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EFFECTIVE FIELD THEORY OF COUPLED DARK ENERGY AND DARK MATTER

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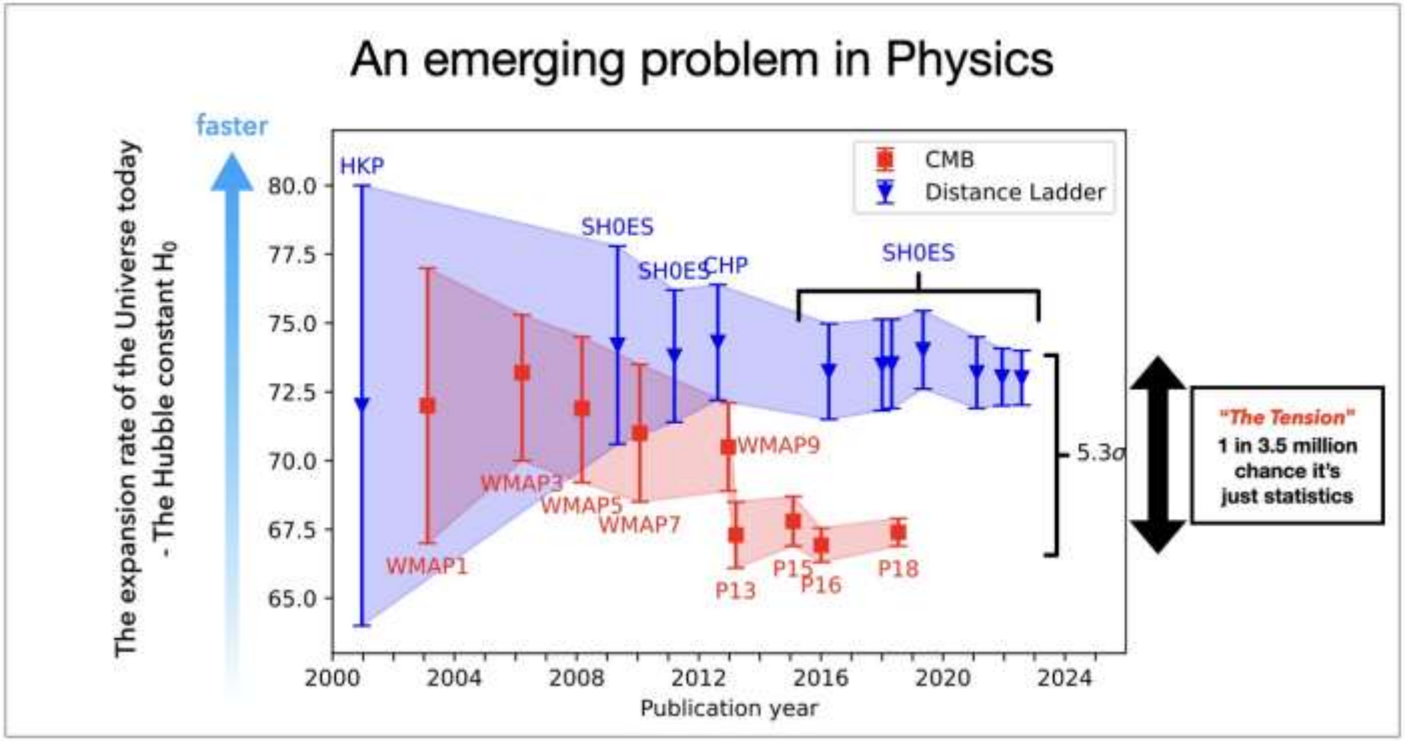
Cosmological tensions

The standard cosmological model is the Λ-Cold-Dark-Matter (ΛCDM) model, but there have been tensions in recent observations.

- H_0 tension

H_0 is today's Hubble expansion rate.



