Holographic QCD

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Holographic applications: from Quantum Realms to the Big Bang — UCAS

12 July 2025

Plan

First lecture

1. Introduction to gauge/gravity duality
   [In part based on lectures by Alfonso Ramallo, 1310.4319]
2. Basics of holographic QCD

Second lecture

1. A top-down example: Witten–Sakai–Sugimoto model
2. Brief introduction to bottom-up approach
3. A bottom-up example: Improved holographic QCD

1. Introduction to gauge/gravity duality

Gauge/gravity duality

Duality relating quantum field theory (QFT) and gravity:

Quantum physics of strongly correlated many-body systems	$\leftrightarrow$	Classical dynamics of gravity in one higher dimension
Difficult!		Easy!

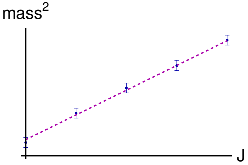
Called by different names — varying slightly in generality

1. AdS/CFT
2. Gauge/gravity correspondence
3. Holographic duality

Applications in QCD, condensed matter, quantum information, hydrodynamics, . . .

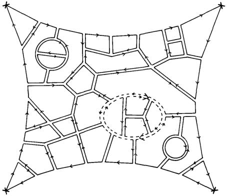
Historical perspective: hints of a duality

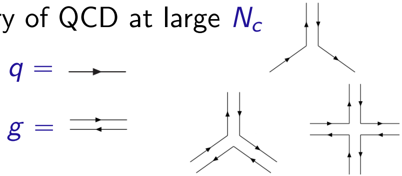
1. String theory emerged as the theory of hadrons



linear "Regge" behavior ↔ dynamics of strings?

2. 't Hooft planar diagram theory of QCD at large N_c





higher order planar diagrams ↔ dynamics of strings? ['t Hooft '74]

3. Advances in black hole physics: they carry temperature and entropy [Hawking, Bekenstein]

4. "Holographic principle": degrees of freedom of a black hole encoded in its surface ['t Hooft '93]

First precise formulation: "AdS/CFT conjecture"

(Strongest form)

$\mathcal{N}=4$ Super-Yang-Mills, $SU(N_c)$ gauge theory in $d=3+1$	$\leftrightarrow$	Full type IIB string theory on $\mathrm{AdS}_5 \times \mathrm{S}^5$

$\downarrow$ large N_c

"Planar" $\mathcal{N}=4$ Super-Yang-Mills	$\leftrightarrow$	Classical string theory on $\mathrm{AdS}_5 \times \mathrm{S}^5$

$\downarrow$ large $\lambda = g^2 N_c$

Strongly coupled planar $\mathcal{N}=4$ Super-Yang-Mills	$\leftrightarrow$	Classical IIB supergravity on $\mathrm{AdS}_5 \times \mathrm{S}^5$

(Weakest, and most useful form)

[Maldacena '97]

Emergence of holography from energy scale

Consider a quantum (field) theory on a lattice with Hamiltonian

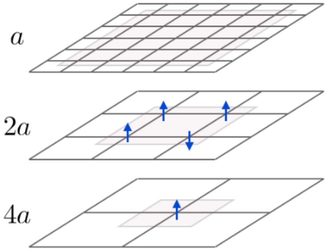
$$H = \sum_{\vec{x},\ i} J_i(\vec{x})\mathcal{O}^i(\vec{x})$$

$$\begin{aligned} \vec{x} &= \text{sites on the lattice} \\ i &= \text{labels of operators} \\ J_i(\vec{x}) &= \text{coupling constants} \end{aligned}$$

Sketch of renormalization group flow

Coarse grain lattice — replace J_i's by their averages

$$J_i(\vec{x}, a)$$
$$\downarrow$$
$$J_i(\vec{x}, 2a)$$
$$\downarrow$$
$$J_i(\vec{x}, 4a)$$



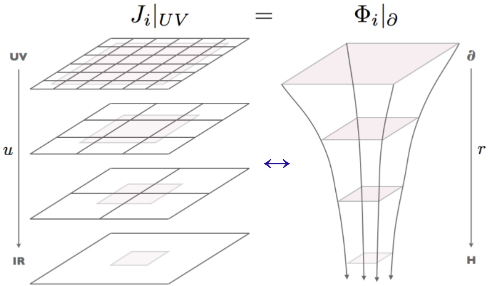
Emergence of holography from energy scale

Generalize the scale a to a continuous variable $u \sim 1/\text{Energy scale}$
$\Rightarrow$ flow equation for generalized $J_i(\vec{x}, u)$

$$u \frac{\partial}{\partial u} J_i(\vec{x}, u) = \beta_i \left(\{J_j(\vec{x}, u)\}, u \right)$$

Holographic (strong coupling) interpretation

- $u \leftrightarrow$ extra "holographic" dimension r
- $J_i(\vec{x}, u) \leftrightarrow$ bulk gravity fields $\phi_i(x, r)$



Dynamics of $\phi_i(\vec{x}, r)$: gravity with some geometry

General properties of the dictionary

gravity fields		field theory operators
▸ scalar ϕ_i	$\leftrightarrow$	scalar operator $\mathcal{O}_i$
▸ vector A^i_μ	$\leftrightarrow$	current operator $\hat{J}^i_\mu$
▸ spin-two $g_{\mu\nu}$	$\leftrightarrow$	energy-momentum tensor $T_{\mu\nu}$

General properties of the dictionary

gravity fields		field theory operators — couplings
scalar ϕ_i	$\leftrightarrow$	scalar operator $\mathcal{O}_i$ — mass m_i
vector A^i_μ	$\leftrightarrow$	current operator $\hat{J}^i_\mu$ — external field $\tilde{A}^i_\mu$
spin-two $g_{\mu\nu}$	$\leftrightarrow$	energy-momentum tensor $T_{\mu\nu}$ — metric of field theory

In general

- Gravity fields at boundary $\leftrightarrow$ field theory couplings (at high energy)
- Field theory "lives at the boundary" of the geometry
- Gauge symmetry in gravity $\leftrightarrow$ global symmetry in field theory (see correspondence $A^i_\mu \leftrightarrow \hat{J}^i_\mu$)

Lightning introduction to QFT (and CFT)

