Secondary Gravitational wave as a new window to the early universe

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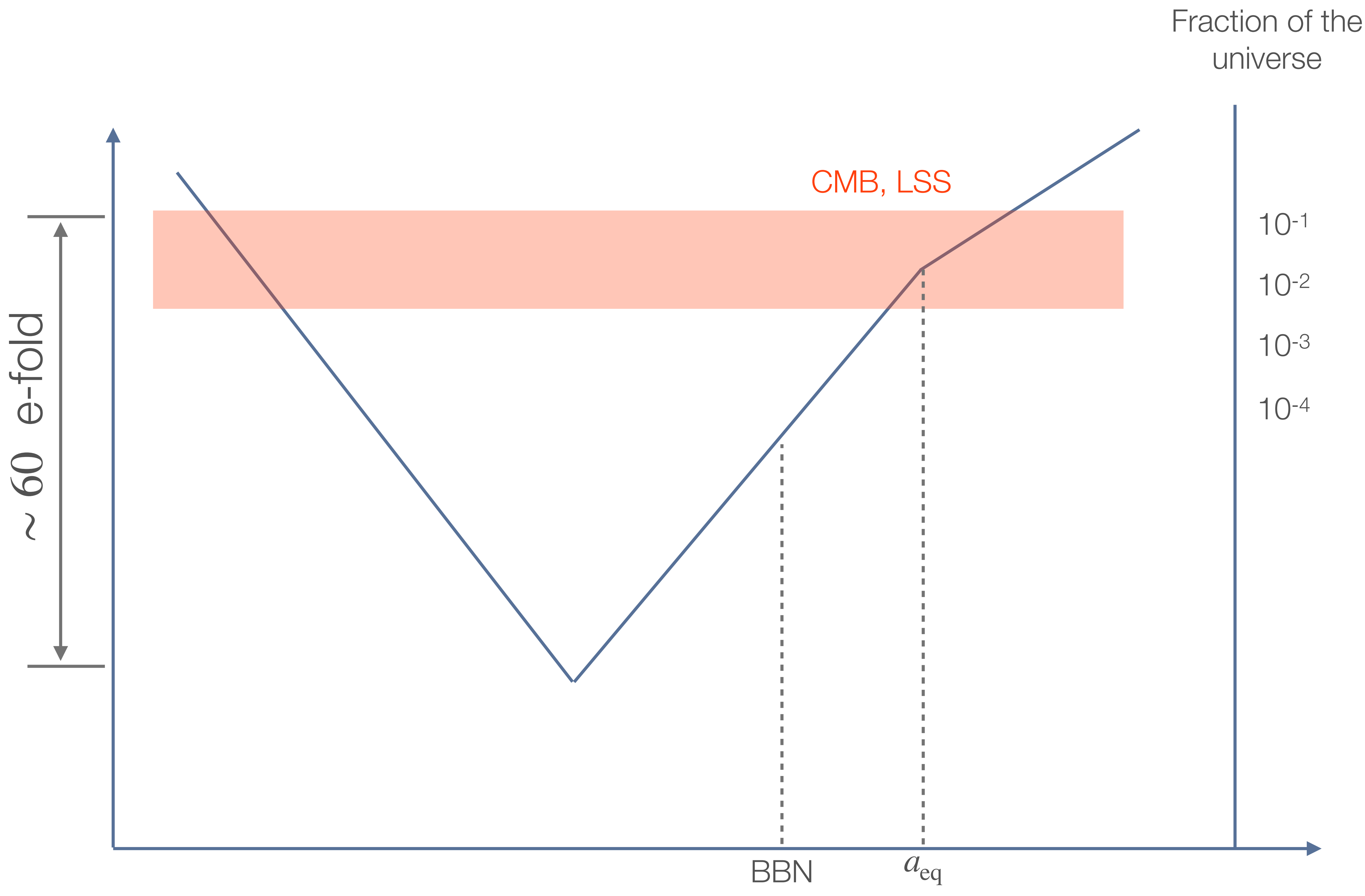
Work in collaboration with
Reza Ebadi, Soubhik Kumar, Amara McCune, Hanwen Tai, LTW, 2307.12048
Soubhik Kumar, Hanwen Tai, LTW, 2410.17291

NOPP Workshop, IHEP, July 18, 2025

Large scale structure



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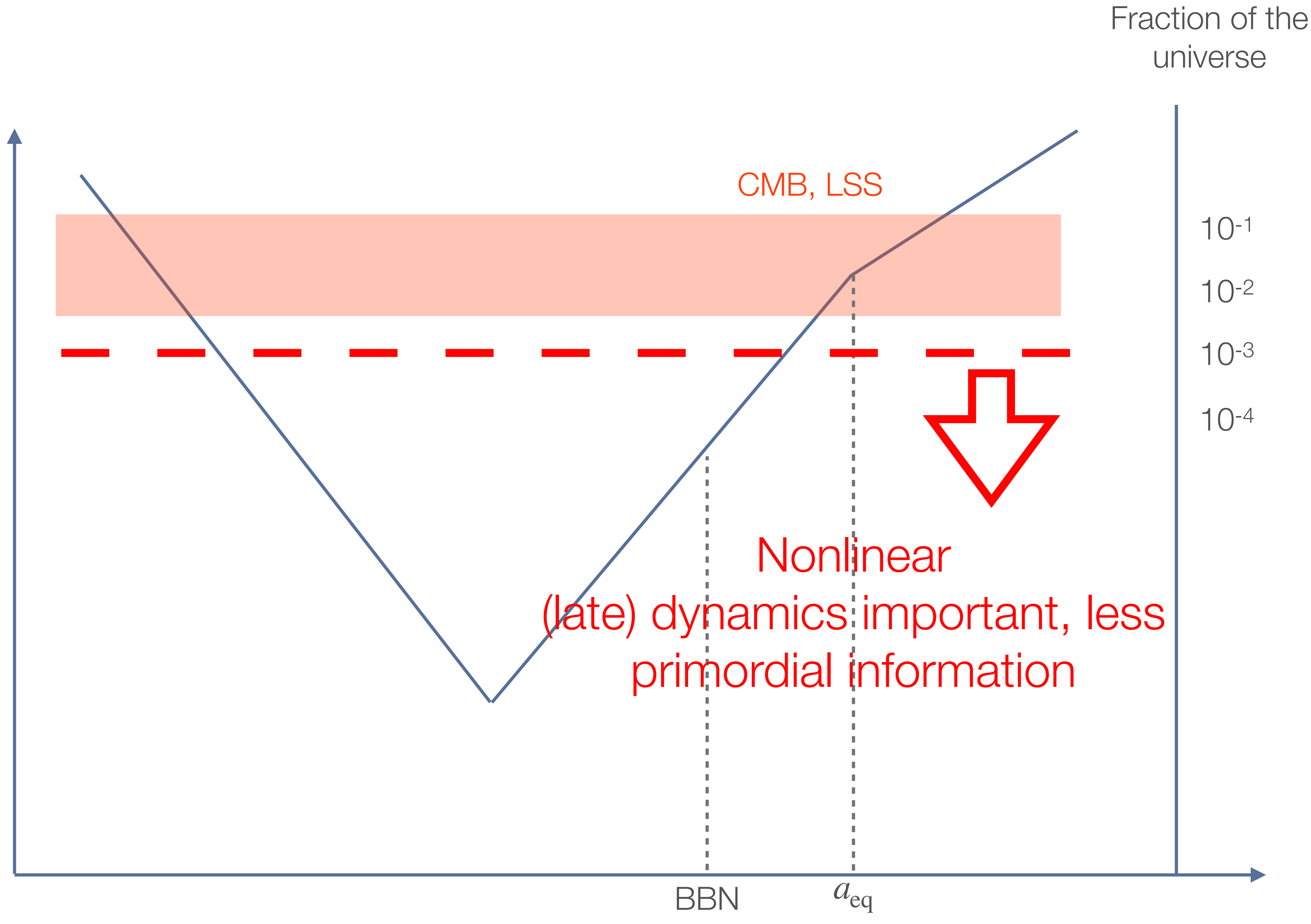
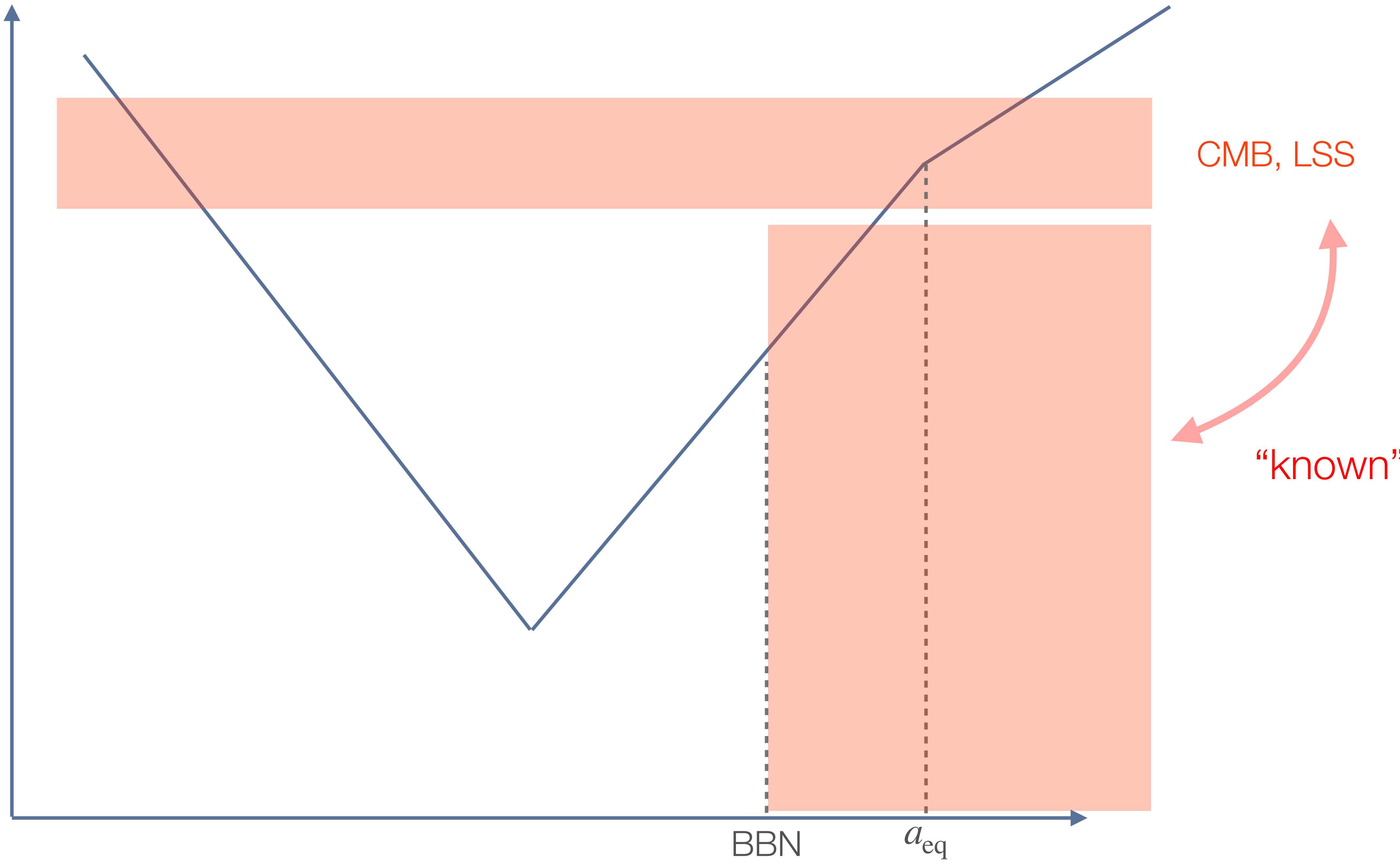


What we know



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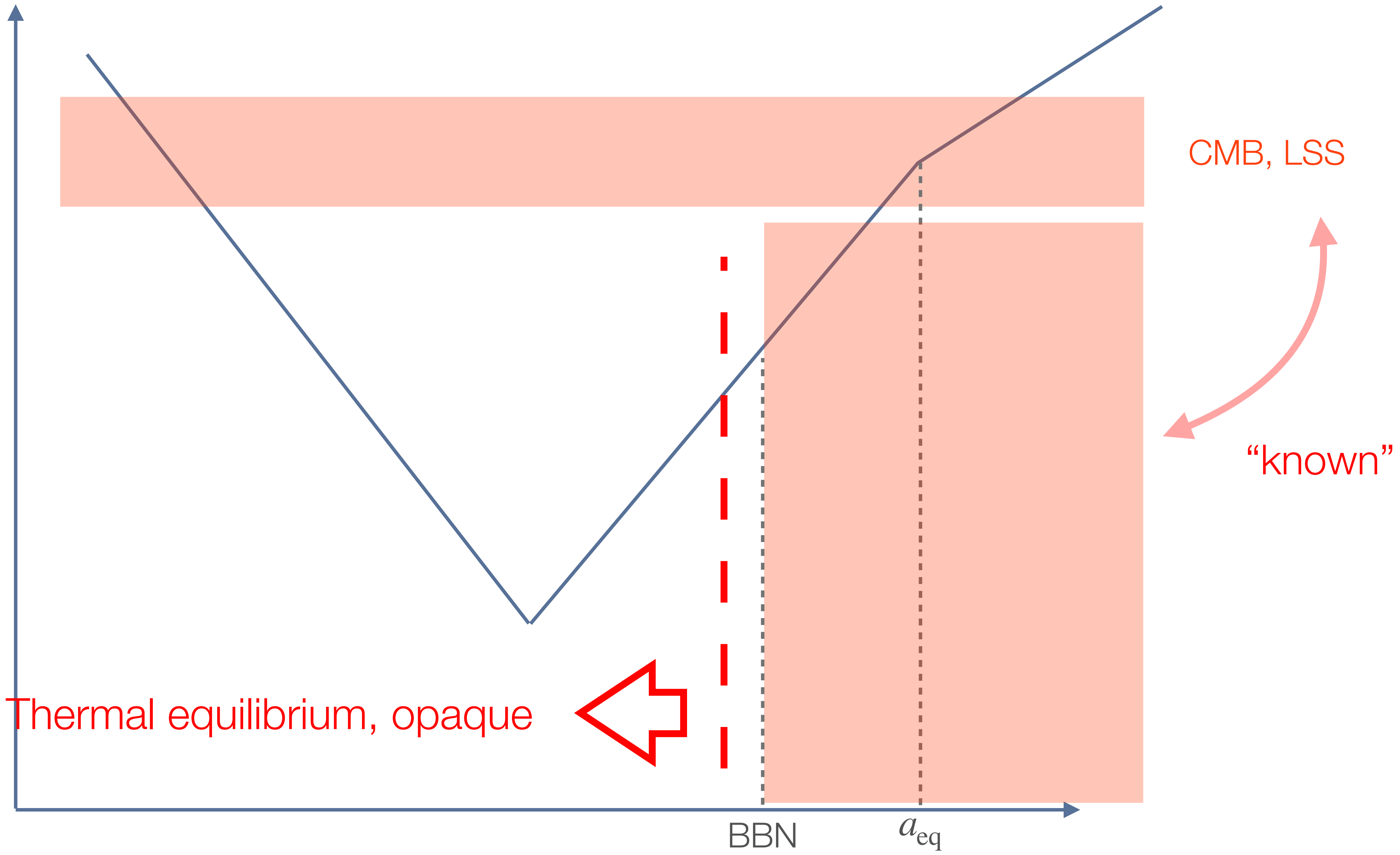
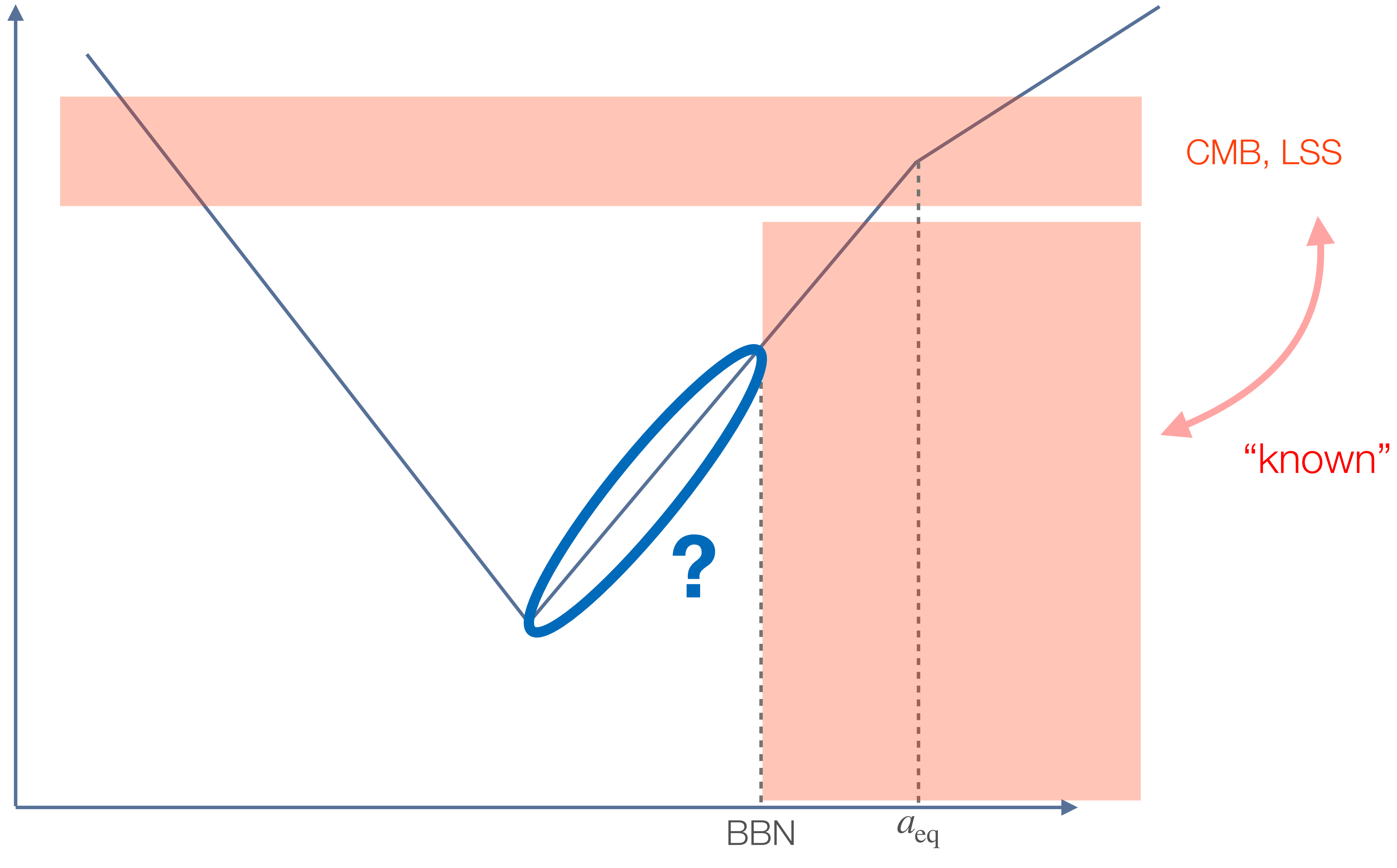
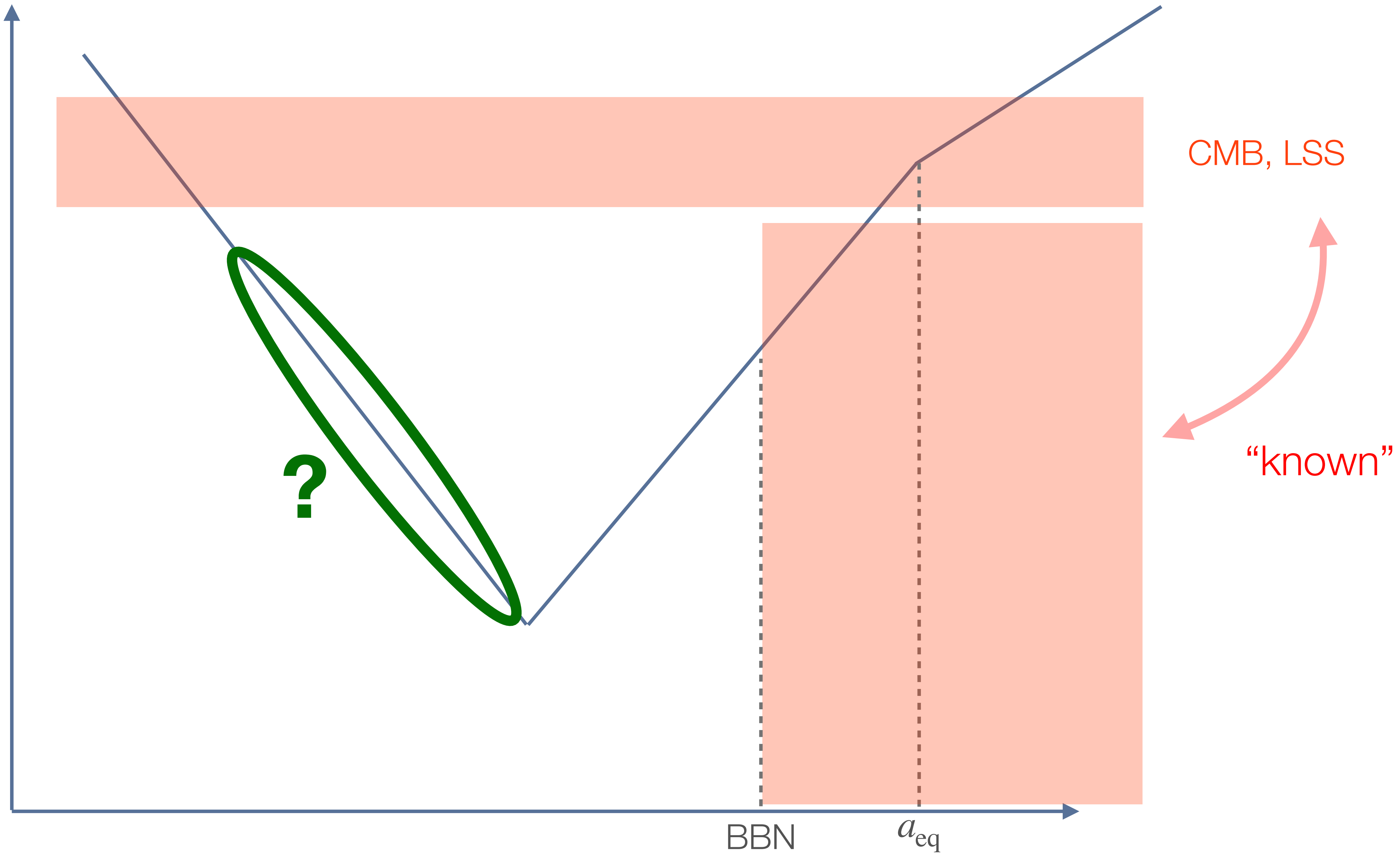


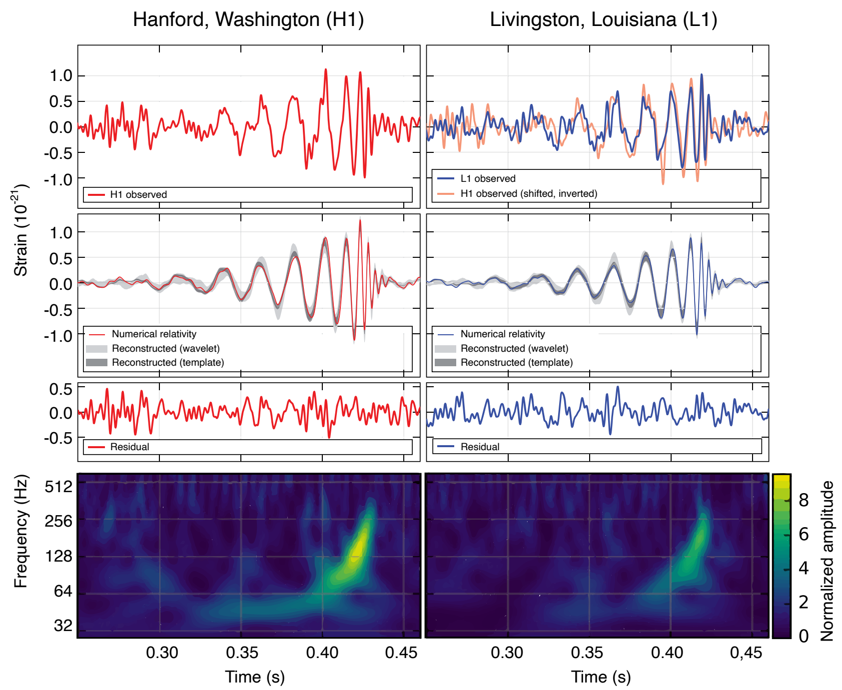
Reheating and after?

