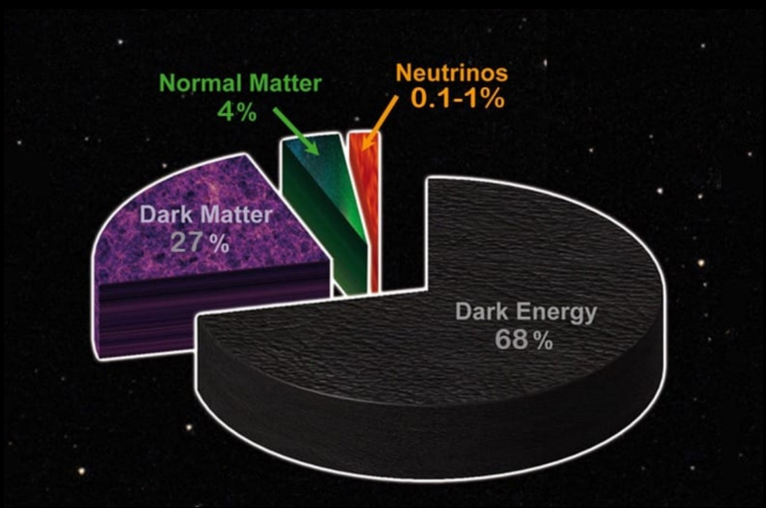
NOPP 2025, July 18-20, 2025

Axion mass prediction from cosmic strings

Minho Son

KAIST

(Korea Adv. Inst. of Sci. and Tech.)



Based on

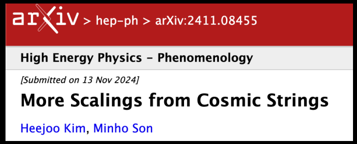

arXiv > hep-ph > arXiv:2402.00741

High Energy Physics – Phenomenology

[Submitted on 1 Feb 2024 (v1), last revised 8 Feb 2024 (this version, v2)]

Axion Dark Matter from Cosmic String Network

Heejoo Kim, Junghyeon Park, Minho Son


arXiv > hep-ph > arXiv:2411.08455

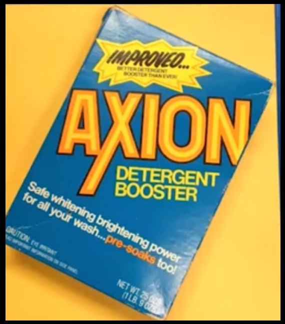
High Energy Physics – Phenomenology

[Submitted on 13 Nov 2024]

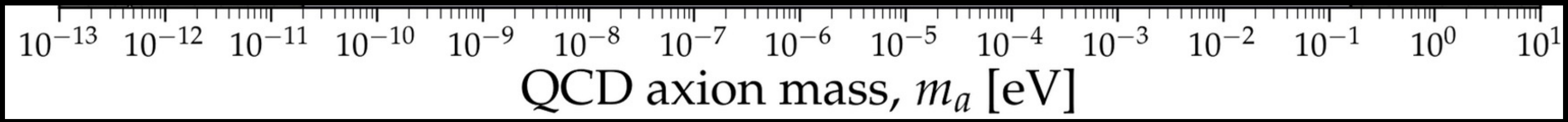
More Scalings from Cosmic Strings

Heejoo Kim, Minho Son

We will focus on



of mass $\sim \mu\text{eV}$



Axion Dark Matter

One of the appealing qualities of the axion is that it can be dark matter

Pre-inflationary scenario

``PQ symmetry breaking before inflation"

VS

Post-inflationary scenario

``PQ symmetry breaking after inflation"

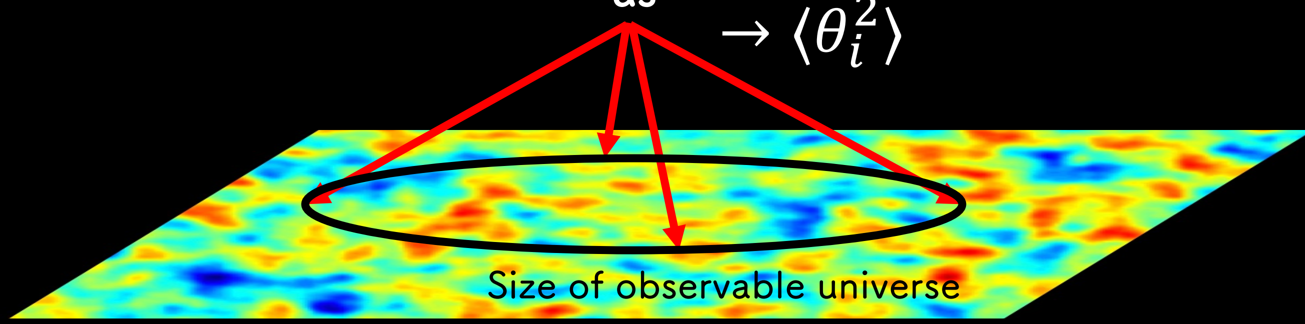
$$\Omega_a h^2 = 0.12 \left(\frac{f_a}{10^{12} GeV}\right)^{7/6} \langle\theta_i^2\rangle$$

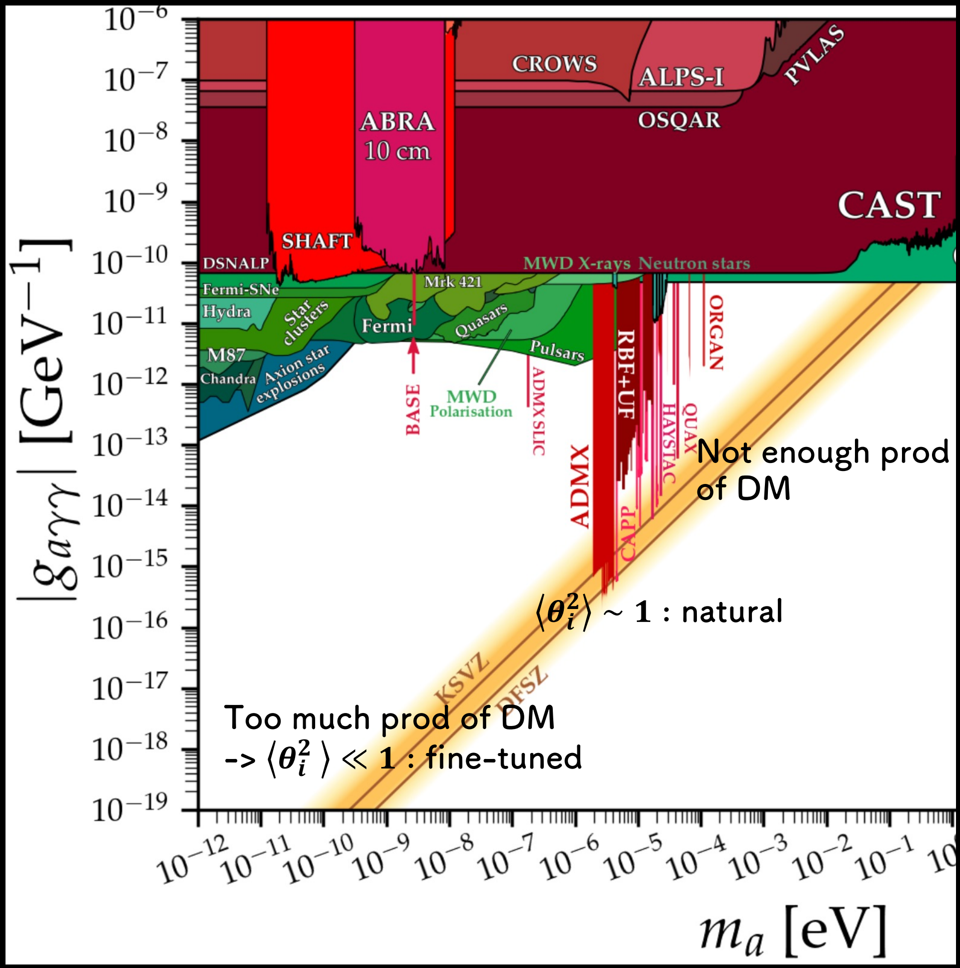
$$m_a \approx 5.7 \mu eV \left(\frac{10^{12}\ GeV}{f_a/N_W}\right)$$

Observable Universe

: includes many causally disconnected patches

us $\to \langle\theta_i^2\rangle$





Misalignment

$$\Omega_a^{\mathrm{mis}} h^2 = 0.12 \left(\frac{f_a}{10^{12} GeV} \right)^{7/6} \langle \theta_i^2 \rangle$$

$$m_a \approx 5.7 \mu eV \left(\frac{10^{12}\ GeV}{f_a / N_W} \right)$$

Topological defects

$$0.12 \sim \Omega_a h^2 = \Omega_a^{\mathrm{mis}} h^2 + \boxed{\Omega_a^{\mathrm{string}} h^2 + \Omega_a^{\mathrm{str-dw}} h^2}$$