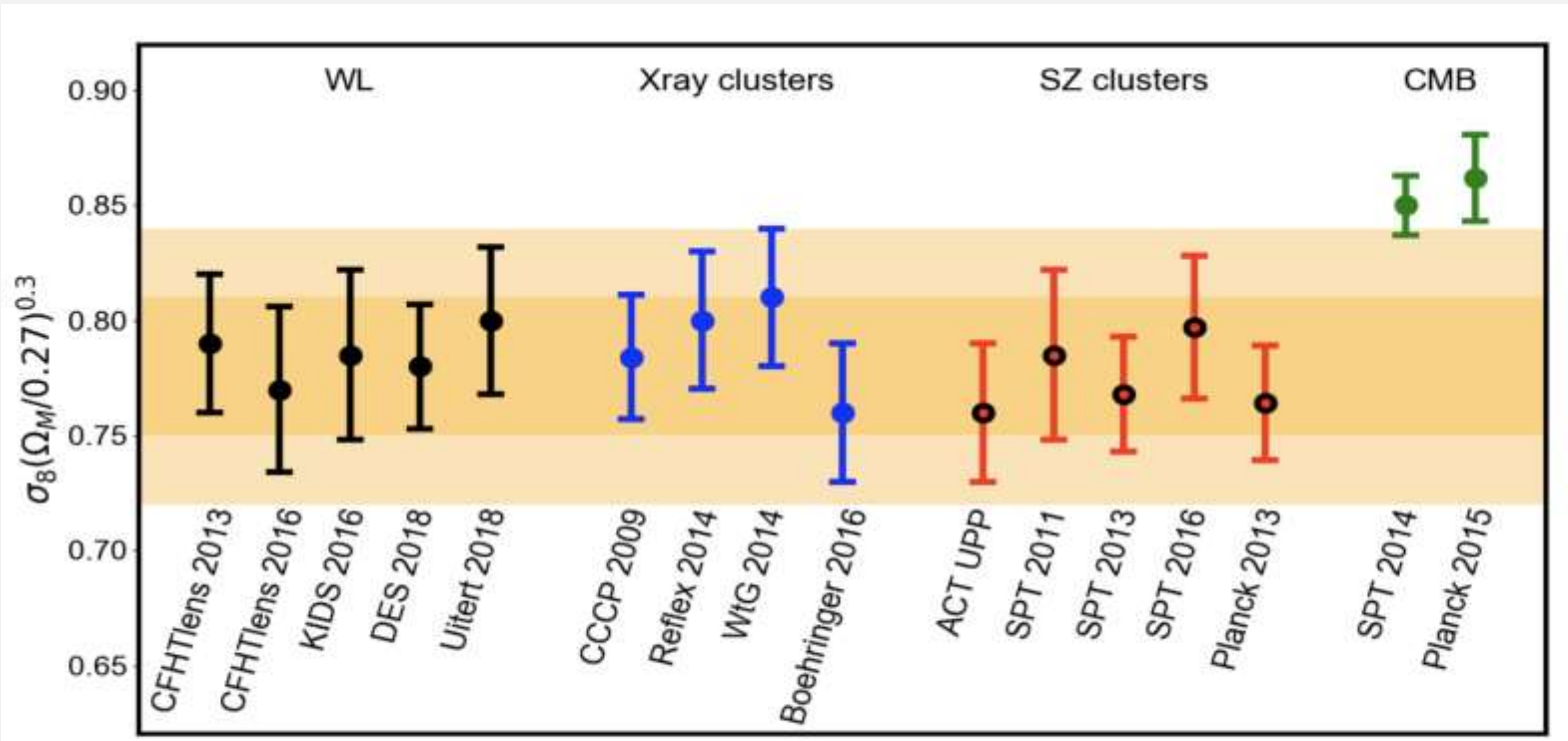
H_0 is different between CMB and low-redshift measurements.

Tension is $> 5.3\sigma$

- σ_8 tension

σ_8 is the root-mean-square mass fluctuations on a scale of $8h^{-1}$ Mpc.

σ_8 is also different between CMB and low-redshift measurements.

