\mathbf{v},\mathbf{u},\tilde{\mathbf{b}}(\lambda),\tilde{\mathbf{a}}(\lambda),\lambda),$$

where $\rho'_{\mathbf{u},\mathbf{v},\mathbf{a},\mathbf{b}}(\lambda) \equiv \rho_{\mathbf{u},\mathbf{v}}(\lambda)\delta(\tilde{\mathbf{a}}(\lambda)-\mathbf{a})\delta(\tilde{\mathbf{b}}(\lambda)-\mathbf{b})$ is a shorthand.

Average over the polarizations:

$$E(\mathbf{a},\mathbf{b}) = \int d\mathbf{u}d\mathbf{v}F(\mathbf{u},\mathbf{v})\overline{AB}(\mathbf{u},\mathbf{v},\mathbf{a},\mathbf{b}),$$

Measurement after evolution (equivalent to measurement using a time-dependent basis, similar to a rotating polarizer.

$$U(\theta_{\mathbf{a}},\rho_{\mathbf{a}}) = \left(\cos\frac{\theta_{\mathbf{a}}}{2} - i\sin\frac{\theta_{\mathbf{a}}}{2}\left(\cos(\rho_{\mathbf{a}})\sigma^x + \sin(\rho_{\mathbf{a}})\sigma^y\right)\right). \tag{10}$$

Suppose that following the evolution $U(\theta_{\mathbf{a}},\rho_{\mathbf{a}})$, a signal A is recorded as $A=+1$ if $|0\rangle$ is detected, while $A=-1$ if $|1\rangle$ is detected. The QM expectation value of A is

$$\bar{A}(\mathbf{u}) = \frac{|\langle 0|U|\mathbf{u}\rangle|^2 - |\langle 1|U|\mathbf{u}\rangle|^2}{|\langle 0|U|\mathbf{u}\rangle|^2 + |\langle 1|U|\mathbf{u}\rangle|^2} = \mathbf{u}\cdot\mathbf{a}, \tag{11}$$

Semileptonic decay of B meson (flavor basis) satisfies Malus Law. Find the "measurement direction"

For a B_d meson, the measurement in the flavor basis $\{|B^0\rangle, |\bar{B}^0\rangle\}$, corresponding to $A_l = \pm 1$, can be made in the semileptonic decay channel, as the direct CP violation or wrong sign decay is negligible [58],

$$\bar{A}_l(\mathbf{u}) = \frac{|\langle B^0|U(t)|\mathbf{u}\rangle|^2 - |\langle \bar{B}^0|U(t)|\mathbf{u}\rangle|^2}{|\langle B^0|U(t)|\mathbf{u}\rangle|^2 + |\langle \bar{B}^0|U(t)|\mathbf{u}\rangle|^2} = \mathbf{u}\cdot\mathbf{a}^l(t), \tag{12}$$

where $\mathbf{a}^l(t) = (\sin(2\beta)\sin(x\Gamma t), -\cos(2\beta)\sin(x\Gamma t), \cos(x\Gamma t))$.

For B介子，the two decay widths are close, β is from (indirect) CP violation 。
xΓis the difference between the eigenvalues of mass-decay width.