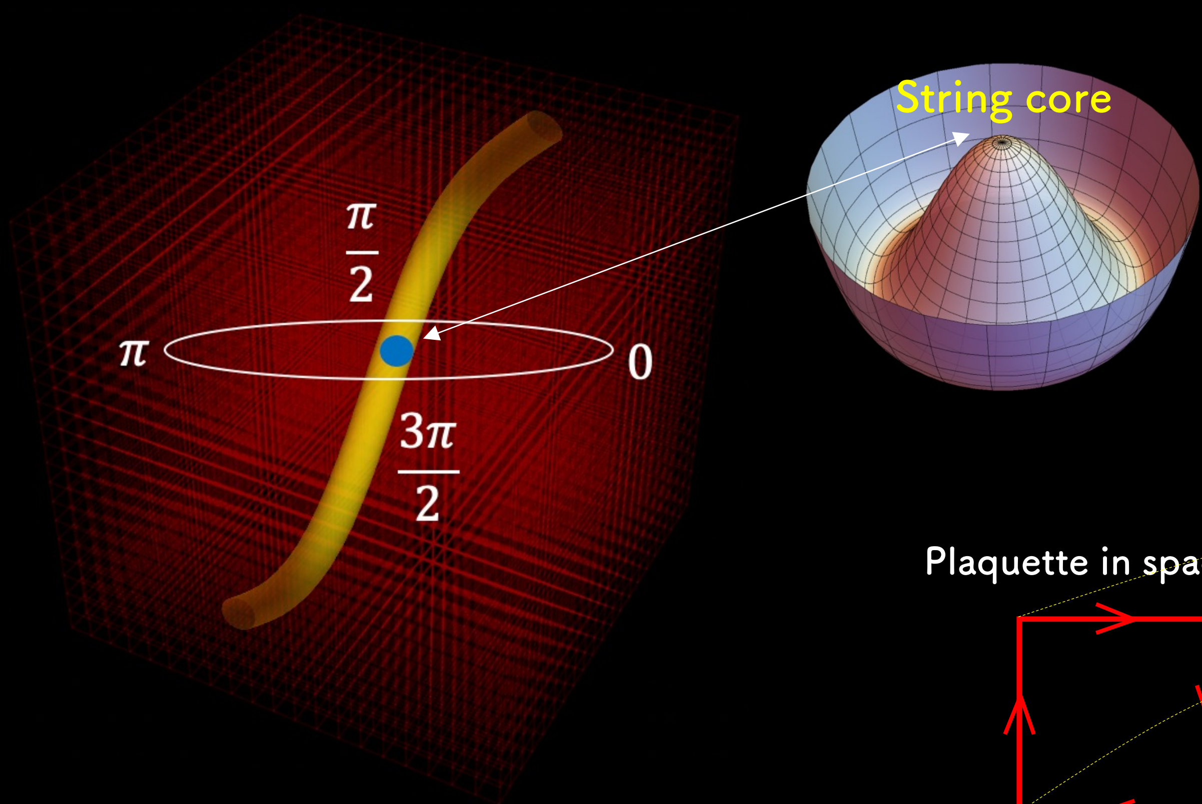
Situation of direct axion search dramatically changes depending on the size of this part

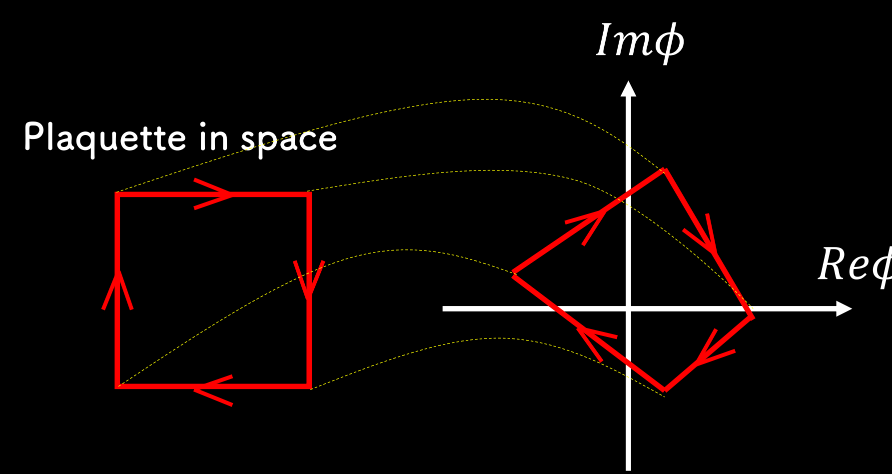
Formation of cosmic string

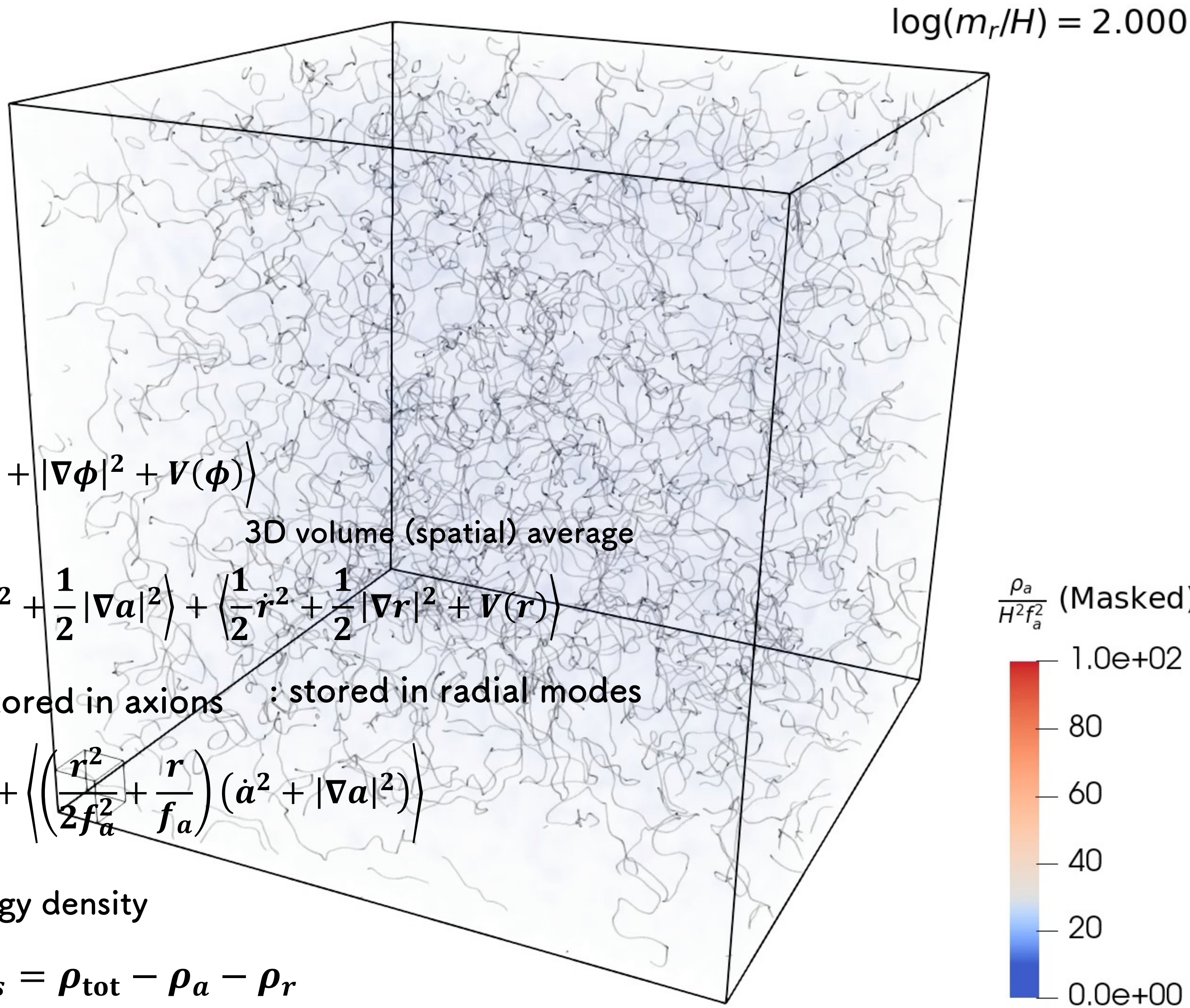
$$ds^2 = dt^2 - R^2(t)d\vec{x}^2$$

$$\ddot{\phi} + 3H\dot{\phi} - \frac{1}{R^2}\nabla^2\phi + \frac{m_r^2}{f_a^2}\left(|\phi|^2 - \frac{f_a^2}{2}\right)\phi = 0$$

Evolution of this complex scalar field is highly non-linear and it can be done only numerically







$$\rho_{\mathrm{tot}} = \left\langle \left|\dot{\phi}\right|^2 + |\nabla\phi|^2 + V(\phi) \right\rangle$$