QFT correlators may be computed from a generating functional

$$Z_{\mathrm{QFT}}\left[\{J_k(x)\}\right] = \int \mathcal{D}e^{iS_{\mathrm{QFT}}+i\int d^4x \sum_k J_k(x)\mathcal{O}^k(x)}$$

Correlators are given by

$$\frac{1}{Z_{\mathrm{QFT}}[0]}\frac{\delta}{\delta J_{i_1}(x_1)}\cdots\frac{\delta}{\delta J_{i_n}(x_n)}Z_{\mathrm{QFT}}\Big|_{J_k=0} = \frac{1}{Z_{\mathrm{QFT}}[0]}\int \mathcal{D}e^{iS_{\mathrm{QFT}}}\mathcal{O}^{i_1}(x_1)\cdots\mathcal{O}^{i_n}(x_n)$$

$$\equiv \langle \mathcal{O}^{i_1}(x_1)\cdots\mathcal{O}^{i_n}(x_n)\rangle$$

Conformal field theories (CFT)s: QFTs with additional scale symmetry

Lightning introduction to CFT

CFTs: scale invariant field theories

- Recall that in QCD, scale invariance is broken by quantum effects, not a CFT
- Scale invariance promoted to conformal symmetry
  - In d-dim Minkowski case,
    $\underbrace{SO(1,d-1)}_{\text{Lorentz}} + \text{scale symm.} + \text{translations} + \cdots \to SO(2,d)$
  - In d-dim Euclidean case,
    $SO(d) + \text{scale symm.} + \text{translations} + \cdots \to SO(1,d+1)$
- CFTs characterized in terms of operators $\mathcal{O}^i$ and their dimensions Δ_i
  - Schematically, $\mathcal{O}^i(\lambda x) = \lambda^{-\Delta_i}\mathcal{O}^i(x)$ for any $\lambda > 0$
- Conformal symmetry implies, e.g.

$$\langle \mathcal{O}_1(x_1)\mathcal{O}_2(x_2)\rangle = 0 \qquad \text{if} \qquad \Delta_1 \neq \Delta_2$$

$$\langle \mathcal{O}_1(x_1)\mathcal{O}_1(x_2)\rangle = \frac{C}{|x_1 - x_2|^{2\Delta_1}}$$

AdS space in $d+1$ dimensions

Consider $\mathbb{R}^{2,d}$ i.e. coordinates $(y^{-1}, y^0, y^1, \dots y^d)$ with metric

$$ds^2 = -\left(dy^{-1}\right)^2 - \left(dy^0\right)^2 + \sum_{i=1}^{d} \left(dy^i\right)^2 \equiv \eta_{MN} dy^M dy^N$$

Take hyperboloid with radius ℓ

$$\eta_{MN} y^M y^N = -\ell^2 \qquad \rightarrow \qquad \mathrm{AdS}_{d+1} \; \textit{space}!$$

Isometry group: $SO(2,d)$ — generalized rotations of $\{y^k\}$ which leave ds^2 and $\eta_{MN} y^M y^N$ invariant

- The same as the conformal group for d-dim CFTs! — motivation for the AdS/CFT correspondence

AdS space in $d+1$ dimensions

Changing variables to

$$r = \frac{\ell^2}{y^{-1} + y^d} \ , \quad x^\alpha = \frac{y^\alpha}{y^{-1} + y^d}\ell \ , \quad (\alpha = 0, \ldots, d-1)$$

the pullback of the $\mathbb{R}^{2,d}$ metric to AdS is (exercise)

$$ds^2 = \frac{\ell^2}{r^2}\left(dr^2 - (dx^0)^2 + \sum_{i=1}^{d-1}(dx^i)^2\right) \qquad \text{(AdS metric)}$$

- Here r is the additional "holographic" coordinate
- x^α are identified as spacetime coordinates of field theory
- Invariant under simulataneous scaling of all coordinates
- These coordinates cover half of the AdS space, the "Poincaré patch"
- The geometry solves the equations of motion of $d+1$ dim Einstein gravity (another exercise),

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = -\Lambda g_{\mu\nu} \ , \qquad \Lambda = -\frac{d(d-1)}{\ell^2}$$

Dictionary for gauge/gravity duality

$$\boxed{Z_{\rm QFT}\left[\{J_k(x)\}\right] = Z_{\rm grav}\Big|_{J_k=\phi_k\ @\ bdry}}$$

where

$$Z_{\rm QFT}\left[\{J_k(x)\}\right] = \int \mathcal{D} e^{iS_{\rm QFT}+i\int d^4x \sum_k J_k(x)\mathcal{O}^k(x)}$$

$$Z_{\rm grav} = e^{iS_{\rm grav}^{\rm on-shell}}, \quad S_{\rm grav} = \frac{1}{16\pi G_{d+1}}\int d^{d+1}x\sqrt{-\det g}\left[R - 2\Lambda + \mathcal{L}_{\rm matter} + \cdots\right]$$

- Gravity action evaluated on the solutions (e.g. AdS spaces) of the equations of motion (Einstein equations ...) with bdry condition $\phi_i(x^\mu, r=0) = J_i(x^\mu)$
- Pressure/free energy: simply compare actions at $J_k = 0$
- Correlators: take functional derivatives with respect to J_k
- Simple example of matter sector:

$$\mathcal{L}_{\rm matter} \ \propto \ \kappa\left(g^{MN}\partial_M\phi\partial_N\phi + m^2\phi^2\right)$$

  - One can show that the dimension of the dual operator satisfies $\Delta(\Delta - d) = m^2\ell^2$ (assuming probe limit, $\kappa \to 0$)

Gauge/gravity duality at finite temperature

The dictionary readily generalizes to nonzero temperature

Field theory: Wick rotate to Euclidean time $\tau = it$, temperature arises from periodicity β of τ ($T = 1/\beta$)

$$Z_{\rm QFT} = \int \mathcal{D}\, e^{-S_{\rm QFT}} \ , \qquad S_{\rm QFT} = \int_0^\beta d\tau \int d^{d-1}x \ \mathcal{L}_{\rm QFT}$$

