Dark Energy and String Theory

Gary Shiu

University of Wisconsin-Madison

Based on work with:

Flavio Tonioni

Hung V. Tran

- STT1: Accelerating universe at the end of time, PRD **108**, no.6, 063527 (2023) [2303.03418].
- STT2: Late-time attractors and cosmic acceleration, PRD **108**, no.6, 063528 (2023) [2306.07327].
- STT3: Collapsing universe before time, JCAP **05**, 124 (2024) [2312.06772].
- STT4: Analytic bounds on late-time axion-scalar cosmologies, JHEP **09**, 158 (2024) [2406.17030].
- STT5: Long-lived SEC violation via DM/DE couplings, [2506.19914].

A plea to the theorists

Nobel Prize 2011

Photo: Roy Kaltschmidt. Courtesy: Lawrence Berkeley National Laboratory

Saul Perlmutter

Photo: Belinda Pratten, Australian National University

Brian P. Schmidt

Photo: Homewood Photography

Adam G. Riess

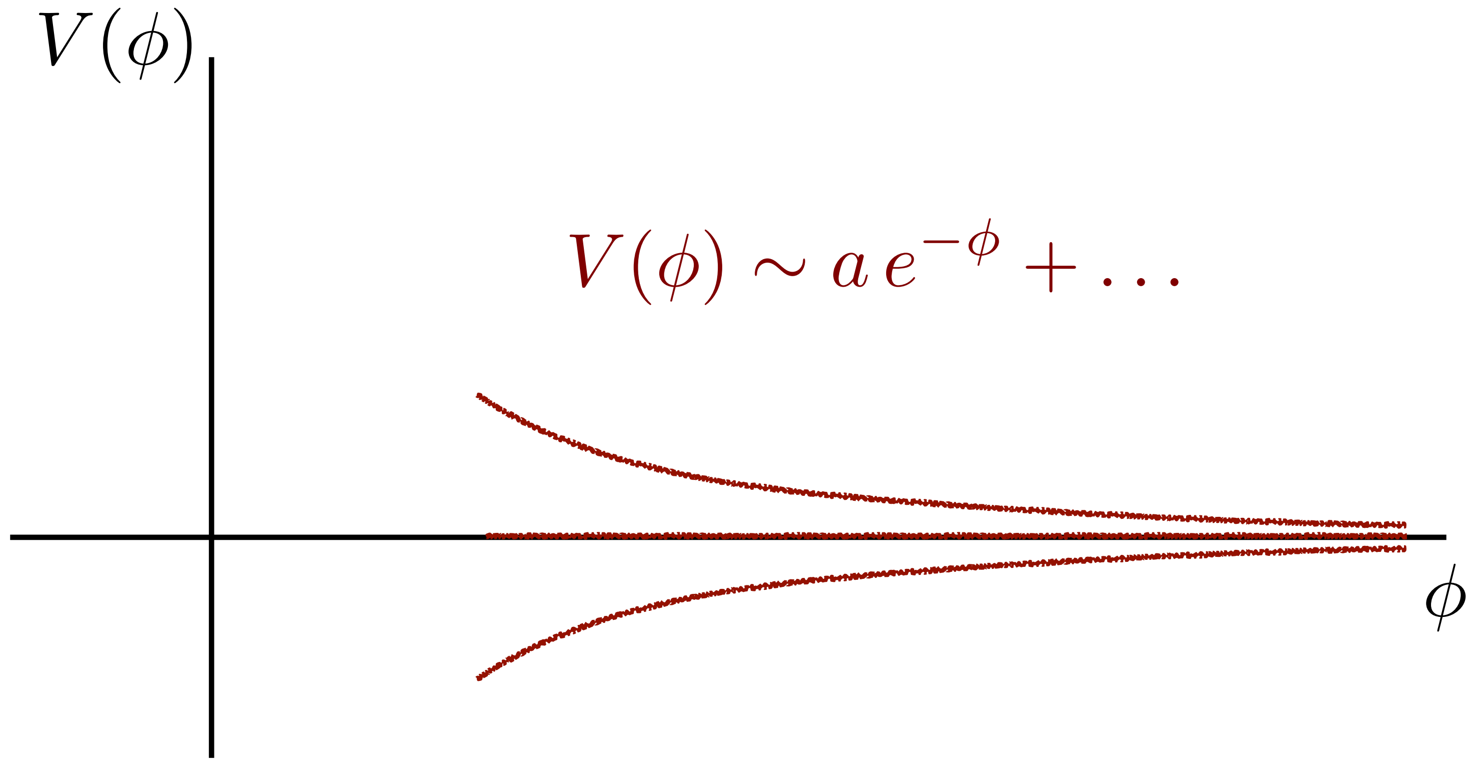
But Riess suspects that the mystery can't be solved by observations alone. "We won't really resolve it until some brilliant person, the next Einstein-like person, is able to get the idea of what's going on," he said.

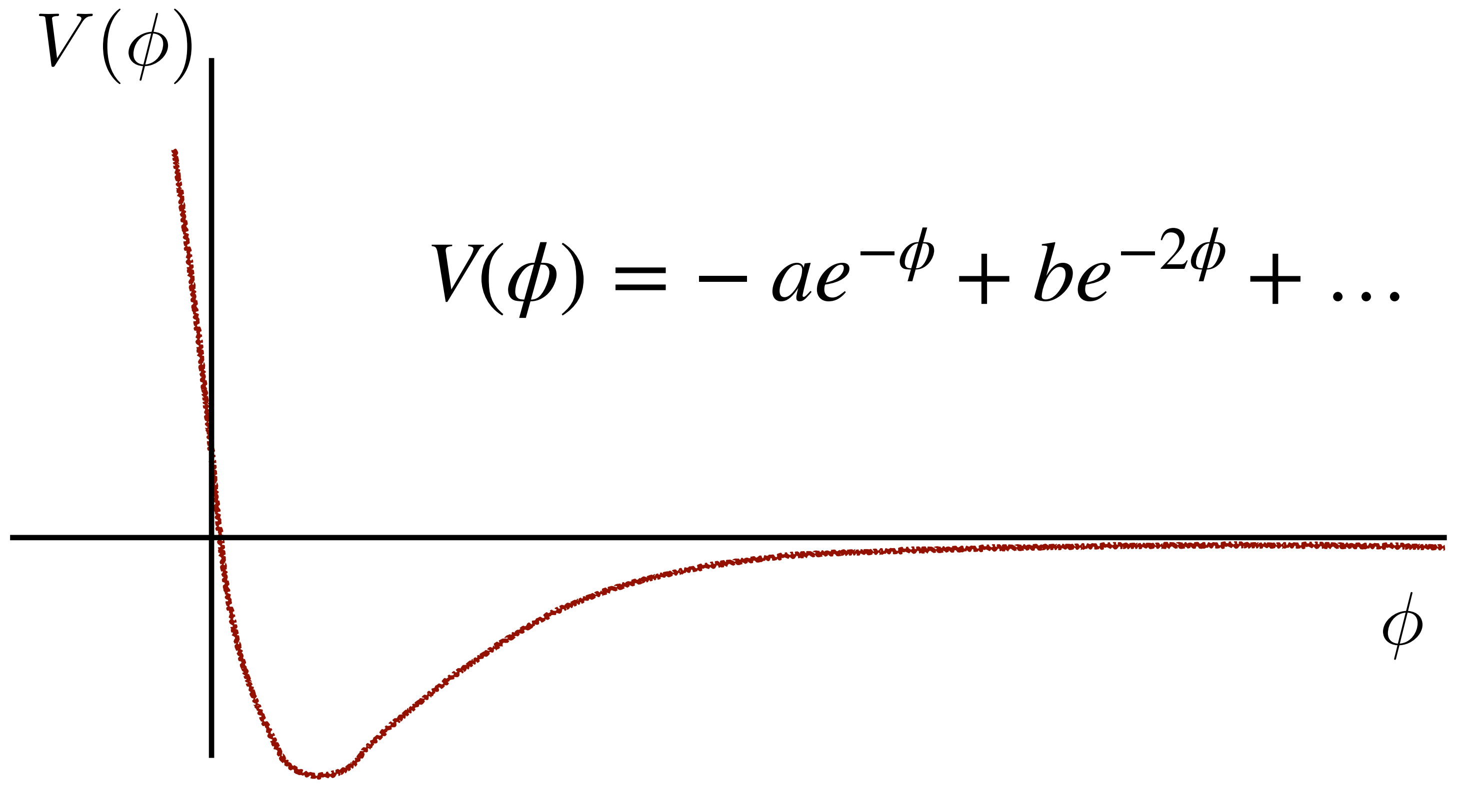
So he issued **a plea to the theorists**: "Keep working," he said. "We need your help. ... It's a very juicy problem, it's hard, and **you'll win a Nobel Prize if you figure it out. In fact, I'll give you mine.**"

