

# 电弱对称性破缺的热历史 与正反物质不对称

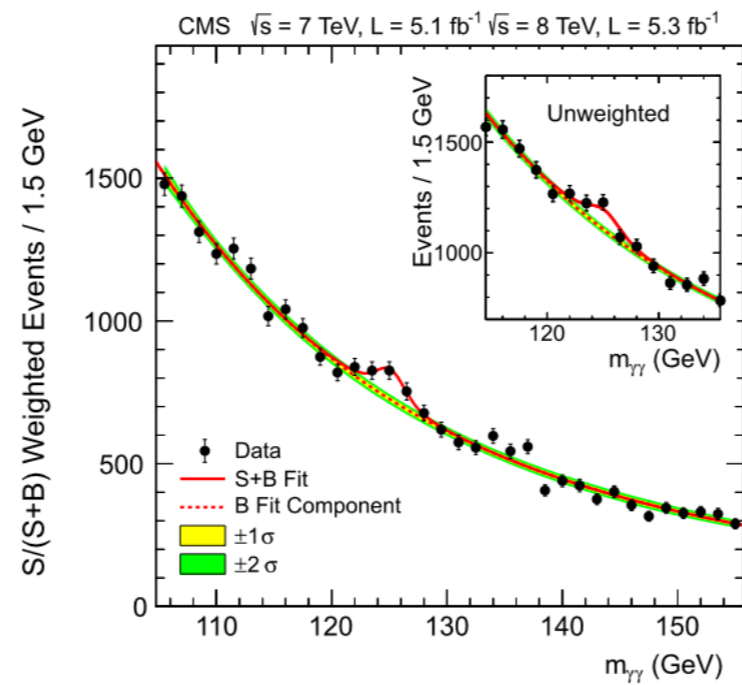
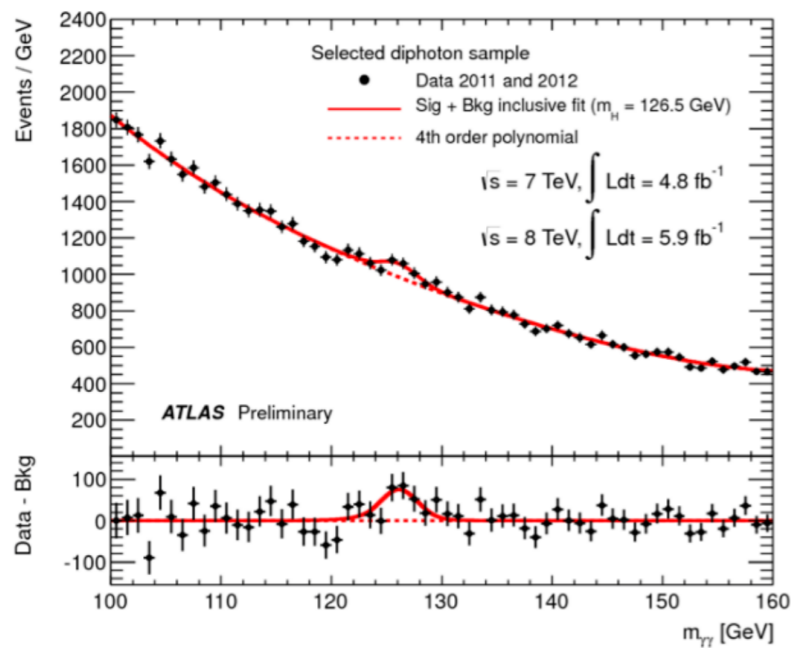
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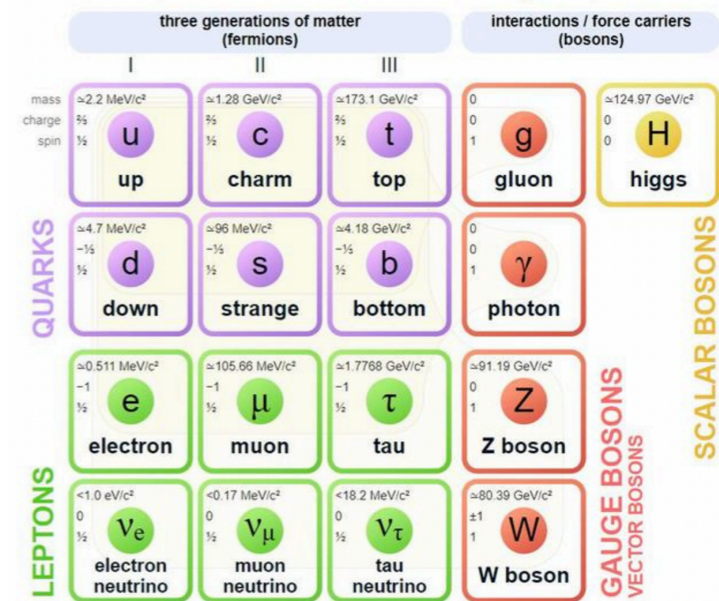
[lgbycl@cqu.edu.cn](mailto:lgbycl@cqu.edu.cn)

**2024/12/12**

# 125 GeV Higgs & Standard Model



## Standard Model of Elementary Particles





# Standard Model is not complete

## Experimental Evidence

Dark Matter

Baryogenesis

Neutrino masses

Origin of flavor

## Theoretical

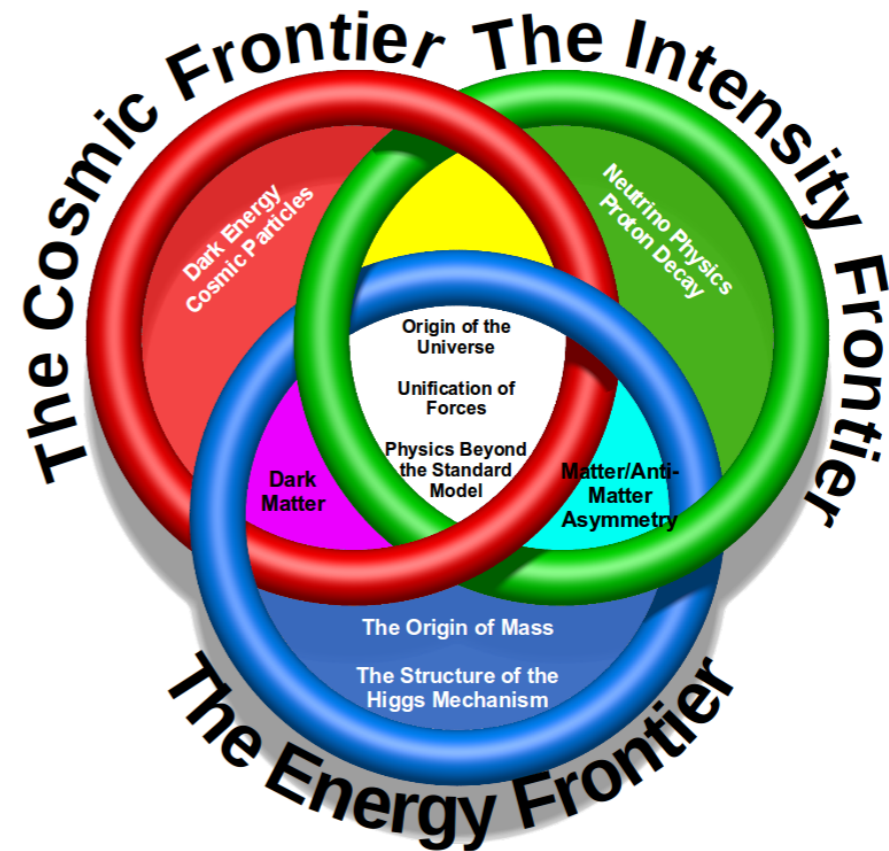
Cosmological constant

Hierarchy problem

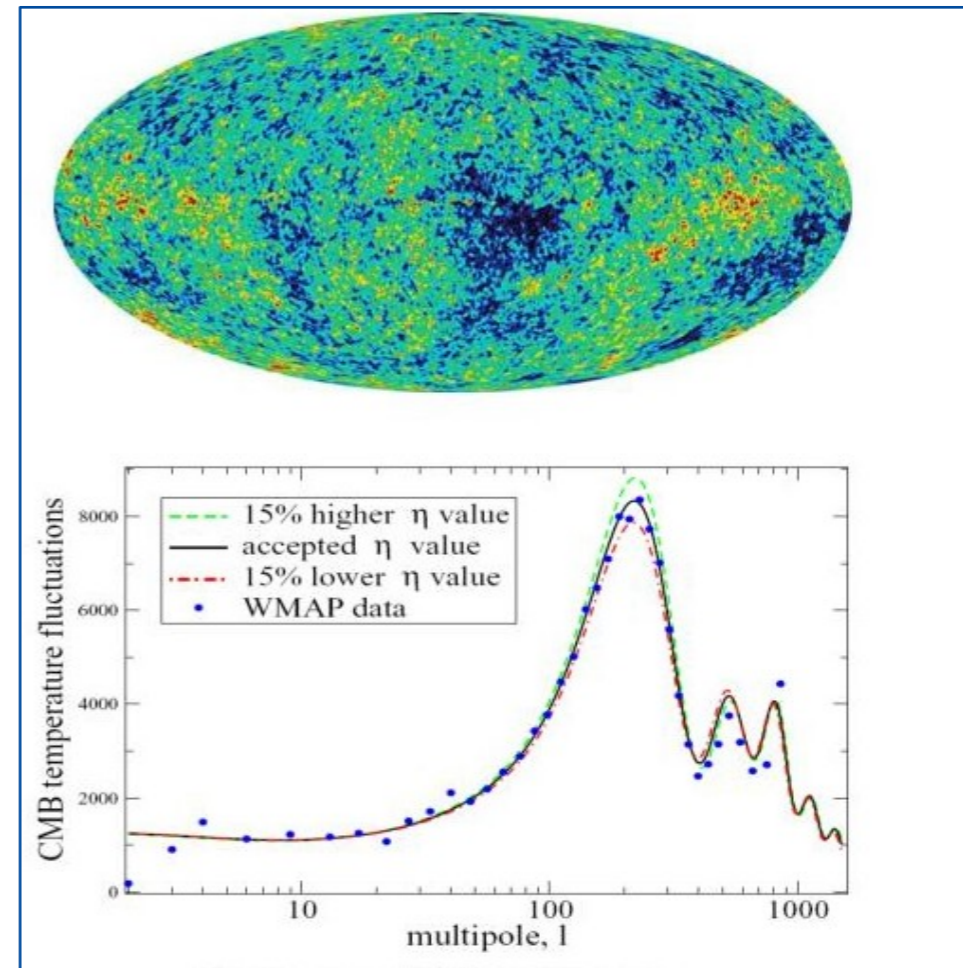
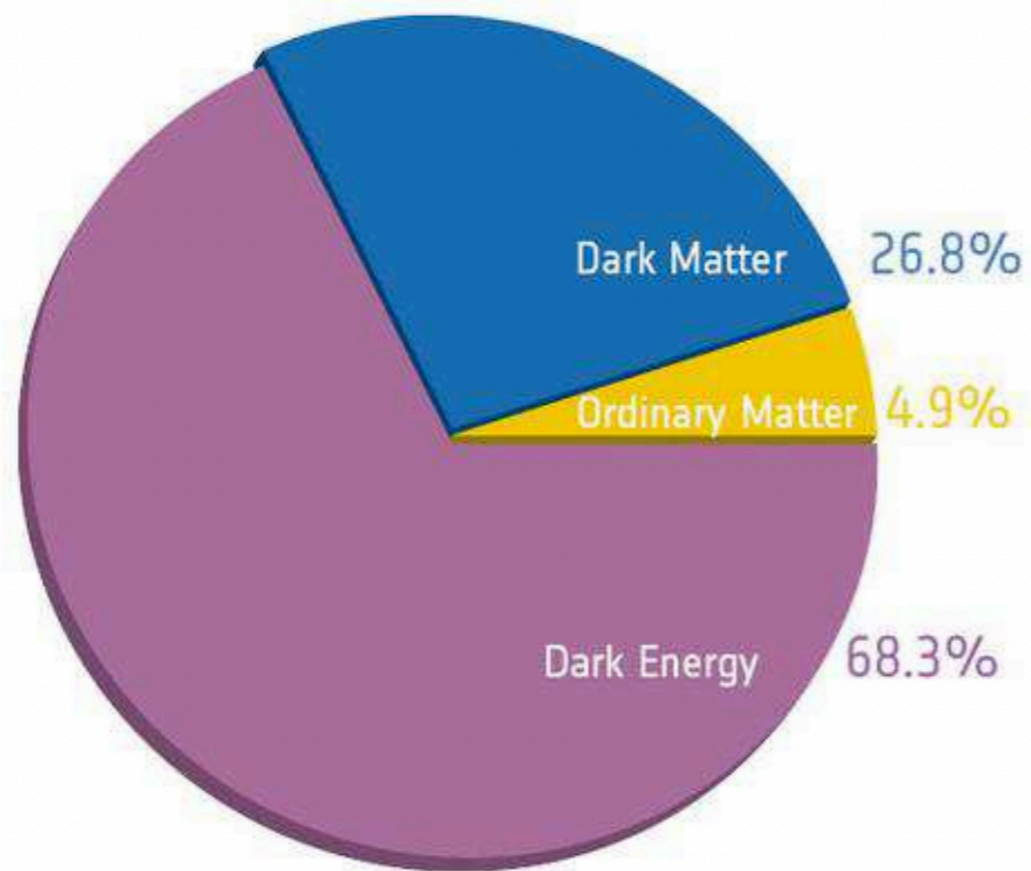
Strong CP problem

Grand Unified Theory (GUT)

## Search for New physics



# Baryon Asymmetry



$$\frac{n_B}{s} = (8.579 \pm 0.109) \times 10^{-11}$$



## Sakharov conditions for Baryon Asymmetry



Nobel Peace Prize in 1975

- Baryon Number Violation  
Weak Sphaleron within SM
- C&CP Violation  
BSM physics
- Out of thermal equilibrium  
BSM physics

CP violation arises naturally in the quark sector of the Standard Model. It's been observed in K, D, and B mesons. **But that's not enough!!!**

$$\text{CKM matrix: } \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{-i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{-i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{-i\delta_{13}} & -c_{12}c_{23} - s_{12}s_{23}s_{13}e^{-i\delta_{13}} & c_{23}c_{13} \end{pmatrix}$$

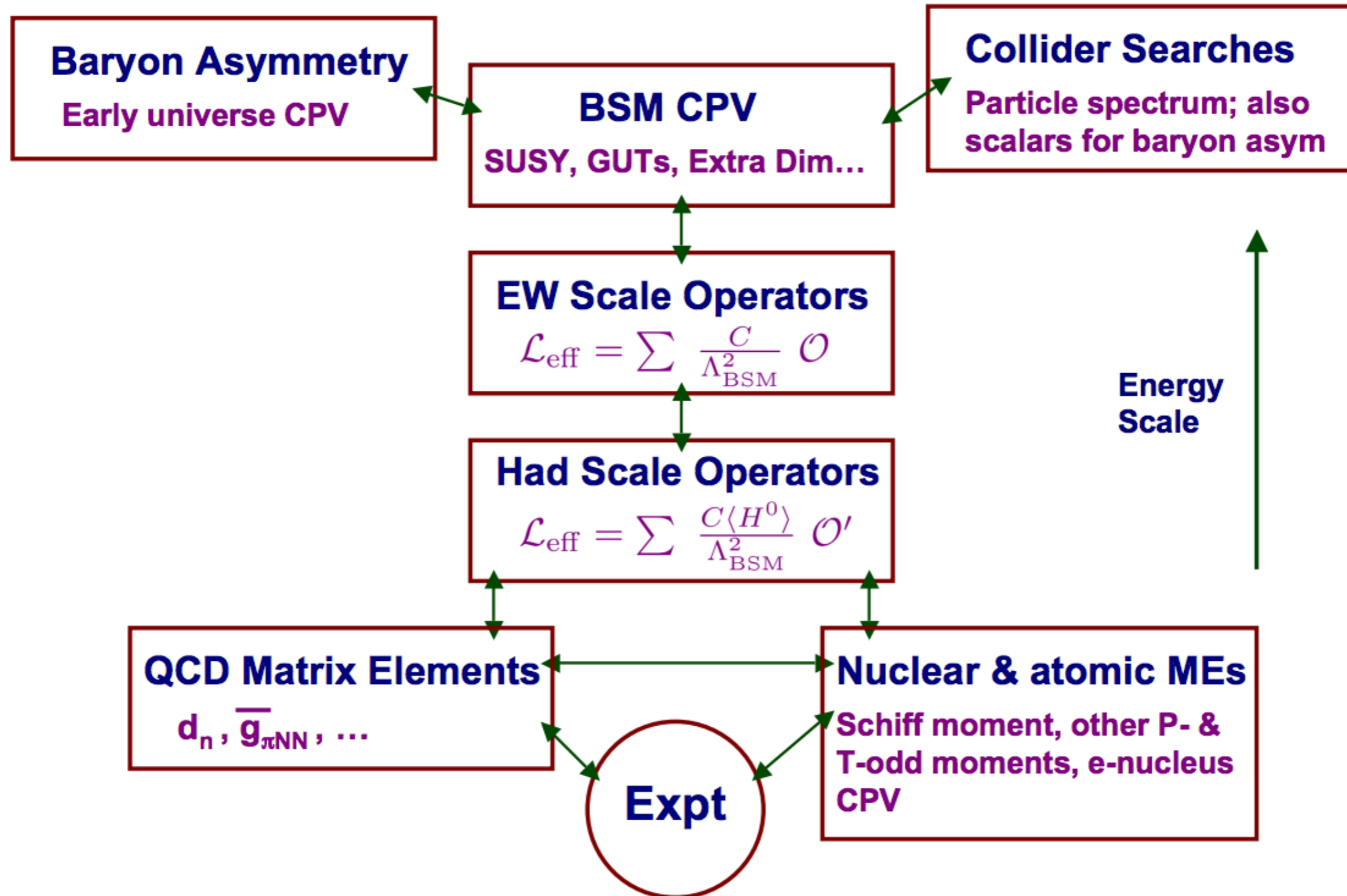
The invariant phase using Jarlskog invariant

$$J_{\text{CKM}} = (m_t^2 - m_c^2)(m_t^2 - m_u^2)(m_c^2 - m_u^2) \\ \times (m_b^2 - m_s^2)(m_b^2 - m_d^2)(m_s^2 - m_d^2)K,$$

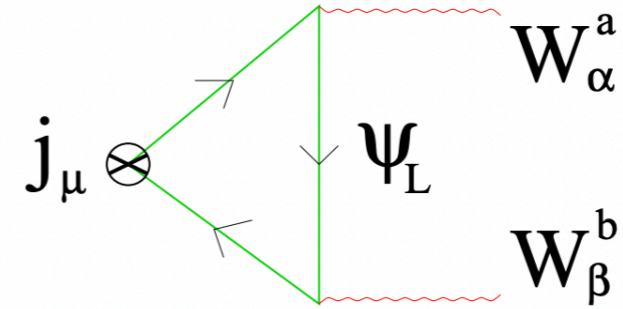
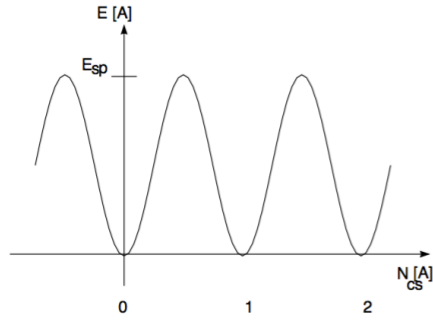
$$K = \text{Im } V_{ii} V_{jj} V_{ij}^* V_{ji}^* = s_1^2 s_2 s_3 c_1 c_2 c_3 \sin \delta$$

$$\frac{J_{\text{CKM}}}{T_c^{12}} \approx 10^{-20} \ll 10^{-11}, \quad T_c \text{ is the SM cross-over temperature}$$





# BAU & Electroweak Sphaleron



$$\partial_\mu J_B^\mu = i \frac{N_F}{32\pi^2} \left( -g_2^2 F^{a\mu\nu} \tilde{F}_{\mu\nu}^a + g_1^2 f^{\mu\nu} \tilde{f}_{\mu\nu} \right),$$

$$\Delta B = N_F (\Delta N_{CS} - \Delta n_{CS}),$$

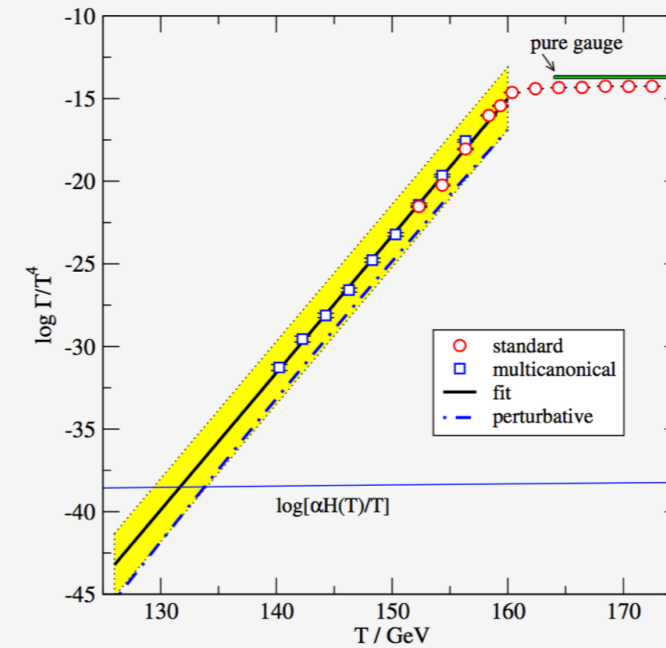
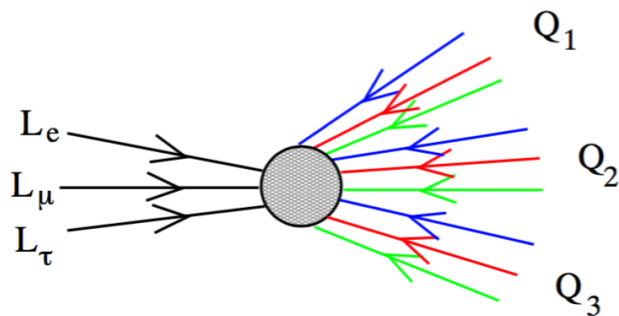
$$N_{CS} = -\frac{g_2^2}{16\pi^2} \int d^3x \, 2\epsilon^{ijk} \text{Tr} \left[ \partial_i A_j A_k + i \frac{2}{3} g_2 A_i A_j A_k \right],$$

$$n_{CS} = -\frac{g_1^2}{16\pi^2} \int d^3x \, \epsilon^{ijk} \partial_i B_j B_k,$$

$$A_i \rightarrow U A_i U^{-1} + \frac{i}{g_2} (\partial_i U) U^{-1},$$

$$\delta N_{CS} = \frac{1}{24\pi^2} \int d^3x \, \text{Tr} \left[ (\partial_i U) U^{-1} (\partial_j U) U^{-1} (\partial_k U) U^{-1} \right] \epsilon^{ijk}.$$

The Standard Model already contains a process that violates B-number. It is known as the electroweak sphaleron ("sphaleron" is Greek for "ready to fall").

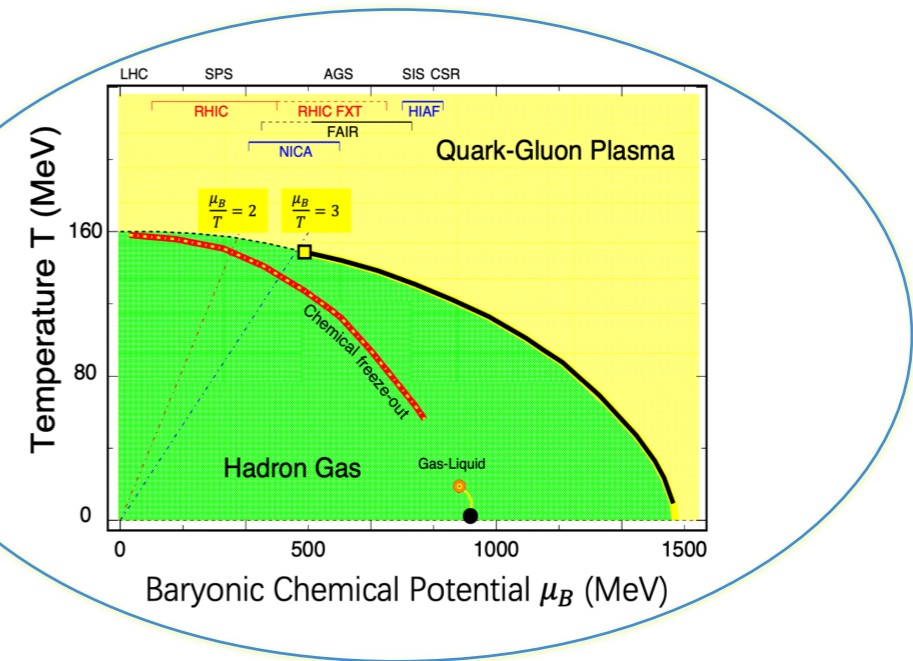
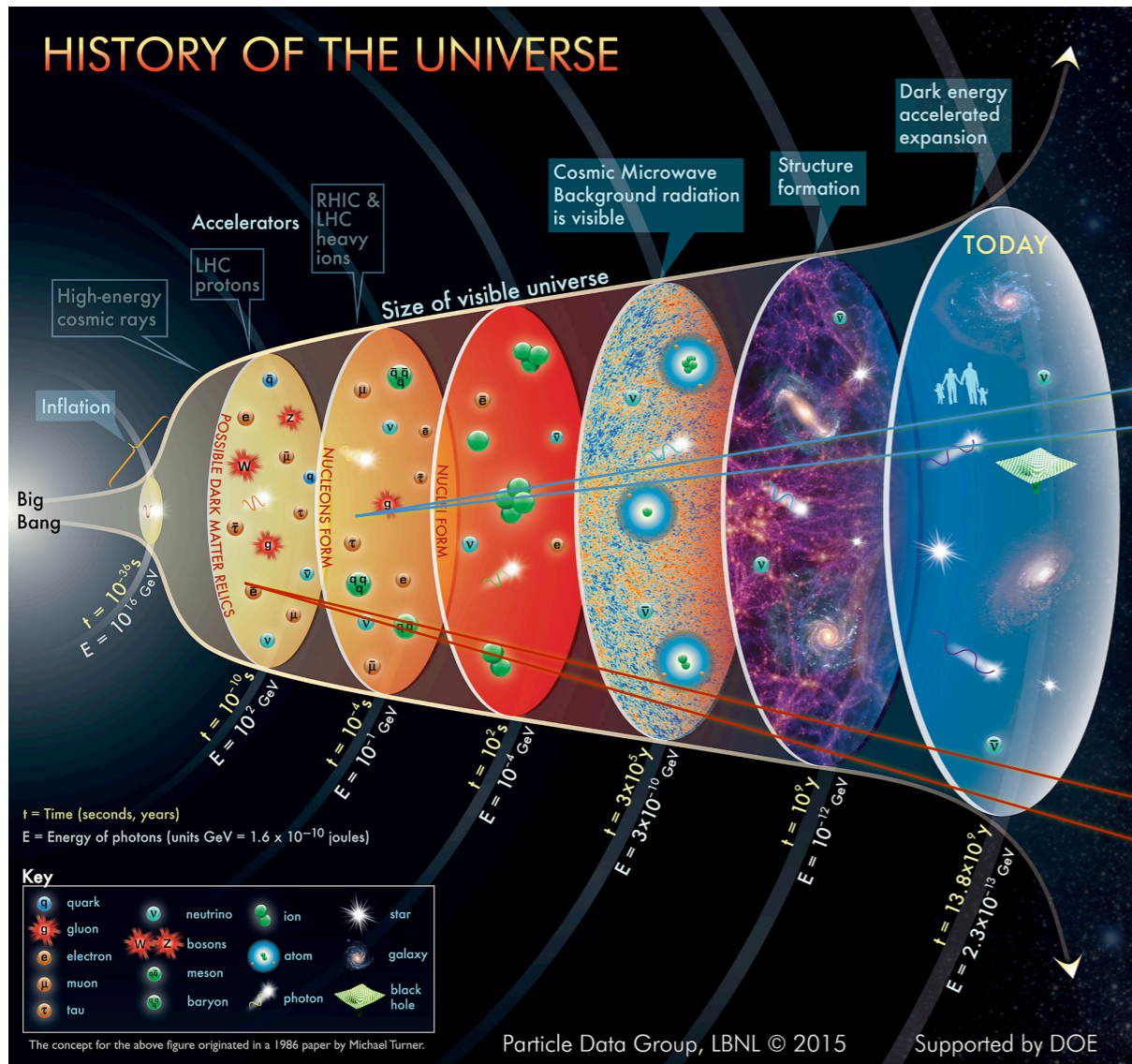


Lattice result,  $T_C = (159.5 \pm 1.5) \text{ GeV}$ , Phys.Rev.Lett,113, 141602 (2014).

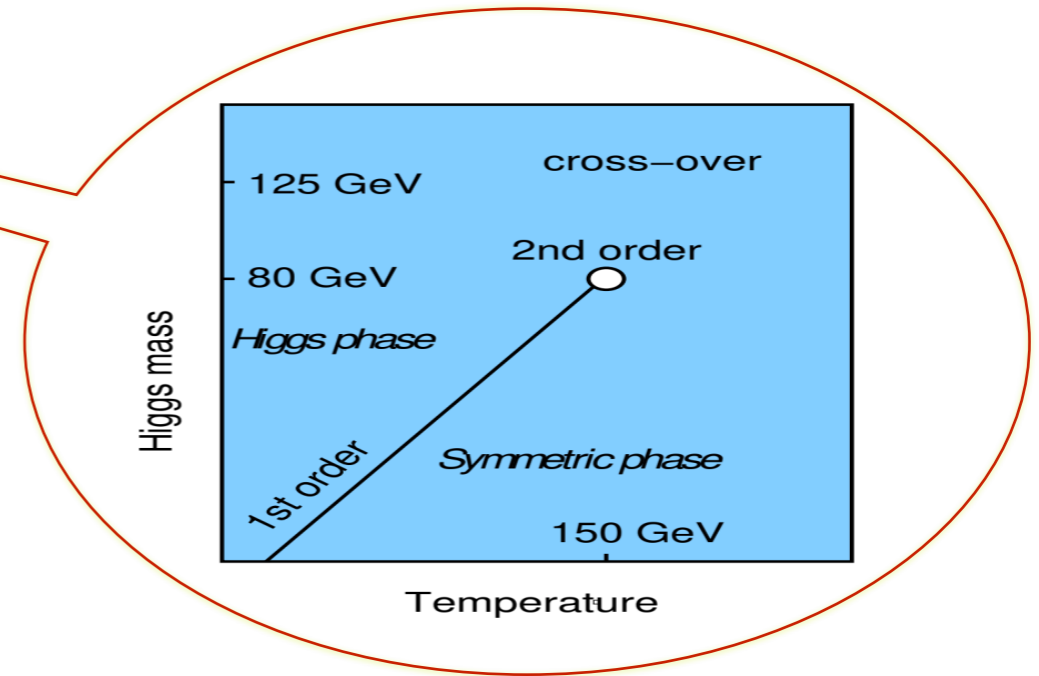
$$\Gamma^{\text{sym}} \approx 6 \times (18 \pm 3) \alpha_W^5 T^4, \quad \Gamma^{\text{brok}} \sim T^4 \exp\left(-\frac{E_{\text{sph}}}{T}\right)$$



# BAU & Non-equilibrium



QCD  
相图



电弱  
相变

## Some Popular Mechanisms

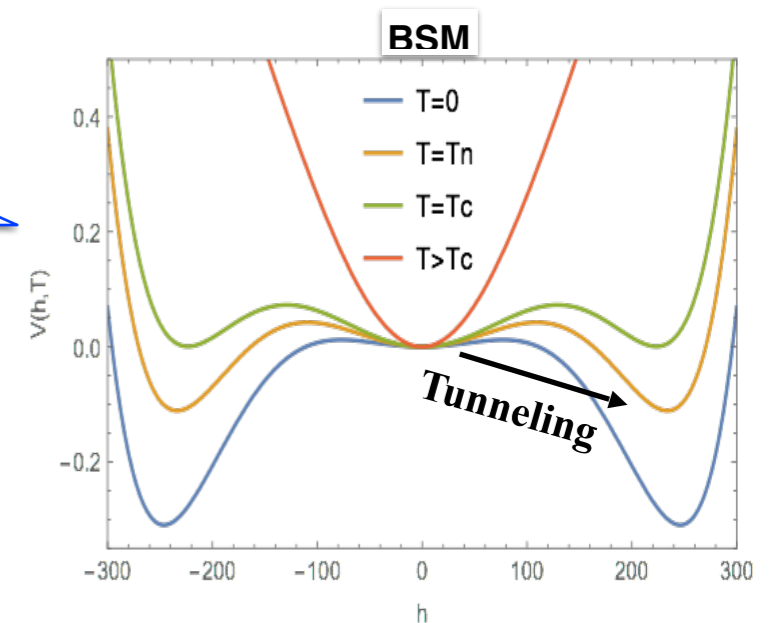
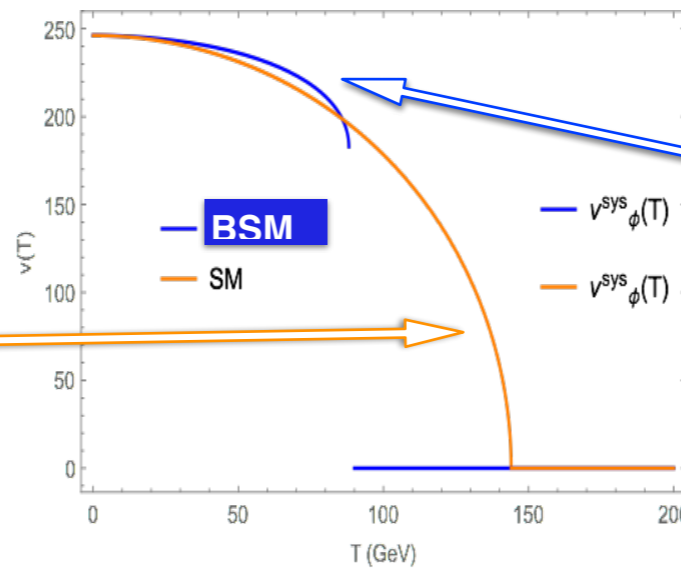
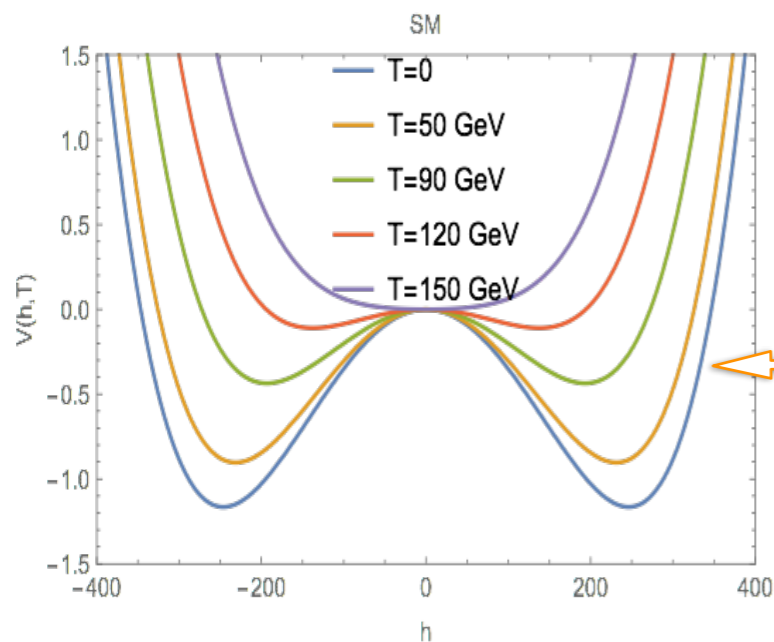
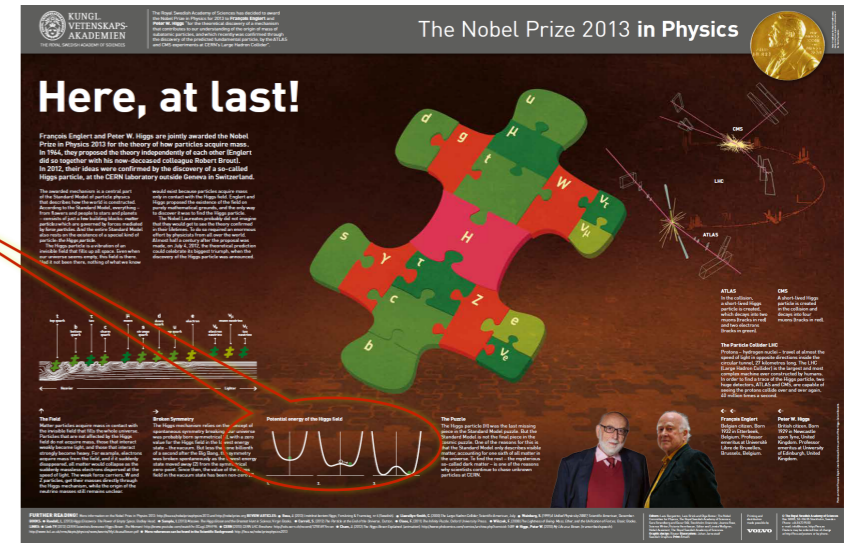
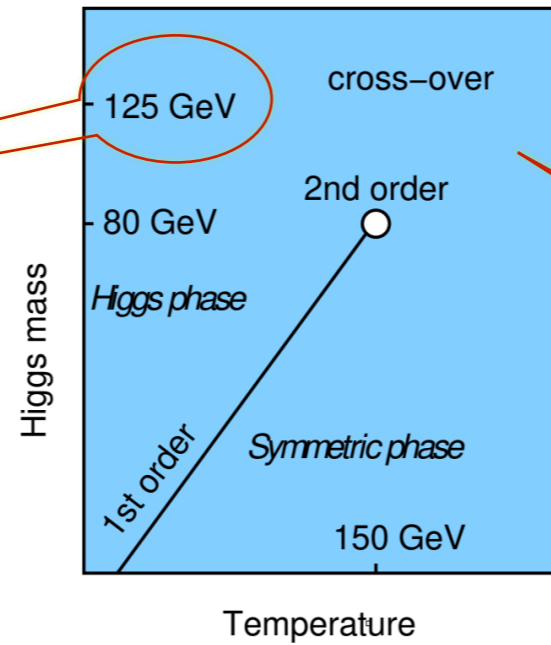
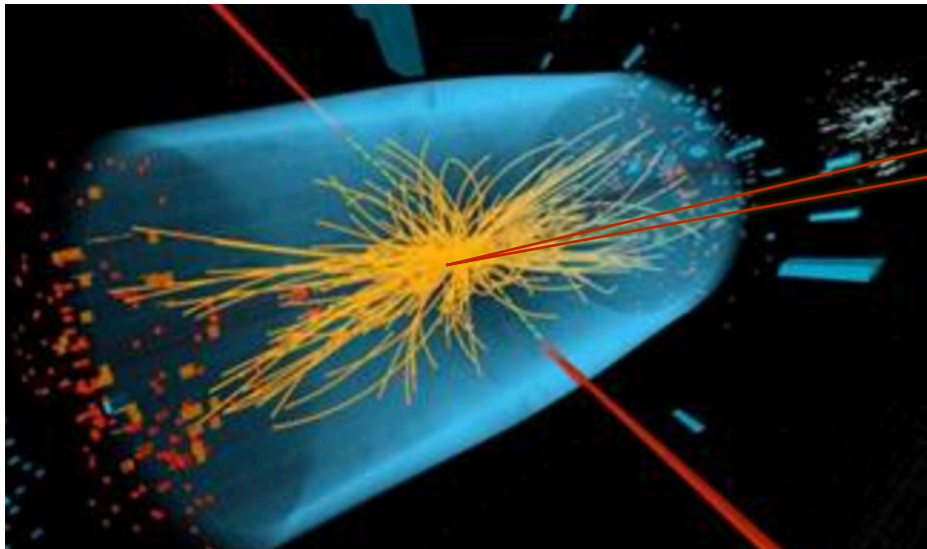
- Leptogenesis  $\rightarrow$  BAU related to the origin of neutrino masses
- Electroweak Baryogenesis  $\rightarrow$  BAU created during the EW phase transition
- GUT Baryogenesis  $\rightarrow$  BAU from B-violating decay of heavy GUT stuff
- Affleck-Dine  $\rightarrow$  BAU from rolling scalars carrying B charges
- Hidden Sector Asymmetric Baryogenesis  $\rightarrow$  BAU in an exotic sector related to dark matter



