International Workshop for New Opportunities for Particle Physics 2025

July 18-20, 2025, IHEP

Entanglement suppression and emergent symmetries in hadron scattering

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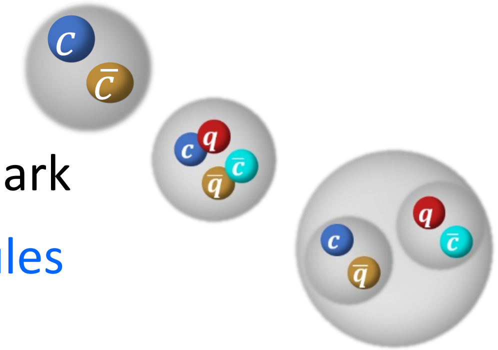
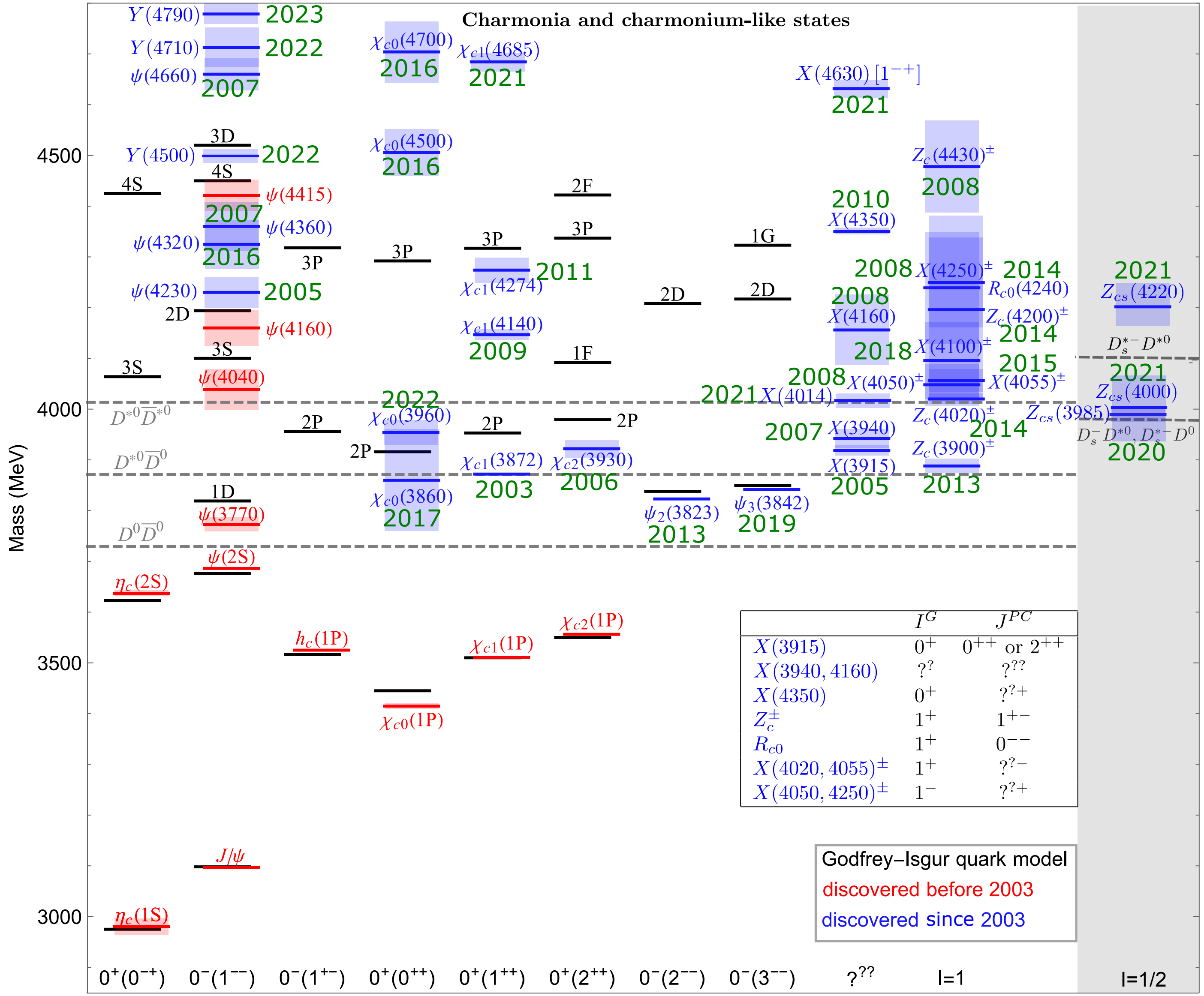
Tao-Ran Hu, S. Chen, FKG, Entanglement suppression and low-energy scattering of heavy mesons, PRD 110 (2024) 014002
Tao-Ran Hu, K. Sone, FKG, T. Hyodo, I. Low, Entanglement suppression, quantum statistics and symmetries in spin-3/2 baryon scatterings, arXiv:2506.08960 [hep-ph]

XYZ charmonium-like structures

- XYZ: charmonium-like states
- Models
  - Charmonium
  - Compact tetraquark
  - Hadronic molecules



- Structures ⟺ symmetries?



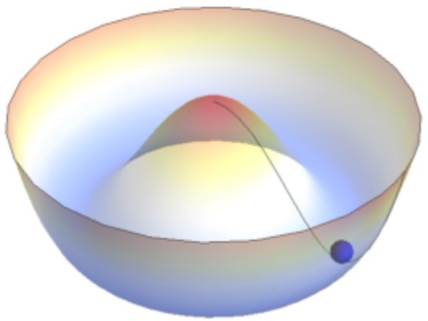
Symmetries

- Symmetries play important role in modern physics
  - Gauge symmetries
  - Global symmetries
- Approximate symmetries in hadron physics

u ~2 MeV | d ~5 MeV | s ~100 MeV $\ll \Lambda_{\rm QCD} \ll$ c ~1.3 GeV | b ~4.2 GeV | t ~173 GeV

Light quarks

☞ Spontaneously broken chiral symmetry:
$\mathrm{SU}(N_f)_L \times \mathrm{SU}(N_f)_R \xrightarrow{\text{SSB}} \mathrm{SU}(N_f)_V$