Gravity: Temperature arises from black hole solutions. The case of plain Einstein gravity

$$ds^2 = \frac{\ell^2}{r^2}\left(\frac{dr^2}{f(r)} + f(r)d\tau^2 + \sum_{i=1}^{d-1}(dx^i)^2\right)$$

with

$$f(r) = 1 \ , \quad (T=0) \ ; \qquad f(r) = 1 - \left(\frac{r}{r_h}\right)^d \ , \quad (T>0)$$

- Temperature = surface gravity by regularity of geometry (separate computation); $T = -\frac{1}{4\pi}f'(r_h) = \frac{d}{4\pi r_h}$
- Entropy = black hole area$/4G_{d+1}$; $s = \frac{1}{4G_{d+1}}\frac{\ell^{d-1}}{r_h^{d-1}} \propto T^{d-1}$
- First law of thermodynamics: $\delta S_{\rm grav}^{\rm on-shell} \propto \delta f = s\,\delta T$ (follows from dictionary)

2. Basics of holographic QCD

QCD — reminder of basic properties

Pure Yang–Mills: $SU(N_c)$ gauge theory

$$\mathcal{L}_{\text{YM}} = -\frac{1}{2g^2} F^{ab}_{\mu\nu} F^{\mu\nu\, ab}, \qquad F^{ab}_{\mu\nu} = \partial_\mu A^{ab}_\nu - \partial_\nu A^{ab}_\mu - i[A_\mu, A_\nu]^{ab}$$

QCD: add N_f (fundamental) flavors

$$\mathcal{L}_{\text{QCD}} = \mathcal{L}_{\text{YM}} + i\bar{\psi}\gamma^\mu D_\mu \psi - \bar{\psi} M \psi$$

$$D_\mu \psi^a_k = \left(\delta^{ab}\partial_\mu - iA^{ab}_\mu\right)\psi^b_k \,, \qquad \bar{\psi} M \psi = \bar{\psi}^a_k m_k \psi^a_k$$

Here color (flavor) indices are $a, b, \ldots$ $(k, l, \ldots)$

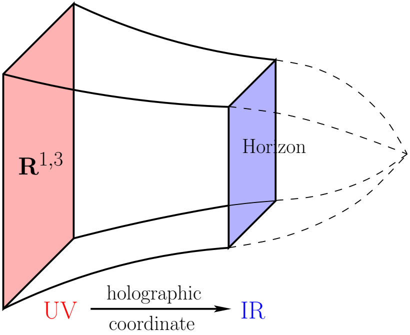
- Coupling g flows in the quantum theory
- Strongly (weakly) coupled at low (high) energies
- Confinement from strong coupling dynamics
- Chiral symmetry $SU(N_f)_L \times SU(N_f)_R$ broken spontaneously down to $SU(N_f)_V$ (and explicitly by the quark masses)
- Regular QCD: $N_c = 3$ and N_f small

Gauge/gravity duality for QCD

- Gauge/gravity duality: at large N_c, strongly coupled field theory $\leftrightarrow$ classical higher dimensional gravity
- Well known example: $\mathcal{N} = 4$ Super Yang-Mills
  $\leftrightarrow$ type IIB supergravity on $AdS_5 \times S^5$
- Relatively easy classical analysis of strongly coupled phenomena $\Rightarrow$ apply to QCD/Yang Mills theory (YM)?
- There are possible issues (QCD not conformal, no SUSY, and $N_c = 3$ is not that large ...) but since solving QCD is hard, it's worth trying
- Turns out to work remarkably well!
  - The "standard" example: description of quark-gluon plasma and its shear viscosity
    $$\frac{\eta}{s} = \frac{1}{4\pi}$$

Gauge/gravity duality for QCD

- Motivated by the original AdS/CFT correspondence for $\mathcal{N} = 4$ Super Yang-Mills
- Instead of conformality, confinement: non-AdS/non-CFT duality
- Field theory lives on the boundary of the 5D geometry



- Operators $O_i(x^\mu) \leftrightarrow$ classical bulk fields $\phi_i(x^\mu, r)$

$$\int \mathcal{D}\, e^{iS_{QCD} + i\int d^4x J^i(x^\mu) O_i(x^\mu)} = Z_{\text{grav}}(\phi_i|_{\text{bdry}} = J_i(x^\mu))$$

- Poles of correlators (at $T = 0$): states of QCD (hadrons) $\leftrightarrow$ normalizable fluctuations around classical gravity solution
- Chiral symmetry of QCD $\leftrightarrow$ gauge symmetry in the bulk
- Thermodynamics of gauge theory $\leftrightarrow$ thermodynamics of a planar bulk black hole

Gauge/gravity duality for QCD: approaches

Top-down: models directly based on string theory

- Concrete, fixed string models in 10/11 d with brane configurations
- Control on what dual field theory is (it's not QCD though)
- E.g., $D3$-$D7$ models and Witten-Sakai-Sugimoto model: $D4$-$D8$-$\overline{D8}$

Bottom-up: models constructed "by hand"

- Follow generic ideas of holography, inspiration from top-down
- Introduce fields for most important operators (marginal)
- Lots of freedom $\rightarrow$ effective 5d description, no link to specific dual theory, comparison with QCD data essential
- Either a fixed geometry (AdS) or dynamical gravity
- Examples: hard/soft wall models, Improved holographic QCD

In the rest of the lectures, I will consider basic properties from models in both classes

Summary

- AdS/CFT realizes explicitly earlier ideas on connections between field theory and gravity/string theory
- Precise duality between strongly coupled $\mathcal{N} = 4$ SYM and type IIB supergravity on $\mathrm{AdS}_5 \times S^5$
- AdS/CFT can be successfully applied to QCD, despite the theory not being conformal or supersymmetric
- Holographic QCD divides to top-down (derived from string theory/supergravity) and bottom-up (engineered) approaches

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1. A top-down example: Witten–Sakai–Sugimoto model
2. Brief introduction to bottom-up approach
3. A bottom-up example: Improved holographic QCD