# 正反物质不对称&强一阶电弱相变



## 电弱对称性破缺的热历史是什么？



平滑过度

一阶相变

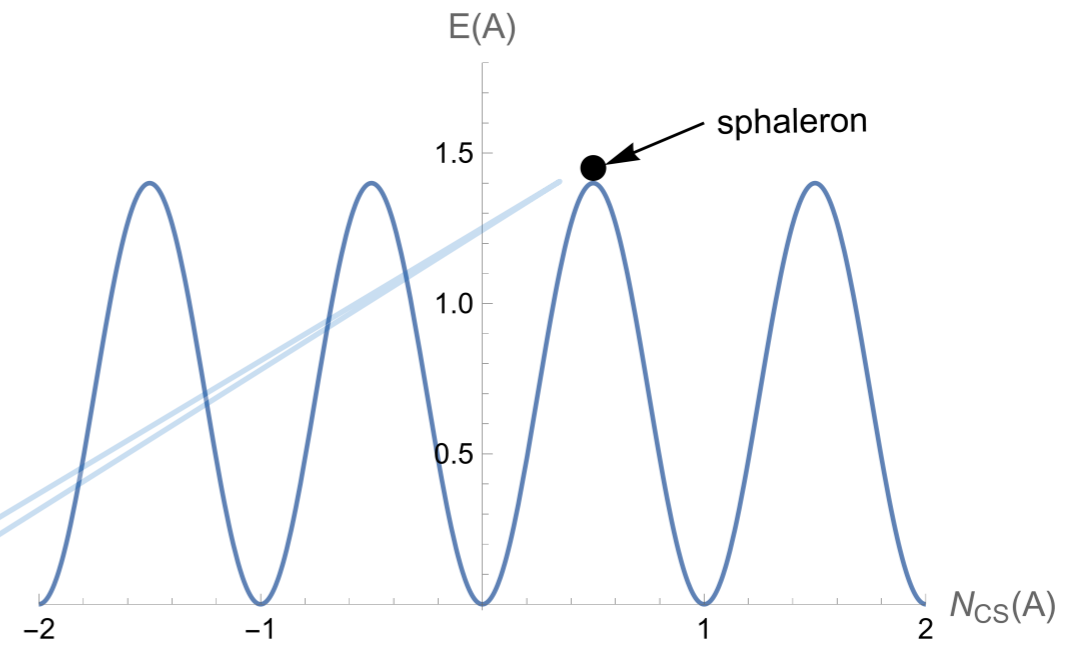


# BNPC & Strongly First-order EWPT

Chern-Simons number

$$\Delta B = \Delta L = 3N_{CS}(t)$$

$$\Gamma_{sph} = \lim_{V, t \rightarrow \infty} \frac{\langle N_{CS}(t)^2 \rangle}{Vt}$$



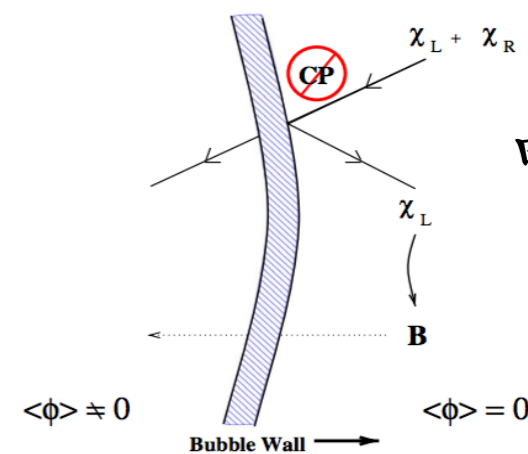
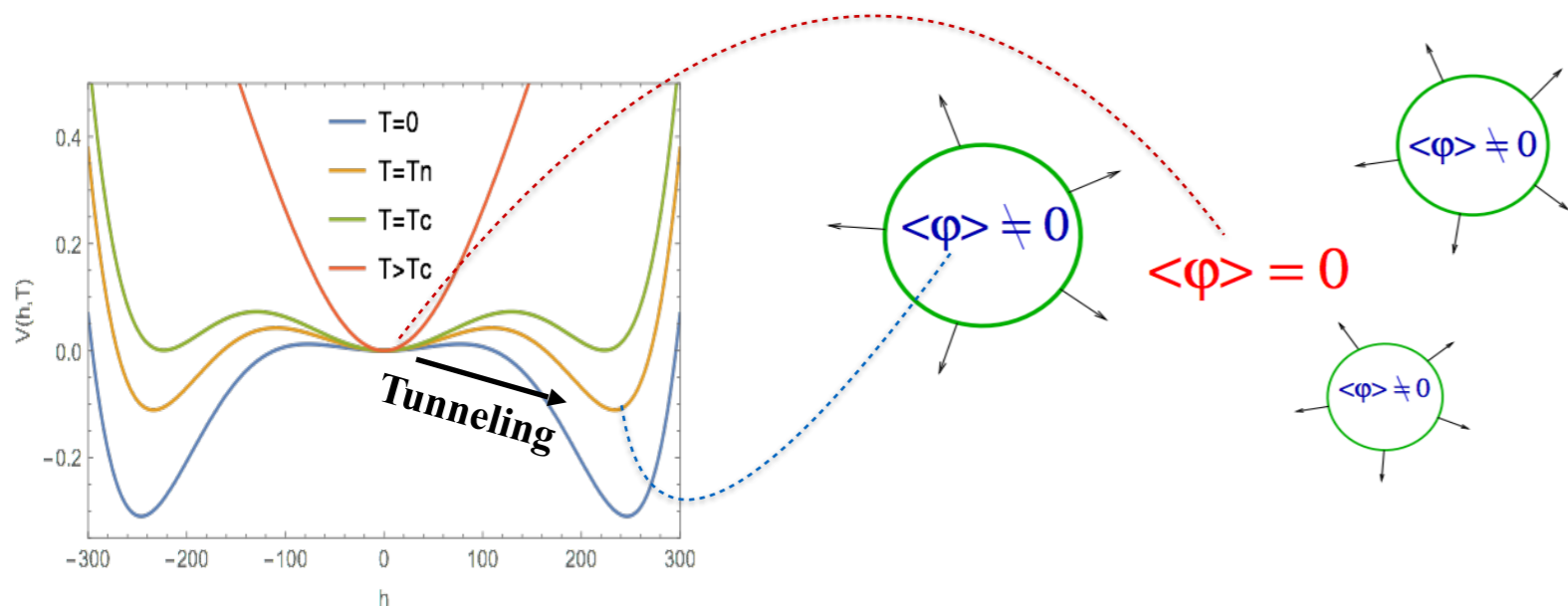
$$\Gamma_{sph} < H$$

$$PT_{sph} = \frac{E_{sph}(T)}{T} - 7 \ln \frac{v(T)}{T} + \ln \frac{T}{100 GeV} > (35.9 - 42.8)$$

1910.00234

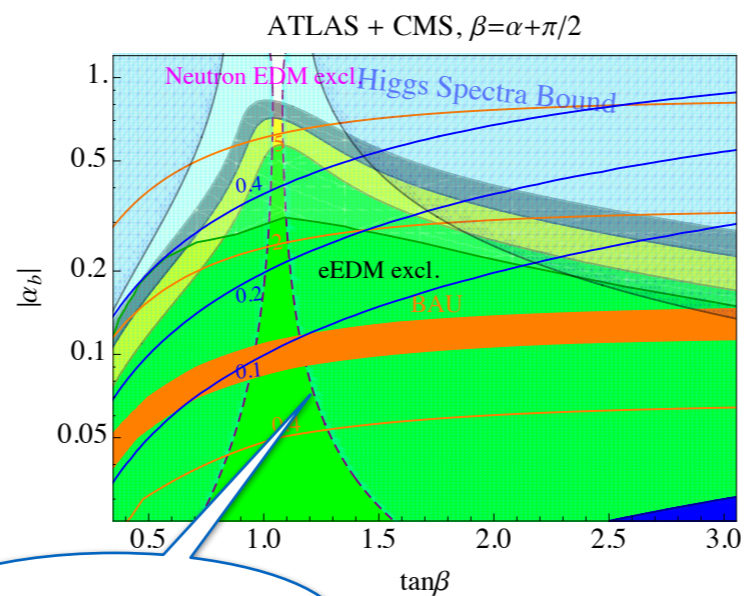
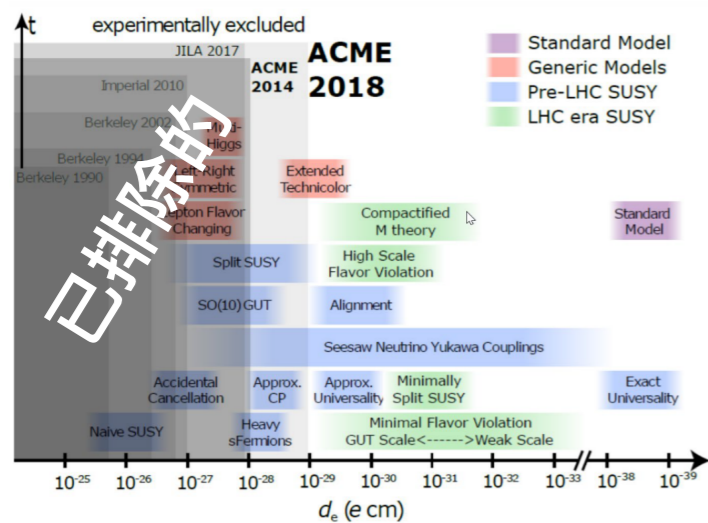
**Baryon Number Preserving Condition (BNPC)**

# 正反物质不对称&强一阶电弱相变

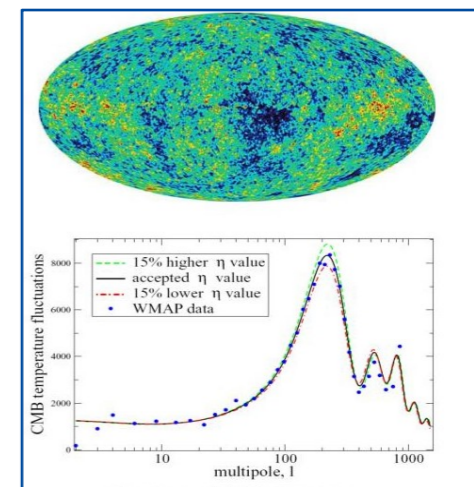


电弱相变中  
解释BAU

Chup et al, Rev Mod Phys.91.015001



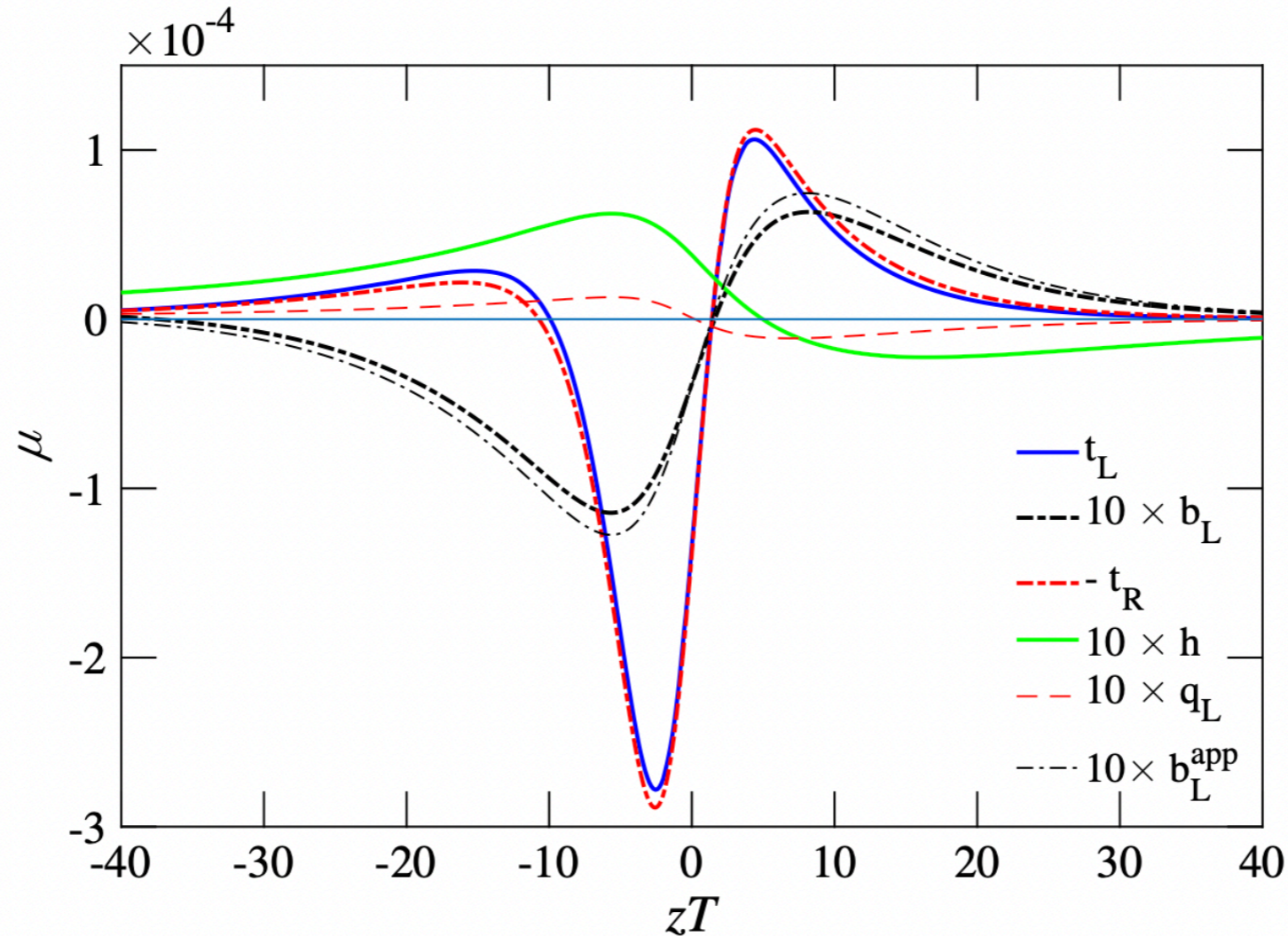
CPV cancellation



$$\frac{n_B}{s} = (8.579 \pm 0.109) \times 10^{-11}$$

Bian, Liu, Shu, PRL115 (2015) 021801

# EWBG with the EW plasma



chemical potential for left handed baryon number

$$\mu_{B_L} = \frac{1}{2}(1 + 4D_0^t)\mu_{t_L} + \frac{1}{2}(1 + 4D_0^b)\mu_{b_L} + 2D_0^t\mu_{t_R}$$

Baryon asymmetry

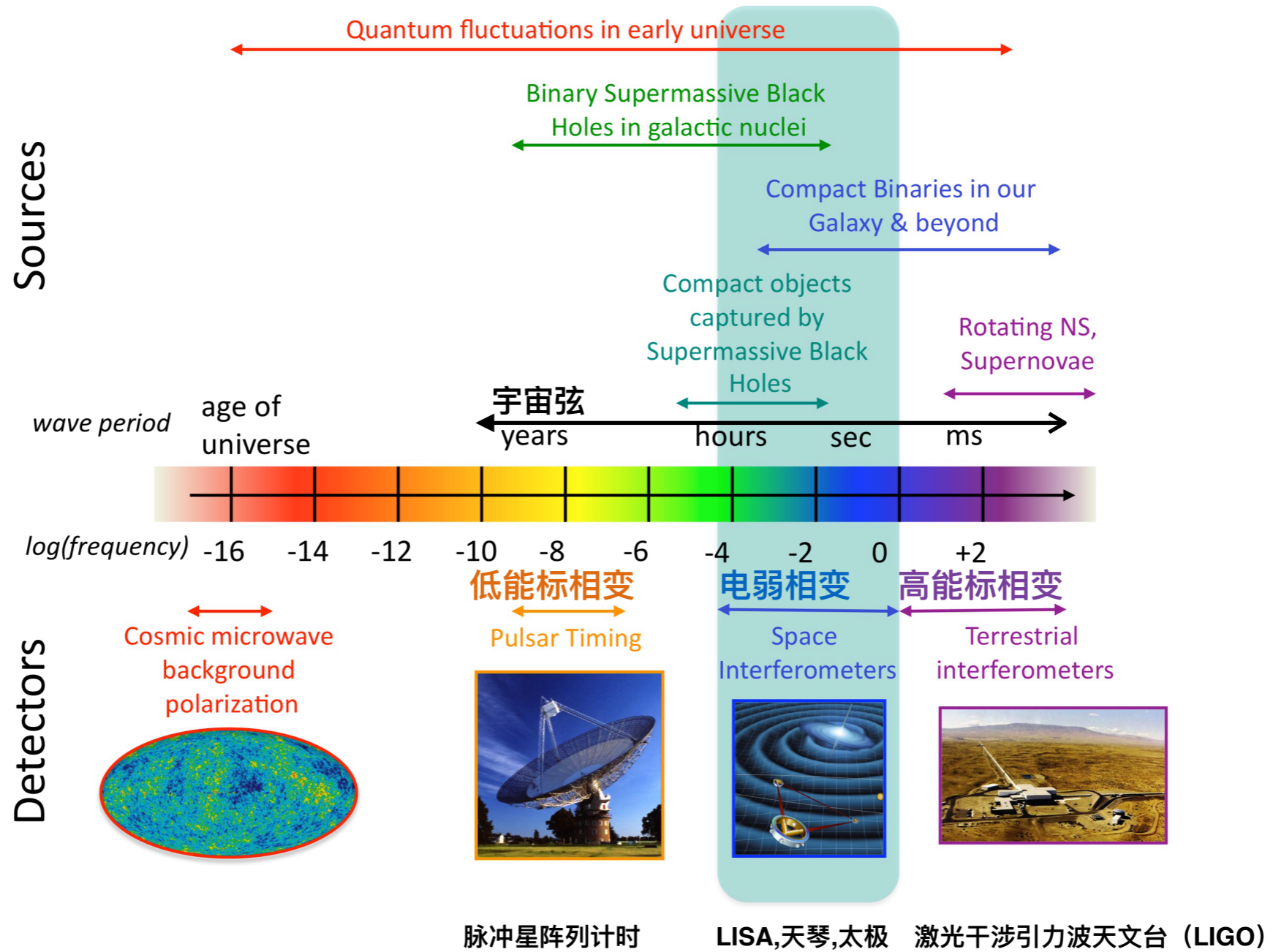
$$\eta_B = \frac{405 \Gamma_{\text{sph}}}{4\pi^2 v_w \gamma_w g_* T} \int dz \mu_{B_L} f_{\text{sph}} e^{-45\Gamma_{\text{sph}}|z|/4v_w},$$

VEV- insertion source tends to predict a larger baryon asymmetry than the WKB source by a factor of  $\sim 10$ .

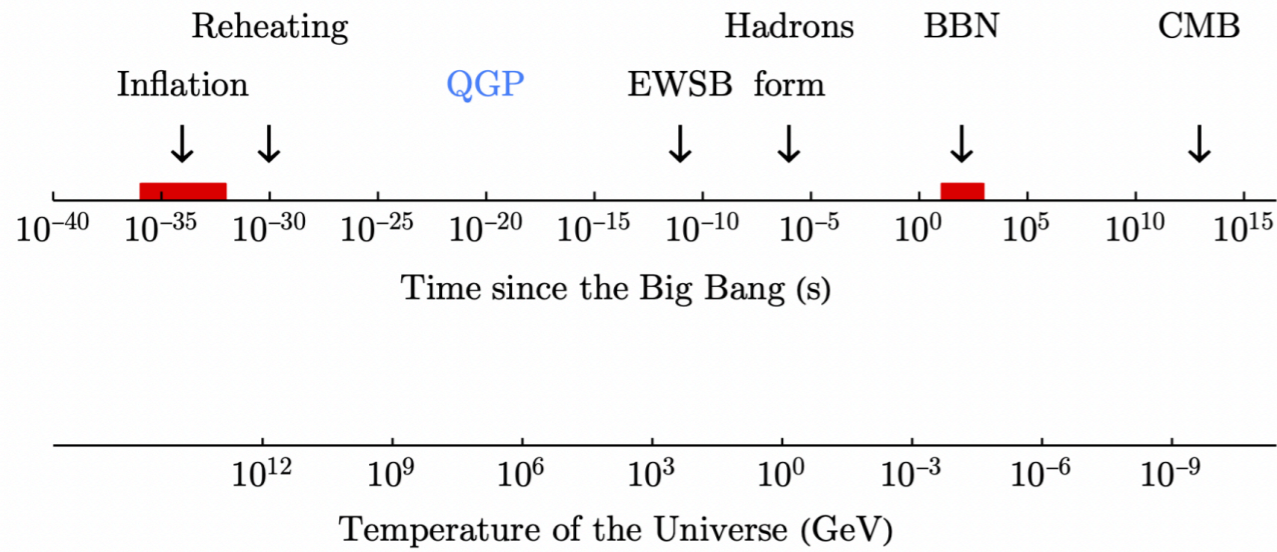


随机引力波探测开启了探索早期宇宙背后新物理的一个新的窗口

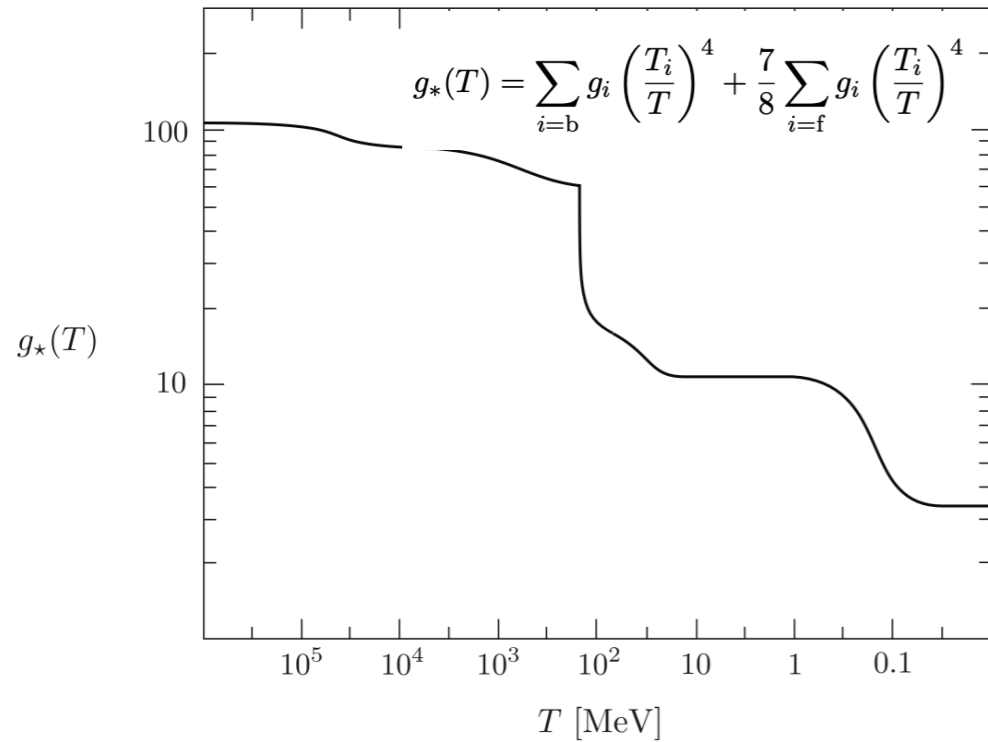
# The Gravitational Wave Spectrum



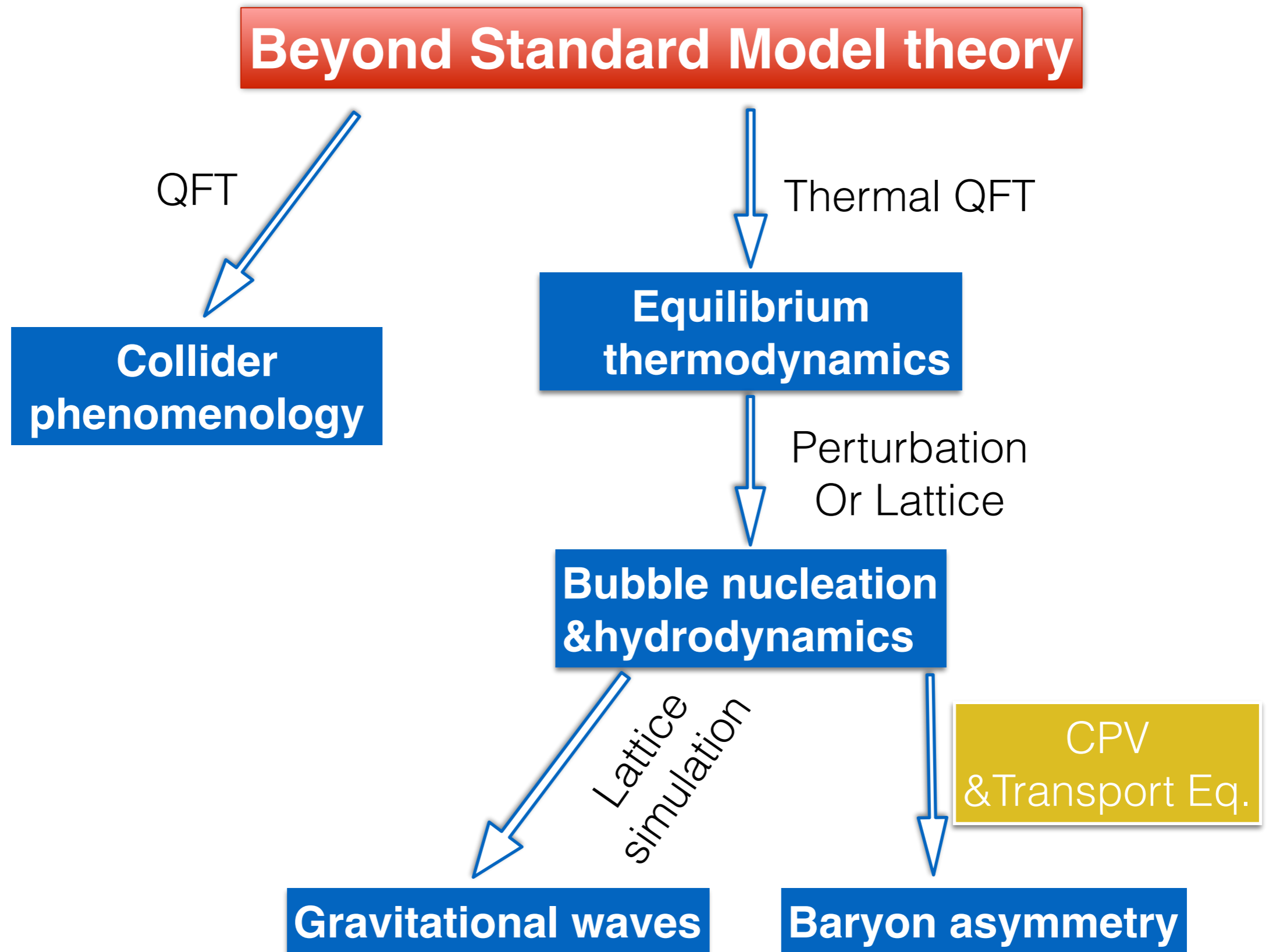
# Key Events in the early Universe



Radiation dominate Universe:  $H = \frac{1}{2t} = \sqrt{\frac{8\pi}{3} \frac{\rho_{\text{rad}}}{M_p^2}} = \sqrt{\frac{8\pi}{3} \frac{\pi^2}{30} \frac{T^4}{g_* M_p^2}} = \sqrt{\frac{8\pi^3}{90} g_*} \frac{T^2}{M_p}$

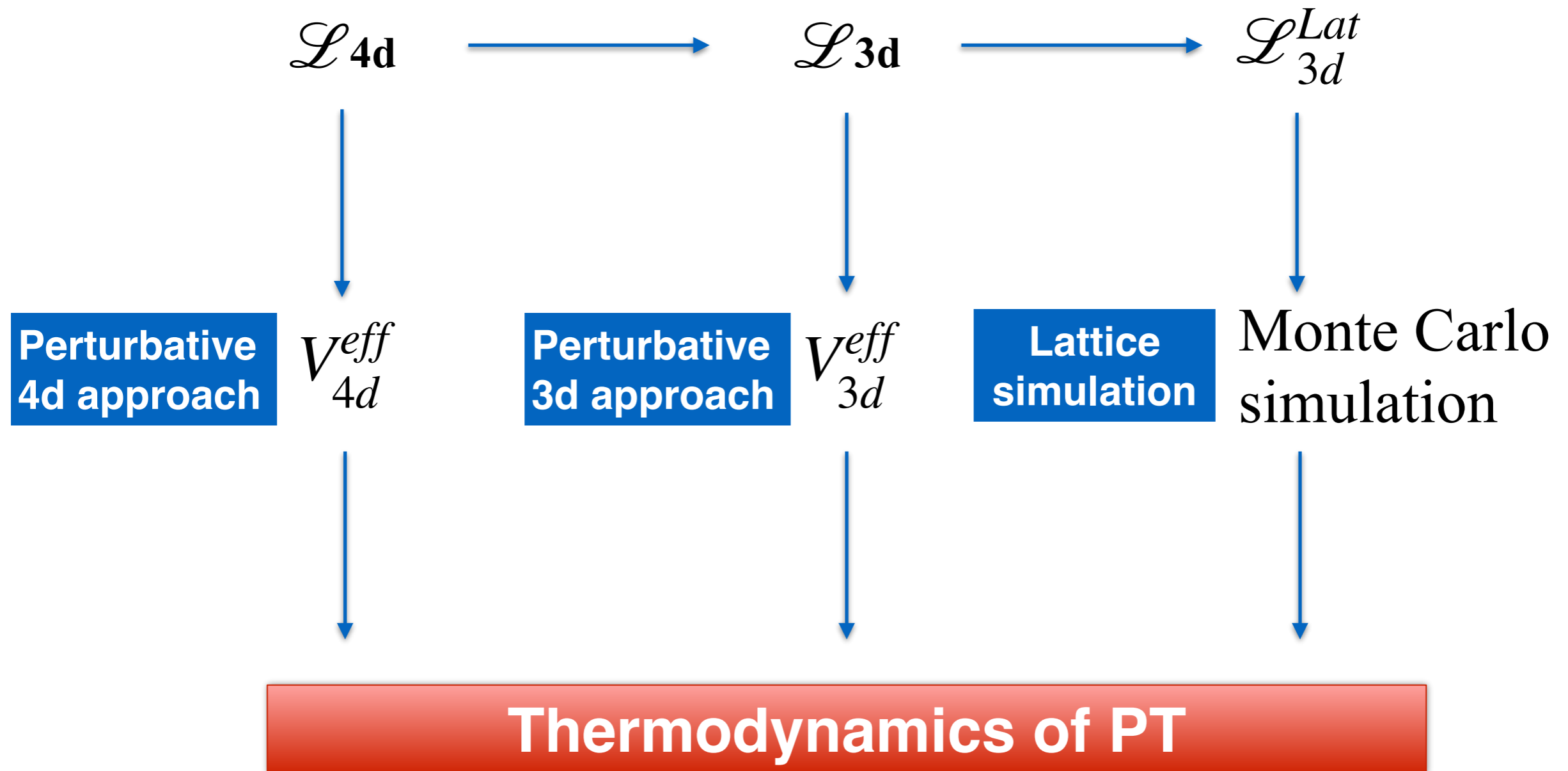


Event	time $t$	redshift $z$	temperature $T$
Inflation	$10^{-34}$ s	–	–
Baryogenesis	?	?	?
EW phase transition	20 ps	$10^{15}$	100 GeV
QCD phase transition	20 $\mu$ s	$10^{12}$	150 MeV
Dark matter freeze-out	?	?	?
Neutrino decoupling	1 s	$6 \times 10^9$	1 MeV
Electron-positron annihilation	6 s	$2 \times 10^9$	500 keV
Big Bang nucleosynthesis	3 min	$4 \times 10^8$	100 keV
Matter-radiation equality	60 kyr	3400	0.75 eV
Recombination	260–380 kyr	1100–1400	0.26–0.33 eV
Photon decoupling	380 kyr	1000–1200	0.23–0.28 eV
Reionization	100–400 Myr	11–30	2.6–7.0 meV
Dark energy-matter equality	9 Gyr	0.4	0.33 meV
Present	13.7 Gyr	0	0.24 meV





# ► Methods for PT dynamics study

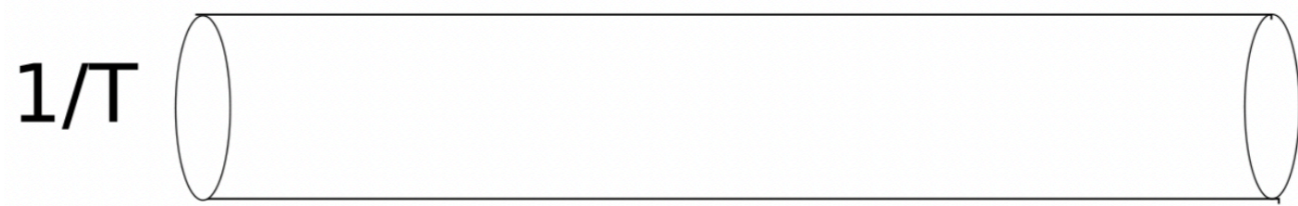


# ► Finite temperature EFT for the 3d Phase transition study

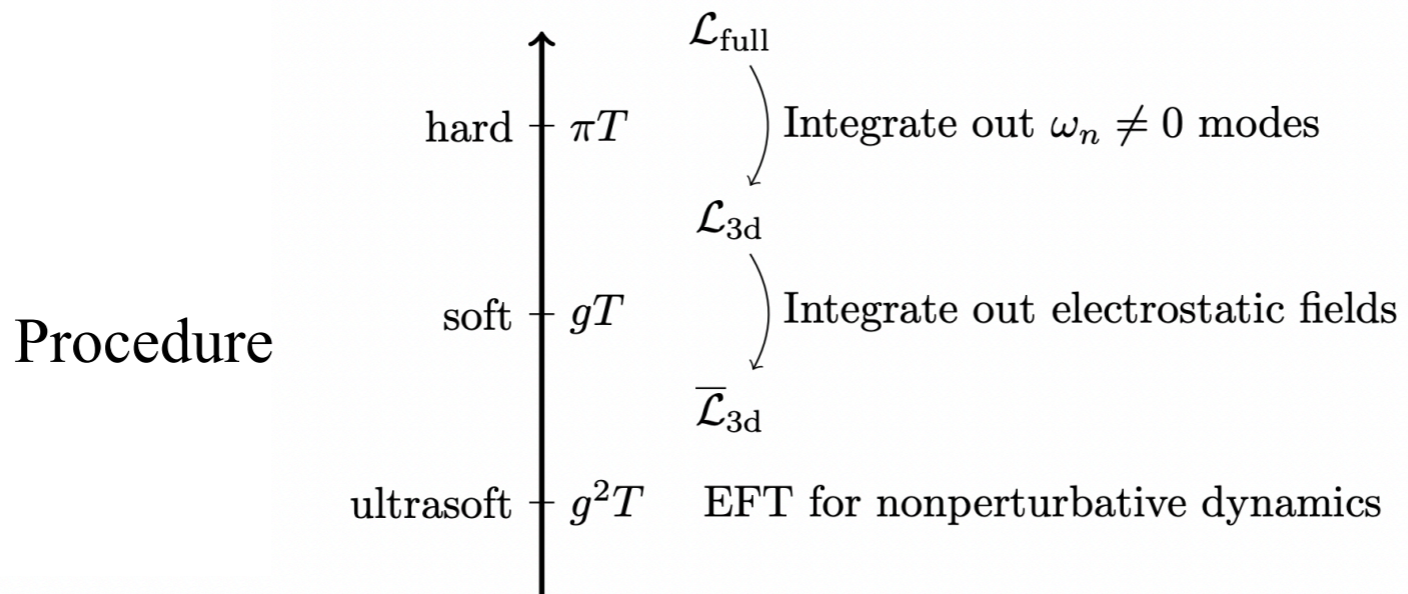
Matsubara decomposition

$$\phi(\tau, \mathbf{x}) = T \sum_n \tilde{\phi}(\mathbf{p}) e^{i\omega_n \tau}, \quad \omega_n = \begin{cases} 2\pi n T & \text{bosons} \\ (2n + 1)\pi T & \text{fermions} \end{cases}$$

$\omega_n \neq 0$  modes are heavy and decouple at distances  $\gg 1/T$ , and can be integrated out

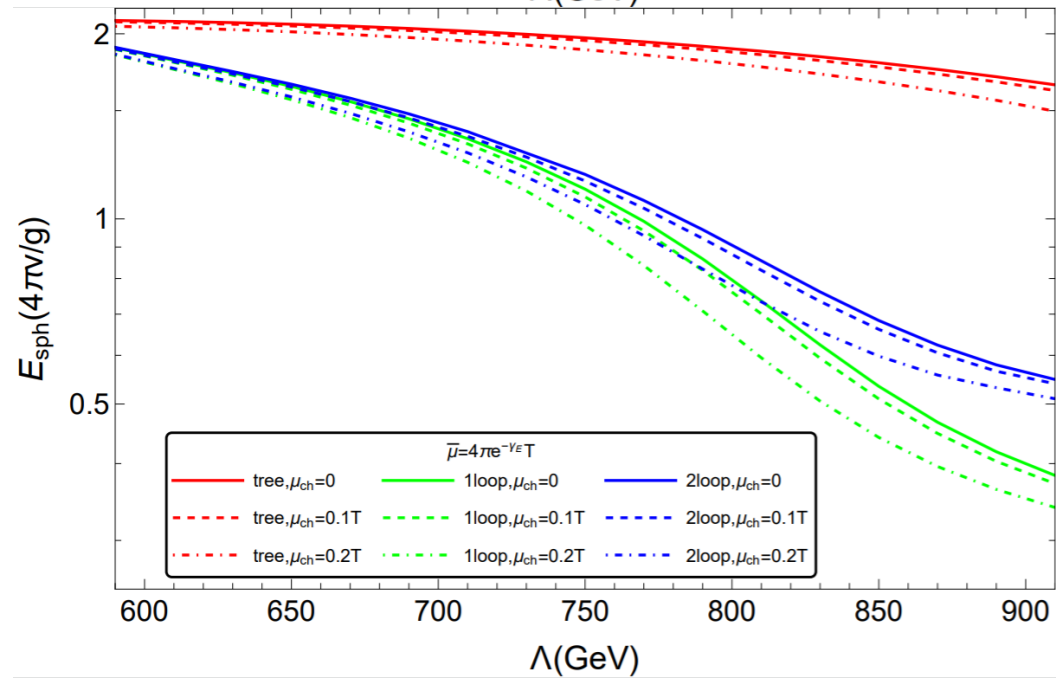
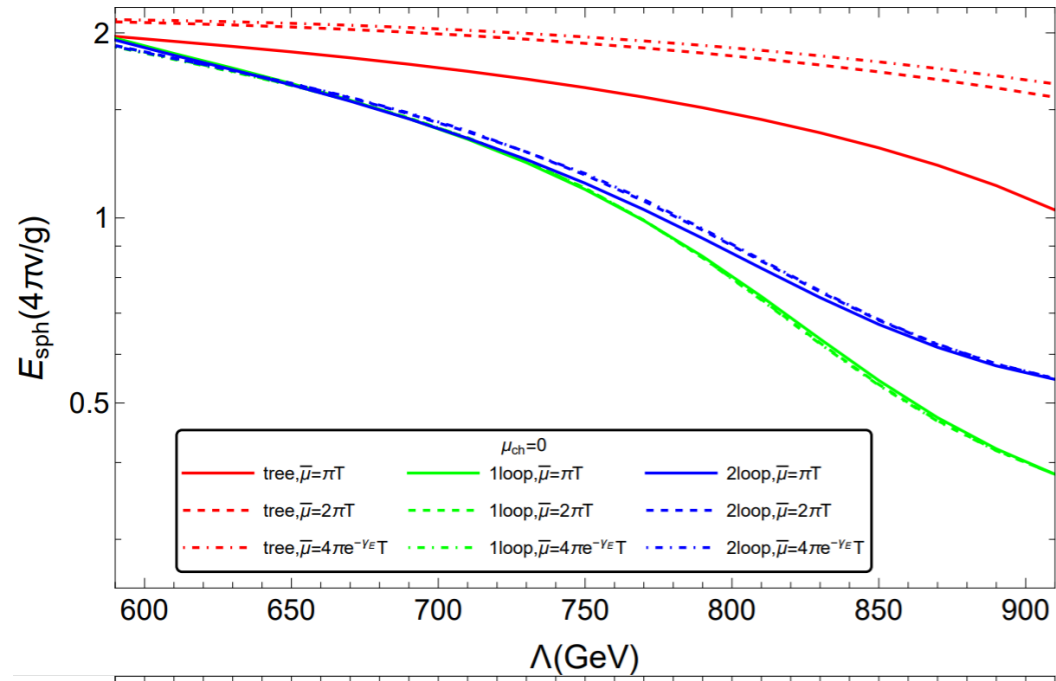


$$S = \int d^4x [\mathcal{L}_{\text{gauge}} + \mathcal{L}_{\text{fermion}} + \mathcal{L}_{\text{scalar}} + \mathcal{L}_{\text{Yukawa}}]$$