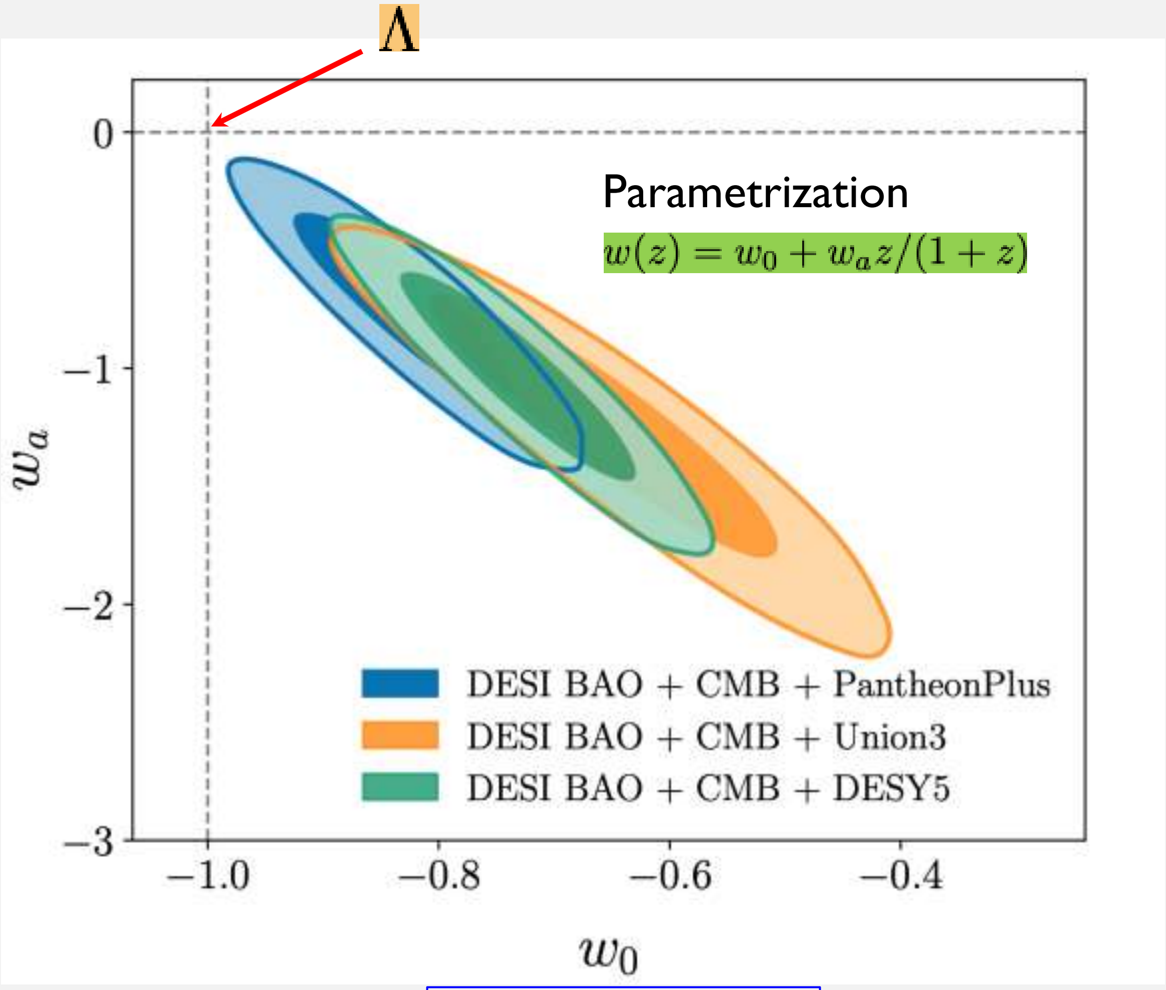


The value of σ_8 constrained from the Planck CMB data is larger than those constrained from low-redshift measurements.

Tension is $> 2\sigma$

Dynamical dark energy?

Recent DESI BAO data favor the dynamical dark energy over Λ.



arXiv: 2404.03002

Dynamical dark energy models

The origin of dark energy could be a scalar field, a vector field, …

- Scalar field ϕ

Most general scalar-tensor theories with second-order equations of motion are known as Horndeski theories.

A subclass of Horndeski theories with a luminal speed of gravitational waves is

$$\mathcal{S} = \int \mathrm{d}^4x \sqrt{-g} \left[G_2(\phi, X) + G_3(\phi, X) \Box \phi + G_4(\phi) R \right]$$

$X = -\partial_\mu \phi \partial^\mu \phi / 2$ and R is the Ricci scalar.

- Vector field A_μ

Vector-tensor theories with a broken $U(1)$ gauge invariance and the equations of motion up to second order are known as generalized Proca (GP) theories.

A subclass of GP theories with a luminal speed of gravitational waves is

$$\mathcal{S} = \int \mathrm{d}^4x \sqrt{-g} \left[F + G_2(\tilde{X}) + G_3(\tilde{X}) \nabla_\mu A^\mu + \frac{M_{\rm Pl}^2}{2} R \right]$$

$F = -F_{\mu\nu} F^{\mu\nu} / 4$,
$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$,
$\tilde{X} = -A_\mu A^\mu / 2$

Dark matter (DM) sector

The DM sector can be described by a dust fluid with negligible pressure and sound speed (CDM).

A fluid description is formulated using three scalar fields ϕ^i (with $i = 1, 2, 3$).

A fluid phase is defined by an invariance under internal volume-preserving diffeomorphisms.

Dubobsky+ (2005), Endrich+ (2013)

We choose the comoving gauge $\phi^i = x^i$, where the perturbations of ϕ^i are eaten by the metric.

Number density: $n := \sqrt{\det g^{\mu\nu}\partial_\mu\phi^i\partial_\nu\phi^j} = \sqrt{\det g^{ij}}$

Four velocity: $u^\mu := -\frac{1}{6n}\varepsilon_{ijk}\epsilon^{\mu\nu\rho\sigma}\partial_\nu\phi^i\partial_\rho\phi^j\partial_\sigma\phi^k = \frac{\delta^\mu_0}{\sqrt{-g_{00}}}$

The action of the DM sector is given by

$$S_{\rm DM} = -\int {\rm d}^4x\sqrt{-g}\,\hat{m}_c n$$

where $\hat{m}_c$ is a constant.

Effective gravitational couplings in Horndeski theories

Let us consider a Horndeski scalar field in the dark energy (DE) sector and a perfect fluid in the dark matter (DM) sector (without direct interactions).

The CDM density contrast δ_m obeys

$$\ddot{\delta}_m + 2H\dot{\delta}_m - 4\pi G_{\rm eff}\rho_m\delta_m = 0$$

with $$\frac{G_{\rm eff}}{G} = \frac{c_t^2}{8\pi G q_t}\left(1 + \frac{4H^2 q_t^2 \Delta_1^2}{q_s c_s^2 c_t^2 \dot{\phi}^2}\right)$$

De Felice, Kobayashi, ST (2011), Kase and ST (2018)

Tensor contribution

Scalar-matter interaction (DM is indirectly coupled to DE through gravity)

Always positive under the absence of ghost and Laplacian instabilities:

$$q_s > 0, \quad q_t > 0, \quad c_s^2 > 0, \quad c_t^2 > 0$$

This gives a stronger gravity, so it is difficult to address the σ_8 tension problem at low redshifts.

q_s, q_t: Associated with no-ghost conditions.

c_s, c_t: Scalar and tensor propagation speeds

The property of strong gravity also holds in GP theories with a luminal speed of gravitational waves. **De Felice + (2016), Amendola + (2018)**

Coupled dark energy (DE) and dark matter (DM)

The direct couplings between DE and DM (especially momentum transfer) can lead to weaker gravity compared to the LCDM model.