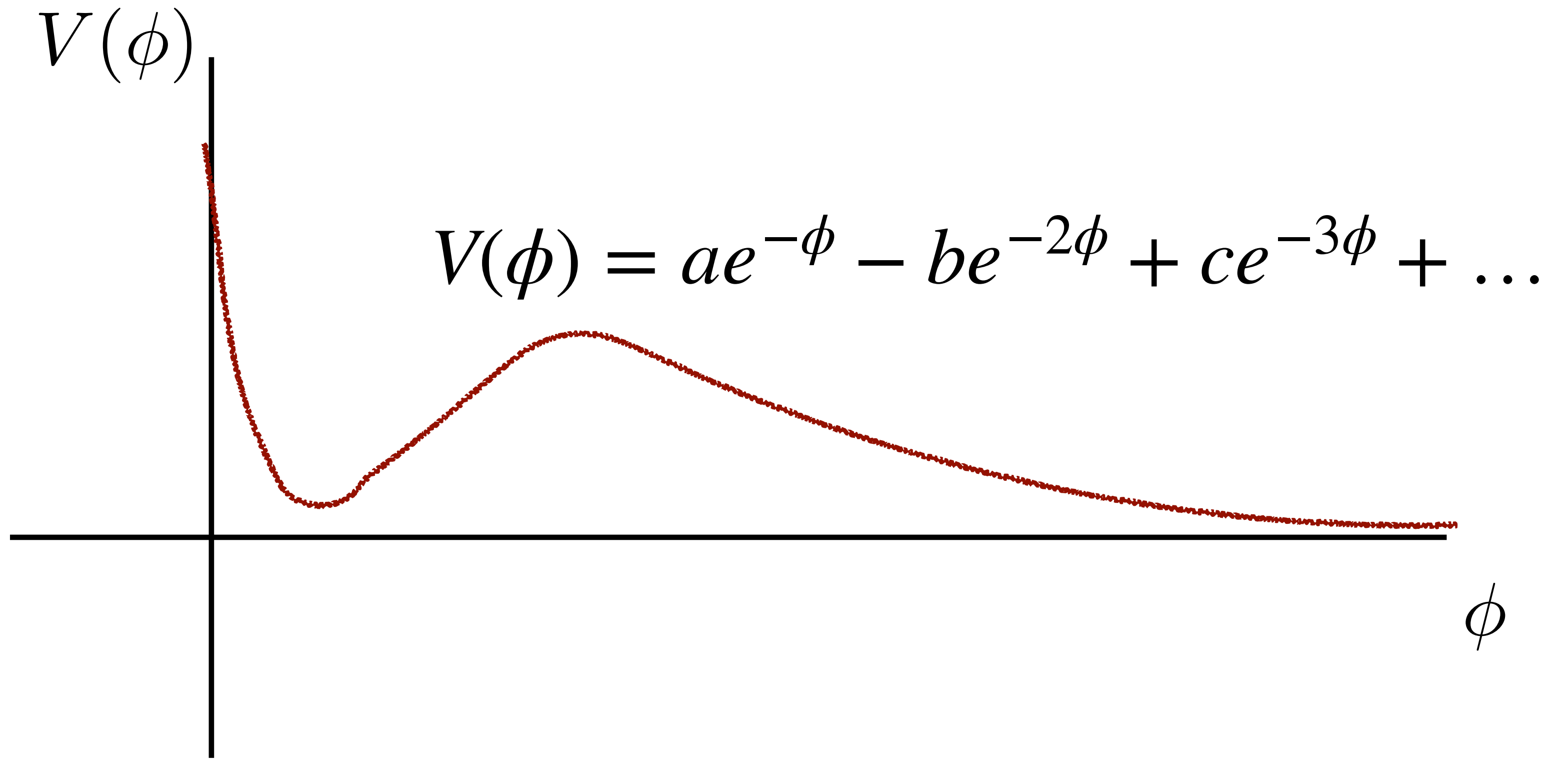
Oct 4, 2011 https://www.nbcnews.com/science/cosmic-log/physics-prize-highlights-puzzles-flna6c10402772

Dark Energy in String Theory

- Simplest possibility is $\Lambda > 0$. Sophisticated string theory scenarios for realizing dS vacua have been developed (KKLT, LVS, …), but a fully explicit construction remains elusive.
- Root of the challenge: source of cosmic acceleration should be ***derived*** (not just postulated) in a UV complete theory of gravity.
- It is a formidable task to demonstrate that the microphysics which stabilizes all moduli would lead to a theoretically controlled metastable de Sitter vacuum.
- The Dine-Seiberg problem highlights the difficulty in finding parametrically weakly-coupled vacua.







To roll or not to roll?

Current cosmic acceleration can be realized by:

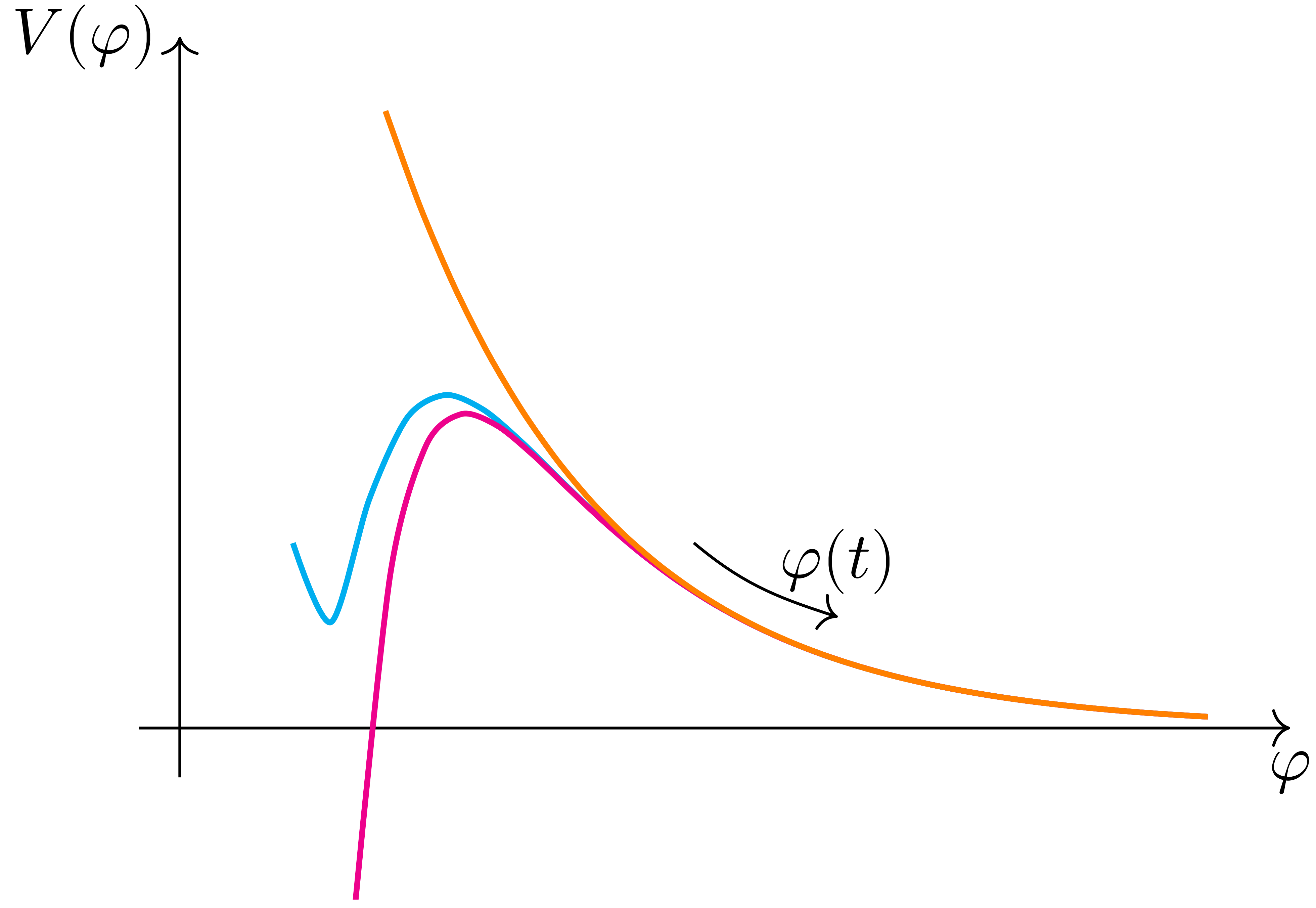
- a de Sitter minimum,
- a de Sitter maximum, or
- a runaway potential with $\epsilon \equiv -\frac{\dot{H}}{H^2} < 1$