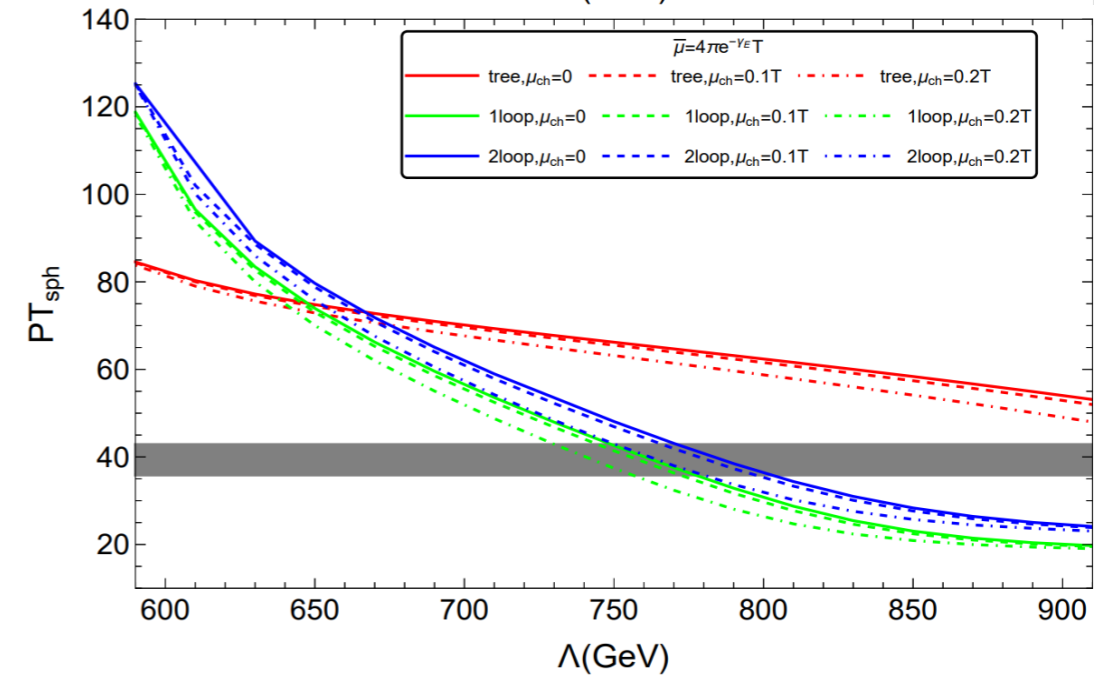
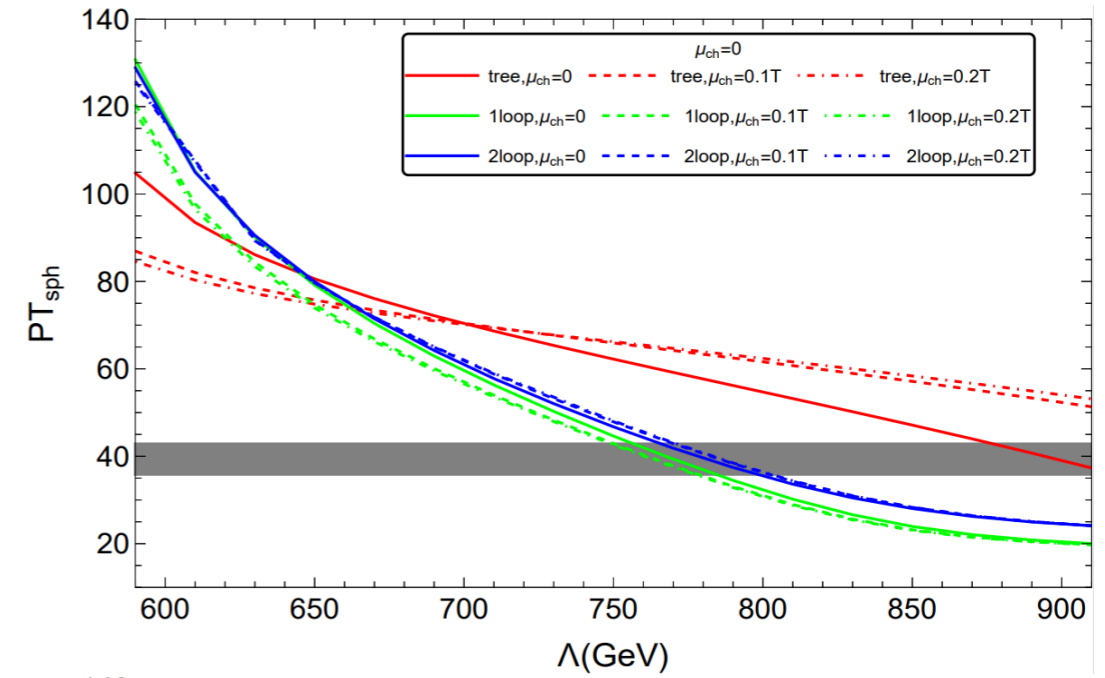


$$S_{3d} = \int d^3x \left[ \frac{1}{4} F_{ij}^a F_{ij}^a + (D_i \phi)^\dagger (D_i \phi) + \bar{m}^2 \phi^\dagger \phi + \bar{\lambda} (\phi^\dagger \phi)^2 + \mathcal{L}_{\text{BSM}} + \text{higher-order operators} \right]$$

## Sphaleron energy



## $PT_{sph}$





# Finite temperature potential and free energy

The grand canonical partition function

$$\mathcal{Z}(T) \equiv \text{Tr}[e^{-\beta(\hat{H}-\mu\hat{N})}], \quad \text{where } \beta \equiv \frac{1}{T} \quad \mu_B/T \ll 1$$

$$\phi(x) = T \sum_{\omega_n} \int_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{x}} \phi(k) = T \sum_{\omega_n} \int_{\mathbf{k}} e^{i(\omega_k\tau - \mathbf{k}\cdot\mathbf{x})} \phi(k)$$

$$\omega_n = 2n\pi T \quad \mathbf{k} = (\omega_n, \mathbf{k})$$

$$\begin{aligned} \mathcal{Z}(T) &= \int \mathcal{D}\phi \exp\left(-T \sum_{\mathbf{k}} \int \frac{1}{2} (\mathbf{k}^2 + \omega_n^2 + m^2) |\phi(k)|^2\right) \\ &= \exp\left[-\frac{V}{T} \int \frac{d^3k}{(2\pi)^3} \left(\frac{\omega}{2} + T \ln(1 - e^{-\omega/T})\right)\right] \end{aligned}$$

The free energy

$$F = -T \ln \mathcal{Z}$$

$$\begin{aligned} \lim_{V \rightarrow \infty} \frac{F}{V} &= \int \frac{d^3k}{(2\pi)^3} \left(\frac{\omega}{2} + T \ln(1 - e^{-\omega/T})\right) \\ &= \int \frac{d^3k}{(2\pi)^3} \left(\frac{\sqrt{m^2 + \mathbf{k}^2}}{2} + T \ln(1 - e^{-\sqrt{m^2 + \mathbf{k}^2}/T})\right) \\ &\equiv J_0(m) + \tilde{J}_B(m, T) \quad \tilde{J}_i = T^4/2\pi^2 J_i. \end{aligned}$$

$$\begin{aligned} \left(\lim_{V \rightarrow \infty} \frac{F}{V}\right)_{\text{fermions}} &= \int \frac{d^3k}{(2\pi)^3} \left(\frac{\omega}{2} + T \ln(1 + e^{-\omega/T})\right) \\ &= \int \frac{d^3k}{(2\pi)^3} \left(\frac{\sqrt{m^2 + \mathbf{k}^2}}{2} + T \ln(1 + e^{-\sqrt{m^2 + \mathbf{k}^2}/T})\right) \\ &\equiv J_0(m) + \tilde{J}_F(m, T) \end{aligned}$$

$$\begin{aligned} \tilde{J}_B(m, T) &= T \int \frac{d^3k}{(2\pi)^3} \ln(1 - e^{-\sqrt{m^2 + \mathbf{k}^2}/T}) \\ &= \frac{T}{2\pi^2} \int d|\mathbf{k}| \mathbf{k}^2 \ln(1 - e^{-\sqrt{m^2 + \mathbf{k}^2}/T}) \\ &= \frac{T^4}{2\pi^2} \int dx x^2 \ln(1 - e^{-\sqrt{(m/T)^2 + x^2}}) \end{aligned}$$

$$\tilde{J}_F = \frac{T^4}{2\pi^2} \left( -\frac{7\pi^4}{360} + \frac{\pi^2 m^2}{24T^2} - \frac{m^4}{32T^4} \left[ \ln\left(\frac{e^{\gamma_E} m^2}{\pi^2 T^2}\right) - \frac{3}{2} \right] + \mathcal{O}\left(\frac{m^6}{T^6}\right) \right)$$

high-T expansion  $m \ll T$

# ► 1-loop Effective potential at finite temperature

1-loop finite-T thermal effective potential

$$V_{\text{eff}}(\phi, T) = V_{\text{tree}} + V_{\text{1-loop}}$$

1-loop

$$\begin{aligned} V_{\text{1-loop}} &= \frac{1}{2} \not\int_P \ln(P^2 + m^2) \\ &= \frac{1}{2} \left( \frac{\bar{\mu}^2 e^{\gamma_E}}{4\pi} \right)^\epsilon \int \frac{d^D p}{(2\pi)^D} \ln(p^2 + m^2) - \int_p T \ln(1 \mp n_{\text{B/F}}(E_p, T)) \\ &\quad V_{\text{CW}}(m) \qquad V_T \sim J_{T,b/f} \left( \frac{m^2}{T^2} \right) \\ &= \frac{T}{2} \int_p \ln(p^2 + m^2) + \frac{1}{2} \not\int'_{P/\{P\}} \ln(P^2 + m^2) \\ &\quad V_{\text{soft}}(m) \qquad V_{\text{hard}}(m) \end{aligned}$$

Daisy/ring resummation  $V_{\text{daisy}} = V_{\text{soft}}^{\text{resummed}} - V_{\text{soft}}$

$$V_{\text{soft}}(m) = -\frac{T}{12\pi} (m^2)^{\frac{3}{2}} \longrightarrow V_{\text{soft}}^{\text{resummed}} = -\frac{T}{12\pi} (m^2 + \Pi_T)^{\frac{3}{2}}$$

Arnold-Espinosa eff potential

$$V_{\text{eff}}^{\text{A-E res.}}(\phi, T, \bar{\mu}) = V_{\text{tree}} + V_{\text{CW}} + V_T + V_{\text{daisy}}$$

$$V_{\text{eff}}^{\text{resummed}}(\phi, T, \bar{\mu}) = V_{\text{tree}} + V_{\text{soft}}^{\text{resummed}} + V_{\text{hard}}$$

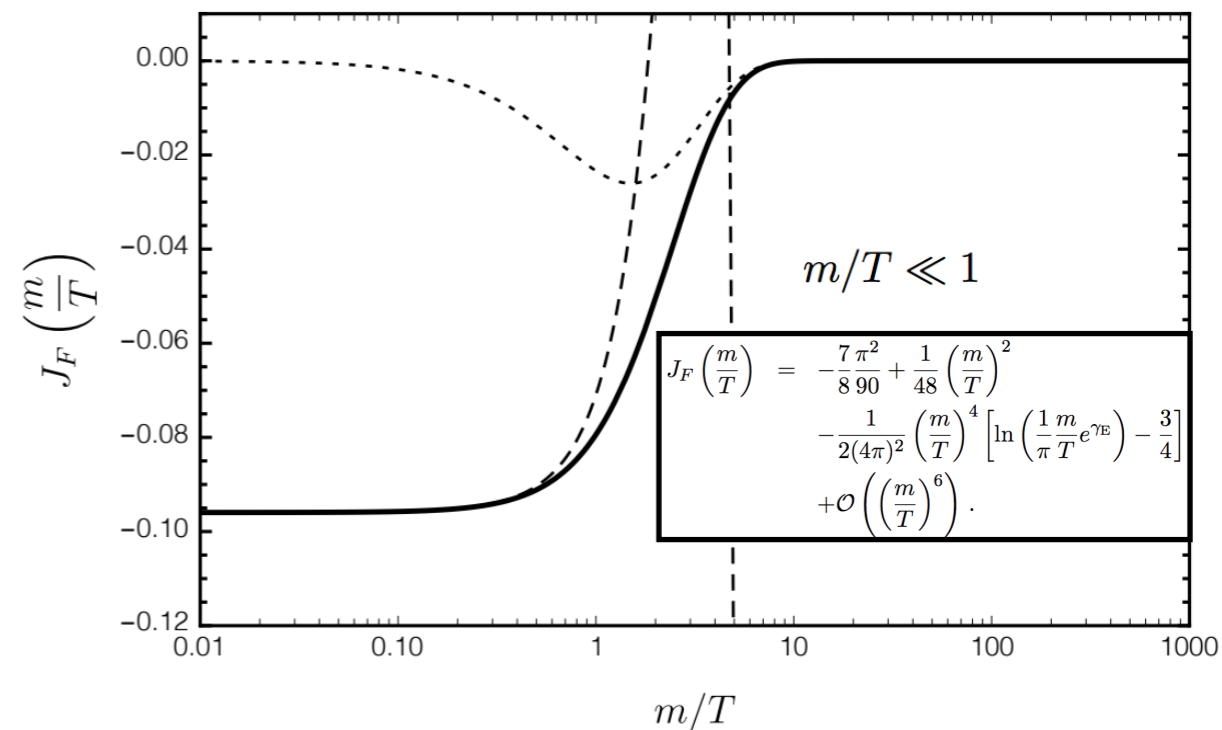
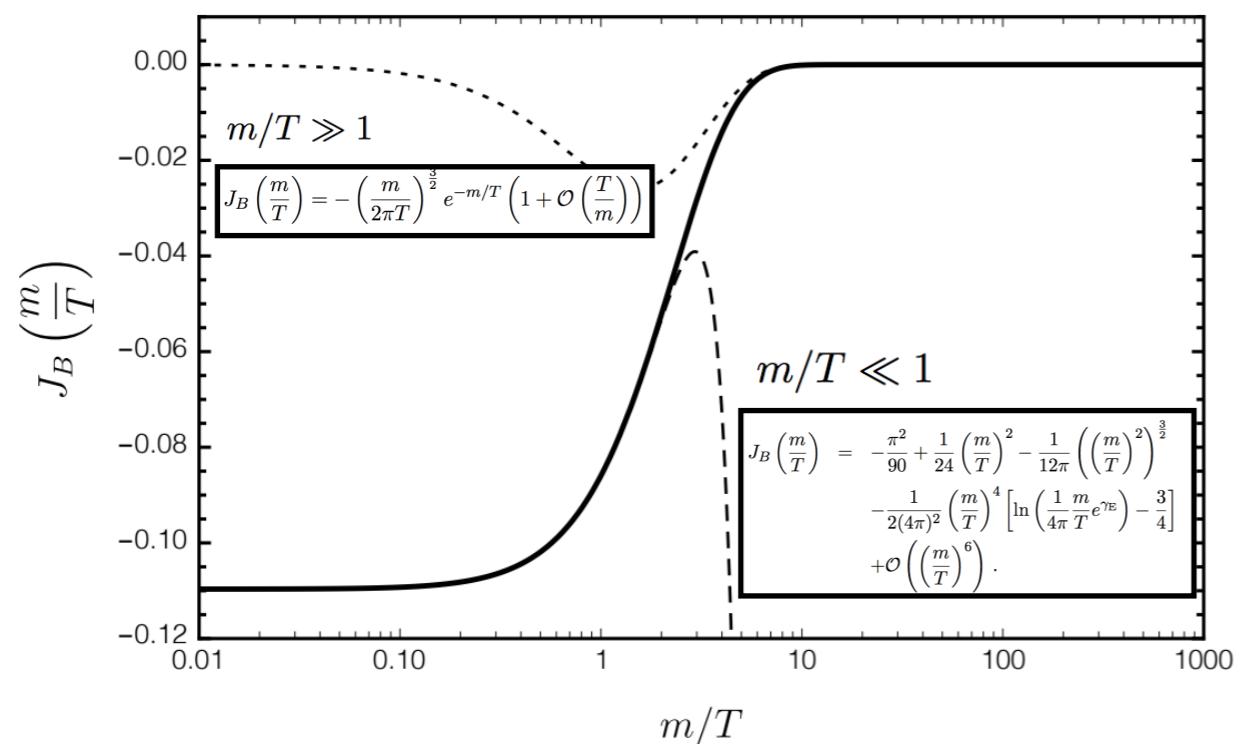
Phys. Rev. D47 (1993) 3546 [hep-ph/9212235]

See also Parwani method in Phys. Rev. D45 (1992) 4695 [hep-ph/9204216]

# Thermal effective scalar potential for PT study

$$V_T(\phi, T) = V_0(\phi) + T^4 \left[ \sum_B J_B \left( \frac{M_B}{T} \right) + \sum_F J_F \left( \frac{M_F}{T} \right) \right]$$

all fermions **F** and bosons **B** that are relativistic at temperature **T**



**High-T expansion**

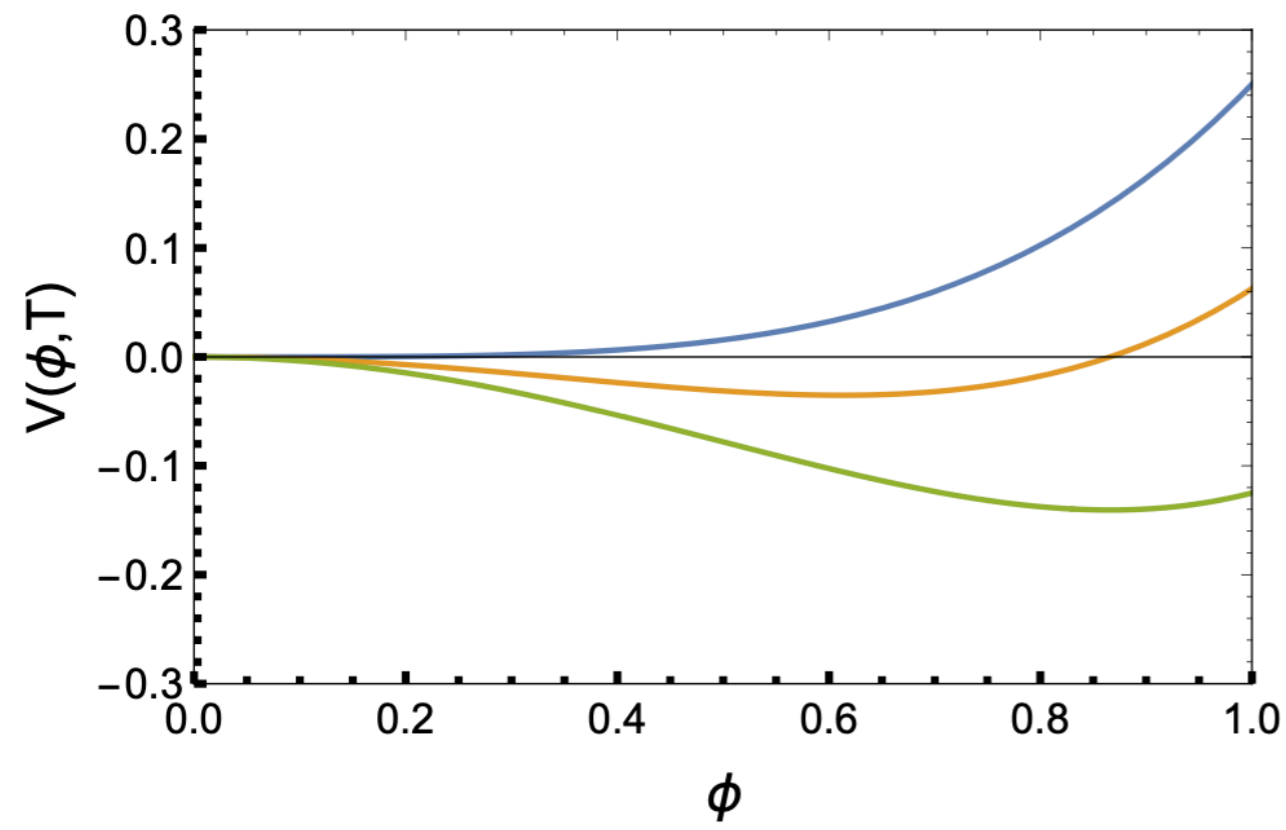
$$m/T \ll 1$$

$$V_T(\phi) = V_0(\phi) + \frac{T^2}{24} \left( \sum_S M_S^2(\phi) + 3 \sum_V M_V^2(\phi) + 2 \sum_F M_F^2(\phi) \right) - \frac{T}{12\pi} \left( \sum_S \left( M_S^2(\phi) \right)^{\frac{3}{2}} + \sum_V \left( M_V^2(\phi) \right)^{\frac{3}{2}} \right) + \text{higher order terms.}$$

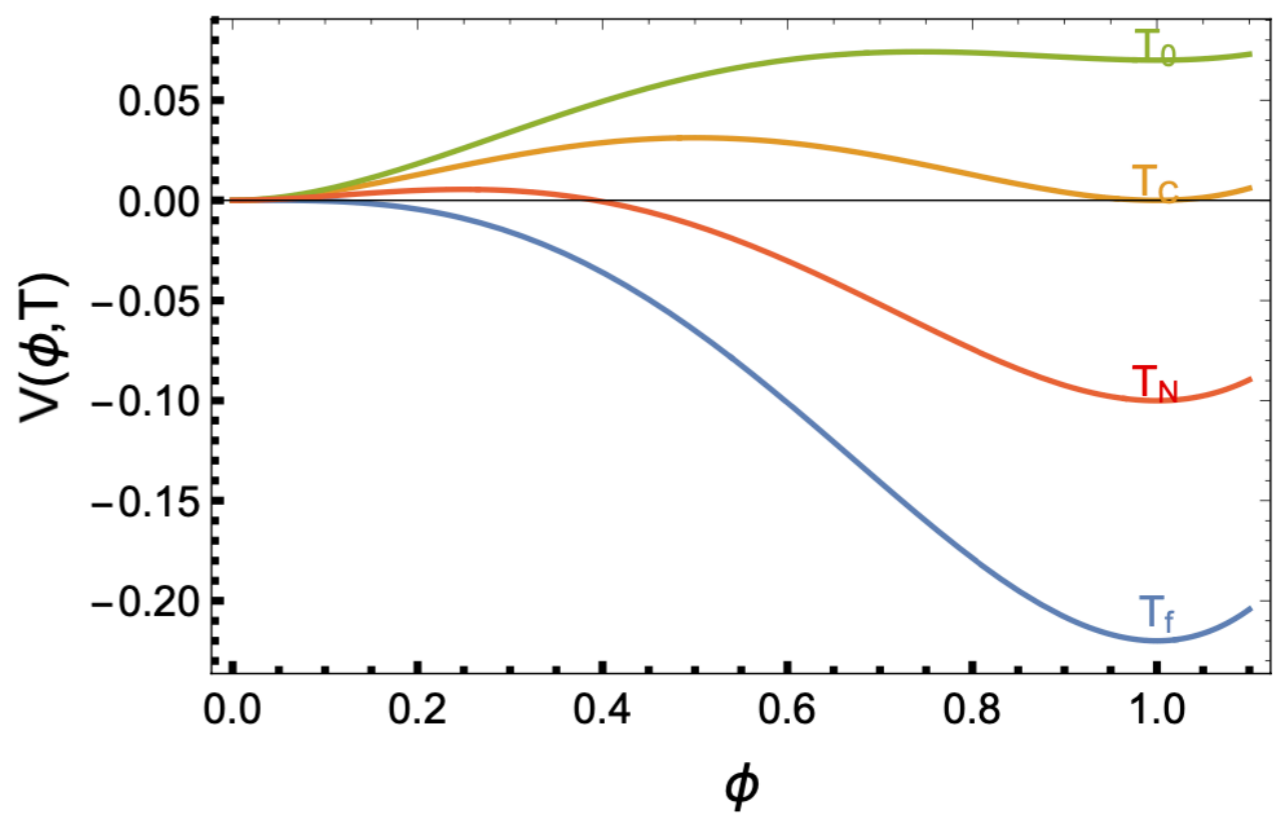
**MS, MV, MF are the masses of the scalar fields S, vector fields V and fermionic fields F**

# Phase transition types

## Second order

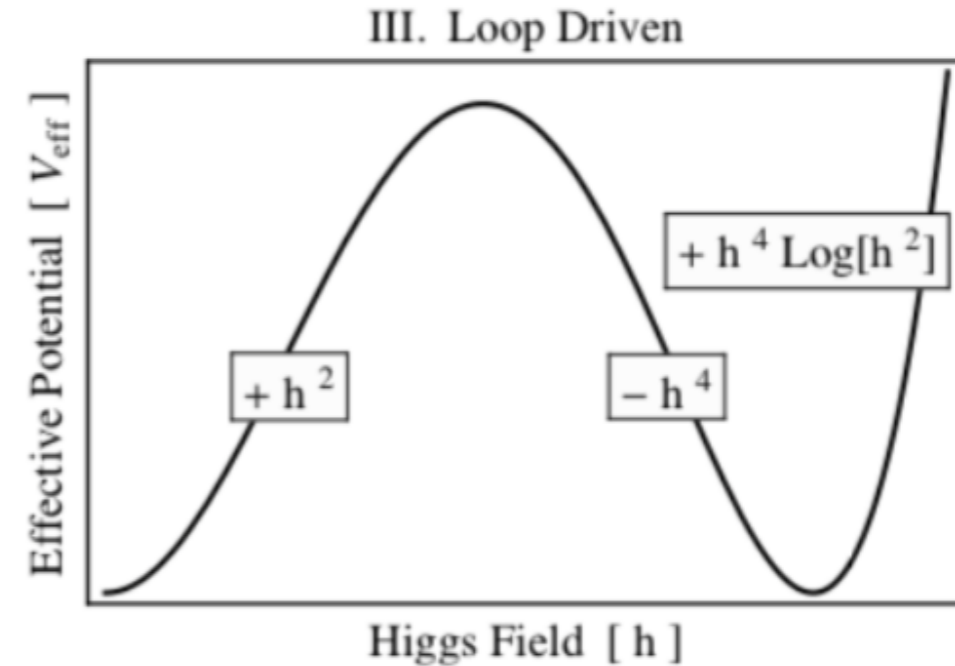
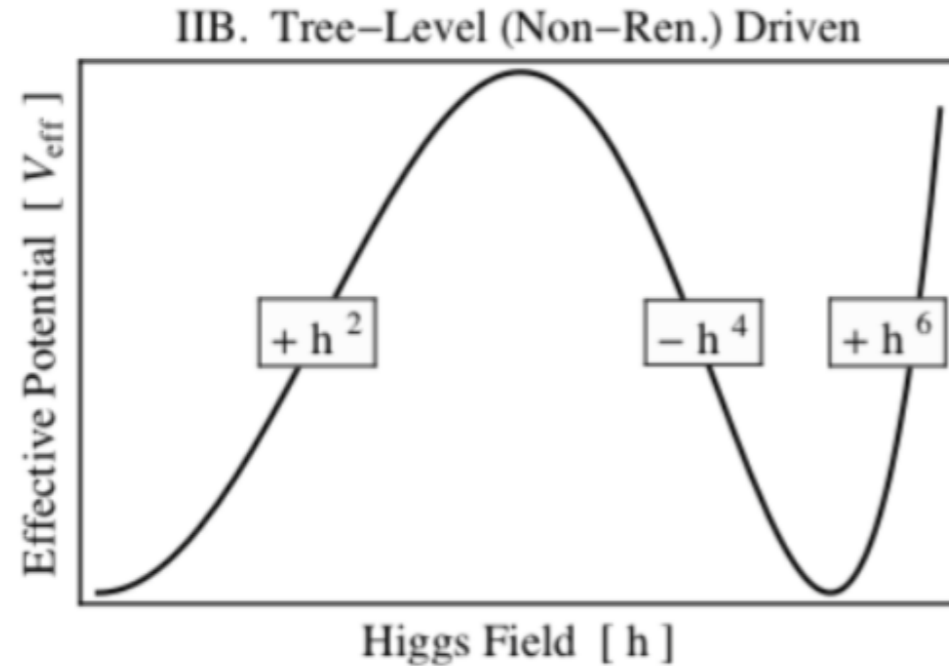
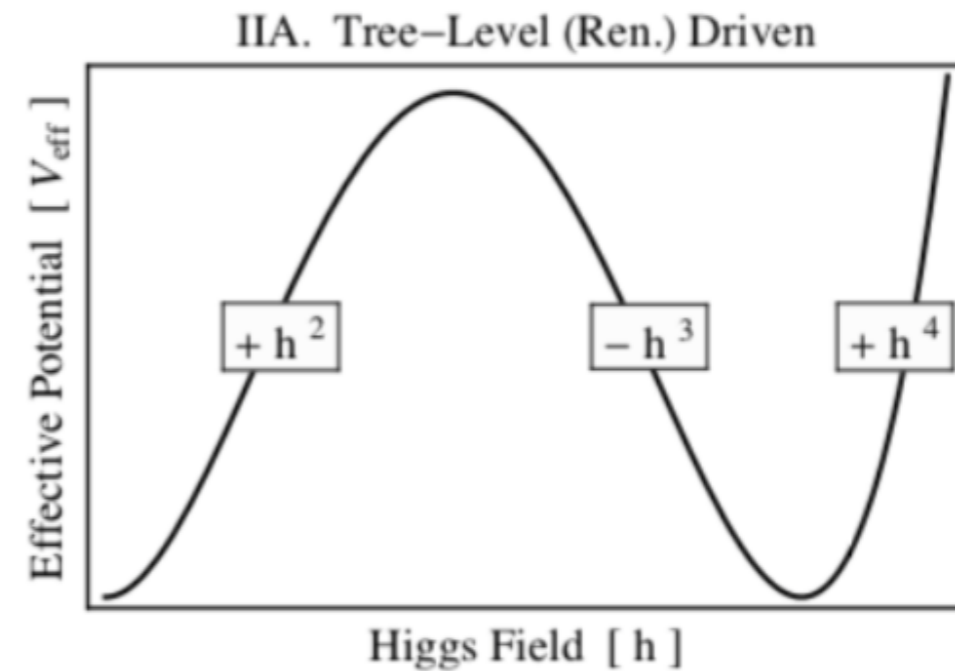
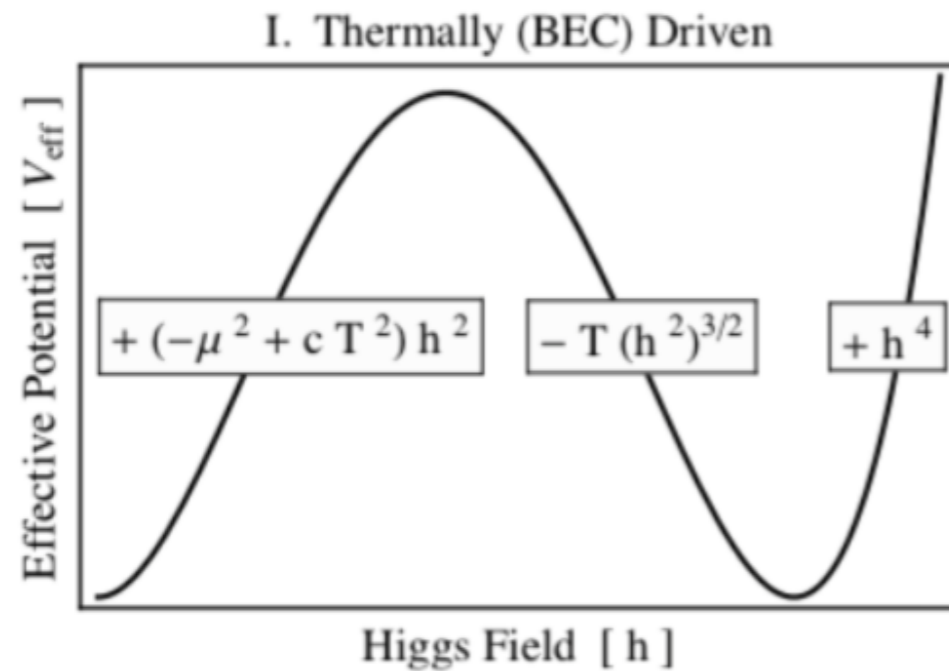


## First order





# Model classes for one-step FOPT



# ► Thermal driven Class-I

$$V_{\text{eff}}(h, T) \approx \frac{1}{2}(-\mu^2 + cT^2)h^2 - \frac{eT}{12\pi}(h^2)^{3/2} + \frac{\lambda}{4}h^4$$

$$e \sim \sum_{\text{light bosonic fields}} (\text{degrees of freedom}) \times (\text{coupling to Higgs})^{3/2}.$$

$$\frac{v(T_c)}{T_c} \approx \frac{e}{6\pi\lambda}$$

TABLE I. Examples of models in the Thermally (BEC) Driven class. The expressions for  $e$  are calculated in the limit that the field-independent contributions to  $m_{\text{eff}}^2(h, T)$  are negligible (e.g., the thermal mass tuning has been performed). Here, the symbol  $\tilde{A}_t$  is  $\tilde{A}_t = A_t - \mu/\tan\beta$  and  $g_s$  is the number of real scalar singlet degrees of freedom coupling to the Higgs.

Model	$-\Delta\mathcal{L}$	$c$	$e$
SM [43]		$c_{\text{SM}} = \frac{6m_t^2 + 6m_W^2 + 3m_Z^2 + \frac{3}{2}m_H^2}{12v^2}$	$e_{\text{SM}} = \frac{6m_W^3 + 3m_Z^3}{v^3}$
MSSM [41]		$c_{\text{SM}} + \frac{6m_t^2}{12v^2} \left(1 - \frac{\tilde{A}_t^2}{m_Q^2}\right)$	$e_{\text{SM}} + \frac{6m_t^3}{v^3} \left(1 - \frac{\tilde{A}_t^2}{m_Q^2}\right)^{3/2}$
Colored scalar [20]	$M_X^2 X ^2 + \frac{K}{6} X ^4 + Q H ^2 X ^2$	$c_{\text{SM}} + \frac{6}{24} \frac{Q}{2}$	$e_{\text{SM}} + 6\left(\frac{Q}{2}\right)^{3/2}$
Singlet scalar [43,44]	$M^2 S ^2 + \lambda_S S ^4 + 2\zeta^2 H ^2 S ^2$	$c_{\text{SM}} + \frac{g_S}{24} \zeta^2$	$e_{\text{SM}} + g_S \zeta^3$
Singlet Majoron [45]	$\mu_s^2 S ^2 + \lambda_s S ^4 + \lambda_{hs} H ^2 S ^2 + \frac{1}{2}y_i S \nu_i \nu_i + \text{H.c.}$	$c_{\text{SM}} + \frac{2}{24} \frac{\lambda_{hs}}{2}$	$e_{\text{SM}} + 2\left(\frac{\lambda_{hs}}{2}\right)^{3/2}$
Two-Higgs doublets [46]	$\mu_D^2 D^\dagger D + \lambda_D (D^\dagger D)^2 + \lambda_3 H^\dagger H D^\dagger D + \lambda_4  H^\dagger D ^2 + (\lambda_5/2)[(H^\dagger D)^2 + \text{H.c.}]$	$c_{\text{SM}} + \frac{2\lambda_3 + \lambda_4}{12}$	$e_{\text{SM}} + 2\left(\frac{\lambda_3}{2}\right)^{3/2} + \left(\frac{\lambda_3 + \lambda_4 - \lambda_5}{2}\right)^{3/2} + \left(\frac{\lambda_3 + \lambda_4 + \lambda_5}{2}\right)^{3/2}$

# ► Tree driven-Class IIA

$$V_{\text{eff}}(\varphi, T) \approx \frac{1}{2}(m^2 + cT^2)\varphi^2 - \mathcal{E}\varphi^3 + \frac{\lambda}{4}\varphi^4.$$

$$T_c \approx \sqrt{\frac{m^2}{c}} \sqrt{\frac{2\mathcal{E}^2}{\lambda m^2} - 1},$$

$$\frac{v(T_c)}{T_c} \approx \sqrt{\frac{2c}{\lambda}} \frac{1}{\sqrt{1 - \frac{\lambda m^2}{2\mathcal{E}^2}}} \cos\alpha.$$

TABLE II. Examples of models that fall into Class IIA. For the non-SUSY models, corrections to the SM Lagrangian are shown, whereas for the SUSY models only the superpotential corrections are given.