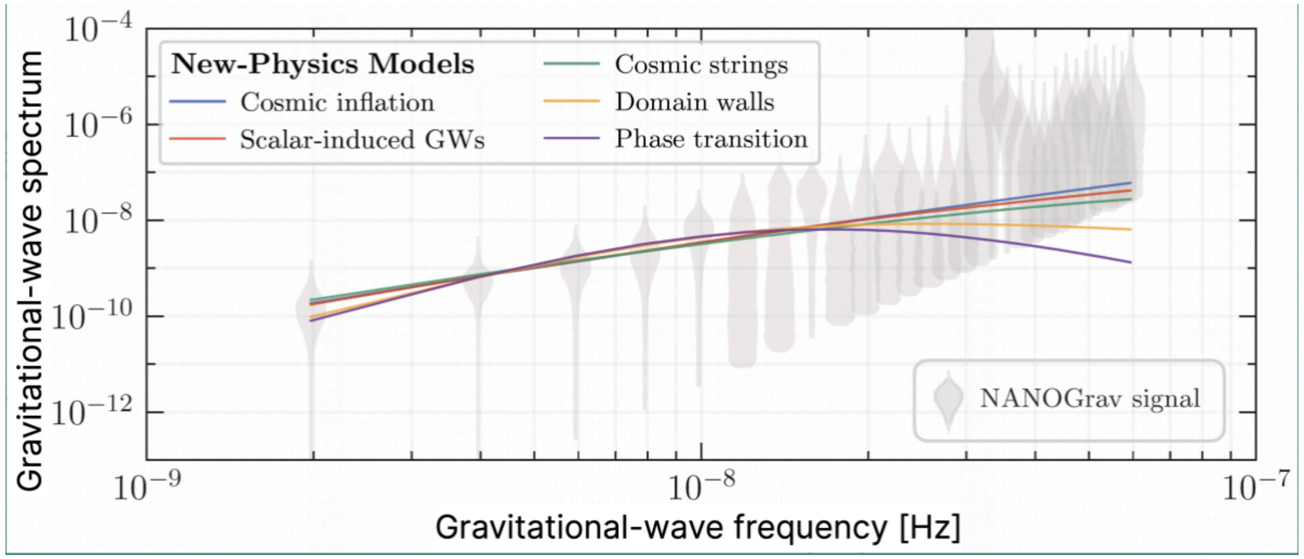


Inflation?



GW Discoveries

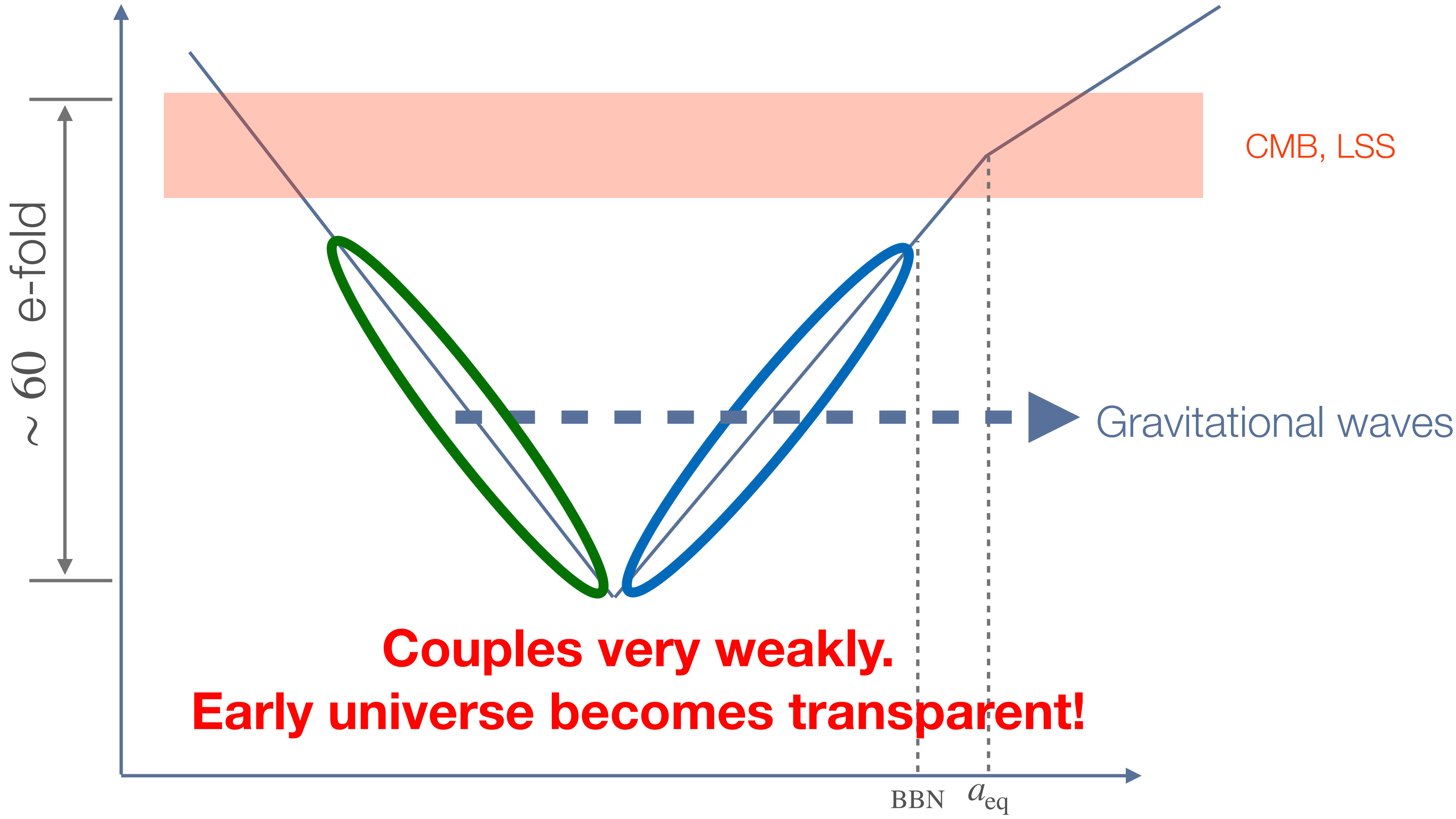




LIGO

NanoGrav

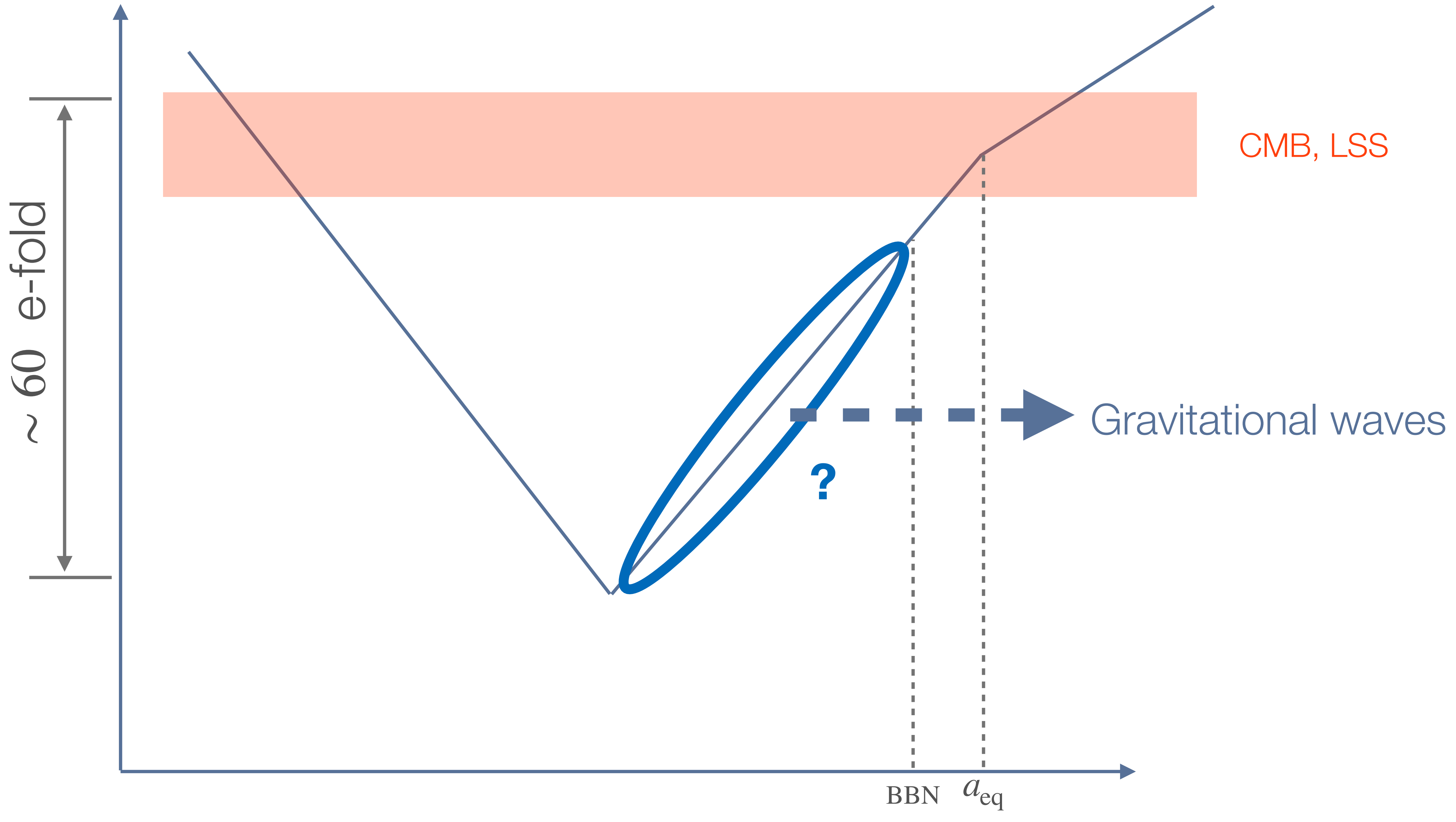
A new window: gravitational waves



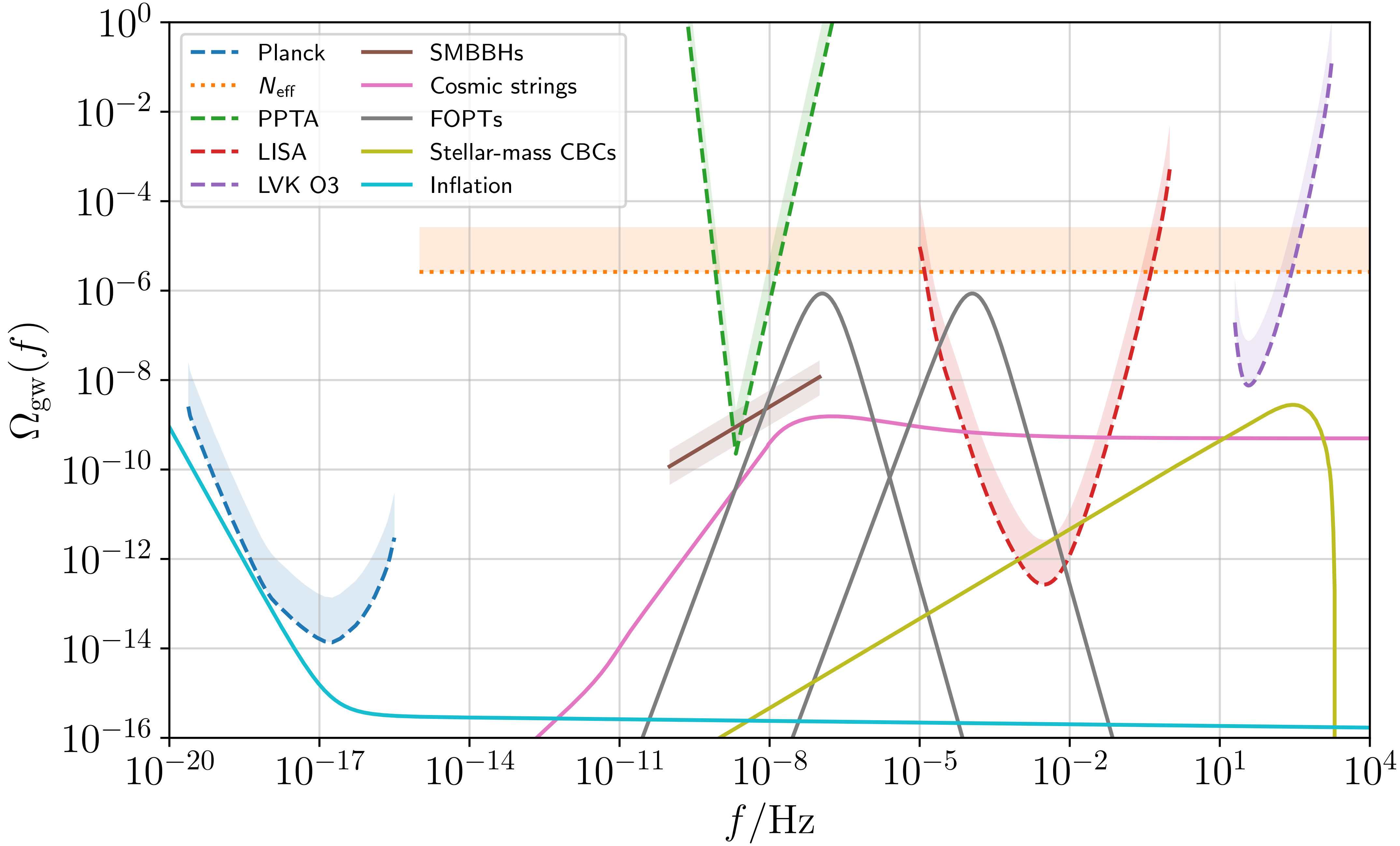
Couples very weakly.
Early universe becomes transparent!

What are we looking for?

Post inflationary

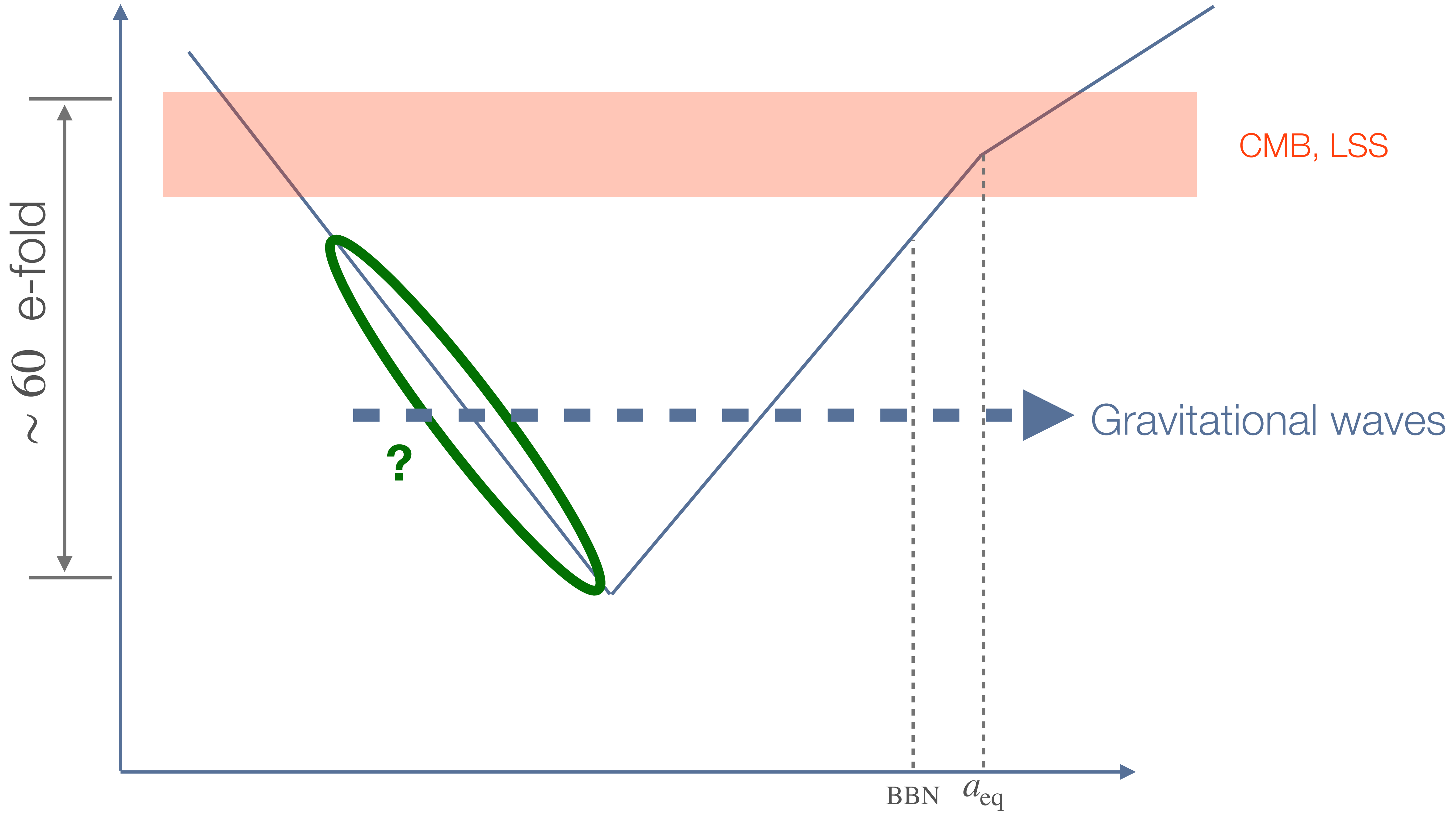


Review by Renzini et al, 2202.00178

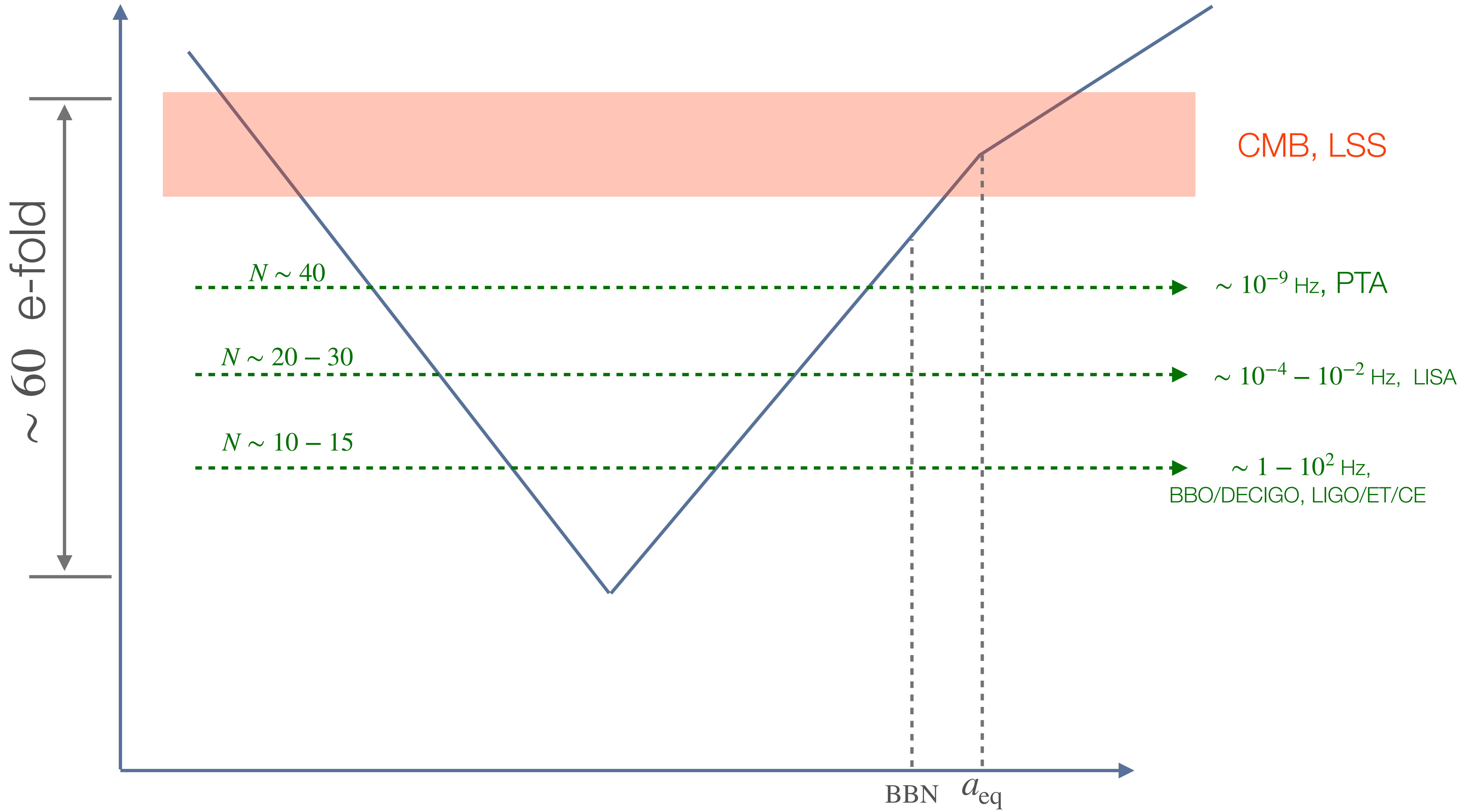


Typically, need something quite dramatic.