● Scalar-tensor (ST) theories

We can construct a product between the CDM four velocity u^μ and a scalar field derivative $\nabla_\mu \phi$, as

$$Z = -u^\mu \nabla_\mu \phi$$

→ Momentum exchange between CDM and DE

The interacting Lagrangian $f(Z)$ can suppress the growth of structures.

Pourtsidou, Skordis, Copeland (2013) $f(Z) = \beta Z^2$

● Vector-tensor (VT) theories

We can construct a product between the CDM four velocity u^μ and a vector field A_μ, as

$$\tilde{Z} = -u^\mu A_\mu$$

→ The interacting Lagrangian $f(\tilde{Z})$ can also realize weak gravity.

De Felice, Nakamura, ST (2020)

Effective field theory (EFT) of coupled DE and DM

We aim to construct a unified framework of coupled DE and DM that encompasses both ST and VT theories.

For this purpose, the EFT approach is useful to extract model-independent predictions. In the uncoupled case, there have been many works so far.

In ST theories, the EFT action is expressed as

$$S = \int d^4x \sqrt{-g} \left[\frac{M_*^2}{2} f(t) R - \Lambda(t) - c(t) g^{00} - \frac{1}{2} \bar{M}_1^3(t) \delta g^{00} \delta K + \cdots \right]$$

Nonminimal coupling | Canonical scalar | Galileon,..

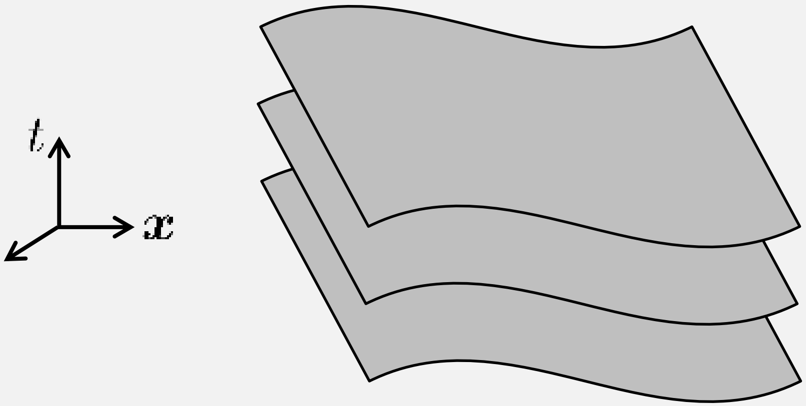
Arkani-Hamed+ 2004, Creminelli+ 2006, Cheung+ 2008, Gubitosi+ 2013, Bloomfield+ 2013, …

EFT coefficients universally characterize the models.

The unified EFT of dark energy, which includes both VT and ST theories, was also constructed (uncoupled case). ⇨ Aoki+ (2021, 2024).

EFT construction

EFT of ST theories



Preferred slices
= scalar condensate

Creminelli+ 2006, Cheung+ 2008,
Gubitosi+ 2013, Bloomfield+ 2013, …

$$\langle \phi(t, \boldsymbol{x}) \rangle = t$$

A clock field

EFT of VT theories

Preferred direction
= vector condensate

Aoki+ 2021

$$\langle v_\mu \rangle = \bar{v}_\mu(t)$$

No clock

In VT theories, a preferred direction determined by a vector field v_μ is different from the one associated with a preferred spacetime slicing.

$$v_\mu = \partial_\mu \tilde{t} + g_M A_\mu$$

g_M is a gauge coupling constant.

Here, $\tilde{t}$ is a Stuckelberg field associated with the combined time and $U(1)$ diffs:

$$\tilde{t} \to \tilde{t}' = \tilde{t} - g_M \theta(t, \mathbf{x}), \qquad A_\mu \to A'_\mu = A_\mu + \partial_\mu \theta(t, \mathbf{x}).$$

We choose the unitary gauge where $\tilde{t} = t$.