Model	$\Delta \mathcal{L}$
xSM [53–56]	$\frac{1}{2}(\partial S)^2 - [\frac{b_2}{2}S^2 + \frac{b_3}{3}S^3 + \frac{b_4}{4}S^4 + \frac{a_1}{2}H^\dagger HS^2 + \frac{a_2}{2}H^\dagger HS^2]$
$\mathbb{Z}_2$ xSM [14,57]	$\frac{1}{2}(\partial S)^2 - [\frac{b_2}{2}S^2 + \frac{b_4}{4}S^4 + \frac{a_2}{2}H^\dagger HS^2]$
Two-Higgs doublets [58]	$\mu_D^2 D ^2 + \lambda_D D ^4 + \lambda_3 H ^2 D ^2 + \lambda_4 H^\dagger D ^2 + (\lambda_5/2)[(H^\dagger D)^2 + \text{H.c.}]$
Model	$\Delta W$
NMSSM [59–61]	$\lambda H_1 H_2 N - \frac{\kappa}{3} N^3 + rN$
nMSSM [62]	$\lambda H_1 H_2 S + \frac{m_{12}^2}{\lambda} S$
$\mu\nu$ MSSM [63]	$-\lambda_i H_1 H_2 \nu_i^c + \frac{\kappa_{ijk}}{3} \nu_i^c \nu_j^c \nu_k^c + Y_\nu^{ij} H_2 L_i \nu_j^c$



# Class IIA (1) no extra EWSB: xSM

For the “xSM” model, the gauge invariant finite temperature effective potential is found to be:

$$V(h, s, T) = -\frac{1}{2}[\mu^2 - \Pi_h(T)]h^2 - \frac{1}{2}[-b_2 - \Pi_s(T)]s^2 + \frac{1}{4}\lambda h^4 + \frac{1}{4}a_1 h^2 s + \frac{1}{4}a_2 h^2 s^2 + \frac{b_3}{3}s^3 + \frac{b_4}{4}s^4, \quad (\text{C1})$$

with the thermal masses given by

$$\Pi_h(T) = \left( \frac{2m_W^2 + m_Z^2 + 2m_t^2}{4v^2} + \frac{\lambda}{2} + \frac{a_2}{24} \right) T^2, \quad (\text{C2})$$

$$\Pi_s(T) = \left( \frac{a_2}{6} + \frac{b_4}{4} \right) T^2,$$

## PT strength

$$v^{\text{xSM}}/T \equiv \frac{v_h(T)}{T} = \frac{\sqrt{v_h^2(T) + v_s^2(T)} \cos \theta(T)}{T},$$

$$\cos \theta(T) \equiv \frac{v_h(T)}{\sqrt{v_h^2(T) + v_s^2(T)}},$$

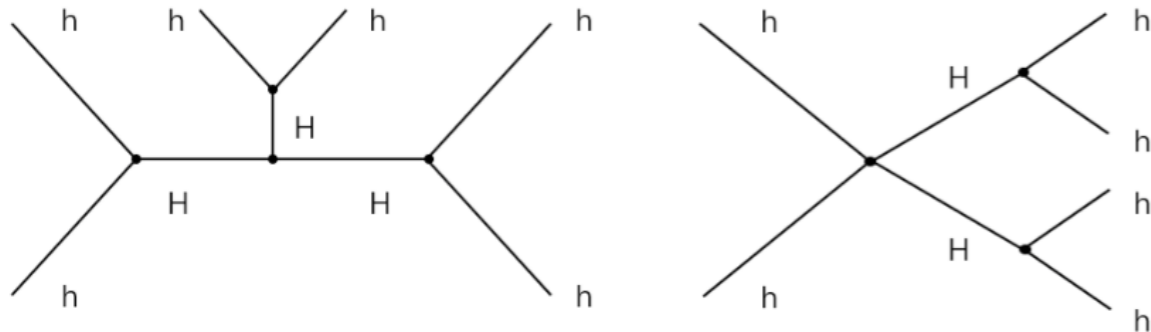
□

For small mixing limit between the extra Higgs and the SM Higgs, one have

$$c_4^{\text{xSM}} = -\frac{a_1^2 - 8b_2\lambda}{32b_2} + \frac{\theta^2(a_1^2(6b_2 - \mu^2) - 8a_1b_2b_3 + 8b_2^2(a_2 - 2\lambda))}{32b_2^2} + O(\theta^3)$$

$$c_6^{\text{xSM}} = -\frac{a_1^2(a_1b_3 - 3a_2b_2)}{192b_2^3} - \frac{\theta^2 a_1}{256b_2^4} (a_1^3b_2 + 4a_1^2b_3(\mu^2 - 3b_2) + 4a_1b_2(a_2(11b_2 - 2\mu^2) - 6b_2(b_4 + \lambda) + 4b_3^2) - 32a_2b_2^2b_3) + O(\theta^3)$$

$$c_8^{\text{xSM}} = \frac{a_1^4b_4}{1024b_2^4} + \frac{a_1^3\theta^2}{1024b_2^5} (a_1(a_2b_2 + 4b_4(\mu^2 - 3b_2)) + 16b_2b_3b_4) + O(\theta^3)$$





# Class IIA (1) with extra EWSB: **GM model**

The most general scalar potential  $V(\Phi, \Delta)$  invariant under  $SU(2)_L \times SU(2)_R \times U(1)_Y$  is given by

$$\begin{aligned}
 V(\Phi, \Delta) = & \frac{1}{2}m_1^2 \text{tr}[\Phi^\dagger \Phi] + \frac{1}{2}m_2^2 \text{tr}[\Delta^\dagger \Delta] + \lambda_1 \left( \text{tr}[\Phi^\dagger \Phi] \right)^2 + \lambda_2 \left( \text{tr}[\Delta^\dagger \Delta] \right)^2 + \lambda_3 \text{tr} \left[ \left( \Delta^\dagger \Delta \right)^2 \right] + \lambda_4 \text{tr}[\Phi^\dagger \Phi] \text{tr}[\Delta^\dagger \Delta] \\
 & + \lambda_5 \text{tr} \left[ \Phi^\dagger \frac{\sigma^a}{2} \Phi \frac{\sigma^b}{2} \right] \text{tr}[\Delta^\dagger T^a \Delta T^b] \\
 & + \mu_1 \text{tr} \left[ \Phi^\dagger \frac{\sigma^a}{2} \Phi \frac{\sigma^b}{2} \right] (P^\dagger \Delta P)_{ab} + \mu_2 \text{tr}[\Delta^\dagger T^a \Delta T^b] (P^\dagger \Delta P)_{ab}, \quad (3)
 \end{aligned}$$

extra EWSB

**Custodial symmetry**

$v_\chi = \sqrt{2}v_\xi$

$v_\phi^2 + 8v_\xi^2 \equiv v^2 \approx (246 \text{ GeV})^2$

$$\Phi \equiv (\varepsilon_2 \phi^*, \phi) = \begin{pmatrix} \phi^{0*} & \phi^+ \\ -\phi^{+*} & \phi^0 \end{pmatrix}, \quad \Delta \equiv (\varepsilon_3 \chi^*, \xi, \chi) = \begin{pmatrix} \chi^{0*} & \xi^+ & \chi^{++} \\ -\chi^{+*} & \xi^0 & \chi^+ \\ \chi^{++*} & -\xi^{+*} & \chi^0 \end{pmatrix}, \quad (1)$$

with

$$\varepsilon_2 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad \varepsilon_3 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & -1 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \quad (2)$$

where the phase convention for the scalar field components is:  $\chi^{--} = \chi^{++*}$ ,  $\chi^- = \chi^{+*}$ ,  $\xi^- = \xi^{+*}$ ,  $\phi^- = \phi^{+*}$ .  $\Phi$  and  $\Delta$  are transformed under  $SU(2)_L \times SU(2)_R$  as  $\Phi \rightarrow U_{2,L} \Phi U_{2,R}^\dagger$  and  $\Delta \rightarrow U_{3,L} \Delta U_{3,R}^\dagger$  with  $U_{L,R} = \exp(i\theta_{L,R}^a T^a)$  and  $T^a$  being the  $SU(2)$  generators.

where summations over  $a, b = 1, 2, 3$  are understood,  $\sigma$ 's and  $T$ 's are the  $2 \times 2$  (Pauli matrices)  $3 \times 3$  matrix representations of the  $SU(2)$  generators, respectively

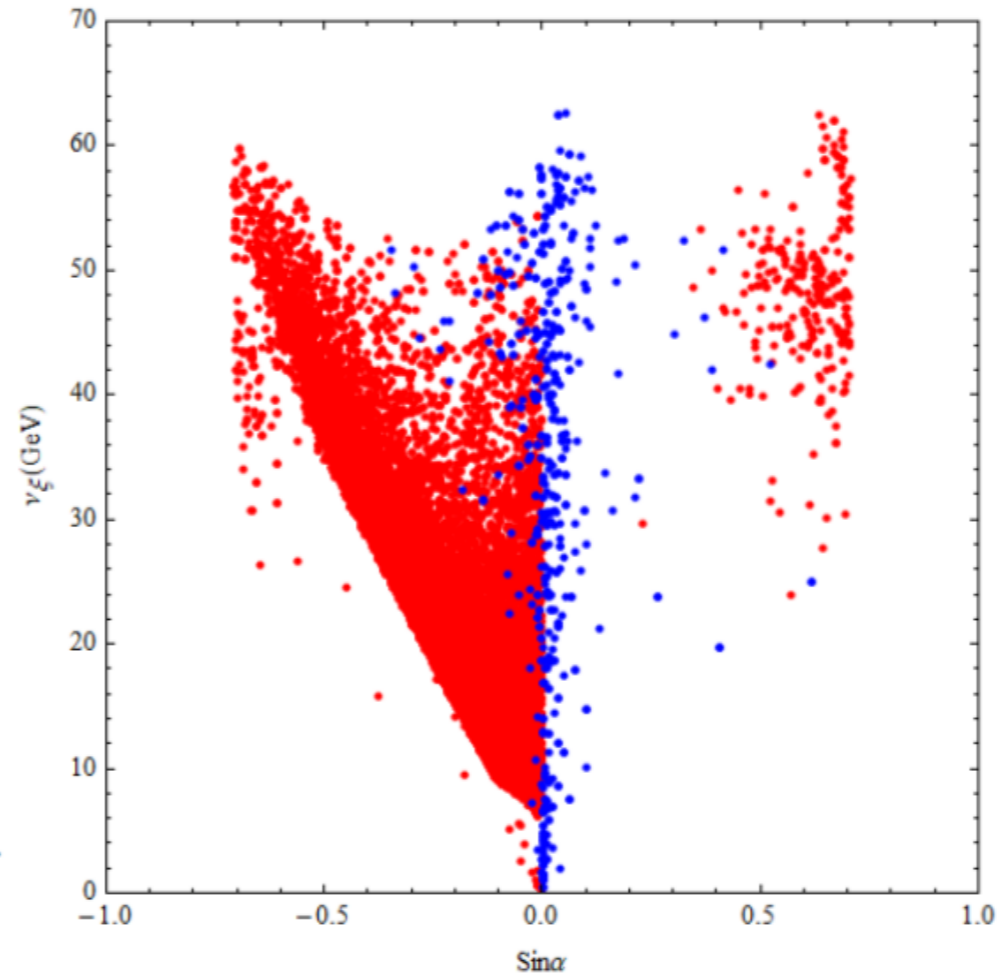
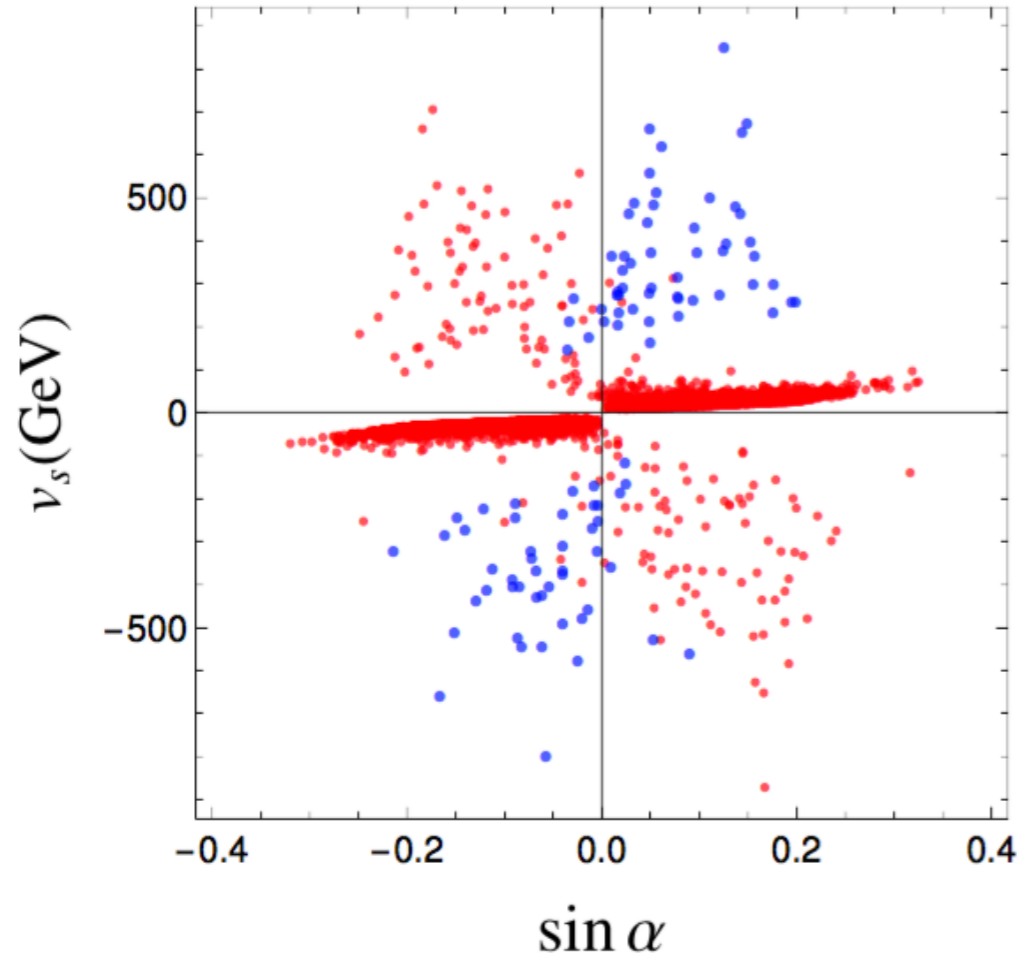
$$T_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad T_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \quad T_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix},$$

The  $P$  matrix, which is the similarity transformation relating the generators in the triplet and adjoint representations, is given by

$$P = \frac{1}{\sqrt{2}} \begin{pmatrix} -1 & i & 0 \\ 0 & 0 & \sqrt{2} \\ 1 & i & 0 \end{pmatrix}.$$

xSM: without extra EWSB

GM: with extra EWSB



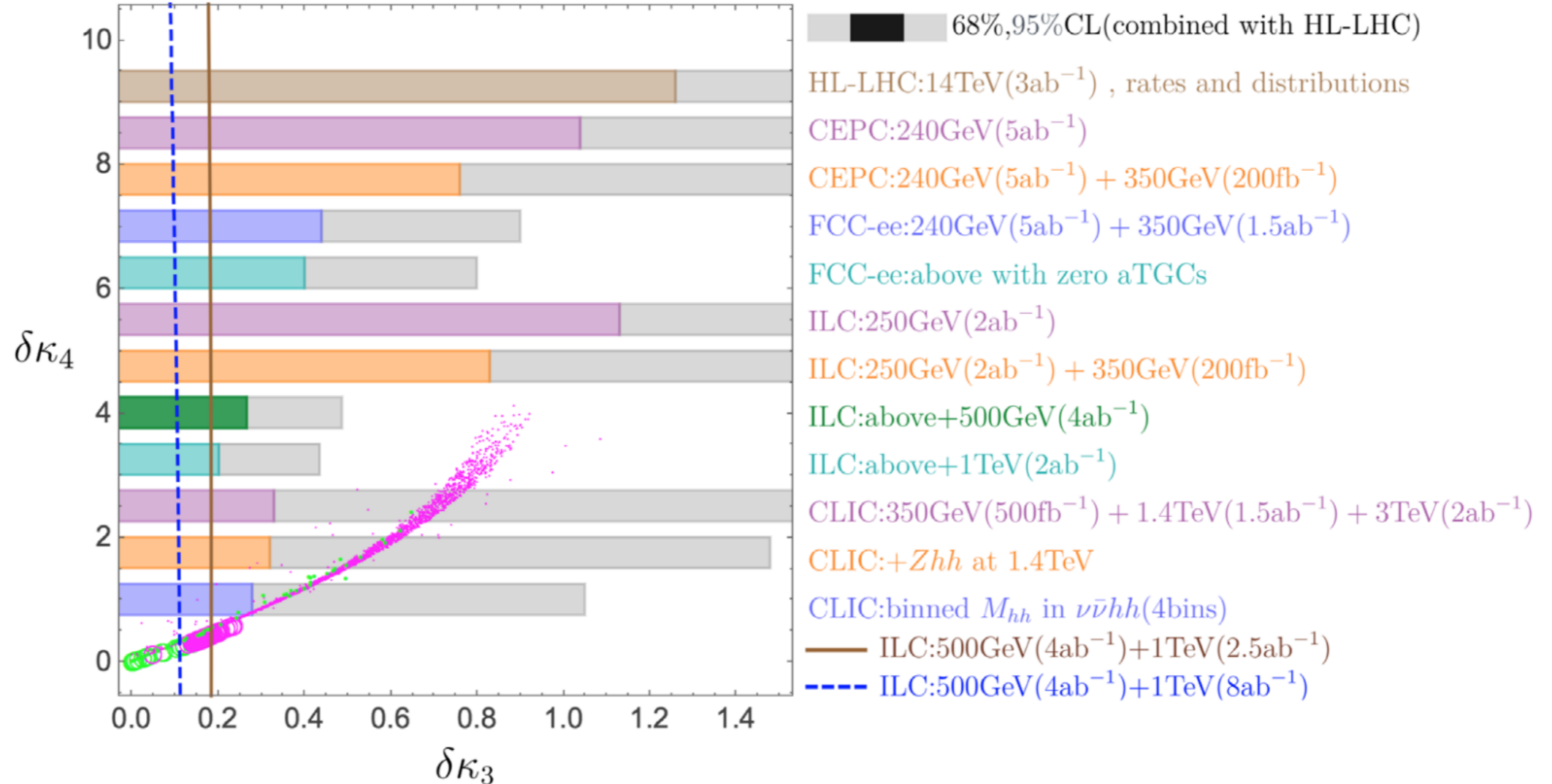
$$g_{hxx} = \cos \alpha g_{hxx}^{SM}$$

$$g_{hf\bar{f}} = \cos \alpha / \cos \theta_H g_{hf\bar{f}}^{SM}, \quad g_{hVV} = (\cos \alpha \cos \theta_H - \sqrt{\frac{8}{3}} \sin \alpha \sin \theta_H) g_{hf\bar{f}}^{SM},$$

$$g_{Hf\bar{f}} = \sin \alpha / \cos \theta_H g_{hf\bar{f}}^{SM}, \quad g_{HVV} = (\sin \alpha \cos \theta_H + \sqrt{\frac{8}{3}} \cos \alpha \sin \theta_H) g_{hVV}^{SM}.$$

# Collider & GW complementary search

SNR > 10 points for **two-step** and **one-step** SFOEWPT



Circles and the dotted points for the GM and xSM scenarios

$$\delta\kappa_3^{\text{xSM}} = \alpha_H^2 \left[ -\frac{3}{2} + \frac{2m_H^2 - 2b_3 v_s - 4b_4 v_s^2}{m_h^2} \right] + \mathcal{O}(\alpha_H^3),$$

$$\delta\kappa_4^{\text{xSM}} = \alpha_H^2 \left[ -3 + \frac{5m_H^2 - 4b_3 v_s - 8b_4 v_s^2}{m_h^2} \right] + \mathcal{O}(\alpha_H^3).$$

$$\delta\kappa_3^{\text{GM}} = -\alpha_H \frac{\sqrt{3}\mu_1 v}{2m_h^2} + \frac{\alpha_H v^2 (4\alpha_H - \sqrt{6}\theta_H)(2\lambda_4 + \lambda_5)}{2m_h^2} - \frac{(3\alpha_H^2 + \theta_H^2)}{2} + \mathcal{O}(\alpha_H^3, \theta_H^3),$$

$$\delta\kappa_4^{\text{GM}} = -2\alpha_H^2 \left( 1 - \frac{2(2\lambda_4 + \lambda_5)v^2}{m_h^2} \right) + \mathcal{O}(\alpha_H^3).$$

# ▶ Tree-level driven-Class II B

< 0 causes the potential to turn over

$$V_{\text{eff}}(h, T) \approx \frac{1}{2}(\mu^2 + cT^2)h^2 + \frac{\lambda}{4}h^4 + \frac{1}{8\Lambda^2}h^6$$

stabilizes the EW-broken vacuum

$$\lambda = \frac{m_H^2}{2v^2} \left(1 - \frac{\Lambda_{\text{max}}^2}{\Lambda^2}\right), \quad \Lambda_{\text{max}} \equiv \sqrt{3}v^2/m_H, \quad T_c = \sqrt{\frac{\mu^2}{c}} \sqrt{\frac{\lambda^2 \Lambda^2}{4\mu^2} - 1},$$

$$\mu^2 = \frac{m_H^2}{2} \left(\frac{\Lambda_{\text{max}}^2}{2\Lambda^2} - 1\right), \quad \Lambda < \Lambda_{\text{max}}, \quad \frac{v(T_c)}{T_c} = \sqrt{-\lambda} \frac{2}{\sqrt{1 - \frac{4\mu^2}{\lambda^2 \Lambda^2}}}$$

$$\lambda_{HHH} \equiv \frac{m_H^2}{v} \left(1 + 2\frac{\Lambda_{\text{min}}^2}{\Lambda^2}\right), \quad \Lambda_{\text{min}} = v^2/m_H$$

Model	Couplings	Wilson coefficient of $H^6$
$\mathbb{R}$ Singlet	$-\frac{1}{2}\lambda_{HS} H ^2 S^2 - g_{HS}H^\dagger HS$	$-\frac{\lambda_{HS}}{2} \frac{g_{HS}^2}{M^4}$
$\mathbb{C}$ Singlet	$-g_{HS} H ^2\Phi - \frac{\lambda_{H\Phi}}{2} H ^2\Phi^2 - \frac{\lambda'_{H\Phi}}{2}H^\dagger H \Phi ^2 + h.c.$	$-\frac{ g_{HS} ^2 \lambda'_{H\Phi}}{2M^4} - \frac{\text{Re}[g_{HS}^2 \lambda_{H\Phi}]}{M^4}$
2HDM	$-Z_6 H_1 ^2 H_1^\dagger H_2 - Z_6^* H_1 ^2 H_2^\dagger H_1$	$\frac{ Z_6 ^2}{M^2}$
$\mathbb{R}$ triplet	$gH^\dagger \tau^a H \Phi^a - \frac{\lambda_{H\Phi}}{2} H ^2 \Phi^a ^2$	$-\frac{g^2}{M^4} \left(\frac{\lambda_{H\Phi}}{8} - \lambda\right)$
$\mathbb{C}$ triplet	$gH^T i\sigma_2 \tau^a H \Phi^a - \frac{\lambda_{H\Phi}}{2} H ^2 \Phi^a ^2$ $-\frac{\lambda'}{4}H^\dagger \tau^a \tau^b H \Phi^a (\Phi^b)^\dagger + h.c.$	$-\frac{g^2}{M^4} \left(\frac{\lambda_{H\Phi}}{4} + \frac{\lambda'}{8} - 2\lambda\right)$
$\mathbb{C}$ 4-plet	$-\lambda_{H3\Phi} H_i^* H_j^* H_k^* \Phi^{ijk} + h.c.$	$\frac{ \lambda_{H3\Phi} ^2}{M^2}$



Class IIB

Dim. six operator, SMEFT

**Higgs potential**

$$V(H) = -m^2(H^\dagger H) + \lambda (H^\dagger H)^2 + \frac{(H^\dagger H)^3}{\Lambda^2}$$

**Finite temperature potential**

$$V_T(h, T) = V(h) + \frac{1}{2}c_{hT}h^2$$

**Thermal correction**

$$c_{hT} = (4y_t^2 + 3g^2 + g'^2 + 8\lambda)T^2/16$$

**Electroweak minimum  
being the global one**

$$\Lambda \geq v^2/m_h$$

**Potential barrier requirement**

$$\Lambda < \sqrt{3}v^2/m_h$$

# ► Loop driven-Class III

$$V_{\text{eff}}(h, T) \approx \frac{1}{2}(\mu^2 + cT^2)h^2 + \frac{\lambda}{4}h^4 + \frac{\kappa}{4}h^4 \ln \frac{h^2}{M^2}$$

$$\lambda = \frac{m_H^2}{2v^2} - \kappa \left( \ln \frac{v^2}{M^2} + \frac{3}{2} \right),$$

$$T_c \approx \frac{m_H}{2\sqrt{c}} \sqrt{\epsilon} \left( 1 + \frac{1}{8}\epsilon + \frac{37}{384}\epsilon^2 + \dots \right),$$

$$\epsilon = 1 - \kappa v^2 / m_H^2$$

$$\mu^2 = -\frac{m_H^2}{2} + \kappa v^2.$$

$$\frac{v(T_c)}{T_c} \approx \frac{2v\sqrt{c}}{m_H} \frac{1}{\sqrt{\epsilon}} \left( 1 - \frac{3}{8}\epsilon - \frac{103}{384}\epsilon^2 + \dots \right).$$

TABLE III. Examples of models in the Loop Driven class.

Model	$-\Delta \mathcal{L}$
Singlet scalars [12,72]	$\sum_i^N M^2  S_i ^2 + \lambda_S  S_i ^4 + 2\zeta^2  H ^2  S_i ^2$
Singlet Majoron [73,74]	$\mu_s^2  S ^2 + \lambda_s  S ^4 + \lambda_{hs}  H ^2  S ^2 + \frac{1}{2} y_i S \nu_i \nu_i + \text{H.c.}$
Two-Higgs doublets [75–78]	$\mu_D^2 D^\dagger D + \lambda_D (D^\dagger D)^2 + \lambda_3 H^\dagger H D^\dagger D + \lambda_4  H^\dagger D ^2 + (\lambda_5/2)[(H^\dagger D)^2 + \text{H.c.}]$

$$V(h_1, h_2, T) = V_0(h_1, h_2) + V_{\text{CW}}(h_1, h_2) + V_{\text{CT}}(h_1, h_2) + V_{\text{th}}(h_1, h_2, T) + V_{\text{daisy}}(h_1, h_2, T)$$

Tree-level

$$V_0(h_1, h_2) = \frac{1}{2} m_{12}^2 t_\beta (h_1 - h_2 t_\beta^{-1})^2 - \frac{v^2}{4} \frac{\lambda_1 h_1^2 + \lambda_2 h_2^2 t_\beta^2}{1 + t_\beta^2} - \frac{v^2}{4} \frac{\lambda_{345} (h_1^2 t_\beta^2 + h_2^2)}{1 + t_\beta^2} + \frac{1}{8} \lambda_1 h_1^4 + \frac{1}{8} \lambda_2 h_2^4 + \frac{1}{4} \lambda_{345} h_1^2 h_2^2$$

One-loop at zero temperature:

$$V_{\text{CW}}(h_1, h_2) = \sum_i (-1)^{2s_i} n_i \frac{\hat{m}_i^4(h_1, h_2)}{64\pi^2} \left[ \ln \left( \frac{\hat{m}_i^2(h_1, h_2)}{Q^2} \right) - C_i \right] \quad [\text{Coleman, Weinberg '73}]$$

One-loop at finite temperature:

$$V_{\text{th}}(h_1, h_2, T) = \frac{T^4}{2\pi^2} \sum_i n_i J_{B,F} \left( \frac{m_i^2(h_1, h_2)}{T^2} \right) \quad [\text{Dolan, Jackiw '74}]$$

$$J_{B,F}(y) = \mp \sum_{l=1}^{\infty} \frac{(\pm 1)^l y}{l^2} K_2(\sqrt{y}l) \quad [\text{Anderson, Halle '92}]$$

$$V_{\text{daisy}}(h_1, h_2, T) = -\frac{T}{12\pi} \sum_i n_i \left[ (M_i^2(h_1, h_2, T))^{\frac{3}{2}} - (m_i^2(h_1, h_2))^{\frac{3}{2}} \right]$$

[Carrington '92; Arnold, Espinosa '93; Delaunay, Grojean, Wells '07]

重要的引力波源，主要科学目标之一

PTA, LIGO, LISA, 天琴, 太极, ...

超出粒子物理模型  
新物理模型

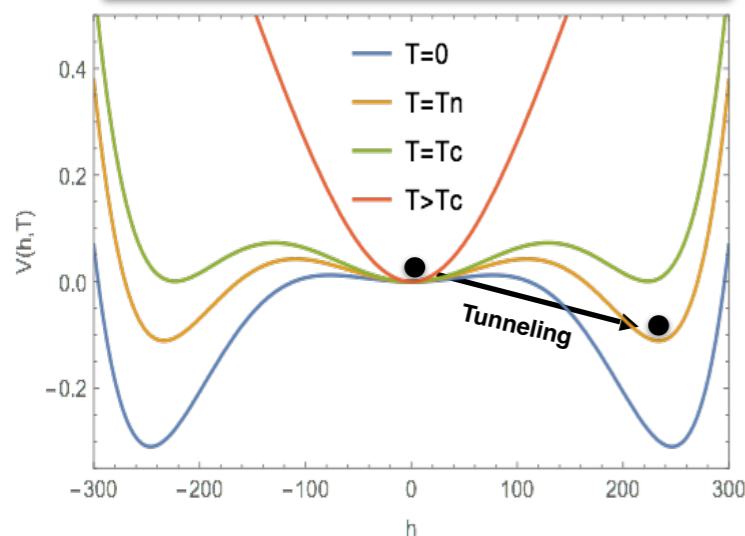
PT parameters

Effective action  $\rightarrow \beta, H_*$

Energy budget  $\rightarrow \alpha, \kappa(\alpha, v_w)$

Bubble wall dynamics  $\rightarrow v_w$

有限温场论计算



GW power spectrum

Numerical simulations  $\rightarrow$   
 $h^2 \Omega_{\text{GW}}(f; H_*, \alpha, \beta, v_w)$

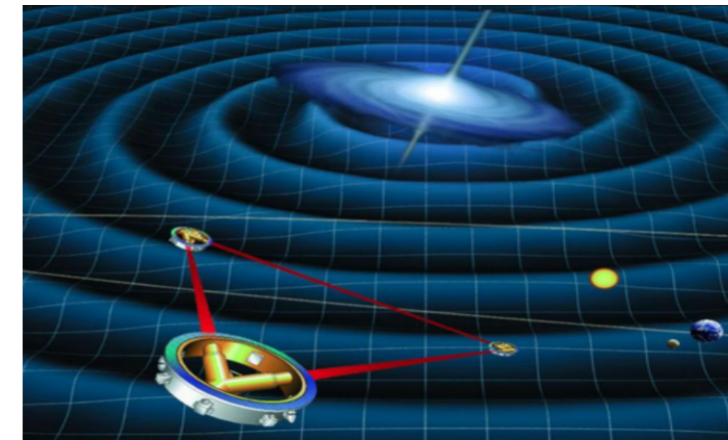
格点场论模拟

LISA sensitivity

Configuration + noise level  $\rightarrow$   
 $h^2 \Omega_{\text{sens}}(f)$

Signal-to-noise ratio

**SNR**





**Bounce solution**

$$S_3(T) = \int 4\pi r^2 dr \left[ \frac{1}{2} \left( \frac{d\phi_b}{dr} \right)^2 + V(\phi_b, T) \right]$$

$$\lim_{r \rightarrow \infty} \phi_b = 0, \quad \left. \frac{d\phi_b}{dr} \right|_{r=0} = 0$$

**Bubble nucleation**

$$\Gamma \approx A(T) e^{-S_3/T} \sim 1$$

**PT strength**

$$\alpha \equiv \frac{1}{\rho_r} \left( \Delta V_{\text{eff}}(\phi, T) - \frac{T}{4} \Delta \frac{\partial V_{\text{eff}}(\phi, T)}{\partial T} \right)$$

**Phase transition  
inverse duration**

$$\frac{\beta}{H_n} = T \left. \frac{d(S_3(T)/T)}{dT} \right|_{T=T_n}$$

# GW parameters and FOPT

The probability, that a randomly chosen point is still in the false vacuum, given by

$$P(t) = e^{-I(t)} \quad I(t) = \frac{4\pi}{3} \int_{t_c}^t dt' \Gamma(t') a(t')^3 r(t, t')^3$$

The fraction of the space which has already been converted to the broken phase

$$r(t, t') = \int_{t'}^t \frac{v_w(\tilde{t}) d\tilde{t}}{a(\tilde{t})}$$

$r(t, t')$  : the comoving radius of a bubble nucleated at  $t'$  propagated until a subsequent time  $t$

$a(t)$ : the scale factor,  $v_w(t)$ : the wall velocity.

Using temperature  $T$  instead of time variable  $t$ , we have

$$I(T) = \frac{4\pi}{3} \int_T^{T_c} \frac{dT'}{H(T')} \Gamma(T') \frac{r(T, T')^3}{T'^4}$$

The transition completes when  $P(t) \approx 0.7$ , which leads to a percolation temperature  $T_p$  when

$$I(T_p) = 0.34.$$

- Bubble collisions**

$$\Omega_{\text{col}} h^2 = 1.67 \times 10^{-5} \left( \frac{H_*}{\beta} \right)^2 \left( \frac{\kappa \alpha}{1 + \alpha} \right)^2 \left( \frac{100}{g_*} \right)^{1/3} \left( \frac{0.11 v_b^3}{0.42 + v_b^2} \right) \frac{3.8 (f/f_{\text{env}})^{2.8}}{1 + 2.8 (f/f_{\text{env}})^{3.8}}$$

peak frequency:  $f_{\text{env}} = 16.5 \times 10^{-6} \left( \frac{f_*}{H_*} \right) \left( \frac{T_*}{100 \text{ GeV}} \right) \left( \frac{g_*}{100} \right)^{1/6} \text{ Hz}$

- Sound Wave**

$$\Omega h_{\text{sw}}^2(f) = 2.65 \times 10^{-6} (H_* \tau_{\text{sw}}) \left( \frac{\beta}{H} \right)^{-1} v_b \left( \frac{\kappa_\nu \alpha}{1 + \alpha} \right)^2 \left( \frac{g_*}{100} \right)^{-1/3} \left( \frac{f}{f_{\text{sw}}} \right)^3 \left( \frac{7}{4 + 3 (f/f_{\text{sw}})^2} \right)^{7/2}$$

phase transition duration:  $\tau_{\text{sw}} = \min \left[ \frac{1}{H_*}, \frac{R_*}{U_f} \right], H_* R_* = v_b (8\pi)^{1/3} (\beta/H)^{-1}$

Root-mean-square four-velocity of the plasma:

$$\bar{U}_f^2 \approx \frac{3}{4} \frac{\kappa_\nu \alpha}{1 + \alpha}$$

peak frequency:  $f_{\text{sw}} = 1.9 \times 10^{-5} \frac{\beta}{H} \frac{1}{v_b} \frac{T_*}{100} \left( \frac{g_*}{100} \right)^{1/6} \text{ Hz}$

- MHD turbulence**

$$\Omega h_{\text{turb}}^2(f) = 3.35 \times 10^{-4} \left( \frac{\beta}{H} \right)^{-1} \left( \frac{\epsilon \kappa_\nu \alpha}{1 + \alpha} \right)^{3/2} \left( \frac{g_*}{100} \right)^{-1/3} v_b \frac{(f/f_{\text{turb}})^3 (1 + f/f_{\text{turb}})^{-11/3}}{[1 + 8\pi f a_0 / (a_* H_*)]}$$

peak frequency:  $f_{\text{turb}} = 2.7 \times 10^{-5} \frac{\beta}{H} \frac{1}{v_b} \frac{T_*}{100} \left( \frac{g_*}{100} \right)^{1/6} \text{ Hz}$

# GW sources

$$\Omega_{\text{GW}}(f) = \begin{cases} \Omega_{\text{GW}*} \left(\frac{f}{f_*}\right)^{n_{\text{GW}1}} & \text{for } f < f_*, \\ \Omega_{\text{GW}*} \left(\frac{f}{f_*}\right)^{n_{\text{GW}2}} & \text{for } f > f_*, \end{cases}$$

**Table 1.** Cosmological GW sources

source	$n_{\text{GW}1}$	$n_{\text{GW}2}$	$f_*$ [Hz]	$\Omega_{\text{GW}}$
Phase transition (bubble collision)	2.8	-2	$\sim 10^{-5} \left(\frac{f_{\text{PT}}}{\beta}\right) \left(\frac{\beta}{H_{\text{PT}}}\right) \left(\frac{T_{\text{PT}}}{100 \text{ GeV}}\right)$	$\sim 10^{-5} \left(\frac{H_{\text{PT}}}{\beta}\right)^2 \left(\frac{\kappa_\phi \alpha}{1+\alpha}\right)^2 \left(\frac{0.11 v_w^3}{0.42 + v_w^2}\right)$
Phase transition (turbulence)	3	-5/3	$\sim 3 \times 10^{-5} \left(\frac{1}{v_w}\right) \left(\frac{\beta}{H_{\text{PT}}}\right) \left(\frac{T_{\text{PT}}}{100 \text{ GeV}}\right)$	$\sim 3 \times 10^{-4} \left(\frac{H_{\text{PT}}}{\beta}\right) \left(\frac{\kappa_{\text{turb}} \alpha}{1+\alpha}\right)^{3/2} v_w$
Phase transition (sound waves)	3	-4	$\sim 2 \times 10^{-5} \left(\frac{1}{v_w}\right) \left(\frac{\beta}{H_{\text{PT}}}\right) \left(\frac{T_{\text{PT}}}{100 \text{ GeV}}\right)$	$\sim 3 \times 10^{-6} \left(\frac{H_{\text{PT}}}{\beta}\right) \left(\frac{\kappa_v \alpha}{1+\alpha}\right)^2 v_w$
Preheating ( $\lambda\phi^4$ )	3	cutoff	$\sim 10^7$	$\sim 10^{-11} \left(\frac{g^2/\lambda}{100}\right)^{-0.5}$
Preheating (hybrid)	2	cutoff	$\sim \frac{g}{\sqrt{\lambda}} \lambda^{1/4} 10^{10.25}$	$\sim 10^{-5} \left(\frac{\lambda}{g^2}\right)^{1.16} \left(\frac{v}{M_{\text{pl}}}\right)^2$
Cosmic strings (loops 1)	[1, 2]	[-1, -0.1]	$\sim 3 \times 10^{-8} \left(\frac{G\mu}{10^{-11}}\right)^{-1}$	$\sim 10^{-9} \left(\frac{G\mu}{10^{-12}}\right) \left(\frac{\alpha_{\text{loop}}}{10^{-1}}\right)^{-1/2}$ (for $\alpha_{\text{loop}} \gg \Gamma G\mu$ )
Cosmic strings (loops 2)	[-1, -0.1]	0	$\sim 3 \times 10^{-8} \left(\frac{G\mu}{10^{-11}}\right)^{-1}$	$\sim 10^{-9.5} \left(\frac{G\mu}{10^{-12}}\right) \left(\frac{\alpha_{\text{loop}}}{10^{-1}}\right)^{-1/2}$ (for $\alpha_{\text{loop}} \gg \Gamma G\mu$ )
Cosmic strings (infinite strings)	[0, 0.2]	[0, 0.2]	—	$\sim 10^{-[11,13]} \left(\frac{G\mu}{10^{-8}}\right)$
Domain walls	3	-1	$\sim 10^{-9} \left(\frac{T_{\text{ann}}}{10^{-2} \text{ GeV}}\right)$	$\sim 10^{-17} \left(\frac{\sigma}{1 \text{ TeV}^3}\right)^2 \left(\frac{T_{\text{ann}}}{10^{-2} \text{ GeV}}\right)^{-4}$
Self-ordering scalar fields	0	0	—	$\sim \frac{511}{N} \Omega_{\text{rad}} \left(\frac{v}{M_{\text{pl}}}\right)^4$
Self-ordering scalar + reheating	0	-2	$\sim 0.4 \left(\frac{T_R}{10^7 \text{ GeV}}\right)$	$\sim \frac{511}{N} \Omega_{\text{rad}} \left(\frac{v}{M_{\text{pl}}}\right)^4$
Magnetic fields	3	$\alpha_B + 1$	$\sim 10^{-6} \left(\frac{T_*}{10^2 \text{ GeV}}\right)$	$\sim 10^{-16} \left(\frac{B}{10^{-10} \text{ G}}\right)$
Inflation+reheating	$\sim 0$	-2	$\sim 0.3 \left(\frac{T_R}{10^7 \text{ GeV}}\right)$	$\sim 2 \times 10^{-17} \left(\frac{r}{0.01}\right)$
Inflation+kination	$\sim 0$	1	$\sim 0.3 \left(\frac{T_R}{10^7 \text{ GeV}}\right)$	$\sim 2 \times 10^{-17} \left(\frac{r}{0.01}\right)$
Particle prod. during inf.	$-2\epsilon$	$-4\epsilon(4\pi\xi - 6)(\epsilon - \eta)$	—	$\sim 2 \times 10^{-17} \left(\frac{r}{0.01}\right)$
2nd-order (inflation)	1	drop-off	$\sim 7 \times 10^5 \left(\frac{T_{\text{reh}}}{10^9 \text{ GeV}}\right)^{1/3} \left(\frac{M_{\text{inf}}}{10^{16} \text{ GeV}}\right)^{2/3}$	$\sim 10^{-12} \left(\frac{T_{\text{reh}}}{10^9 \text{ GeV}}\right)^{-4/3} \left(\frac{M_{\text{inf}}}{10^{16} \text{ GeV}}\right)^{4/3}$
2nd-order (PBHs)	2	drop-off	$\sim 4 \times 10^{-2} \left(\frac{M_{\text{PBH}}}{10^{20} \text{ g}}\right)^{-1/2}$	$\sim 7 \times 10^{-9} \left(\frac{\mathcal{A}^2}{10^{-3}}\right)^2$
Pre-Big-Bang	3	$3 - 2\mu$	—	$\sim 1.4 \times 10^{-6} \left(\frac{H_s}{0.15 M_{\text{pl}}}\right)^4$



# ► One-step FOPT

PHYSICAL REVIEW LETTERS **126**, 251102 (2021)