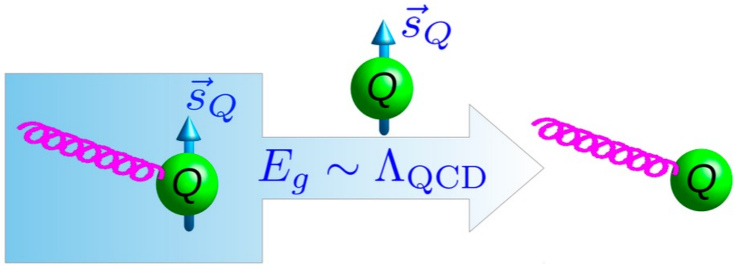


Heavy quarks

☞ Heavy quark spin symmetry (HQSS)

☞ Heavy quark flavor symmetry (HQFS)

☞ Heavy aiquark-diquark symmetry (HADS)

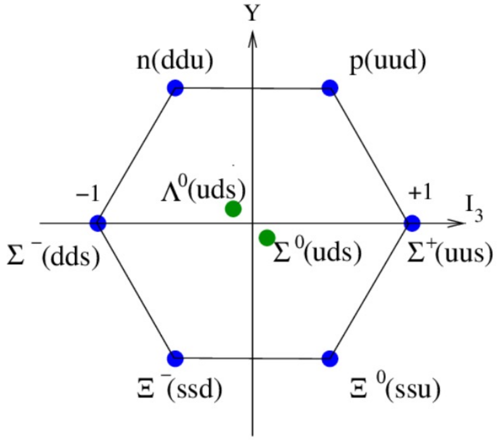


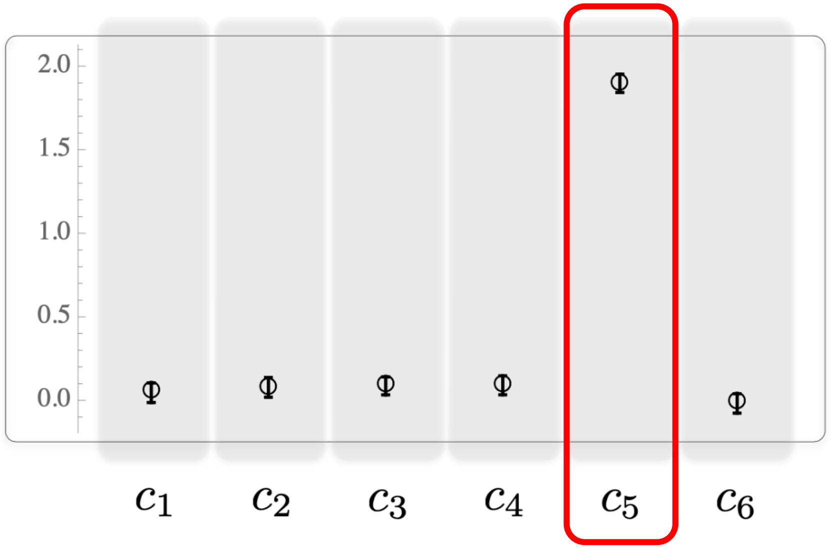
Emergent symmetries

- Emergent symmetries: not explicit in fundamental theory
  - ➢ SU(4), SU(6) spin-flavor symmetry in nonrelativistic quark model
  - ➢ Wigner’s SU(4) among neutrons and protons
  - ➢ Approximate SU(16) symmetry in low-energy baryon-baryon scattering

$$\begin{pmatrix} p\uparrow \\ p\downarrow \\ n\uparrow \\ n\downarrow \end{pmatrix}$$

$$B = \begin{pmatrix} \Sigma^0/\sqrt{2}+\Lambda/\sqrt{6} & \Sigma^+ & p \\ \Sigma^- & -\Sigma^0/\sqrt{2}+\Lambda/\sqrt{6} & n \\ \Xi^- & \Xi^0 & -\sqrt{\frac{2}{3}}\Lambda \end{pmatrix}$$





NPLQCD, PRD 96, 114510 (2017)

- ☐ LO contact-term Lagrangian

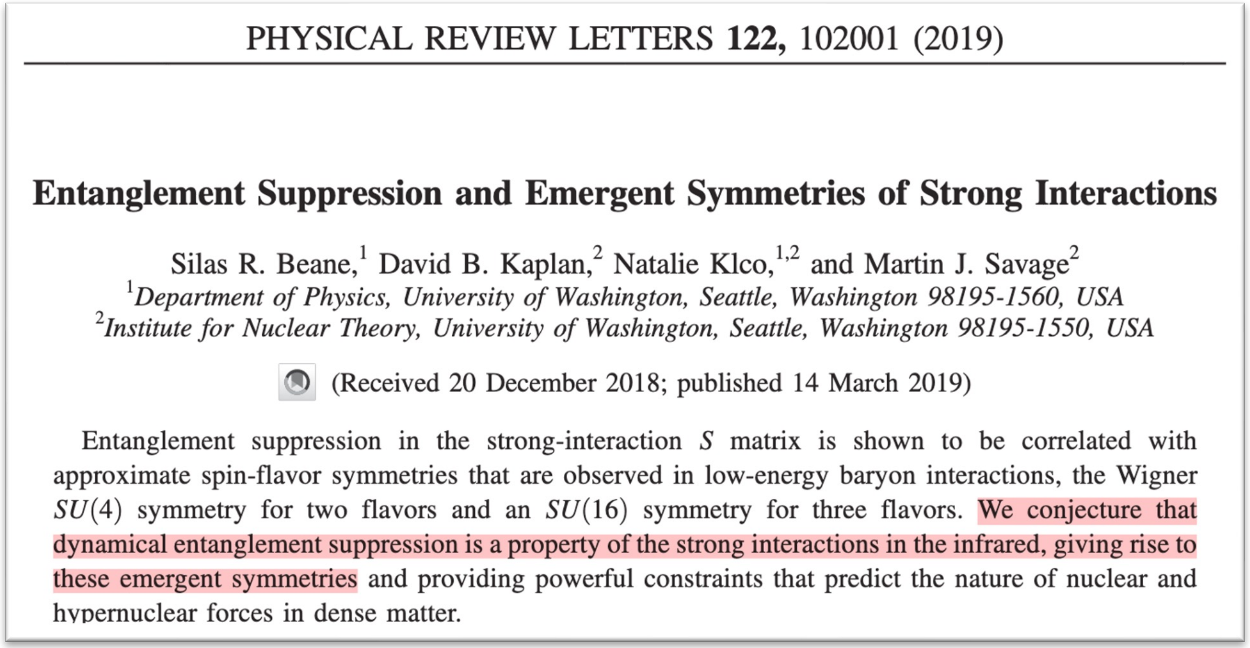
Savage, Wise (1995)