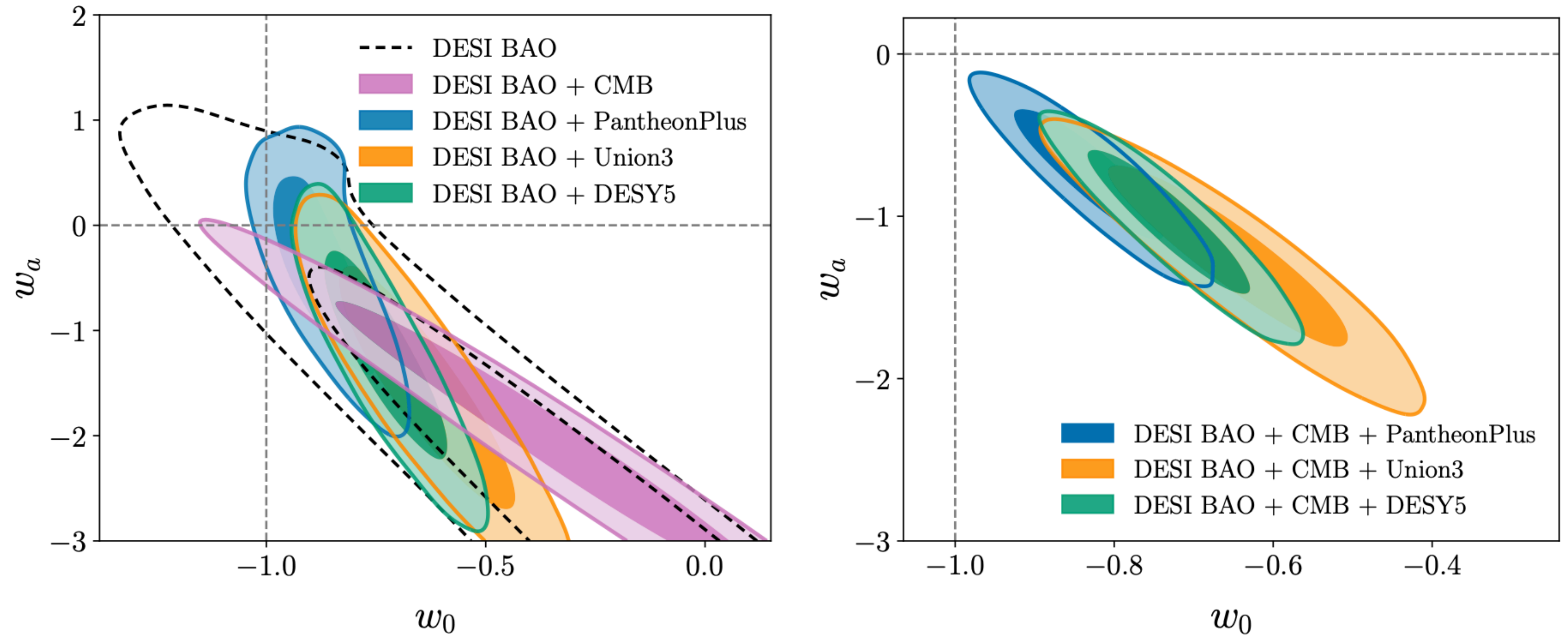
Unlike inflation which needs to last 60 e-folds to solve the flatness & horizon problems, the current acceleration may last only an e-fold or less.

If the universe underwent a rolling phase before, why not again? (main hurdle: 5-th force constraint)

Recent DESI results gave a tantalizing hint of varying dark energy, though it is too early to tell.

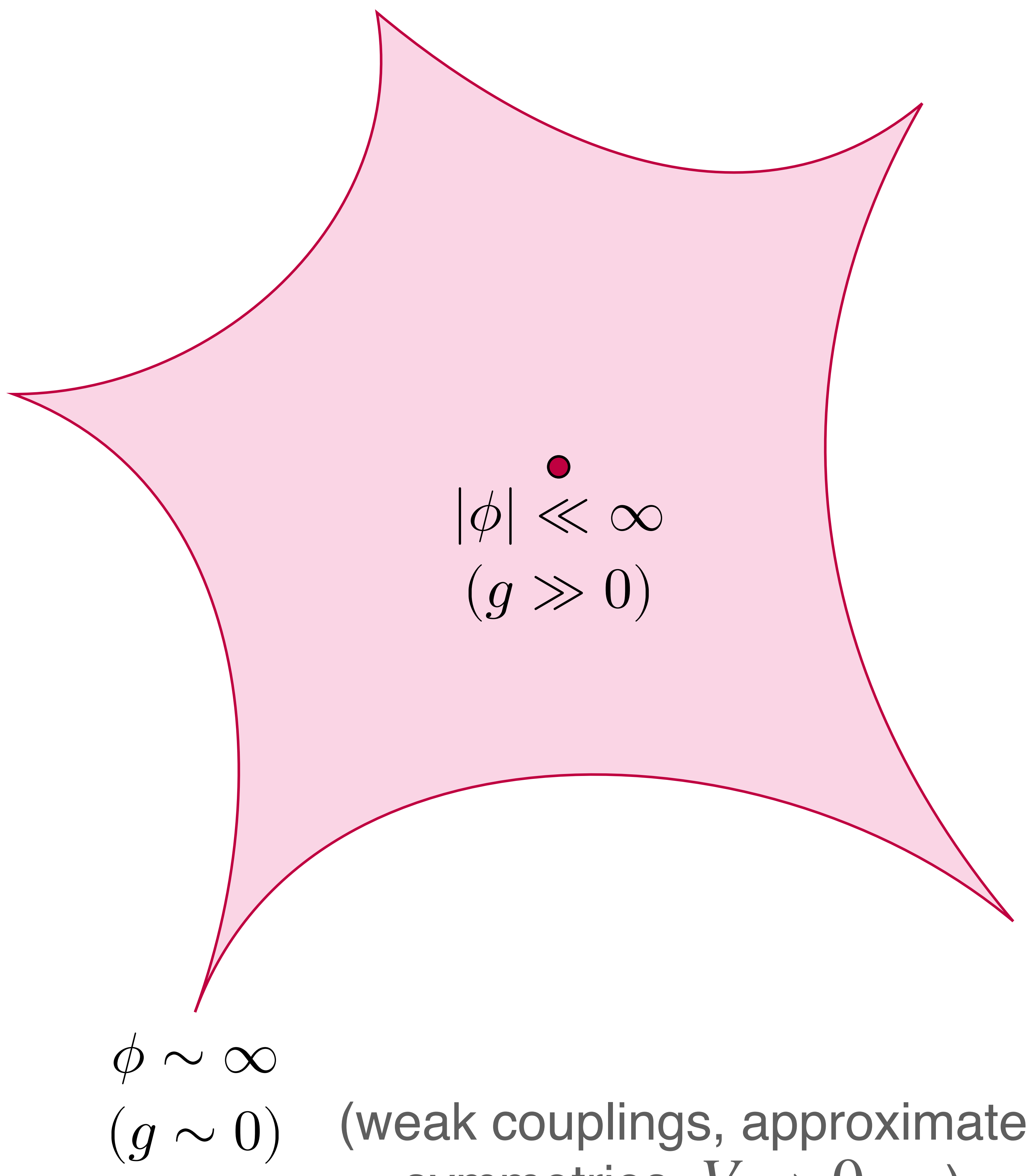
Generally $\epsilon \neq \epsilon_V$ due to non-negligible kinetic energy. How do we bound ϵ w/o knowing on-shell solutions?