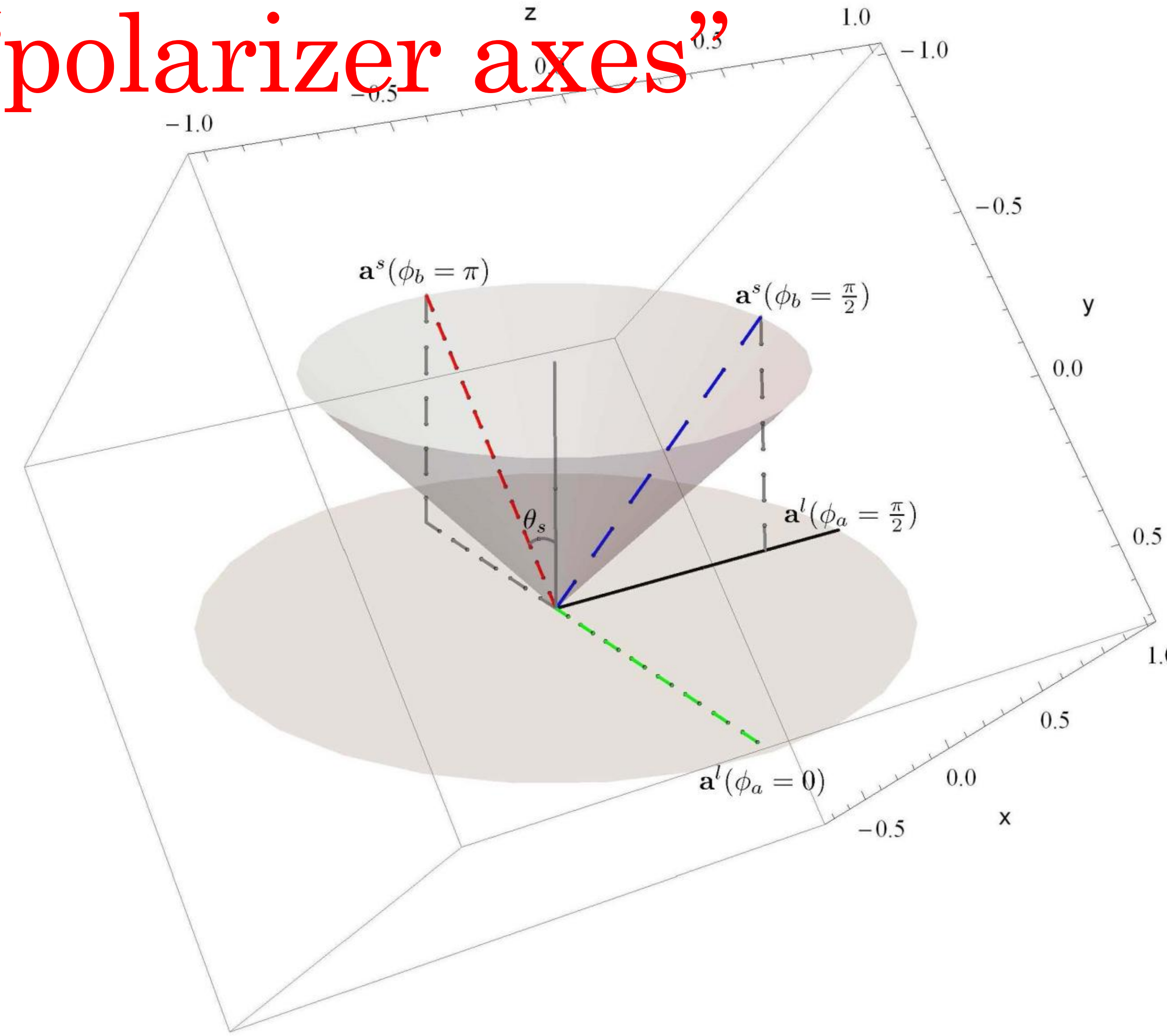
Decay in the CP basis also satisfies Malus Law. Find the "measurement direction".

Likewise, observing the decay product to be CP eigenstates $S_\pm$ effectively measures the meson to be $|B_\pm\rangle \equiv \left(|B^0\rangle \pm |\bar{B}^0\rangle\right)/\sqrt{2}$, as the direct CP violation is negligible [58]. With $B_\pm$ corresponding to $A_s = \pm 1$,

$$\bar{A}_s(\mathbf{u}) = \frac{|\langle B_+|U(t)|\mathbf{u}\rangle|^2 - |\langle B_-|U(t)|\mathbf{u}\rangle|^2}{|\langle B_+|U(t)|\mathbf{u}\rangle|^2 + |\langle B_-|U(t)|\mathbf{u}\rangle|^2} = \mathbf{u}\cdot\mathbf{a}^s(t), \tag{13}$$

where $\mathbf{a}^s(t) = \left(\sin^2(2\beta)\cos(x\Gamma t) + \cos^2(2\beta), \sin(4\beta)\sin^2\left(x\Gamma t/2\right), -\sin(2\beta)\sin(x\Gamma t)\right)$.