3D volume (spatial) average

$$= \left\langle \frac{1}{2}\dot{a}^2 + \frac{1}{2}|\nabla a|^2 \right\rangle + \left\langle \frac{1}{2}\dot{r}^2 + \frac{1}{2}|\nabla r|^2 + V(r) \right\rangle$$

$2\times\frac{1}{2}\dot{a}^2$: stored in axions

: stored in radial modes

$$+ \left\langle \left(\frac{r^2}{2f_a^2} + \frac{r}{f_a}\right)\left(\dot{a}^2 + |\nabla a|^2\right) \right\rangle$$

String energy density

$$\rho_s = \rho_{\mathrm{tot}} - \rho_a - \rho_r$$

$$\rho_{\text{tot}} = \left\langle |\dot{\phi}|^2 + |\nabla\phi|^2 + V(\phi) \right\rangle$$

3D volume (spatial) average

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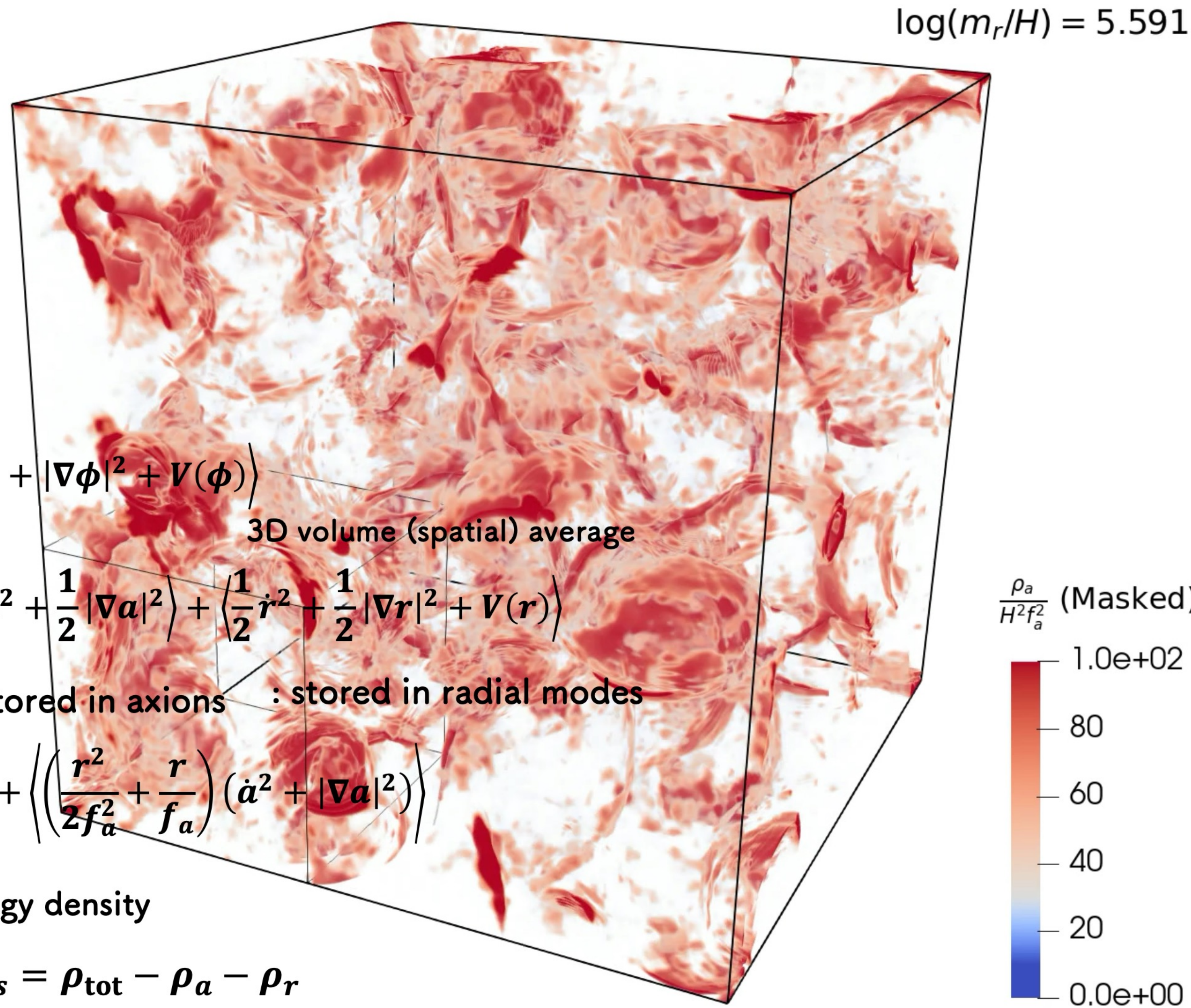
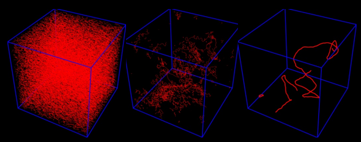


Cosmological evolution of scalar field configuration

$$\log \frac{m_r}{H} = \log \frac{t}{t_0} \lesssim \log N$$



QCD PT

$t \sim 10^{-5}$ sec

~ 1 $\qquad$ ~ 10 $\qquad$ ~ 70 $\qquad$ $\log \frac{m_r}{H}$

Axions decayed from strings around this time contribute to DM

lattice simulation

$H^{-1} \propto t$

$\Delta \propto R(t) = \sqrt{t}$

$m_r^{-1} \propto \mathrm{const}$

Multi-scale problem:
Even numerical simulation is very difficult and progress is very slow

Appearance of Scaling solution

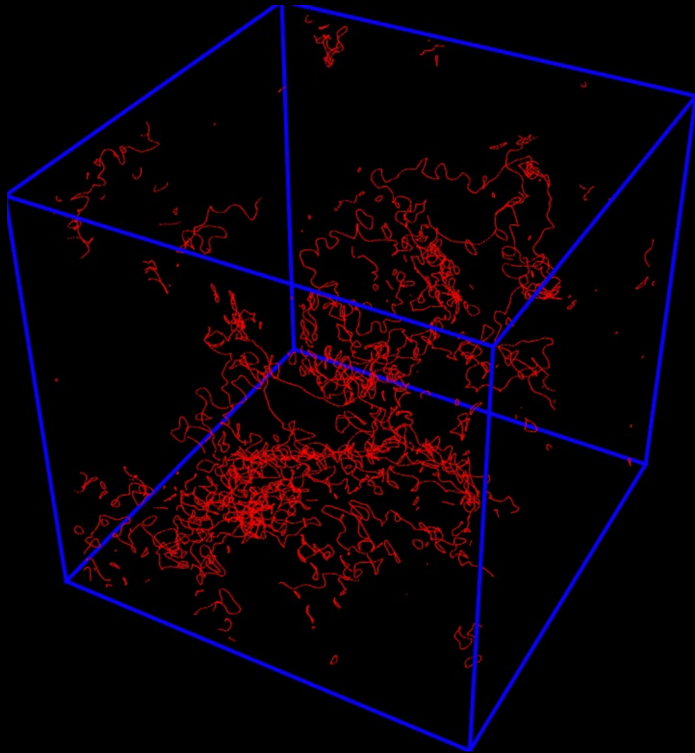
allows us to extrapolate all the way to QCD scale

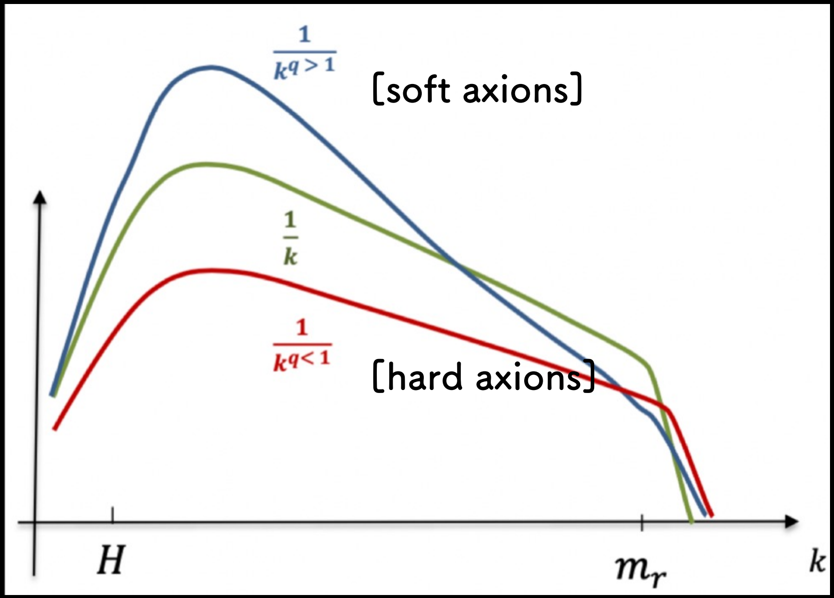
$$\xi \propto \frac{\ell_{\rm tot}(L)}{\left(L^3/(H^{-1})^3\right)} \frac{1}{H^{-1}} = \frac{\ell_{\rm tot}(L) t^2}{L^3}$$