If one is interested in irrotational solutions, we can make an ansatz:

$$A_\mu = [A_0(t, \mathbf{x}), \mathbf{0}]$$

Then, we have

$$v_\mu = \delta^0_\mu + g_M A_\mu = (1 + g_M A_0, \mathbf{0})$$

In this case, v_μ is paralell to a unit vector n_μ orthogonal to constant t hypersurfaces, with the norm:

$$\tilde{g}^{00} = (1 + g_M A_0)^2 g^{00}$$

EFT action in the DE sector

In VT theories, the EFT building blocks are

$$n_\mu, \quad \tilde{g}^{00}, \quad F_\mu\,, \quad K_{\mu\nu}, \quad {}^{(3)}R_{\mu\nu}$$

where $K_{\mu\nu}$ and ${}^{(3)}R_{\mu\nu}$ are extrinsic and intrinsic curvatures, and

$$F_\mu := n^\alpha F_{\mu\alpha}\,, \qquad F_{\mu\alpha} := 2\nabla_{[\mu}A_{\alpha]}$$

Restricting theories to the luminal propagation of gravitational waves, the EFT Lagrangian in the DE sector is given by

$$\mathcal{L}_{\rm DE} = \frac{M_*^2}{2} f(t)\left[{}^{(3)}R + K_{\mu\nu}K^{\mu\nu} - K^2\right] - \hat{\Lambda}(t) - \hat{c}(t)\tilde{g}^{00} - d(t)K$$
$$+ \frac{1}{2}\hat{M}_2^4(t)\left(\frac{\delta\tilde{g}^{00}}{-\tilde{g}^{00}_{\rm BG}}\right)^2 - \frac{1}{2}\bar{M}_1^3(t)\left(\frac{\delta\tilde{g}^{00}}{-\tilde{g}^{00}_{\rm BG}}\right)\delta K + \frac{1}{2}\gamma_1(t)F_\mu F^\mu$$

- The shift-symmetric ST theories can be recovered by taking the limit $g_M \to 0$ and imposing some consistency conditions such as $\dot{f} = 0$.
- The non-shift-symmetric ST theories correspond to the limit $g_M \to 0$ without imposing consistency conditions.

Aoki+ (2021)

The CDM and other matter sectors

We deal with CDM as a perfect fluid described by the Lagrangian

$$\mathcal{L}_{\rm DM} = -\hat{m}_{\rm c} n$$

By using the three-scalar description of the fluid and choosing the comoving gauge $\phi^i = x^i$, the number density and four-velocity are

$$n = \sqrt{\det g^{ij}}\,, \qquad u^\mu = \frac{\delta^\mu_0}{\sqrt{-g_{00}}}$$

We incorporate other matter fields (baryons, radiation) described by the perfect fluids. We can adopt the Schutz-Sorkin action

$$\mathcal{S}_{\rm m} = -\int {\rm d}^4x \left[\sqrt{-g}\,\rho_{\rm m}(n_{\rm m}) + J^\mu \nabla_\mu \ell\right]$$

where the matter density $\rho_{\rm m}$ is a function of its number density $n_{\rm m}$, and ℓ is a Lagrange multiplier, and J^μ is relatd to $n_{\rm m}$, as

$$n_{\rm m} = \sqrt{g_{\mu\nu} J^\mu J^\nu / g}$$

with the four velocity $(u_{\rm m})^\mu = J^\mu / (n_{\rm m}\sqrt{-g})$.

Building blocks of coupled DE and DM

The EFT building blocks associated with the DE and DM interactions are

$$\underbrace{n_\mu, \quad \tilde{g}^{00}, \quad F_\mu,}_{\text{DE}} \qquad \underbrace{n, \quad u^\mu}_{\text{DM}}$$

Energy exchange: $\mathcal{L}_n(\tilde{g}^{00})n$ → This must be linear in n to avoid the nonzero DM sound speed.

Momentum exchange: $n_\mu u^\mu, \quad F_\mu u^\mu$

By introducing $q^\mu := u^\mu + n^\mu(n_\nu u^\nu)$, we have

$$n_\mu q^\mu = 0, \qquad q_\mu q^\mu = -1 + (n_\nu u^\nu)^2.$$

Instead of $n_\mu u^\mu$, $F_\mu u^\mu$, we consider the interaction

$$q_\mu q^\mu, \quad F_\mu q^\mu$$

These scalar products can be further multiplied by $\mathcal{L}_{q^2}(\tilde{g}^{00})$ and $\mathcal{L}_{q\cdot F}(\tilde{g}^{00})$.

→ $\mathcal{L}_{q^2}(\tilde{g}^{00})q_\mu q^\mu, \quad \mathcal{L}_{q\cdot F}(\tilde{g}^{00})F_\mu q^\mu$

EFT Lagrangian for coupled DE and DM

The general interacting Lagrangian is given by

$$\begin{aligned}\mathcal{L}_{\rm int} &= \mathcal{L}_n(\tilde{g}^{00})n + \mathcal{L}_{q^2}(\tilde{g}^{00})q^\mu q_\mu + \mathcal{L}_{q\cdot F}(\tilde{g}^{00})F_\mu q^\mu + \cdots \\ &= -\Delta\Lambda(t) - \Delta c(t)\tilde{g}^{00} - \underline{\Delta m_c(t) n} \\ &\quad + \frac{1}{2}\Delta M_2^4(t)\left(\frac{\delta\tilde{g}^{00}}{-\tilde{g}^{00}_{\rm BG}}\right)^2 \underbrace{- m_1^4(t)\frac{\delta n}{\bar{n}}\left(\frac{\delta\tilde{g}^{00}}{-\tilde{g}^{00}_{\rm BG}}\right)}_{\text{Energy transfer}} \underbrace{- m_2^4(t)q^\mu q_\mu - \bar{m}_1^2(t)q^\mu F_\mu}_{\text{Momentum transfer}} \\ &\quad + \cdots,\end{aligned}$$