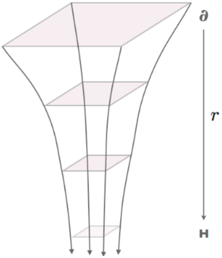
Recap of lecture I

AdS/CFT correspondence

- AdS/CFT maps hard problems in strongly coupled CFT to tractable computations in higher-dimensional classical gravity
- Original example: Precise duality between $\mathcal{N} = 4$ super–Yang–Mills and type IIB supergravity
- CFT lives at the boundary of the AdS space — the conformal group matches with the isometries of the AdS space
- The holographic direction maps to the energy scale of the dual QFT
- Dictionary: the QFT generating functional equals the on-shell partition function in gravity

$$Z_{\text{QFT}}\left[\{J_k(x)\}\right] = Z_{\text{grav}}\Big|_{J_k=\phi_k \text{ @ } bdry}$$

- Thermodynamics of QFT are mapped to black thermodynamics in gravity



General properties of holographic QCD

- Gauge/gravity duality has turned out to be useful to study QCD despite it being non-conformal and non-supersymmetric
- Since QCD not conformal, geometry should deviate from the AdS space
- Holographic QCD models roughly divide into two classes:
  - Top-down approach: models directly based on string theory (or supergravity)
  - Bottom-up approach: models engineered by hand to have desired features

1. Witten–Sakai–Sugimoto model

Top-down holography

Constructions following strictly principles of (super)string theory

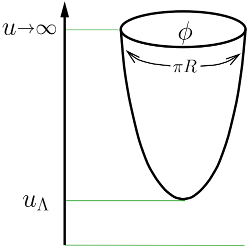
- Recall:
  Large coupling in field theory ↔ classical limit of string theory
  ⇒ 10d or 11d supergravity theories — known explicitly
- Apart from strings, the theory contains D-branes — objects where strings can end
- Gravity description of D-branes well known
  - Large number of coincident branes act as a static source for black-hole-like (super)gravity solutions
  - Dynamics of small number of "probe" branes described by the Dirac–Born–Infeld action
- Basic configuration: stack of N D-branes — transformation of the string end-points implements $SU(N)$ symmetry
- This is the origin for a dual of $SU(N)$ gauge theory

Witten's model

Consider a specific 10d supergravity (type IIA)

- Insert a stack of N_c $D4$-branes — five-dimensional
    - This would give rise to a 5d field theory
- Compactify one of the (spatial) coordinates, ϕ, on a circle
- Impose antiperiodic boundary conditions for fermionic fields $\Rightarrow$ supersymmetry broken!
- The gravity solution sourced by $D4$-branes is a cigar geometry

$$ds^2 = u^{3/2}\left(\eta_{\mu\nu}dx^\mu dx^\nu + f(u)d\phi^2\right)$$
$$+u^{-3/2}\frac{du^2}{f(u)} + u^{1/2}d\Omega_4^2\ , \qquad f(u) = 1 - \frac{u_\Lambda^3}{u^3}$$



- u_Λ related to compactification radius R by regularity at the tip of cigar
- Sets scale of confinement and mass gap
- Dual: $SU(N_c)$ Yang–Mills + extra Kaluza–Klein modes

[Witten '98, . . .]

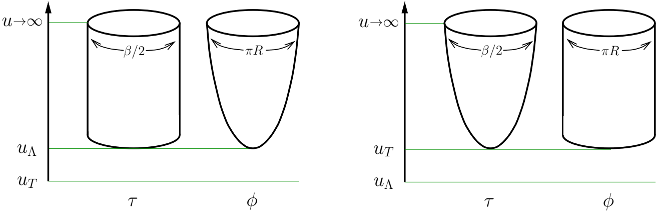
Finite temperature and phase transition

Going to finite temperature, $\tau = it$, two competing geometries

$$ds^2 = u^{3/2}\left(d\tau^2 + d\vec{x}^2 + f(u)d\phi^2\right) + u^{-3/2}\frac{du^2}{f(u)} + u^{1/2}d\Omega_4^2 \qquad \text{(soliton)}$$

$$ds^2 = u^{3/2}\left(\hat{f}(u)d\tau^2 + d\vec{x}^2 + d\phi^2\right) + u^{-3/2}\frac{du^2}{\hat{f}(u)} + u^{1/2}d\Omega_4^2 \qquad \text{(black hole)}$$

$$f(u) = 1 - \frac{u_\Lambda^3}{u^3}\,, \qquad \hat{f}(u) = 1 - \frac{u_T^3}{u^3}$$



low temperature phase   high temperature phase

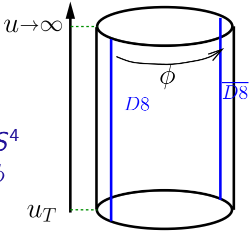
- u_Λ linked to R, and u_T linked to $\beta = 1/T$ by regularity
- As T varies, Hawking–Page (confinement) transition
- Note $\tau \leftrightarrow \phi$ symmetry — transition at $u_\Lambda = u_T$

Adding flavors — chiral symmetry

So far we only described (approximately) Yang–Mills theory

Adding quarks leads Witten–Sakai–Sugimoto (WSS) model:

- Use probe branes — do not affect the geometry
- Two stacks of N_f branes: $D8$ and $\overline{D8}$
- Probe limit means $N_f \ll N_c$
- Branes nine dimensional:
  - Extend over space-time, holographic, and S^4
  - Localized antipodally in the compactified ϕ
- Symmetry imposed by branes:
  $SU(N_f)_L \times SU(N_f)_R$ = chiral symmetry of QCD
- Symmetry intact at high T (picture here)



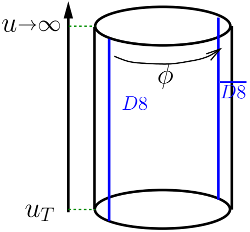
[Sakai, Sugimoto '04]

Adding flavors — chiral symmetry

- Dynamics of the branes: DBI action

$$S_{\mathrm{DBI}} = -T_f \int_{\mathrm{branes}} d^9x\, e^{-\varphi}\, \mathrm{Tr}\left[\sqrt{-\det\left(g_{\mu\nu} + F_{\mu\nu}\right)}\right]$$

- Here T_f is the tension of the brane
- $e^{-\varphi} \propto u^{-3/4}$ is the dilaton, part of supergravity background solution
- Integral over the volume filled by the branes
- Gauge fields represent some of the open strings ending on the branes