$$\mathcal{L}_{\rm LO}^{n_f=3} = -\frac{c_1}{f^2}\langle B_i^\dagger B_i B_j^\dagger B_j\rangle - \frac{c_2}{f^2}\langle B_i^\dagger B_j B_j^\dagger B_i\rangle - \frac{c_3}{f^2}\langle B_i^\dagger B_j^\dagger B_i B_j\rangle - \frac{c_4}{f^2}\langle B_i^\dagger B_j^\dagger B_j B_i\rangle - \frac{c_5}{f^2}\langle B_i^\dagger B_i\rangle\langle B_j^\dagger B_j\rangle - \frac{c_6}{f^2}\langle B_i^\dagger B_j\rangle\langle B_j^\dagger B_i\rangle, \qquad i,j=\uparrow,\downarrow$$

SU(16) symmetry

Entanglement suppression

- Conjecture: entanglement suppression is a low-energy property of strong interactions and gives rise to emergent symmetries.
  - ☐ explains Wigner's SU(4)
  - ☐ explains SU(16) for baryon-baryon interactions



PHYSICAL REVIEW LETTERS **122,** 102001 (2019)

Entanglement Suppression and Emergent Symmetries of Strong Interactions

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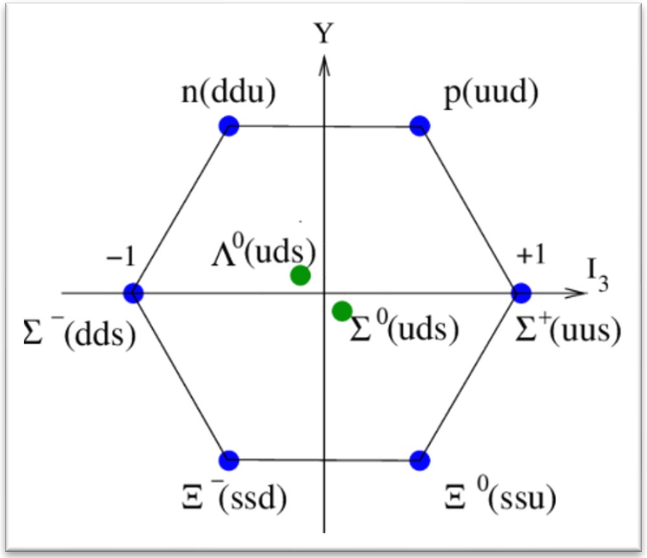
(Received 20 December 2018; published 14 March 2019)

Entanglement suppression in the strong-interaction S matrix is shown to be correlated with approximate spin-flavor symmetries that are observed in low-energy baryon interactions, the Wigner $SU(4)$ symmetry for two flavors and an $SU(16)$ symmetry for three flavors. We conjecture that dynamical entanglement suppression is a property of the strong interactions in the infrared, giving rise to these emergent symmetries and providing powerful constraints that predict the nature of nuclear and hypernuclear forces in dense matter.

- For octet baryon-baryon scattering:
  - ☐ SU(6), SO(8), SU(8) or SU(16) from entanglement minimization for different scattering channels

Q. Liu, I. Low, T. Mehen, PRC 107 (2023) 025204



Following studies:

S. Beane, R. Farrell, Annals Phys. 433 (2021) 168581; S. Beane, R. Farrell, M. Varma, IJMPA 36 (2021) 2150205;
I. Low, T. Mehen, PRD 104 (2021) 074014; Q. Liu, I. Low, T. Mehen, PRC 107 (2023) 025204; Q. Liu, I. Low, PLB 856 (2024) 138899; …

Entanglement measure

- Entanglement measure: measures the degree of entanglement of any given state
  - Many different ways, for bipartite system (density matrix: $\rho = |\psi\rangle\langle\psi|$; partial trace: $\rho_1 = \mathrm{Tr}_2(\rho)$), e.g.,
    - von Neumann entropy:

      $$E(\rho) = -\mathrm{Tr}(\rho_1 \ln \rho_1) = -\mathrm{Tr}(\rho_2 \ln \rho_2)$$

    - linear entropy:

      $$E(\rho) = -\mathrm{Tr}\big(\rho_1(\rho_1 - 1)\big) = 1 - \mathrm{Tr}(\rho_1^2)$$

  - Common property: vanishes for a direct product state $|\psi\rangle = |\psi_1\rangle \otimes |\psi_2\rangle$, maximizes for maximally entangled states

For a system with two spin-1/2 particles, let's define the "computational basis:"

$$\{|\uparrow\uparrow\rangle, |\uparrow\downarrow\rangle, |\downarrow\uparrow\rangle, |\downarrow\downarrow\rangle\}.$$

Then for a general normalized state,

$$|\psi\rangle = \alpha|\uparrow\uparrow\rangle + \beta|\uparrow\downarrow\rangle + \gamma|\downarrow\uparrow\rangle + \delta|\downarrow\downarrow\rangle, \qquad |\alpha|^2 + |\beta|^2 + |\gamma|^2 + |\delta|^2 = 1$$

The reduced density matrix and linear entropy are

$$\rho_1 = \mathrm{Tr}_2 |\psi\rangle\langle\psi| = \begin{pmatrix} |\alpha|^2 + |\beta|^2 & \alpha\gamma^* + \beta\delta^* \\ \alpha^*\gamma + \beta^*\delta & |\gamma|^2 + |\delta|^2 \end{pmatrix},$$

$$E(|\psi\rangle) = 1 - \mathrm{Tr}_1\rho_1^2 = 2|\alpha\delta - \beta\gamma|^2.$$

Easy to check that

1. It vanishes for a product state.
2. Maximal entanglement is 1/2, which is the case for the Bell states:

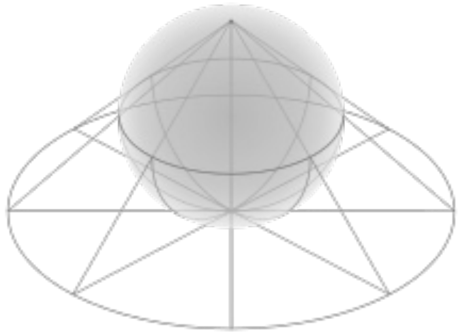
$$(|\uparrow\uparrow\rangle \pm |\downarrow\downarrow\rangle)/\sqrt{2} \qquad (|\uparrow\downarrow\rangle \pm |\downarrow\uparrow\rangle)/\sqrt{2}$$