Asymptotic Dark Energy

- Could the current acceleration be realized by rolling towards the asymptotic regions of the landscape? Andriot, Cremonini, Calderon-Infante, Hebecker, Rajaguru, Revello, Ruiz, Schreyer, GS, Tang, Tonioni, Tran, Tsimpsis, Valenzuela, Van Riet, Venken, Wrase, …
- Does not require terms of different order to compete, in contrast to the Dine-Seiberg problem for vacua.
- A tower of states becomes light as we approach the asymptotic. Entropy bound $\Rightarrow$ potential has an exponential falloff [Ooguri, Palti, GS, Vafa]
- But solving multi-field dynamics is much more difficult than taking derivatives of potential!
- As in many dynamical systems, the late-time regime exhibits some universal behaviors. This allows us to prove bounds on acceleration [GS, Tonioni, Tran].



(weak couplings, approximate symmetries, $V \to 0$, …)

explain small numbers in Nature?

Multi-field Quintessence

- String theoretical potentials generically take the form (also argument by [Ooguri, Palti, GS, Vafa]):

$$V = \sum_{i=1}^{m} \Lambda_i \, e^{-\kappa_d \gamma_{ia} \phi^a} \, .$$

after canonically normalizing the scalar fields to ϕ^a, $a = 1,\ldots,n$.

- Λ_i, γ_{ia} depend on the microscopic origin of V_i, κ_d = d-dim. gravitational coupling. Potentials from e.g. internal curvature, fluxes, branes/O-planes, Casimir-energy, etc take this form.

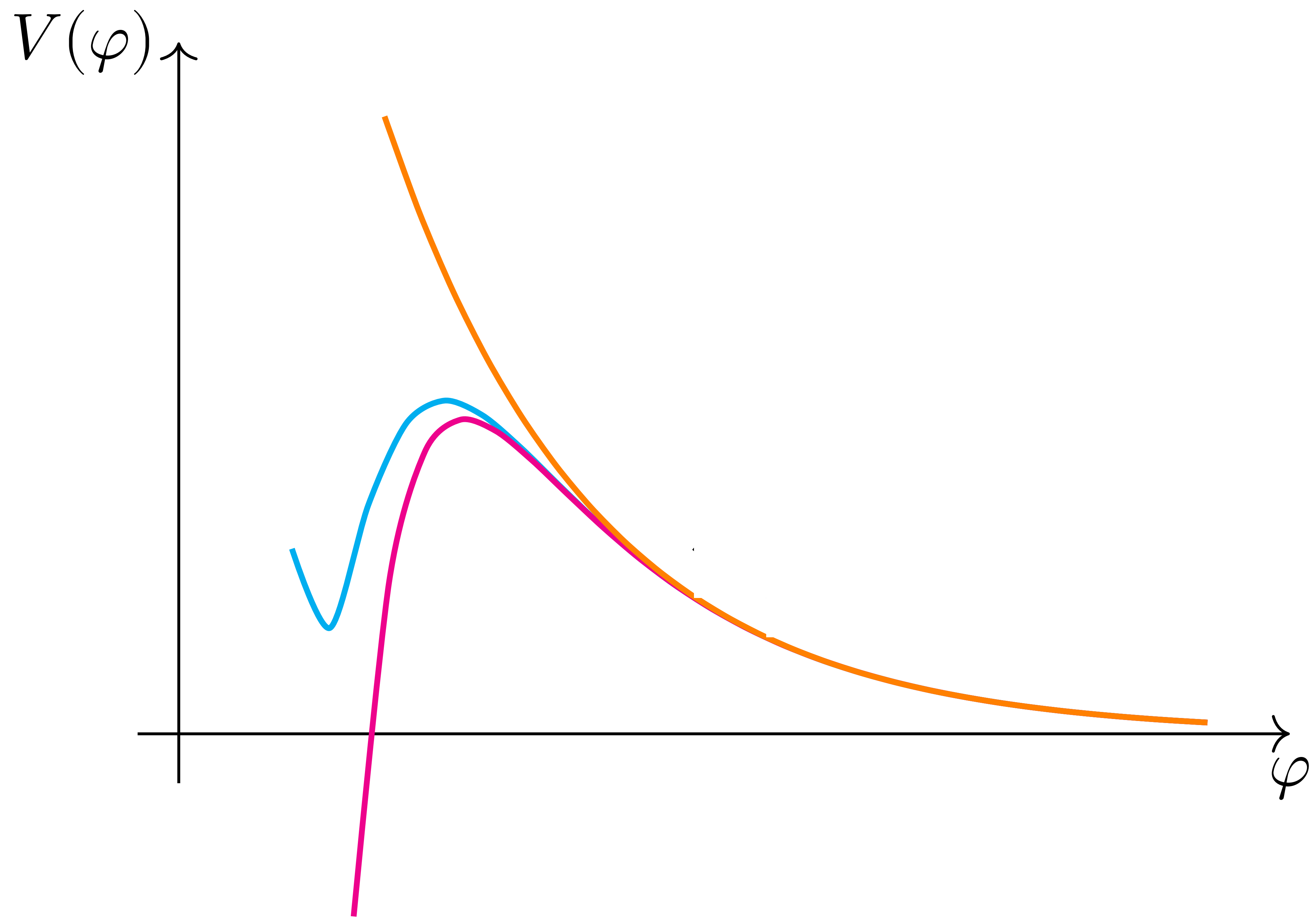
- Given a multi field quintessence model, how do we diagnose if it can support acceleration without solving for the time-dependent solutions? [STT1, STT2].

- We consider scalars rolling towards the field space boundary: axions with a compact field space are assumed to be stabilized above. The saxions can then be canonically normalized.

- In the presence of dynamical axions, the field space metric is curved but in certain classes of models, the bounds we derived continue to apply [STT4].

Multi-field Quintessence

- We are **not** considering transient acceleration even though it is all we need observationally.



- Our aim is to identify the roots of the apparent obstruction to an $\epsilon < 1$ phase that lasts; lessons learned can potentially point us to viable models with shorter accelerating periods.

Cosmological Equations

- Non-compact d-dim. spacetime is characterized by the FLRW metric:

$$d\tilde{s}_d^2 = -\mathrm{d}t^2 + a^2(t)\,\mathrm{d}l^2_{\mathbb{R}^{d-1}}\,,$$

- Hubble parameter: $H \equiv \frac{\dot{a}}{a}$. The proper diagnostic for cosmic **acceleration** is $\epsilon \equiv -\frac{\dot{H}}{H^2} < 1$ to be **distinguished from the slow-roll parameter** $\epsilon_V = \frac{d-2}{4}\kappa_d^2\left(\frac{\nabla V}{V}\right)^2$.

- Scalar field equations and Friedmann equations:

$$\ddot{\phi}^a + (d-1)H\dot{\phi}^a + \frac{\partial V}{\partial \phi_a} = 0,$$

$$\frac{(d-1)(d-2)}{2}\,H^2 - \kappa_d^2\left[\frac{1}{2}\,\dot{\phi}_a\dot{\phi}^a + V\right] = 0,$$

$$\dot{H} = -\frac{\kappa_d^2}{d-2}\left[\frac{1}{2}\,\dot{\phi}_a\dot{\phi}^a - V\right] - \frac{d-1}{2}H^2,$$

Cosmology as a Dynamical System

- It is convenient to work with the rescaled variables:

$$x^a = \frac{\kappa_d}{\sqrt{d-1}\sqrt{d-2}} \frac{\dot{\phi}^a}{H}, \; y_i = \frac{\kappa_d \sqrt{2}}{\sqrt{d-1}\sqrt{d-2}} \frac{\sqrt{V_i}}{H}$$

- The cosmological equations can be formulated in terms of an autonomous system of ODEs given schematically as follows:

$$\frac{d\vec{z}}{dt} = g(\vec{z}) \; , \qquad \text{where } \vec{z} \equiv (x^1, \ldots, x^n, y^1, \ldots, y^m, H)$$

- Among the above ODEs is $\epsilon = -\dot{H}/H^2 = (d-1)x^2$; strategy is to bound the kinetic energy.