Inflationary era



Early universe



Interesting stories

Secondary GW from spectator scalar

First order phase transition during inflation

GW from inflated string domain wall network

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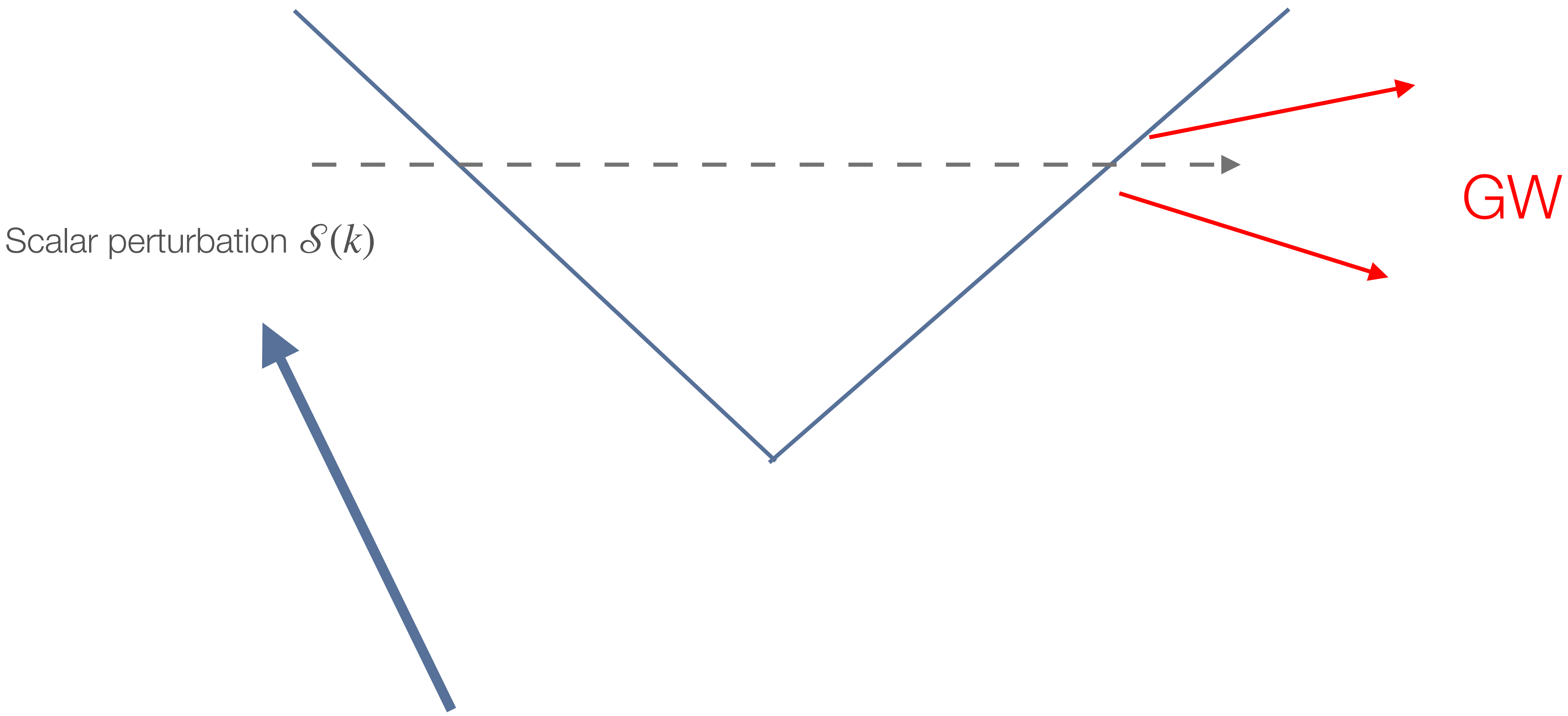
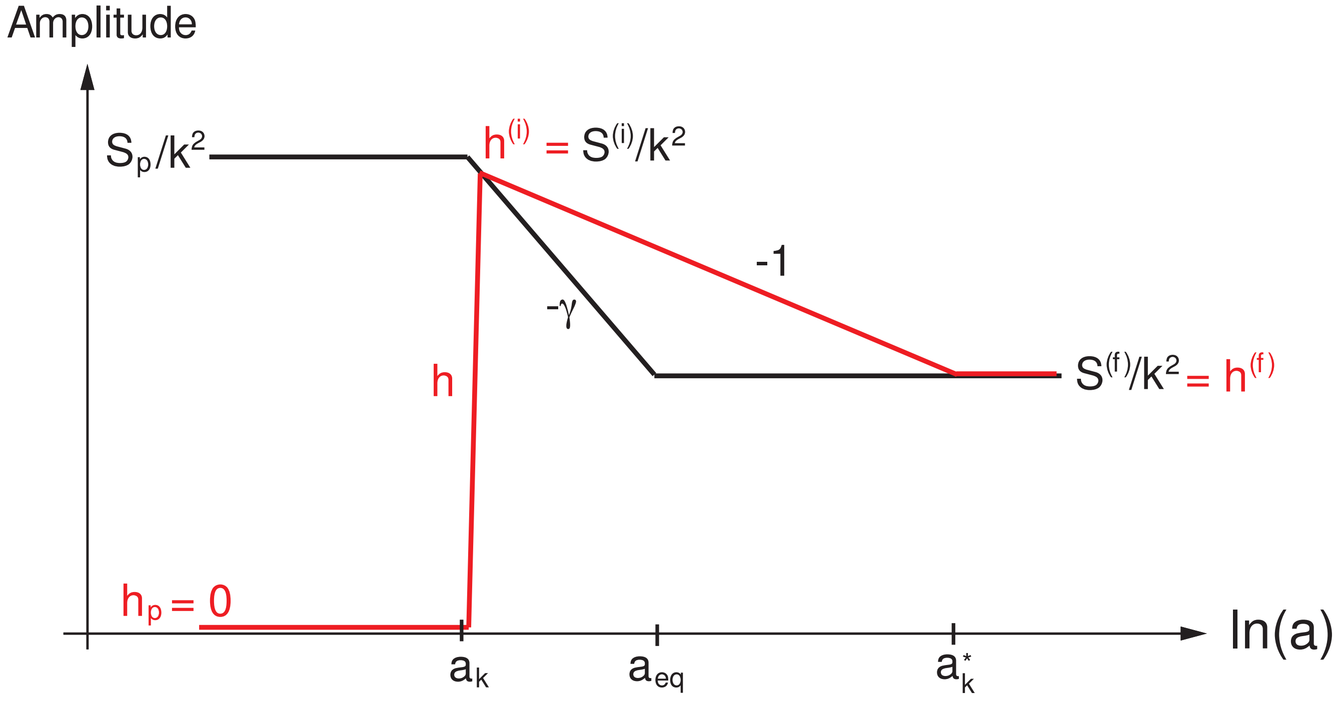
GW from inflated string domain wall network

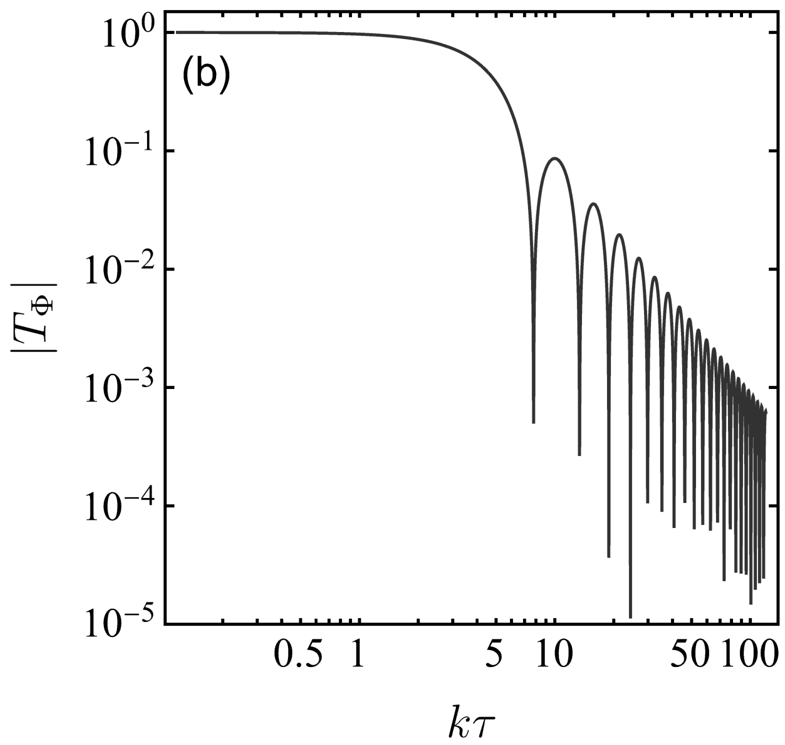
Example: secondary GW



In addition to the inflaton, many other fields have quantum fluctuations

Example: secondary GW





Baumann, Steinhardt, Takahashi, hep-th/0703290

Modes enter horizon during RD, starts oscillate, and generates GW

Size of the signal

Curvature perturbation Φ

$$\mathrm{d}s^2 = -(1+2\Phi)\,\mathrm{d}t^2 + a^2\left((1-2\Phi)\,\delta_{ij} + \frac{1}{2}h_{ij}\right)\mathrm{d}x^i\mathrm{d}x^j$$

Einstein equation:

$$h'' + 2Hh' + k^2h = \Phi\partial^2\Phi + \ldots$$

Gravitational wave abundance:

$$\Omega_{\mathrm{gw}} \propto (\dot{h})^2 \propto \Phi^4 \sim \Omega_{\mathrm{rad}} P_\zeta^2$$

On large (CMB, LSS) scales: $\Omega_{\mathrm{rad}} \sim 10^{-5}$, $P_\zeta \sim 10^{-9}$

Clearly, to have observable signal, need much larger curvature perturbation on smaller scales.

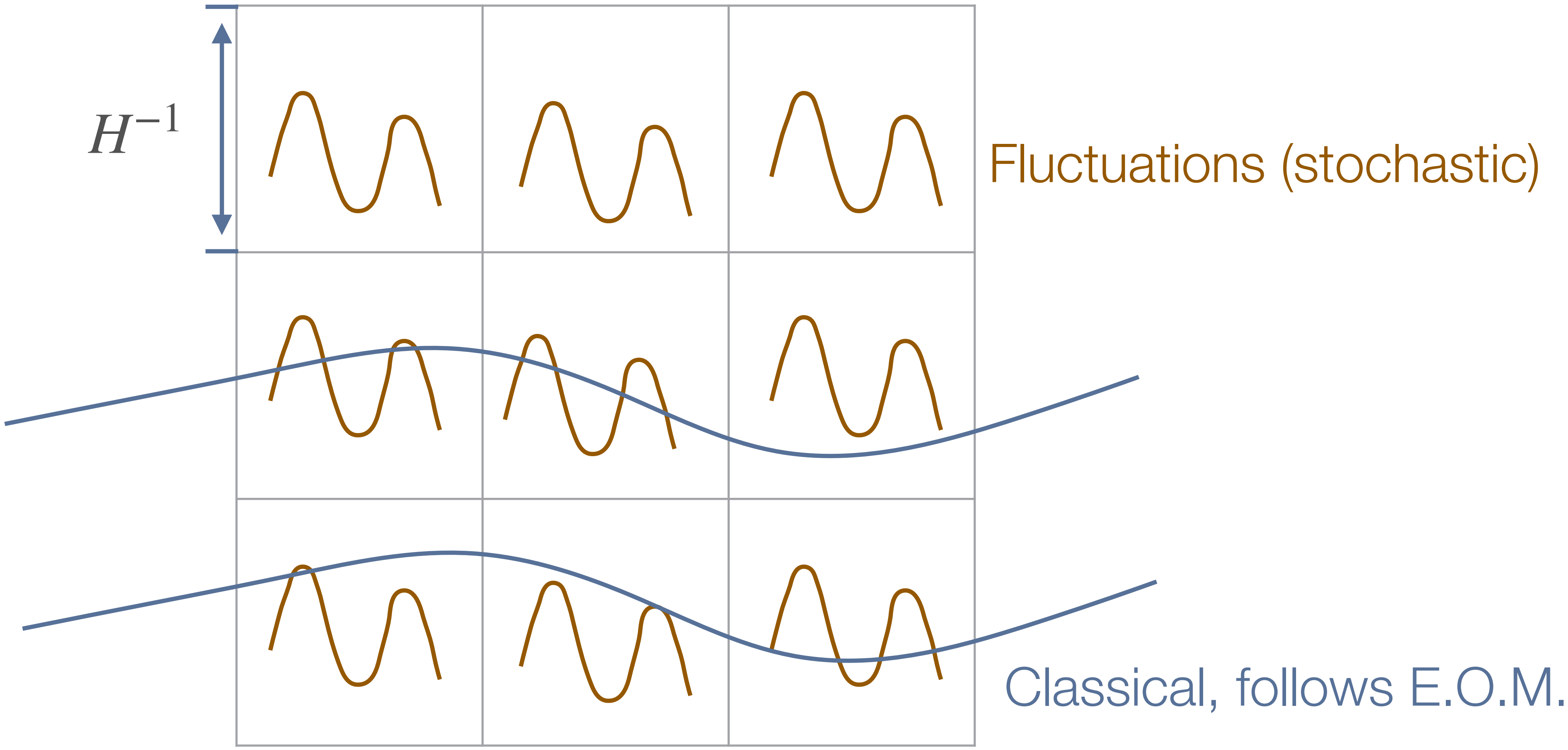
A spectator light scalar

R. Ebadi, S. Kumar, A. McCune, H. Tai, LTW 2023

$$\mathcal{L} = \frac{1}{2}(\partial\sigma)^2 - \frac{1}{2}m^2\sigma^2 - \frac{\lambda}{4}\sigma^4$$

with $m < H$

Evolution of fluctuations: small vs large scales



Stochastic method

The spectrum of its fluctuation on large scales
can be studied by stochastic method

Starobinsky and Yokoyama, 1994

Fokker-Planck
$$\frac{\partial P_{\mathrm{FP}}(t,\sigma)}{\partial t} = \left(\frac{V''(\sigma)}{3H} + \frac{V'(\sigma)}{3H}\frac{\partial}{\partial \sigma} + \frac{H^3}{8\pi^2}\frac{\partial^2}{\partial^2 \sigma} \right) P_{\mathrm{FP}}(t,\sigma)$$

$P_{\mathrm{FP}}(t,\sigma)$: 1-pt PDF for field σ

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Classical evolution, drift

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Classical evolution, drift   Stochastic, diffusion

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Starobinsky and Yokoyama, 1994; Markkanen, Rajantie, Stopyra, Tenkanen, 1904.11917

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Classical evolution, drift — Stochastic, diffusion

$P_{\rm FP}(t,\sigma)$: 1-pt PDF

Light field during inflation

$$m_\sigma^2 < H^2$$

1. Massless. "Stuck" at large field value.
    - Example: misaligned axion.
2. Massive but light.

Light field during inflation

- Massive but light. (Free field for simplicity)

$$\dot{\sigma} = -\frac{m_\sigma^2 \sigma}{3H} \quad \rightarrow \quad \sigma = \exp\left(-m_\sigma^2 \int^t \frac{dt'}{3H(t')}\right) \boxed{\cdot \, \sigma_i}$$

Initial field value

Light field during inflation

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Initial field value

- Roughly, $-\int^t \frac{dt'}{3H(t')} \sim \frac{1}{\dot{H}}$

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Initial field value

- Roughly, $-\int^t \frac{dt'}{3H(t')} \sim \frac{1}{\dot{H}}$

- If $m_\sigma^2 > \epsilon H^2$ $(\epsilon = \dot{H}/H^2)$,
  - Initial value of field does not matter. Amplitude of field dominated by stochastic fluctuation around origin

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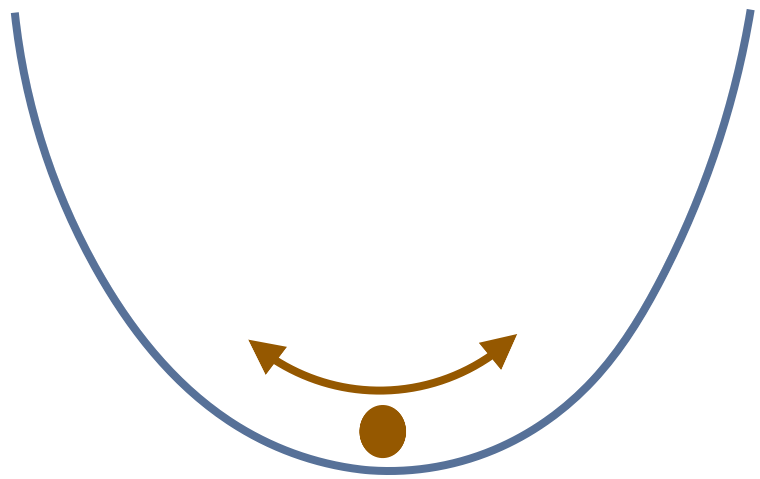
R. Ebadi, S. Kumar, A. McCune, H. Tai, LTW 2023

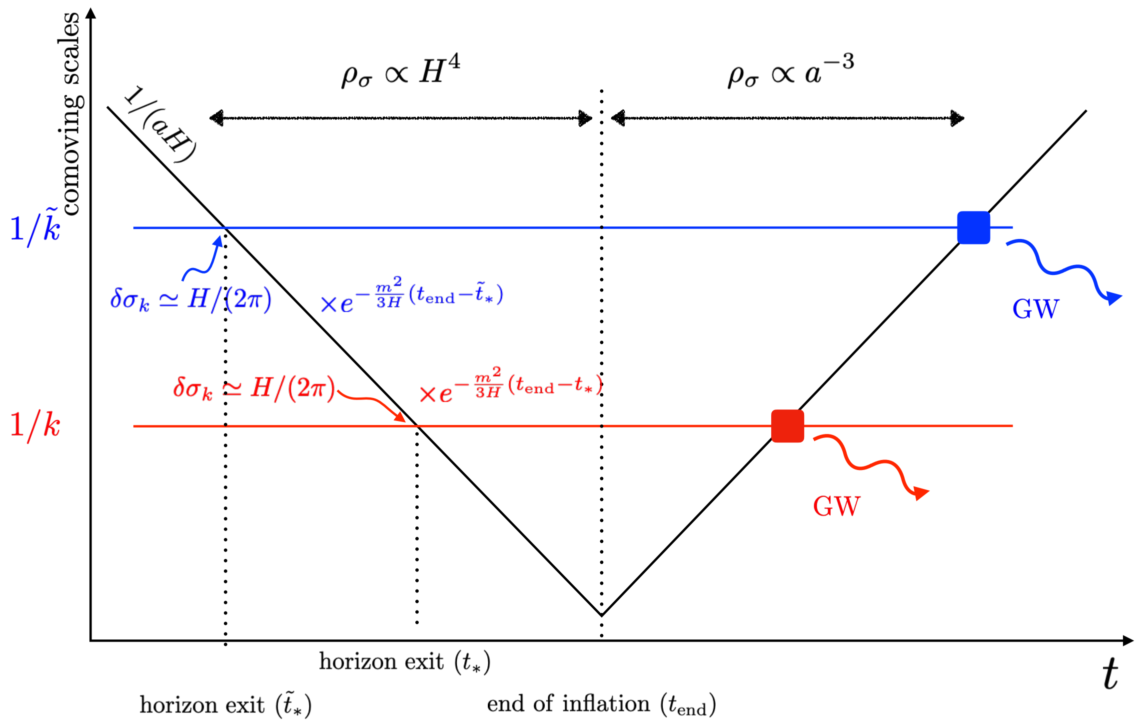
$$\mathcal{L} = \frac{1}{2}(\partial\sigma)^2 - \frac{1}{2}m^2\sigma^2 - \frac{\lambda}{4}\sigma^4$$ with $m < H$



$$\mathcal{P}_f(k) = \sum_n \frac{2}{\pi} f_n^2 \Gamma\left(2 - 2\frac{\Lambda_n}{H}\right) \sin\left(\frac{\Lambda_n \pi}{H}\right)\left(\frac{k}{H}\right)^{2\Lambda_n/H} \quad \to \mathscr{A}\left(\frac{k}{H}\right)^{\frac{2\Lambda_{\text{lowest}}}{H}}$$ for $k \ll H$

Starobinsky and Yokoyama, 1994; Markkanen, Rajantie, Stopyra, Tenkanen, 1904.11917