: number of strings per Hubble patch

$$\frac{\partial^2 \rho_a}{\partial t \partial k} \propto \frac{1}{k^q}$$

: no new scale between m_r and H





$$\xi = \frac{\ell_{\text{tot}}(L)t^2}{L^3}$$

: number of strings
per Hubble patch

$$\frac{\partial^2 \rho_a}{\partial t \partial k} \propto \frac{1}{k^q}$$

: no new scale
between m_r and H

Interesting situation would be $\left.\frac{n_a^{\text{str}, q>1}}{n_a^{\text{mis},\theta_0=1}}\right|_{t_\ell} > 1$

Axion abundance

$$n_a = \int \frac{dk}{k} \frac{\partial \rho_a}{\partial k}$$

: We want to evaluate axion number density as a function of time

: should follow a **power law** due to absence of other non-trivial scales and sampling has to be done in scaling regime

$$\frac{\partial \rho_a}{\partial k} = \int^t dt' \frac{\Gamma[t']}{H(t')} \left(\frac{R(t')}{R(t)}\right)^3 F\left[\frac{k'}{H(t')}, \frac{m_r}{H(t')}\right]$$

: **instantaneous emission function**

$$\Gamma[t] \sim \frac{\xi \mu_{\rm eff}}{t^3} \sim 8\pi H^3 f_a^2 \, \xi \log \frac{m_r}{H}$$

$$F \sim \frac{1}{k^q}$$

Two most important scalings in ξ and spectral index q in cosmic string

E.g. $$\left.\frac{n_a^{q>1}}{n_a^{\rm mis}}\right|_{t_\ell} \propto \left(\xi_\star \log \frac{m_r}{H_\star}\right)^{\frac{1}{2}+\cdots}$$

Gorghetto, Hardy, Villadoro 20'
Kim, **SON**, Park 2024 (redone)

Recent intriguing observations are

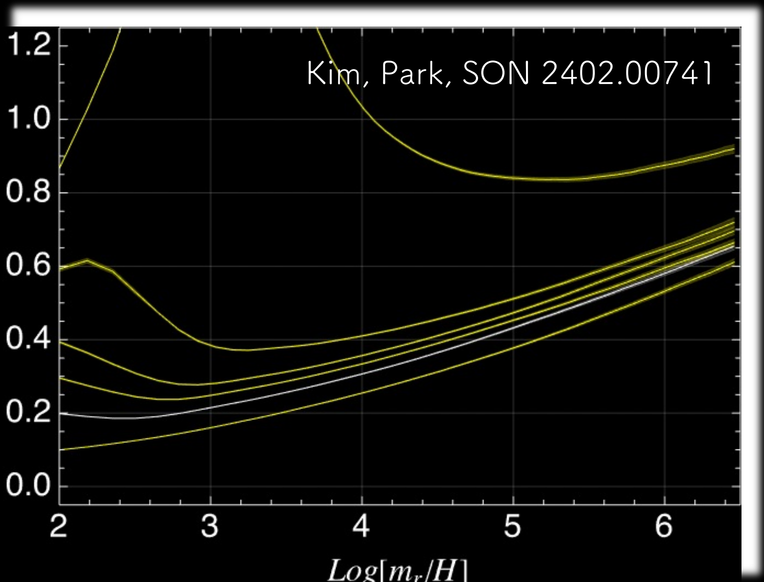
This is where simulation frontiers leads the theory.
These findings are not explained by theory

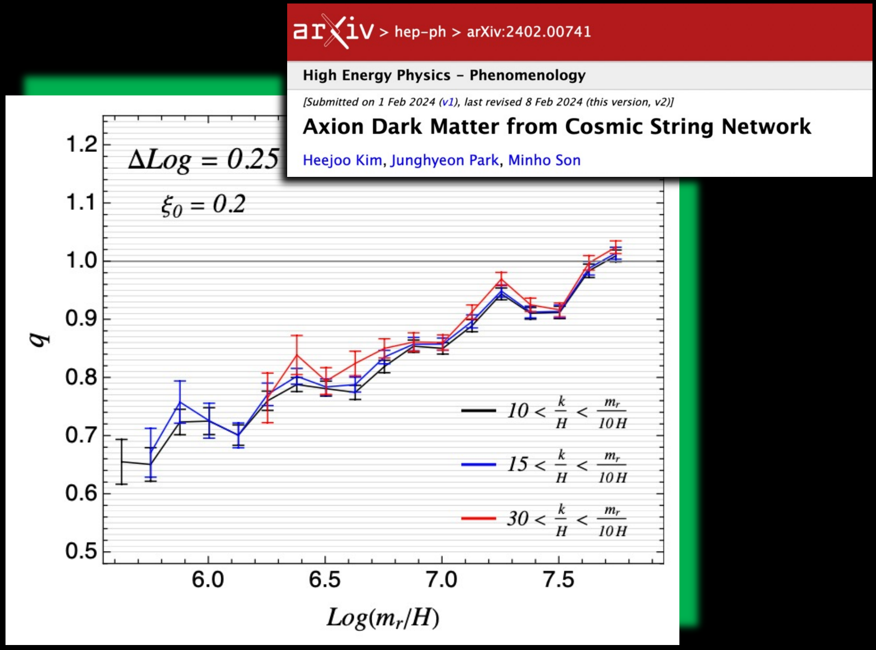
$$\xi = \beta + \alpha \log \frac{m_r}{H}$$

Gorghetto, Hardy, Villadoro 18′,20′
Buschmann, Foster, Hook, Peterson, Willcox, Zhang, Safdi 21′, 24′
Saikawa, Redondo, Vaquero, Kaltshmidt 24′

$$\frac{\partial^2 \rho_a}{\partial t \partial k} \propto \frac{1}{k^q} \qquad \left.\frac{n_a^{\mathrm{str},\, q>1}}{n_a^{\mathrm{mis},\theta_0=1}}\right|_{t_\ell} > 1$$

$$q = q_0 + \varepsilon \log \frac{m_r}{H}$$





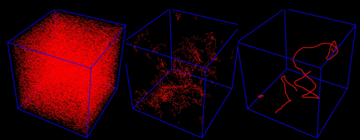
Kim, Park, **SON** 2402.00741

Extrapolates to

$$\xi \sim \mathcal{O}(10)$$

$$q > 1$$

$$@\, m_r t = e^{70}$$



: soon after this time, axions become non-relativistic dark matter

QCD PT

$t \sim 10^{-5}$ sec

~ 1 ~ 10 ~ 70 $\log \frac{m_r}{H}$

Strongly supported by scaling solution

t_* ⟷ t_ℓ

lattice simulation → Extrapolation →

Nonlinearity may or may not kick in

Gorghetto, Hardy, Villadoro 20′

$$\frac{\sqrt{\langle a^2 \rangle}}{f_a} \geq 1 \text{ vs } \leq 1$$

$$\Omega_a^{\rm mis} h^2 = 0.12 \left(\frac{f_a}{10^{12} GeV}\right)^{7/6} \langle \theta_i^2 \rangle$$

$$0.12 \sim \Omega_a h^2 = \Omega_a^{\rm mis} h^2 + \Omega_a^{\rm str} h^2 + \Omega_a^{\rm str-dw} h^2$$

Axions from strings in scaling regime @QCD PT

Gorghetto, Hardy, Villadoro 18′,20′
Saikawa, Redondo, Vaquero, Kaltshmidt 24′
Kim, Park, **SON** 24′

Axions from string-domain wall network @QCD phase transition

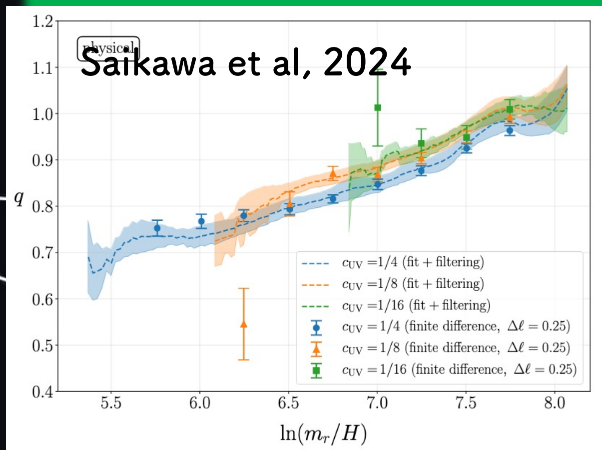
Gorghetto, Hardy, Villadoro 20′
Benabou, Buschmann, Foster, Safdi 24′