Ian Low, HEP Seminar at Nanjing Normal University, Oct. 12, 2022

Entanglement power

- Entanglement power: quantifies the ability of an operator U to generate entanglement by averaging over all states obtained by acting it on tensor-product states

$$E(U) = \overline{E\left(U\left|\psi\right\rangle\right)}, \quad \left|\psi\right\rangle = \left|\psi_1\right\rangle \otimes \left|\psi_2\right\rangle .$$



- Entanglement power of S-matrix for 2-body scattering

  - For (iso)spin-1/2 particle, qubit, 2 real parameters for $\mathbb{CP}^1$ manifold, $|\psi\rangle = \left(\cos\frac{\theta}{2}, e^{i\phi}\sin\frac{\theta}{2}\right)^T$, with $\theta \in [0,\pi), \phi \in [0,2\pi)$. For 2-body scattering

$$E(S) = 1 - \int \frac{\mathrm{d}\Omega_1}{4\pi}\frac{\mathrm{d}\Omega_2}{4\pi}\mathrm{Tr}_1[\rho_1^2]$$

I. Bengtsson and K. Życzkowski, Geometry of Quantum States: An Introduction to Quantum Entanglement, 2nd ed. (2017)

  - In general, a $(m+1)$-dim. qudit quantum state, $2m$ parameters: $\mathbb{CP}^m$ manifold:

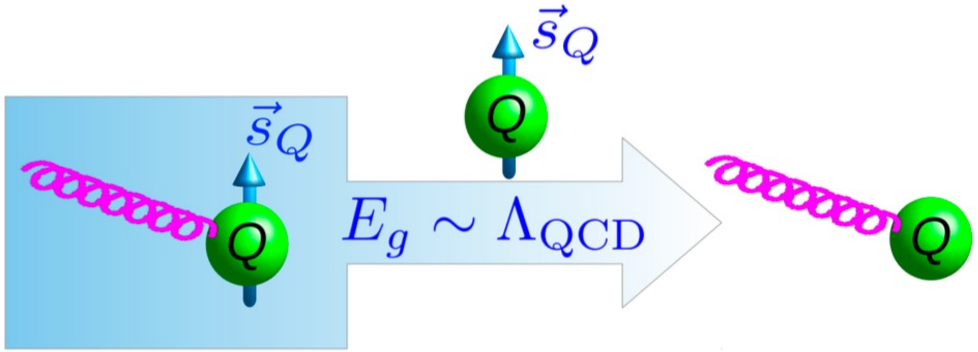
$$|\psi_{\boldsymbol{m+1}}\rangle = \left(\cos\theta_1 \prod_{i=2}^{m}\sin\theta_i, \prod_{i=1}^{m}\sin\theta_i\, e^{i\nu_1}, \ldots, \cos\theta_m e^{i\nu_m}\right), \quad \theta_i \in \left[0, \frac{\pi}{2}\right), \nu_i \in [0, 2\pi)$$

$$E(S) = 1 - \int \mathrm{d}\omega_1 \mathrm{d}\omega_2 \mathrm{Tr}_1[\rho_1^2],$$

Fubini-Study measure: $\mathrm{d}\omega_{\boldsymbol{m+1}} = \frac{m!}{\pi^m}\prod_{i=1}^{m}\mathrm{d}\theta_i\ \mathrm{d}\nu_i \cos\theta_i \sin^{2i-1}\theta_i$

HQSS

- Heavy quark spin symmetry (HQSS)
  - In the heavy quark limit, heavy quark spin decouples
  - Good quantum number s_ℓ: light quark spin + orbital AM



- Consider S-wave interaction between a pair of $s_\ell^P = \frac{1}{2}^-$ (anti-)heavy mesons:

$$0^{++}: \quad D\bar{D}, \quad D^*\bar{D}^*$$

$$1^{+-}: \quad \frac{1}{\sqrt{2}}\left(D\bar{D}^* + D^*\bar{D}\right), \quad D^*\bar{D}^*$$

$$1^{++}: \quad \frac{1}{\sqrt{2}}\left(D\bar{D}^* - D^*\bar{D}\right); \qquad 2^{++}: \quad D^*\bar{D}^*$$

here, phase convention: $D \overset{C}{\to} +\bar{D},\ D^* \overset{C}{\to} -\bar{D}^*$

HQSS for hadronic molecules

- For the HQSS consequences, convenient to use the basis of states: $s_L^{PC} \otimes s_{c\bar{c}}^{PC}$
  - ☞ S-wave: $s_L^{PC}, s_{c\bar{c}}^{PC} = 0^{-+}$ or 1^{--}
  - ☞ multiplet with $s_L = 0$:

$$0_L^{-+} \otimes 0_{c\bar{c}}^{-+} = 0^{++}, \qquad 0_L^{-+} \otimes 1_{c\bar{c}}^{--} = 1^{+-}$$

  - ☞ multiplet with $s_L = 1$:

$$1_L^{--} \otimes 0_{c\bar{c}}^{-+} = 1^{+-}, \qquad 1_L^{--} \otimes 1_{c\bar{c}}^{--} = 0^{++} \oplus \boxed{1^{++}} \oplus 2^{++}$$

Two parameters at LO for each isospin!

- Multiplets in strict heavy quark limit:
  - ☞ $X(3872)$ has three partners with 0^{++}, 2^{++} and 1^{+-}

Hidalgo-Duque et al., PLB 727 (2013) 432; Baru et al., PLB 763 (2016) 20

  - ☞ Z_b, Z_b' as $B^{(*)}\bar{B}^*$ molecules would imply 6 $I=1$ hadronic molecules:

$Z_b[1^{+-}], Z_b'[1^{+-}]$ and $W_{b0}[0^{++}], W_{b0}'[0^{++}], W_{b1}[1^{++}]$ and $W_{b2}[2^{++}]$

Bondar et al., PRD 84 (2011) 054010; Voloshin, PRD 84 (2011) 031502; Mehen, Powell, PRD 84 (2011) 114013

Light quark spin symmetry (emergent) in Z_b resonances? M. Voloshin, PRD 93 (2016) 074011