Two decay modes' "polarizer axes"



$\theta_s = 2\beta$

FIG. 1: The effective measuring directions $\mathbf{a}^l$ and $\mathbf{a}^s$. In a certain coordinate system, $\mathbf{a}^l(\phi_l)$ is on xy plane, $\mathbf{a}^s(\theta_s, \phi_s)$ is on a cone. For B_d mesons, $\phi_l = x\Gamma t$,

Yu Shi, Quantum Entanglement in Particle Physics: from Physics to Quantum Information

Correlation function

We first consider correlation functions of various combinations of $\mathbf{a}^l$ and $\mathbf{a}^s$. Define $\hat{E}^{\pm}(\mathbf{a},\mathbf{b}) \equiv E(\mathbf{a},\mathbf{b})+E(\mathbf{b},\pm\mathbf{b})$, and rewrite $\hat{E}^{\pm}(\mathbf{a}^s(\theta_s,\phi_a),\mathbf{a}^l(\phi_b))$ as $\hat{E}^{\pm}_{sl}(\theta_s,\xi,\varphi)$, where $\xi \equiv (\phi_a+\phi_b)/2$, $\varphi \equiv \phi_a-\phi_b$. $\hat{E}^{\pm}_{ll}(\theta_s,\xi,\varphi)$ and $\hat{E}^{+}_{ss}(\theta_s,\xi,\varphi)$ are similarly defined. Furthermore, we consider the averages over ξ, $\hat{E}^{-}_{sl}(\theta_s,\varphi) \equiv \int \frac{d\xi}{2\pi}\hat{E}^{-}_{sl}(\theta_s,\xi,\varphi)$ and so on.

Upper bound:

$$\hat{E}^{-}_{sl}(\theta_s,\varphi_1)+\frac{\pi\cos(\theta_1(\theta_s,\varphi_1))L_1(\theta_s,\varphi_1)}{4\cos(\frac{\varphi_2}{2})}\hat{E}^{-}_{ll}(\theta_s,\varphi_2)$$
$$\le 2\left(1+\frac{\pi\cos(\theta_1(\theta_s,\varphi_1))L_1(\theta_s,\varphi_1)}{4\cos(\frac{\varphi_2}{2})}\right)-\cos(\theta_1(\theta_s,\varphi_1))L_1(\theta_s,\varphi_1), \tag{26}$$

where $L_1(\theta_s,\varphi)\equiv|\mathbf{a}^s+\mathbf{a}^l|=\sqrt{2+2\cos(\varphi)\sin(\theta_s)}$, $\theta_1(\theta_s,\varphi)=\cos^{-1}\frac{\cos(\theta_s)}{\sqrt{2+2\cos(\varphi)\sin(\theta_s)}}$. With $0<\theta_s<\pi/2$, we have $\sin(\theta_1)>0$, $\cos(\theta_1)>0$.

Lower bound

We find two lower bounds. The first is given as

$$\hat{E}^{+}_{sl}(\theta_s,\varphi_1)+\frac{\pi\cos(\theta_2(\theta_s,\varphi_1))L_2(\theta_s,\varphi_1)}{4\left|\sin(\frac{\varphi_2}{2})\right|}\hat{E}^{+}_{ll}(\theta_s,\varphi_2)$$
$$\geq -2\left(1+\frac{\pi\cos(\theta_2(\theta_s,\varphi_1))L_2(\theta_s,\varphi_1)}{4\left|\sin(\frac{\varphi_2}{2})\right|}\right)+\cos(\theta_2(\theta_s,\varphi_1))L_2(\theta_s,\varphi_1).$$