where dots represent terms for high-order perturbations, and

$$\Delta\Lambda(t) = \bar{\mathcal{L}}_{n\tilde{g}^{00}}\,\bar{n}\,\tilde{g}^{00}_{\rm BG}\,, \qquad \Delta c(t) = -\bar{\mathcal{L}}_{n\tilde{g}^{00}}\,\bar{n}\,, \qquad \Delta m_c(t) = -\bar{\mathcal{L}}_n\,,$$
$$\Delta M_2^4(t) = \bar{\mathcal{L}}_{n\tilde{g}^{00}\tilde{g}^{00}}\,\bar{n}\,(\tilde{g}^{00}_{\rm BG})^2\,, \qquad m_1^4(t) = \bar{\mathcal{L}}_{n\tilde{g}^{00}}\,\tilde{g}^{00}_{\rm BG}\,\bar{n}\,,$$
$$m_2^4(t) = -\bar{\mathcal{L}}_{q^2}\,, \qquad \bar{m}_1^2(t) = -\bar{\mathcal{L}}_{q\cdot F}\,,$$

and

$$\delta n = n - \bar{n}(t)\,, \qquad \mathcal{L}_{n\tilde{g}^{00}} = \frac{{\rm d}\mathcal{L}_n}{{\rm d}\tilde{g}^{00}}, \qquad \mathcal{L}_{n\tilde{g}^{00}\tilde{g}^{00}} = \frac{{\rm d}\mathcal{L}_{n\tilde{g}^{00}}}{{\rm d}\tilde{g}^{00}}\,.$$

Note that CDM acquires a time-dependent mass term, $-\Delta m_c(t)n$, through the energy transfer.

Full EFT action

EFT action for coupled DE and DM

$$\mathcal{S} = \int \mathrm{d}^4x\sqrt{-g}\left(\mathcal{L}_{\mathrm{D}}^{\mathrm{NL}} + \mathcal{L}_{\mathrm{D}}^{(2)}\right) + \mathcal{S}_{\mathrm{m}}\,, \tag{2.56}$$

where

$$\mathcal{L}_{\mathrm{D}}^{\mathrm{NL}} = \frac{M_*^2}{2}f(t)\left({}^{(3)}R + K_{\mu\nu}K^{\mu\nu} - K^2\right) - \Lambda(t) - \tilde{c}(t)\tilde{g}^{00} - d(t)K - m_{\mathrm{c}}(t)n\,, \tag{2.57}$$

$$\begin{aligned}\mathcal{L}_{\mathrm{D}}^{(2)} = {} & \frac{1}{2}M_2^4(t)\left(\frac{\delta\tilde{g}^{00}}{-\tilde{g}^{00}_{\mathrm{BG}}}\right)^2 - \frac{1}{2}\bar{M}_1^3(t)\left(\frac{\delta\tilde{g}^{00}}{-\tilde{g}^{00}_{\mathrm{BG}}}\right)\delta K + \frac{1}{2}\gamma_1(t)F_\mu F^\mu \\ & - m_1^4(t)\frac{\delta n}{\bar{n}}\left(\frac{\delta\tilde{g}^{00}}{-\tilde{g}^{00}_{\mathrm{BG}}}\right) - m_2^4(t)q^\mu q_\mu - \bar{m}_1^2(t)q^\mu F_\mu\,,\end{aligned} \tag{2.58}$$

$$\mathcal{S}_{\mathrm{m}} = -\int \mathrm{d}^4x\left[\sqrt{-g}\,\rho_{\mathrm{m}}(n_{\mathrm{m}}) + J^\mu\nabla_\mu\ell\right]. \tag{2.59}$$

where

$$\Lambda(t) = \hat{\Lambda}(t) + \Delta\Lambda(t)\,, \qquad \tilde{c}(t) = \hat{c}(t) + \Delta c(t)\,,$$

$$m_{\mathrm{c}}(t) = \hat{m}_{\mathrm{c}} + \Delta m_{\mathrm{c}}(t)\,, \qquad M_2^4(t) = \hat{M}_2^4(t) + \Delta M_2^4(t)$$