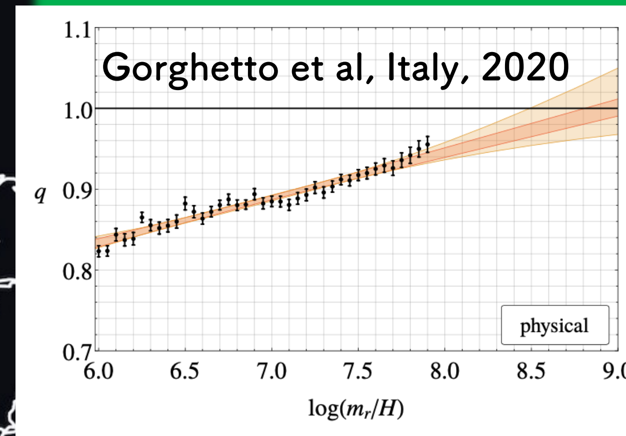
If $\Omega_a^{\rm mis} h^2, \; \Omega_a^{\rm str-dw} h^2 \ll \Omega_a^{\rm str} h^2$, $\quad \Omega_a^{\rm str} h^2 \approx \dfrac{n_a^{\rm str}}{n_a^{{\rm mis},\theta_0=1}} \Omega_a^{{\rm mis},\theta_0=1} h^2 \leq 0.12$

$\rightarrow m_a \geq$ ☐ μeV

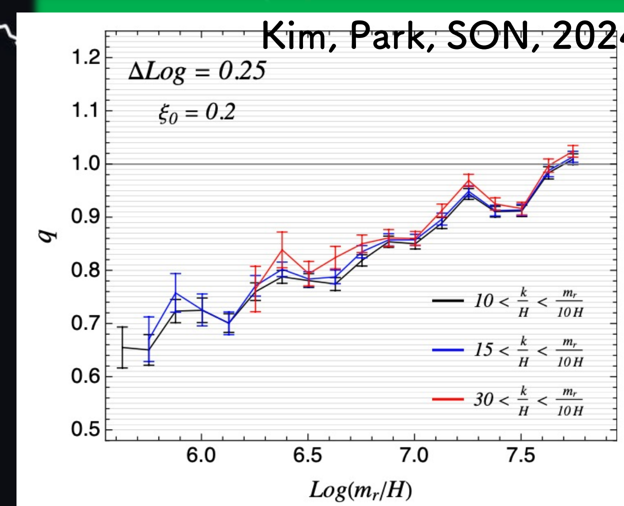
Recent results in cosmic string frontier

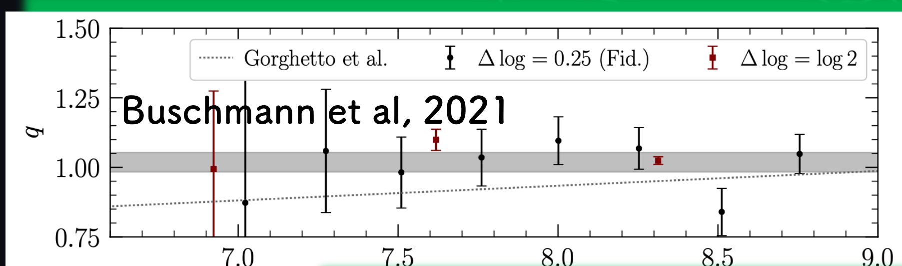
IR dominant axions, q > 1

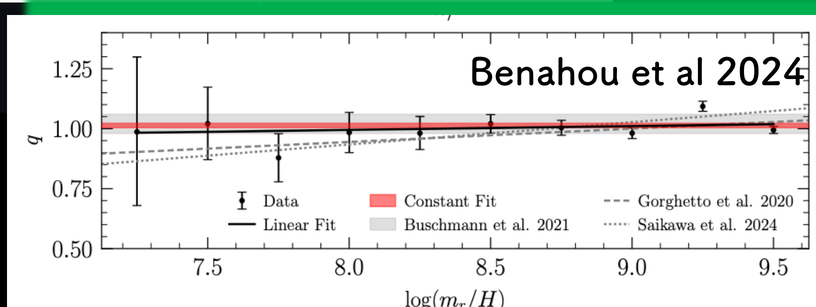




Nearly
conformal strings ,q ~ 1

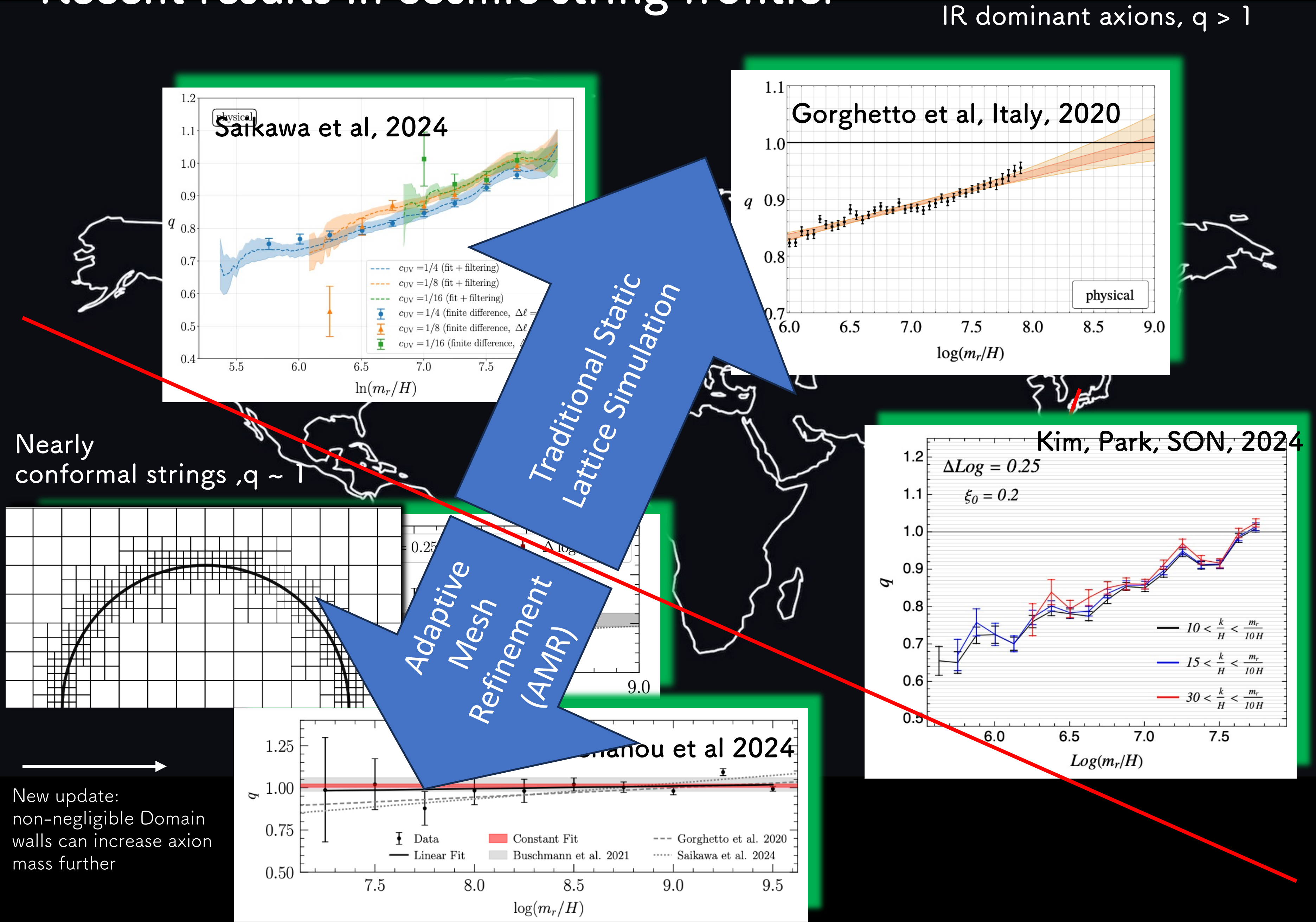






non-negligible Domain walls can increase axion mass further

Recent results in cosmic string frontier



Axion mass prediction

Axion Dark Matter from Cosmic String Network

Heejoo Kim[†], Junghyeon Park[†], and Minho Son[†]

[†] *Department of Physics, Korea Advanced Institute of Science and Technology, 291 Daehak-ro, Yuseong-gu, Daejeon 34141, Republic of Korea*

Kim, Park, **SON** 2402.00741

$$m_a \geq 420\ \mu eV$$

Fat-string pre-evolution

$$m_a \geq 470\ \mu eV$$

Thermal pre-evolution

More Axions from Strings

Marco Gorghetto[a], Edward Hardy[b], and Giovanni Villadoro[c]

[a] *Department of Particle Physics and Astrophysics, Weizmann Institute of Science, Herzl St 234, Rehovot 761001, Israel*

[b] *Department of Mathematical Sciences, University of Liverpool, Liverpool, L69 7ZL, United Kingdom*

[c] *Abdus Salam International Centre for Theoretical Physics, Strada Costiera 11, 34151, Trieste, Italy*

Gorghetto et al 2020

$$m_a \geq 450\ \mu eV$$

Fat-string pre-evolution

Spectrum of global string networks and the axion dark matter mass

Ken'ichi Saikawa,[1] Javier Redondo,[2,3] Alejandro Vaquero,[3] and Mathieu Kaltschmidt[3]