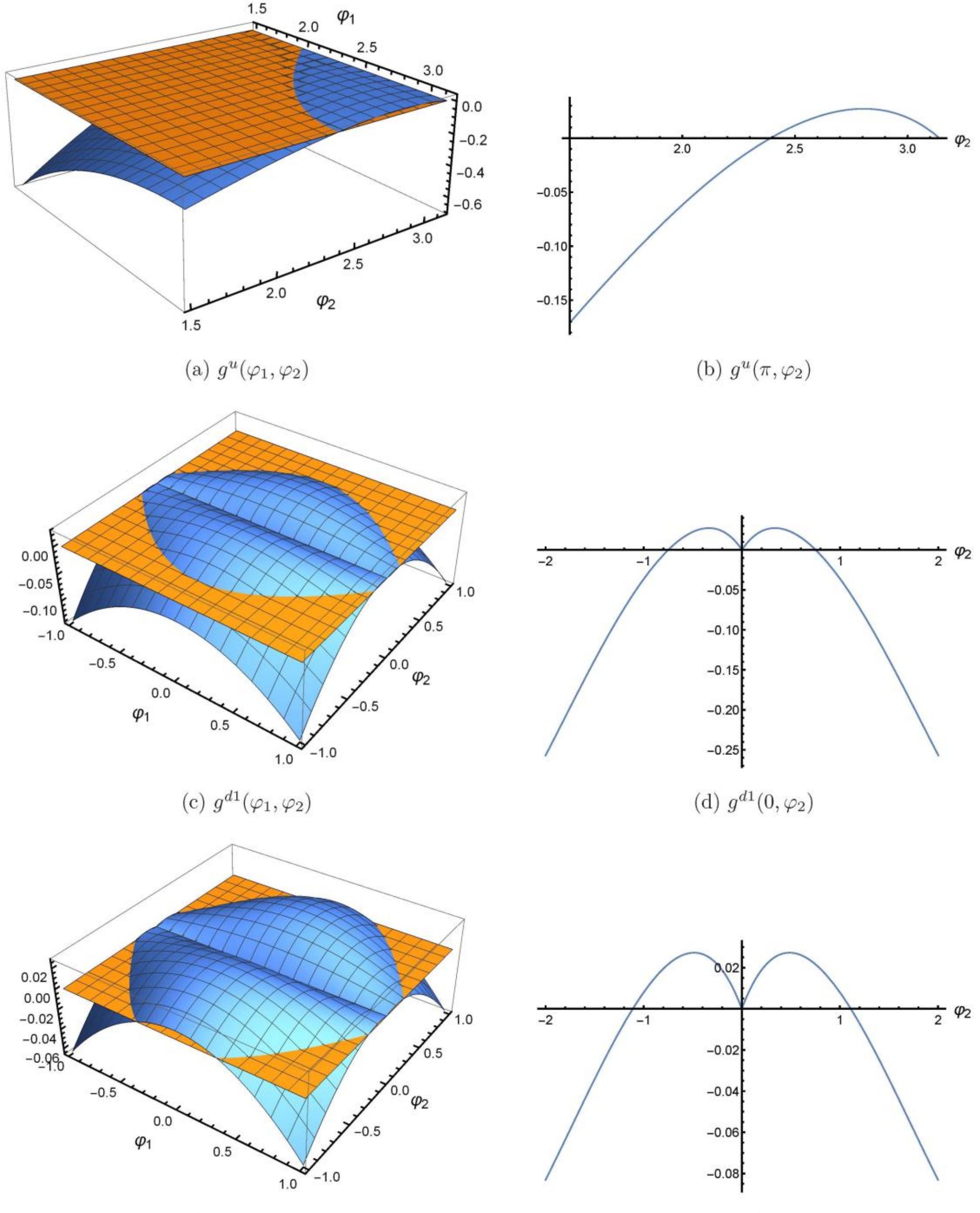
where $L_2(\theta_s,\varphi)=\sqrt{2-2\cos(\varphi)\sin(\theta_s)}$, $\theta_2(\theta_s,\varphi)=\cos^{-1}\frac{\cos(\theta_s)}{\sqrt{2-2\cos(\varphi)\sin(\theta_s)}}$.

The second lower bound is given as

$$\hat{E}^{+}_{sl}(\theta_s,\varphi_1)+\frac{\pi\cos(\theta_2(\theta_s,\varphi_1))L_2(\theta_s,\varphi_1)}{4\sin(\theta_s)\left|\sin(\frac{\varphi_2}{2})\right|}\hat{E}^{+}_{ss}(\varphi_2)$$
$$\geq -2\left(1+\frac{\pi\cos(\theta_2(\theta_s,\varphi_1))L_2(\theta_s,\varphi_1)}{4\sin(\theta_s)\left|\sin(\frac{\varphi_2}{2})\right|}\right)+\cos(\theta_2)L_2(\theta_s,\varphi_1).$$

LI is violated by particle physics

- g=（left−right）/|left|。
- condition: θs≠0 （CP violation）
- Indeed, CP is violated.



(a) $g^u(\varphi_1, \varphi_2)$ (b) $g^u(\pi, \varphi_2)$

(c) $g^{d1}(\varphi_1, \varphi_2)$ (d) $g^{d1}(0, \varphi_2)$

(e) $g^{d2}(\varphi_1, \varphi_2)$ (f) $g^{d2}(0, \varphi_2)$

6. Recent work: Minimal concurrence to violate LI

- YS and JC Yang, in preparation

Not every entangled state violate LI

- Every (pure) entangled state violate CHSHI.
- We have derived the minimum entanglement to violate LI.
- So LI violation is stronger.

- Wilczek Quantum Center (http://www.wilczekqc.net/english) recruits faculty, postdocs, students.

Thank you!