Blue tilt

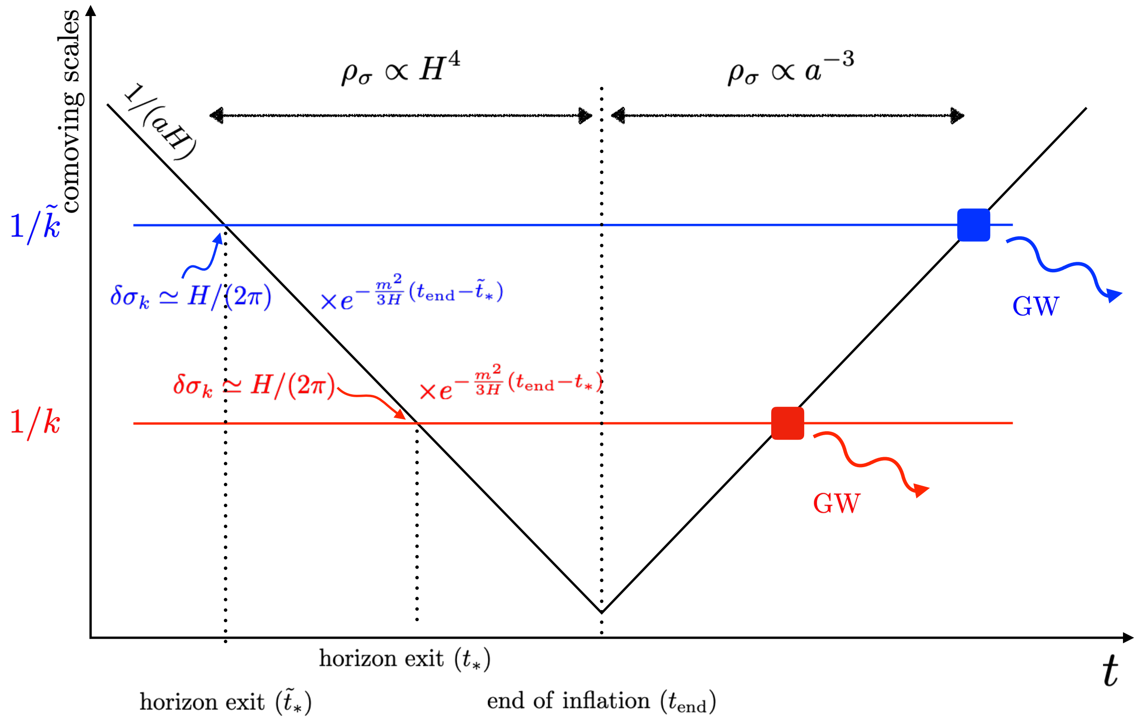


At horizon exit:
Amplitude ≈ H

After exit, damping

$$\dot{\sigma} = -\frac{m_\sigma^2 \sigma}{3H}$$

Blue tilt



At horizon exit:
Amplitude ≈ H

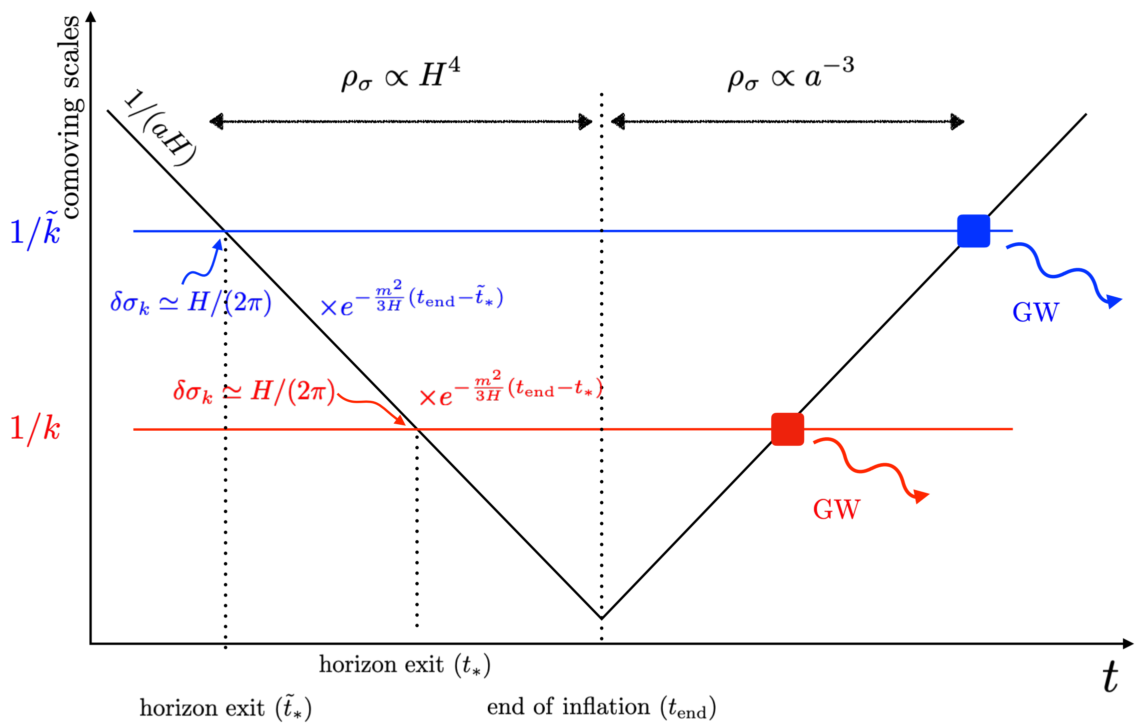
After exit, damping

$$\dot{\sigma} = -\frac{m_\sigma^2 \sigma}{3H}$$

$$\sigma_k(t) = \sigma(t_*) \exp\left(-\frac{m_\sigma^2}{3H}(t-t_*)\right) = \sigma(t_*) \left[\exp\left(-H(t-t_*)\right)\right]^{\frac{m_\sigma^2}{3H^2}} = \sigma(t_*) \left[\frac{k(t)}{H}\right]^{\frac{m_\sigma^2}{3H^2}}$$

More damping for longer wave-length (earlier exit)

Blue tilt



At horizon exit:
Amplitude ≈ H

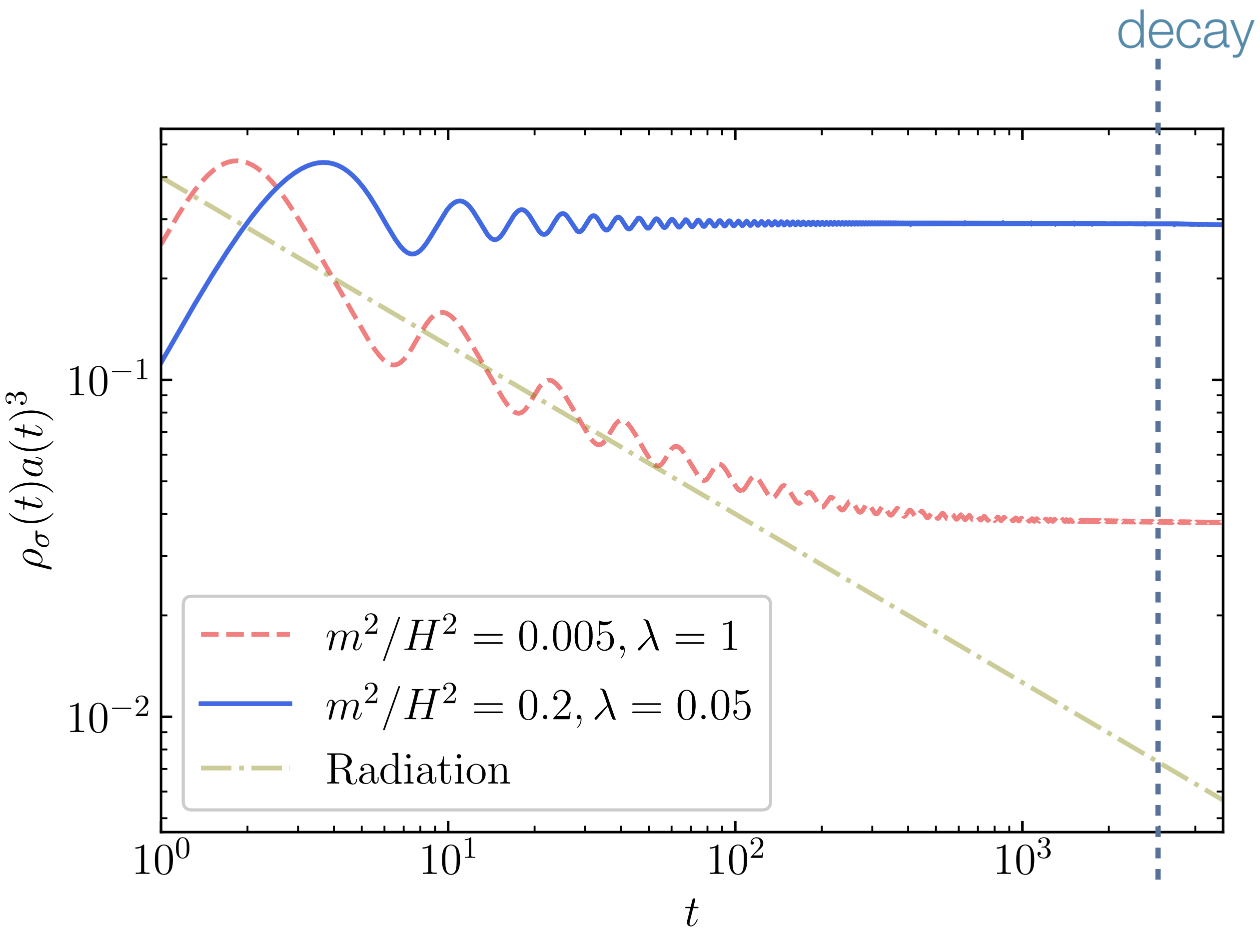
After exit, damping

$$\dot{\sigma} = -\frac{m_\sigma^2 \sigma}{3H}$$

For more general scalar theory

$$\mathcal{P}_f(k) = \sum_n \frac{2}{\pi} f_n^2 \Gamma\left(2 - 2\frac{\Lambda_n}{H}\right) \sin\left(\frac{\Lambda_n \pi}{H}\right)\left(\frac{k}{H}\right)^{2\Lambda_n/H} \quad \to \mathscr{A}\left(\frac{k}{H}\right)^{\frac{2\Lambda_{\rm lowest}}{H}} \text{ for } k \ll H$$

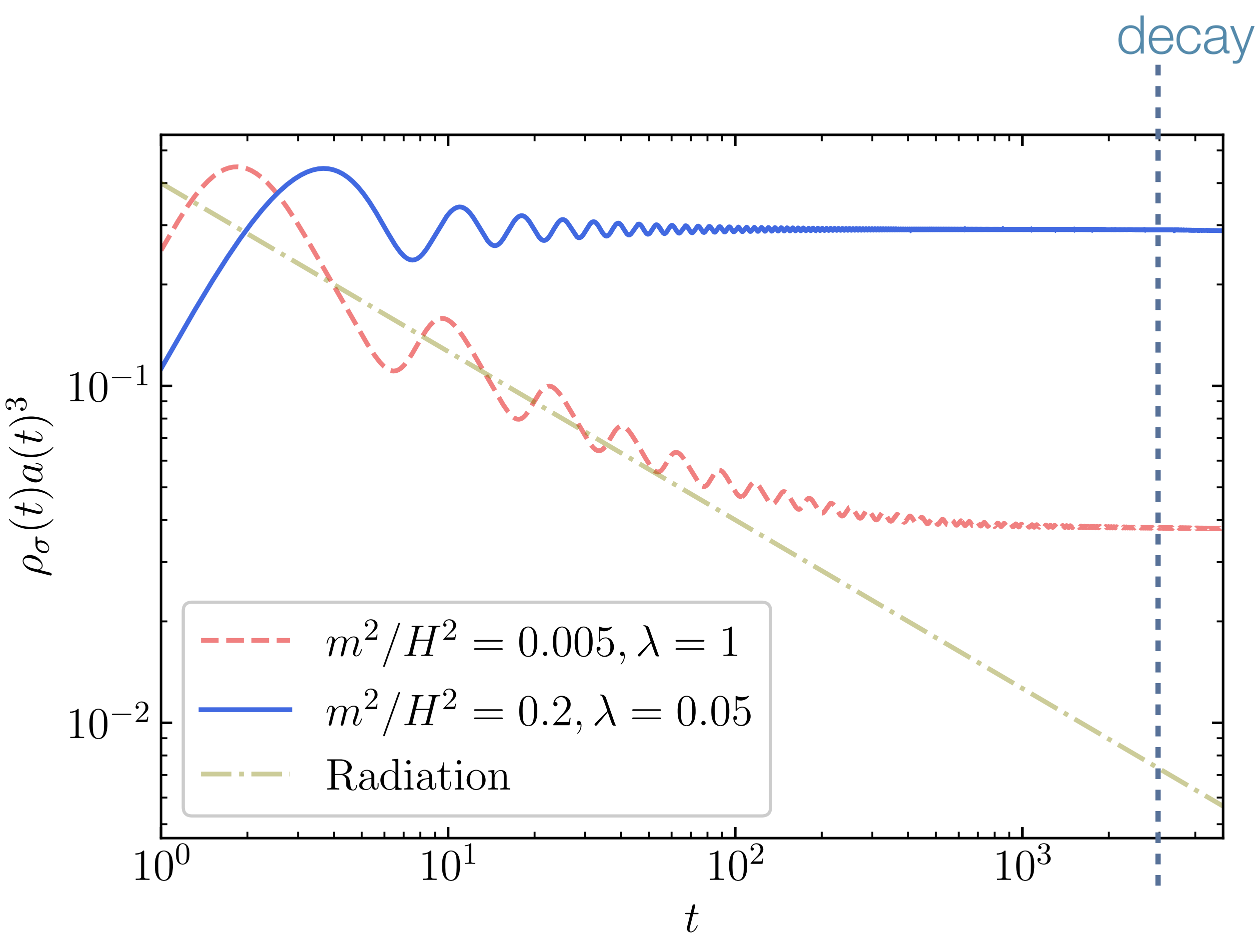
After inflation



Eventually,
evolve like matter

Can become important

After inflation



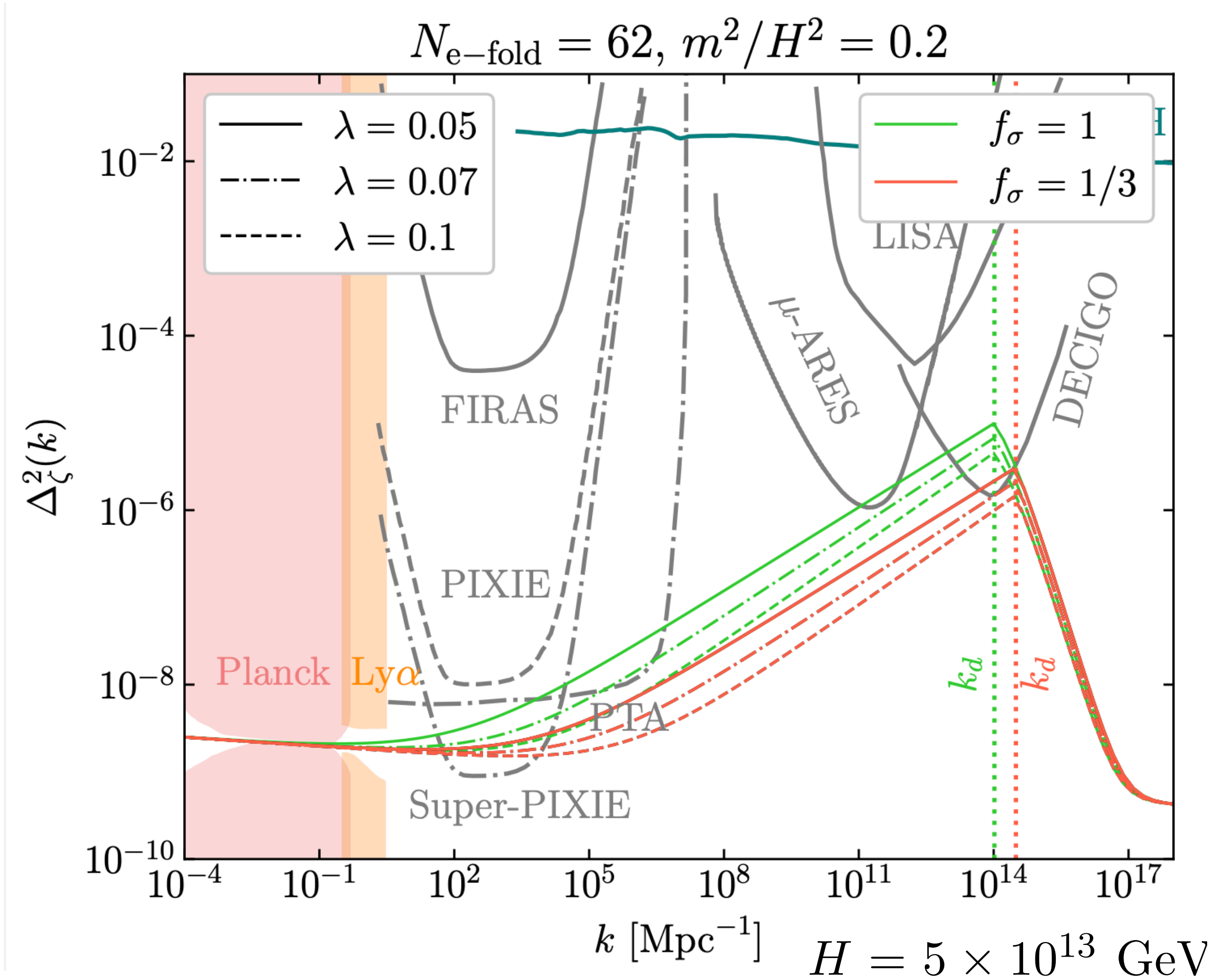
Eventually,
evolve like matter

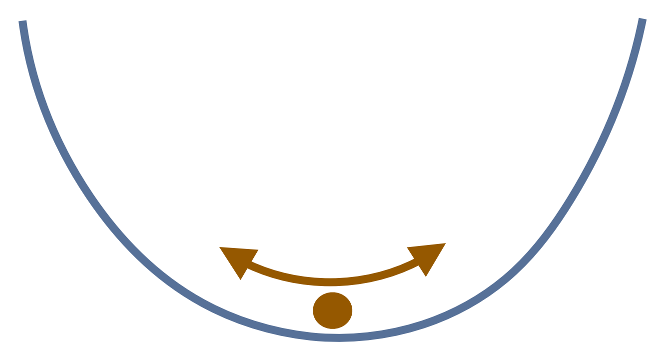
Can become important

$$\Delta_\zeta^2(k) = \begin{cases} \Delta_{\zeta_r}^2(k) + \left(\frac{f_\sigma(t_d)}{4+3f_\sigma(t_d)}\right)^2 \Delta_{S_\sigma}^2(k), & k < k_d, \\ \Delta_{\zeta_r}^2(k) + \left(\frac{f_\sigma(t_d)(k_d/k)}{4+3f_\sigma(t_d)(k_d/k)}\right)^2 \Delta_{S_\sigma}^2(k), & k > k_d \end{cases}$$

Power spectrum

Reza Ebadi, Soubhik Kumar, Amara McCune, Hanwen Tai, LTW, 2307.12048





Assuming the scalar behave similar to curvaton.

Becoming important before decay.

Assumption: scalar field does not dominate (more later)

Power spectrum

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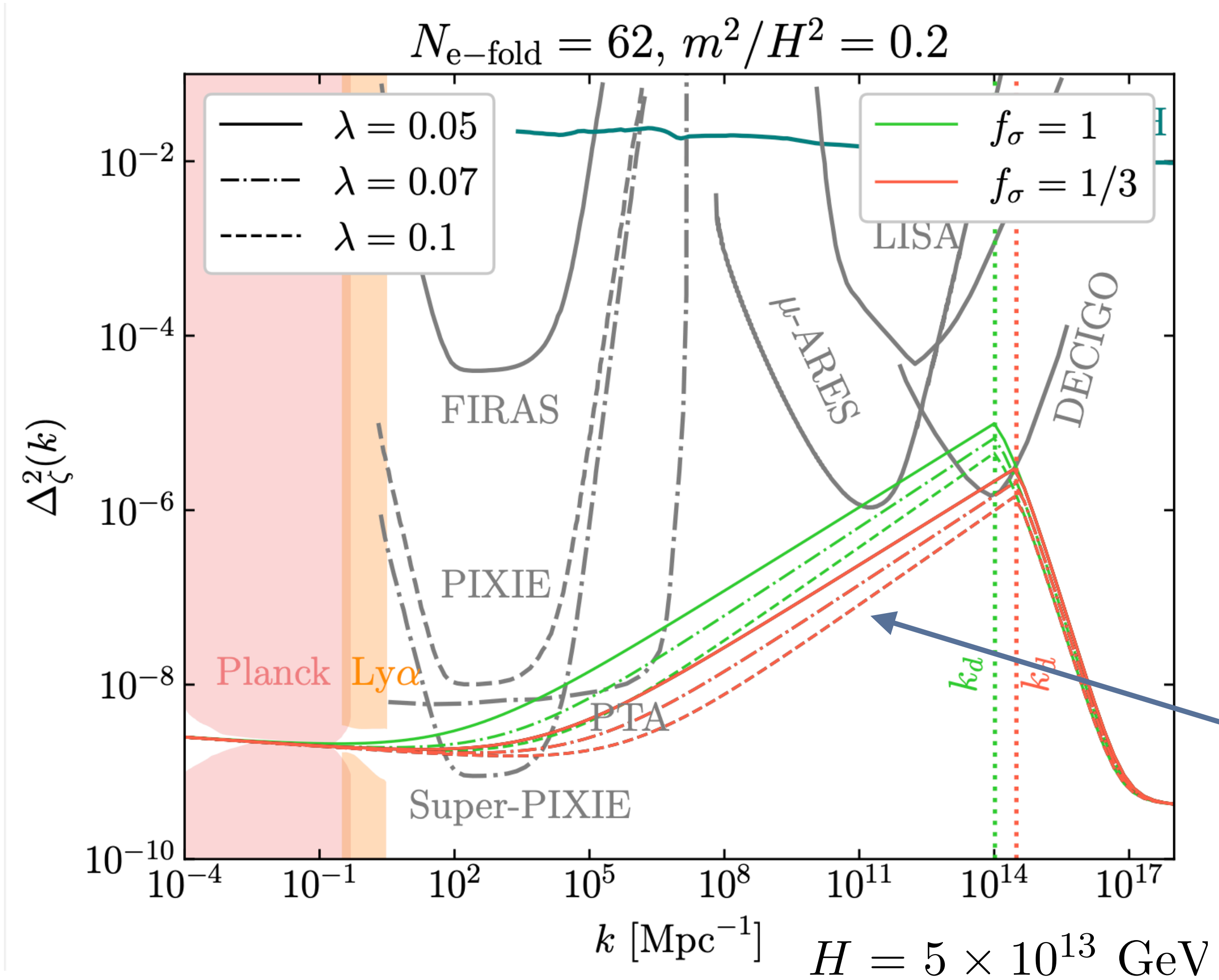
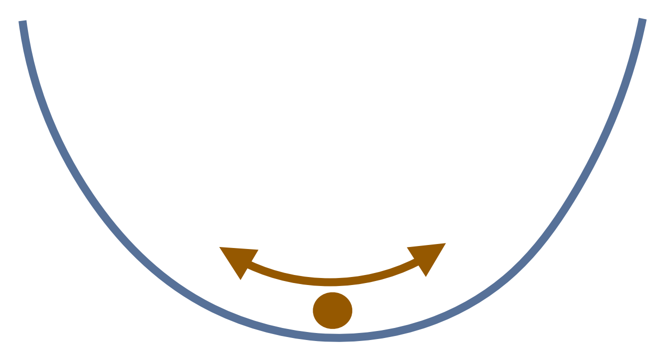