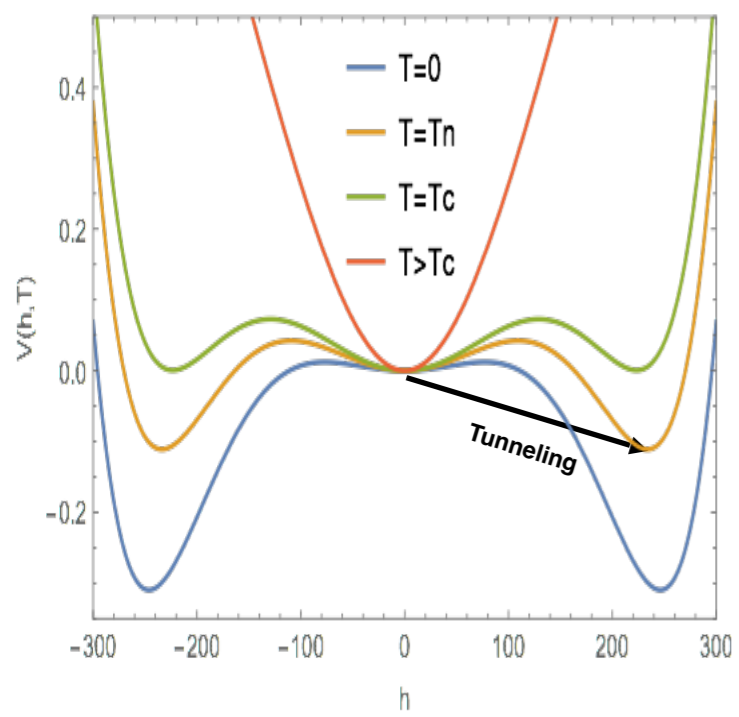
## Magnetic Field and Gravitational Waves from the First-Order Phase Transition

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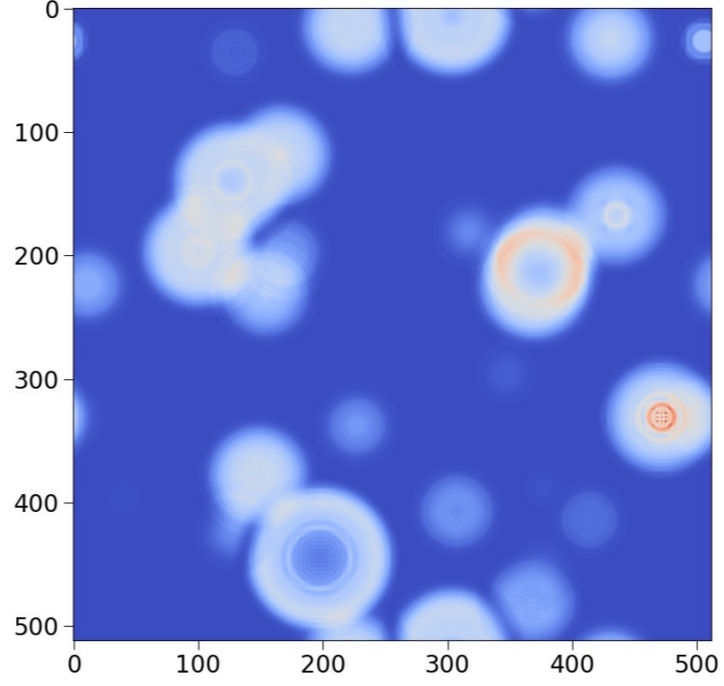
Jing Liu<sup>‡</sup>  
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### Finite-T Veff



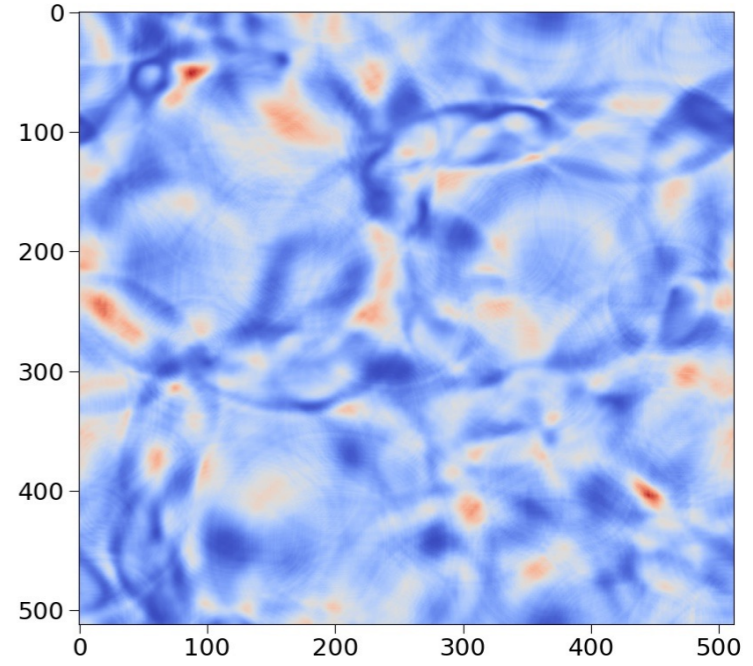
**Finite-T calculation**

### Nucleation



**Lattice Simulation**

### Expansion & Percolation



# Lattice EW field foundation

$\Phi(t, \mathbf{x})$  : Higgs field doublet defined on sites;

$U_i(t, \mathbf{x})$  and  $V_i(t, \mathbf{x})$  : SU(2) and U(1) link fields, defined on the link between the neighboring sites  $\mathbf{x}$  and  $\mathbf{x} + \mathbf{i}$ ,  $\Phi(t, \mathbf{x})$ ,  $U_i(t, \mathbf{x})$  and  $V_i(t, \mathbf{x})$  are defined at time steps  $t + \Delta t, t + 2\Delta t, \dots$ ;

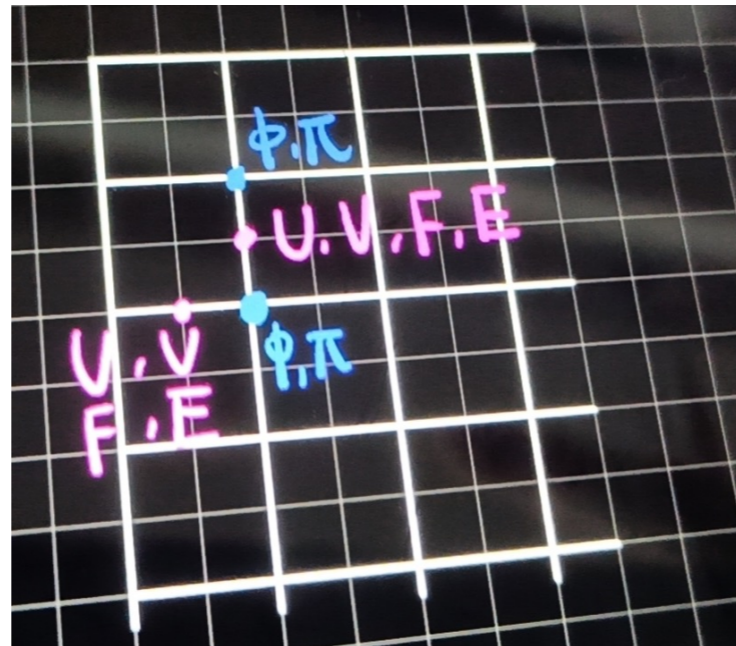
Conjugate momentum fields:  $\Pi(t+\Delta t/2, \mathbf{x})$ ,  $F(t+\Delta t/2, \mathbf{x})$  and  $E(t+\Delta t/2, \mathbf{x})$ , are defined at time steps  $t + \Delta t/2, t + 3\Delta t/2$ .

$$U_i(t, x) = \exp\left(-\frac{i}{2}g\Delta x\sigma^a W_i^a\right)$$

$$U_0(t, x) = \exp\left(-\frac{i}{2}g\Delta t\sigma^a W_0^a\right)$$

$$V_i(t, x) = \exp\left(-\frac{i}{2}g\Delta x B_i\right)$$

$$V_0(t, x) = \exp\left(-\frac{i}{2}g\Delta t B_0\right).$$



$$D_i\Phi = \frac{1}{\Delta x} [U_i(t, x)V_i(t, x)\Phi(t, x + i) - \Phi(t, x)]$$

$$D_0\Phi = \frac{1}{\Delta t} [U_0(t, x)V_0(t, x)\Phi(t + \Delta t, x) - \Phi(t, x)].$$

$$\Phi(t + \Delta t, x) = \Phi(t, x) + \Delta t\Pi(t + \Delta t/2, x)$$

$$V_i(t + \Delta t, x) = \frac{1}{2}g'\Delta x\Delta t E_i(t + \Delta t/2, x)V_i(t, x)$$

$$U_i(t + \Delta t, x) = g\Delta x\Delta t F_i(t + \Delta t/2, x)U_i(t, x),$$

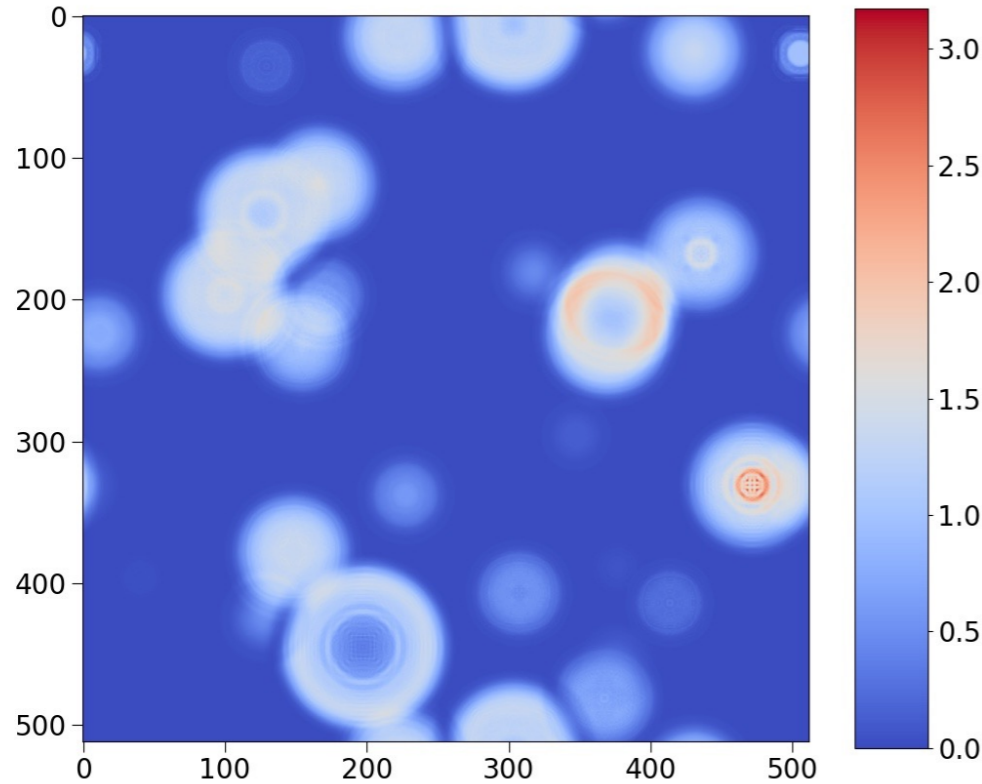
Temporal gauge  
 $U_0(t, \mathbf{x}) = \mathbf{I}_2, V_0(t, \mathbf{x}) = 1$

leapfrog



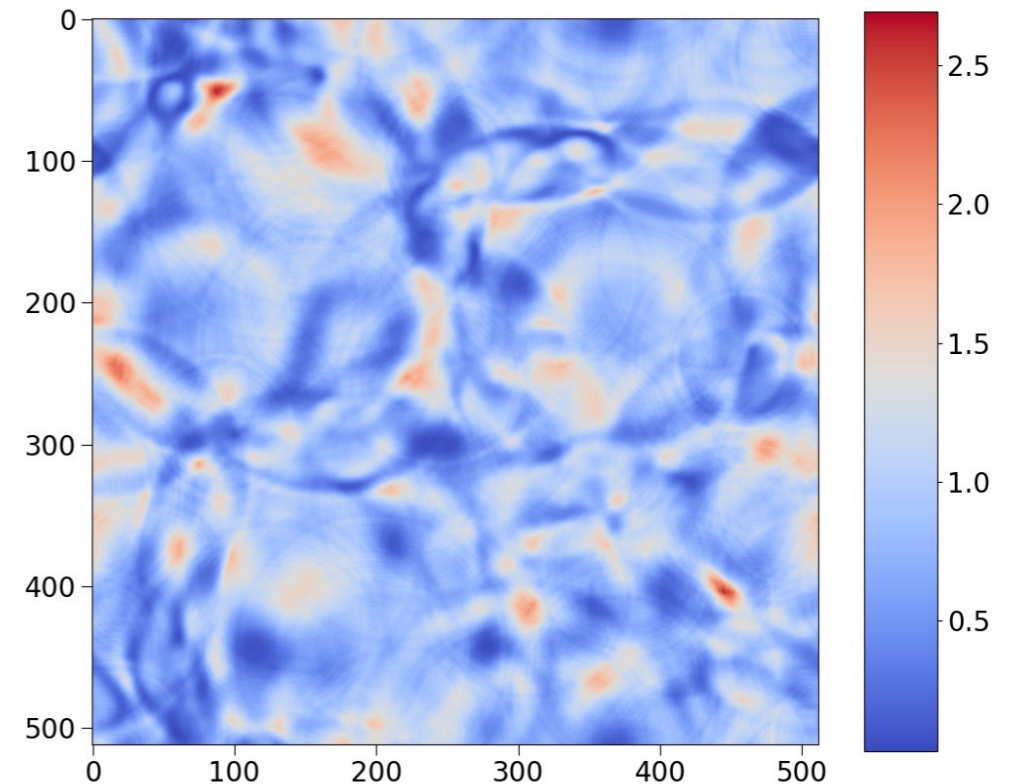
## Field basis

$$\begin{aligned}\partial_0^2 \Phi &= D_i D_i \Phi - 2\lambda(|\Phi|^2 - \eta^2)\Phi - 3(\Phi^\dagger \Phi)^2 \Phi / \Lambda^2, \\ \partial_0^2 B_i &= -\partial_j B_{ij} + g' \text{Im}[\Phi^\dagger D_i \Phi], \\ \partial_0^2 W_i^a &= -\partial_k W_{ik}^a - g \epsilon^{abc} W_k^b W_{ik}^c + g \text{Im}[\Phi^\dagger \sigma^a D_i \Phi], \\ \partial_0 \partial_j B_j - g' \text{Im}[\Phi^\dagger \partial_0 \Phi] &= 0, \\ \partial_0 \partial_j W_j^a + g \epsilon^{abc} W_j^b \partial_0 W_j^c - g \text{Im}[\Phi^\dagger \sigma^a \partial_0 \Phi] &= 0.\end{aligned}$$



## Lattice implementation

$$\begin{aligned}\Pi(t + \Delta t/2, x) &= \Pi(t - \Delta t/2, x) + \Delta t \left\{ \frac{1}{\Delta x^2} \sum_i [U_i(t, x) V_i(t, x) \Phi(t, x + i) \right. \\ &\quad \left. - 2\Phi(t, x) + U_i^\dagger(t, x - i) V_i^\dagger(t, x - i) \Phi(t, x - i)] - \frac{\partial U}{\partial \Phi^\dagger} \right\} \\ \text{Im}[E_k(t + \Delta t/2, x)] &= \text{Im}[E_k(t - \Delta t/2, x)] + \Delta t \left\{ \frac{g'}{\Delta x} \text{Im}[\Phi^\dagger(t, x + k) U_k^\dagger(t, x) V_k^\dagger(t, x) \Phi(t, x)] \right. \\ &\quad \left. - \frac{2}{g' \Delta x^3} \sum_i \text{Im}[V_k(t, x) V_i(t, x + k) V_k^\dagger(t, x + i) V_i^\dagger(t, x) \right. \\ &\quad \left. + V_i(t, x - i) V_k(t, x) V_i^\dagger(t, x + k - i) V_k^\dagger(t, x - i)] \right\} \\ \text{Tr}[i\sigma^m F_k(t + \Delta t/2, x)] &= \text{Tr}[i\sigma^m F_k(t - \Delta t/2, x)] + \Delta t \left\{ \frac{g}{\Delta x} \text{Re}[\Phi^\dagger(t, x + k) U_k^\dagger(t, x) V_k^\dagger(t, x) i\sigma^m \Phi(t, x)] \right. \\ &\quad \left. - \frac{1}{g \Delta x^3} \sum_i \text{Tr}[i\sigma^m U_k(t, x) U_i(t, x + k) U_k^\dagger(t, x + i) U_i^\dagger(t, x) \right. \\ &\quad \left. + i\sigma^m U_k(t, x) U_i^\dagger(t, x + k - i) U_k^\dagger(t, x - i) U_i(t, x - i)] \right\},\end{aligned}$$



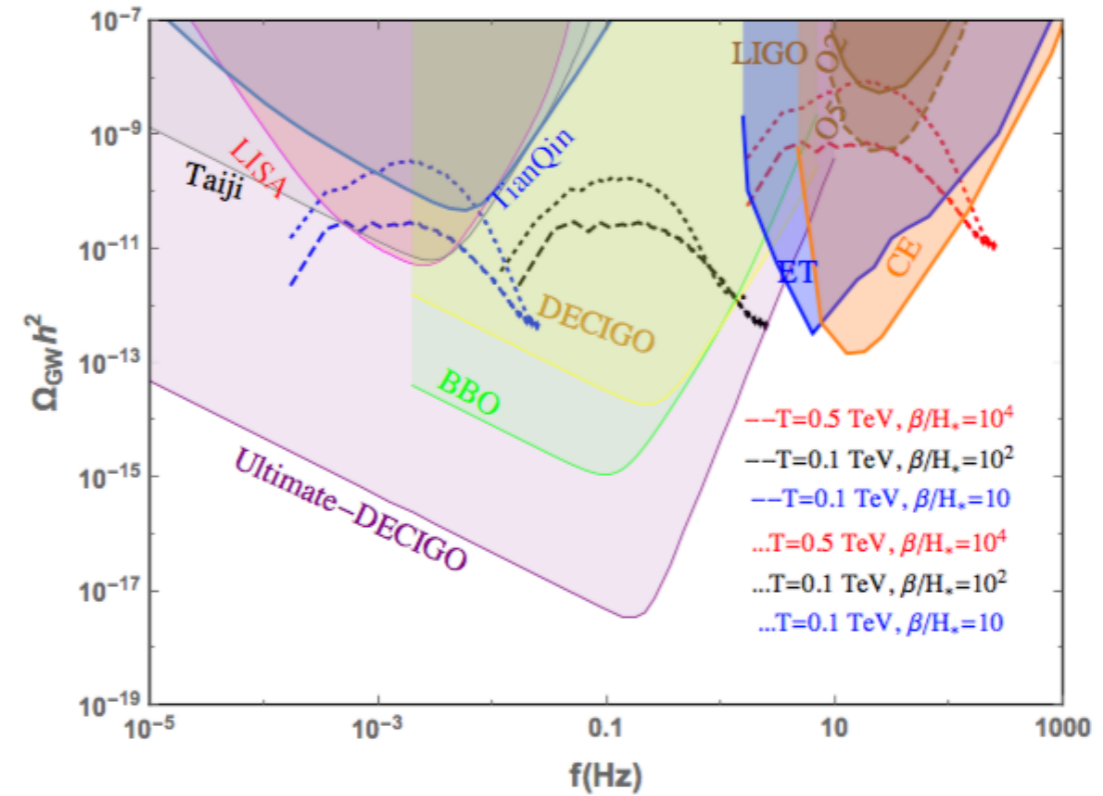
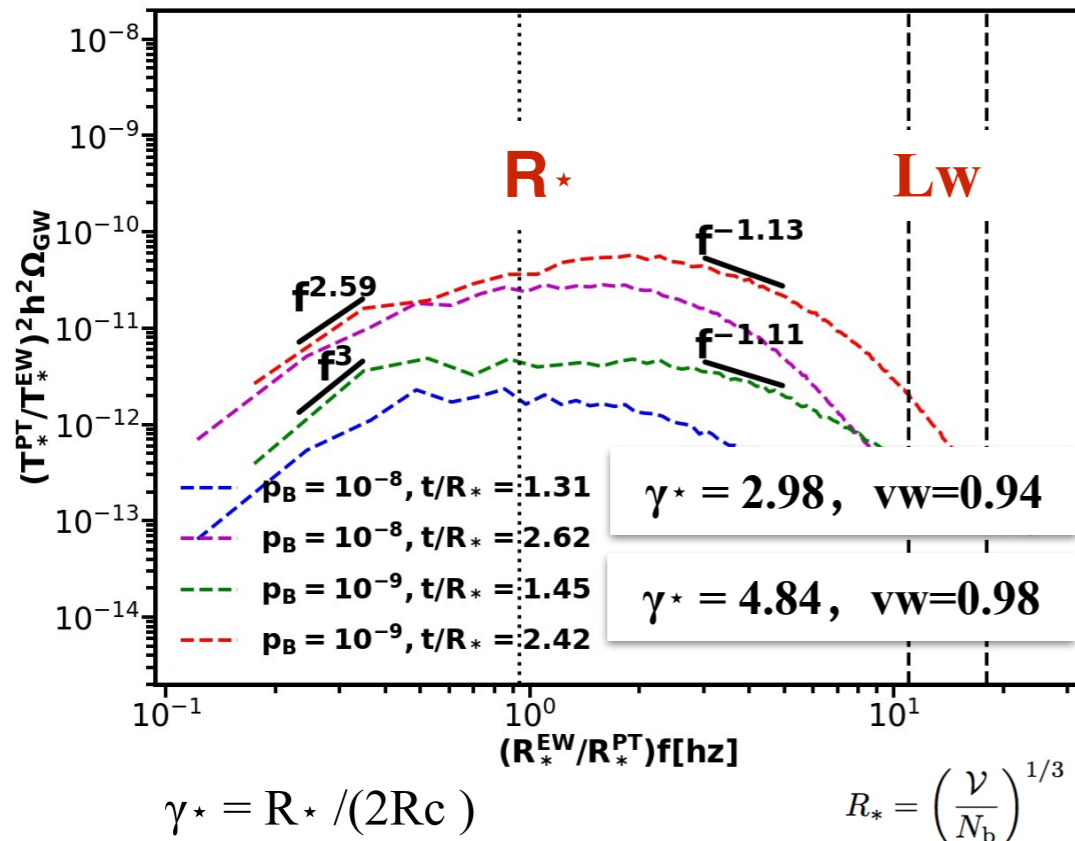
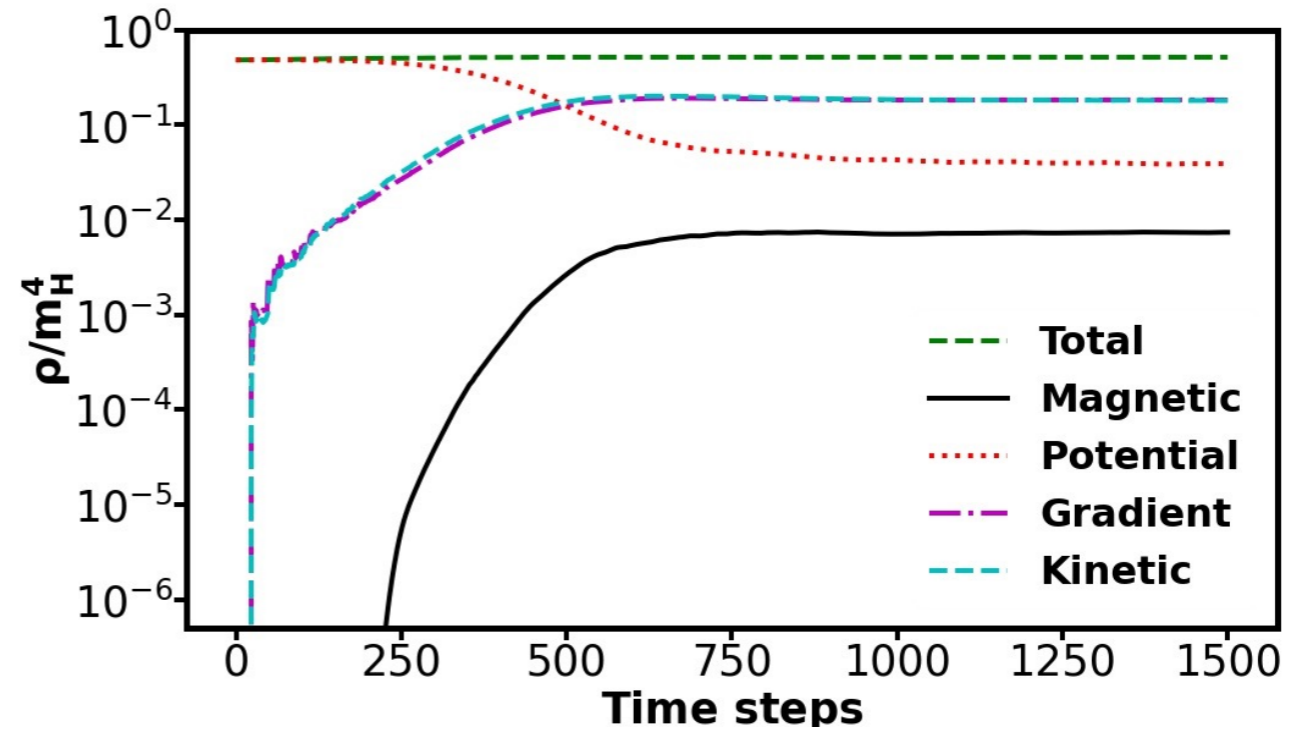
# GW from Bubble collisions

$$\ddot{h}_{ij} - \nabla^2 h_{ij} = 16\pi G T_{ij}^{TT}$$

$$T_{\mu\nu} = \partial_\mu \Phi^\dagger \partial_\nu \Phi - g_{\mu\nu} \frac{1}{2} \text{Re}[(\partial_i \Phi^\dagger \partial^i \Phi)^2]$$

$$\langle \dot{h}_{ij}^{TT}(\mathbf{k}, t) \dot{h}_{ij}^{TT}(\mathbf{k}', t) \rangle = P_h(\mathbf{k}, t) (2\pi)^3 \delta(\mathbf{k} + \mathbf{k}')$$

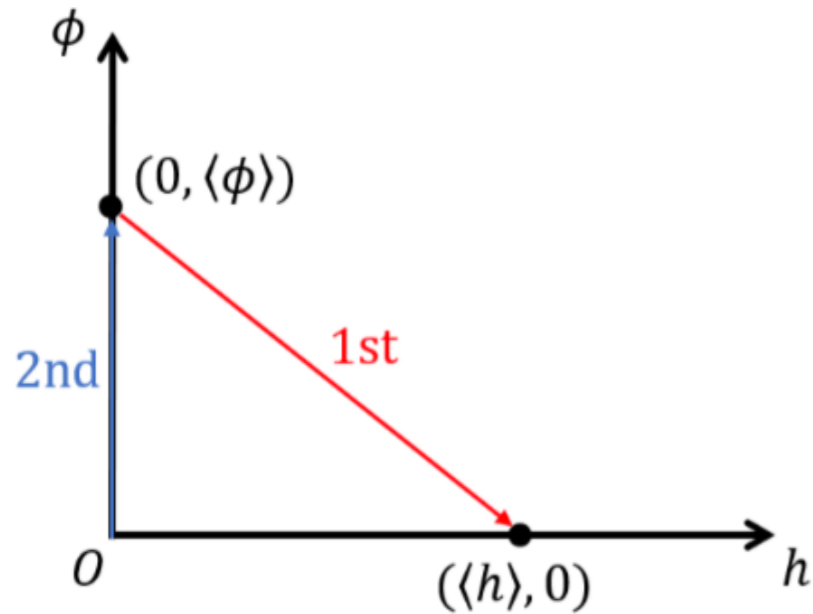
$$\frac{d\Omega_{\text{gw}}}{d\ln(k)} = \frac{1}{32\pi G \rho_c} \frac{k^3}{2\pi^2} P_h(\mathbf{k}, t)$$





# Two-step FOPT potential

## Type-a

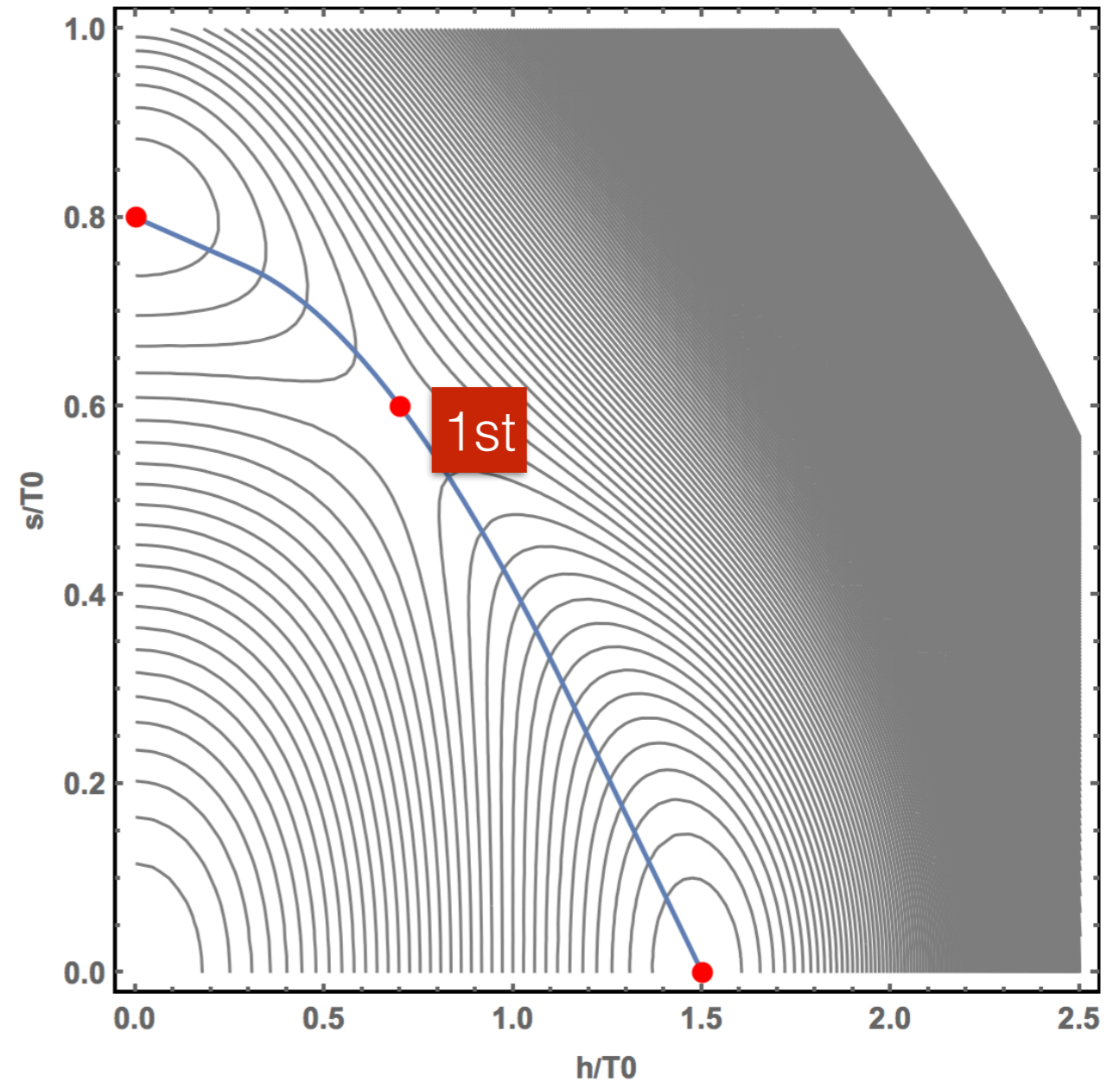


$$V_a(\phi, h, T) = \frac{1}{2}(\mu_\phi^2 + c_\phi T^2)\phi^2 + \frac{1}{2}\lambda_{h\phi}h^2\phi^2 + \frac{1}{4}\lambda_\phi\phi^4$$

$$+ \frac{1}{2}(-\mu_h^2 + c_h T^2)h^2 + \frac{1}{4}\lambda_h h^4$$

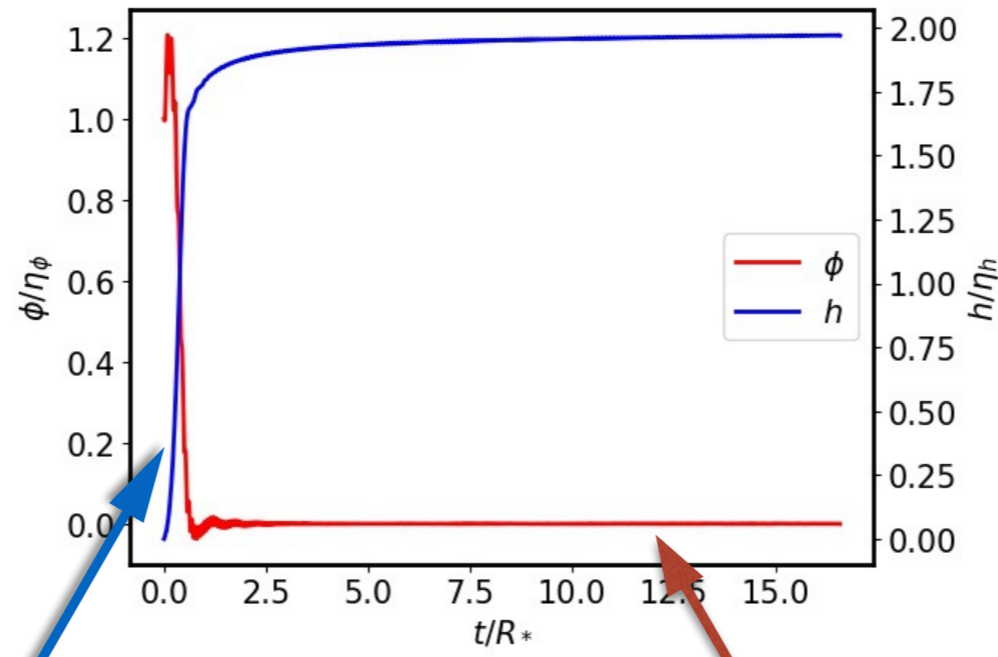
$$c_\phi = \lambda_\phi/4 + \lambda_{h\phi}/3$$

$$c_h = (2m_W^2 + m_Z^2 + 2m_t^2)/(4v^2) + \lambda_h/2 + \lambda_{h\phi}/12$$

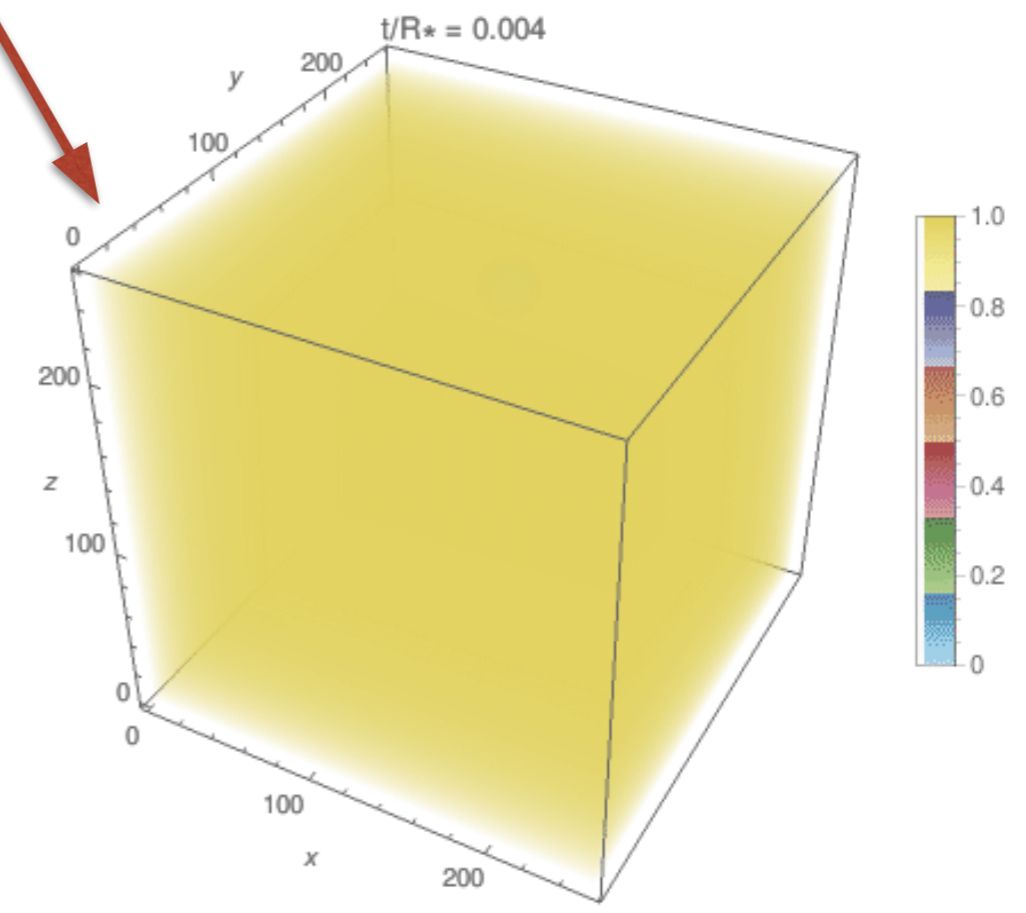
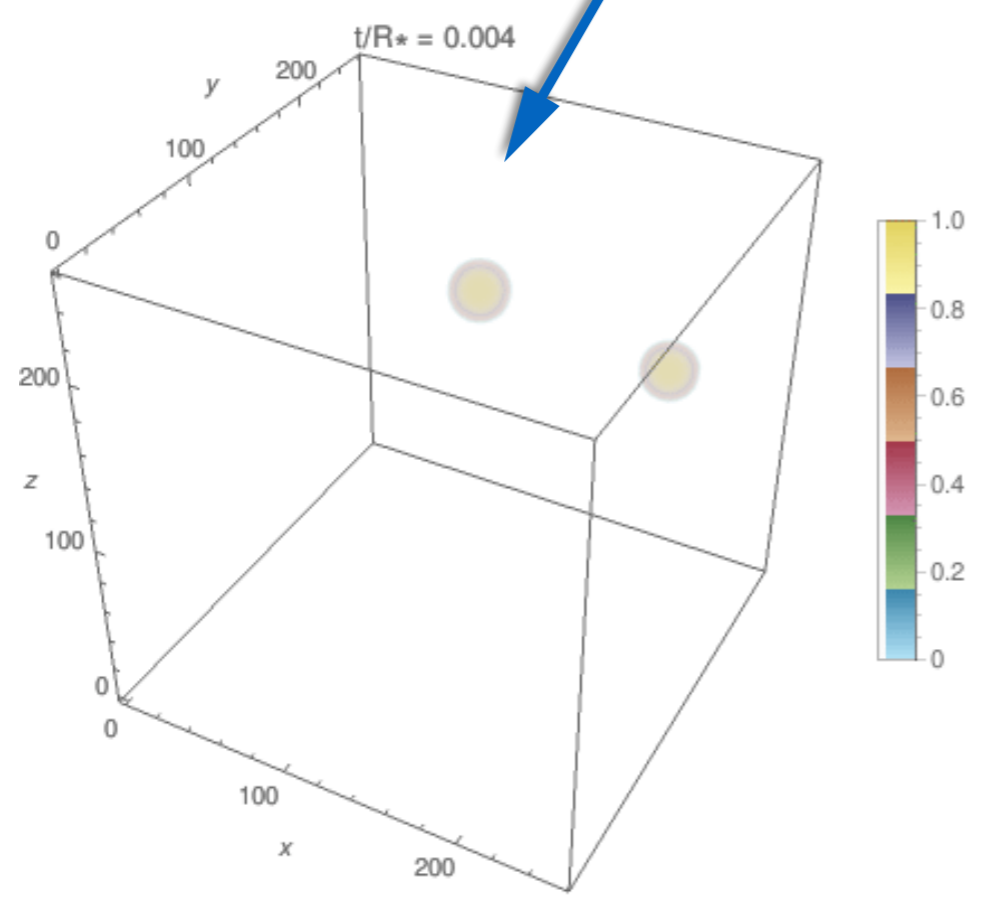


