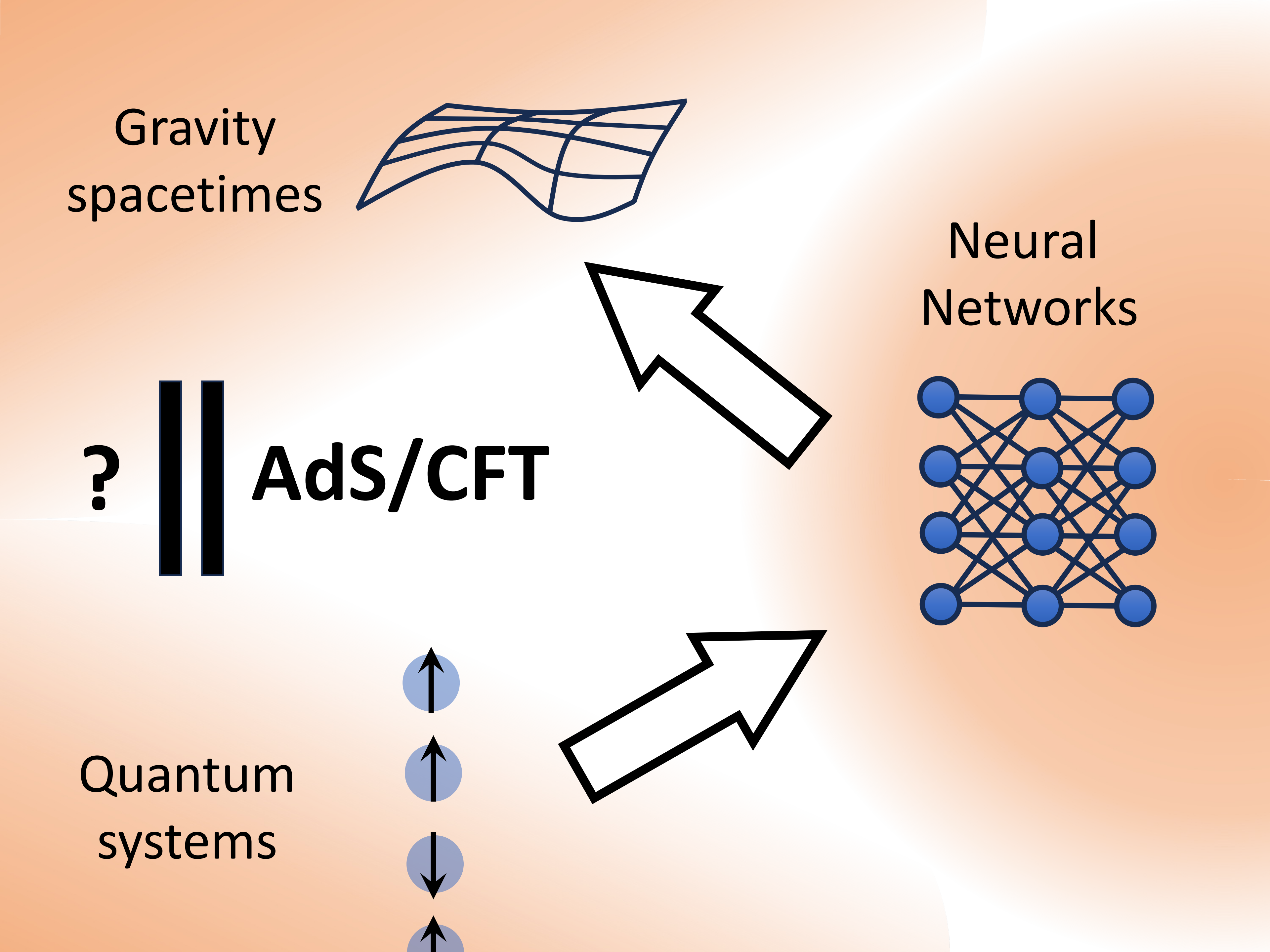
Invited lectures at “Holographic applications: from Quantum Realms to the Big Bang”, 12 Jul. 2025

Machine Learning Holographic QCD

Koji Hashimoto (Kyoto U, MLPhys)

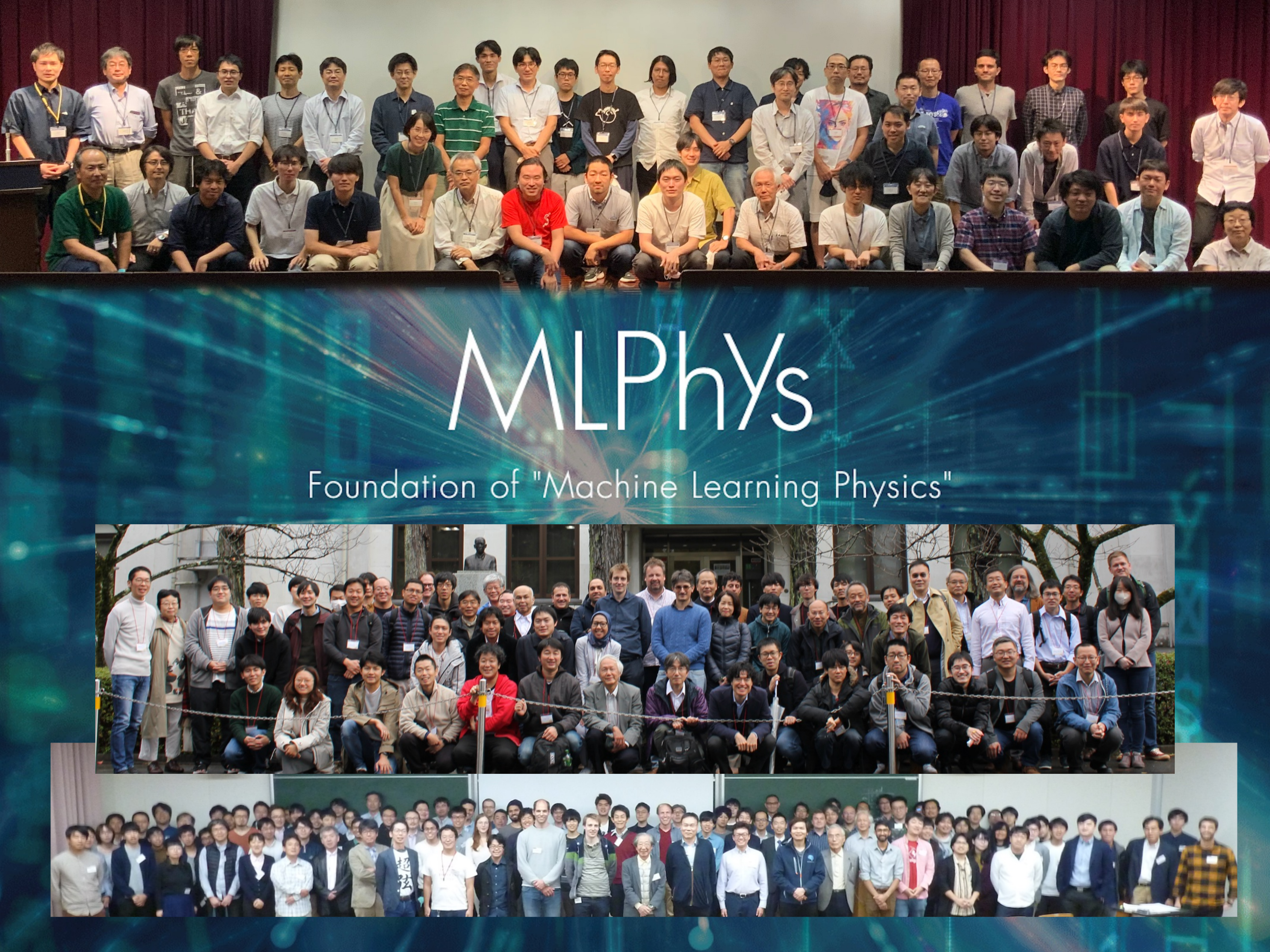

Gravity
spacetimes
Neural
Networks
?
AdS/CFT
Quantum
systems

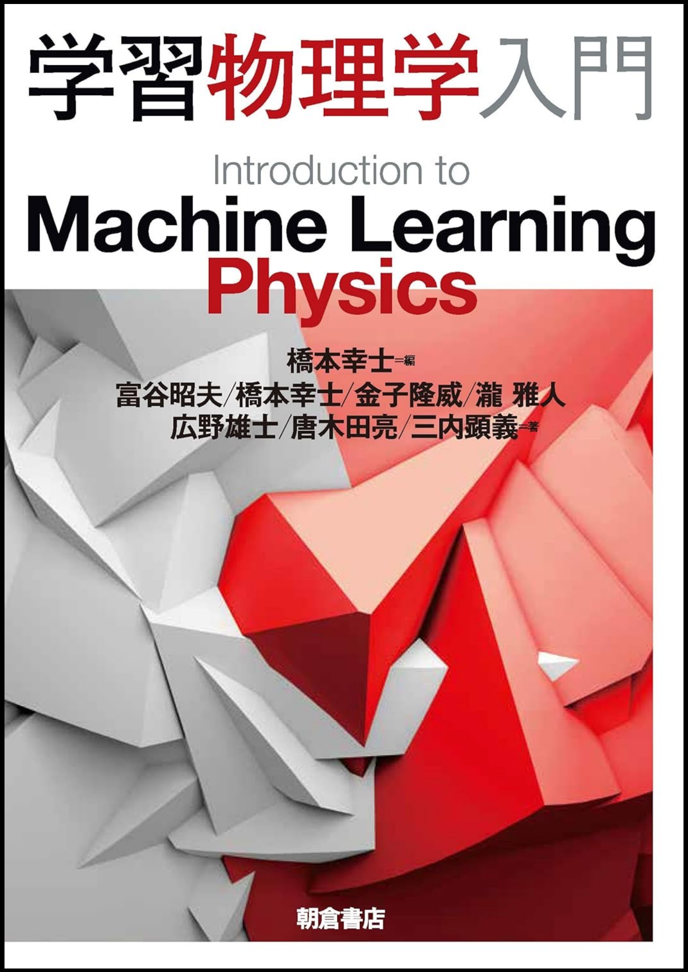
1. AI and physics.

1-1. Recent results in MLPhys.

1-2. How NNs work.

2. AI-QCD.


MLPhYs
Foundation of "Machine Learning Physics"



【目次】

0. イントロダクション

A 機械学習と物理学
- A1. 線形モデル
- A2. ニューラルネットワーク(NN)
- A3. 対称性と機械学習:畳み込み・同変 NN
- A4. 古典力学と機械学習: NN と微分方程式
- A5. 量子力学と機械学習: NN 波動関数

B 機械学習模型と物理学
- B1. トランスフォーマー
- B2. 拡散モデルと経路積分
- B3. 機械学習の仕組み: 統計力学的アプローチ
- B4. 大規模言語モデルと科学

学習物理学解説bot

学習物理学解説bot

AKIYOSHI SANNAI が作成

学習物理学（教科書）の解説を行います。

この本の概要を教えてください

対称性を持つニューラルネットワークとは

"PINNs"とは何か教えてください

拡散モデルと経路積分について

学習物理学解説bot にメッセージを送信する

ChatGPT の回答は必ずしも正しいとは限りません。重要な情報は確認するようにしてください。

MLPhys in the last year.

AI Phys

AI comp. phys. : Spin MC by transformer

AI particle phys. : Jet recogn. by transformer

AI cond-mat phys. : Wave fn. of superconductors

AI molc. dyn. : Siml. of quasicrystals

Phys AI

Stat-mech AI： Wetting transition in deep NN

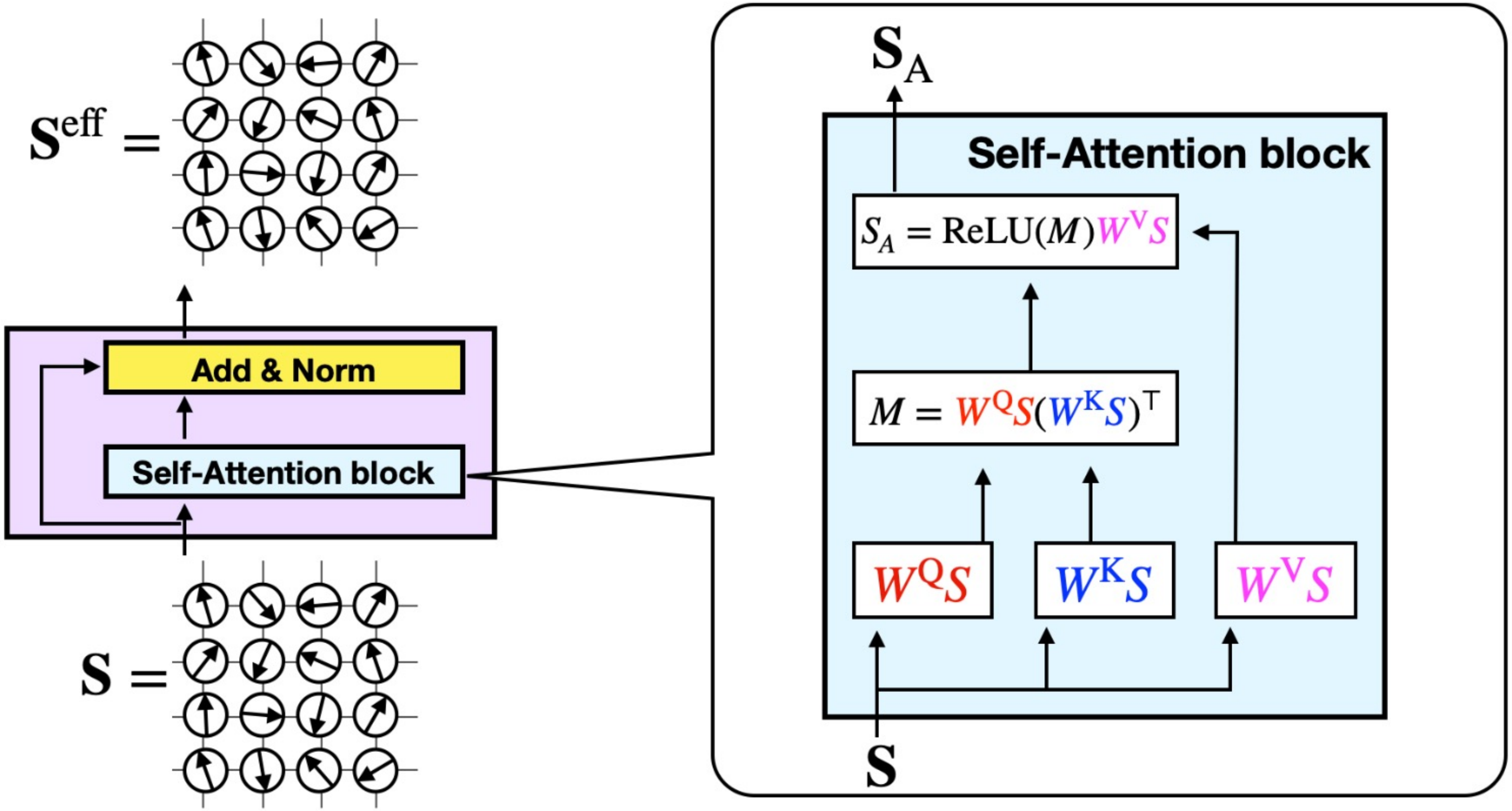
Path-integral AI： Difusion models via QM