[1] *Institute for Theoretical Physics, Kanazawa University, Kakuma-machi, Kanazawa, Ishikawa 920-1192, Japan*
[2] *Max-Planck-Institut für Physik (Werner-Heisenberg-Institut), Boltzmannstr. 8, 85748 Garching, Germany*
[3] *CAPA & Departamento de Física Teórica, Universidad de Zaragoza, C. Pedro Cerbuna 12, 50009 Zaragoza, Spain*

(Dated: October 16, 2024)

Saikawa et al 2024

$$450\ \mu eV \geq m_a \geq 95\ \mu eV$$

Fat-string pre-evolution

Axion mass prediction from adaptive mesh refinement cosmological lattice simulations

Joshua N. Benabou,[1,2] Malte Buschmann,[3] Joshua W. Foster,[4,5] and Benjamin R. Safdi[1,2]

[1] *Berkeley Center for Theoretical Physics, University of California, Berkeley, CA 94720, U.S.A.*
[2] *Theoretical Physics Group, Lawrence Berkeley National Laboratory, Berkeley, CA 94720, U.S.A.*
[3] *GRAPPA Institute, Institute for Theoretical Physics Amsterdam, University of Amsterdam, Science Park 904, 1098 XH Amsterdam, The Netherlands*
[4] *Astrophysics Theory Department, Theory Division, Fermilab, Batavia, IL 60510, USA*
[5] *Kavli Institute for Cosmological Physics, University of Chicago, Chicago, IL 60637*

(Dated: December 13, 2024)

Benahou et al 2024

$$280 \leftarrow 65\ \mu eV \geq m_a \geq 45\ \mu eV$$

Buschmann et al 2021

$$180\ \mu eV \geq m_a \geq 40\ \mu eV$$

Thermal pre-evolution

Axion mass prediction

Static Lattice Simulation

Axion Dark Matter from Cosmic String Network

partment of Particle Physics and Astrophysics, Weizmann Institute of Science,
Herzl St 234, Rehovot 761001, Israel
[b] Department of Mathematical Sciences, University of Liverpool,
Liverpool, L69 7ZL, United Kingdom
[c] Abdus Salam International Centre for Theoretical Physics,
Strada Costiera 11, 34151, Trieste, Italy

Do not exceed current bound,
Only axion from strings in scaling regime

Kim, Park, **SON** 2402.00741

$m_a \geq 420\ \mu eV$

Fat-string pre-evolution

$m_a \geq 470\ \mu eV$

Thermal pre-evolution

Gorghetto et al 2020

$m_a \geq 450\ \mu eV$

Fat-string pre-evolution

Spectrum of global string networks and the axion dark matter mas

Account for 100% DM abundance,
Only axion from strings in scaling regime,
Mass range due to multiple ansatz for ξ and q + stat. + syst. errors

(Dated: October 16, 2024)

Saikawa et al 2024

$450\ \mu eV \geq m_a \geq 95\ \mu eV$

Fat-string pre-evolution

AMR

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Joshua N. Benabou,[1,2] Malte Buschmann,[3] Joshua W. Foster,[4,5] and Benjamin R. Safdi[1,2]

Account for 100% DM abundance,
Mass range due to the range of q from fitting,
Axions from strings scaling regime + string-domain walls,

Benahou et al 2024

$280 \leftarrow 65\ \mu eV \geq m_a \geq 45\ \mu eV$

Buschmann et al 2021

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Thermal pre-evolution

Exact scaling behavior under debates
It has a dramatic impact on axion mass

$$\xi \propto \frac{\ell_{\rm tot}(L)}{\left(L^3/_{(H^{-1})^3}\right)}\frac{1}{H^{-1}} = \frac{\ell_{\rm tot}(L)t^2}{L^3} = \beta + \alpha \log\frac{m_r}{H} \quad \text{vs} \quad \xi \sim \mathcal{O}(1)$$

: number of strings per Hubble patch

A counter-study claiming a constant scaling of a unity
Correia, Hindmarsh, Lizarraga, Lopes-Eiguren, Rummukainen, Urrestilla, 2410.18064

Moving frame vs Rest frame

$$\xi_r = \xi\langle\gamma^{-1}\rangle$$

$$q = \mathcal{O}(1)$$

$$, q = q_0 + q_1 \log\frac{m_r}{H} \quad , q = q_0 + \frac{q_1}{\log^2\frac{m_r}{H}} \quad \text{etc}$$

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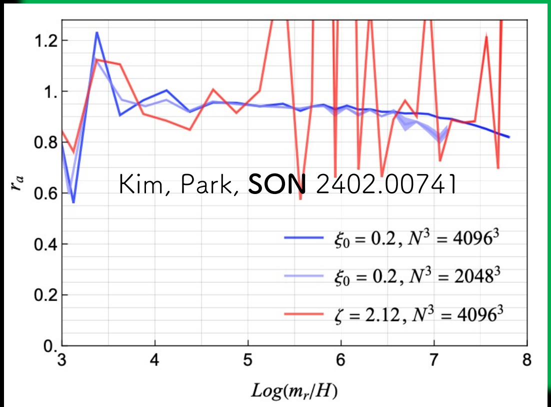
$$\Gamma = \Gamma_a + \Gamma_r \sim \Gamma_a$$

$$r_a = \frac{\Gamma_a}{\Gamma_a + \Gamma_r}$$

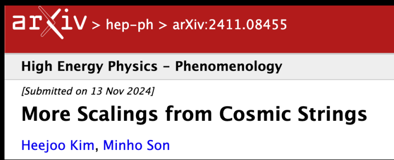
$$R^{-4}\partial_t (R^4 \rho_a) \sim \Gamma_a$$



We speculate that axion power-law spectrum may be originated from strings's power-law behavior

Looking into String power spectrum

Kim, **SON**, 2411.08455


arXiv > hep-ph > arXiv:2411.08455

High Energy Physics – Phenomenology

[Submitted on 13 Nov 2024]

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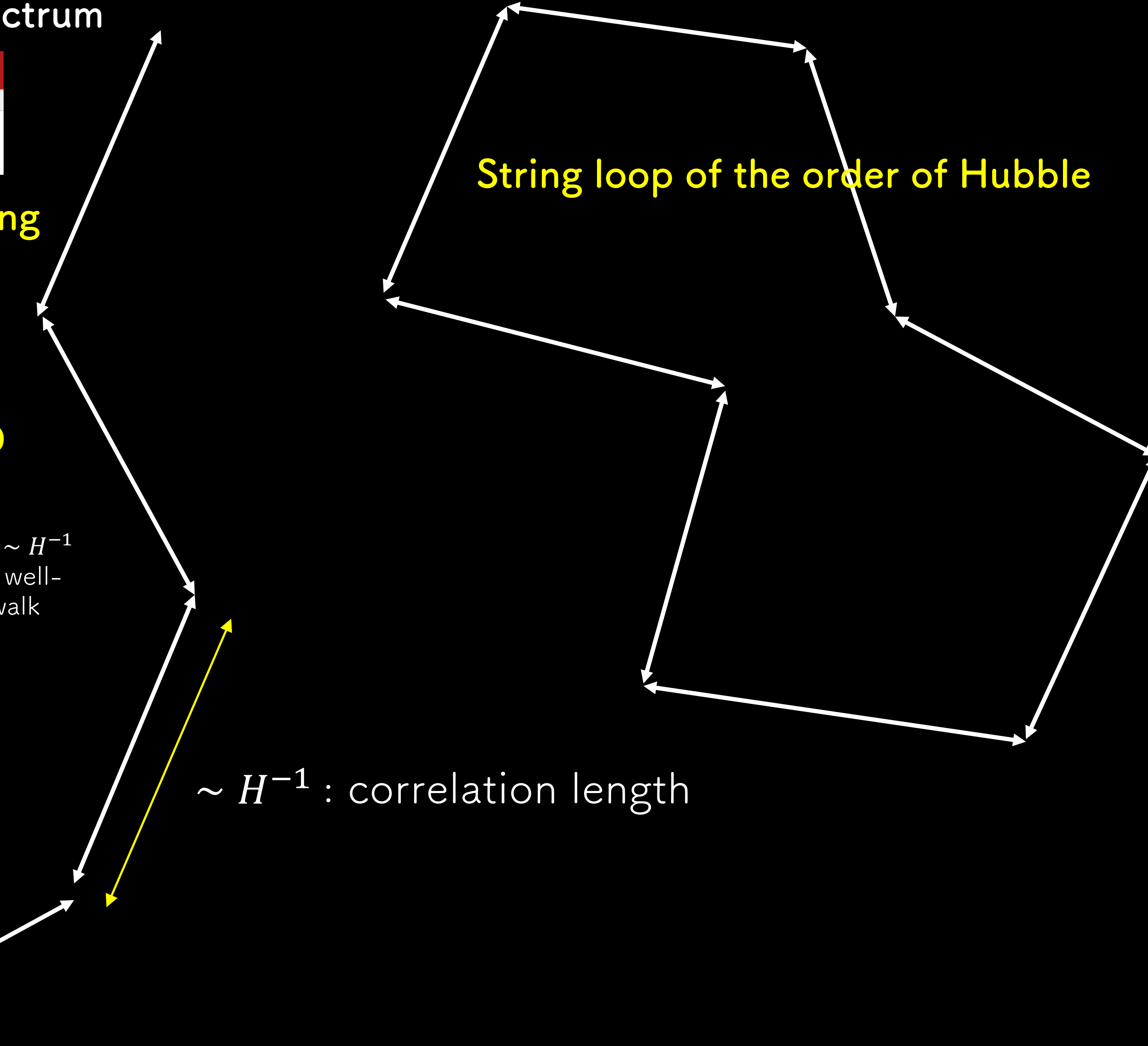