# Two-step PT with the second-step being FOPT

Type-a

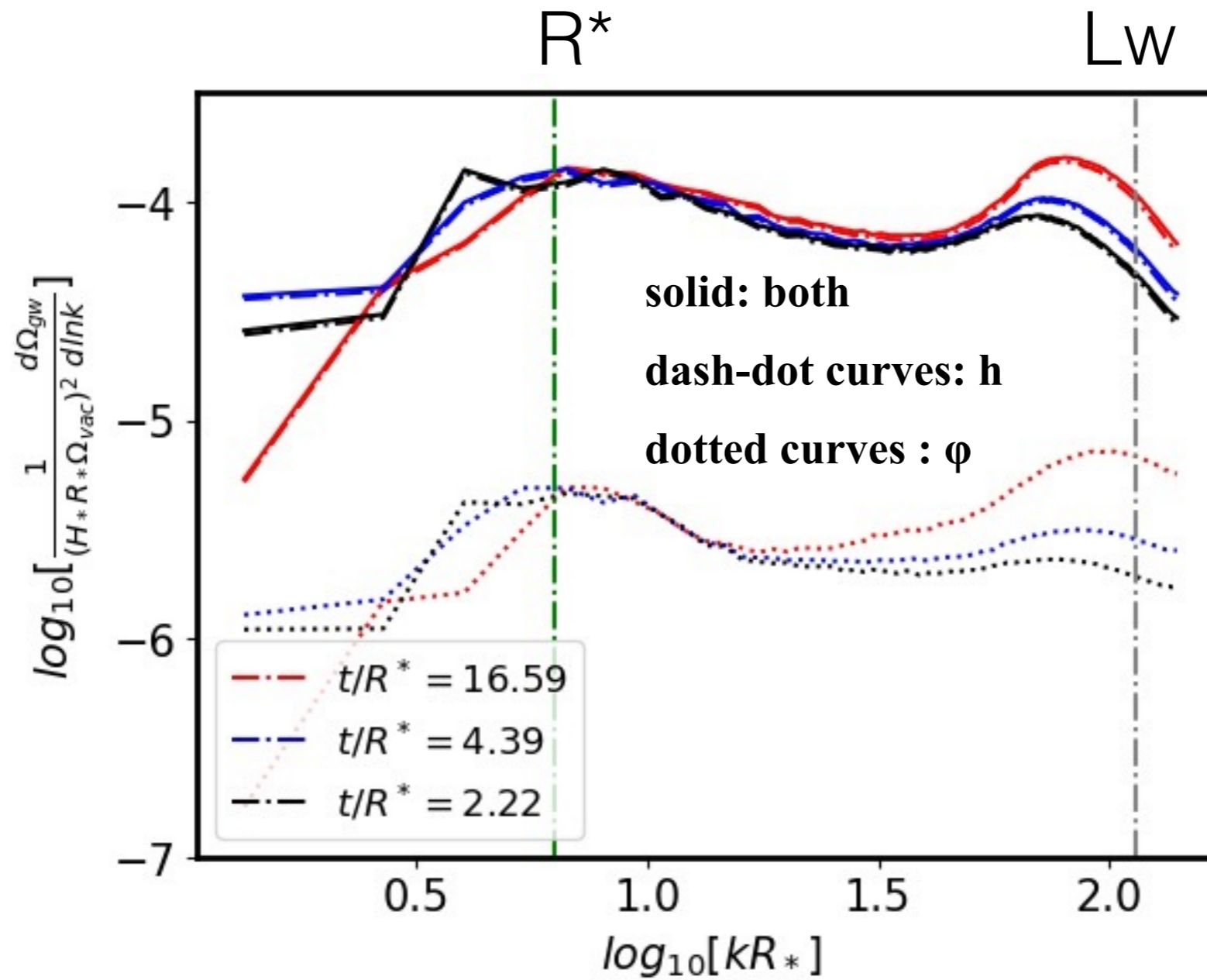


$$h(t=0, r) = \eta_h/2 \left[ 1 - \tanh\left(\frac{r - R_0}{L_w}\right) \right]$$
$$\phi(t=0, r) = \eta_\phi/2 \left[ 1 + \tanh\left(\frac{r - R_0}{L_w}\right) \right]$$



# Two-step PT with the second-step being FOPT

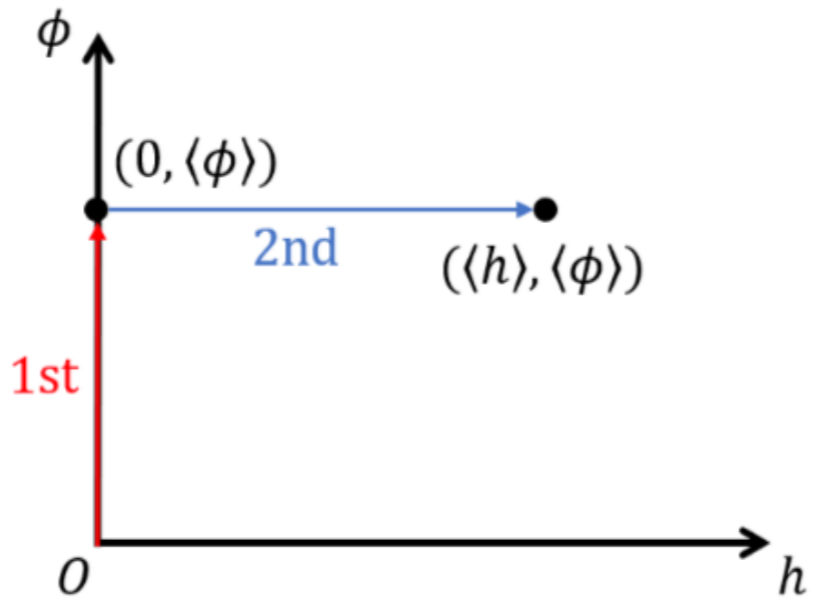
Type-a



# Two-step PT with first-step being FOPT

**Type-b**

Without Global U(1)



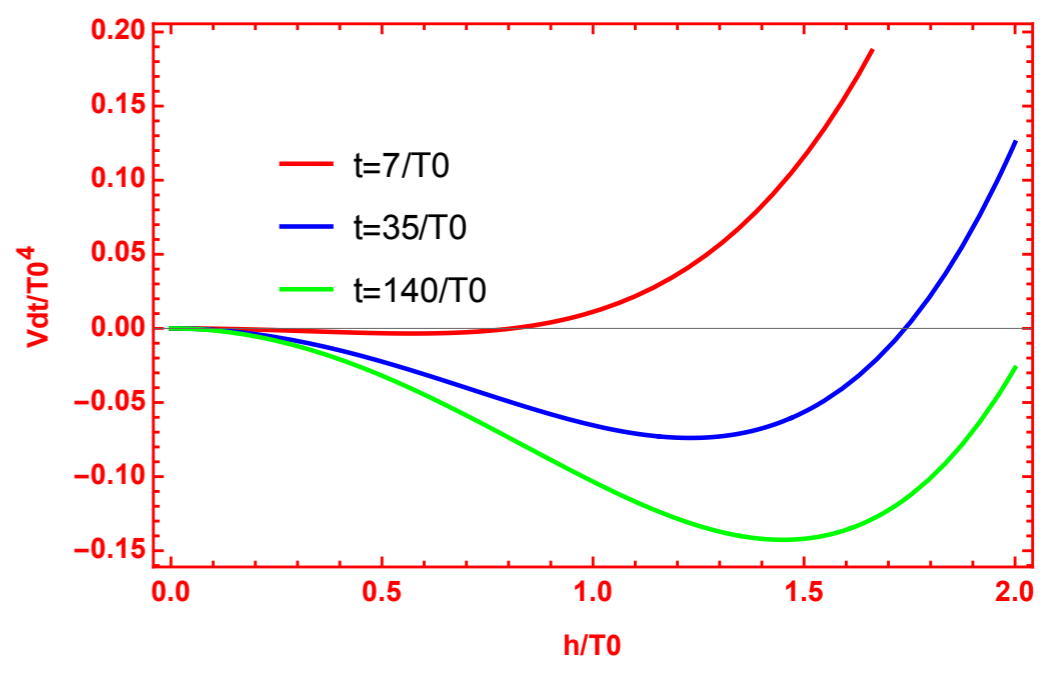
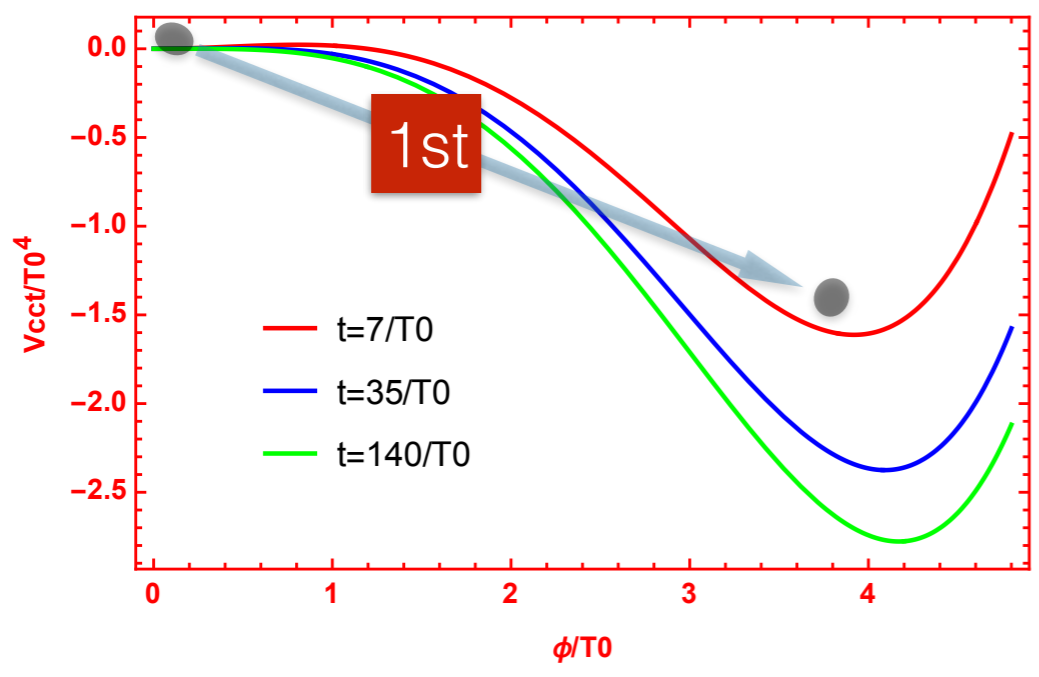
$$V_{cct}(\phi, T) = a\phi^4(\log[|\phi|^2/v_\phi^2] - 1/4) + bT^2|\phi|^2$$

$$V_{dt}(\phi, h, T) = \frac{1}{2}c'_h T^2 h^2 + \frac{1}{4}\lambda_h h^4 - \frac{\lambda_p}{4} h^2 \phi^2$$

$$c'_h = (2m_W^2 + m_Z^2 + 2m_t^2)/(4v^2) + \lambda_h/2 + \lambda_p/24$$

$$\langle h \rangle = \sqrt{(\lambda_p \eta^2 - 2c'_h T^2)/(2\lambda_h)}$$

Classical conformal + Dimensional transmutation

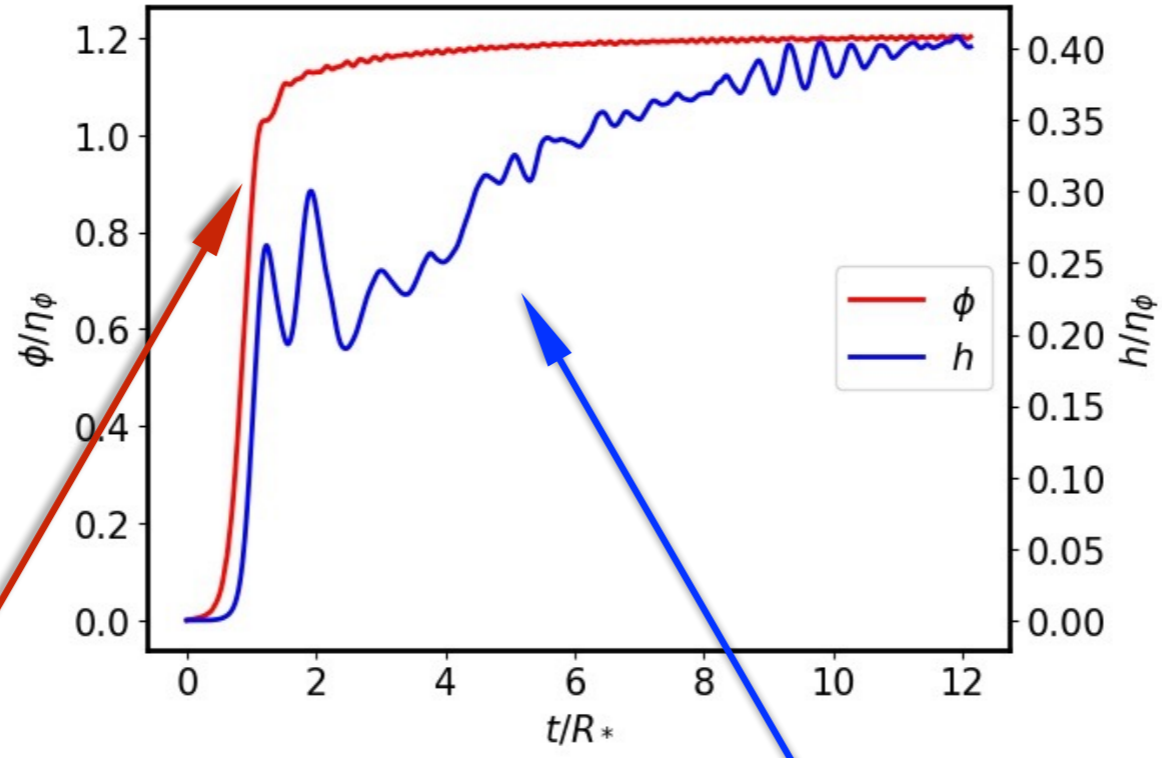




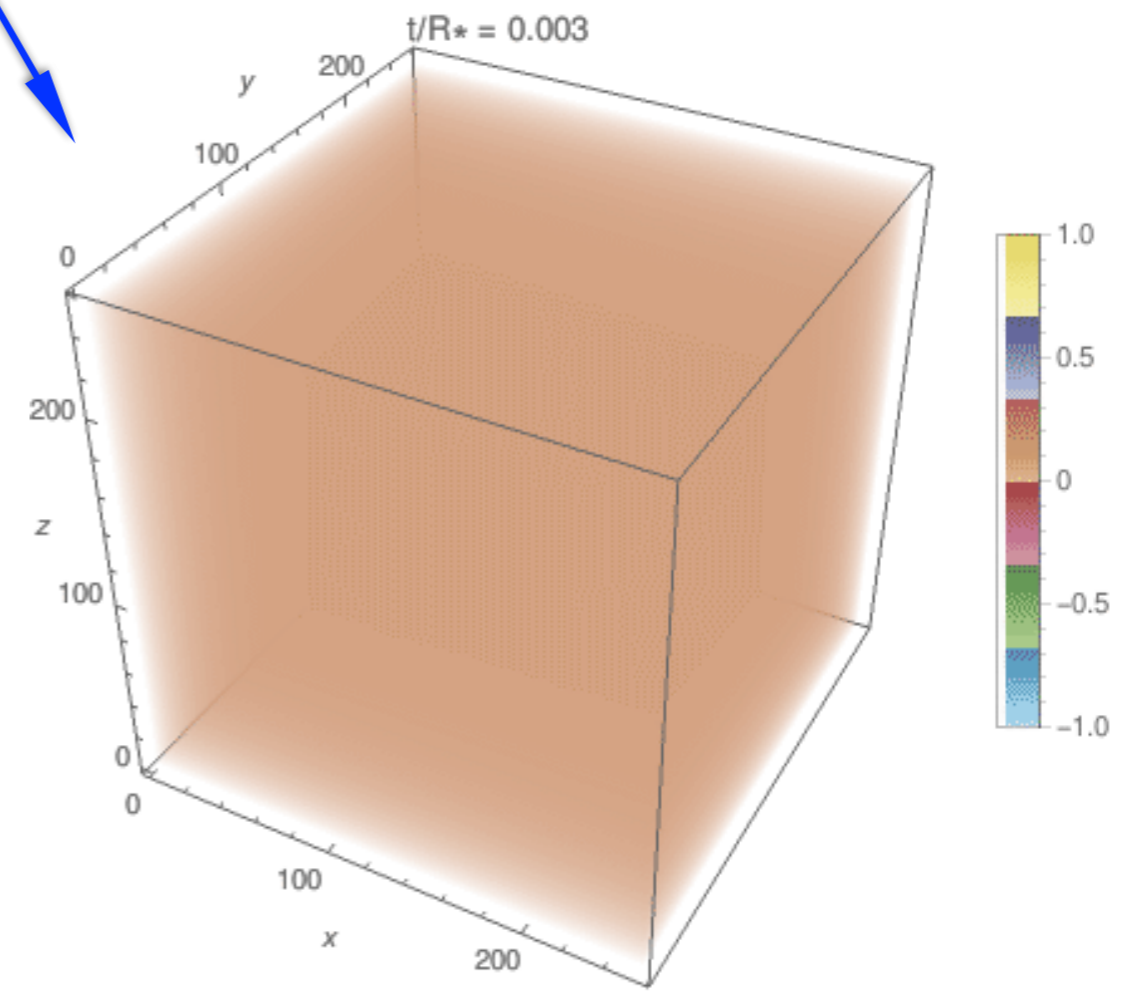
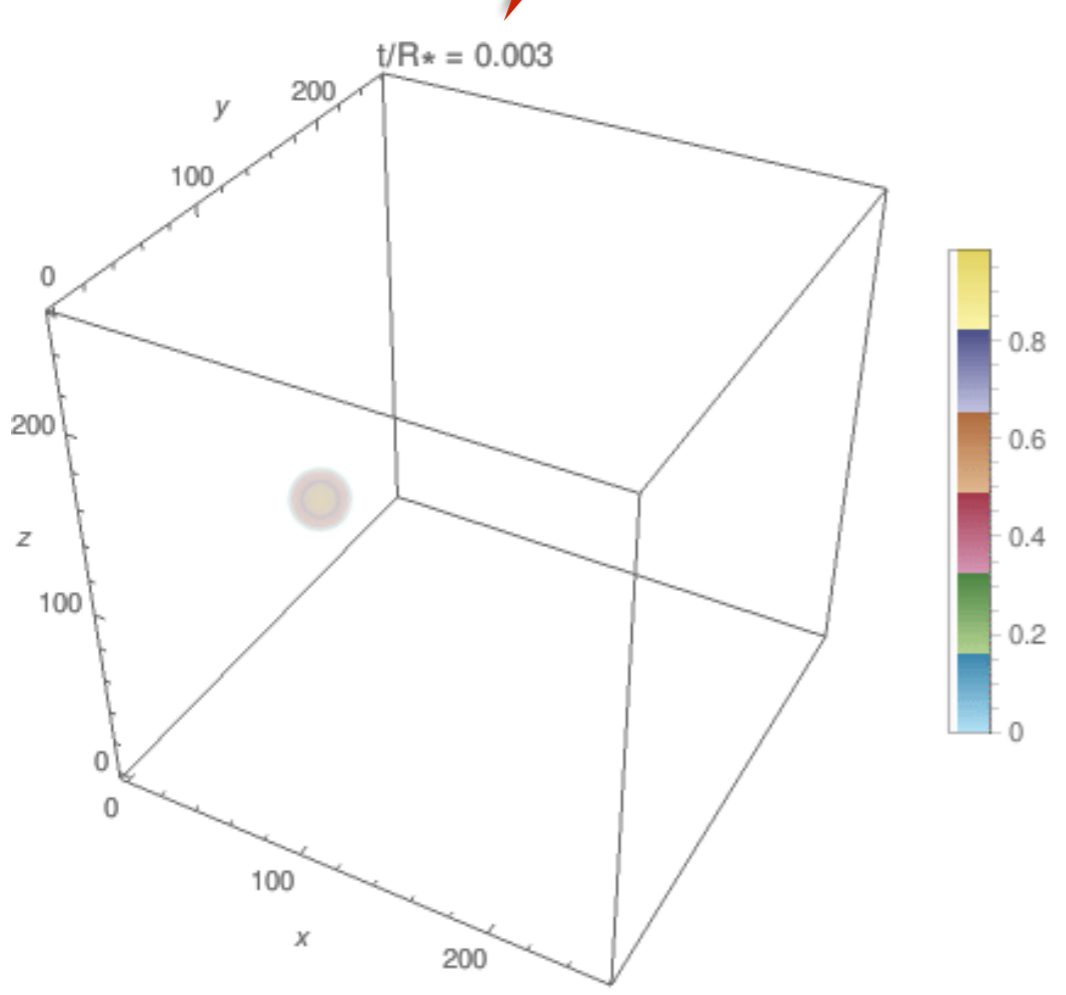
# Two-step PT with first-step being FOPT

**Type-b**

Without Global U(1)



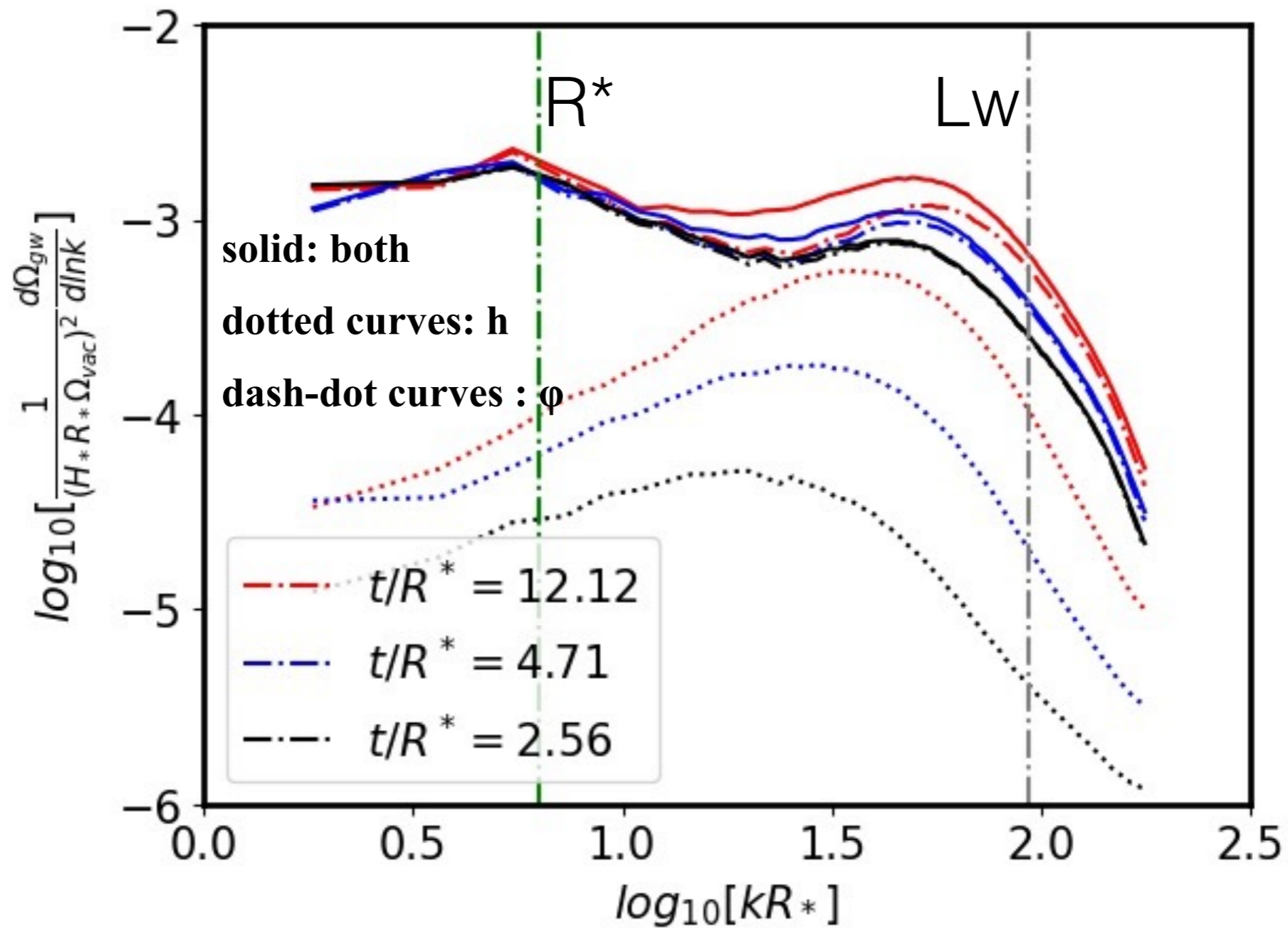
$$\phi(t=0, r) = \eta_\phi/2 \left[ 1 - \tanh\left(\frac{r-R_0}{L_w}\right) \right]$$
$$\langle h \rangle = \sqrt{(\lambda_p \eta^2 - 2c'_h T^2)/(2\lambda_h)}$$



# Two-step PT with first-step being FOPT

Type-b

Without Global U(1)



# 真空泡碰撞、合并、流体演化产生引力波

## 有限温度有效势能

$$V(\phi, T) = \frac{1}{2}\gamma(T^2 - T_0^2)\phi^2 - \frac{1}{3}AT\phi^3 + \frac{1}{4}\lambda\phi^4$$

新物理

## 标量场-相对论流体运动方程

$$-\ddot{\phi} + \nabla^2\phi - \frac{\partial V}{\partial\phi} = \eta W(\dot{\phi} + V^i\partial_i\phi) \quad \eta: \text{粒子和真空泡壁相互作用}$$

$$\begin{aligned} \dot{E} + \partial_i(EV^i) + p[\dot{W} + \partial_i(WV^i)] - \frac{\partial V}{\partial\phi}W(\dot{\phi} + V^i\partial_i\phi) \\ = \eta W^2(\dot{\phi} + V^i\partial_i\phi)^2 \end{aligned}$$

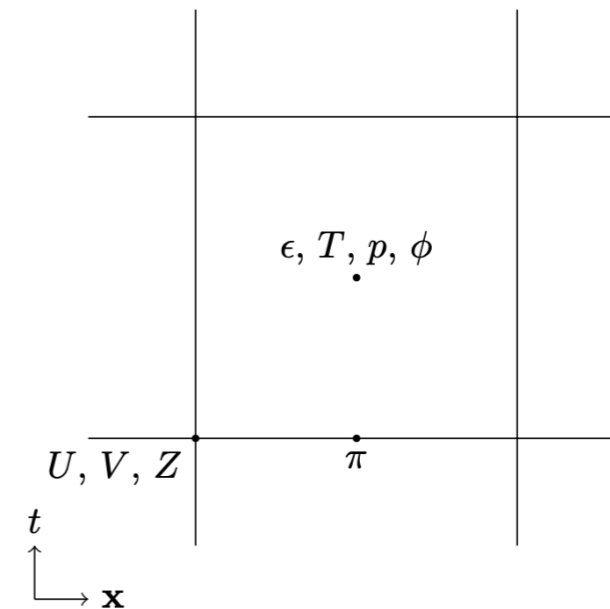
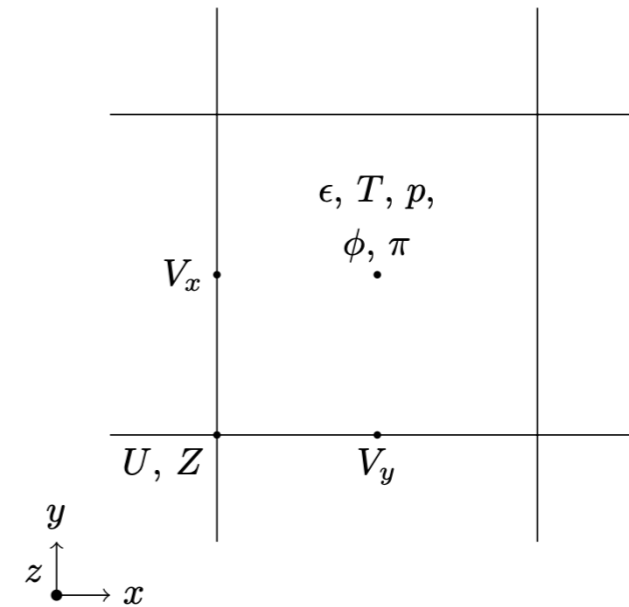
$$\dot{Z}_i + \partial_j(Z_iV^j) + \partial_i p + \frac{\partial V}{\partial\phi}\partial_i\phi = -\eta W(\dot{\phi} + V^j\partial_j\phi)\partial_i\phi$$

equation of state  $\epsilon(T, \phi) = 3aT^4 + V(\phi, T) - T\frac{\partial V}{\partial T},$

$$p(T, \phi) = aT^4 - V(\phi, T)$$

fluid momentum density  $Z_i = W(\epsilon + p)U_i$

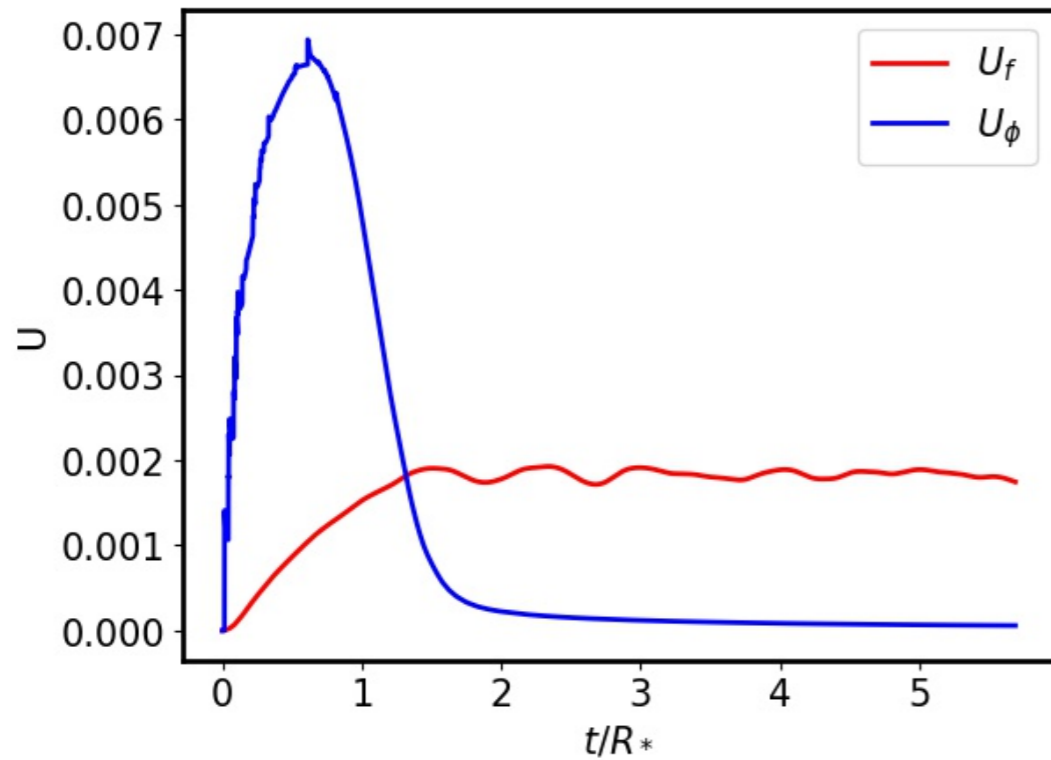
fluid energy density  $E = W\epsilon$



$V^i$  is the fluid 3-velocity

$U^i = WV^i$ ,  $W$ : relativistic  $\gamma$ -factor

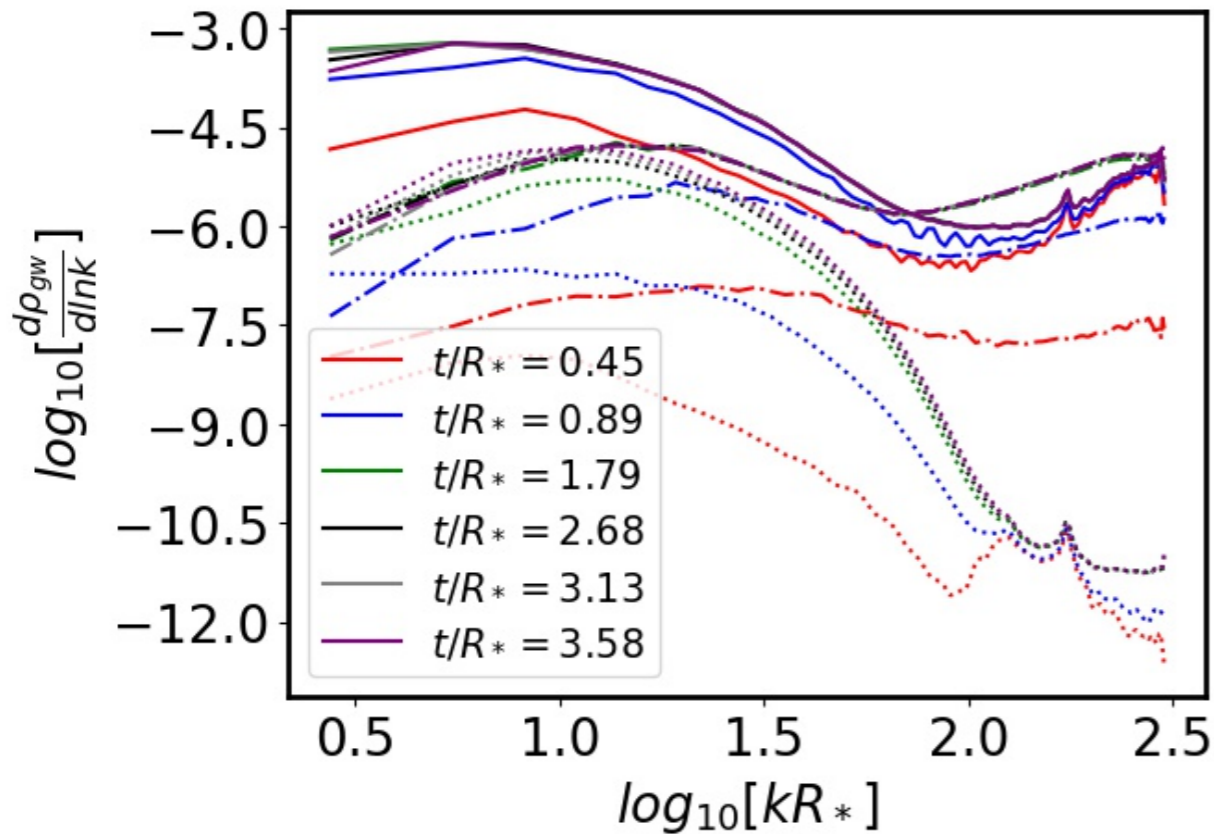
# 真空泡碰撞、合并、流体演化产生引力波



$$\tau_{ij}^{\phi} = \partial_i \phi \partial_j \phi, \quad \tau_{ij}^f = W^2 (\epsilon + p) V_i V_j$$

$$(\bar{\epsilon} + \bar{p}) \bar{U}_f^2 = \frac{1}{V} \int_V d^3x \tau_{ii}^f$$

$$(\bar{\epsilon} + \bar{p}) \bar{U}_{\phi}^2 = \frac{1}{V} \int_V d^3x \tau_{ii}^{\phi}$$



$$\ddot{u}_{ij} - \nabla^2 u_{ij} = 16\pi G (\tau_{ij}^{\phi} + \tau_{ij}^f)$$

$$h_{ij}(\mathbf{k}) = \lambda_{ij,lm}(\hat{\mathbf{k}}) u_{lm}(t, \mathbf{k})$$

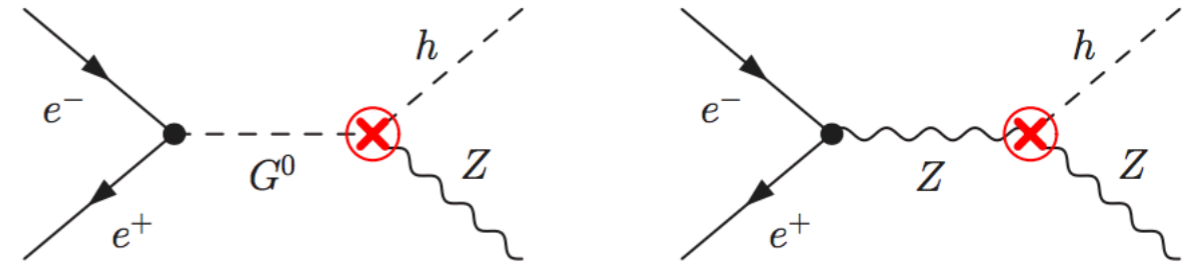


# Collider search for 2step FOPT

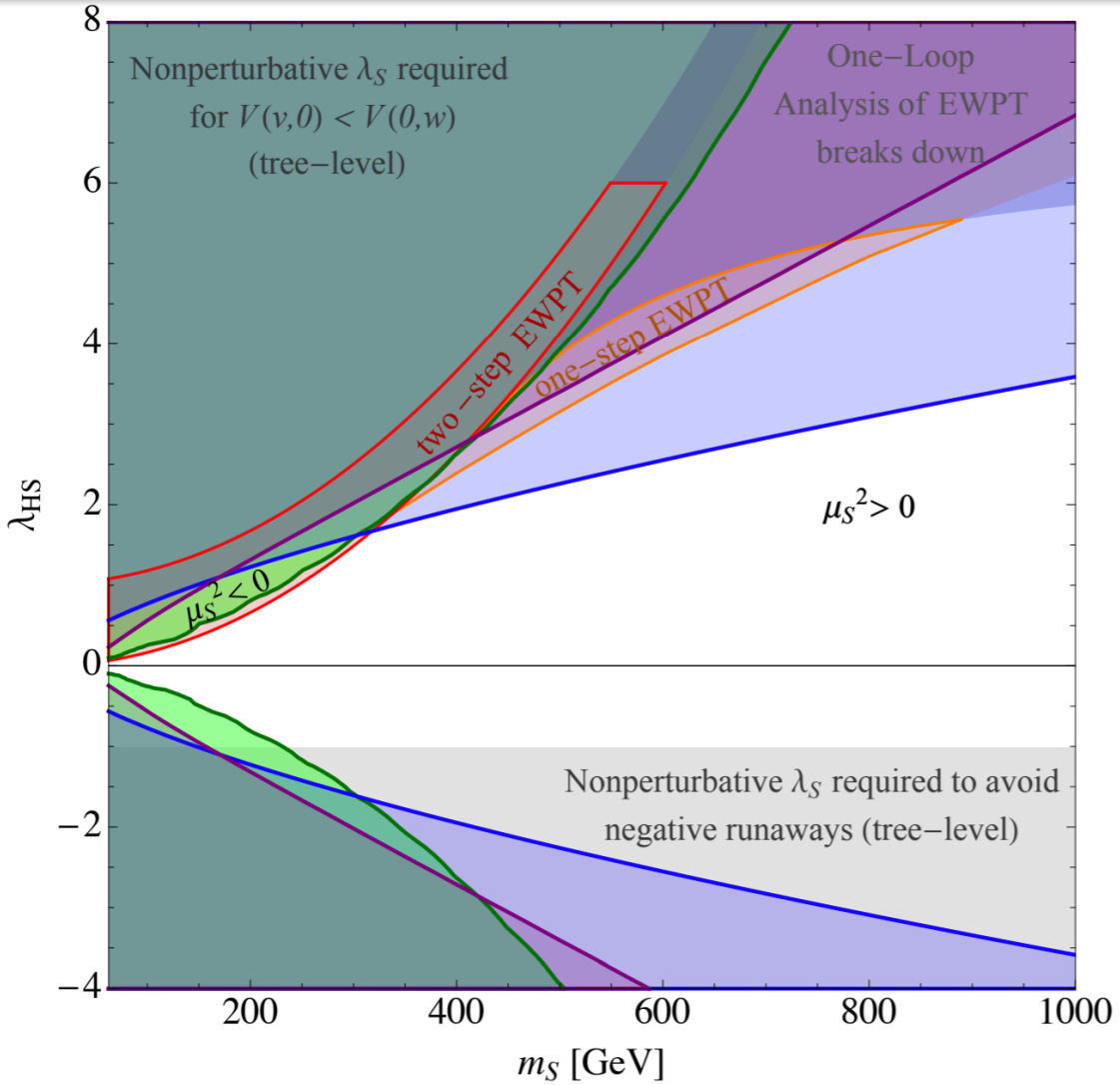
## Zh@ILC/CEPC

$$V_0 = -\mu^2|H|^2 + \lambda|H|^4 + \frac{1}{2}\mu_S^2 S^2 + \lambda_{HS}|H|^2 S^2 + \frac{1}{4}\lambda_S S^4$$