Gravity AI： Symmetry in neurons

AI comp. phys. : Spin MC by transformer

[Tomiya Nagai *PoS* LATTICE2023 (2024) 001]

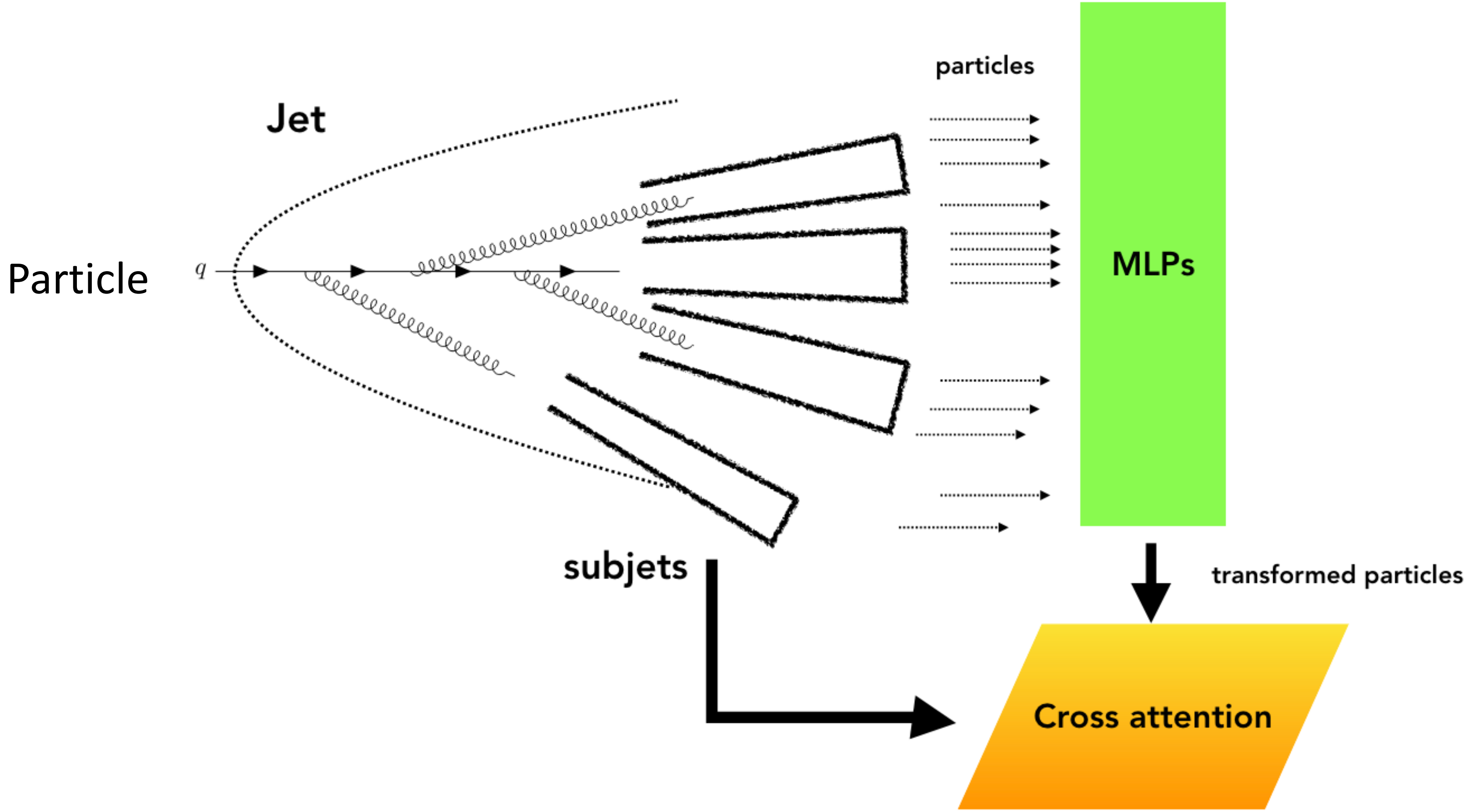
Attention mechanism
describes nonlocality



AI particle phys. : Jet recogn. by transformer

[Hammad, Nojiri, JHEP 06(2024)176, JHEP 03 (2024) 114]

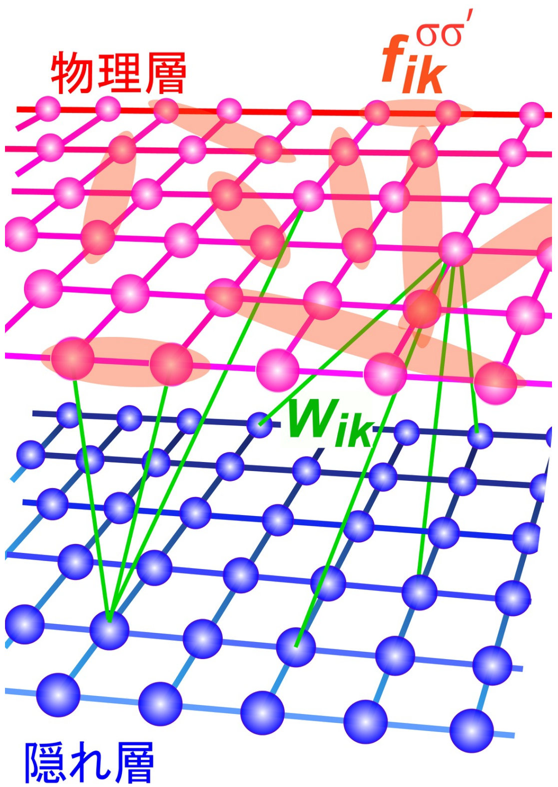
Cross-attention learns
Physical energy scales

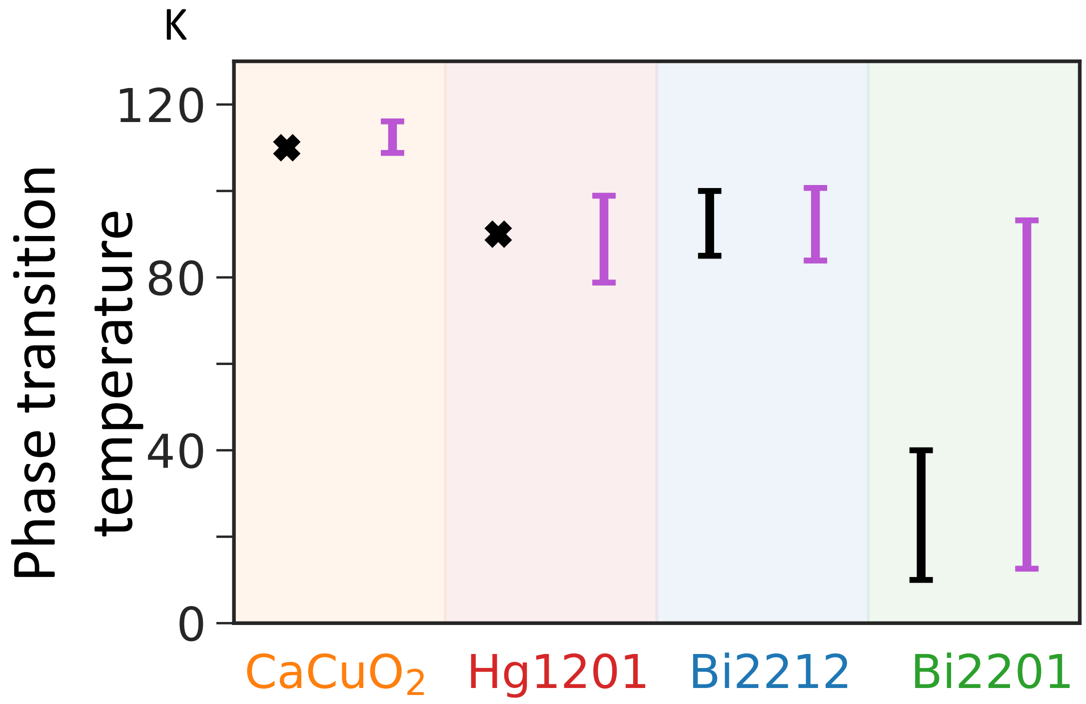


AI cond-mat phys. : Wave fn. of superconductors

Quantum many-body wave functions by Boltzmann machine

[Schmid Morée Kaneko Yamaji Imada, Phys. Rev. X 13, 041036 (2023)]





Exp. vs. AI

AI molc. dyn. : Siml. of quasicrystals

[Nagai et al., Phys.Rev. Lett. 102, 041124 (2024)]

Higher-dim. structure
learned by Machine

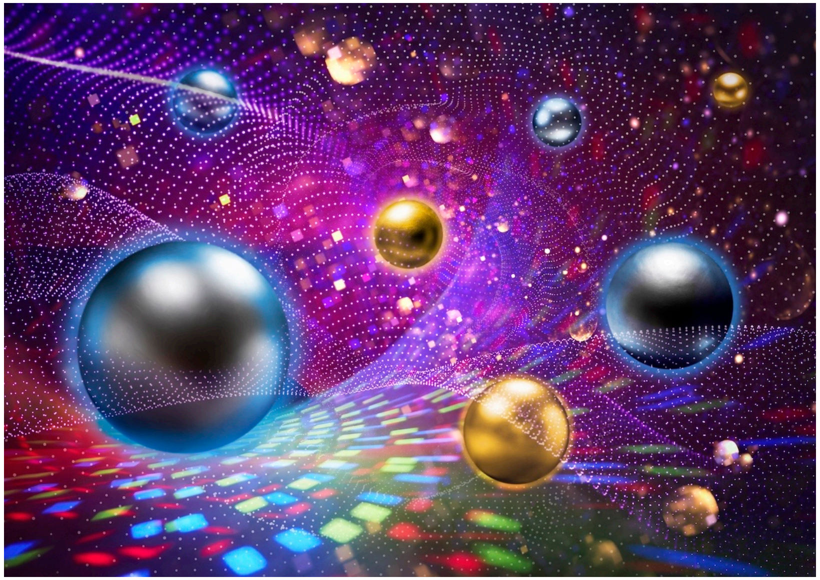
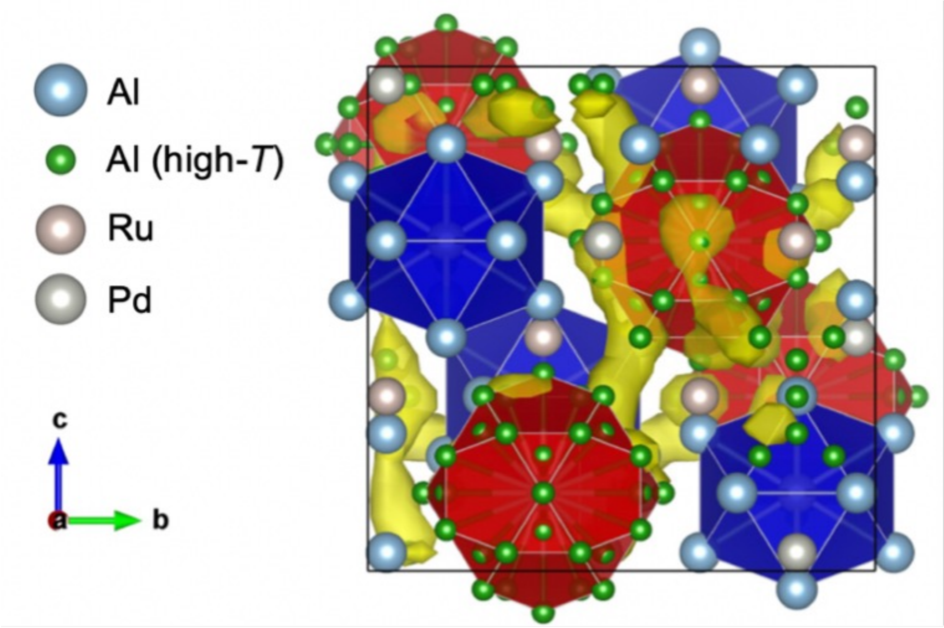


Image of higher-dimensional atoms
affecting the 3-dim. world
Credit: UTokyo ITC/Shinichiro Kinoshita

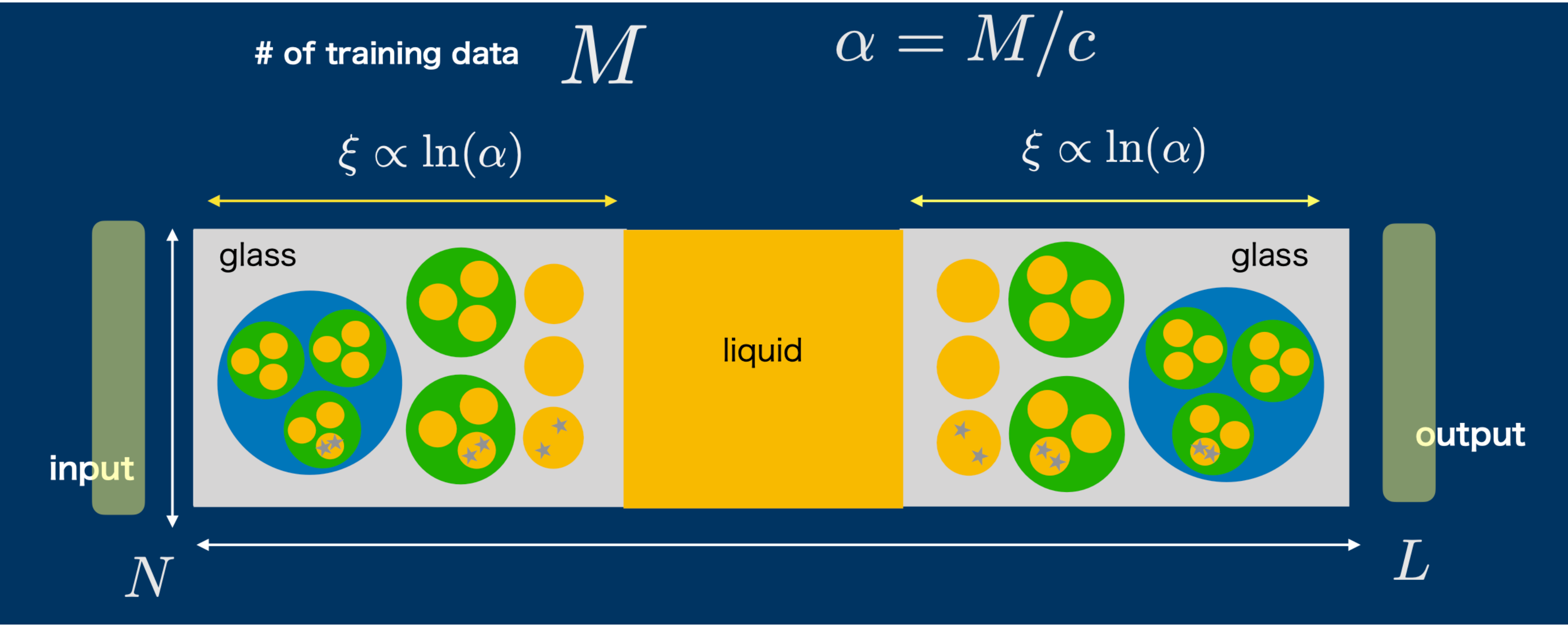


Alminium atoms floating through
high-dim. paths

Stat-mech AI：Wetting transition in deep NN

Inside deep NN is liquid

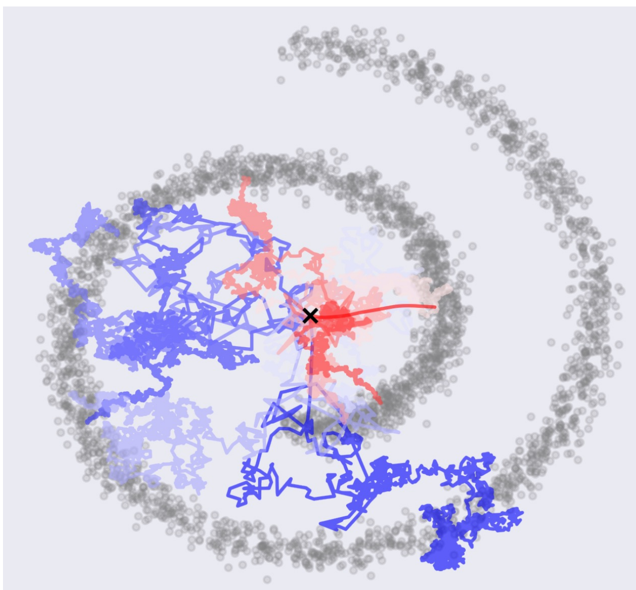
[Yoshino, Phys. Rev. Research, 5, 033068 (2023), SciPostPhys. Core 2, 005 (2020).]