Self-avoiding 3D random walks

Oscillations larger than $\sim H^{-1}$ is frozen, and those are well-described by Random walk strings

Looking into String power spectrum

Self-avoiding 3D random walks

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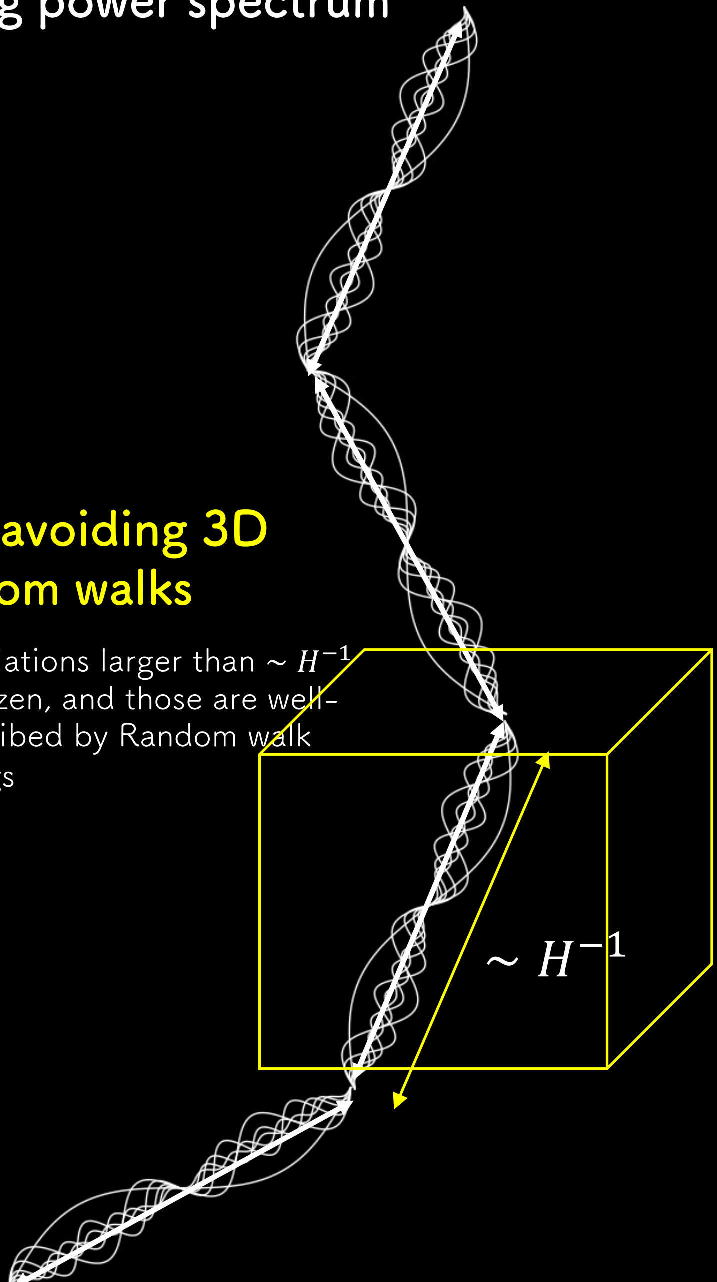

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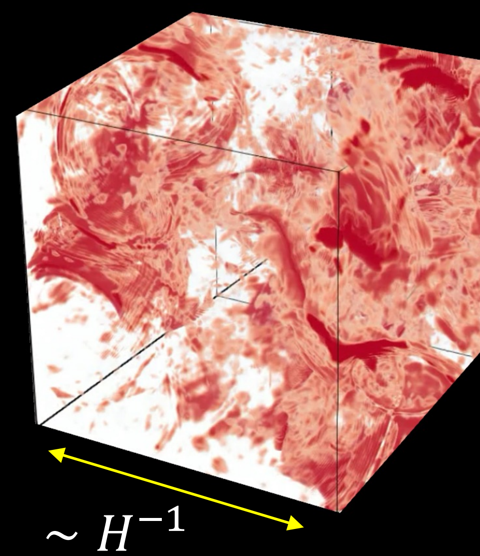
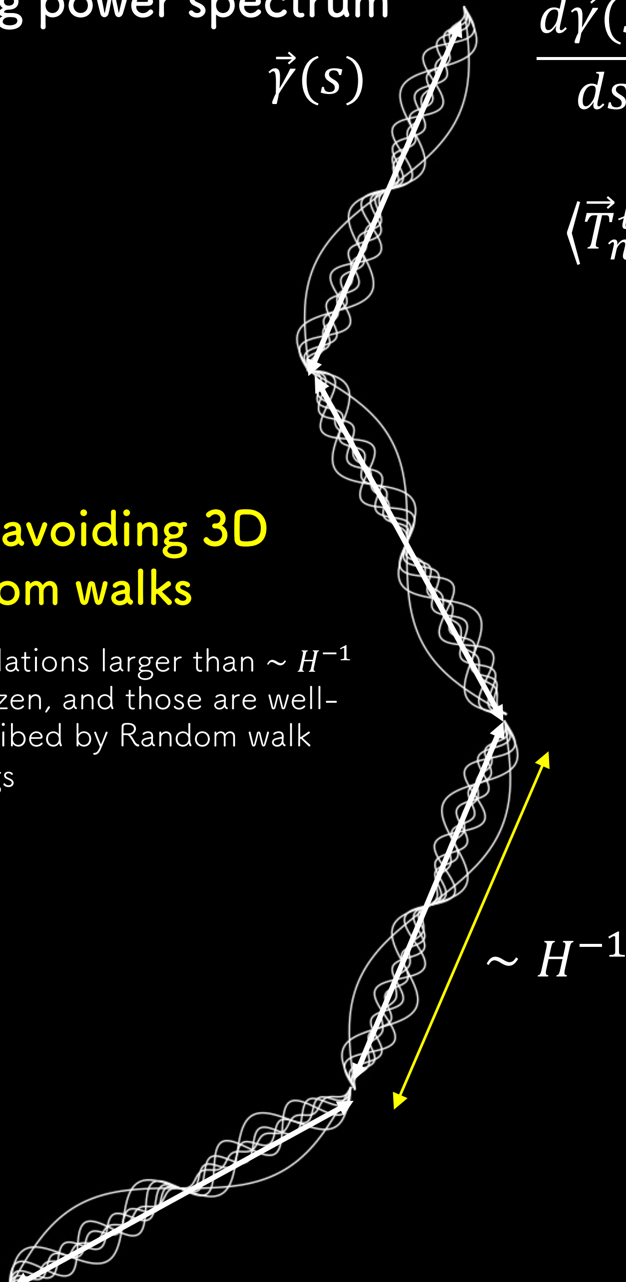


Illustration (from real simulation) for axion emission from strings within the Hubble volume

Looking into String power spectrum

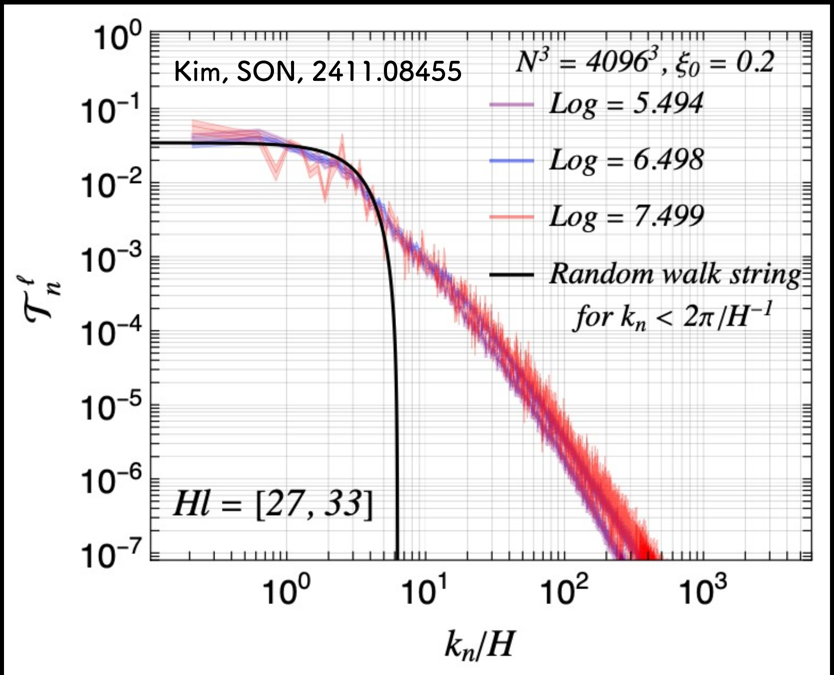


$$\frac{d\vec{\gamma}(s)}{ds} = \vec{t}(s) = \sum_{n=-\infty}^{\infty} \vec{T}_n^{\ell} e^{i\frac{2\pi n}{\ell}s}$$

$$\langle \vec{T}_n^{\ell} \cdot \vec{T}_{n'}^{\ell} \rangle = \mathcal{T}_n^{\ell} \delta_{(n+n')0}$$