blue tilt



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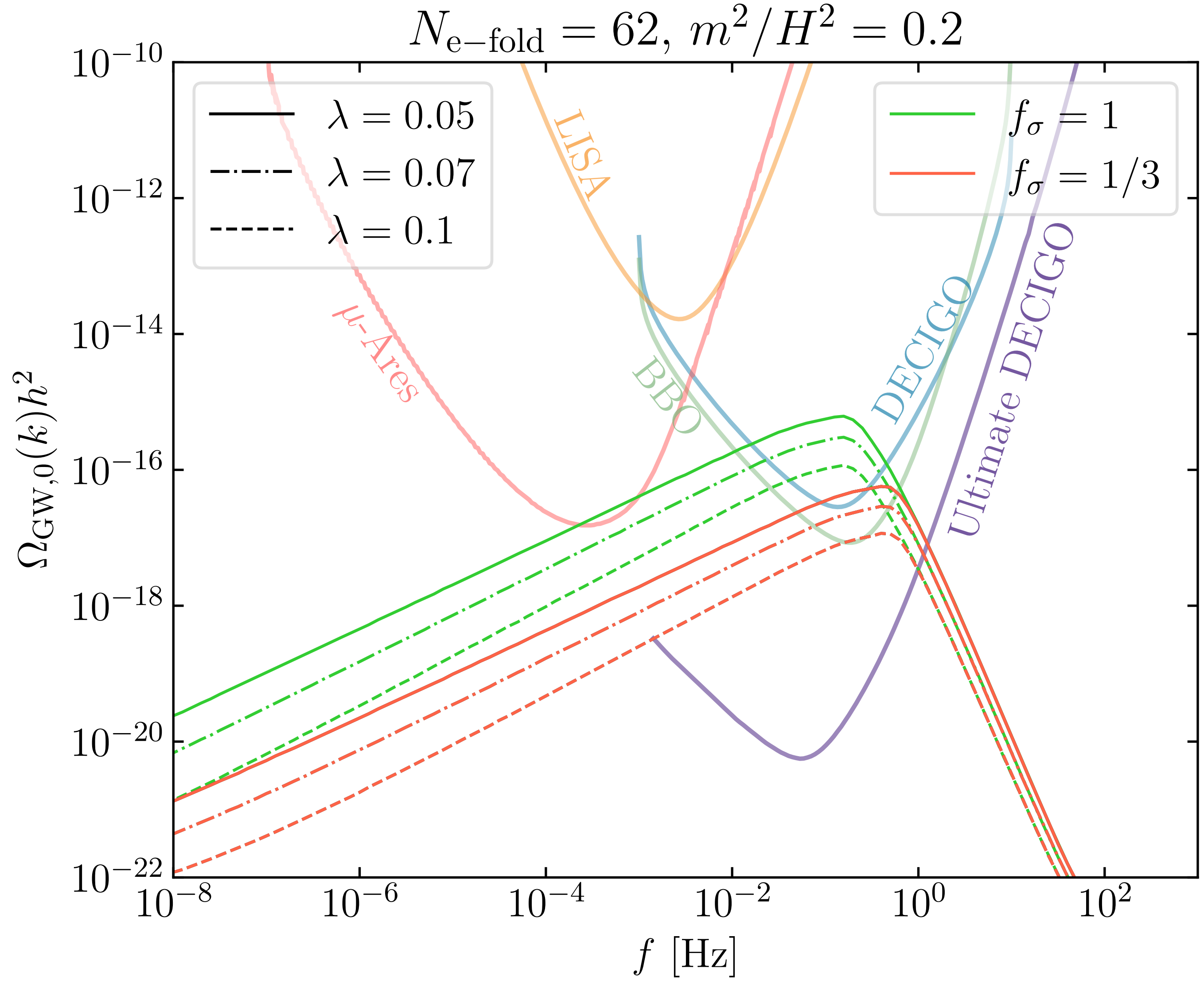
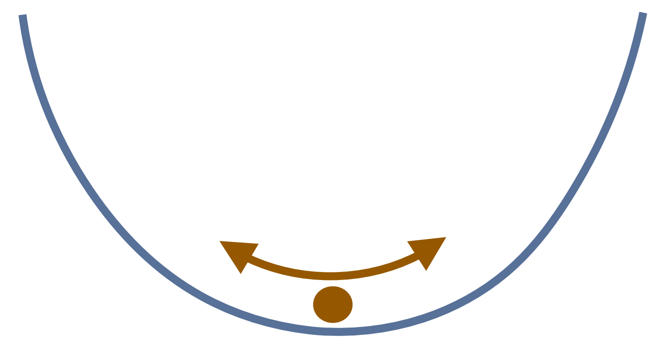
Gravitational wave

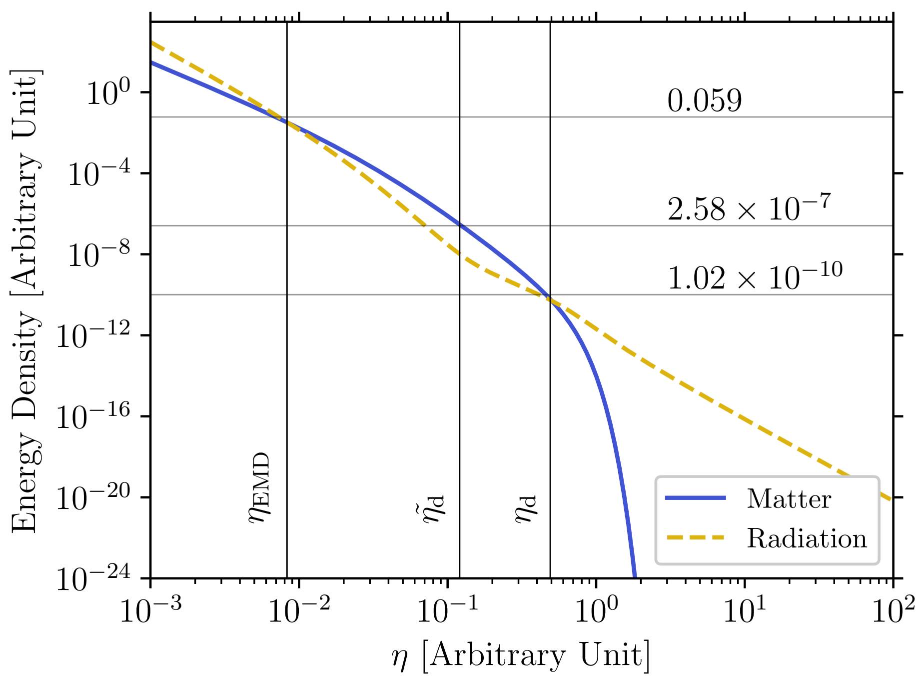
Reza Ebadi, Soubhik Kumar, Amara McCune, Hanwen Tai, LTW, 2307.12048

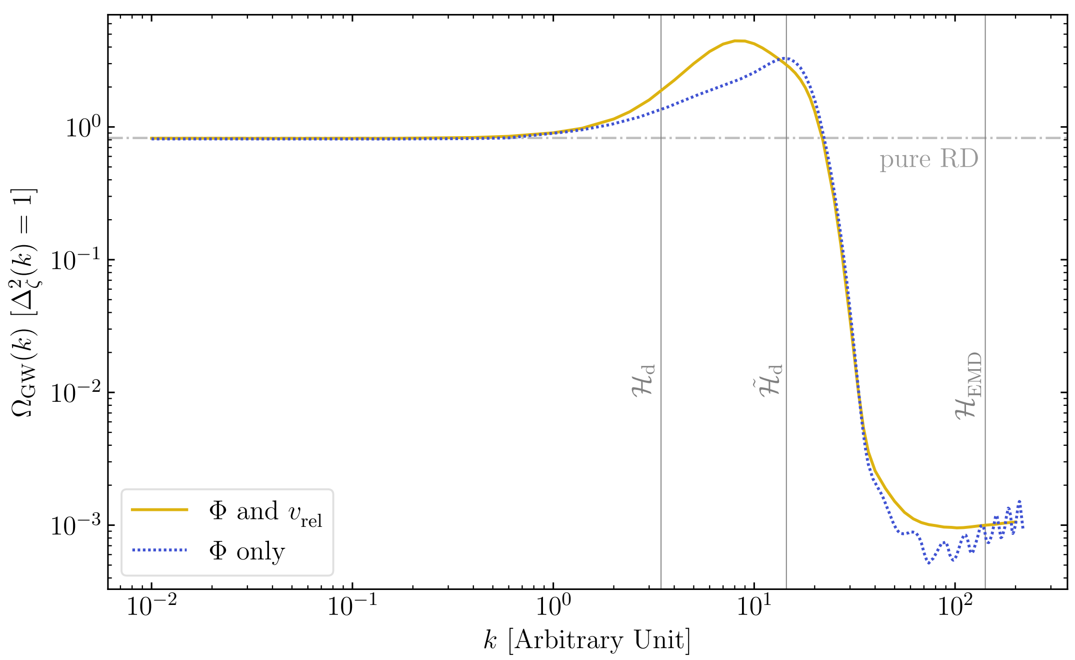




More general scenario

Soubhik Kumar, Hanwen Tai, LTW, 2410.17291



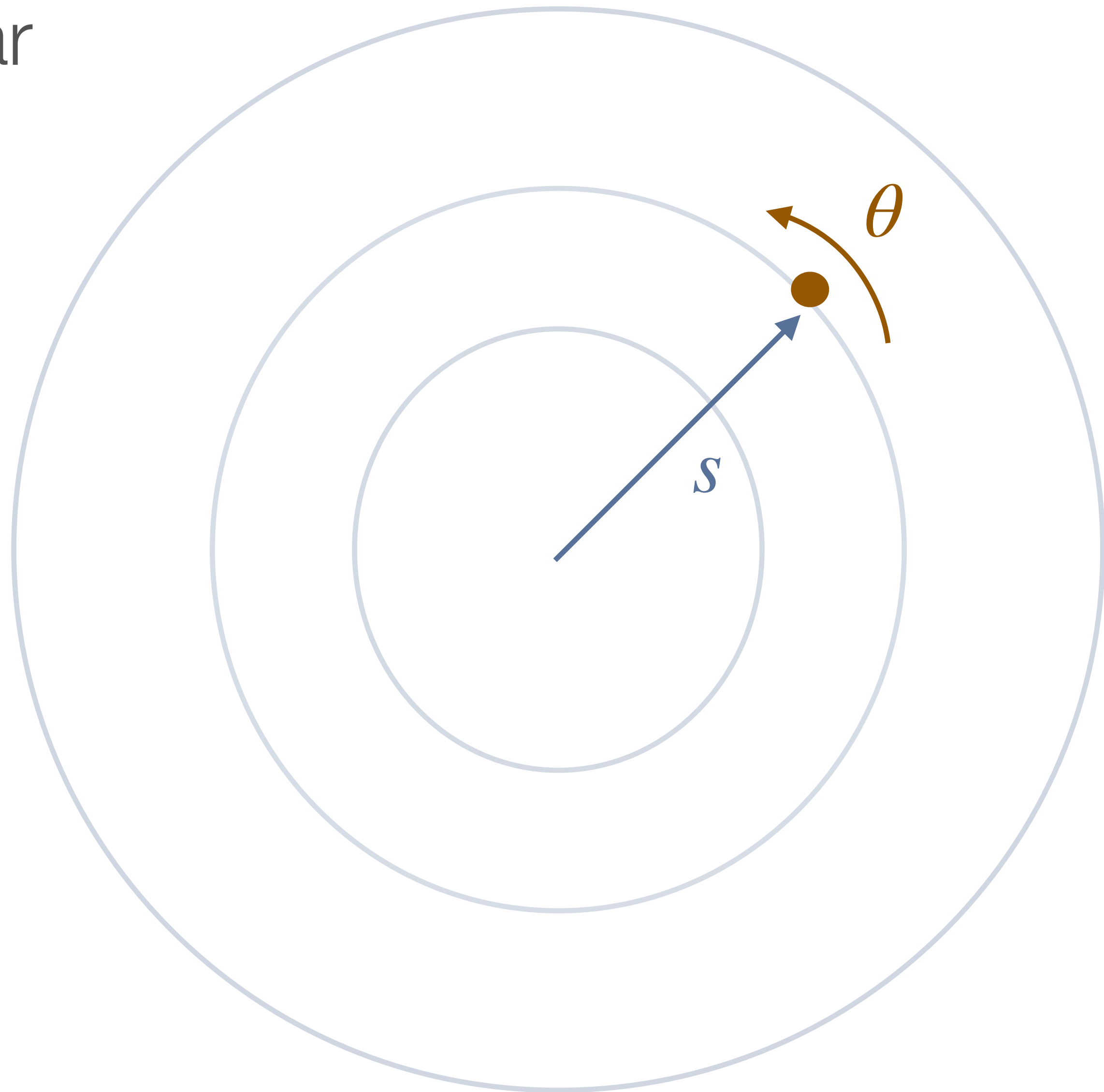


More generally, can consider the case scalar perturbation dominates (curvaton-like).
Larger signal, interesting spectral shape.
To treat this properly, much care is needed, numerically challenging.

Complex scalar

Soubhik Kumar, Hanwen Tai, LTW, 2410.17291

Complex scalar

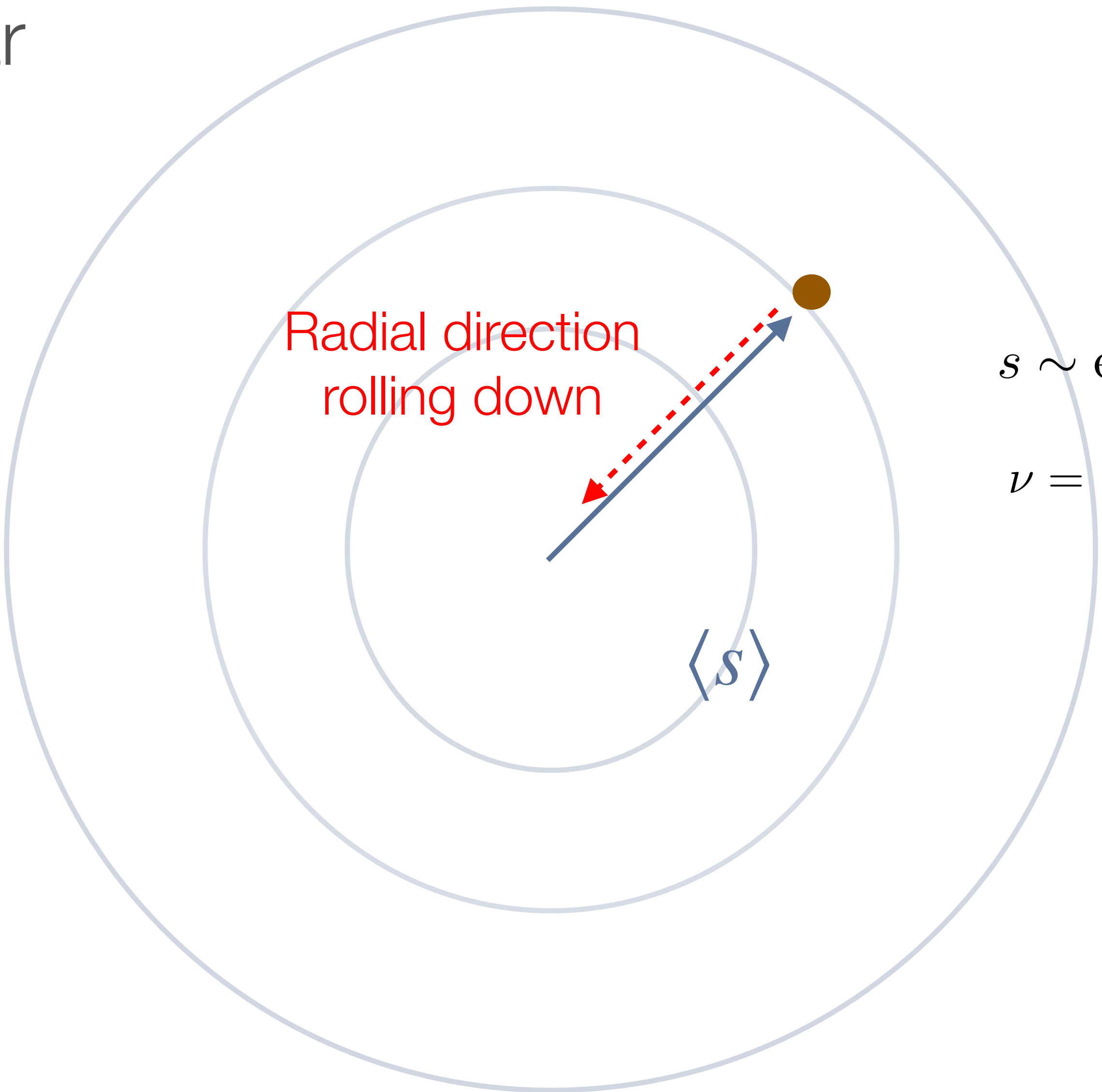


$$\mathcal{L} \supset \frac{1}{2}(\partial_\mu s)^2 + \frac{1}{2}s^2(\partial_\mu \theta)^2 - \lambda_\Phi(s^2 - f_a^2)^2/4 + \frac{1}{2}m^2 s^2 \theta^2.$$

Rolling radial mode

Soubhik Kumar, Hanwen Tai, LTW, 2410.17291

Complex scalar



$$s \sim \exp(-(3/2 - \nu)Ht)$$

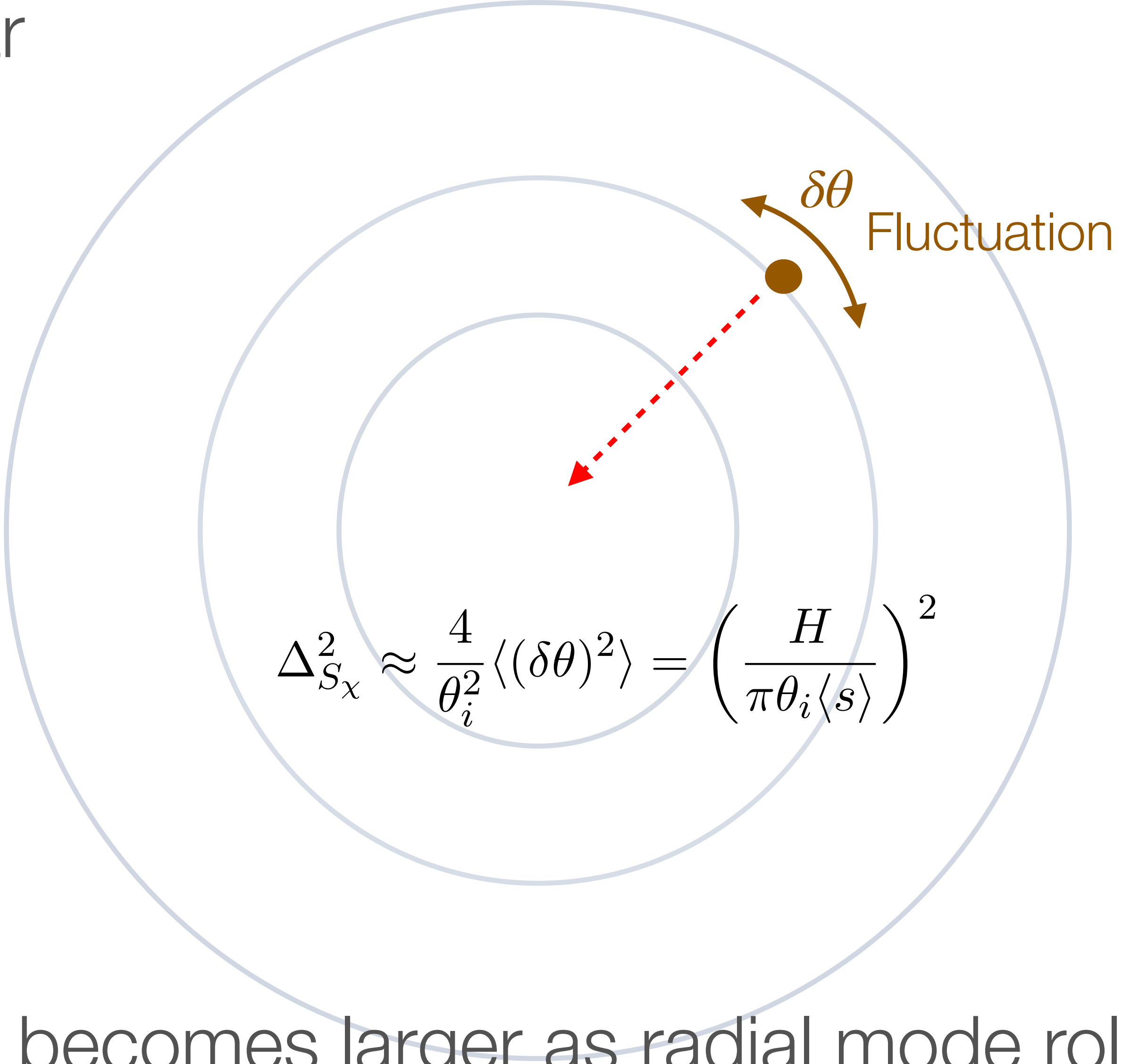
$$\nu = \sqrt{9/4 - m_s^2/H^2}$$

$$\mathcal{L} \supset \frac{1}{2}(\partial_\mu s)^2 + \frac{1}{2}s^2(\partial_\mu \theta)^2 - \lambda_\Phi (s^2 - f_a^2)^2/4 + \frac{1}{2}m^2 s^2 \theta^2.$$