Heavy-meson scattering

- LO Lagrangians for heavy-meson scattering with HQSS
  - For $D^{(*)}D^{(*)}$ scattering S. Fleming, R. Hodges, T. Mehen, PRD 104 (2021) 116010; M.-L. Du et al., PRD 105 (2022) 014024

$$\mathcal{L}_{HH} = -\frac{D_{00}}{8}\mathrm{Tr}\left[H^{a\dagger}H_b H^{b\dagger}H_a\right] - \frac{D_{01}}{8}\mathrm{Tr}\left[H^{a\dagger}H_b\sigma^m H^{b\dagger}H_a\sigma^m\right] - \frac{D_{10}}{8}\mathrm{Tr}\left[H^{a\dagger}H_b H^{c\dagger}H_d\right]\boldsymbol{\tau}_a^d\cdot\boldsymbol{\tau}_c^b - \frac{D_{11}}{8}\mathrm{Tr}\left[H^{a\dagger}H_b\sigma^m H^{c\dagger}H_d\sigma^m\right]\boldsymbol{\tau}_a^d\cdot\boldsymbol{\tau}_c^b$$

  - For $D^{(*)}\bar{D}^{(*)}$ scattering

M. T. AlFiky, F. Gabbiani, A. A. Petrov, PLB 640 (2006) 238; J. Nieves, M. P. Valderrama, PRD 86 (2012) 056004; T. Ji et al., PRD 106 (2022) 094002

$$\mathcal{L}_{H\bar{H}} = -\frac{1}{4}\mathrm{Tr}\left[H^{a\dagger}H_b\right]\mathrm{Tr}\left[\bar{H}^c\bar{H}_d^\dagger\right]\left(F_A\delta_a^b\delta_c^d + F_A^\tau\boldsymbol{\tau}_a^b\cdot\boldsymbol{\tau}_c^d\right) + \frac{1}{4}\mathrm{Tr}\left[H^{a\dagger}H_b\sigma^m\right]\mathrm{Tr}\left[\bar{H}^c\bar{H}_d^\dagger\sigma^m\right]\left(F_B\delta_a^b\delta_c^d + F_B^\tau\boldsymbol{\tau}_a^b\cdot\boldsymbol{\tau}_c^d\right)$$

Heavy mesons and anti-heavy mesons: $H_a = P_a + \boldsymbol{P}_a^*\cdot\boldsymbol{\sigma}$, $\bar{H}^a = \bar{P}^a + \bar{\boldsymbol{P}}^{*a}\cdot\boldsymbol{\sigma}$

- Then we compute the entanglement power of the S-matrix, find solutions vanishing it
  - E.g., for the isospin subspace: $E(S_J) = \frac{1}{6}\sin^2[2(\delta_{0J} - \delta_{1J})]$, vanishes for
    - $|\delta_{0J} - \delta_{1J}| = 0$
    - or $|\delta_{0J} - \delta_{1J}| = \frac{\pi}{2}$

Heavy-meson scattering from entanglement suppression

T.-R. Hu, S. Chen, FKG, PRD 110 (2024) 014002

- Input: $X(3872)$ as isoscalar $D\bar{D}^*$ molecule with $J^{PC} = 1^{++}$: $\bar{\delta}_{01+} = \pi/2$
  - Two possible solutions

TABLE II. Partners of the $X(3872)$ predicted by HQSS or the two solutions of entanglement suppression given in Eqs. (53) and (54). The symbol "$\odot$" denotes the input $X(3872)$, "$\otimes$" represents its predicted partners, "$\oslash$" indicates no near-threshold state is allowed, and "–" signifies that no prediction can be made without further inputs. Moreover, "$\oplus$" means that the corresponding meson pair needs to be mixed with another one to get a spin partner of $X(3872)$, see Eqs. (57) and (58).

Channel	HQSS		Eq. (53) predictions		Eq. (54) predictions	
	$I=0$	$I=1$	$I=0$	$I=1$	$I=0$	$I=1$
$D\bar{D}(0^{++})$	$\oplus$	–	$\otimes$	$\oslash$	$\otimes$	$\otimes$
$D\bar{D}^*(1^{++})$	$\odot$	–	$\odot$	$\oslash$	$\odot$	$\otimes$
$D\bar{D}^*(1^{+-})$	$\oplus$	–	$\otimes$	$\oslash$	$\otimes$	$\otimes$
$D^*\bar{D}^*(0^{++})$	$\oplus$	–	$\otimes$	$\oslash$	$\otimes$	$\otimes$
$D^*\bar{D}^*(1^{+-})$	$\oplus$	–	$\otimes$	$\oslash$	$\otimes$	$\otimes$
$D^*\bar{D}^*(2^{++})$	$\otimes$	–	$\otimes$	$\oslash$	$\otimes$	$\otimes$

Heavy-meson scattering from entanglement suppression

- Input: $T_{cc}(3875)$ as isoscalar DD^* molecule: $\delta_{01} = \pi/2$
  - Two possible solutions

TABLE I. Partners of the $T_{cc}(3875)^+$ predicted by HQSS or the two solutions of entanglement suppression given in Eqs. (49) and (50). The symbol "$\odot$" denotes the input $T_{cc}(3875)^+$ state, "$\otimes$" represents its predicted partners, "$\oslash$" indicates that no near-threshold state is allowed (the red ones are forbidden by Bose-Einstein statistics), and "–" signifies that no prediction can be made without further inputs.

Channel	HQSS		Eq. (49) predictions		Eq. (50) predictions	
	$I=0$	$I=1$	$I=0$	$I=1$	$I=0$	$I=1$
$DD(0^+)$	$\oslash$	–	$\oslash$	$\oslash$	$\oslash$	$\otimes$
$D^*D(1^+)$	$\odot$	–	$\odot$	$\oslash$	$\odot$	$\otimes$
$D^*D^*(0^+)$	$\oslash$	–	$\oslash$	$\oslash$	$\oslash$	$\otimes$
$D^*D^*(1^+)$	$\otimes$	$\oslash$	$\otimes$	$\oslash$	$\otimes$	$\oslash$
$D^*D^*(2^+)$	$\oslash$	–	$\oslash$	$\oslash$	$\oslash$	$\otimes$