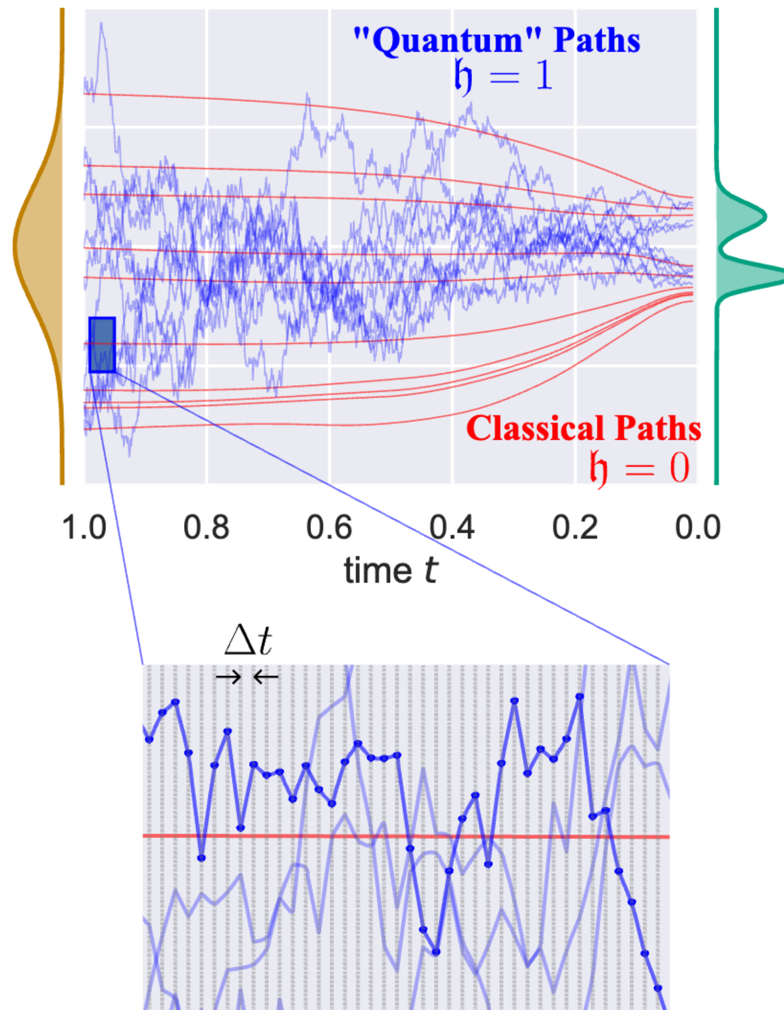


Path-integral AI：Difusion models via QM

[Hirono Tanaka Fukushima ICML 2024, arXiv:2403.11262]

Diffusion model for generative AI is rewritten by path-integral

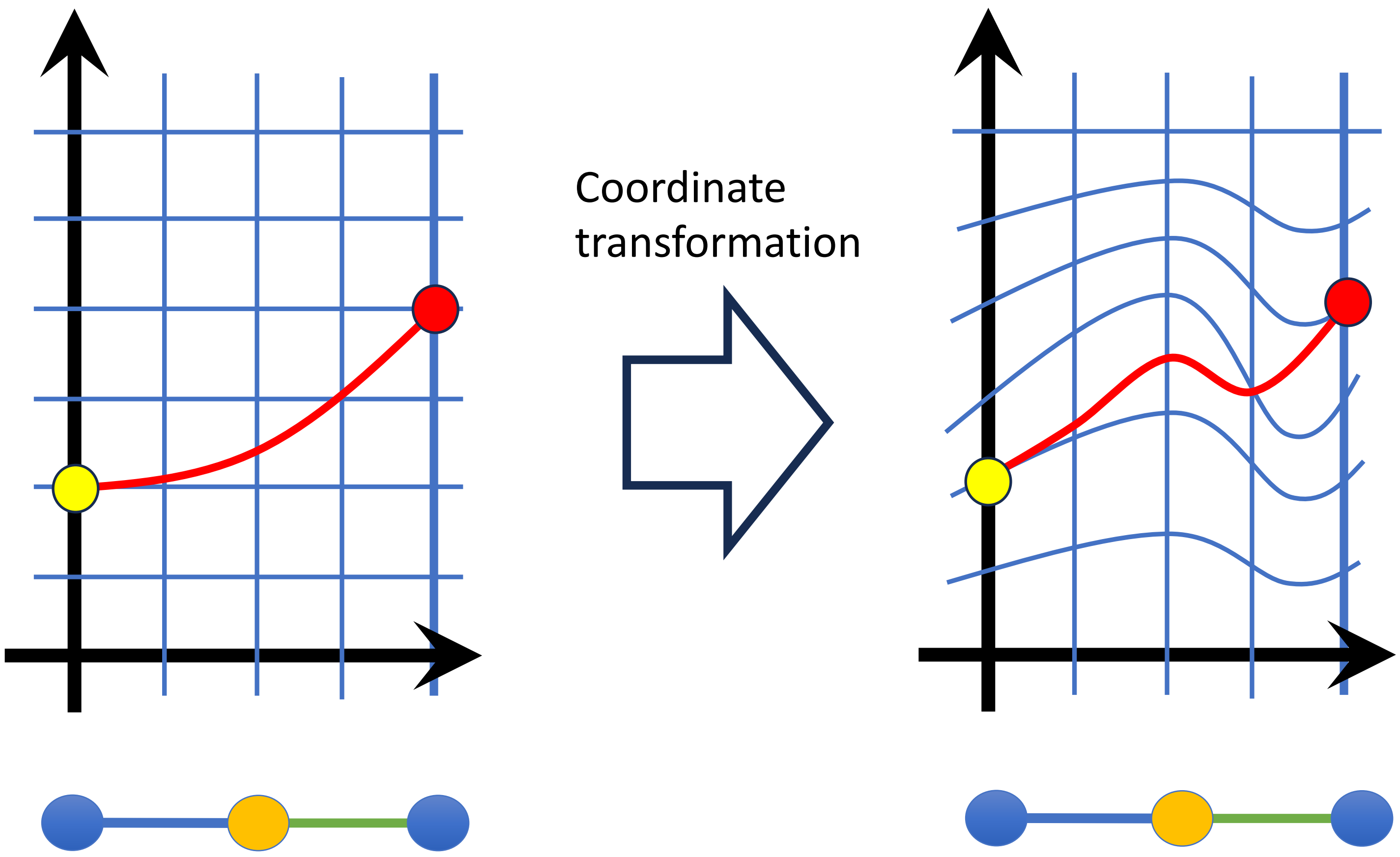




Gravity AI：Symmetry in neurons

[Hashimoto, Hirono, Sannai, Mach. Learn.: Sci. Technol. 5 025079 (2024)]

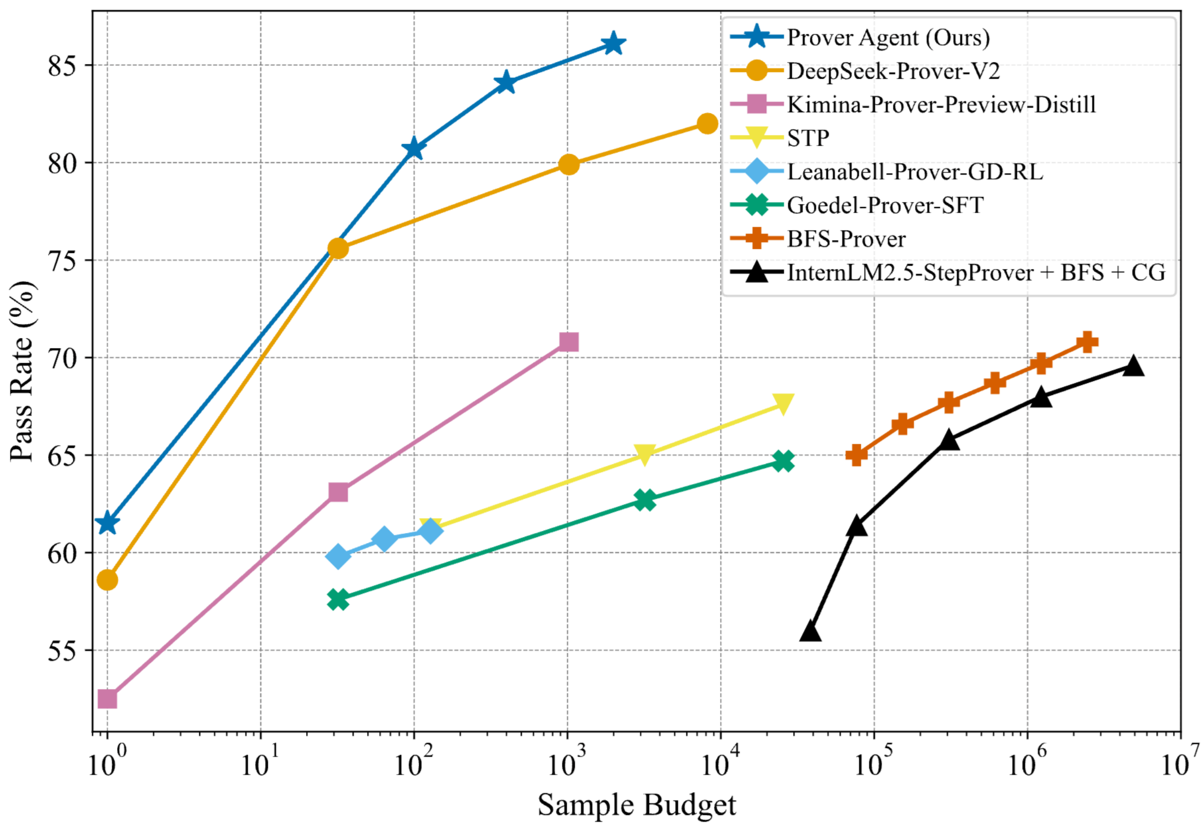
Gravitational symmetry
in transformers and neural ODEs



AI prover agent for auto-math

"Prover Agent: An Agent-based Framework for Formal Mathematical Proofs"
K. Baba, C. Liu, S. Kurita, A. Sannai, ArXiv:2506.19923

86% success rate in miniF2F benchmark (world record)

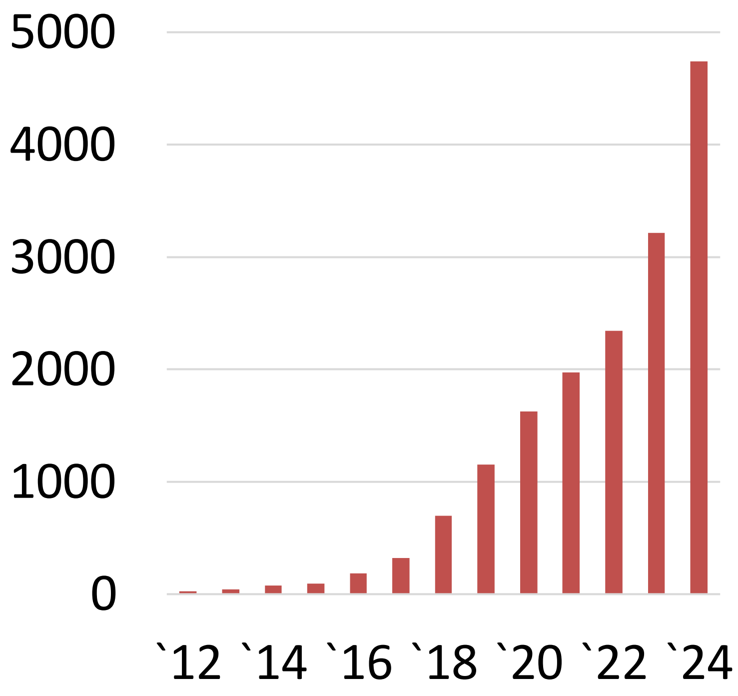


Expanding research field in the world

arXiv papers of ML+Phys

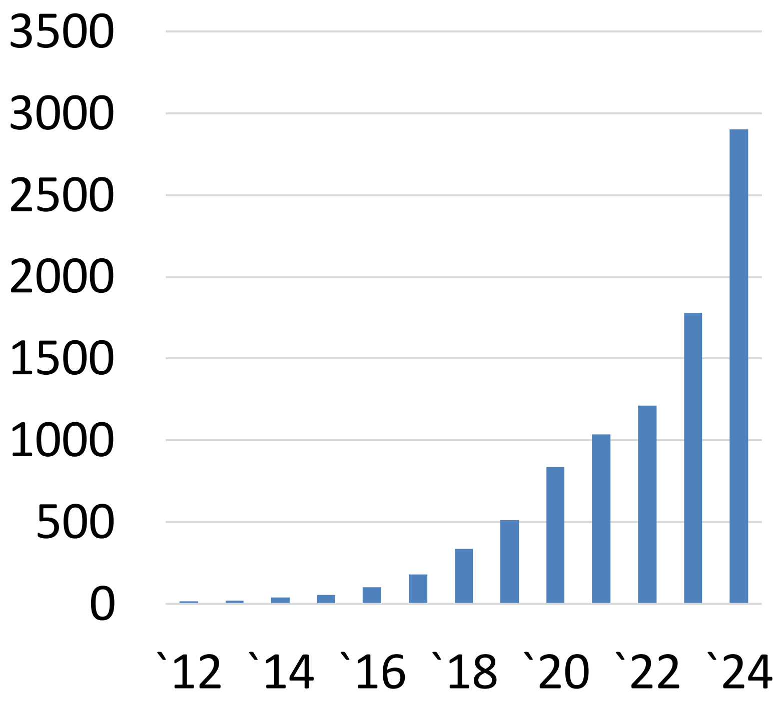
Phys category

(abstract includes "machine (deep) learning")



CS category

(abstract includes "physics" and "learning")