- Friedmann equation also takes a simple form:

$$(x)^2 + (y)^2 = 1$$

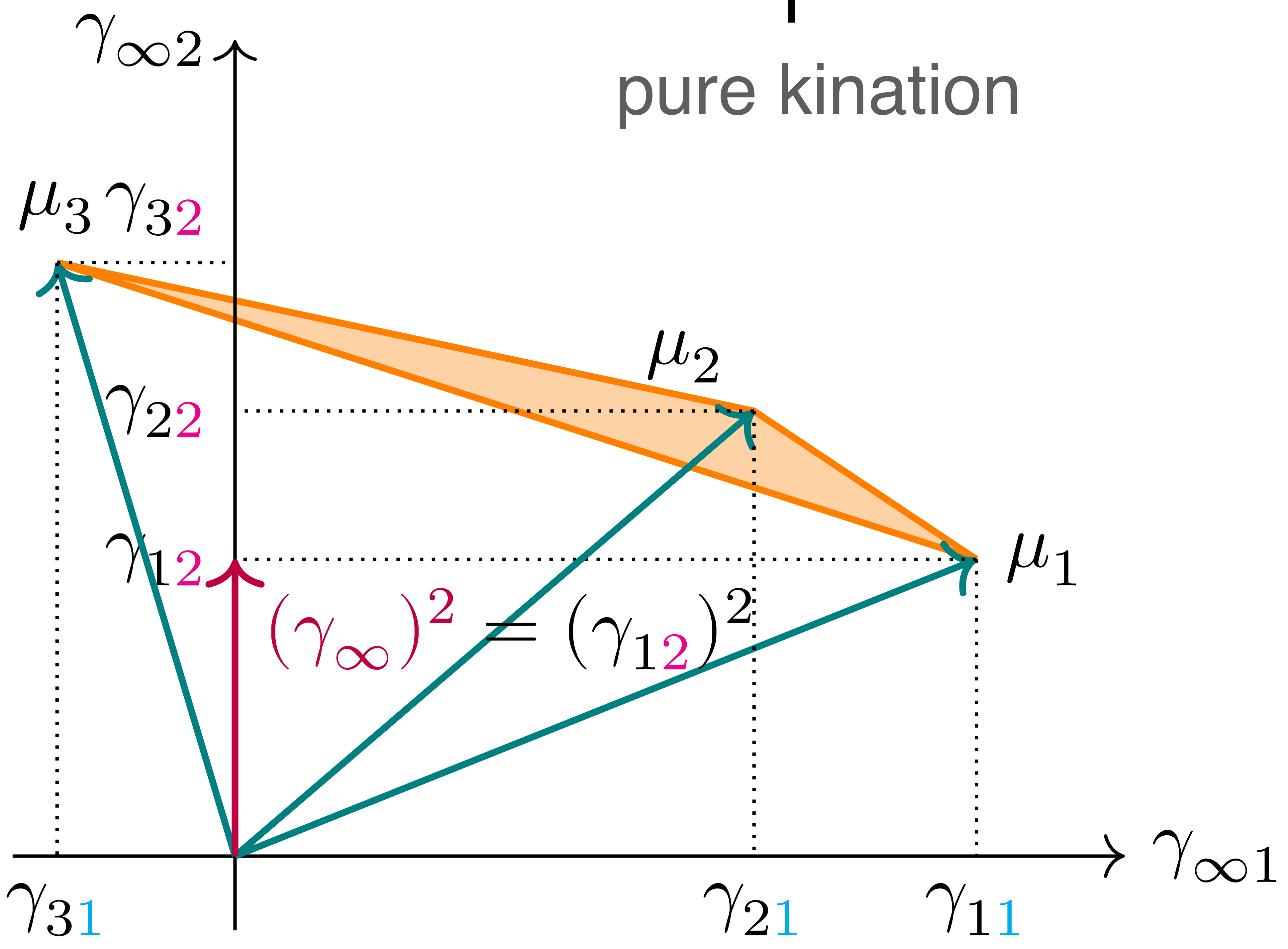
Geometric Bound on Cosmic Acceleration

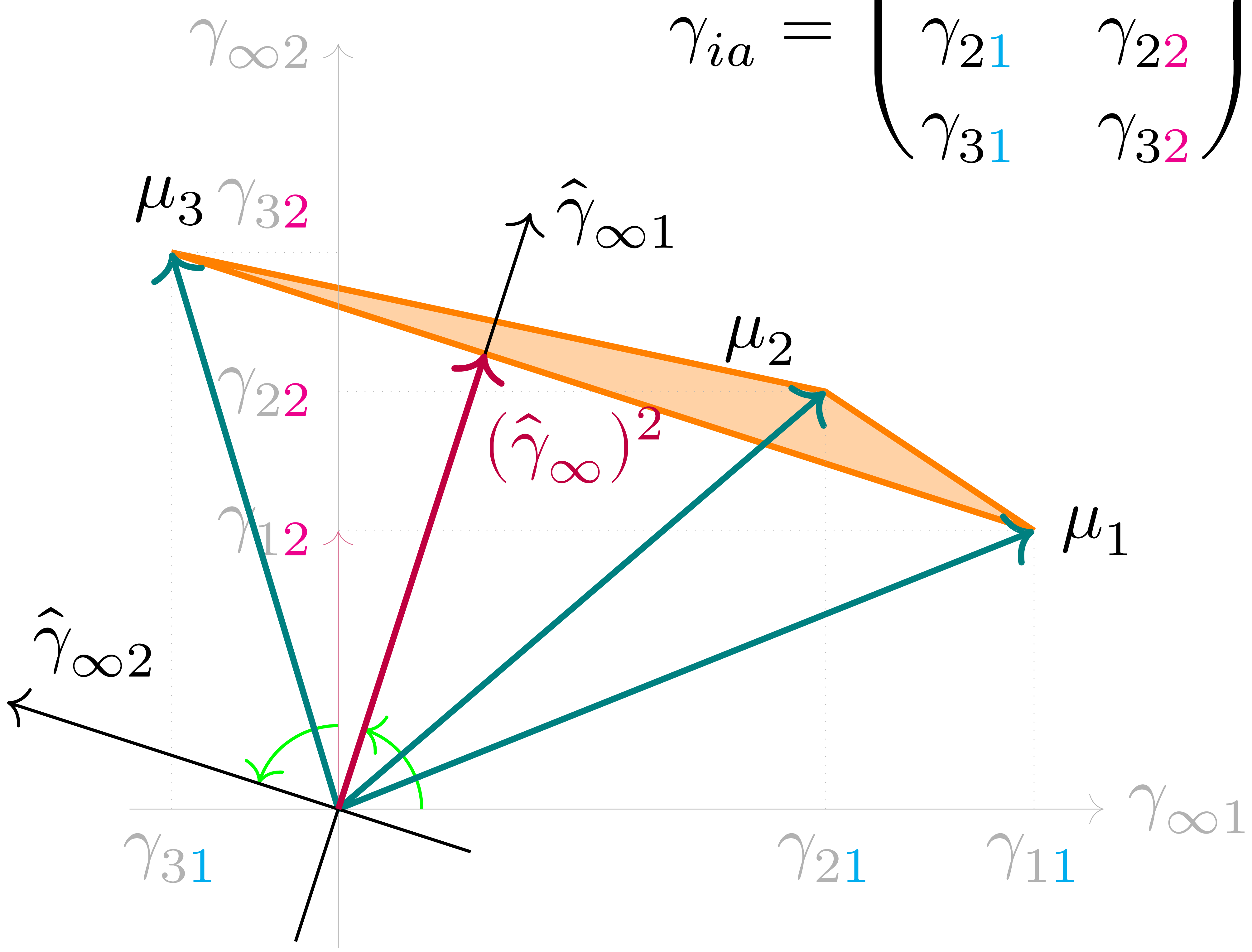
- Define m vectors μ_i, one for each potential term with components $(\mu_i)_a = \gamma_{ia}$

$$d - 1 \geq \; \epsilon \geq \frac{d-2}{4} \max_{R \in O(n)} \left[\gamma_\infty(R)\right]^2$$

↑ pure kination

$$\gamma_{ia} = \begin{pmatrix} \gamma_{11} & \gamma_{12} \\ \gamma_{21} & \gamma_{22} \\ \gamma_{31} & \gamma_{32} \end{pmatrix}$$





A Universal Obstruction

- String-theoretical potentials take the form:

$$S = -\int_{X_{1,9}} [A_r \wedge \star_{1,9} A_r]\, \Lambda_{10,r}\, \mathrm{e}^{-k\sigma - \chi_{\mathrm{E}}\Phi} = -\int_{X_{1,d-1}} \tilde{*}_{1,d-1} \Lambda\, \mathrm{e}^{\kappa_d[\gamma_{\tilde{\delta}}(\chi_{\mathrm{E}})\tilde{\delta} - \gamma_{\tilde{\sigma}}(\chi_{\mathrm{E}},r,k)\tilde{\sigma}]}$$

RR fields are not weighed by $e^{-\chi_E \Phi}$ (effectively set $\chi_E = 0$) but would not affect our argument.