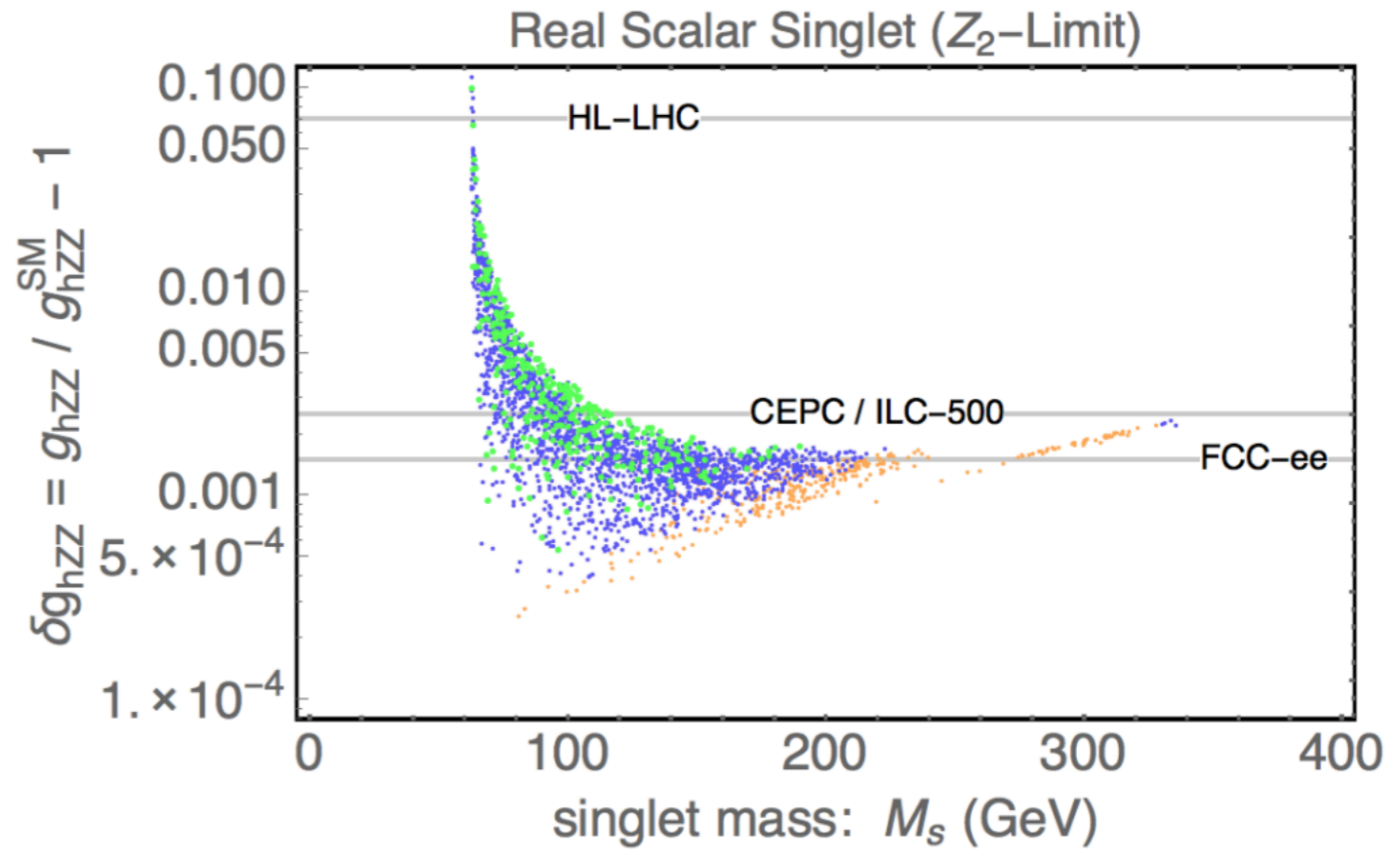
$$V_{\text{eff}}(h, T) = V_0(h) + V_0^{CW}(h) + V_T(h, T) + V_r(h, T)$$



Craig, Englert, and McCullough, 1305.5251



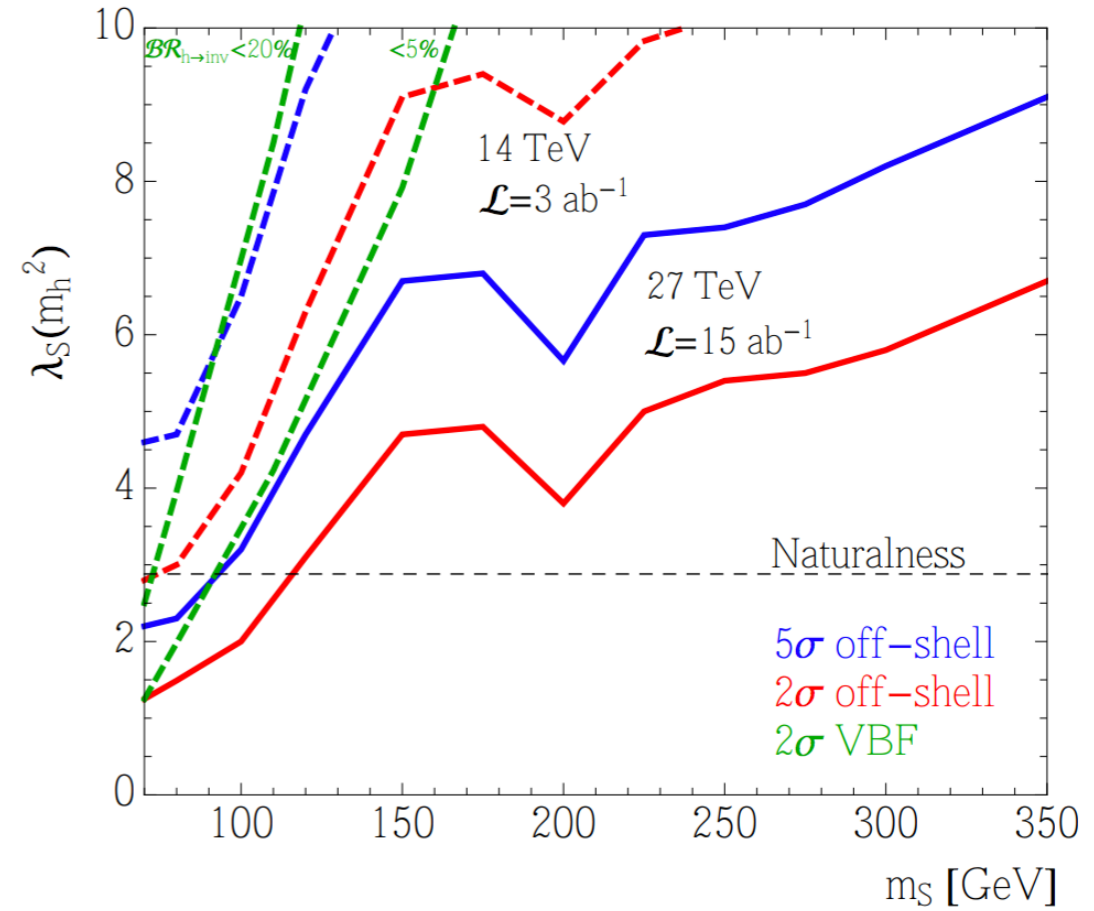
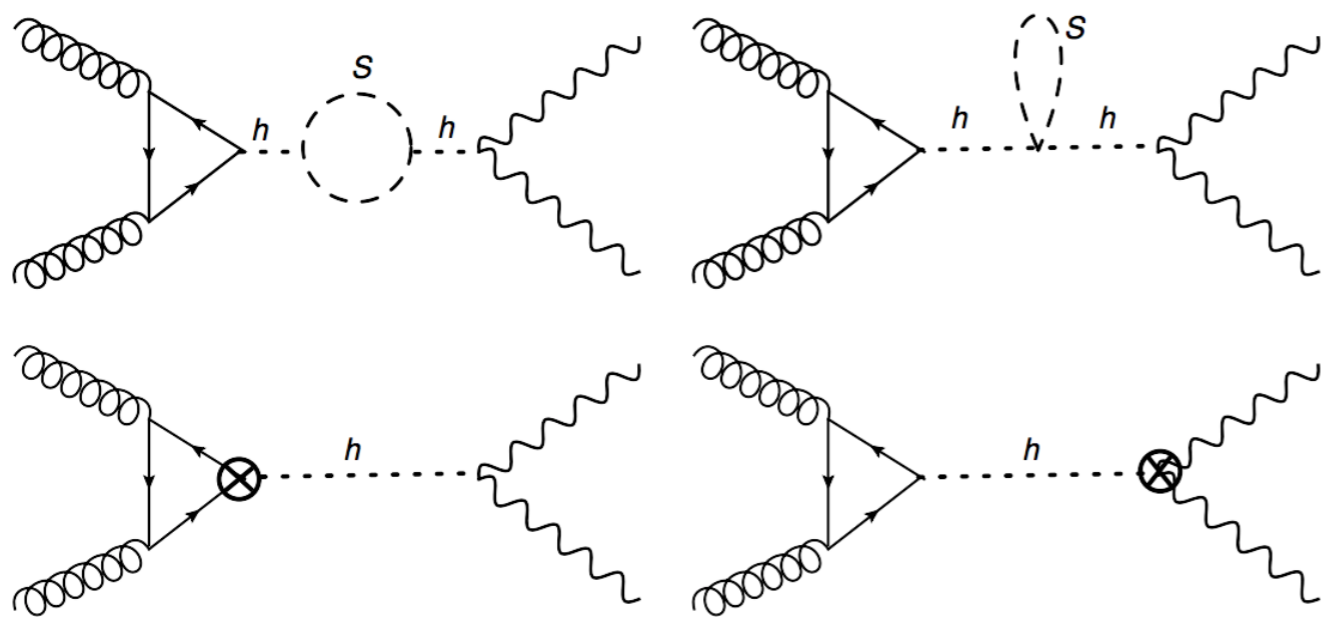
Curtin, Meade, Yu, 1409.0005



Huang, Long, and Wang, 1608.06619

# ► Collider search for 2 step FOPT

## ● Off-shell Higgs@LHC



Goncalves, Han, and Mukhopadhyay, 1710.02149

See also: Lee, Park, and Qian, 1812.02679

# Beyond SM models for FOPT

## Higgs&GWs

### SM+Scalar Singlet

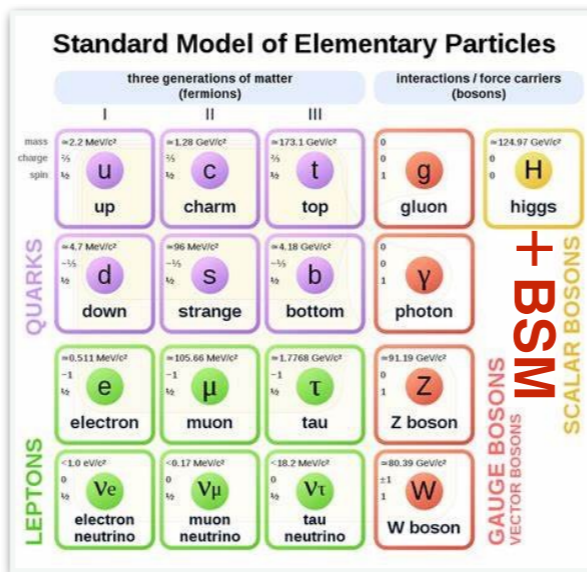
Bian, Huang, Shu 15, Cheng, Bian 17, Bian, Tang 18, Chen, Li, Wu, Bian, 19

### SM+Scalar Doublet

Bernon, Bian, Jiang 17, Bian, Liu 18

### SM + Scalar Triplet

Zhou, Cheng, Deng, Bian, Wu 18, Zhou, Bian, Guo, Wu 19, Zhou, Bian, Du, 22



### Composite Higgs

Bian, Wu, Xie 19, Bian, Wu, Xie 20

### NMSSM

Bi, Bian, Huang, Shu, Yin 15, Bian, Guo, Shu 17

### SMEFT

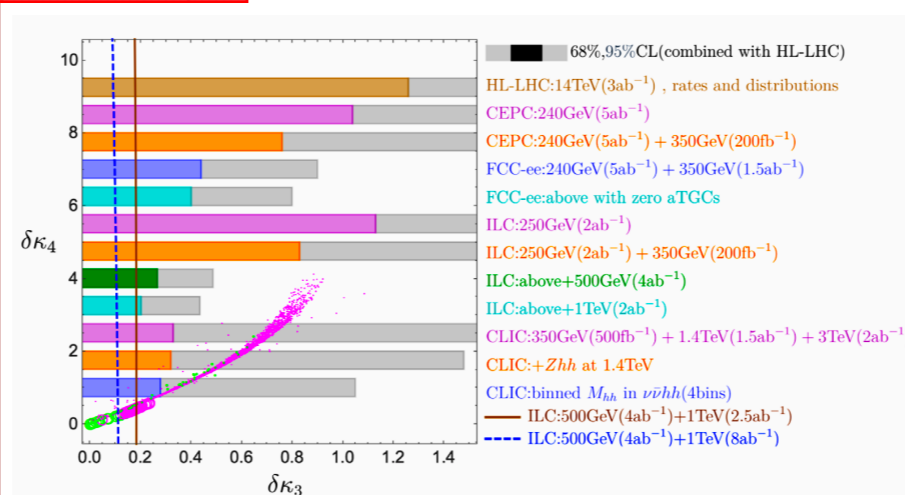
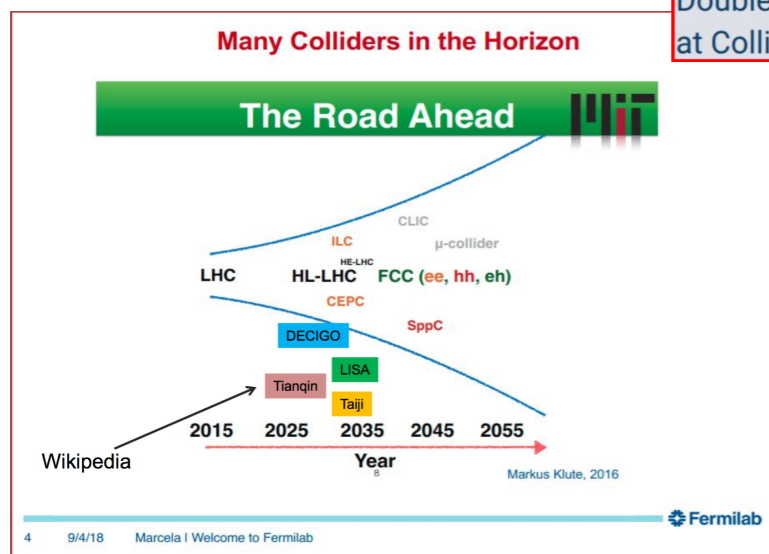
Zhou, Bian, Guo 19

Symmetry breakdown

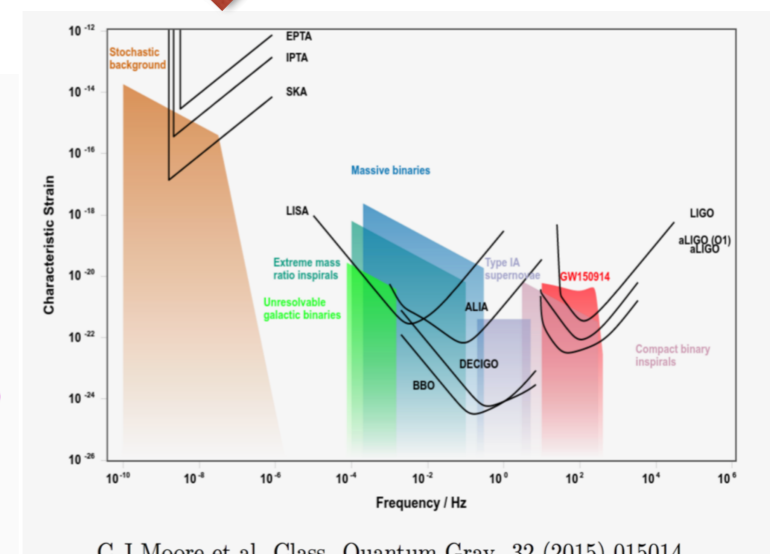
Symmetry breaking process

Double Higgs Production at Colliders Workshop

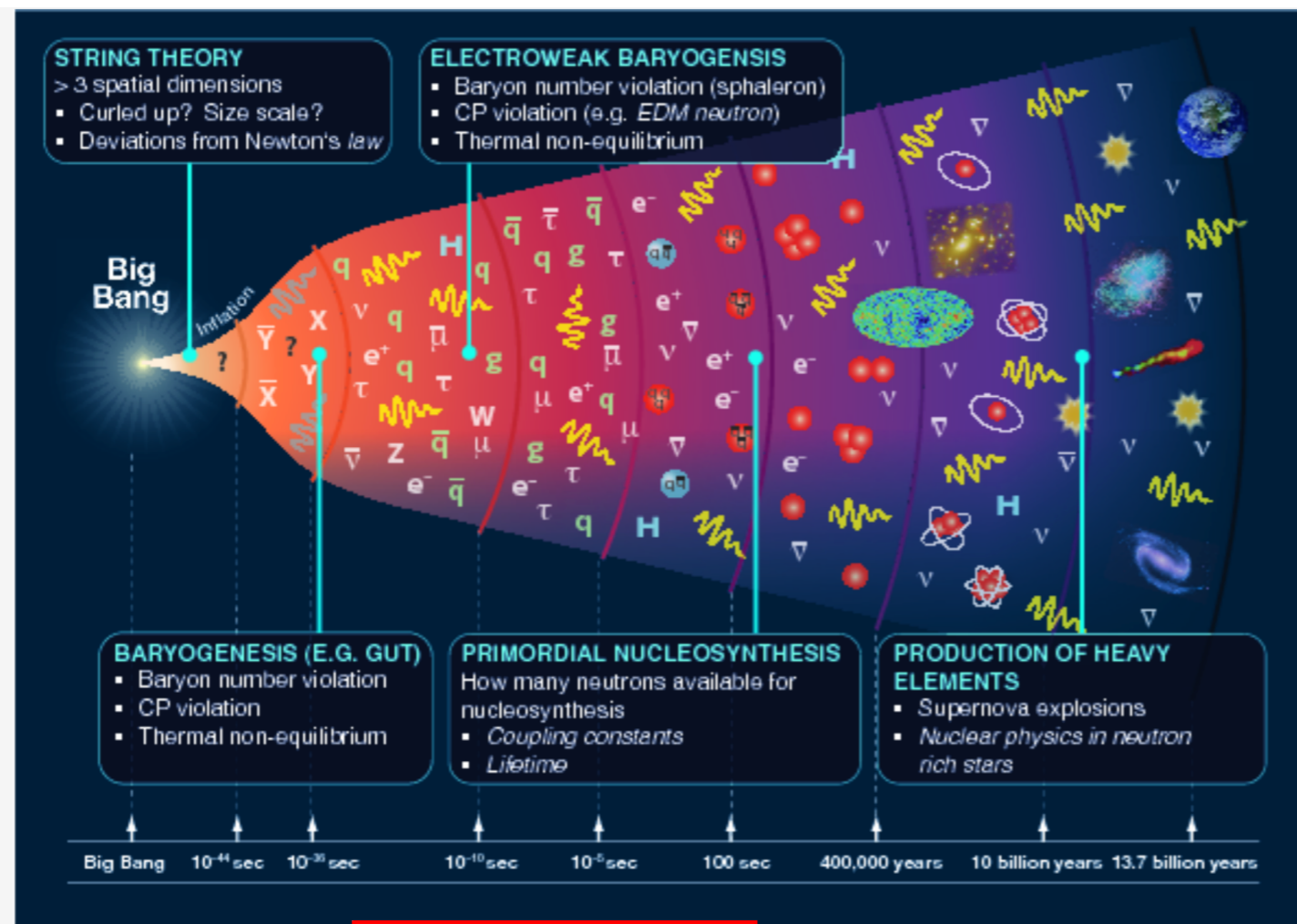
$$\Delta\mathcal{L} = -\frac{1}{2} \frac{m_h^2}{v} (1 + \delta\kappa_3) h^3 - \frac{1}{8} \frac{m_h^2}{v^2} (1 + \delta\kappa_4) h^4$$



SNR > 10 for two-step and one-step SFOEWPT



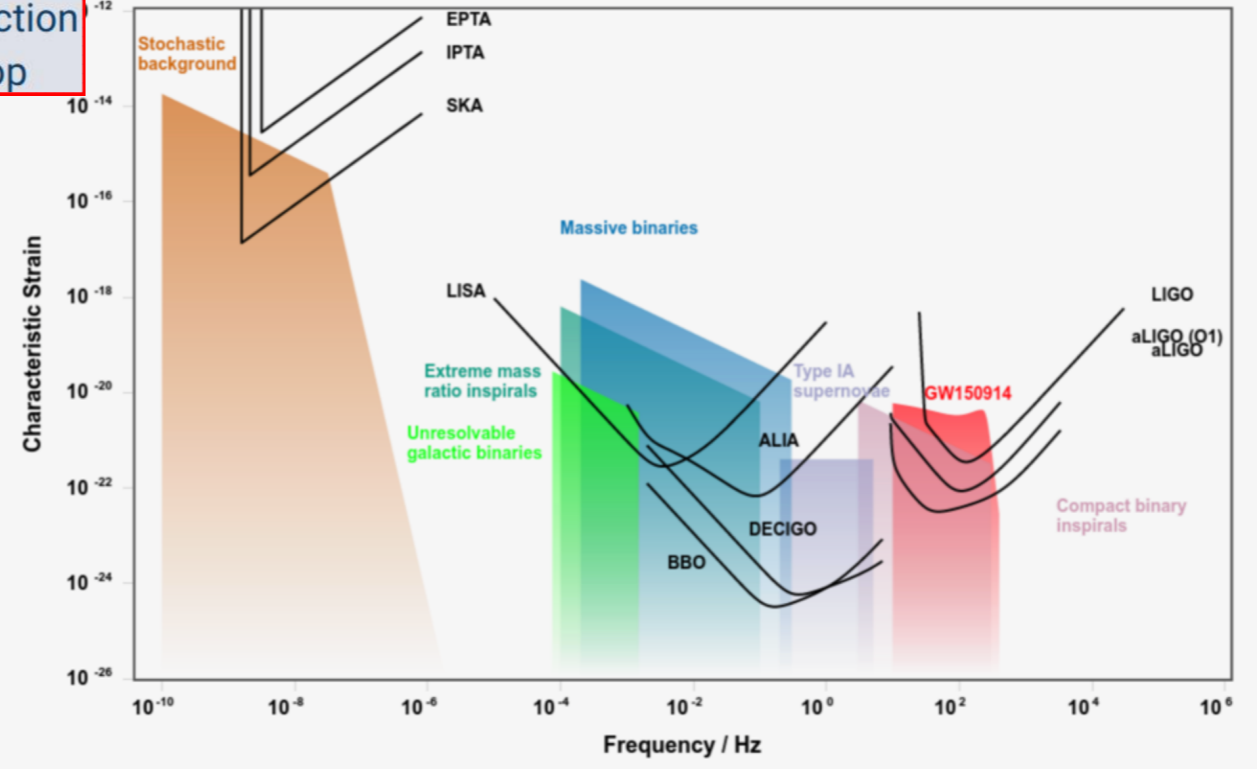
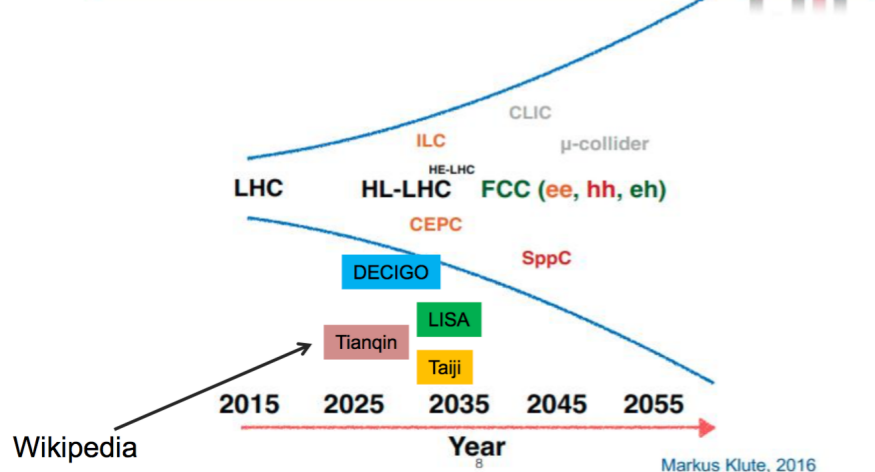
PTA, LISA, TianQin, Taiji, LIGO, ...



**Many Colliders in the Horizon**

**Double Higgs Production at Colliders Workshop**

**The Road Ahead**



C J Moore et al. Class. Quantum Grav. 32 (2015) 015014.



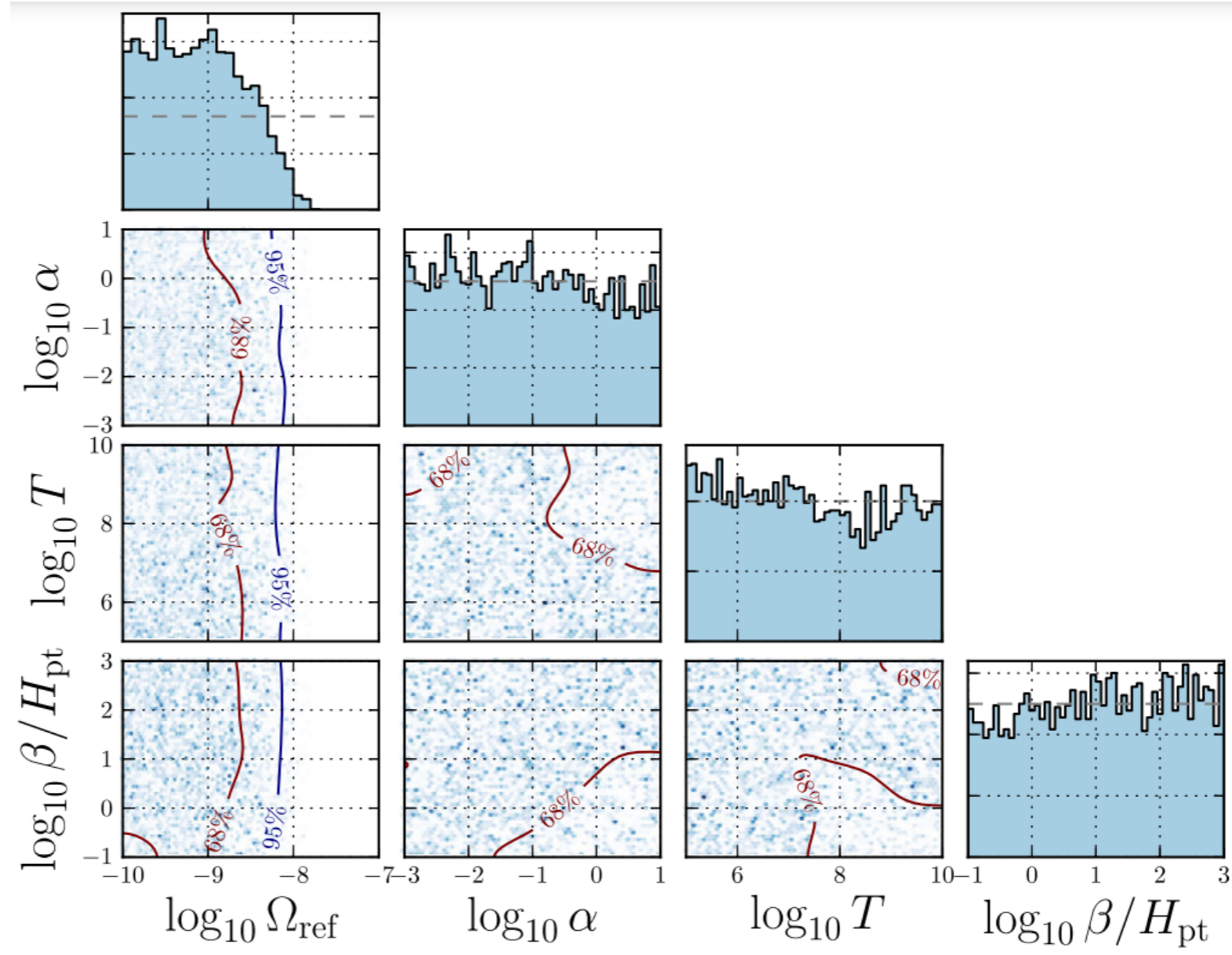


# LIGO-Virgo search for FOPT

## High-scale PT

Romero, Martinovic, Callister, Guo, et al., Phys.Rev.Lett. 126 (2021) 15, 151301

LIGO-Virgo O3



# ▶ PPTA search for FOPT

## ■ PPTA DR2 dataset constrain low-scale phase transition, dark sector and QCD scale FOPT

PHYSICAL REVIEW LETTERS 127, 251303 (2021)

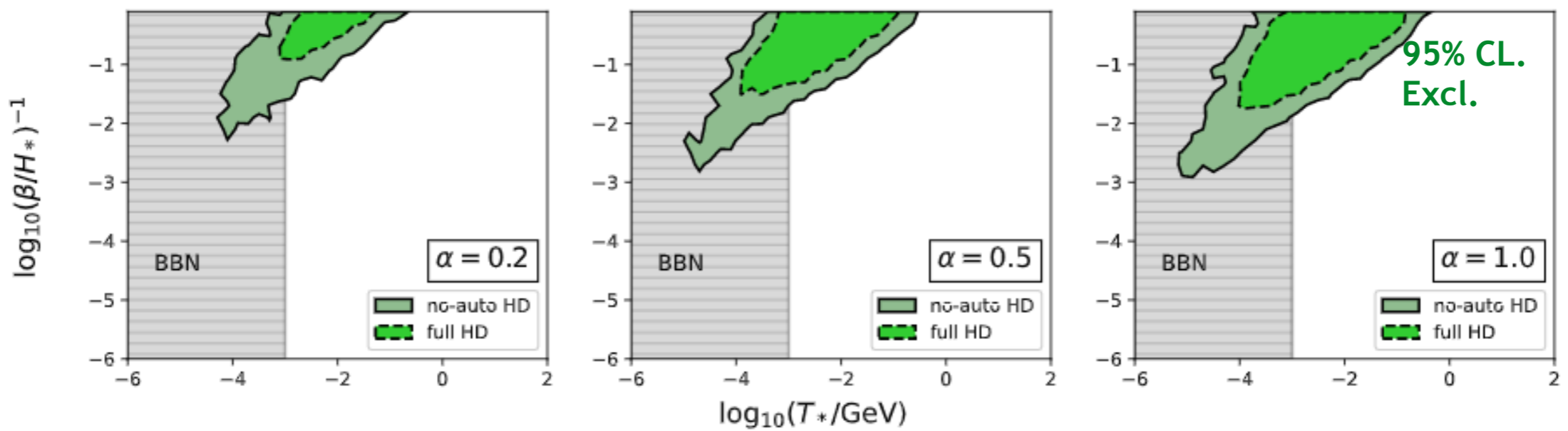
Editors' Suggestion    Featured in Physics

### Constraining Cosmological Phase Transitions with the Parkes Pulsar Timing Array

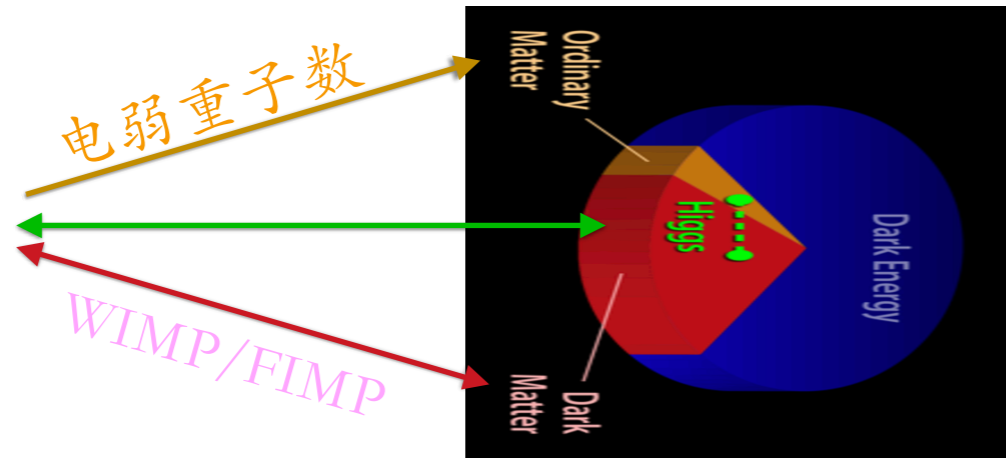
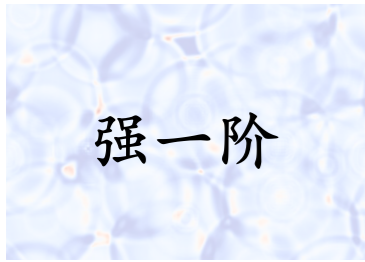
Xiao Xue<sup>1,2,3</sup>, Ligong Bian<sup>4,5,\*</sup>, Jing Shu<sup>1,2,6,7,8,†</sup>, Qiang Yuan<sup>9,10,7,‡</sup>, Xingjiang Zhu<sup>11,12,13,§</sup>, N. D. Ramesh Bhat<sup>14</sup>,  
 Shi Dai<sup>15</sup>, Yi Feng<sup>16</sup>, Boris Goncharov<sup>11,12</sup>, George Hobbs<sup>17</sup>, Eric Howard<sup>17,18</sup>, Richard N. Manchester<sup>17</sup>,  
 Christopher J. Russell<sup>19</sup>, Daniel J. Reardon<sup>12,20</sup>, Ryan M. Shannon<sup>12,20</sup>, Renée Spiewak<sup>21,20</sup>,  
 Nithyanandan Thyagarajan<sup>22</sup> and Jingbo Wang<sup>23</sup>

TABLE I: Description of hypotheses tested in this work and the Bayes factors between them.

Hypothesis	Pulsar noise	Common red process	HD process FOPT spectrum	Bayes Factors	Parameter Estimation (median and 1- $\sigma$ interval)	
					$T_*/\text{MeV}, \alpha \times 10^3, \beta/H_*$	$A_{\text{comred}}, \gamma_{\text{comred}}$
H0:Pulsar Noise	yes	no	no			
H1:Common Red	yes	yes	no	$10^{3.5}$ (against H0)		$-14.45^{+0.62}_{-0.64}, 3.31^{+1.36}_{-1.53}$
H2:FOPT	yes	no	yes (full HD)	$10^{1.8}$ (against H0)	$7.4^{+11.9}_{-4.7}, 271^{+165}_{-92}, 9.9^{+11.4}_{-5.4}$	
H3:FOPT1	yes	yes	yes (full HD)	1.04 (against H1)	$9.6^{+232.2}_{-9.2}, 3.8^{+27.9}_{-3.4}, 854^{+9622}_{-782}$	$-14.51^{+0.64}_{-0.68}, 3.36^{+1.39}_{-1.54}$
H4:FOPT2	yes	yes	yes (no-auto HD)	0.96 (against H1)	$10.9^{+290.5}_{-10.6}, 3.2^{+19.9}_{-2.8}, 1053^{+11256}_{-962}$	$-14.45^{+0.62}_{-0.64}, 3.27^{+1.37}_{-1.54}$



# 一阶相变效应



## Impact of a complex singlet: Electroweak baryogenesis and dark matter

Minyuan Jiang (Beijing, Inst. Theor. Phys. and Beijing, KITPC and Nanjing U.), Ligong Bian (Beijing, Inst. Theor. Phys. and Beijing, KITPC), Weicong Huang (Beijing, Inst. Theor. Phys. and Beijing, KITPC), Jing Shu (Beijing, Inst. Theor. Phys. and Beijing, KITPC) (Feb 26, 2015)

Published in: *Phys.Rev.D* 93 (2016) 6, 065032 • e-Print: 1502.07574 [hep-ph]

pdf DOI cite claim reference search 112 citations

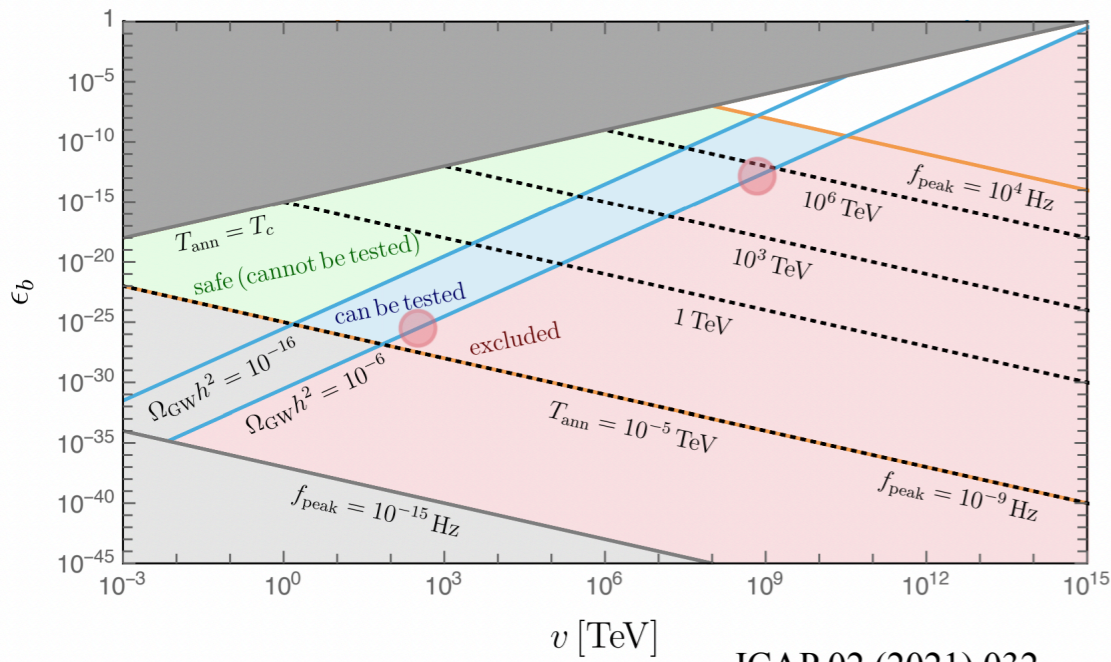
## Thermally modified sterile neutrino portal dark matter and gravitational waves from phase transition: The Freeze-in case

Ligong Bian (Chongqing U. and Chung-Ang U.), Yi-Lei Tang (Korea Inst. Advanced Study, Seoul) (Oct 7, 2018)

Published in: *JHEP* 12 (2018) 006 • e-Print: 1810.03172 [hep-ph]

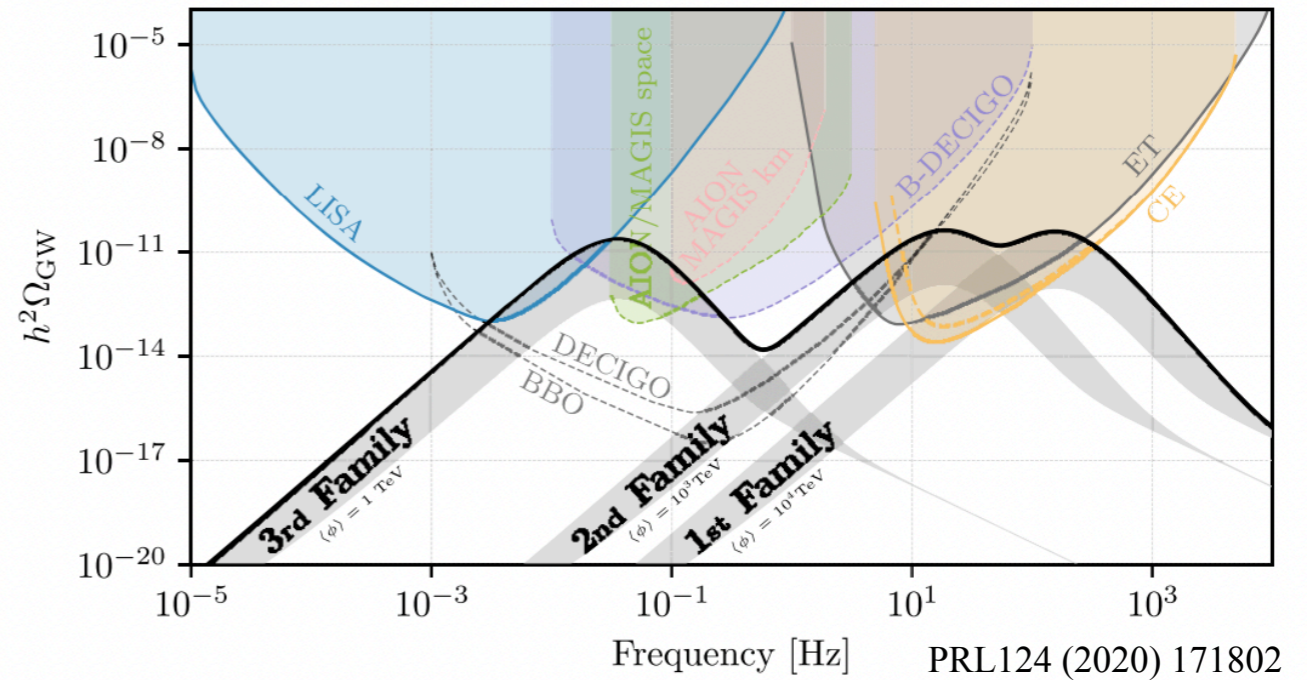
pdf DOI cite claim reference search 52 citations