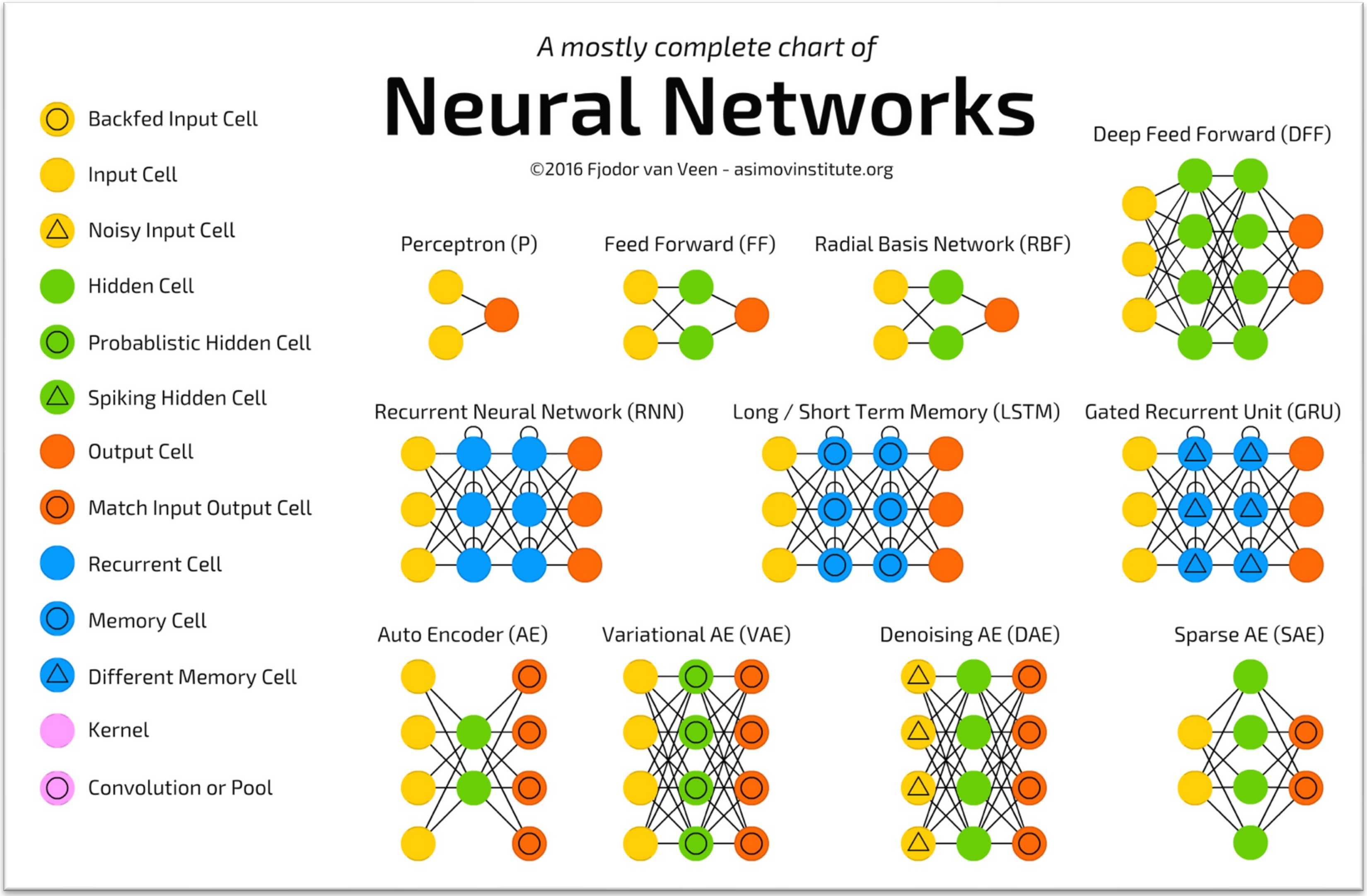
1. AI and physics.

1-1. Recent results in MLPhys.

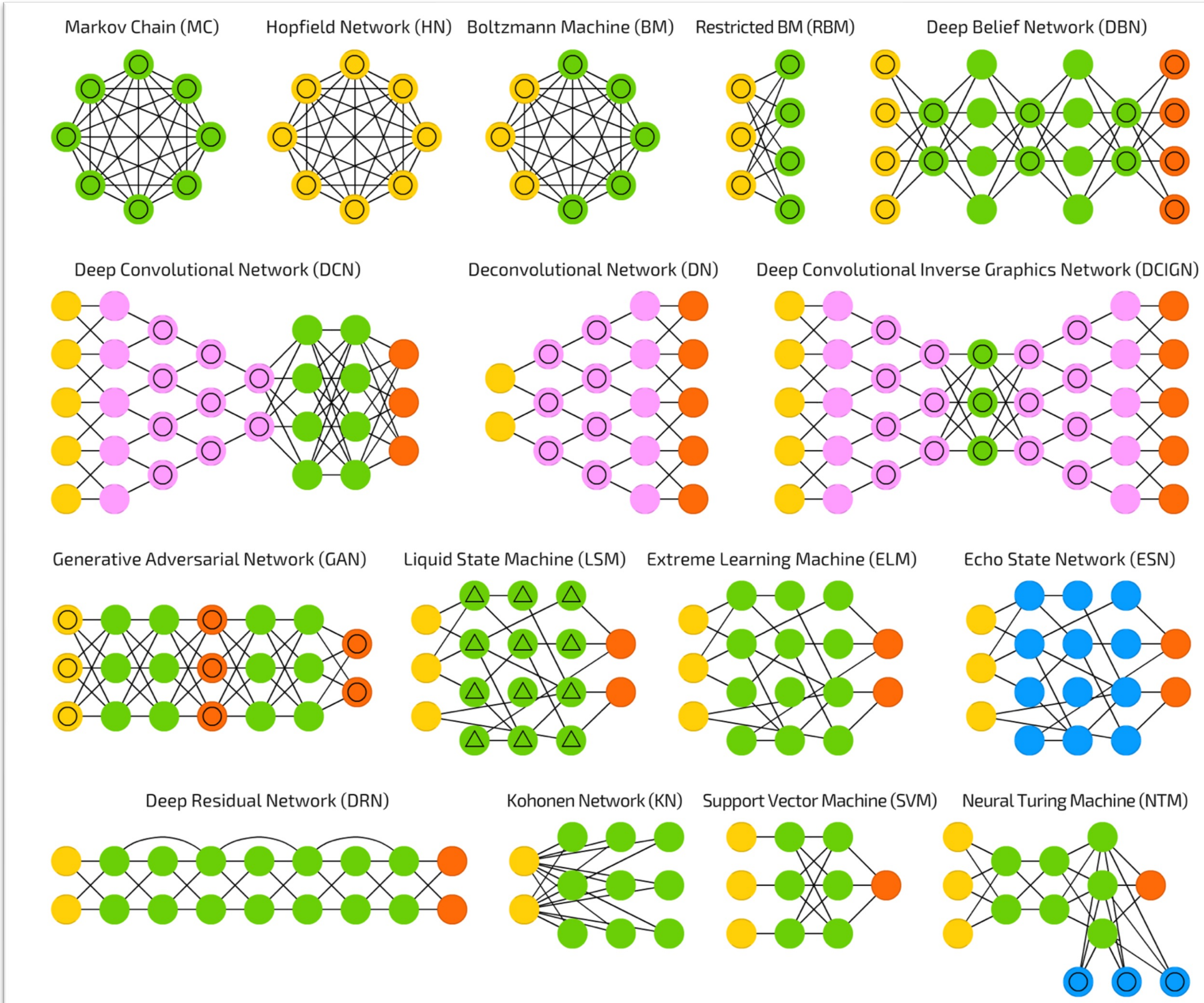
1-2. How NNs work.

2. AI-QCD.

Machine learning = Optimizing networks



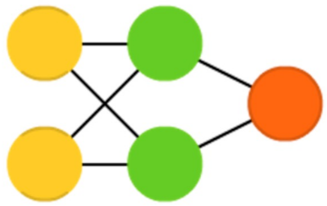
http://www.asimovinstitute.org/neural-network-zoo/


Markov Chain (MC)
Hopfield Network (HN)
Boltzmann Machine (BM)
Restricted BM (RBM)
Deep Belief Network (DBN)
Deep Convolutional Network (DCN)
Deconvolutional Network (DN)
Deep Convolutional Inverse Graphics Network (DCIGN)
Generative Adversarial Network (GAN)
Liquid State Machine (LSM)
Extreme Learning Machine (ELM)
Echo State Network (ESN)
Deep Residual Network (DRN)
Kohonen Network (KN)
Support Vector Machine (SVM)
Neural Turing Machine (NTM)

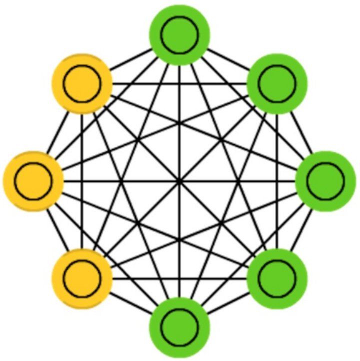
Machine learning = function approximator

Input: a vector $(v_1, v_2, v_3, \ldots)$
Output: a value $f(v_1, v_2, v_3, \ldots)$

Network architecture = Function ansatz



Perceptron model
[Rosenblatt 1958]
[Rumelhart, McClelland 1986]



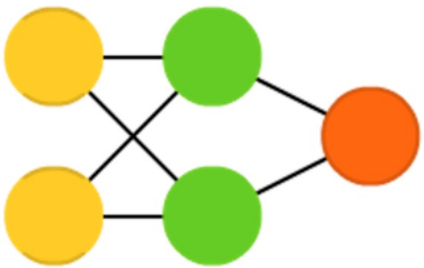
Boltzmann machine
[Ackley, Hinton, Sejnowski 1985]

Universal approximation theorem :

Any function can be approximated with more hidden units [Cybenko 1989] [Roux, Bengio 2008]

Neural network for classification

Perceptron model



$$f = W_i^{(2)} \varphi \left(W_{ij}^{(1)} x_j \right)$$

"Unit" (circles) : Vector components

"Weight" (lines) : Linear transformation to be optimized

"Activation function" (hidden line-end) :
Nonlinear component-wise transf.

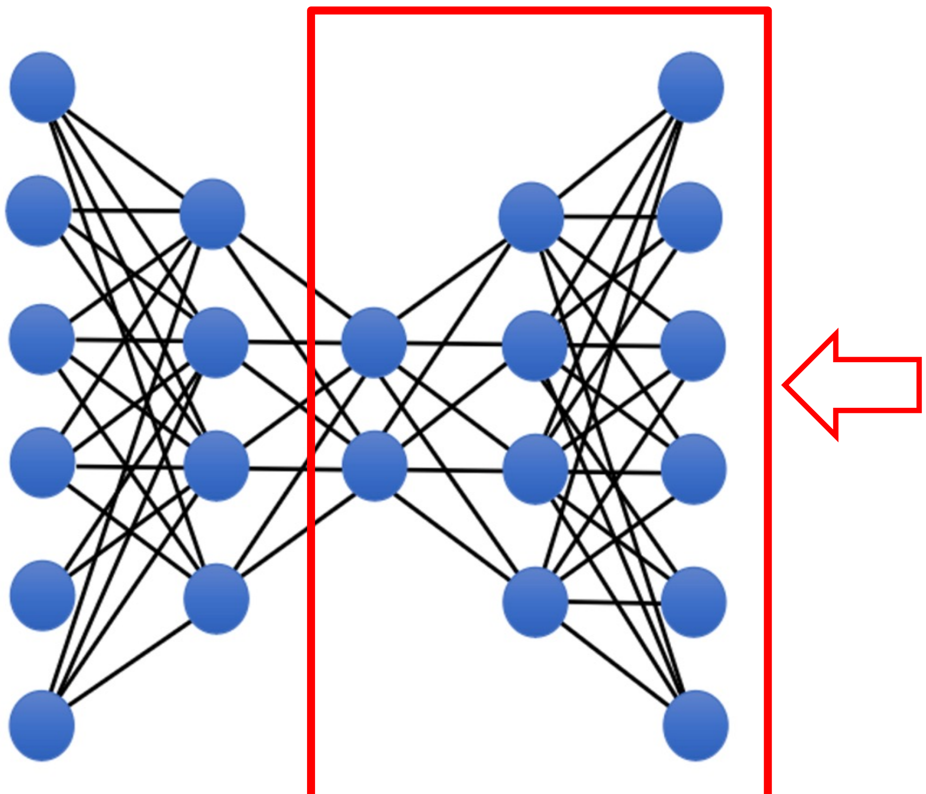
$$\varphi(x) \equiv \frac{1}{1+e^{-x}}$$

- Training protocol :

1) Prepare many sets $\{(x_j, f)\}$: input + output
2) Train the network (adjust W) by lowering "Loss function" $E \equiv \sum_{\text{data}} \left| f - W_i^{(2)} \varphi \left(W_{ij}^{(1)} x_j \right) \right|$

A path to generative AI

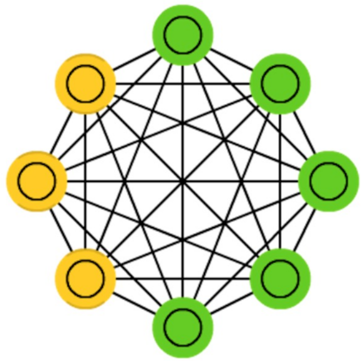
NN finds features among complicated phenomena.



VAE (Variational autoencoder)

Neural network for probability distribution

Boltzmann machine



"Unit" (circles) : Spins
"Visible units" (yellow)
"Hidden units" (green)

"Weight" (lines) : Spin-spin coupling
to be optimized

$$P(v_i) = \sum_{h_i \in \{0,1\}} \exp\left[-\mathcal{E}(v_i, h_i)\right] \qquad \mathcal{E}(v_i, h_i) \equiv \sum_{ij} w_{ij} v_i h_j$$

- Training protocol :

1) Prepare many sets $\{(v_i, P_{\mathrm{ex}}(v_i))\}$: input + output
2) Train the network (adjust W) by lowering "Loss function" $E \equiv D_{\mathrm{KL}}\left(P_{\mathrm{ex}}(v_i) \,||\, P(v_i)\right)$

AdS/CFT is a Nobel prize

OU-HET-1003

AdS/CFT as a deep Boltzmann machine

Koji Hashimoto[1]

[1] *Department of Physics, Osaka University, Toyonaka, Osaka 560-0043, Japan*

(Dated: March 13, 2019)

We provide a deep Boltzmann machine (DBM) for the AdS/CFT correspondence. Under the philosophy that the bulk spacetime is a neural network, we give a dictionary between those, and obtain a restricted DBM as a discretized bulk scalar field theory in curved geometries. The probability distribution as training data is the generating functional of the boundary quantum field theory, and it trains neural network weights which are the metric of the bulk geometry. The deepest layer implements black hole horizons, and an employed regularization for the weights is an Einstein action. A large N_c limit in holography reduces the DBM to a folded feed-forward architecture. We also neurally implement holographic renormalization into an autoencoder. The DBM for the AdS/CFT may serve as a platform for studying mechanisms of spacetime emergence in holography.

04951v1 [hep-th] 12 Mar 2019

I. INTRODUCTION

Deep Boltzmann machines [1] are a particular type of neural networks in deep learning [2–4] for modeling probabilistic distribution of data sets. They are equipped with deep layers of units in their neural network architecture, and are a generalization of Boltzmann machines [5] which are one of the fundamental models of neural networks. Deepening the architecture enlarges the representation power of the models, and recent advances in training deep models in machine learning were initiated by analogues of the deep Boltzmann machines.