- The d-dim. dilaton $\tilde{\delta}$ is a linear combination of the 10d dilaton Φ and Einstein frame volume.

- While the field basis choice is not unique, d-dimensional dilaton $\tilde{\delta}$ has **universal properties**:

$$\gamma_{\tilde{\delta}} = \frac{d}{\sqrt{d-2}} - \frac{1}{2}\chi_{\mathrm{E}}\sqrt{d-2} \geq \frac{2}{\sqrt{d-2}} \quad \Rightarrow \quad \epsilon \geq \frac{d-2}{4}(\gamma_\infty)^2 \geq \frac{d-2}{4}\gamma_{\tilde{\delta}}^2 \geq 1$$

- Ways out: 1) $\tilde{\delta}$ is stabilized; 2) $\tilde{\delta}$ is rolling but not in the asymptotic regions; 3) V contains at least three terms, not all of the same sign (e.g., from loop corrections).

- **Living dangerously:** structure of string theory couplings puts us on the borderline.

Scaling Solutions

- The cosmological autonomous system admits **scaling solutions** ($\epsilon = \text{constant} > 0$):
  - scale factor takes a power law form: $a(t) \sim t^p$
  - critical points of the autonomous system: $\dot{x}^a = 0$
- **Analytic solution:** for $\text{rank}\, \gamma_{ia} = m$
  - field space trajectory: $$\phi_*^a(t) = \phi_0^a + \frac{2}{\kappa_d}\left[\sum_{i=1}^{m}\sum_{j=1}^{m} \gamma_i{}^a (M^{-1})^{ij}\right] \ln\frac{t}{t_0}, \qquad M_{ij} = \gamma_{ia}\gamma_j{}^a$$
  - scale factor: $$p = \frac{4}{d-2}\sum_{i=1}^{m}\sum_{j=1}^{m}(M^{-1})^{ij}.$$

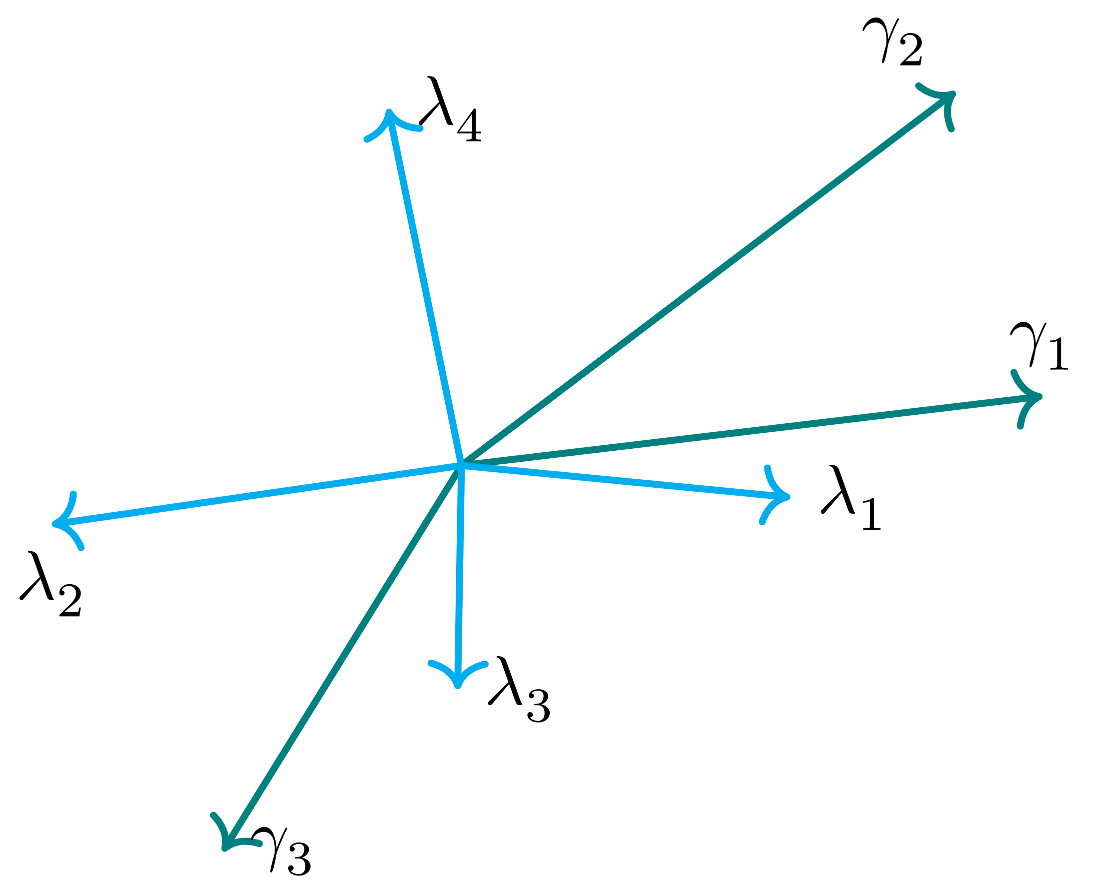
[Copeland, Liddle, Wands, '97]
[Collinucci, Nielsen, Van Riet, '04]

- The kinetic term & every potential term have the same parametric dependence in time:

No slow-roll: $$T(t) = T(t_0)\left(\frac{t_0}{t}\right)^2, \quad V_i(t) = V_i(t_0)\left(\frac{t_0}{t}\right)^2$$

Late-time attractor behavior **proved** in [STT2, STT4], going beyond earlier analysis of linear stability.

Field Space Curvature



- In the presence of axions, the field space metric is curved:

$$T[\phi,\zeta] = \frac{1}{2}\sum_{a=1}^{n}(\dot{\phi}^a)^2 + \frac{1}{2}\sum_{r=1}^{p} \mathrm{e}^{-\kappa_d \sum_a \lambda_{ra}\phi^a}\,(\dot{\zeta}^r)^2,$$

- It has been conjectured that the field space metric is generically negatively curved in the asymptotic limits [Ooguri, Vafa, '06]; though ∃ exceptions [Trenner, Wilson, '09];[Marchesano, Melotti, Paloni, '23];[Raman, Vafa, '24].

- The Kahler potential for a chiral multiplet $\xi = \theta + ie^{l\varphi}$ in $N=1,\ D=4$ SUGRA typically takes this form:

$$K = -\,n\ln[-i(\xi-\bar{\xi})] \qquad \Rightarrow \qquad T[\phi,\zeta] = \frac{n}{4}\left[l^2\dot{\varphi}^2 + e^{-2l\varphi}\dot{\theta}^2\right] = \frac{1}{2}\dot{\phi}^2 + \frac{1}{2}e^{-\frac{2\sqrt{2}}{\sqrt{n}}\phi}\dot{\zeta}^2$$

- This kind of negatively curved field space metric is common in string compactifications. Irrespective of the UV origin, it has been used for quintessence model-building, offering rich pheno possibilities.

- But the multi-field dynamics is much more complicated, allowing for spirals in field space. There are two sets of coupling convex hulls, how do we bound ϵ and find late-time attractors?

- We identified several geometric configurations for the potential and kinetic couplings for which the universal bounds for flat field spaces found earlier are still in place [STT4].

DM/DE Coupling & SEC Violation

DM/DE Coupling and SEC Violation

- A coupling of DM to DE induces an effective transient violation of NEC [Das, Corasaniti, Khoury, '05]. This DM/DE coupling has recently been revisited [Chakraborty, Chanda, Das, Dutta, '25];[Khoury, Lin, Trodden, '25]; [Andriot, '25] in light of the 2025 DESI results.

- The energy density of a cosmological fluid with constant state parameter w evolves as

$$\rho = \rho_0 \left(\frac{a_0}{a} \right)^{(d-1)(1+w)/2}$$

- A NEC-violating fluid thus has a growing energy density. If there is a second fluid decaying into DE, then effectively the DE density may increase over time while still fulfilling the NEC.

- STT5: the DM/DE coupling may also induce an effective SEC violation, even though each component of the universe satisfies the SEC.

- This coupling dates back to the 90s [Wetterich, '94];[Amendola, '99] though with a different purpose.