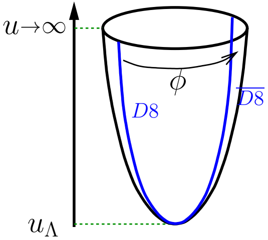


- Chiral symmetry $SU(N_f)_L \times SU(N_f)_R$ promoted to gauge symmetry in gravity

Breaking of chiral symmetry

At low (or zero) temperature, the holographic u and compactified ϕ form the cigar geometry

- $D8$ and $\overline{D8}$ branes join at the tip of the cigar
- Breaks chiral symmetry to vectorial subgroup, $SU(N_f)_L \times SU(N_f)_R \to SU(N_f)_V$
- That is, chiral symmetry breaking triggered by confinement
- Matches with the pattern in QCD!
- Meson spectrum can be computed from the fluctuations of the brane action — turns out to be in good agreement with the experimental results



- Massless pions arise from specific components of the gauge fields — Goldstone bosons of the broken chiral symmetry

2. Bottom-up holographic QCD

Bottom-up approach

Bottom-up holographic models for QCD: models engineered by hand to mimic the properties of QCD

- One might think that this gives a huge amount of freedom ...
- However one needs to follow general principles, and lessons from top-down ⇒ for example, two-derivative or DBI actions
- Also, strongly constrained by (chiral) symmetry

Various variations:

1. Simple models where one considers probe quark sector on a fixed geometry, e.g.
   - Hard-wall models
   - Soft-wall models
   - $D3$-$D7$-inspired models with effects of running coupling
   - ...
2. More involved models with dynamical gravity and (potentially backreacted) quark sectors
   - Improved holographic QCD
   - Einstein–Maxwell–dilaton models
   - V-QCD

Holography setup for QCD: expected dictionary

Most important (relevant and marginal) operators — gluon sector

- $T_{\mu\nu}$, dual to the metric $g_{\mu\nu}$ — source (or coupling) is the field theory metric
- Gluon operator $(F^{ab}_{\mu\nu})^2$, dual to a scalar (the dilaton) ϕ — source is the coupling in field theory

Flavor sector: additional operators

- Flavor currents $\bar{\psi}_i\gamma_\mu(1\pm\gamma_5)\psi_j$, dual to the gauge fields $(A^{L/R}_\mu)_{ij}$ (with $i,j=1\ldots N_f$) — sources are chemical potentials and external fields
- Flavor bilinears $\bar{\psi}_i\psi_j$ dual to a complex scalar X_{ij} — sources are quark masses

NB chiral symmetry promoted to gauge symmetry in the bulk

Very often one restricts to flavor independent configurations and vectorial fields

$$A_L = A_R = \hat{A}\,\mathbb{1}\ , \qquad X = \hat{X}\,\mathbb{1}$$

with $\mathbb{1}$ being the $N_f \times N_f$ unit matrix

Simple bottom-up models

Hard-wall model

- Gravity sector: AdS_5 geometry with a hard cutoff to impose confinement — similar effect as the cigar in WSS
- Probe matter sector with quadratic action (Tr over flavors)
  $\mathcal{L}_{\text{matter}} \sim \text{Tr}\left[|DX|^2 + 3|X|^2 - \frac{1}{4g_5^2}(F_L^2 + F_R^2)\right]$
- Fluctuations of this action produce an excellent description of QCD spectrum! [Erlich et al.; Da Rold, Pomarol '05]

Soft-wall model — adjustment of the hard-wall

- Take AdS_5 geometry, but instead of hard-wall add by hand a background dilaton field, imposing a "soft" cutoff in the IR
- Improved description of higher meson excitations — however the dilaton background doesn't solve any simple gravity theory [Karch et al. '06]

Focus of the rest of the lectures: More complicated models using dynamical (Einstein-dilaton) gravity

- In particular, the Improved Holographic QCD (IHQCD) model

3. Improved holographic QCD

IHQCD action and dictionary

"Improved holographic QCD" (IHQCD): string-inspired bottom-up model for pure Yang–Mills (YM) theory
[Reviews: Gürsoy, Kiritsis, Mazzanti, Michalogiorgakis, Nitti arXiv:1006.5461; Gürsoy 1612.00899]

Dictionary

- $F^a_{\mu\nu} F^{\mu\nu\, a}$ dual to the dilaton ϕ
- $T_{\mu\nu}$ dual to the metric g_{MN}

Gravity action: 5d Einstein-dilaton gravity

$$\mathcal{S} = -\mathcal{N} \int d^5x \sqrt{-\det g} \left[R - \frac{4}{3} (\partial\phi)^2 + V_g(\phi) \right]$$

Choice of V_g: compare with data ... however, in general (not only IHQCD), there are different ways to do this

How to compare the model with data?

Potentials are determined (among other things) by comparing with lattice results for QCD thermodynamics. Two main strategies:

Strategy I (IHQCD): Include confined phase, and the transition to a deconfined phase

- Fit lattice data above $T = T_c$

  [Gürsoy, Kiritsis, Mazzanti, Nitti 0903.2859]
- Faithful to the behavior in large-N_c QCD or pure Yang–Mills (first order transition as a function of T at low density)

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Strategy II: Only deconfined black holes: no phase transition

- Fit lattice data at all temperatures

  [Gubser, Nellore, Pufu, Rocha 0804.1950; Gubser, Nellore 0804.0434; ...]
- Follows the behavior in full QCD (crossover at low density)
- Does not reproduce confinement or realistic glueball spectrum

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These lectures: focus on first approach, implemented in the IHQCD model

Improved Holographic QCD — more details

$$\mathcal{S}_{\text{IHQCD}} = -M^3 N_c^2 \int d^5x \sqrt{-\det g}\left[R - \frac{4}{3}\frac{(\partial\lambda)^2}{\lambda^2} + V_g(\lambda)\right]$$

- $\lambda = e^{\phi}$
- Near boundary $\lambda \approx$ source, i.e., the 't Hooft coupling $g^2 N_c$
- Computation in 5d noncritical string theory gives $V_g \sim \lambda^{4/3}(c_1 - c_2\lambda^2)$
- However this choice does not give good phenomenology

Therefore switch to full bottom-up and determine V_g (only) by comparing to QCD physics