## 离散的味对称性A4 与畴壁 (DW)



JCAP 02 (2021) 032

## 三代夸克和轻子质量等级问题与FOPT



PRL124 (2020) 171802



# 一阶相变与 Seesaw scale

**Gravitational waves from first-order phase transitions in Majoron models of neutrino mass** #9  
 Pasquale Di Bari (Southampton U.), Danny Marfatia (Hawaii U.), Ye-Ling Zhou (Southampton U. and HIAS, UCAS, Hangzhou and ICTP-AP, Beijing) (May 31, 2021)  
 Published in: *JHEP* 10 (2021) 193 • e-Print: 2106.00025 [hep-ph]  
[pdf](#) [DOI](#) [cite](#) [claim](#) [reference search](#) [23 citations](#)

**Gravitational waves from neutrino mass and dark matter genesis** #16  
 Pasquale Di Bari (Southampton U.), Danny Marfatia (Hawaii U.), Ye-Ling Zhou (Southampton U.) (Jan 21, 2020)  
 Published in: *Phys.Rev.D* 102 (2020) 9, 095017 • e-Print: 2001.07637 [hep-ph]  
[pdf](#) [DOI](#) [cite](#) [claim](#) [reference search](#) [14 citations](#)

**Gravitational Waves from First-Order Phase Transitions: LIGO as a Window to Unexplored Seesaw Scales** #1  
 Vedran Brdar (Heidelberg, Max Planck Inst.), Alexander J. Helmboldt (Heidelberg, Max Planck Inst.), Jisuke Kubo (Heidelberg, Max Planck Inst. and Toyama U.) (Oct 29, 2018)  
 Published in: *JCAP* 02 (2019) 021 • e-Print: 1810.12306 [hep-ph]  
[pdf](#) [DOI](#) [cite](#) [claim](#) [reference search](#) [93 citations](#)

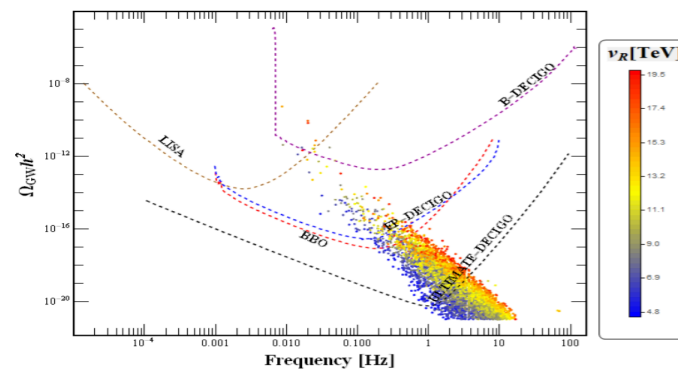
**Gravitational wave pathway to testable leptogenesis** #2  
 Arnab Dasgupta (Pittsburgh U.), P.S. Bhupal Dev (Washington U., St. Louis and McDonnell Ctr. Space Sci.), Anish Ghoshal (Warsaw U.), Anupam Mazumdar (U. Groningen, VSI) (Jun 14, 2022)  
 Published in: *Phys.Rev.D* 106 (2022) 7, 075027 • e-Print: 2206.07032 [hep-ph]  
[pdf](#) [DOI](#) [cite](#) [claim](#) [reference search](#) [23 citations](#)

**Gravitational wave imprints of left-right symmetric model with minimal Higgs sector** #1  
 Lukáš Gráf (Heidelberg, Max Planck Inst. and UC, Berkeley and UC, San Diego), Sudip Jana (Heidelberg, Max Planck Inst.), Ajay Kaladharan (Oklahoma State U.), Shaikh Saad (Basel U.) (Dec 22, 2021)  
 Published in: *JCAP* 05 (2022) 05, 003 • e-Print: 2112.12041 [hep-ph]  
[pdf](#) [DOI](#) [cite](#) [claim](#) [reference search](#) [7 citations](#)

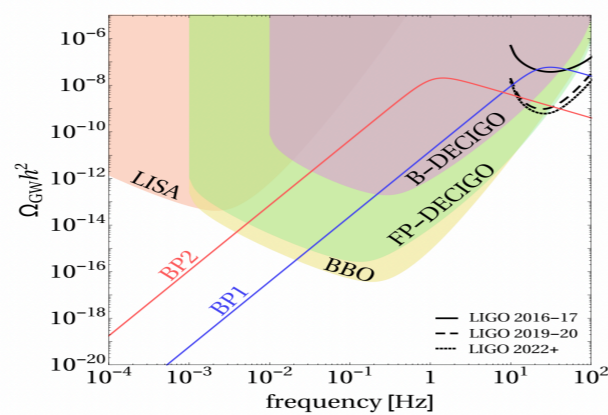
**Cosmological implications of a B – L charged hidden scalar: leptogenesis and gravitational waves** #5  
 Ligong Bian (Chongqing U.), Wei Cheng (Beijing, Inst. Theor. Phys.), Huai-Ke Guo (Oklahoma U.), Yongchao Zhang (Washington U., St. Louis and Peking U., CHEP) (Jul 31, 2019)  
 Published in: *Chin.Phys.C* 45 (2021) 11, 113104 • e-Print: 1907.13589 [hep-ph]  
[pdf](#) [DOI](#) [cite](#) [claim](#) [reference search](#) [27 citations](#)

**Prospects of gravitational waves in the minimal left-right symmetric model** #19  
 Mingqiu Li (Beijing, GUCAS), Qi-Shu Yan (Beijing, GUCAS and Beijing, Inst. High Energy Phys.), Yongchao Zhang (Southeast U., Nanjing and Washington U., St. Louis), Zhijie Zhao (Beijing, Inst. High Energy Phys.) (Dec 26, 2020)  
 Published in: *JHEP* 03 (2021) 267 • e-Print: 2012.13686 [hep-ph]  
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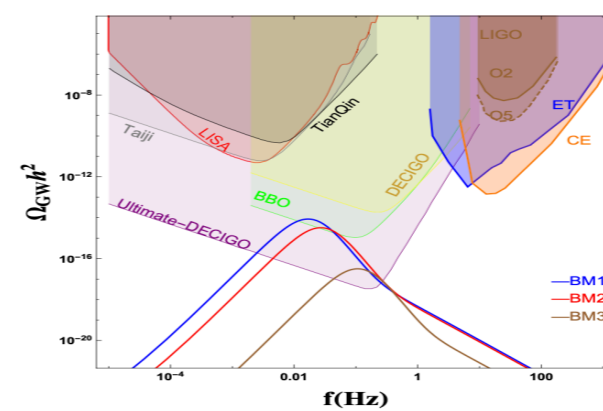
**Electroweak phase transition and gravitational waves in the type-II seesaw model** #12  
 Ruiyu Zhou (CUPT, Chongqing), Ligong Bian (Chongqing U. and Peking U., CHEP), Yong Du (Beijing, Inst. Theor. Phys.) (Mar 3, 2022)  
 Published in: *JHEP* 08 (2022) 205 • e-Print: 2203.01561 [hep-ph]  
[pdf](#) [DOI](#) [cite](#) [claim](#) [reference search](#) [15 citations](#)



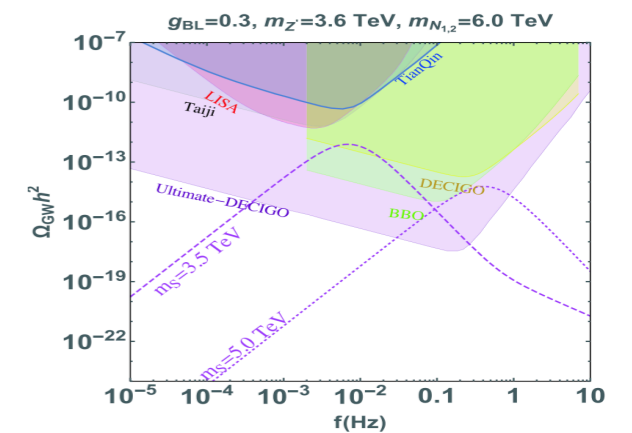
L-R



CSB



Type-II

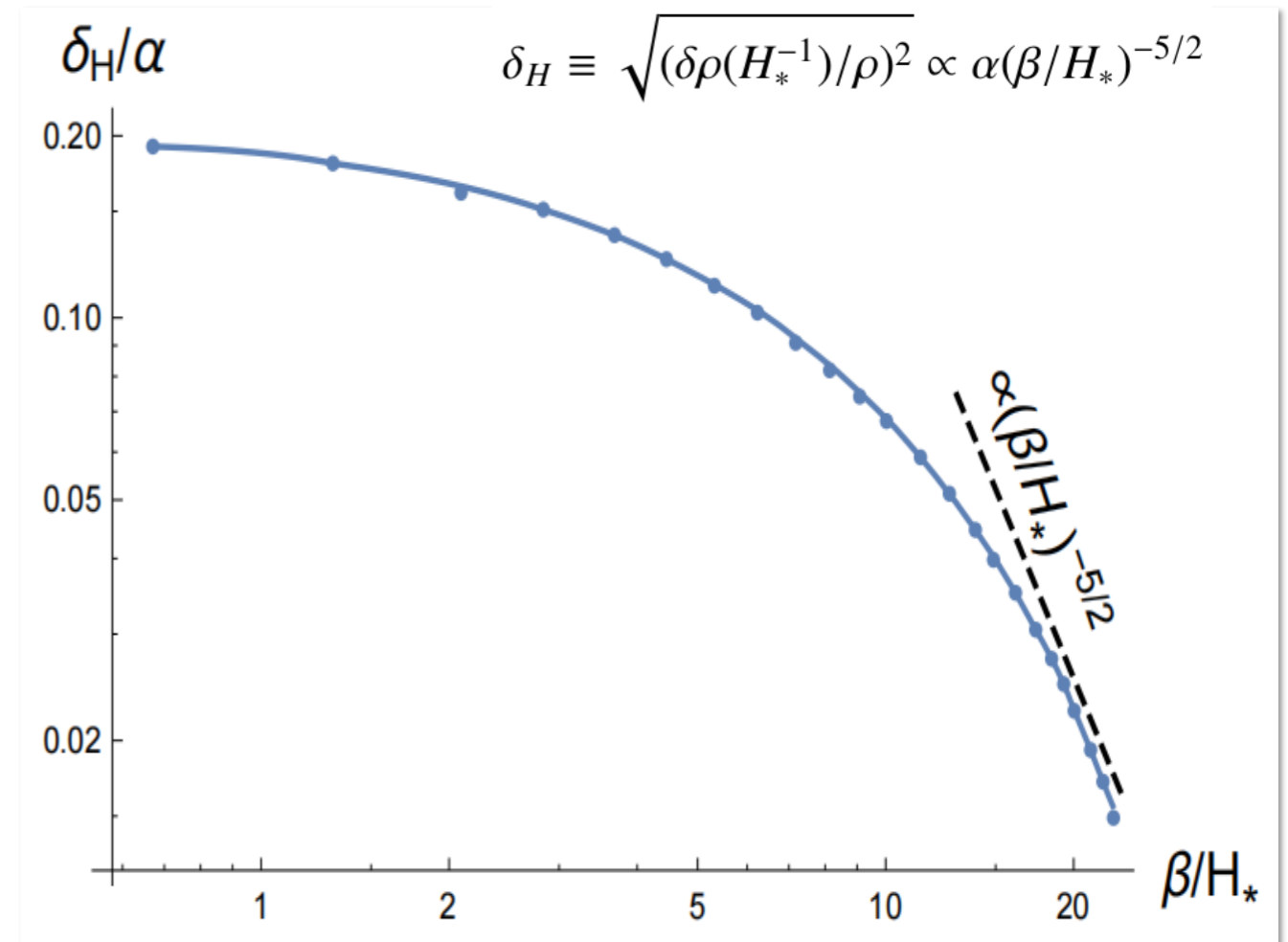
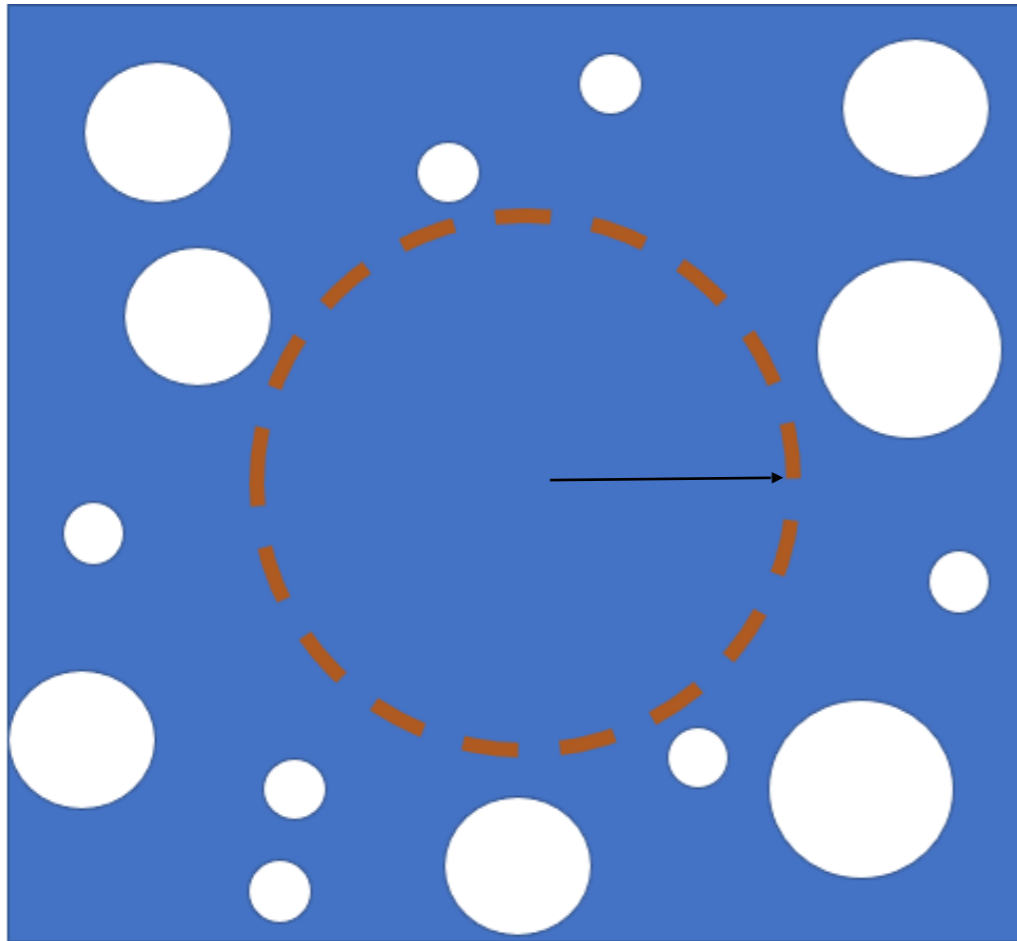


Type-I



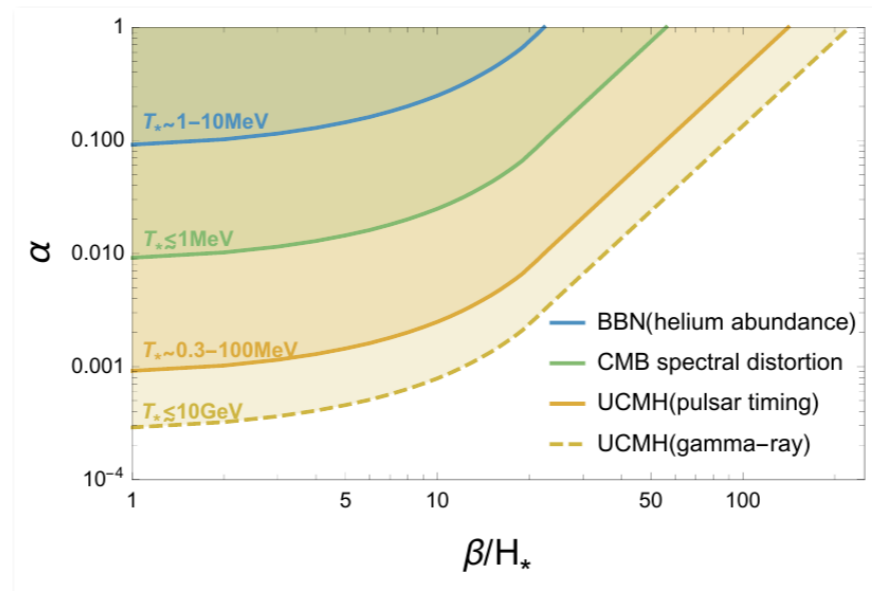
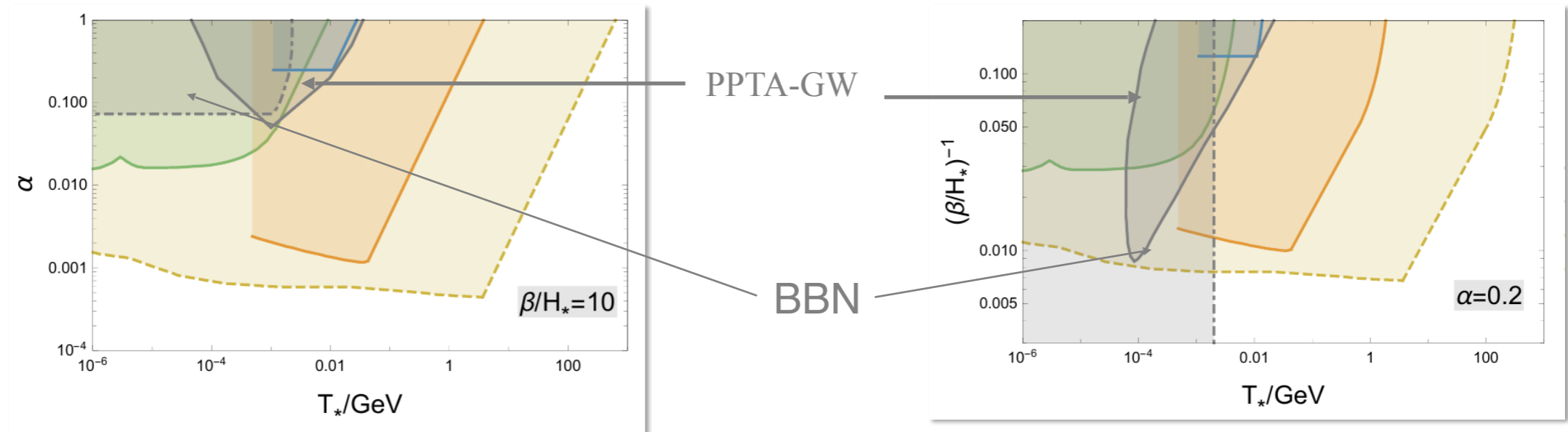
# 真空延迟衰变与曲率扰动限制相变

Hubble-sized perturbations



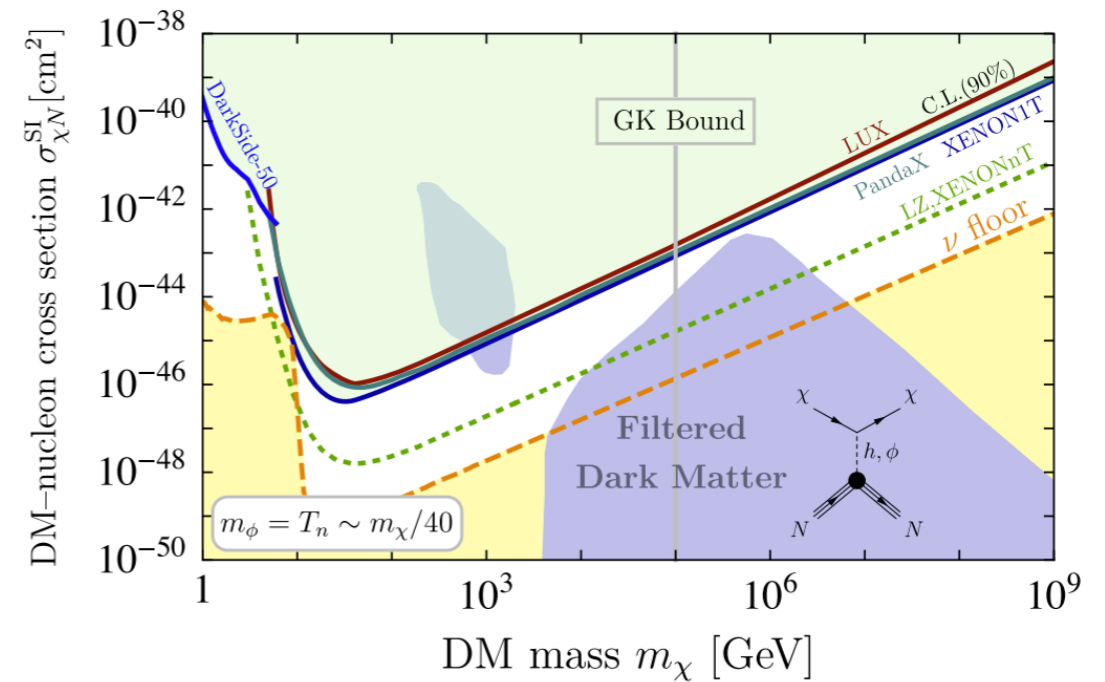
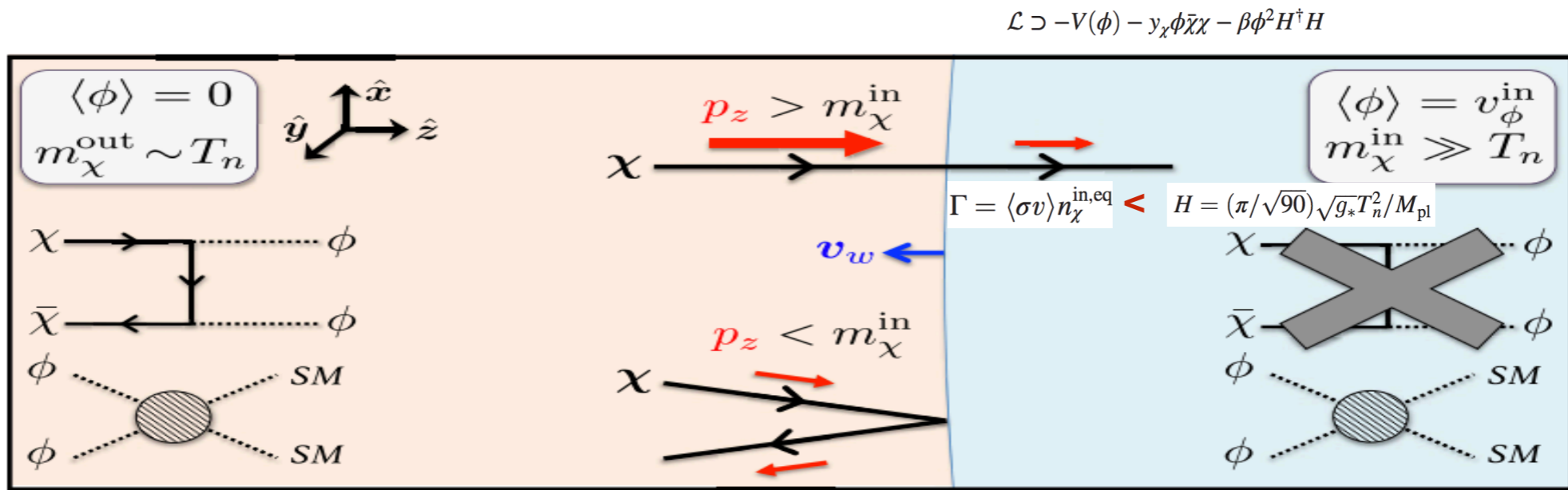
# 真空延迟衰变与曲率扰动限制相变

low-scale and slow 1st PTs motivated for dark PT and BAU



# WIMP 暗物质与强一阶相变

过  
滤  
暗  
物  
质

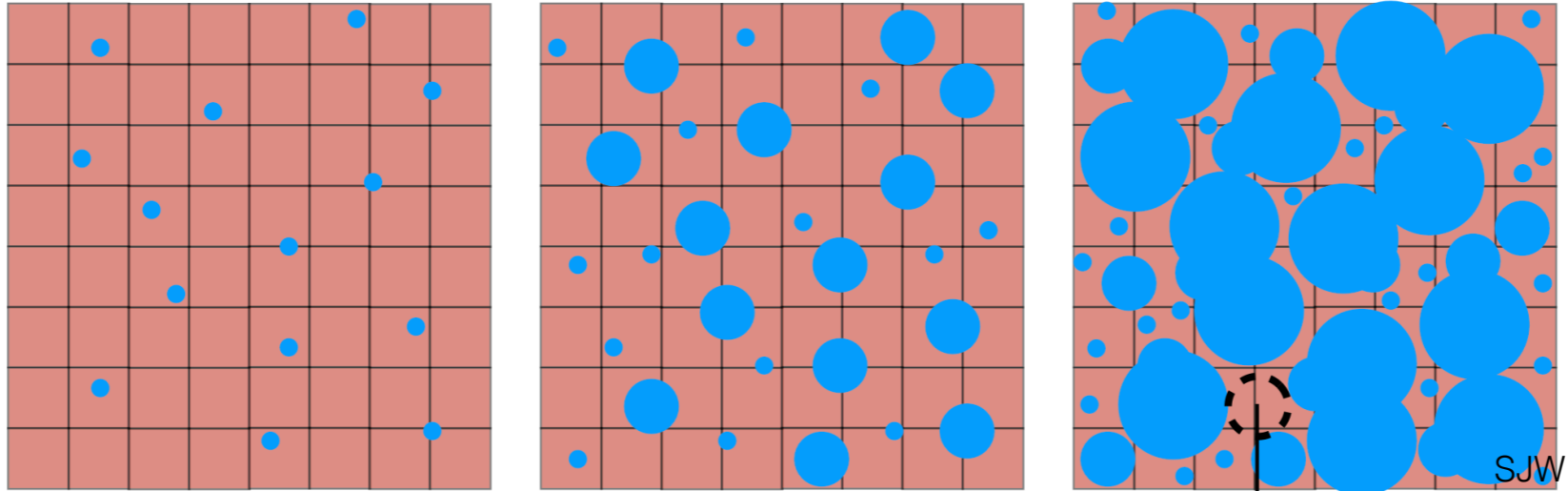


暗区一阶相变

Baker , Kopp, and Long, Phys.Rev.Lett. 125 (2020) 15, 151102

see also: Chao, Li, Wang, JCAP 06 (2021) 038

## PBH from postponed vacuum decay

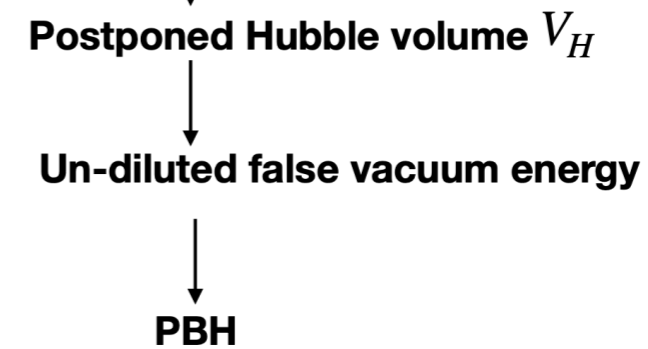


**Probability for a Hubble volume not to decay until time  $t_n$**

$$V_H(t) = \frac{4}{3}\pi H(t_{\text{PBH}})^{-3} \frac{a(t)^3}{a(t_{\text{PBH}})^3}$$

$$P(t_n) = \exp \left[ -\frac{4\pi}{3} \int_{t_i}^{t_n} \frac{a^3(t)}{a^3(t_{\text{PBH}})} H^{-3}(t_{\text{PBH}}) \Gamma(t) dt \right]$$

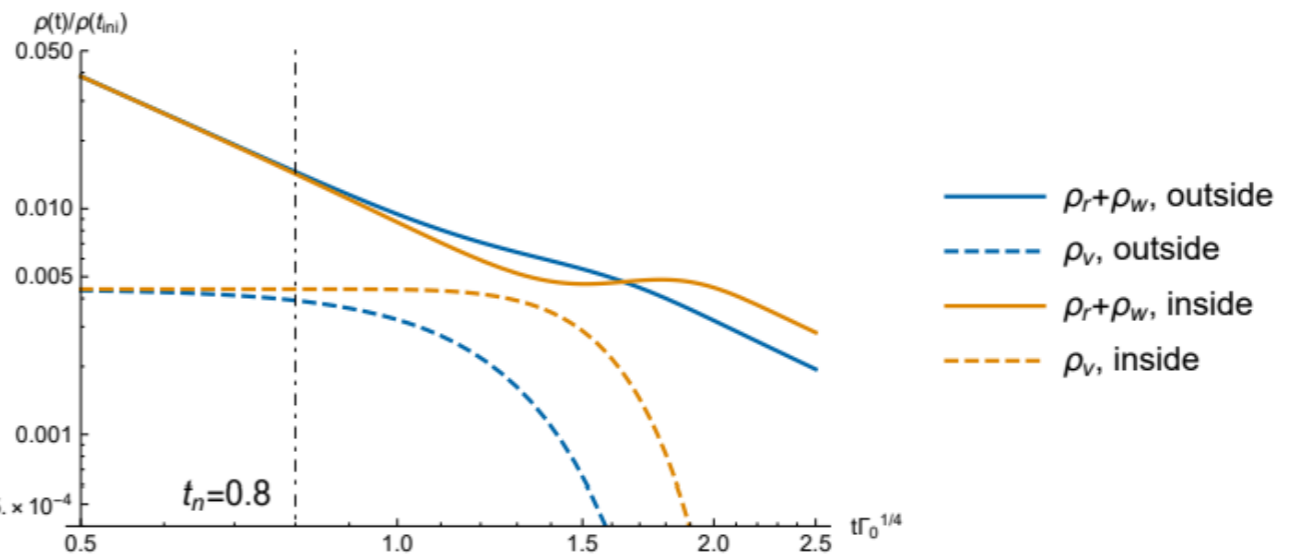
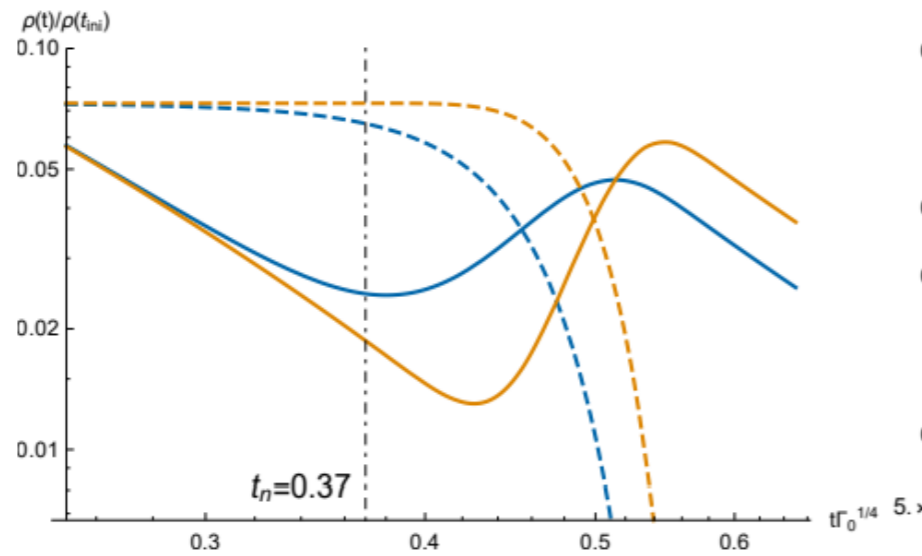
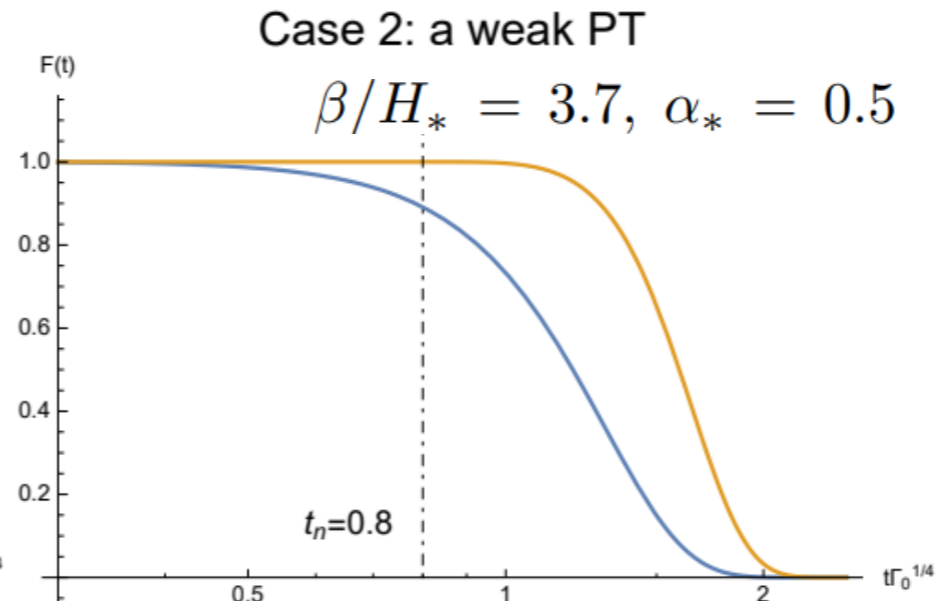
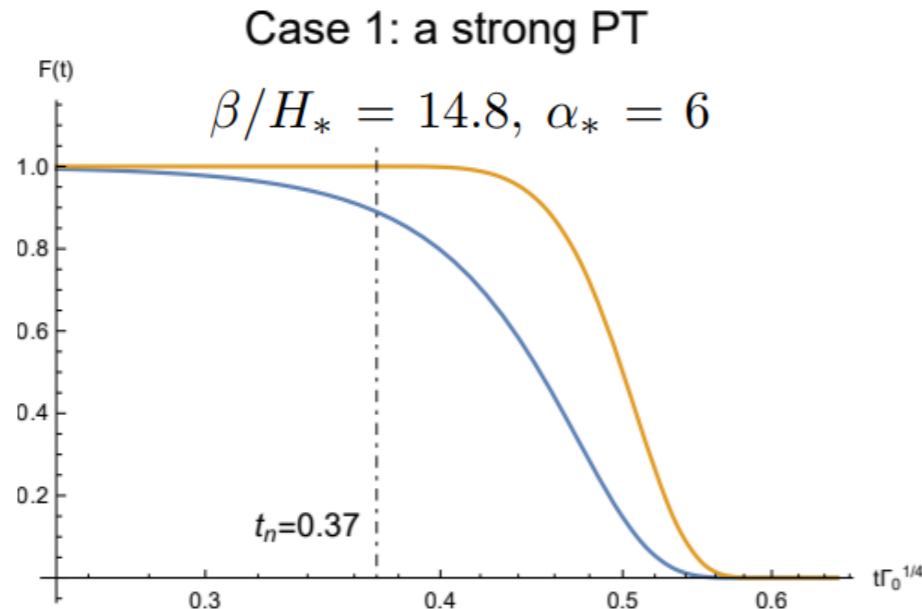
**PBH abundance**  $\Omega_{\text{PBH}}^{\text{form}} = P(t_n)$



**Collapse of the Hubble horizon**



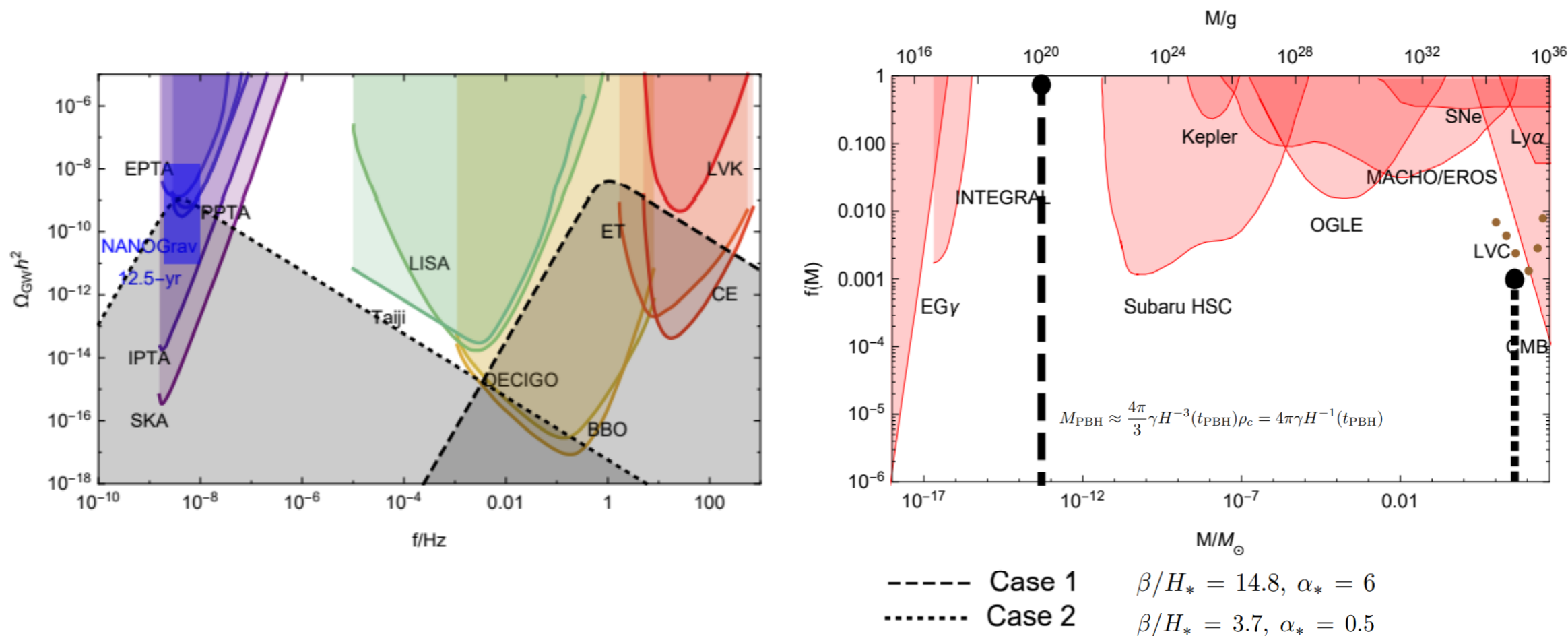
# PBH 暗物质和一阶相变



$$\delta(t_{\text{PBH}}) = \frac{\rho_v(t_{\text{PBH}}; t_n) + \rho_r(t_{\text{PBH}}; t_n)}{\rho_v(t_{\text{PBH}}; t_i) + \rho_r(t_{\text{PBH}}; t_i)} - 1 \geq \delta_c \Rightarrow t_{\text{PBH}}$$

# ● PBH 暗物质和一阶相变

PBH is more abundant in **strong and slow** first-order PTs.



- Case 1: PBHs constitute all dark matter,  $\Omega_{\text{GW}}$  to be probed CE,ET
- Case 2: GWs explain the CPL observed by NANOGrav, PBHs explain the coalescence events observed by the LIGO-Virgo collaboration

### ❖ Lattice simulation

- **PT GW simulation, Electroweak sphaleron, PT dynamics**
- **Topological defects: Magnetic monopoles, cosmic strings, domain walls**

### ❖ Pheno

1. EWSB and GW from FOPT
- **Probing the Higgs Potential shape and EWPT patterns with GW production and Colliders complementarily**
2. BAU and GW from FOPT
- **Sphaleron process, bubble dynamics**
3. DM and GW from FOPT
- **DM and high/low-scale PT, DM out-of-equilibrium & FOPT, PBH DM&FOPT**

谢谢！