The neural network of a deep Boltzmann machine consists of visible units and hidden units. On those units binary variables live, and they interact with each other under a Hamiltonian called an energy function. Thus basically the deep Boltzmann machine is an Ising model in which spins only at a boundary layer are visible (observable), and the Hamiltonian allows inhomogeneity and nonlocality. For a given probability distribution of the

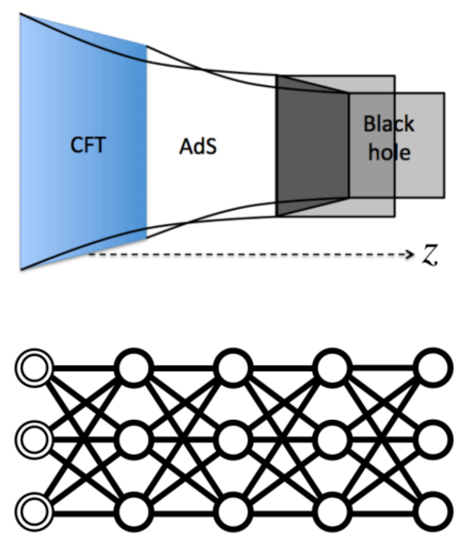


FIG. 1: Top: the AdS/CFT correspondence. The horizontal

Combinatrics (1): Research fields.

MLPhys is a math penetrating various physics fields.

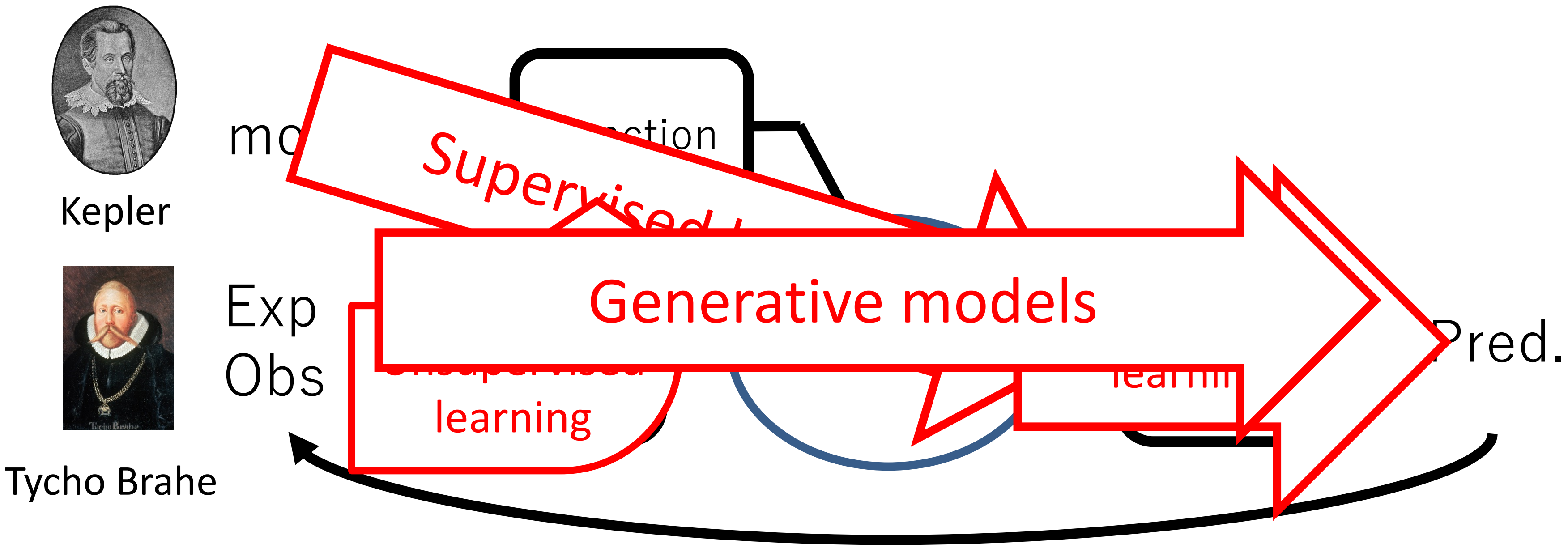
Comp. phys.	Quantum path-integrate!
Particle phys.	Find theories!
Nuclear phys.	Precision calculus! Quantum many body!
Cosmology, astro.	Describe phenomena and evolution!
Cond-mat phys.	Find emergent phenomena and effective DoF!

Maths penetrate.

Ex) Topology.	**2016 Nobel prize**
Ex) Quantum info.	**2022 Nobel prize.**
Ex) ML.	**2024 Nobel prize.**

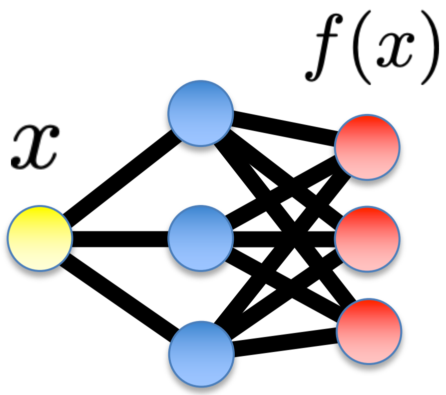
Combinatrics (2): Methods.

MLPhys accelerates scientific processes.



Combinatrics (3): Observables

MLPhys adopts new func. basis for inverse problems.



Input	Output	Loss function	Physics
t	$x(t)$	$(F(x) - m\ddot{x})^2$	Classical mech.
x	$\psi(x)$	$\langle\psi\|\mathcal{H}\|\psi\rangle$	Quantum mech.
x^μ	$\phi(x^\mu)$	$S[\phi]$	Classical FT.
s_i	$\mathcal{H}(s_i)$	(data - VEV)^2	Quantum inverse problem
$x(t_0)$	$x(t_1)$	(data - pred)^2	Classical inverse problem

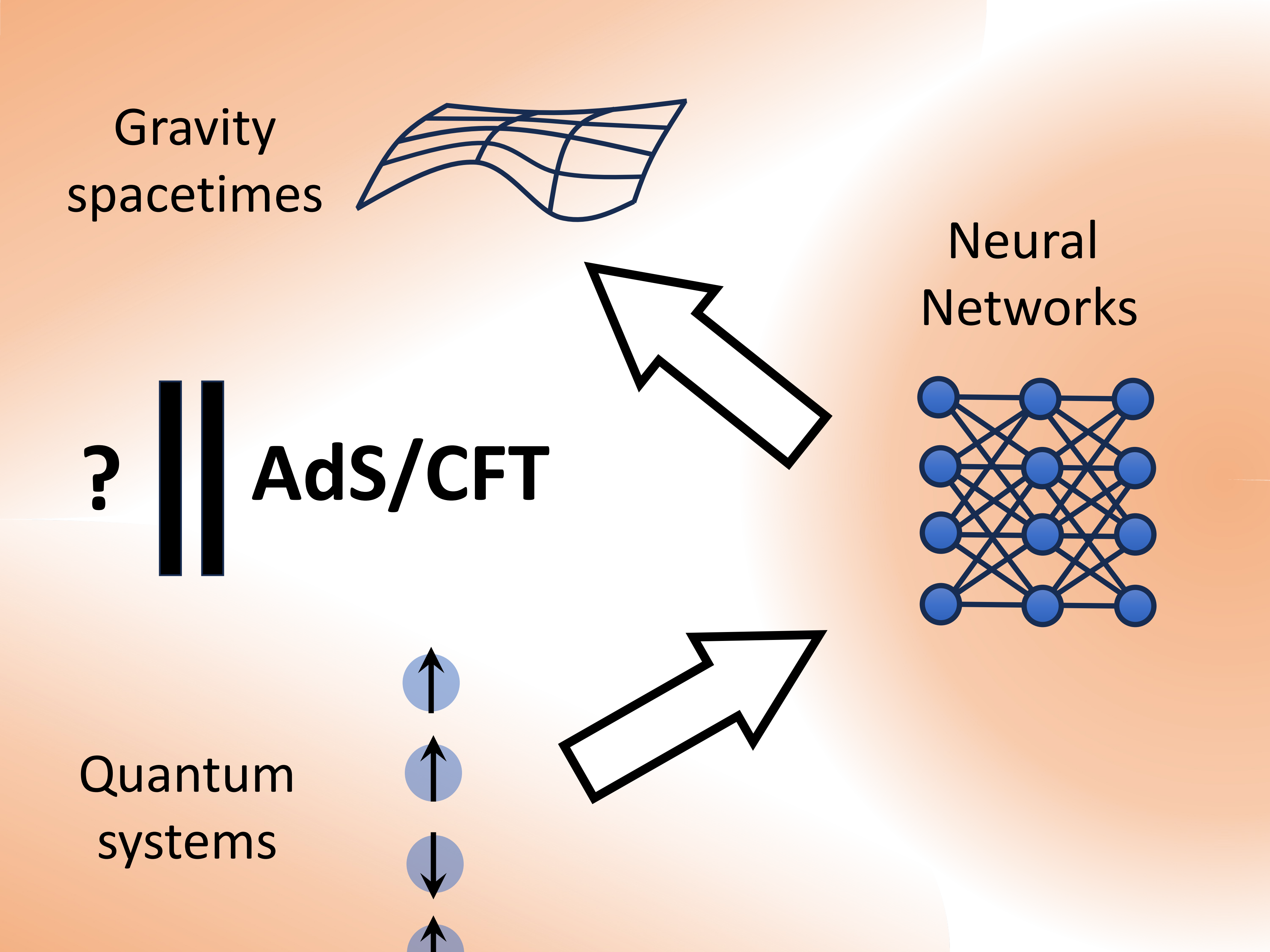
Genetarive Model (LLM)

1. AI and physics.

1-1. Recent results in MLPhys.

1-2. How NNs work.

2. AI-QCD.


Gravity
spacetimes
Neural
Networks
? = AdS/CFT
Quantum
systems

Our group papers include

“Machine-learning emergent spacetime from linear response in future tabletop quantum gravity experiments” 2411.16052 w/ K.Matsuo, M. Murata, G.Ogiwara (Saitama Tech), D.Takeda (Kyoto)
“Spacetime-emergent ring toward tabletop quantum gravity experiments” 2211.13863 w/ D.Takeda, K.Tanaka, S.Yonezawa (Kyoto)
“Deriving dilaton potential in improved holographic QCD from chiral condensate” 2209.04638 w/ K.Ohashi (Keio), T.Sumimoto (Osaka u)
“Deriving dilaton potential in improved holographic QCD from meson spectrum” 2108.08091 w/ K.Ohashi (Keio), T.Sumimoto (Osaka u)
“Neural ODE and Holographic QCD” 2006.00712 w/ H.Y.Hu, Y.Z.You (UCSD)
“Deep Learning and AdS/QCD” 2005.02636 w/ T. Akutagawa, T. Sumimoto (Osaka u)
“Deep Boltzmann Machine and AdS/CFT” 1903.04951
“Deep Learning and Holographic QCD” 1809.10536 w/ S. Sugishita (Kentucky), A. Tanaka, A. Tomiya (RIKEN)
“Deep Learning and AdS/CFT” 1802.08313 w/ S. Sugishita (Kentucky), A. Tanaka, A. Tomiya (RIKEN)

AdS/DL method explored by many friends:

Yi-Zhuang You, Zhao Yang, and Xiao-Liang Qi. Physical Review B, 97(4):045153, 2018.
Romain Vasseur, Andrew C Potter, Yi-Zhuang You, and Andreas WW Ludwig. Physical Review B, 100(13):134203, 2019.
Jing Tan and Chong-Bin Chen. International Journal of Modern Physics D, 28(12):1950153, 2019.
Hong-Ye Hu, Shuo-Hui Li, Lei Wang, and Yi-Zhuang You. Physical Review Research, 2(2):023369, 2020.
Yu-Kun Yan, Shao-Feng Wu, Xian-Hui Ge, and Yu Tian. Physical Review D, 102(10):101902, 2020.
Jonathan Lam and Yi-Zhuang You. Physical Review Research, 3(4):043199, 2021.
Mugeon Song, Maverick SH Oh, Yongjun Ahn, and Keun-Young Kim. Chinese Physics C, 45(7):073111, 2021.
Emad Yaraie, Hossein Ghaffarnejad, and Mohammad Farsam. Iranian Journal of Astrophysics and Astronomy, 10(4):335, 2023.
Kai Li, Yi Ling, Peng Liu, and Meng-He Wu. Physical Review D, 107(6):066021, 2023.
Byoungjoon Ahn, Hyun-Sik Jeong, Keun-Young Kim, and Kwan Yun. Journal of High Energy Physics, 2024(3):1–30, 2024.
Mahdi Mansouri, Kazem Bitaghsir Fadafan, and Xun Chen. arXiv preprint arXiv:2406.06285.
Rong-Gen Cai, Song He, Li Li, Hong-An Zeng. arXiv preprint arXiv:2406.12772.

And many more…

Other AI bulk reconstruction includes:

Xun Chen and Mei Huang. Physical Review D, 109(5):L051902, 2024.
Ou-Yang Luo, Xun Chen, Fu-Peng Li, Xiao-Hua Li, and Kai Zhou. arXiv preprint arXiv:2408.03784.
Xun Chen and Mei Huang. arXiv preprint arXiv:2405.06179.
Byoungjoon Ahn, Hyun-Sik Jeong, Keun-Young Kim, and Kwan Yun. arXiv preprint arXiv:2406.07395.
Zhuo-Fan Gu, Yu-Kun Yan, and Shao-Feng Wu. arXiv preprint arXiv:2401.09946.
Yago Bea, Raul Jimenez, David Mateos, Shuheng Liu, Pavlos Protopapas, Pedro Taranć oń Alvarez, and Pablo Tejerina-Ṕ erez. arXiv preprint arXiv:2403.14763.

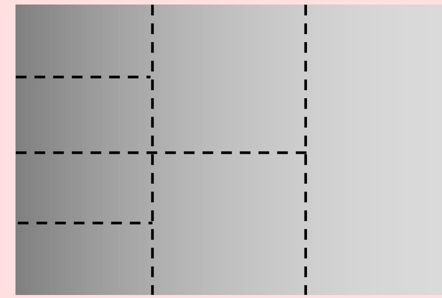
And many more…

Roadmap

Quantum gravity in (d+1)-dim.

General spacetime

Anti de Sitter spacetime



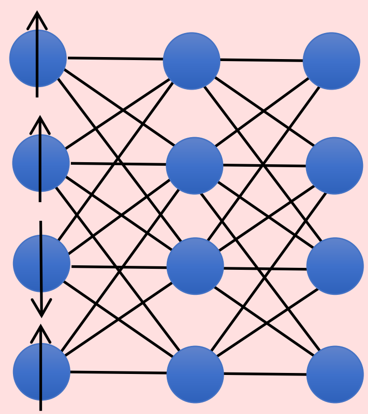
'tHooft `93
Susskind `94
Maldacena `97

||

|| ?