- Strong coupling (IR) physics: qualitative properties of QCD, e.g., confinement
- Weak coupling (UV) physics: perturbation theory
- Remaining degrees of freedom: lattice data

Asymptotic constraints

1. Weak coupling: perturbative Yang-Mills — require agreement of with the β-function of YM at two loops
   $\Rightarrow$ asymptotic freedom and logarithmic flow of the coupling, as in YM and QCD

$$V_g = \frac{12}{\ell^2}(1 + c_1\lambda + c_2\lambda^2 + \cdots) \qquad (\lambda \to 0)$$

   - Holography does not work at weak coupling — however we still match with YM to achieve good "boundary conditions" for the strongly coupled part
   - Similar adjustments are are typical also in other bottom-up models

Geometry = asymptotic AdS_5 (as $r \to 0$) with log-corrections:

$$ds^2 = \frac{\ell^2}{r^2}\left(dr^2 + d\tau^2 + d\vec{x}^2\right)\left[1 + \mathcal{O}\left(\frac{1}{\log r}\right)\right]$$

$$\lambda(r) = -\frac{8c_1}{9\log(r\Lambda)} + \mathcal{O}\left(\frac{1}{(\log r)^2}\right)$$

Asymptotic constraints

2. Strong coupling: require linear confinement, discrete spectrum, linear glueball trajectories $\Rightarrow$

$$V_g \sim \lambda^{4/3}\sqrt{\log \lambda}\,, \qquad (\lambda \to \infty)$$

   - Note: confinement for $V(\lambda) \sim \lambda^p$ with $p \geqslant 4/3$, so this choice is (somewhat weakly) confining
   - Similar with result from noncritical string theory, up to the log-term!

   Vacuum geometry (zero T) = "good" IR singularity (as $r \to \infty$)

$$ds^2 = C\, r\, e^{-2r^2}\left(dr^2 + d\tau^2 + d\vec{x}^2\right)\left[1 + \mathcal{O}\left(\frac{1}{r^2}\right)\right]$$

Choose an Ansatz which satisfies the constraints, e.g.

$$V_g = \frac{12}{\ell^2}\left[1 + c_1\lambda + V_1\lambda^{4/3}\sqrt{\log(1 + V_2\lambda^{4/3} + V_3\lambda^2)}\right]$$

and compare V_i with lattice results!

Finite temperature analysis

Two geometries at nonzero T

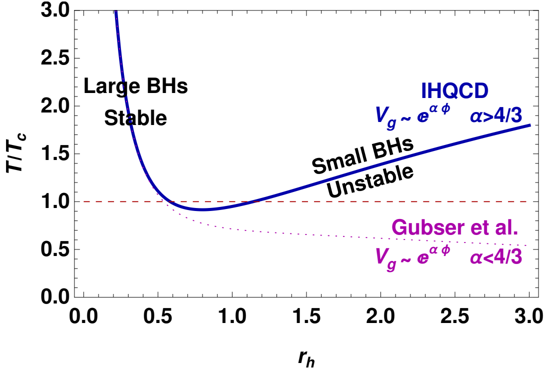
1. "Thermal gas" geometry (low T): $ds^2 = e^{2A(r)}(dr^2 + d\tau^2 + d\vec{x}^2)$
   - Obtained from $T = 0$ vacuum by compactifying τ (period $\beta = 1/T$)
   - Confining
   - No BH, so entropy and pressure vanish
2. Black hole geometry (high T): $ds^2 = e^{2A(r)}\left(\frac{dr^2}{f(r)} + f(r)d\tau^2 + d\vec{x}^2\right)$
   - Horizon at some $r = r_h$ with $f(r_h) = 0$
   - Temperature related to surface gravity = Hawking temperature (regularity of geometry at horizon) $T = -f'(r_h)/4\pi$
   - Deconfining
   - Nontrivial $O(N_c^2)$ (black hole) entropy and pressure

Hawking-Page transition between the geometries at some $T = T_c$

Solve geometries numerically, compute thermodynamics . . .

Finite temperature analysis

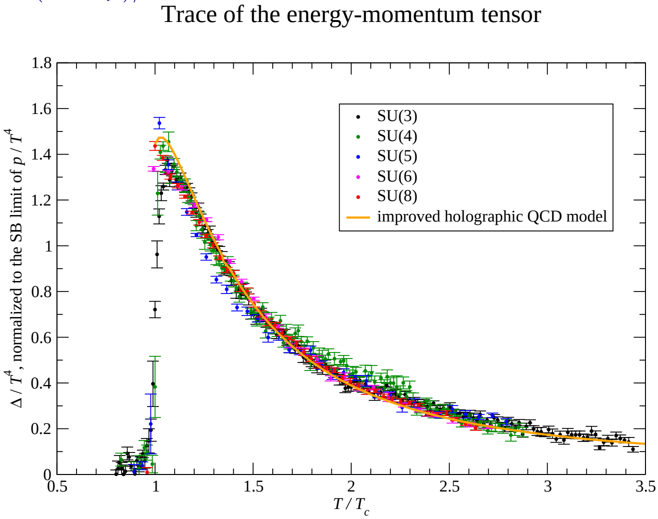
Black hole branches



- First order transition $\leftrightarrow$ unstable phase of small black holes
- IHQCD (Strategy I) qualitatively different from the approach of Gubser et al. (Strategy II) without a phase transition
- Recall: thermal gas solutions exist at all T, physical phase (lowest free energy) for $T < T_c$

Comparison to lattice data

Equation of state from lattice: interaction measure in Yang-Mills
$\Delta/T^4 = (\epsilon - 3p)/T^4$



[Panero]

Glueball spectrum (from fluctuations around vacuum geometry)

	IHQCD	Lattice ($N_c = 3$)	Lattice ($N_c = \infty$)
m_{0*++}/m_{0++}	1.61	1.56(11)	1.90(17)
m_{2++}/m_{0++}	1.36	1.40(4)	1.46(11)

And critical temperature

$$\left(\frac{T_c}{m_{0^{++}}}\right)_{\text{IHQCD}} = 0.167, \qquad \left(\frac{T_c}{m_{0^{++}}}\right)_{\text{lattice}} = 0.177(7)$$

[Gürsoy, Kiritsis, Mazzanti, Nitti arXiv:0903.2859]

- Good simultaneous agreement with spectrum, T_c, and finite temperature EoS
- That is, predictions from both finite-T geometries and the transition between them agree with data — highly nontrivial!