Fluctuations

Soubhik Kumar, Hanwen Tai, LTW, 2410.17291

Complex scalar

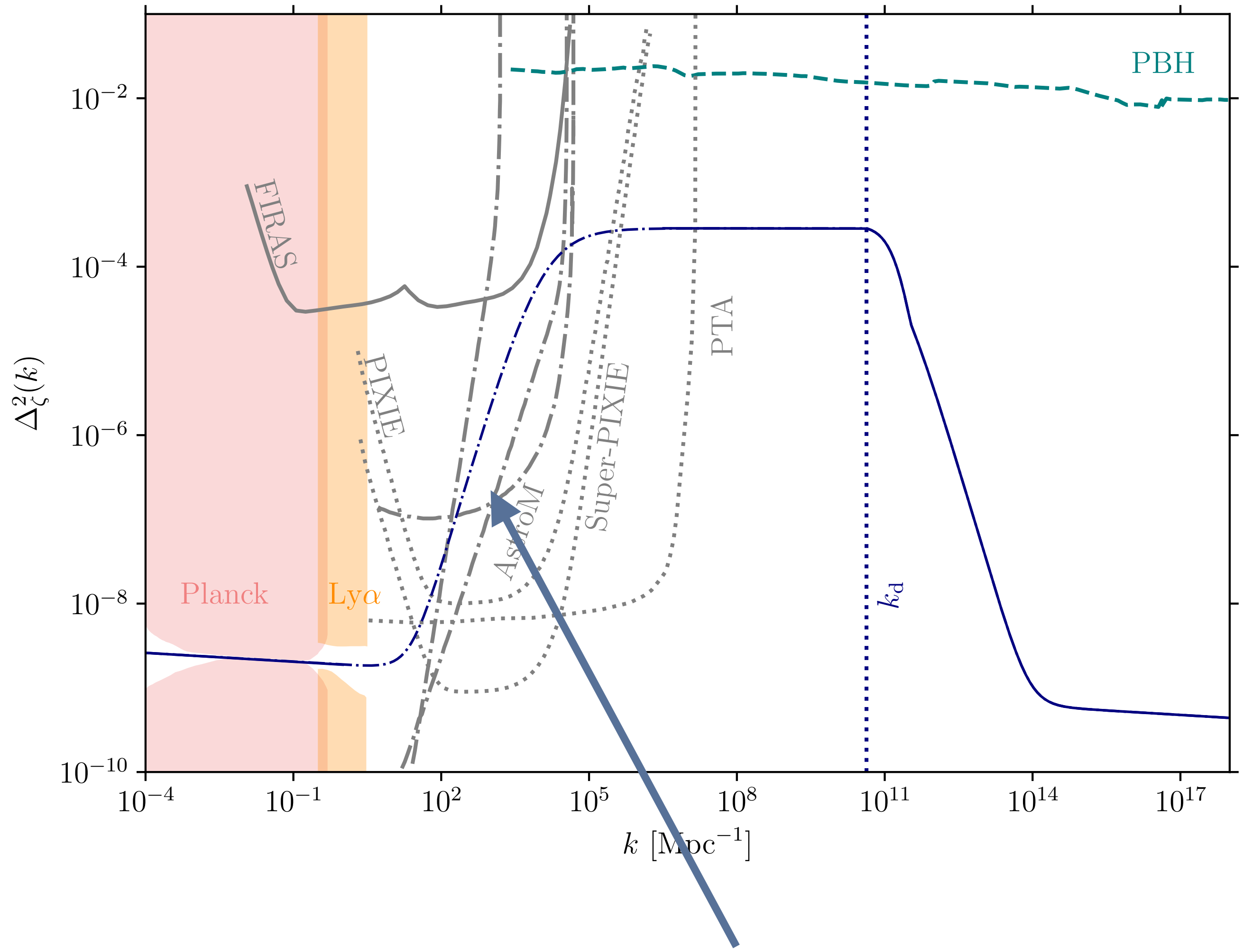


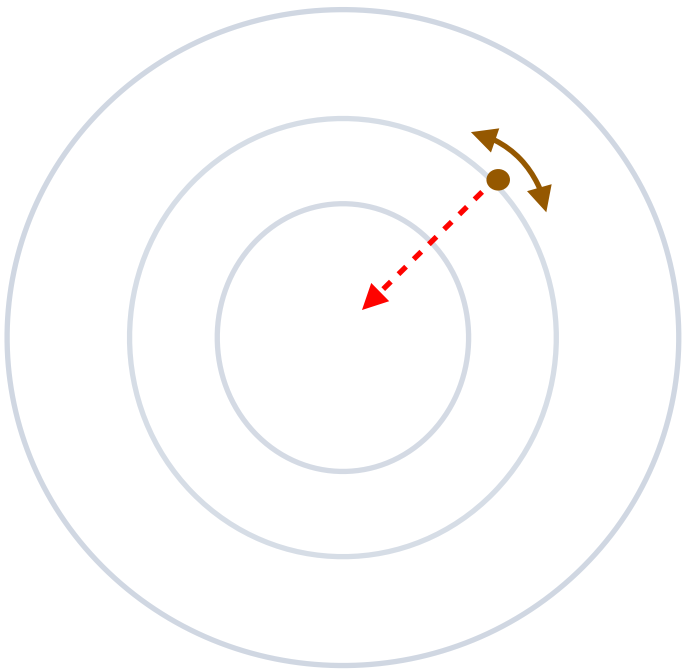
$$\Delta^2_{S_\chi} \approx \frac{4}{\theta_i^2}\langle(\delta\theta)^2\rangle = \left(\frac{H}{\pi\theta_i\langle s\rangle}\right)^2$$

Fluctuation becomes larger as radial mode rolling down

Perturbation spectrum

Complex scalar

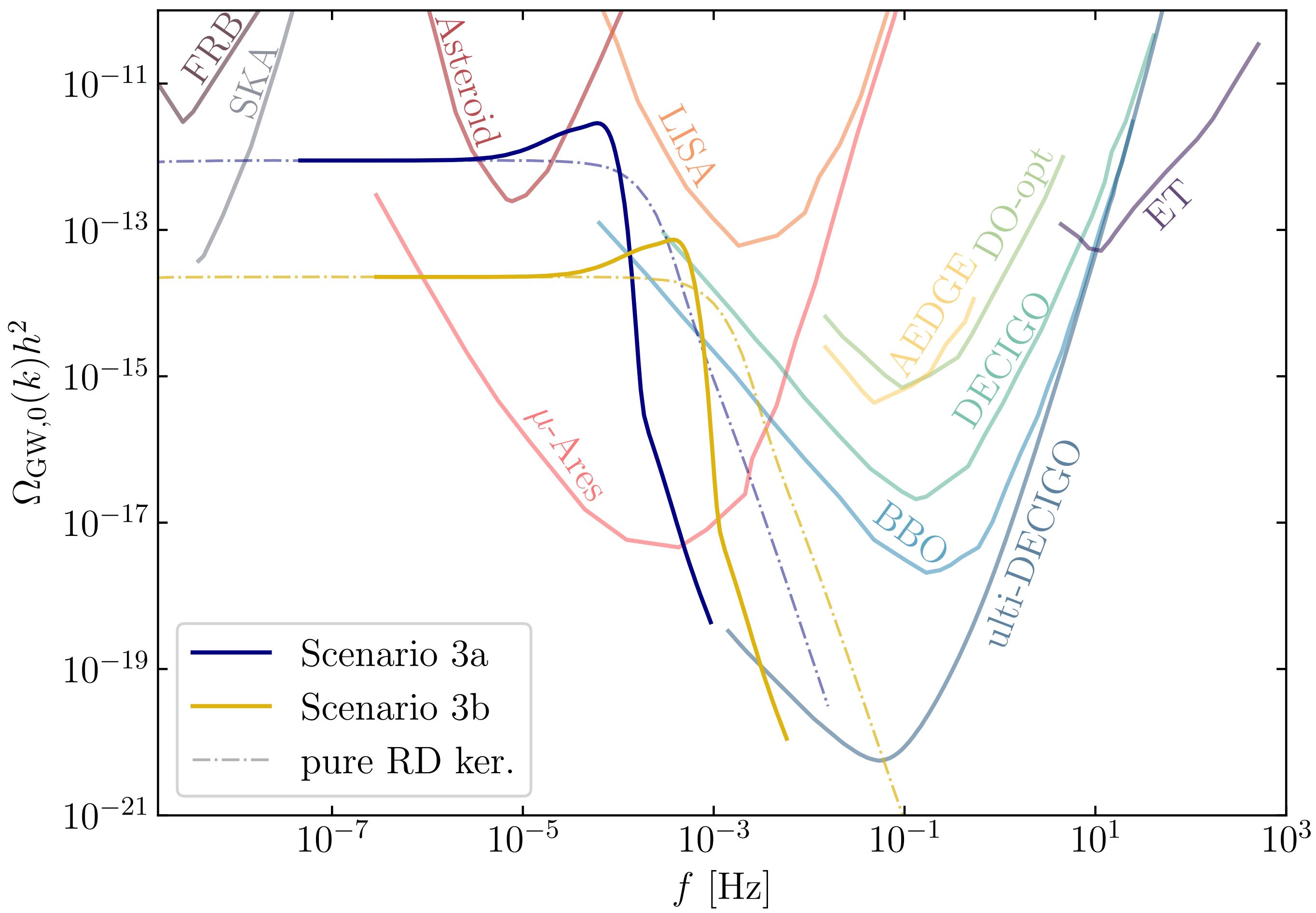


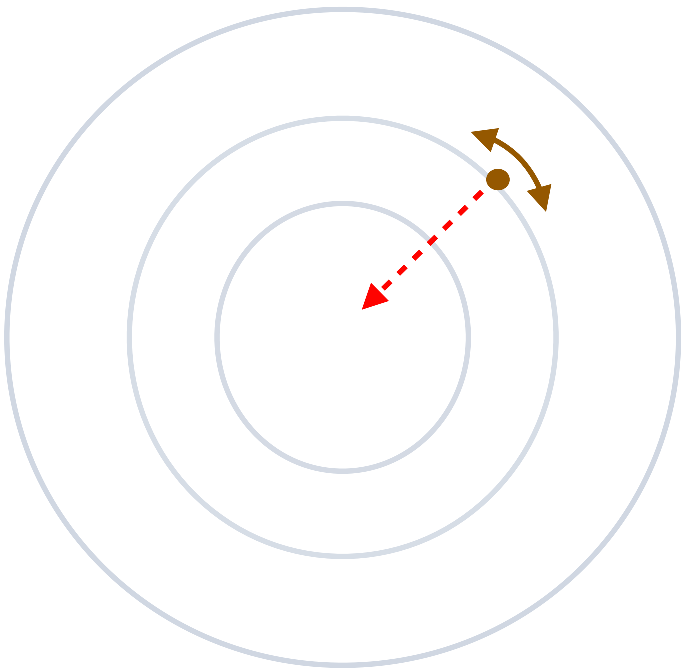


Steeper blue tilt than the previous case

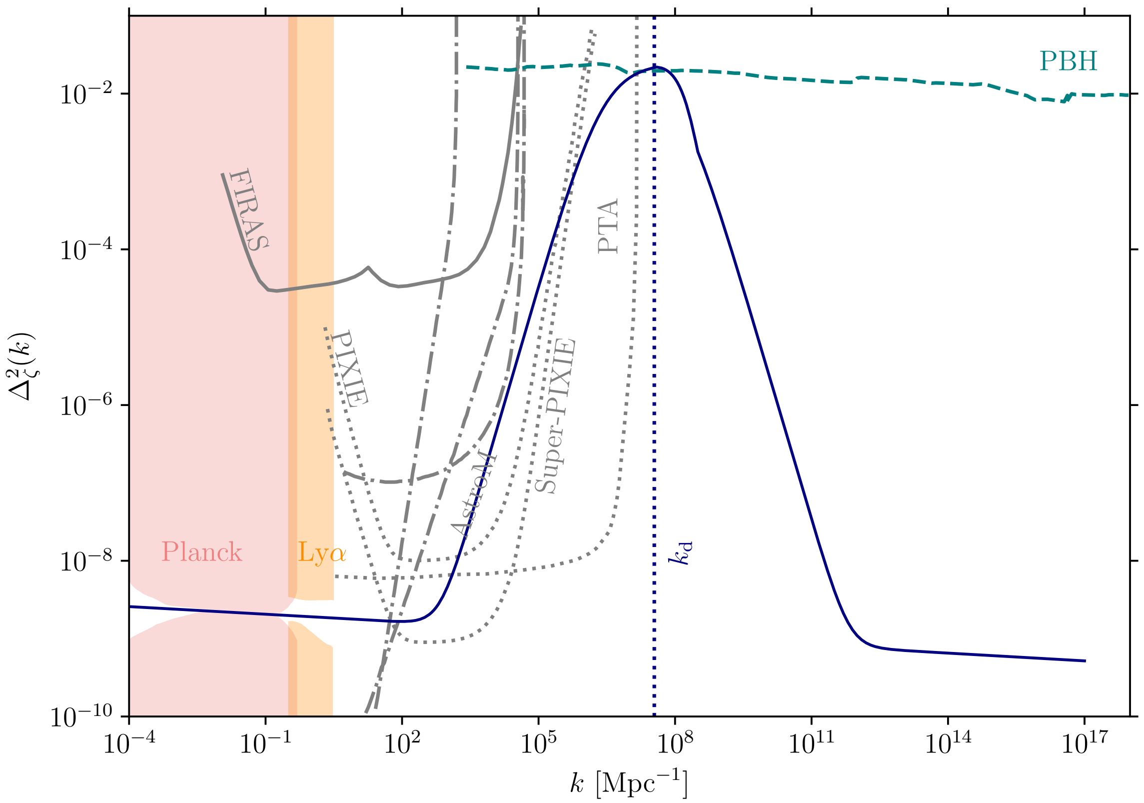
GW prediction

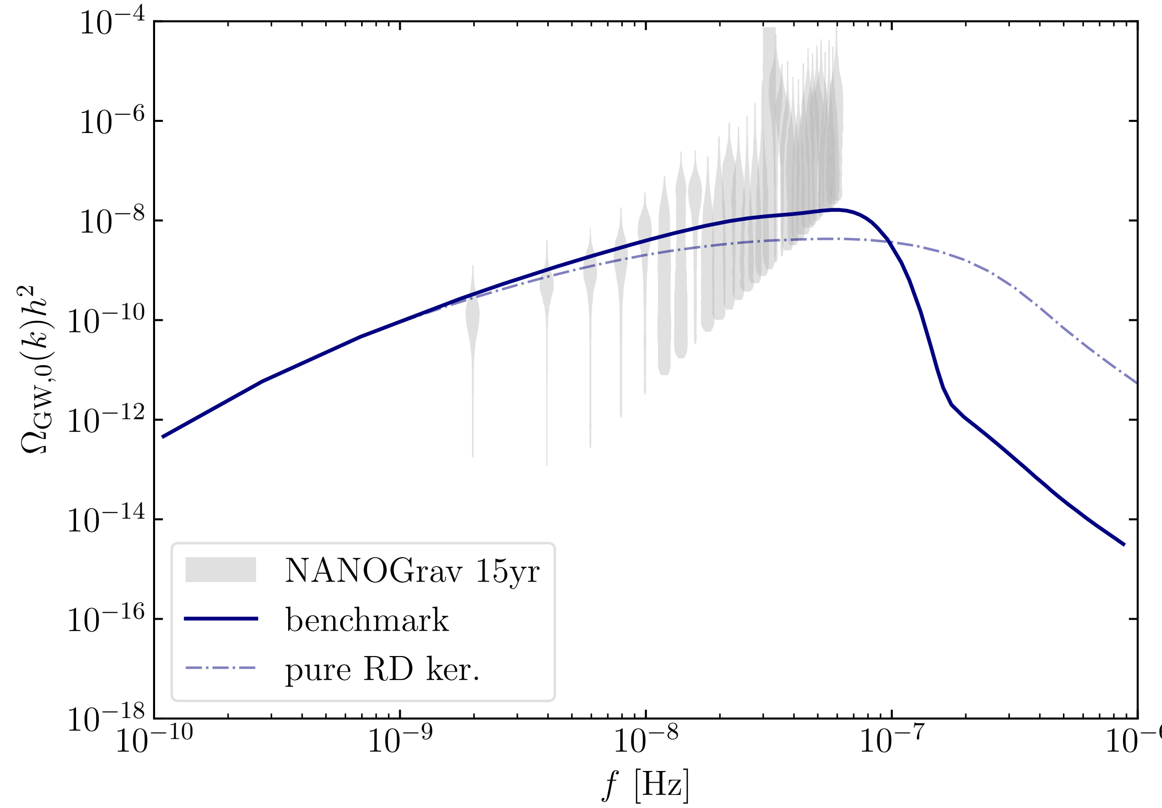
Complex scalar





Another benchmark





$$\chi_{0,\rm end} = f_a = 0.6H,\ H = 1.9 \times 10^{12}\ \text{GeV},\ m = 0.05H,\ \lambda_\Phi = 0.75$$

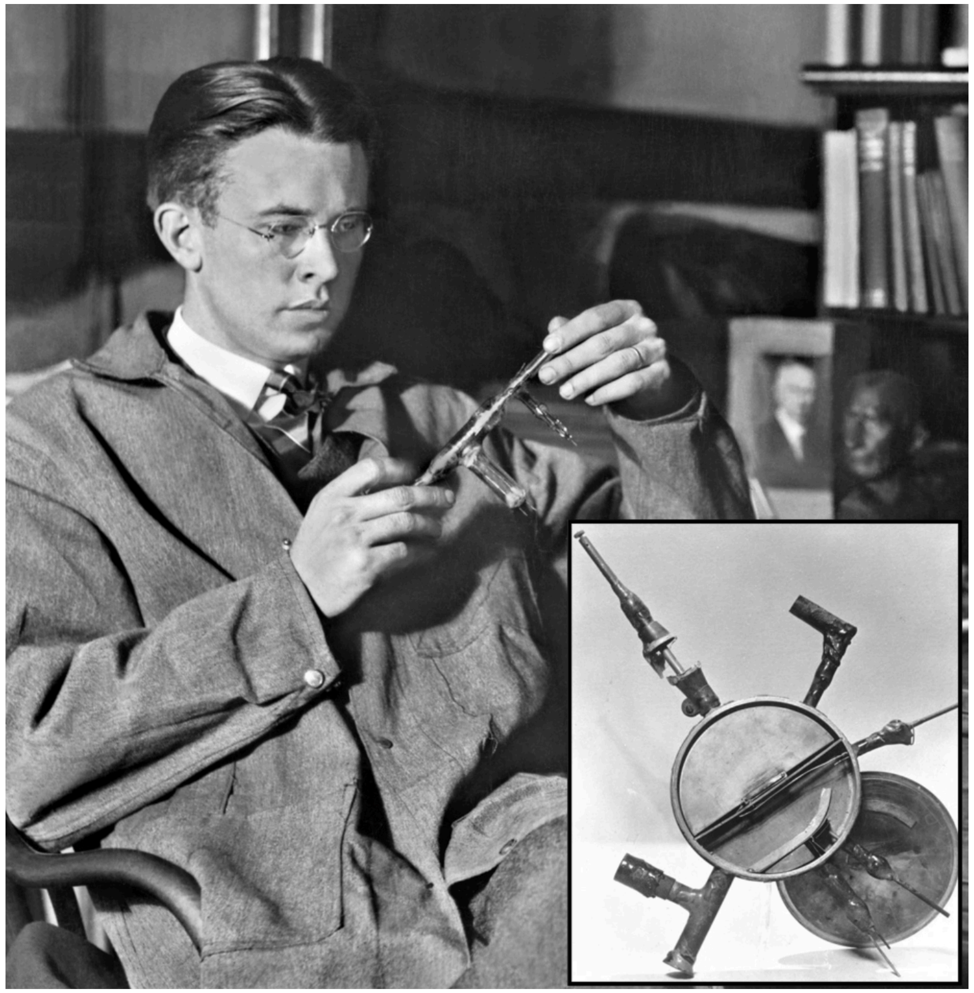
N	$k_{\rm end}$ [Mpc^{-1}]	$k_{\rm EMD}$ [Mpc^{-1}]	$k_{\rm d}$ [Mpc^{-1}]
59.2	1.18×10^{22}	3.14×10^{8}	4.0×10^{7}

Conclusions

- We are at the beginning of a new era, gravitational wave as a new window to early universe.
- More observations of stochastic gravitational wave in the coming decades.
- Can reveal important dynamics in the early universe
- I focused on the question of new dynamics during inflation:
  - Light field fluctuations → secondary GW
- A fast advancing field with many opportunities.