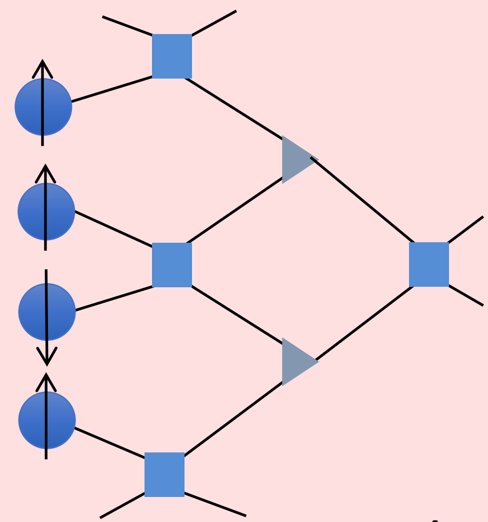
|| Swingle `10

Quantum mechanics in d-dim.



Neural network

Carleo, Troyer `17



Tensor network

Roadmap

1.

Quantum gravity in (d+1)-dim.

'tHooft `93
Susskind `94
Maldacena `97

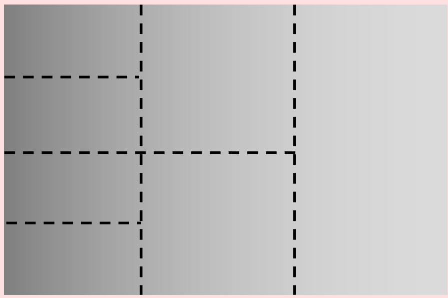
||

Quantum mechanics in d-dim.

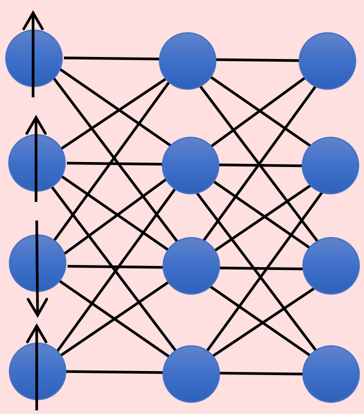
General spacetime

|| ?

Anti de Sitter spacetime

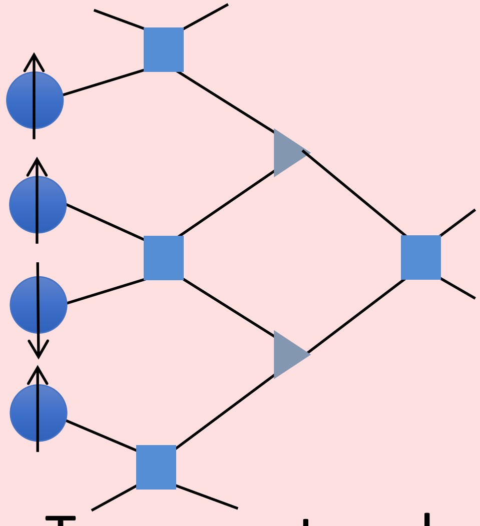


|| Swingle `10



Neural network

Carleo, Troyer `17

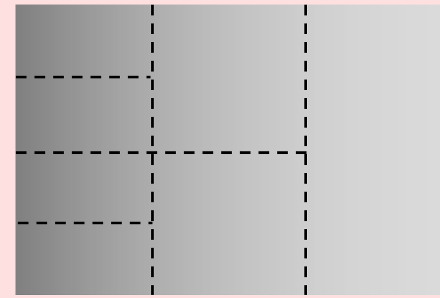


Tensor network

Roadmap

General spacetime

Anti de Sitter spacetime



Quantum gravity in (d+1)-dim.

'tHooft `93
Susskind `94
Maldacena `97

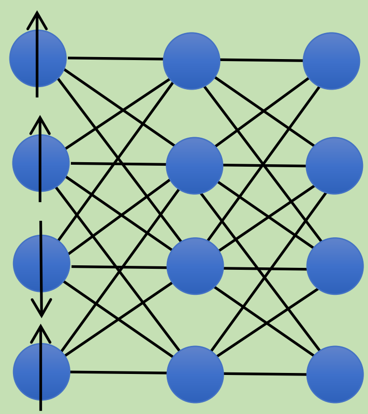
||

?

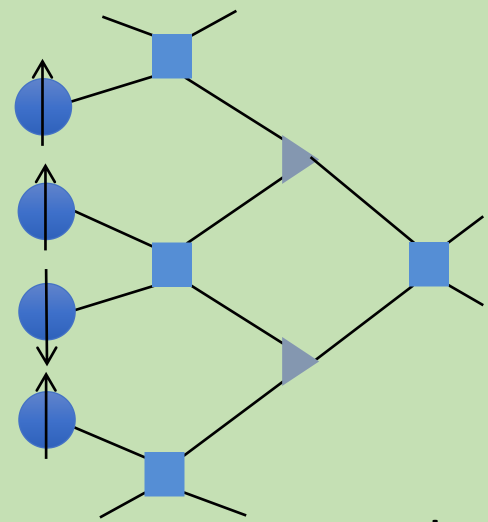
Swingle `10

Quantum mechanics in d-dim.

2.



Carleo, Troyer `17



Neural network

Tensor network

Roadmap

Quantum gravity in (d+1)-dim.

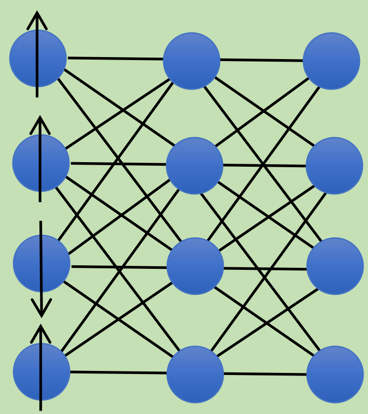
'tHooft `93
Susskind `94
Maldacena `97

Quantum mechanics in d-dim.

3.

General spacetime

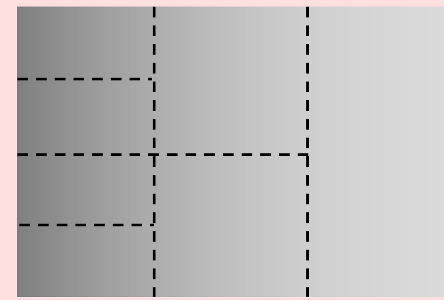
?



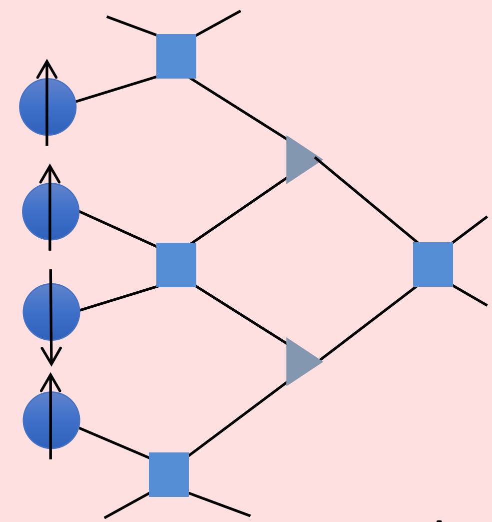
Neural network

Carleo, Troyer `17

Anti de Sitter spacetime

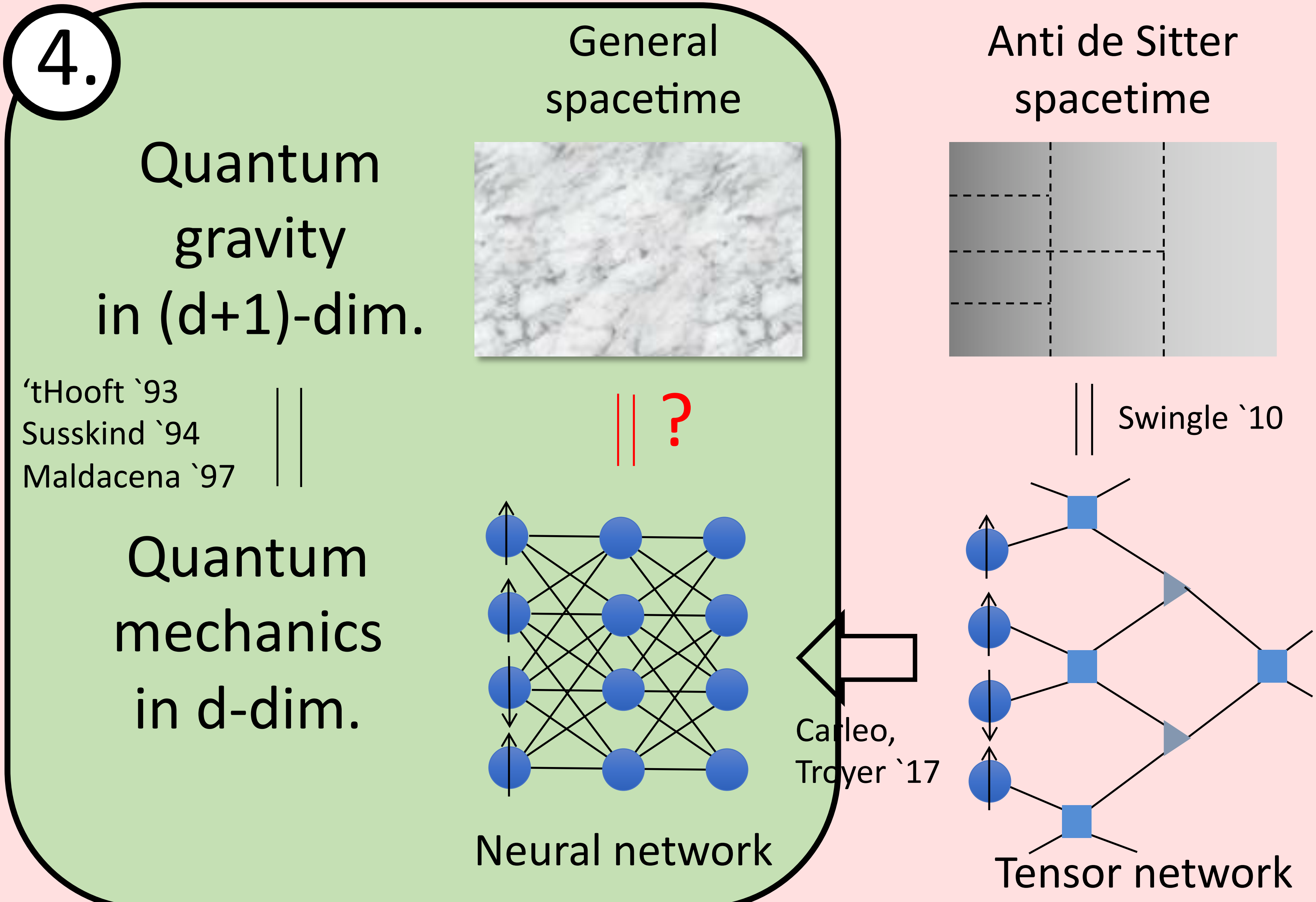


Swingle `10



Tensor network

Roadmap


4.
Quantum gravity in (d+1)-dim.
General spacetime
'tHooft `93
Susskind `94
Maldacena `97
?
Quantum mechanics in d-dim.
Neural network
Anti de Sitter spacetime
Swingle `10
Carleo, Troyer `17
Tensor network

Machine Learning Holographic QCD

1. Quantum gravity — 4 pages
2. Neural network quantum states — 6 pages
3. When is NN a spacetime? — 5 pages
4. Spacetime emergent from data — 15 pages

Discussion: Quantum gravity $\subset$ ML ?

Roadmap

1.

Quantum gravity in (d+1)-dim.

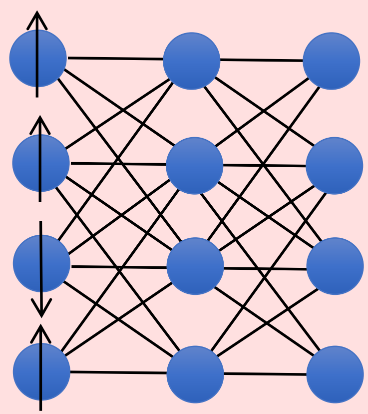
'tHooft `93
Susskind `94
Maldacena `97

||

Quantum mechanics in d-dim.

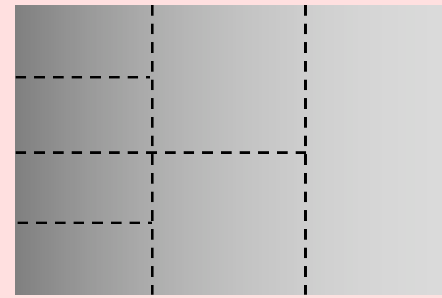
General spacetime

|| ?

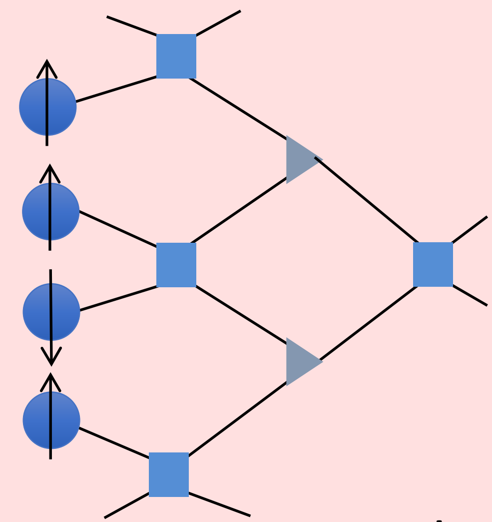


Neural network

Anti de Sitter spacetime



|| Swingle `10



Tensor network

Carleo, Troyer `17

① Quantum Gravity

Brief History of quantum gravity

1974 'tHooft, Veltman:
Perturbation fails in Einstein gravity.

1970 Nambu, Susskind, Nielsen:
String theory of hadrons.

1974 Yoneya, Scherk, Schwarz:
String is quantum gravity.

1971 Bekenstein:
Black hole entropy.

1993 'tHooft, Susskind:
Holographic principle.

1997 Maldacena:
AdS/CFT correspondence.

① Quantum Gravity

AdS/CFT correspondence, no proof

[Maldacena, Adv.Theor.Math.Phys. 2 (1998) 231]

"CFT"		"AdS"
"Large N" Quantum mechanics in d-dim. spacetime	=	Classical ~~Quantum~~ gravity in (d+1)-dim. spacetime

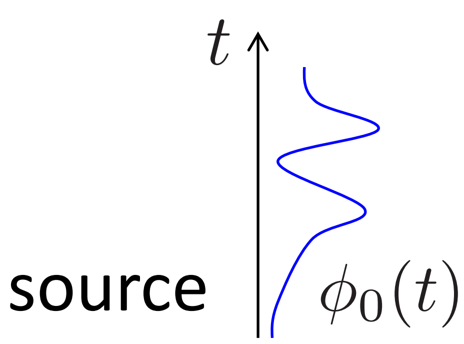
- Vast amount of examples known
- No proof! How does it work?
- Given Left, how can one get Right?