4. Adding flavors

Setting up holography for QCD: dictionary

We want to describe holographically QCD (N_f massless flavors)

Recall the expected dictionary in the flavor sector

- Flavor currents $\bar{\psi}_i \gamma_\mu (1 \pm \gamma_5) \psi_j$, dual to the gauge fields $(A_\mu^{L/R})_{ij}$ (with $i, j = 1 \ldots N_f$) — sources are chemical potentials and external fields
- Flavor bilinears $\bar{\psi}_i \psi_j$ dual to a complex scalar T_{ij} — sources are quark masses

For the Yang–Mills, or pure gravity sector, we use 5d Einstein-dilaton gravity (IHQCD)

What kind of action should one choose for the flavors?

Choosing the flavor action

Simplest choice: write an action quadratic in the new fields

$$\mathcal{L}_{\rm matter} \sim -{\rm Tr}\left[Z(\lambda)\left(F_L^2 + F_R^2\right) - |DX|^2 + V(XX^\dagger)\right]$$

- If one also takes Abelian vectorial Ansatz, $A_L = A_R = \hat{A}\,\mathbb{1}$ and $X = 0 \Rightarrow$ Einstein-Maxwell-dilaton model

[DeWolfe, Gubser, Rosen '10, ...]

A choice motivated by a $D4$-$\overline{D4}$ configuration

$$\mathcal{L}_{\rm m} \sim -{\rm Tr}\left[V(\lambda, X)\sqrt{-\det(g_{MN} + \kappa(\lambda) D_M X D_N X^\dagger + w(\lambda) F^L_{MN})} + (L \leftrightarrow R)\right]$$

with $V(\lambda, X) \sim V(\lambda) e^{-XX^\dagger}$ "rolling tachyon" potential

[Bigazzi,Casero,Cotrone,Kiritsis,Paredes '05, ...]

- Backreacting this with the dilaton gravity leads to the V-QCD models

[MJ,Kiritsis '11]

V-QCD models

V-QCD: A class of holographic bottom-up models for QCD in the Veneziano limit (large N_f, N_c; fixed N_f/N_c)

- Bottom-up, but trying to follow principles from string theory closely
- Many parameters: effective description of QCD
- Comparison with QCD data essential
- Data fitted typically at $N_c = 3$ and $N_f = 2$ or 3
- Relatively complicated model (because QCD is complicated)

Full backreaction between the gluon (dilaton gravity) and flavor (DBI) sectors in the Veneziano limit!

[Review: MJ 2110.08281]

V-QCD models

Take an Ansatz only including components $\propto \mathbb{1}$ in flavor space and vectorial gauge fields

Two bulk scalars: $\lambda \leftrightarrow g^2 N_c, \qquad Tr(X) = \tau \leftrightarrow \bar{\psi}\psi$

$$\mathcal{S}_{\mathrm{V-QCD}} = -N_c^2 M^3 \int d^5x \sqrt{-\det g}\left[R - \frac{4}{3}\frac{(\partial\lambda)^2}{\lambda^2} + V_g(\lambda)\right]$$
$$+N_f N_c M^3 \int d^5x \; V_{f0}(\lambda) e^{-\tau^2}$$
$$\times \sqrt{\det(g_{ab} + \kappa(\lambda)\partial_a\tau\partial_b\tau + w(\lambda)\hat{F}_{ab})}$$

$$\hat{F}_{rt} = \hat{A}_t'(r) \qquad \hat{A}_t(0) = \mu$$

- Condensation of τ in the bulk $\leftrightarrow$ chiral condensate in QCD and broken chiral symmetry
- Finite density: temporal gauge field A_t sources the quark number chemical potential (= baryon chemical potential$/N_c$)
- Four functions V_g, V_{f0}, κ, w to be fitted to QCD data

Constraining the potentials

Similar approach as with IHQCD only:

In the UV ($\lambda \to 0$):

- UV expansions of potentials matched with perturbative QCD beta functions $\Rightarrow$ asymptotic freedom and logarithmic flow of the coupling and quark mass, as in QCD

In the IR ($\lambda \to \infty$): various qualitative constraints

- Linear confinement, discrete glueball & meson spectrum, linear radial trajectories
- Existence of a “good” IR singularity
- Correct behavior at large quark masses
- Working potentials often string-inspired power-laws, multiplied by logarithmic corrections (i.e, first guesses usually work!)

Final task: determine the potentials in the middle, $\lambda = \mathcal{O}(1)$

- Qualitative comparison to lattice/experimental data

Ansatz for potentials

$$V_g(\lambda) = 12\left[1 + V_1\lambda + \frac{V_2\lambda^2}{1+\lambda/\lambda_0} + V_{\rm IR}\, e^{-\lambda_0/\lambda}(\lambda/\lambda_0)^{4/3}\sqrt{\log(1+\lambda/\lambda_0)}\right]$$

$$V_{f0} = W_0 + W_1\lambda + \frac{W_2\lambda^2}{1+\lambda/\lambda_0} + W_{\rm IR}\, e^{-\lambda_0/\lambda}(\lambda/\lambda_0)^2$$

$$\frac{1}{\kappa(\lambda)} = \kappa_0\left[1 + \kappa_1\lambda + \bar{\kappa}_0\, e^{-\lambda_0/\lambda}\frac{(\lambda/\lambda_0)^{4/3}}{\sqrt{\log(1+\lambda/\lambda_0)}}\right]$$

$$\frac{1}{w(\lambda)} = w_0\left[1 + \frac{w_1\lambda/\lambda_0}{1+\lambda/\lambda_0} + \bar{w}_0\, e^{-\lambda_0/\lambda w_s}\frac{(w_s\lambda/\lambda_0)^{4/3}}{\log(1+w_s\lambda/\lambda_0)}\right]$$

$$V_1 = \frac{11}{27\pi^2}\ , \quad V_2 = \frac{4619}{46656\pi^4}\ ; \quad W_1 = \frac{8+3\,W_0}{9\pi^2}$$

$$W_2 = \frac{6488+999\,W_0}{15552\pi^4}\ ; \quad \kappa_0 = \frac{3}{2} - \frac{W_0}{8}\ , \qquad \kappa_1 = \frac{11}{24\pi^2}$$