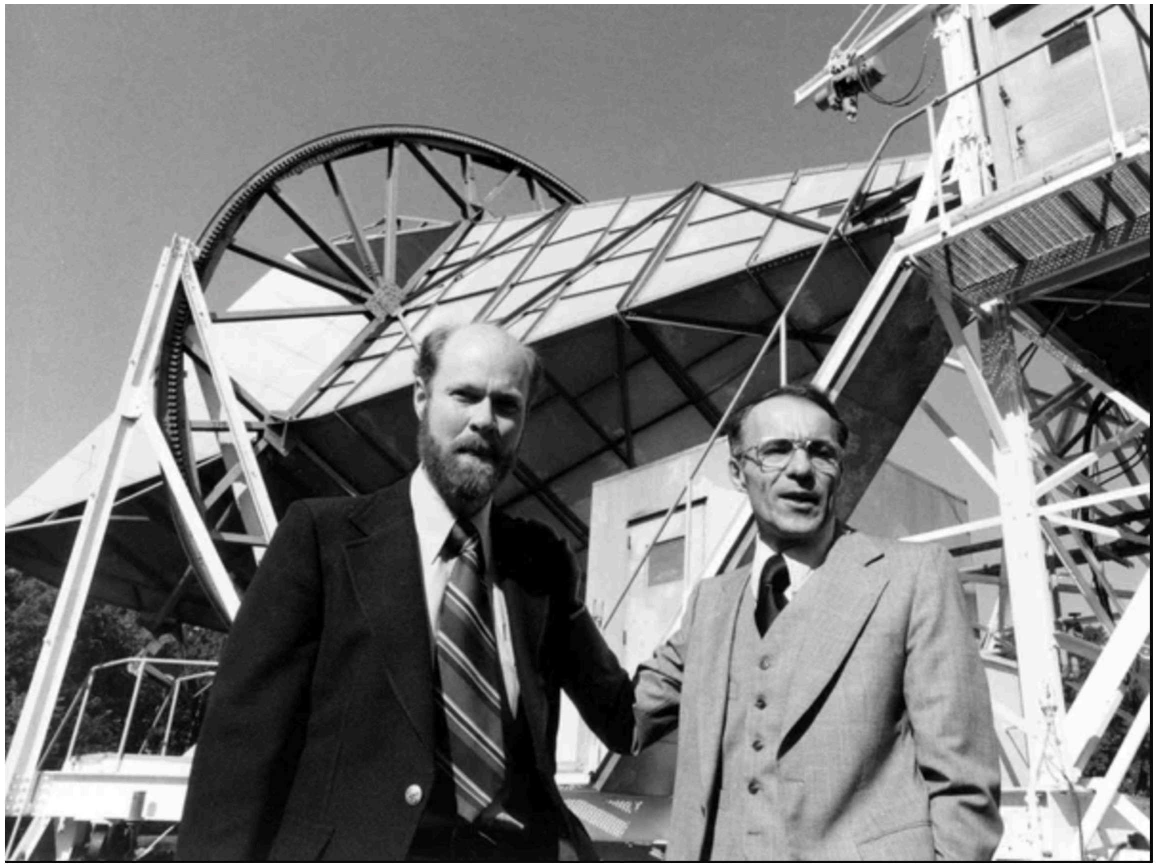
Beginnings of exciting times

E. Lawrence



LBNL

A. Penzias and R. Wilson



AP

Beginnings of exciting times

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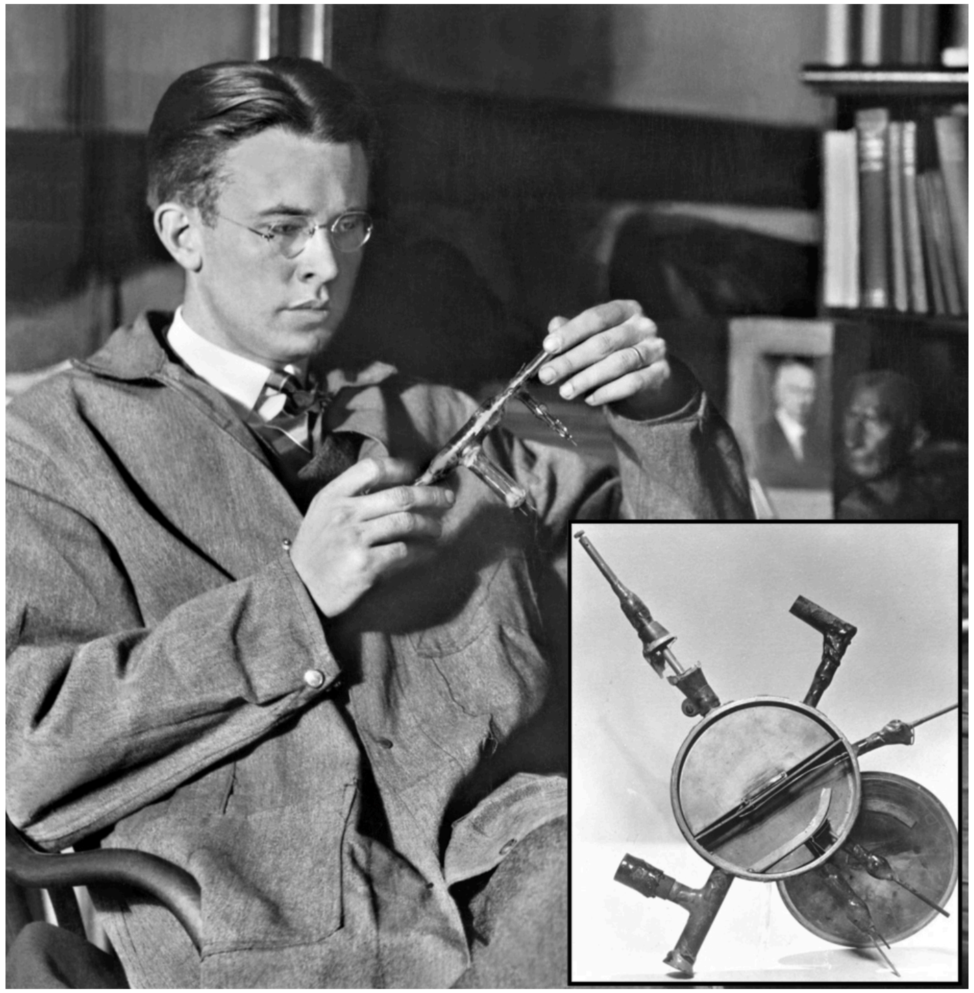

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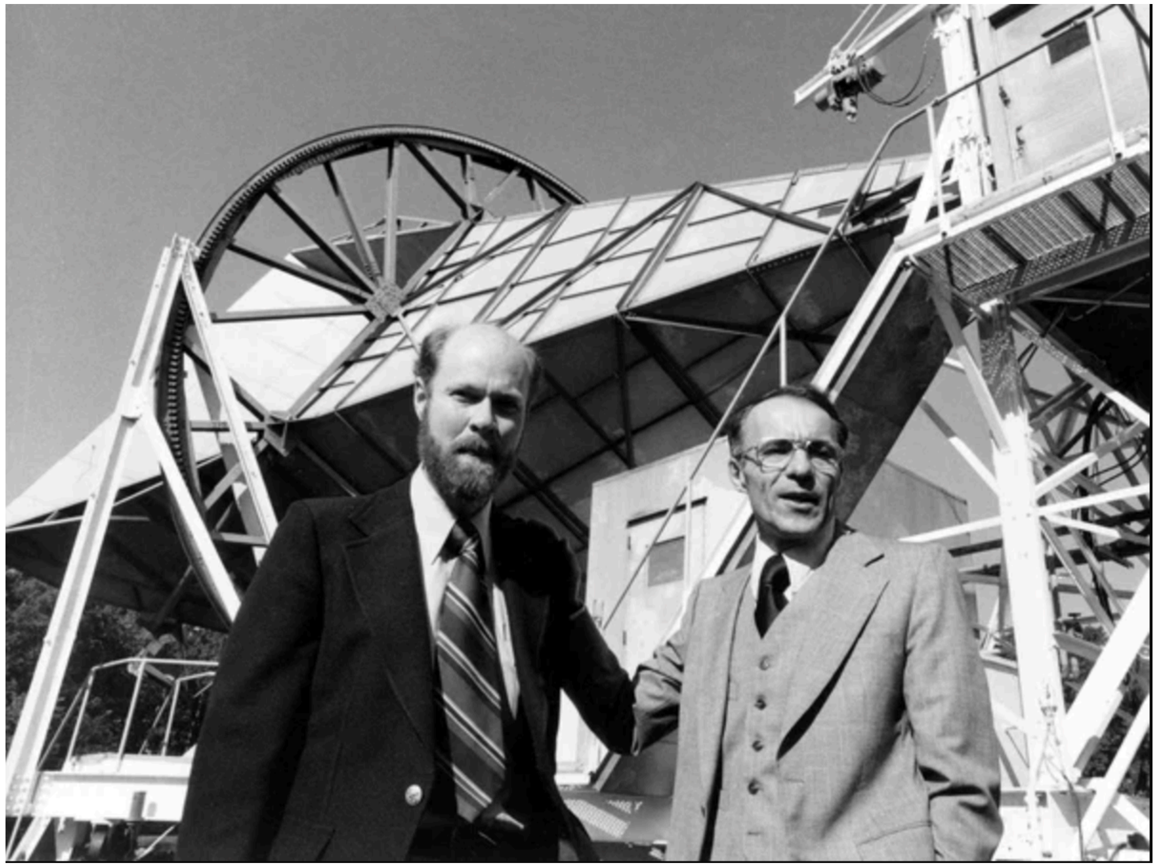
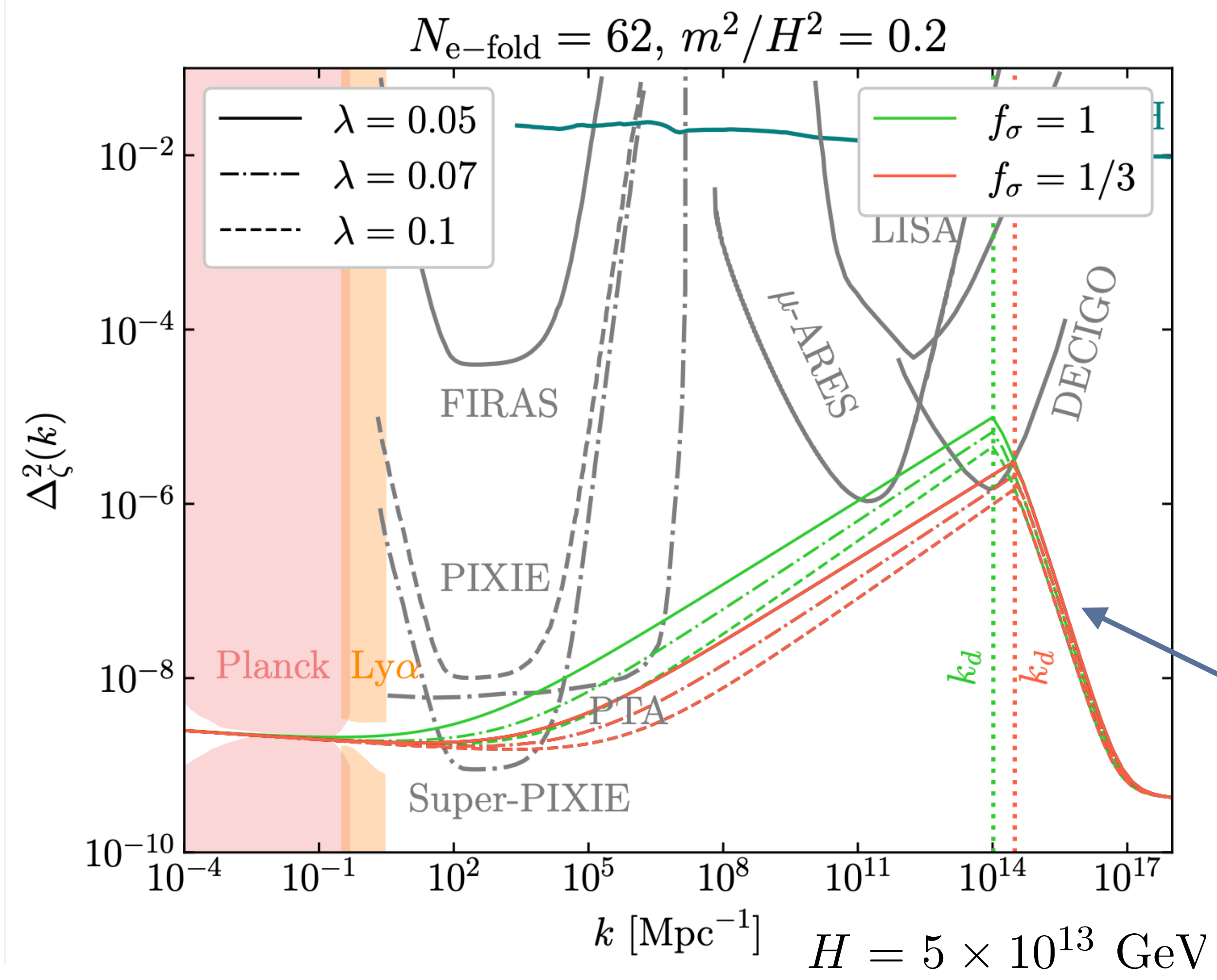


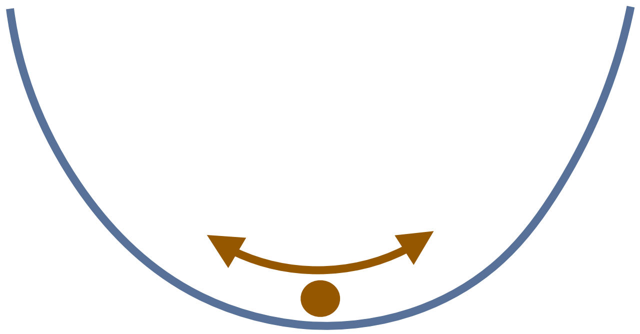
AP

We are at a similar historical juncture for gravitational waves

Power spectrum

Reza Ebadi, Soubhik Kumar, Amara McCune, Hanwen Tai, LTW, 2307.12048





Entering horizon before decaying, Smaller energy fraction.

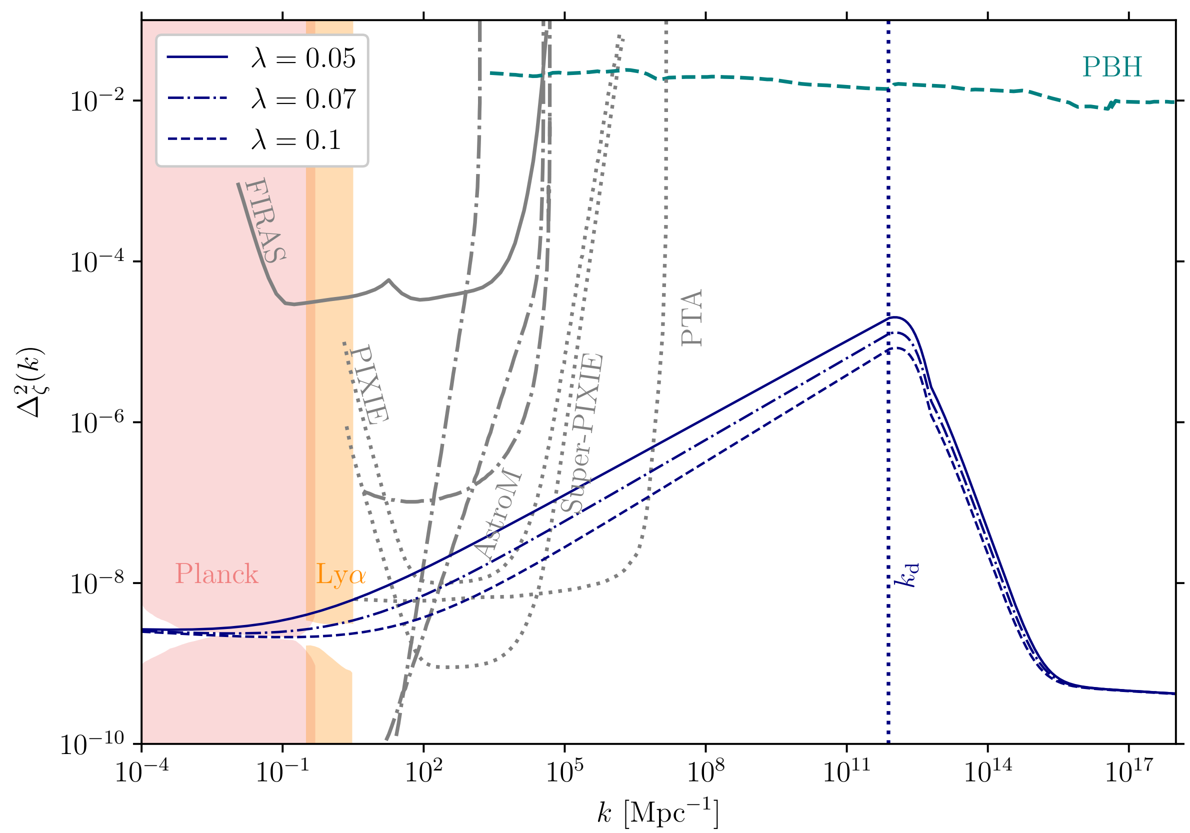
Assuming the scalar behave similar to curvaton.
Becoming important before decay.

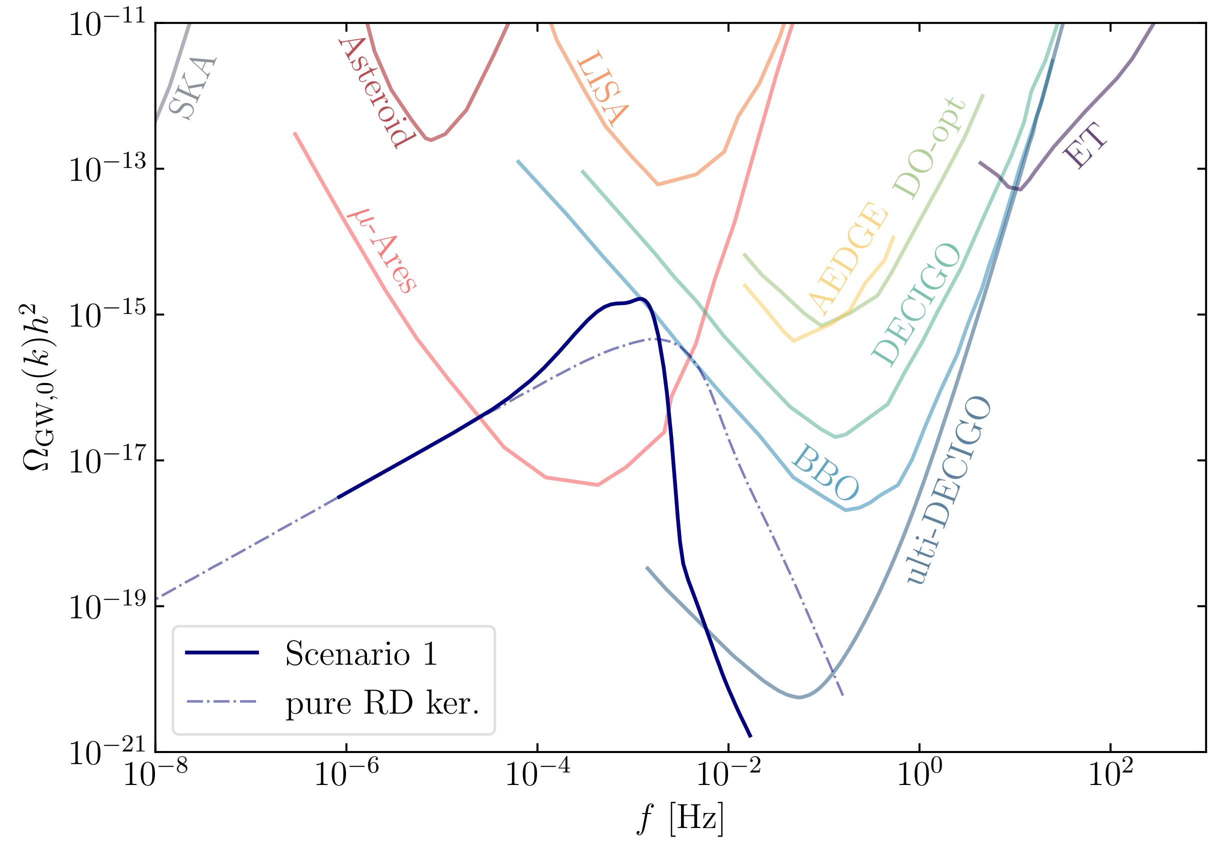
Blue tilt

m^2/H^2	λ	Λ_2/H	g_2^2	Λ_4/H	g_4^2
0.2	0.05	0.16	1.99	0.37	0.03
0.2	0.07	0.17	1.98	0.40	0.05
0.2	0.1	0.18	1.98	0.44	0.07
0.25	0.05	0.19	1.99	0.42	0.02
0.25	0.07	0.20	1.99	0.45	0.03
0.25	0.1	0.21	1.98	0.49	0.05
0.3	0.05	0.22	1.99	0.48	0.01
0.3	0.07	0.23	1.99	0.51	0.02
0.3	0.1	0.24	1.99	0.54	0.03

Generic to have sizable blue tilt

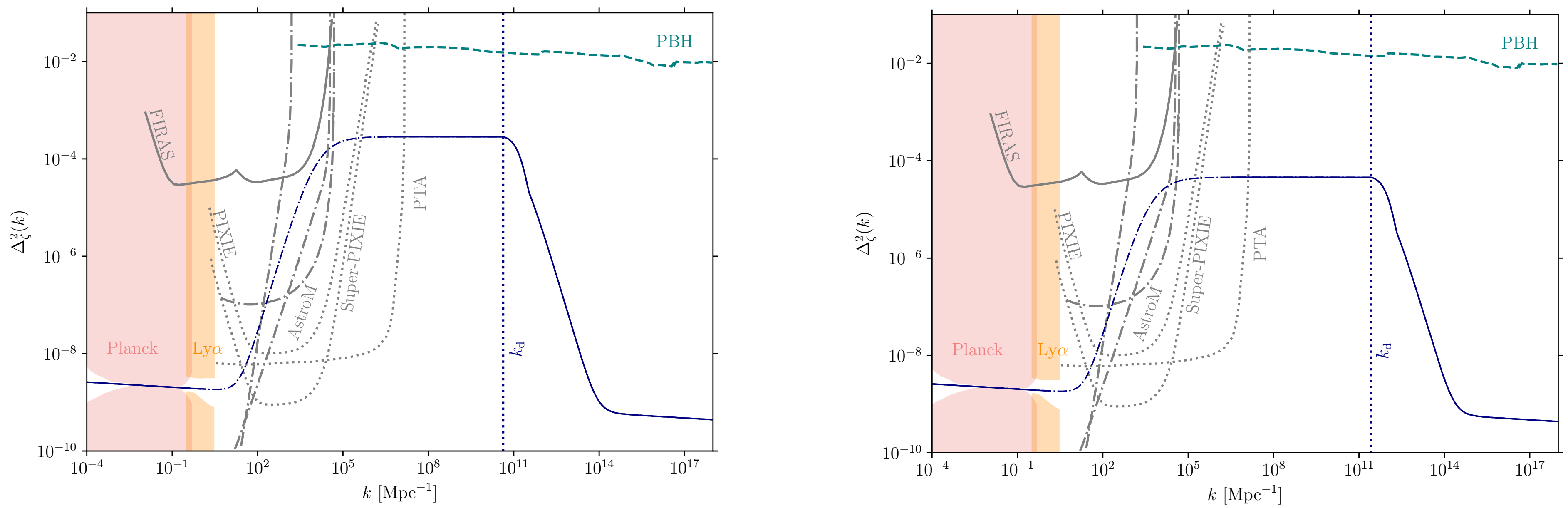
$$\mathcal{P}_f(k) = \sum_n \frac{2}{\pi} f_n^2 \Gamma\left(2 - 2\frac{\Lambda_n}{H}\right) \sin\left(\frac{\Lambda_n \pi}{H}\right) \left(\frac{k}{H}\right)^{2\Lambda_n/H} \quad \rightarrow \mathscr{A}\left(\frac{k}{H}\right)^{\frac{2\Lambda_{\text{lowest}}}{H}} \quad \text{for } k \ll H$$





Benchmark. For $m^2 = 0.2H^2$ and $\lambda = 0.05$, $\langle V(\chi)\rangle \approx 0.02H^4$. We also fix $H = 4 \times 10^{13}$ GeV, slightly below the current upper limit [87] and target of future B-mode experiments, $\rho_{\rm end} \simeq V_k/100$ (see, e.g., [84–86]), and a reheat temperature after inflation $T_{\rm RH} = 10^{15}$ GeV. With our choice of $\rho_{\rm d}/\rho_{\rm EMD} \approx 1.7 \times 10^{-9}$, $\tilde{\rho}_{\rm d}/\rho_{\rm EMD} \approx 4.3 \times 10^{-6}$, and $\rho_\chi(t_{\rm d})/\rho_r(t_{\rm d}) \approx 29$, corresponding to Fig. 2, we get

N	$k_{\rm end}$ [Mpc^{-1}]	$k_{\rm EMD}$ [Mpc^{-1}]	$k_{\rm d}$ [Mpc^{-1}]
60.6	4.6×10^{22}	6×10^{12}	7.6×10^{11}