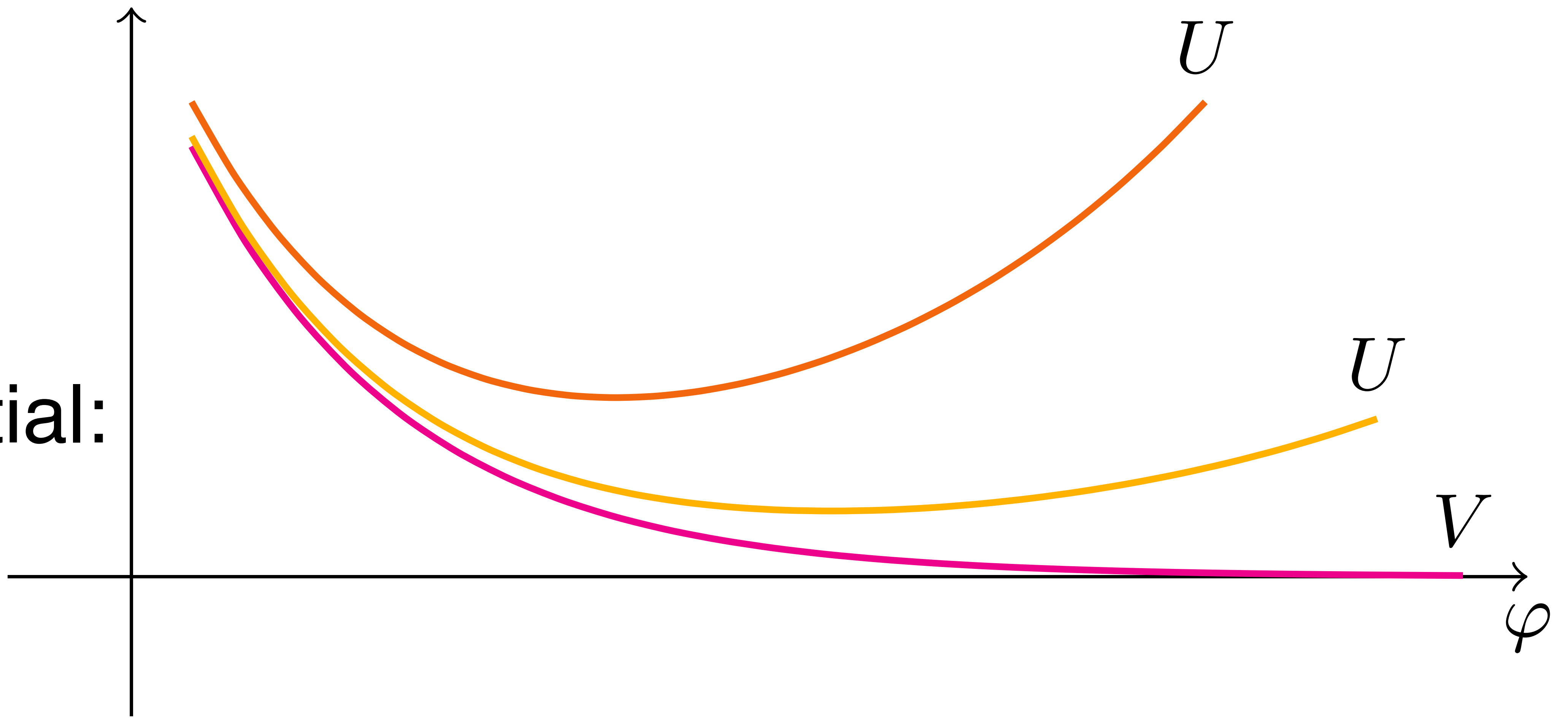
Time-Dependent Potential Minimum

- The energy density of non-relativistic DM scales inversely with the volume and proportionally to the mass. For a field-dependent DM mass:

$$\rho_{\mathrm{DM}} = \rho_{\mathrm{DM},0} \left(\frac{a_0}{a}\right)^{d-1} \frac{m(\varphi)}{m(\varphi_0)},$$

- The effect of the DM/DE coupling is an effective potential:

$$U(\varphi) = V(\varphi) + \rho_{\mathrm{DM},0} \left(\frac{a_0}{a}\right)^{d-1} \frac{m(\varphi)}{m(\varphi_0)}.$$



- Effective potential minimum shifts with time as universe expands (from darker to lighter orange).
- Exponential dependence is generic in string theory, consider for illustration the functional form:

$$m = \mu\, \mathrm{e}^{\kappa_d \beta \varphi}, \qquad V = \Lambda\, \mathrm{e}^{-\kappa_d \gamma \varphi}$$

New Critical Point

- The dynamical system involving the DM/DE coupling:

$$\ddot{\varphi} + (d-1)H\dot{\varphi} + V' = -\rho\,\frac{m'}{m},$$

$$\dot{\rho} + (d-1)(1+w)H\rho = \rho\,\dot{\varphi}\,\frac{m'}{m},$$

$$H^2 = \frac{2\kappa_d^2}{(d-1)(d-2)}\left[\frac{1}{2}\,\dot{\varphi}^2 + V + \rho\right]$$

has a new linearly stable critical point (late-time attractor is typically a critical point with smallest ϵ):

$$\varphi(t) = \varphi_0 + \frac{2}{\kappa_d\gamma}\,\ln\frac{t}{t_0},$$

$$\rho(t) = \frac{4}{(1+w)\gamma^2}\frac{1}{\kappa_d^2 t^2}\left[\frac{1}{2}\frac{d-2}{d-1}\frac{\gamma(\gamma+\beta)}{1+w} - 1\right]$$

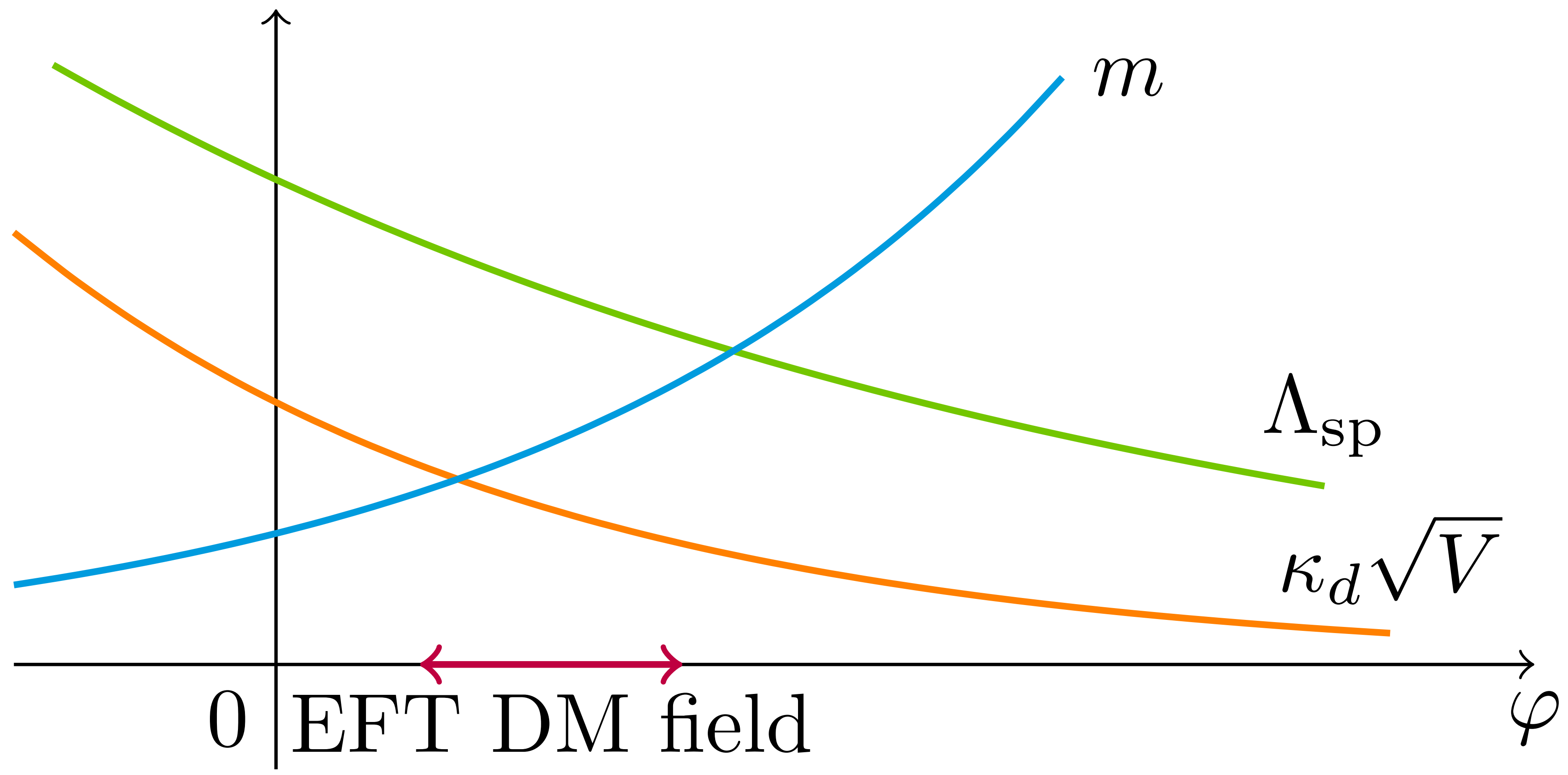
- Acceleration is possible if $\beta > 0$, $\gamma > 0$ and for moderately large β/γ :