Heavy-meson scattering from entanglement suppression

- In both cases, for a given isospin, all possible heavy-meson pairs allowed by Bose-Einstein statistics either at the unitary limit (molecules at threshold) or at the noninteracting limit
  - Meson pairs with different s_ℓ have the same interaction strengths
  - $\Rightarrow$ light quark spin symmetry !
    - $\mathrm{SU}(2)\times\mathrm{SU}(2)$ is enlarged to $\mathrm{SU}(4)$

$$\begin{pmatrix} |0,0\rangle \\ |1,+1\rangle \\ |1,0\rangle \\ |1,-1\rangle \end{pmatrix}$$

    - X(3872) has 5 isoscalar partners (v.s. 3 partners predicted with only HQSS)

☞ multiplet with $s_L = 1$:

$$1_L^{--} \otimes 0_{c\bar{c}}^{-+} = 1^{+-}, \qquad 1_L^{--} \otimes 1_{c\bar{c}}^{--} = 0^{++} \oplus \boxed{1^{++}} \oplus 2^{++}$$

    - Experimental evidence for X_2 in $\gamma\gamma \to \psi'\gamma$

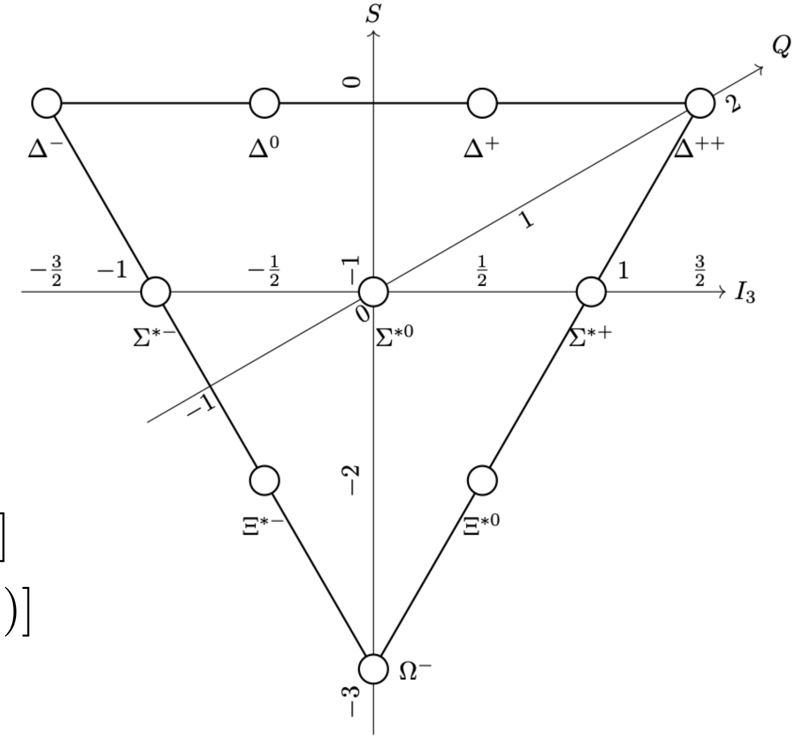
Belle, PRD 105 (2022) 112011

Decuplet baryon scatterings

T.-R. Hu, K. Sone, FKG, T. Hyodo, I. Low, arXiv:2506.08960

- Consider the scattering of two SU(3) flavor-decuplet baryons. Spin = 3/2
- For scattering of distinguishable spin-3/2 particles

$$
\begin{aligned}
E(\hat{S}) = \frac{1}{200000} \{ & 77482 - 2100\cos[2(\delta_0+\delta_1-\delta_2-\delta_3)] \\
& - 2100\cos[2(\delta_0-\delta_1+\delta_2-\delta_3)] - 2100\cos[2(\delta_0-\delta_1-\delta_2+\delta_3)] \\
& - 1200\cos[2(\delta_0-2\delta_1+\delta_2)] - 4200\cos[2(\delta_0+\delta_2-2\delta_3)] \\
& - 8400\cos[2(\delta_1-2\delta_2+\delta_3)] \\
& - 375\cos[4(\delta_0-\delta_1)] - 10800\cos[2(\delta_0-\delta_2)] - 625\cos[4(\delta_0-\delta_2)] \\
& - 875\cos[4(\delta_0-\delta_3)] - 2175\cos[4(\delta_1-\delta_2)] - 26376\cos[2(\delta_1-\delta_3)] \\
& - 5481\cos[2(\delta_1-\delta_3)] - 10675\cos[2(\delta_2-\delta_3)] \}
\end{aligned}
$$



Vanishing entanglement solution: $\delta_0 = \delta_2 \equiv \delta_{\text{even}}, \quad \delta_1 = \delta_3 \equiv \delta_{\text{odd}}; \quad |\delta_{\text{even}} - \delta_{\text{odd}}| = 0 \text{ or } \frac{\pi}{2}$

- ☐ To be compared with scattering of distinguishable spin-1/2 particles:

$$E(\hat{S}) = \frac{1}{6}\sin^2[2(\delta_0-\delta_1)]$$

Vanishing entanglement solution: $|\delta_0 - \delta_1| = 0 \text{ or } \frac{\pi}{2}$

S. Beane et al., PRL 122 (2019) 102001;
Q. Liu, I. Low, T. Mehen, PRC 107 (2023) 025204

a general pattern

Decuplet baryon scatterings

- S-matrix:

$$\hat{S} = \sum_J \mathcal{J}_J e^{2i\delta_J}$$ $\mathcal{J}_J$: projector into irrep with a total spin J

  - ☐ Phase shifts are all equal, $\hat{S} \propto e^{i\delta} \Rightarrow$ Identity gate
  - ☐ Difference of $\pi/2$, $|\delta_{\text{even}} - \delta_{\text{odd}}| = \pi/2 \Rightarrow$ SWAP gate: $\text{SWAP}|i\rangle \otimes |j\rangle = |j\rangle \otimes |i\rangle$

$$\hat{S} \propto -(\mathcal{J}_0 - \mathcal{J}_1 + \mathcal{J}_2 - \mathcal{J}_3) = -\sum_{\text{even } J} \mathcal{J}_J + \sum_{\text{odd } J} \mathcal{J}_J$$

$$\frac{3}{2} \otimes \frac{3}{2} = \underbrace{0 \oplus 2}_{\text{antisymmetric}} \oplus \underbrace{1 \oplus 3}_{\text{symmetric}}$$

SWAP can be defined from symmetric and antisymmetric projectors: $\text{SWAP} = \sum_i \mathcal{S}_i - \sum_j \mathcal{A}_j$

Similarly, Identity gate: $\sum_i \mathcal{S}_i + \sum_j \mathcal{A}_j$

- General result:

For bipartite system with same dimensions, entanglement power vanishes iff the operator ∝ Identity or SWAP

E. Alfsen, F. Shultz, JMP 51 (2010) 052201; N. Johnston, arXiv:1008.3633; S. Friedland et al., JMP 52 (2011) 042203; I. Low, T. Mehen, PRD 104 (2021) 074014

Decuplet baryon scatterings

- Consider generalized identical particles in SU(3) flavor space

  S-matrix:
$$\hat{S} = \sum_{JF} \mathcal{J}_J \otimes \mathcal{F}_F e^{2i\delta_{JF}}$$
  projectors into spin, flavor irreps

- Quantum statistics ⇒ spin and flavor spaces totally symmetric (Bose-Einstein) or antisymmetric (Fermi-Dirac)

$$\mathrm{SWAP}_{\mathrm{spin}} \otimes \mathrm{SWAP}_{\mathrm{flavor}} \, |s_1, f_1\rangle \otimes |s_2, f_2\rangle = |s_2, f_2\rangle \otimes |s_1, f_1\rangle$$

$$|s_2, f_2\rangle \otimes |s_1, f_1\rangle = \begin{cases} +\,|s_1, f_1\rangle \otimes |s_2, f_2\rangle\,, & \mathrm{BE} \\ -\,|s_1, f_1\rangle \otimes |s_2, f_2\rangle\,, & \mathrm{FD} \end{cases}$$

Thus,

$$\mathrm{SWAP}_{\mathrm{spin}} \otimes \mathrm{SWAP}_{\mathrm{flavor}} = \begin{cases} +1_{\mathrm{spin}} \otimes 1_{\mathrm{flavor}}\,, & \mathrm{BE} \\ -1_{\mathrm{spin}} \otimes 1_{\mathrm{flavor}}\,, & \mathrm{FD} \end{cases} \qquad \mathrm{SWAP}_{\mathrm{spin}} \otimes 1_{\mathrm{flavor}} = \begin{cases} +1_{\mathrm{spin}} \otimes \mathrm{SWAP}_{\mathrm{flavor}}\,, & \mathrm{BE} \\ -1_{\mathrm{spin}} \otimes \mathrm{SWAP}_{\mathrm{flavor}}\,, & \mathrm{FD} \end{cases}$$

the ability of an operator to generate entanglement in the spin space is equivalent to its ability to entangle in the flavor space

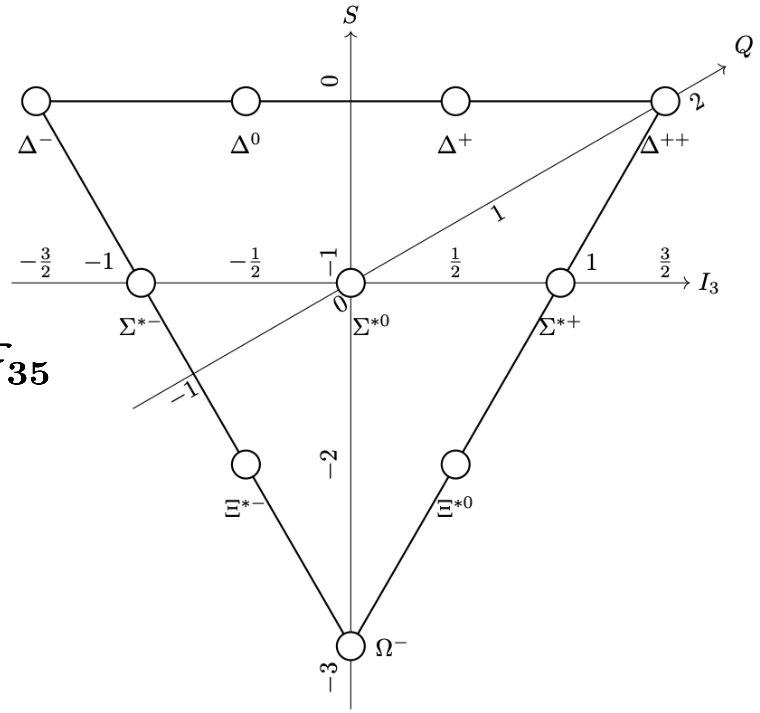
Decuplet baryon scatterings

- For two decuplet baryons, spin-flavor structures:

$$\frac{3}{2}\otimes\frac{3}{2} = \underbrace{0\oplus 2}_{\text{antisymmetric}} \oplus \underbrace{1\oplus 3}_{\text{symmetric}}$$

$$\mathbf{10}\otimes\mathbf{10} = \underbrace{\mathbf{27}\oplus\mathbf{28}}_{\text{symmetric}} \oplus \underbrace{\overline{\mathbf{10}}\oplus\mathbf{35}}_{\text{antisymmetric}}$$

$$\mathrm{SWAP}_{\text{spin}} = -\left(\mathcal{J}_0 - \mathcal{J}_1 + \mathcal{J}_2 - \mathcal{J}_3\right), \quad \mathrm{SWAP}_{\text{flavor}} = \mathcal{F}_{\mathbf{27}} + \mathcal{F}_{\mathbf{28}} - \mathcal{F}_{\overline{\mathbf{10}}} - \mathcal{F}_{\mathbf{35}}$$



- The S-matrix

$$\begin{aligned}\hat{S} = \; & \mathcal{J}_0 \otimes \left(\mathcal{F}_{\mathbf{27}} e^{2i\delta_{0,27}} + \mathcal{F}_{\mathbf{28}} e^{2i\delta_{0,28}}\right) \\ & + \mathcal{J}_1 \otimes \left(\mathcal{F}_{\overline{\mathbf{10}}} e^{2i\delta_{1,\overline{\mathbf{10}}}} + \mathcal{F}_{\mathbf{35}} e^{2i\delta_{1,\mathbf{35}}}\right) \\ & + \mathcal{J}_2 \otimes \left(\mathcal{F}_{\mathbf{27}} e^{2i\delta_{2,\mathbf{27}}} + \mathcal{F}_{\mathbf{28}} e^{2i\delta_{2,\mathbf{28}}}\right) \\ & + \mathcal{J}_3 \otimes \left(\mathcal{F}_{\overline{\mathbf{10}}} e^{2i\delta_{3,\overline{\mathbf{10}}}} + \mathcal{F}_{\mathbf{35}} e^{2i\delta_{3,\mathbf{35}}}\right)\end{aligned}$$