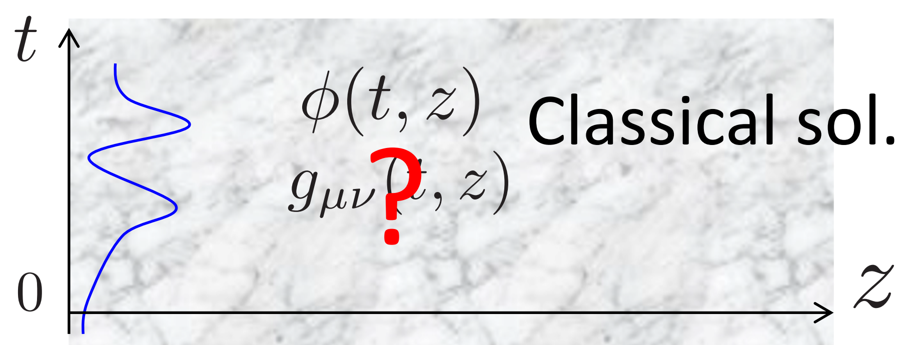
1. Quantum Gravity

Dictionary : equating partition functions

[Gubser, Klebanov, Polyakov, Phys.Lett.B428(1998)105]
[Witten, Adv.Theor.Math.Phys. 2 (1998) 253]

Partition function of Quantum mechanics		Partition function of Classical gravity
$Z[\phi_0]$	$=$	$Z[\phi_0]$
‖		‖
$\int [\mathcal{D}q(t)]e^{-\int dt(\mathcal{L}[q,\dot{q}]+\phi_0(t)\mathcal{O}[q])}$		$e^{-\int dtdz\sqrt{-g}(R[g]+\mathcal{L}[\phi]+\cdots)}$?





$\phi_0(t)=\phi(t,z=0)$

① Quantum Gravity

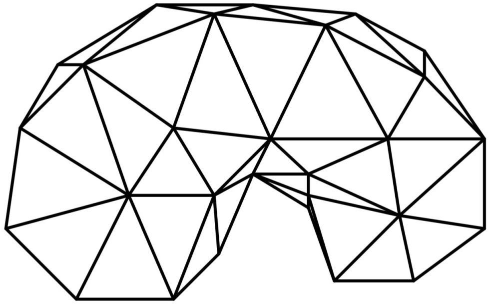
Quantum geometry is a network

Regge calculus

[Regge 1961]

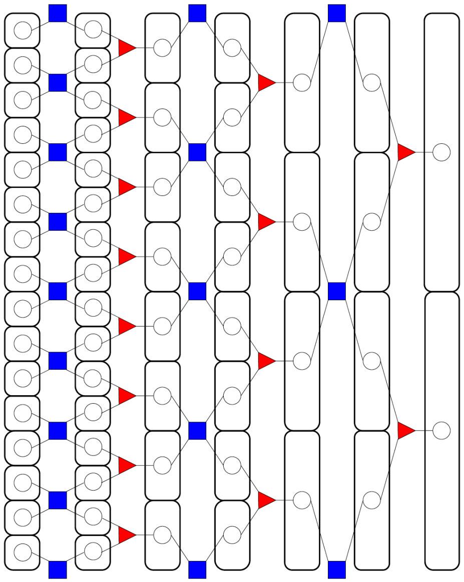
Causal dynamical triangulation

[Ambjorn, Loll 1998]



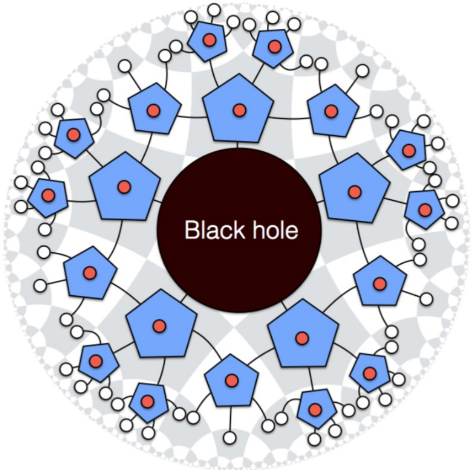
AdS/MERA (Tensor Network)

[Swingle `09]



Quantum codes for holography

[Pastawski, Yoshida, Harlow, Preskill `15]



Roadmap

1.

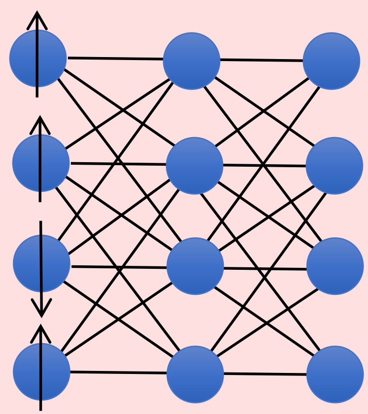
Quantum gravity in (d+1)-dim.

‘tHooft `93
Susskind `94
Maldacena `97

Quantum mechanics in d-dim.

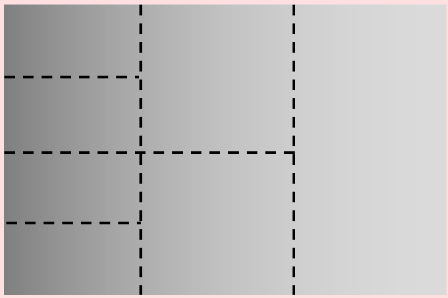
General spacetime

?

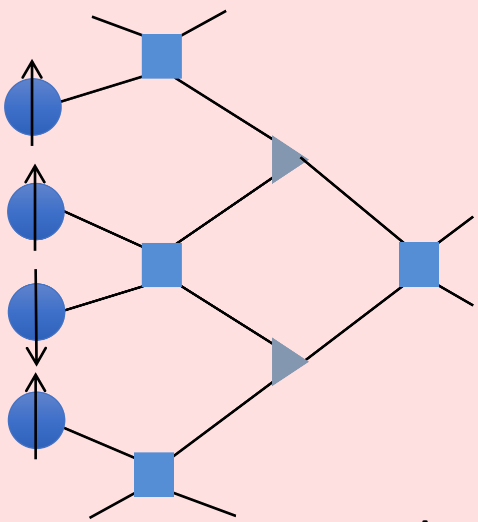


Neural network

Anti de Sitter spacetime



Swingle `10



Carleo, Troyer `17

Tensor network

Machine Learning Holographic QCD

1. Quantum gravity — 4 pages
2. Neural network quantum states — 6 pages
3. When is NN a spacetime? — 5 pages
4. Spacetime emergent from data — 15 pages

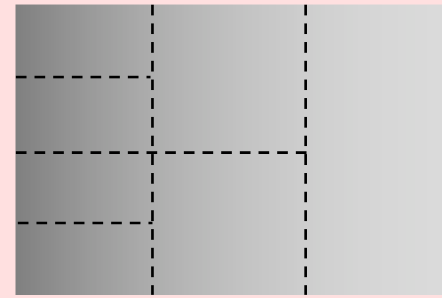
Discussion: Quantum gravity $\subset$ ML ?

Roadmap

Quantum gravity in (d+1)-dim.

General spacetime

Anti de Sitter spacetime

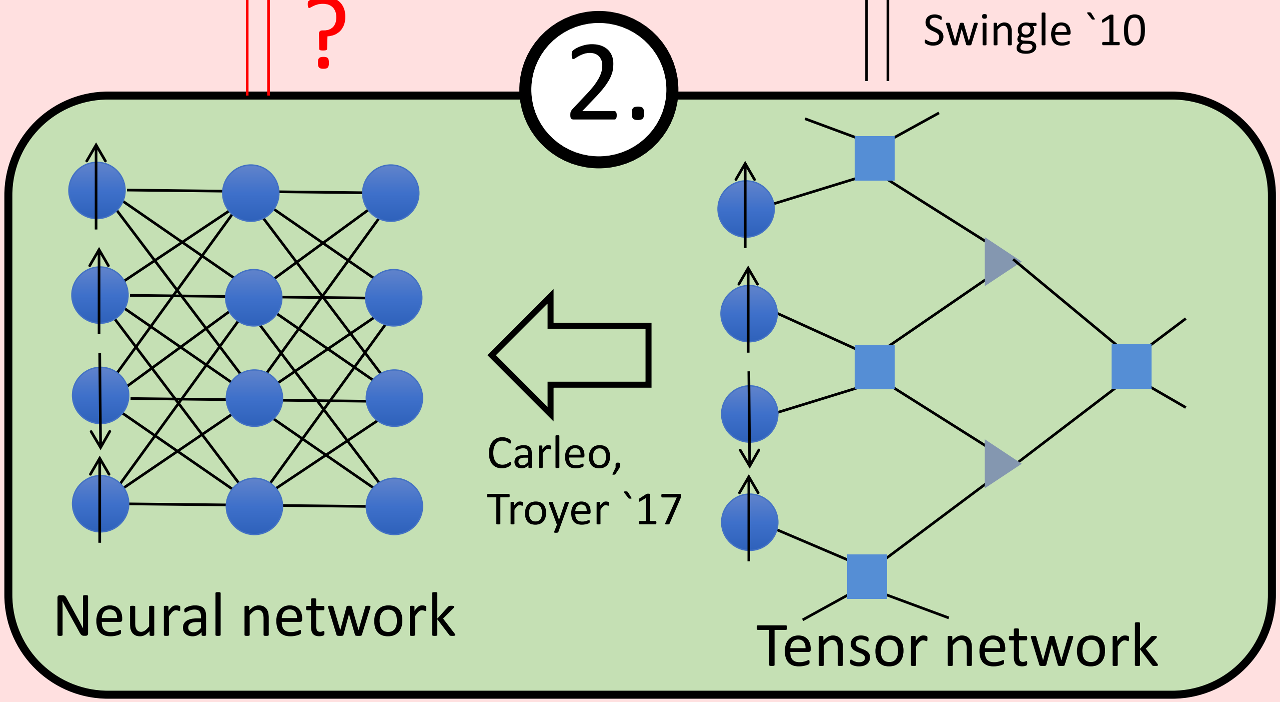


'tHooft `93
Susskind `94
Maldacena `97

?

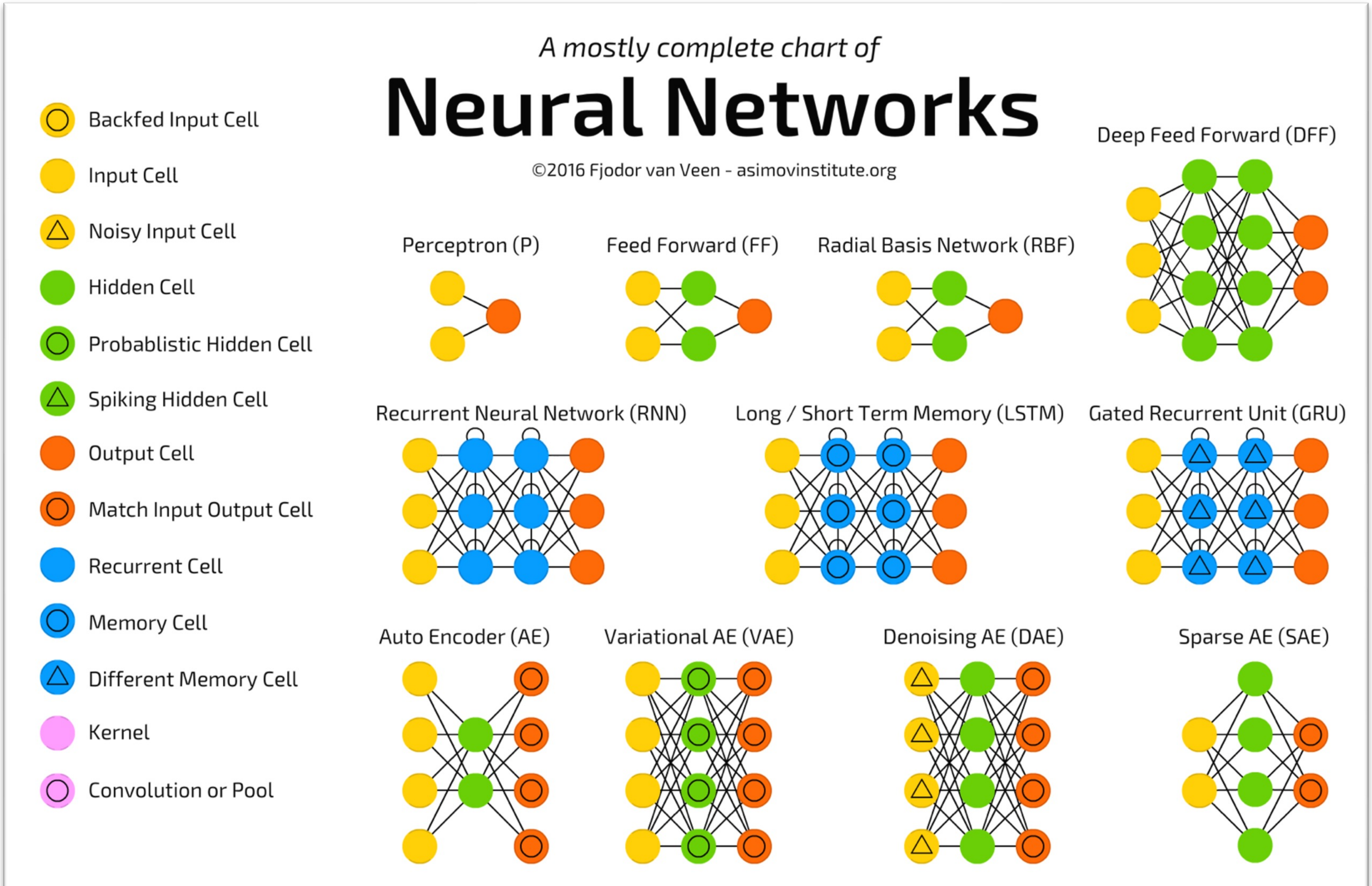
Swingle `10

Quantum mechanics in d-dim.