$$\epsilon = \frac{d-1}{2}\frac{1+w}{1+\beta/\gamma}$$

Seeking String Realizations

- In string compactifications, the cutoff must lie below the species scale [Veneziano, '01]; [Dvali, '07]. which is expected to fall as:

$$\Lambda_{\rm sp} = m_{{\rm P},d}\, {\rm e}^{-\kappa_d \lambda \varphi} \,, \qquad \lambda \sim \mathcal{O}(1)$$



- However, since we are treating DM as a cosmological fluid made out of non-relativistic matter constituents (classical source), it is not necessary for the DM mass to be below the cutoff.

- For example, consider a DM star made up of DM particles. If the DM particles have a field-dependent mass, it is not unreasonable to expect a DM star with $m(\varphi)$.

- Moreover, although the DM mass grows exponentially, the universe expansion is still sufficient to dilute the DM energy density over time.

Seeking String Realizations

- Distance conjecture [Ooguri, Vafa, '06]: towers of states that become light as one approaches the field-space asymptotics $\varphi \to \infty$:

$$m_{\rm DC}(\varphi) = \mu_{\rm DC}\, e^{-\kappa_d \alpha \varphi},$$

for order 1 constant $\alpha > 0$. Evidence for $\alpha \geq 1/\sqrt{d-2}$ [Etheredge, Heidenreich, Kaya, Qiu, Rudelius, '22].

- Due to string dualities, towers of light states emerge also in the opposite regime $\varphi \to -\infty$ and thus there are towers of heavy states with growing mass $m = \mu\, e^{\kappa_d \beta \varphi}$, with $\beta \geq 1/\sqrt{d-2}$.

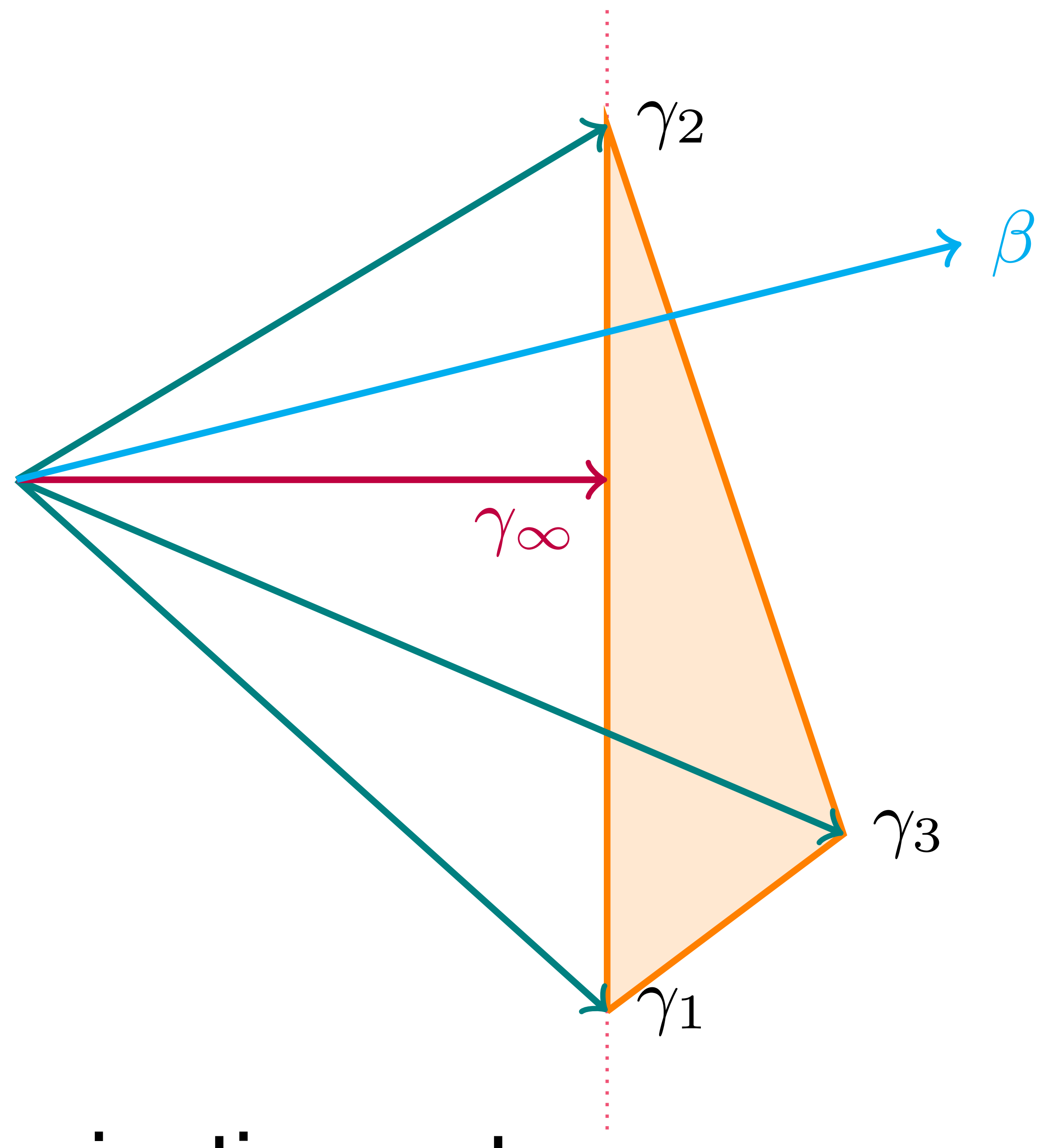
- Curiously, with the saturating values of the exponents of the potential and DM/DE coupling:

$$\gamma \geq 2/\sqrt{d-2}, \quad \beta \geq 1/\sqrt{d-2}, \quad \Rightarrow \quad \epsilon = (d-1)/3$$

Seeking String Realizations

- It is not necessary for the bounds on β and γ to be saturated (and at the same time).
- As a specific example, a curvature-induced potential $V(\varphi)$ & a KK-monopole $m(\varphi)$ give $\epsilon = 6/7$.
- As before, we can generalize our results to multiple fields with potential and DM/DE coupling:

$$V = \sum_i \Lambda_i \, e^{-\kappa_d \gamma_i \cdot \varphi} \,, \quad m = \mu \, e^{\kappa_d \beta \cdot \varphi}$$



- There exists a solution to this dynamical system with

$$\epsilon = \frac{d-1}{2} \frac{1+w}{1+\frac{\beta \cdot \gamma_\infty}{(\gamma_\infty)^2}},$$

- The multi-field problem can be reduced to a single field problem by projecting onto γ_∞.

Summary of Results

- Treating the universe as a dynamical system, we bound the rate of time variation of the Hubble parameter at late time [STT1]. The bound provides a useful diagnostic for dark energy models.
- Our bound when applied to string theoretic constructions identifies a generic obstacle to acceleration if the d-dim. dilation is one of the rolling fields. We also suggest several ways out.
- We prove conditions under which scaling solutions are **late-time attractors**. Moreover, we prove that scaling solutions **saturate** our bound on ϵ [STT2].
- Our results apply irrespective of whether the potential is generated classically or quantum mechanically, whether the kinetic term is negligible, & whether some potential term dominates.
- This program can be extended to quintessence models with dynamical axions as well [STT4].
- As a spinoff, we derived analogous bounds on ekpyrosis [STT3].
- DM/DE coupling relaxes these bounds. Some features can be realized in string theory, though a fully UV complete model that explains the current cosmic acceleration remains to be constructed. [STT5].