Finite temperature analysis

Again two geometries at nonzero T

1. Confining "thermal gas" geometry
   - $ds^2 = e^{2A(r)}(dr^2 + dt_E^2 + d\vec{x}^2)$
   - Gauge field $\hat{A}_t = \text{const.} = \mu$
     $\Rightarrow$ still trivial thermodynamics, zero density
2. Black hole geometries (high T)
   - $ds^2 = e^{2A(r)}(dr^2/f(r) + f(r)dt_E^2 + d\vec{x}^2)$
   - Gauge field $\hat{A}_t(r)$ now a nontrivial function
   - Resulting (black hole) entropy, pressure, and density are all $O(N_c^2)$

Also, two possibilities for the tachyon

A. Zero tachyon $\tau = 0$: chirally symmetric
B. Nonzero tachyon $\tau \neq 0$, $\langle\bar{\psi}\psi\rangle \neq 0$: chirally broken

All four configurations ($1A$, $1B$, $2A$, $2B$) exist!

- However, often only confined chirally broken ($1B$) and deconfined chirally symmetric ($2A$) appear in phase diagram
- Fit to lattice data in the deconfined phase ($2A$)

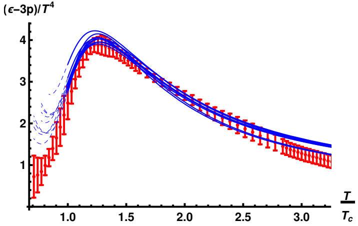
Constraining the flavors at $\mu \approx 0$

Stiff fit to lattice data near $\mu = 0$ (several parameters, but results insensitive to them) [MJ, Jokela, Remes, arXiv:1809.07770]

- Many of the parameters already fixed by requiring qualitative agreement with QCD
- Good description of lattice data — nontrivial result!

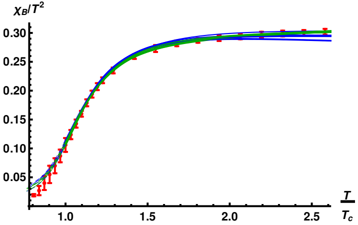
Interaction measure, 2+1 flavors

Lattice data: Borsanyi et al. arXiv:1309.5258

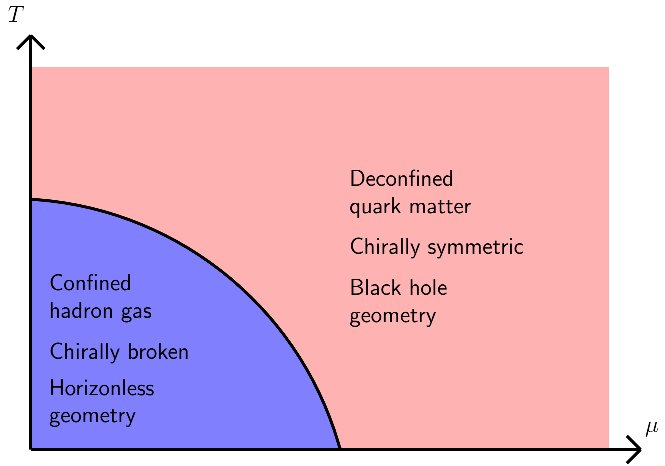


Baryon number susceptibility

Lattice data: Borsanyi et al. arXiv:1112.4416



Holographic phase diagram (V-QCD)



- Only two of the four possible phases appear: confinement triggers chiral symmetry breaking
- Critical chemical potential reasonable!
- Note: 1st order transition natural due to large N_c limit

A final comment: CS terms

Any complete holographic model of QCD must also contain Chern–Simons (CS) terms — this is required by anomalies of QCD

Example: the five form term, which reads for $X = 0$ (chirally symmetric case)

$$S_{CS} = \frac{iN_c}{24\pi^2} \int \mathrm{Tr}\Big[- iA_L \wedge F_L \wedge F_L + \frac{1}{2} A_L \wedge A_L \wedge A_L \wedge F_L + \\ + \frac{i}{10} A_L \wedge A_L \wedge A_L \wedge A_L \wedge A_L - (L \leftrightarrow R) \Big]$$

- Reproduce global chiral anomalies of QCD

Relevant for

- Baryon solutions (instantons in gravity)
- Modulated instabilities

Summary

- I gave a (biased) review of gauge/gravity duality and various holographic approaches to QCD
- Covered some models (both top-down and bottom-up) but ignored many others suggested in the extensive literature ...
- I presented only basics for each model, extensions are found in the literature

Main points of the second lecture:

- Witten–Sakai–Sugimoto model is a successful top-down model: it implements a geometric picture of confinement, chiral symmetry, and its breaking in QCD
- IHQCD gives an excellent simultaneous description of glueball spectra and Yang–Mills thermodynamics

Literature

See https://www.stringwiki.org/wiki/The_AdS/CFT_correspondence for list of reviews on AdS/CFT

1. *Introduction to the AdS/CFT correspondence*, Alfonso V. Ramallo, Springer Proc.Phys. 161 (2015) 411-474, e-Print: 1310.4319 [hep-th]
2. *The Witten-Sakai-Sugimoto model: A brief review and some recent results*, Anton Rebhan, EPJ Web Conf. 95 (2015) 02005, e-Print: 1410.8858 [hep-th]
3. *Improved Holographic QCD*, Umut Gursoy, Elias Kiritsis, Liuba Mazzanti, Georgios Michalogiorgakis, Francesco Nitti, Lect.Notes Phys. 828 (2011) 79-146, e-Print: 1006.5461 [hep-th]
4. *Improved Holographic QCD and the Quark-gluon Plasma*, Umut Gursoy, Acta Phys.Polon.B 47 (2016) 2509, e-Print: 1612.00899 [hep-th]
5. *Holographic modeling of nuclear matter and neutron stars*, Matti Järvinen, Eur.Phys.J.C 82 (2022) 4, 282, e-Print: 2110.08281 [hep-ph]

Thank you for your attention!