Decuplet baryon scatterings

- For two decuplet baryons, spin-flavor structures:

$$\frac{3}{2}\otimes\frac{3}{2}=\underbrace{0\oplus 2}_{\text{antisymmetric}}\oplus\underbrace{1\oplus 3}_{\text{symmetric}}$$

$$\mathbf{10}\otimes\mathbf{10}=\underbrace{\mathbf{27}\oplus\mathbf{28}}_{\text{symmetric}}\oplus\underbrace{\overline{\mathbf{10}}\oplus\mathbf{35}}_{\text{antisymmetric}}$$

- Identity gate is achieved with:
  - Symmetry: $\delta_{0,27}=\delta_{2,27}=\delta_{0,28}=\delta_{2,28}=\delta_{1,\overline{10}}=\delta_{3,\overline{10}}=\delta_{1,35}=\delta_{3,35}$

$\mathrm{SU}(40)_{\text{spin+flavor}}$: $\mathbf{40}\otimes\mathbf{40}=\underbrace{\mathbf{820}}_{\text{symmetric}}\oplus\underbrace{\mathbf{780}}_{\text{antisymmetric}}$

$(27+28)\times(1+5)+(10+35)\times(3+7)$
$=780$

  - Effective Lagrangian for decuplet baryons $\boldsymbol{T}=(\Delta^{++},\Delta^{+},\Delta^{0},\Delta^{-},\Sigma^{*+},\Sigma^{*0},\Sigma^{*-},\Xi^{*0},\Xi^{*-},\Omega^{-})$:

$$\begin{aligned}\mathcal{L}=c_1&\left(T^{\dagger}_{abc}T_{abc}\right)\left(T^{\dagger}_{def}T_{def}\right)+c_2\left(T^{\dagger}_{abc}\Sigma^{\alpha}T_{abc}\right)\left(T^{\dagger}_{def}\Sigma^{\alpha}T_{def}\right)\\&+c_3\left(T^{\dagger}_{abc}\Sigma^{\alpha\beta}T_{abc}\right)\left(T^{\dagger}_{def}\Sigma^{\alpha\beta}T_{def}\right)+c_4\left(T^{\dagger}_{abc}\Sigma^{\alpha\beta\gamma}T_{abc}\right)\left(T^{\dagger}_{def}\Sigma^{\alpha\beta\gamma}T_{def}\right)\\&+c_5\left(T^{\dagger}_{abc}T_{abd}\right)\left(T^{\dagger}_{def}T_{cef}\right)+c_6\left(T^{\dagger}_{abc}\Sigma^{\alpha}T_{abd}\right)\left(T^{\dagger}_{def}\Sigma^{\alpha}T_{cef}\right)\\&+c_7\left(T^{\dagger}_{abc}\Sigma^{\alpha\beta}T_{abd}\right)\left(T^{\dagger}_{def}\Sigma^{\alpha\beta}T_{cef}\right)+c_8\left(T^{\dagger}_{abc}\Sigma^{\alpha\beta\gamma}T_{abd}\right)\left(T^{\dagger}_{def}\Sigma^{\alpha\beta\gamma}T_{cef}\right)\end{aligned}$$

$\Rightarrow$ $c_1\left(\boldsymbol{T}^{\dagger i}\cdot\boldsymbol{T}^{i}\right)^2$ has an SU(40) symmetry

Decuplet baryon scatterings

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$$\mathbf{10}\otimes\mathbf{10}=\underbrace{\mathbf{27}\oplus\mathbf{28}}_{\text{symmetric}}\oplus\underbrace{\overline{\mathbf{10}}\oplus\mathbf{35}}_{\text{antisymmetric}}$$

- $\mathrm{SWAP}_{\mathrm{spin}}$ gate is achieved with: $\delta_{0,27}=\delta_{2,27}=\delta_{0,28}=\delta_{2,28},\ \delta_{1,\overline{\mathbf{10}}}=\delta_{3,\overline{\mathbf{10}}}=\delta_{1,35}=\delta_{3,35},\ \left|\delta_{0,27}-\delta_{1,\overline{\mathbf{10}}}\right|=\frac{\pi}{2},$

  - Symmetry: $\mathrm{SU}(4)_{\mathrm{spin}}\times\mathrm{SU}(10)_{\mathrm{flavor}}$

$$\mathrm{SU}(10)_{\mathrm{flavor}}:\quad \mathbf{10}\otimes\mathbf{10}=\underbrace{\mathbf{55}}_{\text{symmetric}}\oplus\underbrace{\mathbf{45}}_{\text{antisymmetric}}$$

$(27+28)=55;\ (10+35)=45$

$$\mathrm{SU}(4)_{\mathrm{spin}}:\quad \mathbf{4}\otimes\mathbf{4}=\underbrace{\mathbf{6}}_{\text{antisymmetric}}\oplus\underbrace{\mathbf{10}}_{\text{symmetric}}$$

  - Justified by effective Lagrangian, remaining terms with $\mathrm{SU}(4)_{\mathrm{spin}}\times\mathrm{SU}(10)_{\mathrm{flavor}}$:

$$\propto -\frac{1}{4}\left(\boldsymbol{T}^{\dagger i}\cdot\boldsymbol{T}^{i}\right)^{2}\pm\frac{1}{4}\left(\boldsymbol{T}^{\dagger i}\cdot\boldsymbol{T}^{j}\right)\left(\boldsymbol{T}^{\dagger j}\cdot\boldsymbol{T}^{i}\right)$$

Summary

- Entanglement suppression (conjectured) leads to emergent symmetries
  - For for low-energy heavy-meson scattering, entanglement suppression conjecture + $X(3872)$ and T_{cc} as molecules:
    - emergent light quark spin symmetry
  - For spin-3/2 decuplet baryon scattering:
    - $\hat{S} \propto \mathbb{I} \Rightarrow \mathrm{SU}(40)_{\mathrm{spin+flavor}}$
    - $\hat{S} \propto \mathrm{SWAP} \Rightarrow \mathrm{SU}(4)_{\mathrm{spin}} \times \mathrm{SU}(10)_{\mathrm{flavor}}$
  - Not discussed here (see T.-R. Hu, K. Sone, FKG, T. Hyodo, I. Low, arXiv:2506.08960): Scattering of spin-s identical particles, non-unitary S-matrix in restricted Hilbert space (symmetric for BE or antisymmetric for FD)

Thank you for your attention!