Self-avoiding 3D random walks

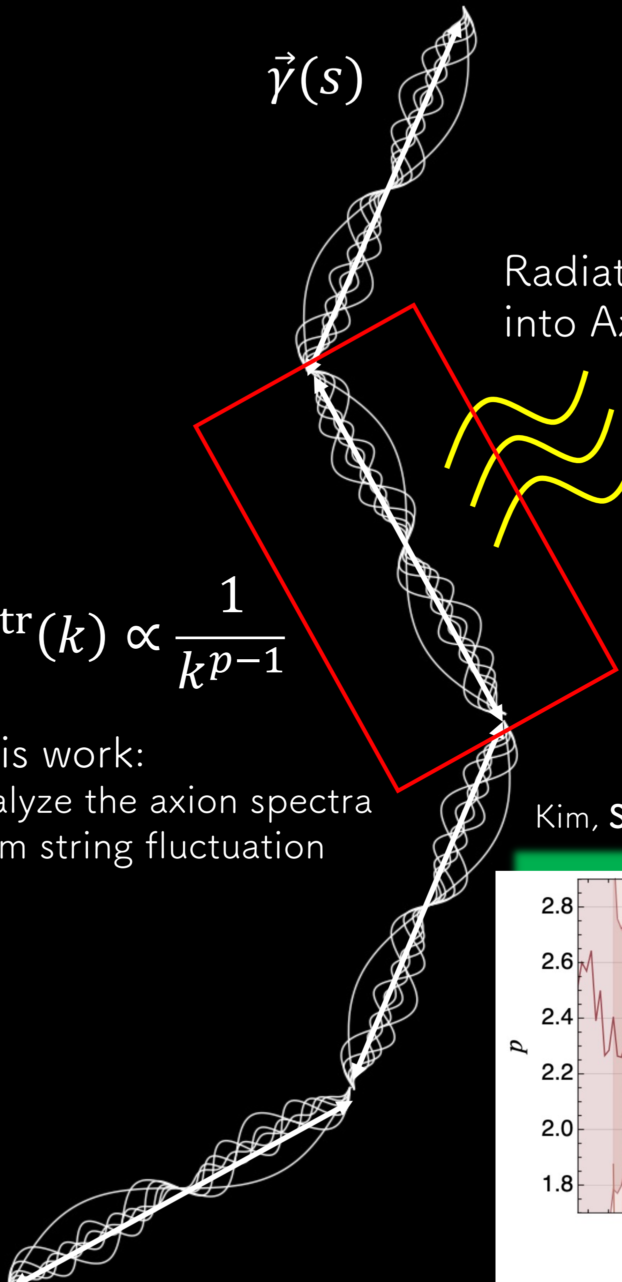
Oscillations larger than $\sim H^{-1}$ is frozen, and those are well-described by Random walk strings



Assume string fluctuation follows the power law similarly to axions

$$\mathcal{T}_n^{\ell} \sim \frac{1}{k^p}$$

String power spectrum



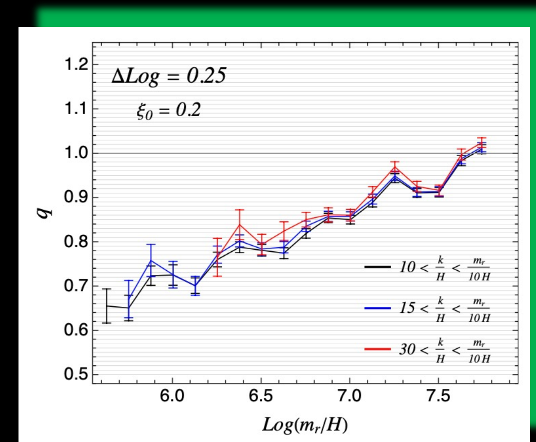
$$\Gamma_a^{\rm str}(k) \propto \frac{1}{k^{p-1}}$$

This work:
analyze the axion spectra
from string fluctuation

$$\Gamma_a \sim \frac{1}{k^q}$$

Traditional:
All literature analyze the axion spectra away from string cores

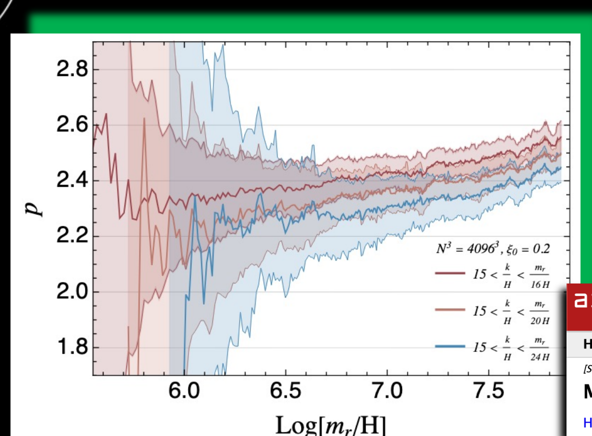
Kim, Park, SON 2402.00741

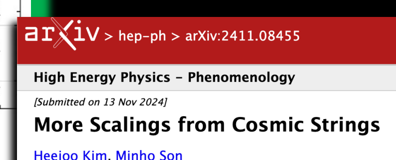


$$q = p - 1 - \Delta^{\rm str.}_{\rm non-pert}$$

Part missed in our derivation

Kim, **SON**, 2411.08455




arXiv > hep-ph > arXiv:2411.08455

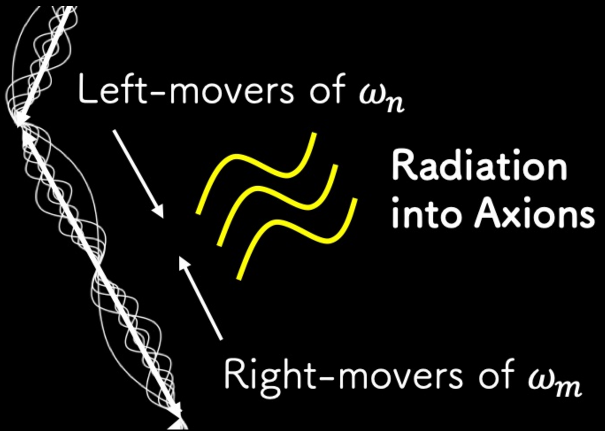
High Energy Physics – Phenomenology

[Submitted on 13 Nov 2024]

More Scalings from Cosmic Strings

Heejoo Kim, Minho Son

Connection between strings and axions



1. No Expansion of Universe
2. Almost straight string
3. Kalb-Ramond interaction in thin string Limit

$$\frac{\Gamma_a^{\mathrm{str.}}(k)}{m_r^2 f_a^2} = 8\pi^3 \xi \left(\frac{m_r}{H}\right)^{-2} \int_0^\infty \frac{d(H\ell)}{H\ell_{\mathrm{ave.}}} \frac{\rho(\ell)}{H} \sum_{n=1}^{\infty} nH\delta\left(k - \frac{2\pi n}{\ell}\right) \sum_{\substack{|m|<n,\\ m+n\ \mathrm{even}}} \mathcal{T}^{\ell}_{\frac{n+m}{2}} \mathcal{T}^{\ell}_{\frac{n-m}{2}} \ ,$$

$$\Gamma_a^{\mathrm{str}}\left(k = \frac{2\pi n}{\ell}\right) \propto n \sum_{m=1}^{n-1} \frac{1}{(n+m)^p (n-m)^p}$$

$$\mathcal{T}_n^{\ell} \sim \begin{cases} B_\ell \left(\dfrac{k_n}{H}\right)^{-p} & : k_{IR} < k < k_{UV} \\ 0 & : \text{otherwise} \end{cases}$$

$$\rightarrow \Gamma_a^{\mathrm{str}}(k) \propto k^{2-2p} \times k^{p-1} = \frac{1}{k^{p-1}}$$

$: \text{a few} \times (2k_{IR}) < \ k < k_{UV}$

$$q = p - 1 - \Delta^{\mathrm{str.}}_{\mathrm{non-pert}}$$

Part missed in our derivation

More update in near future