Consistency conditions

In VT theories, there are the following consistency conditions

Consistency conditions

$$\dot{\Lambda} + 3H\dot{d} + \dot{\tilde{c}}\tilde{g}^{00}_{\rm BG} - m_1^4 \frac{\rm d}{\bar{N}{\rm d}t} \ln(-\tilde{g}^{00}_{\rm BG}) = 0\,, \tag{2.61}$$

$$2M_2^4 \frac{\rm d}{\bar{N}{\rm d}t} \ln(-\tilde{g}^{00}_{\rm BG}) + 3\bar{M}_1^3 \dot{H} + 2\dot{\tilde{c}}\tilde{g}^{00}_{\rm BG} - 6Hm_1^4 = 0\,, \tag{2.62}$$

$$\dot{m}_c\bar{n} + m_1^4 \frac{\rm d}{\bar{N}{\rm d}t} \ln(-\tilde{g}^{00}_{\rm BG}) = 0\,, \tag{2.63}$$

$$\dot{d} + \frac{1}{2}\bar{M}_1^3 \frac{\rm d}{\bar{N}{\rm d}t} \ln(-\tilde{g}^{00}_{\rm BG}) = 0\,, \tag{2.64}$$

$$\dot{f} = 0\,, \tag{2.65}$$

The shift-symmetric ST theories (with the coupling to DM) are obtained by taking the limit

$$g_M \to 0 \quad \text{and} \quad \bar{m}_1^2 \to 0,$$

and by imposing the above consistency conditions.

The generic non-shift-symmetric scalar-tensor theories are obtained by omitting the consistency conditions.

Cosmological perturbations

To study the evolution of scalar perturbations, we consider the ADM line element

$$ds^2 = -N^2 dt^2 + h_{ij}(dx^i + N^i dt)(dx^j + N^j dt)$$

where

$$N = \bar{N}(t)(1+\alpha)\,, \qquad N_i = \bar{N}(t)\partial_i\chi\,, \qquad h_{ij} = a^2(t)\left[(1+2\zeta)\delta_{ij} + 2\partial_i\partial_j E\right]$$

The density contrasts of CDM and ordinary matter are given by

$$\delta_c := \frac{\delta\rho_c}{\bar{\rho}_c} = \frac{\delta n}{\bar{n}} = -(3\zeta + \nabla^2 E)\,, \qquad \delta_m := \frac{\delta\rho_m}{\bar{\rho}_m}$$

The dynamical scalar degrees of freedom are given by

- ζ: DE perturbation
- δ_c: CDM density perturbation
- δ_m: Ordinary matter density perturbation

From the EFT action, we can derive the second-order action of scalar perturbations and linear perturbation equations of motion.

Second-order action of scalar perturbations

$$\mathcal{S}_{\rm S}^{(2)} = \int {\rm d}^4x \bar{N} a^3 \frac{M^2}{2}\Big\{ 2\left(\dot{\zeta} - H\alpha\right)^2 + \left(2\dot{\zeta} + \dot{\delta}_{\rm c} + H\alpha + \frac{1}{a^2}\nabla^2\chi\right)^2 + \frac{2}{a^2}(\partial_i\zeta)^2$$
$$- \left(4\tilde{\alpha}_B H\alpha + \alpha_{\mu_3} H\delta_{\rm c} - \alpha_{\mu_4}\frac{1}{a^2}\nabla^2\chi\right)\left(\dot{\delta}_{\rm c} + 3H\alpha + \frac{1}{a^2}\nabla^2\chi\right) + \tilde{\alpha}_K H^2\alpha^2$$
$$+ (\alpha_{m_2} + 3\Omega_{\rm c})\frac{H^2}{a^2}(\partial_i\chi)^2 + \frac{\tilde{\alpha}_{\bar{m}_1}}{a^4}(\nabla^2\chi)^2 + \alpha_{\mu_1}H^2\delta_{\rm c}^2 + \alpha_{\mu_5}\frac{H}{a^2}\partial_i\alpha\partial_i\chi$$
$$+ (\alpha_{\mu_2} - 1)\left(\dot{\delta}_{\rm c} + 3H\alpha + \frac{1}{a^2}\nabla^2\chi\right)^2 + H\delta_{\rm c}\left[(\tilde{\alpha}_{m_1} - 6\Omega_{\rm c})H\alpha - \frac{\alpha_{\mu_6}}{a^2}\nabla^2\chi\right]$$
$$+ \frac{4}{a^2}\partial_i\alpha\partial_i\zeta + \frac{2}{M^2}\Big[\left(\dot{v}_{\rm m} - 3Hc_{\rm m}^2 v_{\rm m} - \alpha\right)\rho_{\rm m}\delta_{\rm m} - \frac{c_{\rm m}^2}{2(\bar{\rho}_{\rm m} + \bar{p}_{\rm m})}\rho_{\rm m}^2\delta_{\rm m}^2$$
$$- \frac{\bar{\rho}_{\rm m} + \bar{p}_{\rm m}}{2a^2}\left((\partial_i v_{\rm m})^2 + 2\partial_i v_{\rm m}\partial_i\chi\right) - (\bar{\rho}_{\rm m} + \bar{p}_{\rm m})\,\delta_{\rm c}\left(\dot{v}_{\rm m} - 3Hc_{\rm m}^2 v_{\rm m}\right)\Big]\Big\}. \quad (5.28)$$

The new dimensionless EFT parameters relevant to the DE-DM interactions are

Energy transfer: $\tilde{\alpha}_{m_1} := \alpha_{m_1}(1 - \mathcal{G})\,, \qquad \alpha_{m_1} := -\frac{4m_1^4}{H^2M^2}\,,$