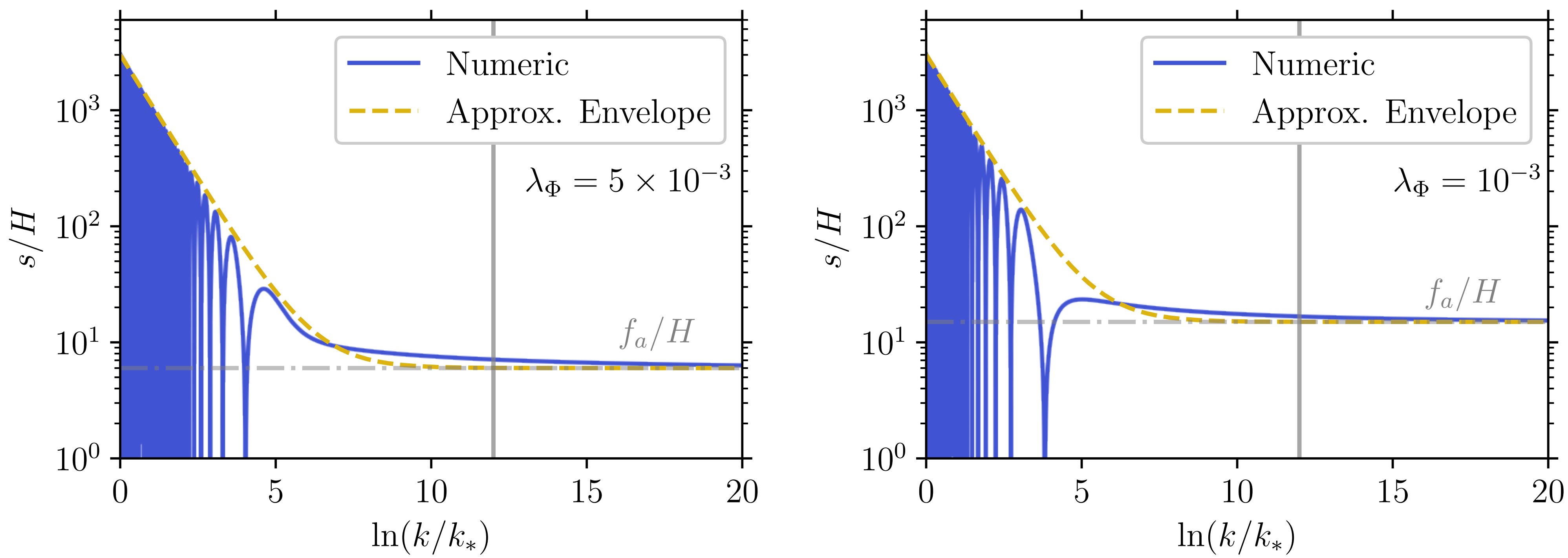
Benchmark (a). We choose $H = 5 \times 10^{12}$ GeV during inflation, which implies $T_{\rm RH} \approx 5 \times 10^{14}$ GeV, along with $m = 0.05H$, $\chi_{0,\rm end} = 6H$, and $\lambda_\Phi = 5 \times 10^{-3}$. This implies

N	$k_{\rm end}$ [Mpc^{-1}]	$k_{\rm EMD}$ [Mpc^{-1}]	$k_{\rm d}$ [Mpc^{-1}]
59.6	1.8×10^{22}	3.3×10^{11}	4.2×10^{10}

The resulting spectrum is shown in Fig. 5. This corresponds to the left panel of Fig. 1.

Benchmark (b). We describe another benchmark with all the parameters identical to the above, except $\chi_{0,\rm end} = 15H$, and $\lambda_\Phi = 10^{-3}$. This implies

N	$k_{\rm end}$ [Mpc^{-1}]	$k_{\rm EMD}$ [Mpc^{-1}]	$k_{\rm d}$ [Mpc^{-1}]
59.6	1.8×10^{22}	2.1×10^{12}	2.7×10^{11}

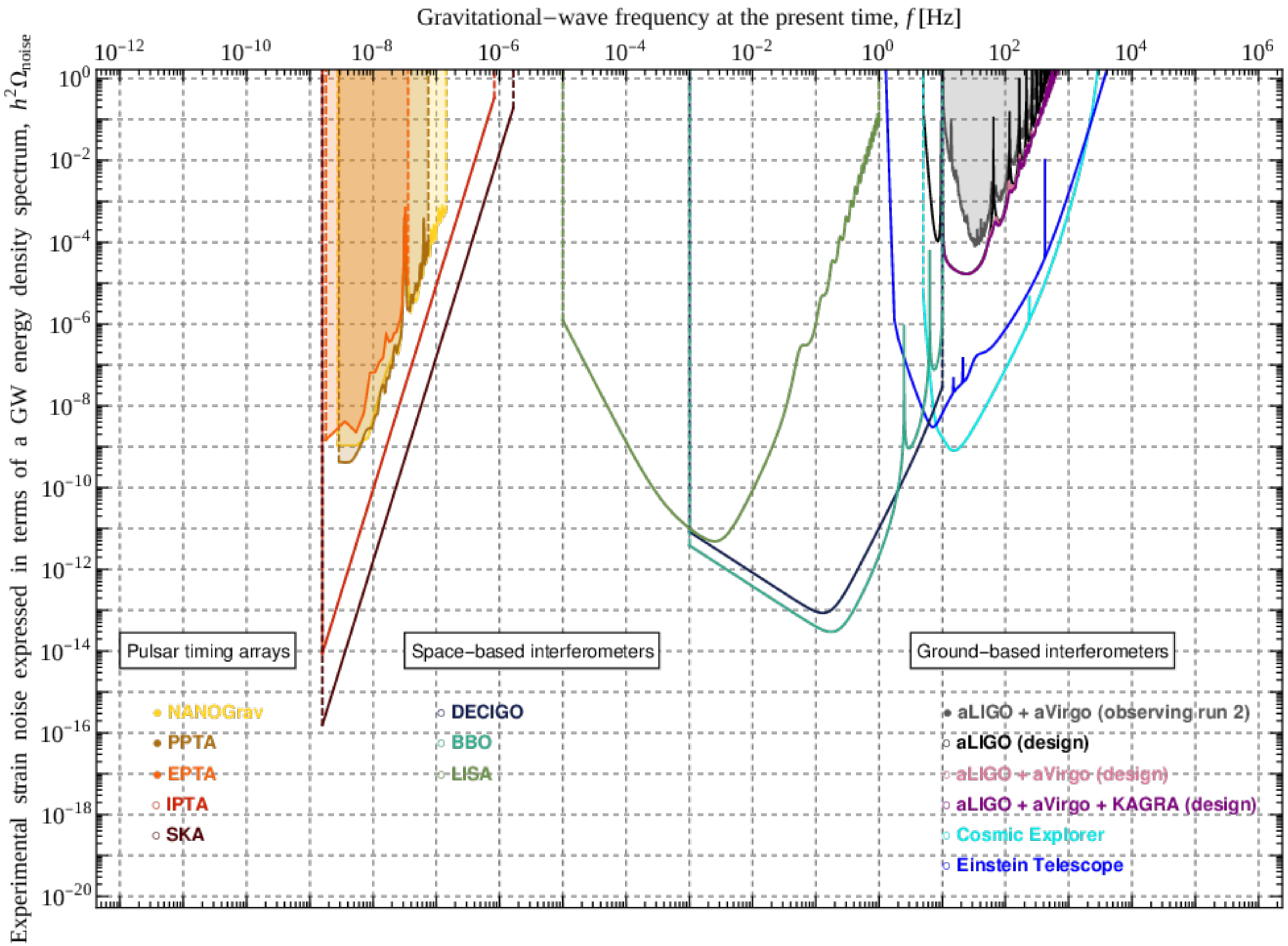


Numeric
Approx. Envelope
$\lambda_\Phi = 5 \times 10^{-3}$
f_a/H
s/H
$\ln(k/k_*)$
Numeric
Approx. Envelope
$\lambda_\Phi = 10^{-3}$
f_a/H
s/H
$\ln(k/k_*)$

Comparisons

source	spectral shape
gauge str. + inf. + wall	$f^3 \to f^{3/2} \to f^{-1}$
global str. ($w_i \gtrsim k_{\rm NGB}$) + inf. + wall	$f^3 \to f^{3/2} \to f^{-1} \to f^{-3}$
global str. ($w_i \lesssim k_{\rm NGB}$) + inf. + wall	$f^3 \to f^{3/2} \to f^{-3}$
primordial metric perturbation	$f^{n_T} \to f^{n_T-2}$
secondary GW (log-normal P_ζ)	$f^3 \ln^2 f \to$ cutoff
secondary GW (Dirac delta P_ζ)	$f^2 \ln^2 f \to$ cutoff
secondary GW ($k^{n_{\rm IR}} \to k^{-n_{\rm UV}}$)	$f^3 \ln^2 f \to f^{-2n_{\rm UV}}$
phase transition, turbulence, analytical	$f^3 \to f^{-7/2}$
phase transition, turbulence, numerical	$f^1 \to f^{-8/3}$
phase transition, sound wave	$f^9 \to f^{-3}$
domain wall	$f^3 \to f^{-1}$
cosmic gauge string	$f^{3/2} \to f^0 \to f^{-1}$
gauge string in kination domination	$f^1 \to f^{-2}$ bump
supermassive black hole binary	$f^{2/3}$

Gravitational wave signal



K. Schmitz, 2002.04615

NanoGrav? No.

Blue tilt in the case not large enough to give rise to the signal.

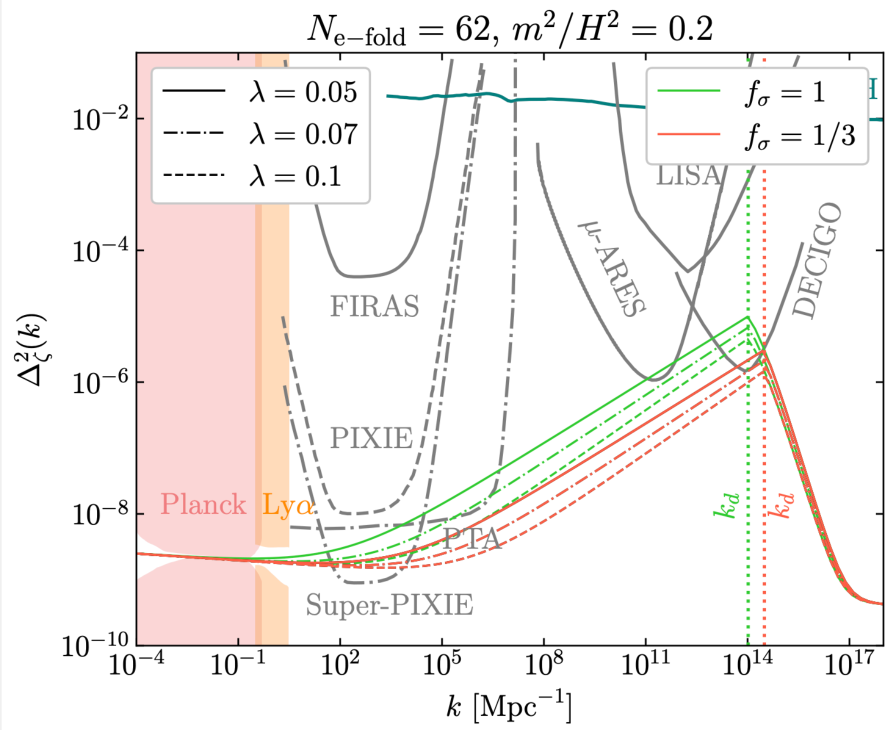
Larger tilt?

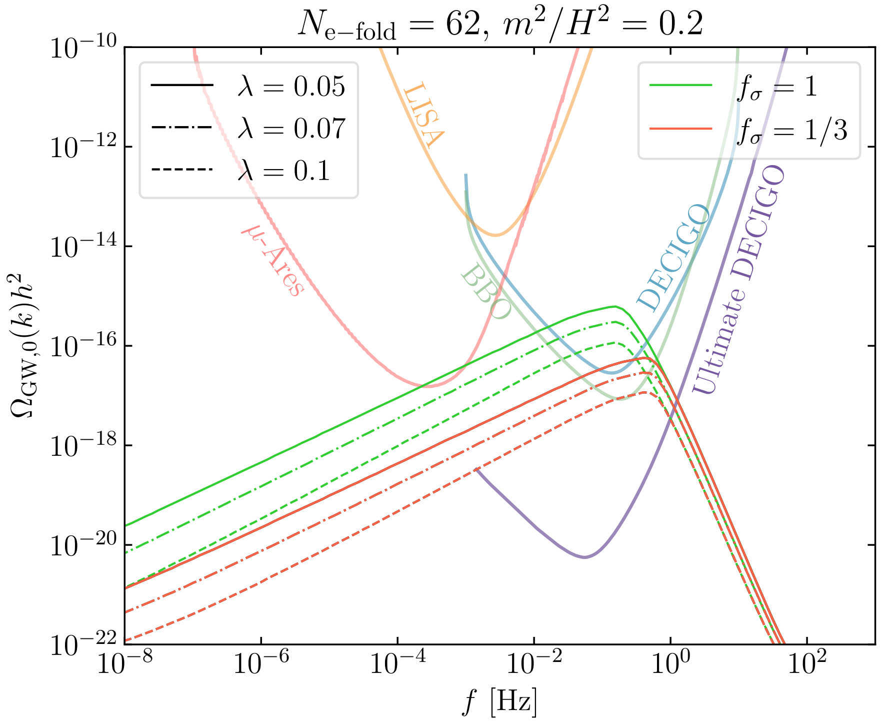
$$\mathcal{P}_f(k) = \sum_n \frac{2}{\pi} f_n^2 \Gamma\left(2 - 2\frac{\Lambda_n}{H}\right) \sin\left(\frac{\Lambda_n \pi}{H}\right) \left(\frac{k}{H}\right)^{2\Lambda_n/H} \quad \to \mathscr{A} \left(\frac{k}{H}\right)^{\frac{2\Lambda_{\text{lowest}}}{H}} \text{ for } k \ll H$$

Where $\frac{\Lambda}{H} \sim \frac{m^2}{H^2}$

Larger tilt needs $m > H$, not a light field, fluctuation suppressed.

NanoGrav? No.





Blue tilt in the case not large enough to give rise to the signal.

Larger tilt?

$$\mathcal{P}_f(k) = \sum_n \frac{2}{\pi} f_n^2 \Gamma\left(2 - 2\frac{\Lambda_n}{H}\right) \sin\left(\frac{\Lambda_n \pi}{H}\right)\left(\frac{k}{H}\right)^{2\Lambda_n/H} \quad \to \mathscr{A}\left(\frac{k}{H}\right)^{\frac{2\Lambda_{\rm lowest}}{H}} \quad \text{for } k \ll H$$

Power spectrum, GW

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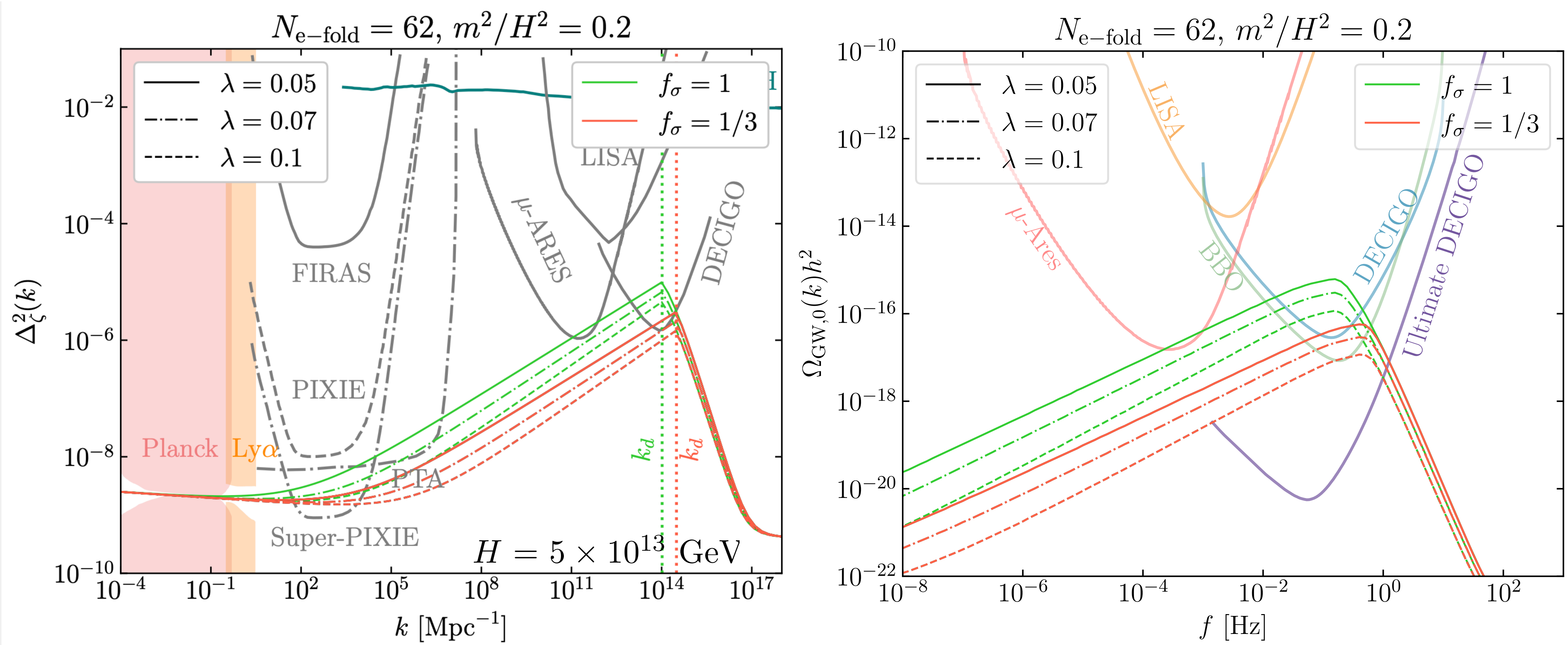
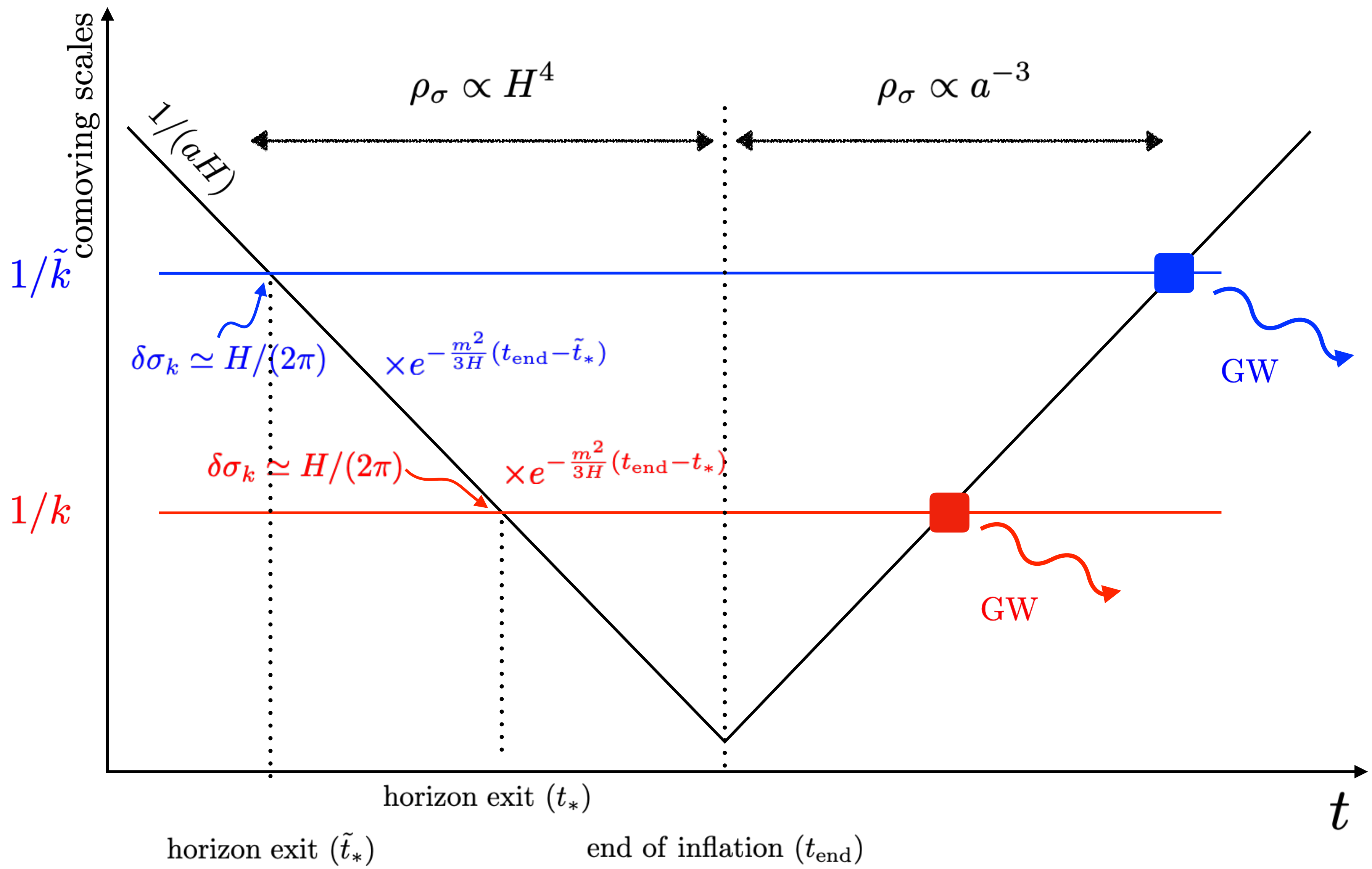


Assuming the scalar behave similar to curvaton.
Becoming important before decay.

2nd GW


comoving scales
$1/(aH)$
$\rho_\sigma \propto H^4$
$\rho_\sigma \propto a^{-3}$
$1/\tilde{k}$
$1/k$
$\delta\sigma_k \simeq H/(2\pi)$
$\times e^{-\frac{m^2}{3H}(t_{\rm end}-\tilde{t}_*)}$
$\delta\sigma_k \simeq H/(2\pi)$
$\times e^{-\frac{m^2}{3H}(t_{\rm end}-t_*)}$
GW
GW
t
horizon exit (t_*)
horizon exit ($\tilde{t}_*$)
end of inflation ($t_{\rm end}$)