Momentum transfer: $\alpha_{m_2} := -\frac{2m_2^4}{H^2M^2}\,, \qquad \tilde{\alpha}_{\bar{m}_1} := -\frac{\mathcal{G}}{\alpha_K\alpha_g^2}\alpha_{\bar{m}_1}^2\,, \qquad \alpha_{\bar{m}_1} := \frac{\bar{m}_1^2}{\sqrt{\gamma_1}HM}\,,$

with $\mathcal{G} = \frac{\alpha_K\alpha_g^2}{-H^{-2}\nabla_i\nabla^i + \alpha_K\alpha_g^2}\,, \qquad \alpha_g := \frac{M}{2}g_{\rm eff}\,, \qquad \alpha_K := \frac{4M_2^4}{H^2M^2}$

$\propto g_M$

➡ $\mathcal{G} \neq 0$ in VT theories.

CDM effective gravitational coupling

For the modes deep inside the DE sound horizon, the CDM density contrast obeys

$$\ddot{\delta}_c + \mathcal{C}\dot{\delta}_c - 4\pi G_{\rm eff}\bar{\rho}_c\delta_c \simeq 0\,,$$

with

$$G_{\rm eff} = \frac{1}{16\pi\bar{\rho}_c(q_c\nu_s^2 + b_{12}^2\nu_s)}\Big\{4\mu_{11}\nu_s^2 + 4g_{12}^2\nu_s - \left[3H^2(7-2\epsilon_H)b_{12}^2 + \dot{b}_{12}^2\right]\nu_s$$
$$- 2(\ddot{b}_{12} + 8H\dot{b}_{12} + 2\dot{g}_{12} + 4Hg_{12})b_{12}\nu_s + 2(\dot{b}_{12} + 3Hb_{12} + 2g_{12})b_{12}\dot{\nu}_s\Big\}\,,$$

Negative

where $\nu_s = q_s\hat{c}_s^2$ (with $\hat{c}_s^2$ being a part of the squared scalar DE speed), and

$$b_{12} = -b_{21} = \frac{M^2 H}{4(1+\alpha_B)}\left(\alpha_{m_1} + 2\alpha_{m_2} - 2\alpha_{\bar{m}_1}^2 - 4\alpha_g\alpha_B\alpha_{\bar{m}_1}\right)$$

Energy transfer

Momentum transfer

If at least one of α_{m_1}, α_{m_2}, and $\alpha_{\bar{m}_1}$ is nonzero, $G_{\rm eff}$ can be suppressed by the DE pressure induced by $\hat{c}_s^2$ at low redshifts.

Example of suppressed cosmic growth

See Appendix in 2504.17293

There are mappings between concrete interacting Lagrangians and EFT parameters.

In ST theories with the interacting Lagrangian

$$\mathcal{L}_{\rm int} = -f_1(\phi, X, Z)\rho_{\rm DM} + f_2(\phi, X, Z)$$

where

$$X = -(1/2)\nabla_\mu\phi\nabla^\mu\phi,$$
$$Z = -u^\mu\nabla_\mu\phi$$

we have

- Energy transfer: $m_c = f_1\,, \qquad m_1^4 = -\bar{n}\left(f_{1,X}X_{\rm BG} + \frac{1}{2}\sqrt{2X_{\rm BG}}f_{1,Z}\right)\,,$
- Momentum transfer: $m_2^4 = \frac{1}{2}\sqrt{2X_{\rm BG}}\,(f_{1,Z}\,\bar{n} - f_{2,Z})\,, \quad \bar{m}_1^2 = 0$

Let us consider a model by Pourtsidou et al (2013):

$$\mathcal{S} = \int {\rm d}^4x\sqrt{-g}\left[X - V_0 e^{-\lambda\phi/M_{\rm Pl}} + \frac{M_{\rm Pl}^2}{2}R - \hat{m}_c n + \beta Z^2\right] + \mathcal{S}_{\rm m}$$

$\Rightarrow$ $f_2(Z) = \beta Z^2$ with $f_1(Z) = 0$

In this case, there is a momentum transfer with $\alpha_{m_2} \neq 0$.

$$G_{\rm eff} = G_N \frac{\Omega_c}{\Omega_c + 4\beta(1+2\beta)x^2}$$

where $x := \dot{\phi}/(\sqrt{6}HM_{\rm Pl})$

$\Rightarrow$ At low redshifts, $G_{\rm eff} < G_{\rm N}$ is realized.

Summary

We constructed the EFT of coupled DE and DM that encompasses both ST and VT theories in the DE sector.

Besides the CDM effective mass $m_c(t)$, the DE-DE interactions are weighed by three EFT parameters:

Energy transfer: α_{m_1}

Momentum transfer: α_{m_2}, $\alpha_{\bar{m}_1}$

They allow the possibility for reducing the σ_8 tension.

It is straightforward to implement our EFT of coupled to DE and DM to a cosmological MCMC code.

J. Beltran Jimenez, F. Teppa Pannia, and ST, in preparation

We hope to find the observational evidence for the interaction of DE and DM !