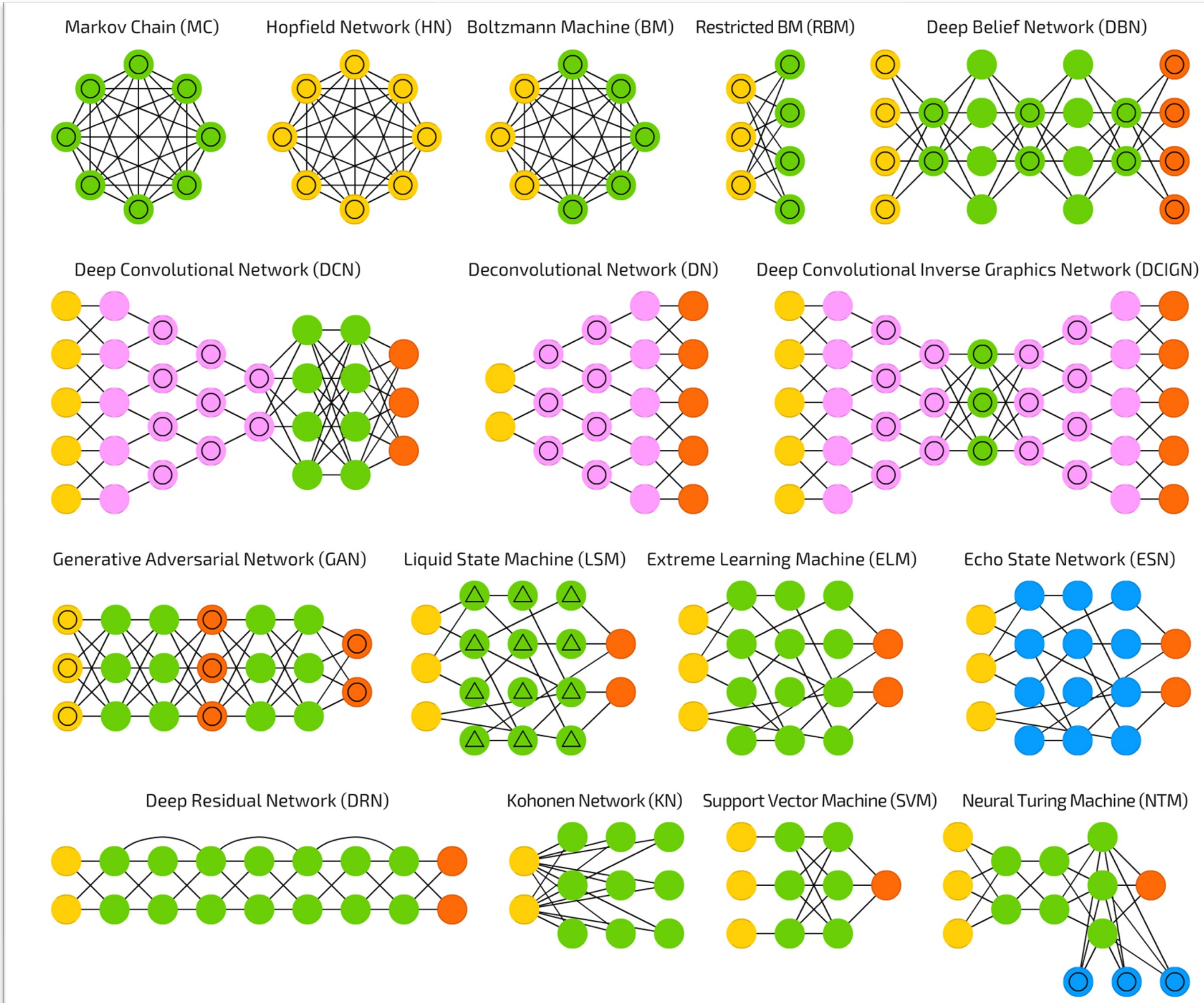


② Neural Network Quantum States

Various neural networks were invented



http://www.asimovinstitute.org/neural-network-zoo/

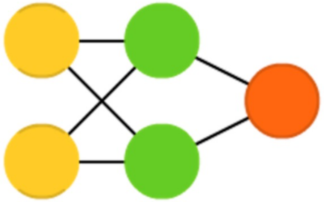

Markov Chain (MC)
Hopfield Network (HN)
Boltzmann Machine (BM)
Restricted BM (RBM)
Deep Belief Network (DBN)
Deep Convolutional Network (DCN)
Deconvolutional Network (DN)
Deep Convolutional Inverse Graphics Network (DCIGN)
Generative Adversarial Network (GAN)
Liquid State Machine (LSM)
Extreme Learning Machine (ELM)
Echo State Network (ESN)
Deep Residual Network (DRN)
Kohonen Network (KN)
Support Vector Machine (SVM)
Neural Turing Machine (NTM)

② Neural Network Quantum States

Machine learning = function approximator

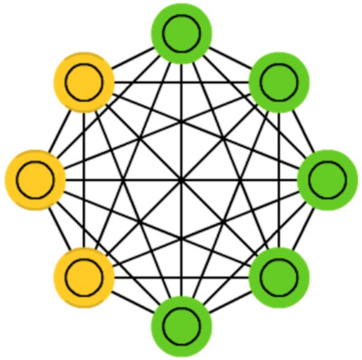
Input: a vector $(x_1, x_2, x_3, \cdots)$
Output: a value $f(x_1, x_2, x_3, \cdots)$

Network architecture is the function ansatz



Perceptron model
[Rosenblatt 1958]
[Rumelhart, McClelland 1986]

$$f = W_i^{(2)} \varphi \left(W_{ij}^{(1)} x_j \right)$$

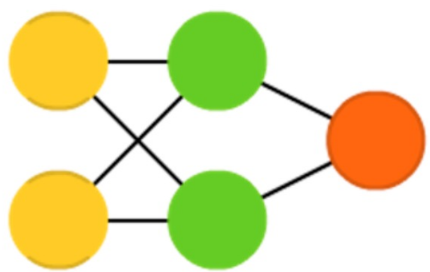


Boltzmann machine
[Ackley, Hinton, Sejnowski 1985]

② Neural Network Quantum States

Neural network for classification

Perceptron model



$$f = W_i^{(2)} \varphi \left(W_{ij}^{(1)} x_j \right)$$

"Unit" (circle) : Vector component

"Weight" (line) : Linear transformation to be optimized

"Activation function" (hidden line-end) : Nonlinear component-wise transf.

$$\varphi(x) \equiv \frac{1}{1+e^{-x}}$$

- Training protocol :

1) Prepare many sets $\{(x_j, f)\}$: (input, output)
2) Train the network (adjust W) by lowering "Loss function" $E \equiv \sum_{\text{data}} \left| f - W_i^{(2)} \varphi \left(W_{ij}^{(1)} x_j \right) \right|$

② Neural Network Quantum States

Find ground state wave function $\psi(s_1, s_2, \cdots, s_N)$

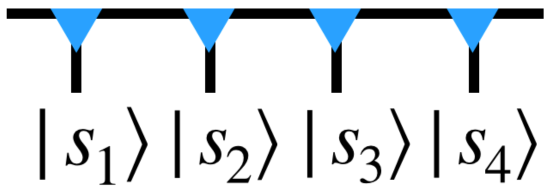
Q : Minimize its energy E for a given Hamiltonian H,

$$E = \frac{\sum_{s_1,\cdots s_N, s'_1,\cdots,s'_N} \psi^\dagger(s'_1,\cdots,s'_N)\hat{H}_{s'_1,\cdots s'_N,s_1,\cdots,s_N}\psi(s_1,\cdots,s_N)}{\sum_{s_1,\cdots s_N}\psi^\dagger(s_1,\cdots,s_N)\psi(s_1,\cdots,s_N)}$$

A : Use ansatz and optimize parameters!

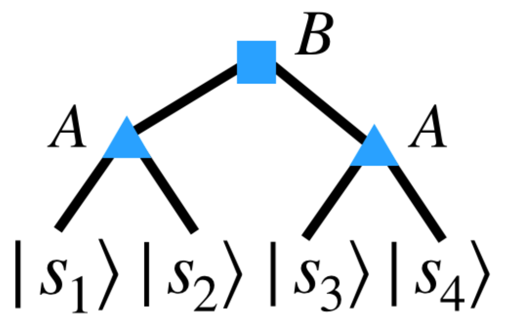
- Matrix product states

$$\psi(s_1, s_2, \cdots) = \mathrm{tr}[A^{(s_1)}A^{(s_2)}\cdots]$$



- Tensor network states

$$\psi(s_1, s_2, \cdots) = \sum_{m,n} B_{mn}A_{ms_1s_2}A_{ns_3s_4}$$

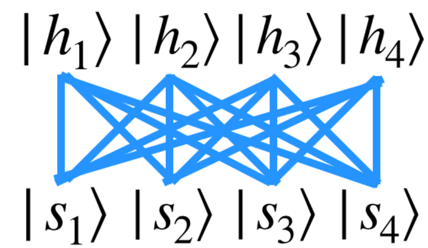


② Neural Network Quantum States

Neural network can be wave functions

- Boltzmann machine states

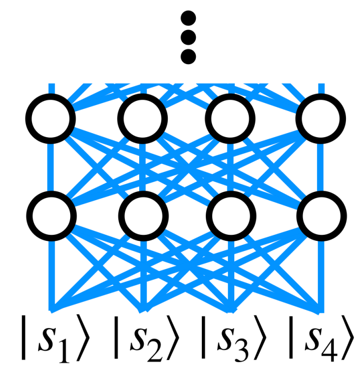
  [Carleo, Troyer `17],
  [Nomura, Darmawan, Yamaji, Imada `17], ..



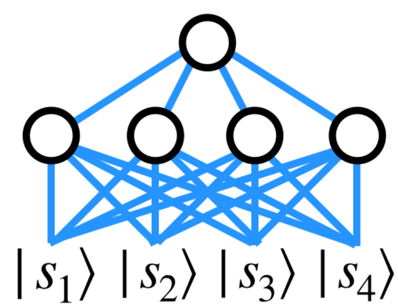
$$\psi(s_1,\cdots,s_N)=\sum_{h_A}\exp\left[\sum_a a_a s_a+\sum_A b_A h_A+\sum_{a,A} J_{aA} s_a h_A\right]$$

- Deep Boltzmann machine states

  [Carleo, Nomura, Imada `18], ..



- Feedfoward network states [Saito `18], ..



$$\psi(s_1,\cdots,s_N)=\sum_i f_i\,\sigma\left(\sum_j W_{ij}s_j+b_i\right)$$

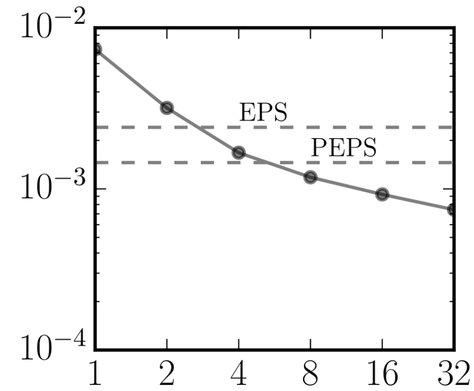
② Neural Network Quantum States

Better? and Why?

Neural states may beat conventional ones.

Ex) 2-dimensional antiferromagnetic Heisenberg model [Carleo, Troyer `17]

Energy with RBM states



# of hidden units

Discovered intimate relations are there.

1) Boltzmann machine states are tensor network states [Chen, Cheng, Xie, Wang, Xiang `18]

2) Tensor states are deep Boltzmann [Gao, Duan `17] [Huang, Moore `17]

3) Tensor states are feedforward with "product pooling" [Cohen, Shashua `18]

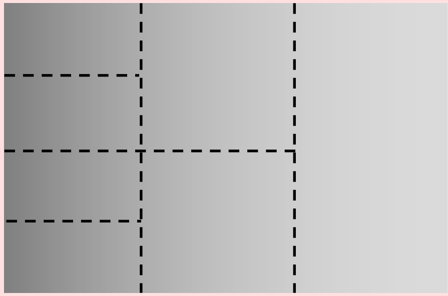
Ex) Unified approach: MPO-Net [Gao, Cheng, He, Xie, Zhao, Lu, Xiang `19]

Roadmap

General spacetime

Anti de Sitter spacetime

Quantum gravity in (d+1)-dim.



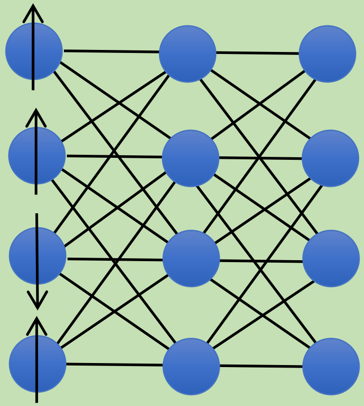
'tHooft `93
Susskind `94
Maldacena `97

?

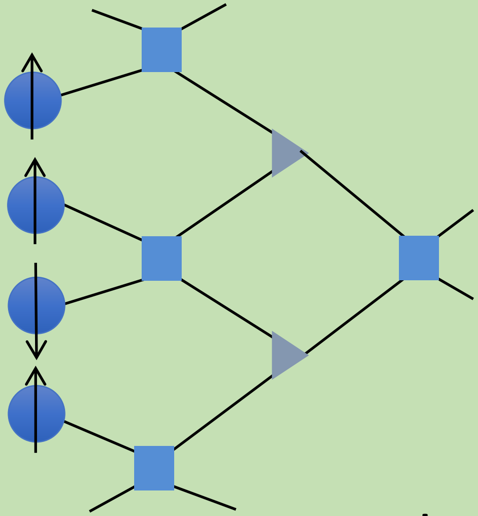
Swingle `10

2.

Quantum mechanics in d-dim.



Carleo, Troyer `17



Neural network

Tensor network

Machine Learning Holographic QCD

1. Quantum gravity — 4 pages
2. Neural network quantum states — 6 pages
3. When is NN a spacetime? — 5 pages
4. Spacetime emergent from data — 15 pages

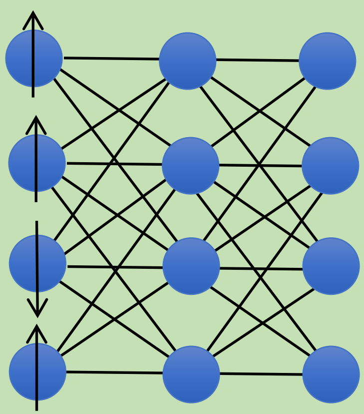
Discussion: Quantum gravity $\subset$ ML ?

Roadmap

3.

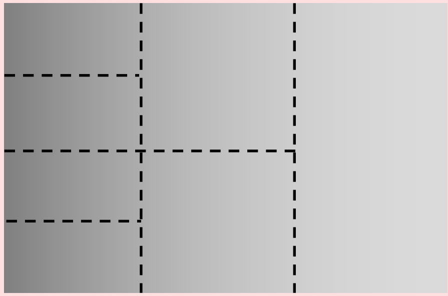
General spacetime

|| ?

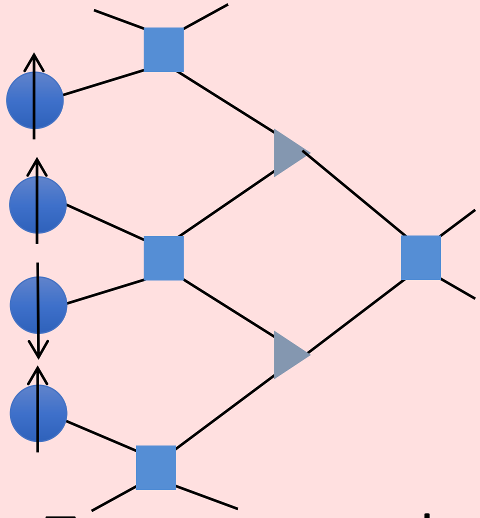


Neural network

Anti de Sitter spacetime



|| Swingle `10



Tensor network

Carleo, Troyer `17

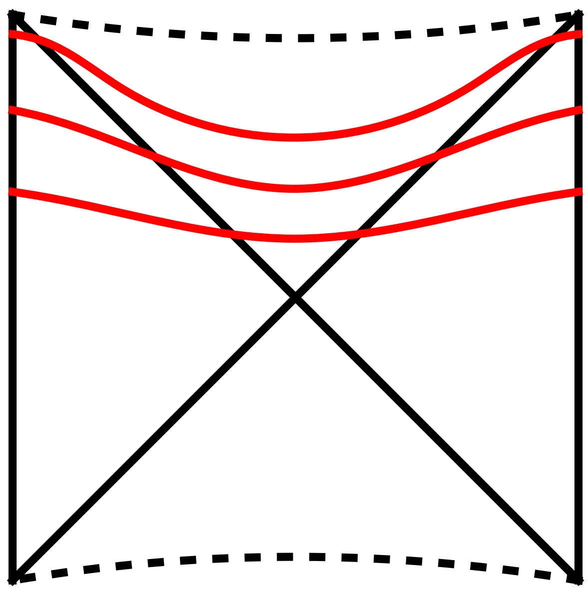
Quantum gravity in (d+1)-dim.

‘tHooft `93
Susskind `94
Maldacena `97

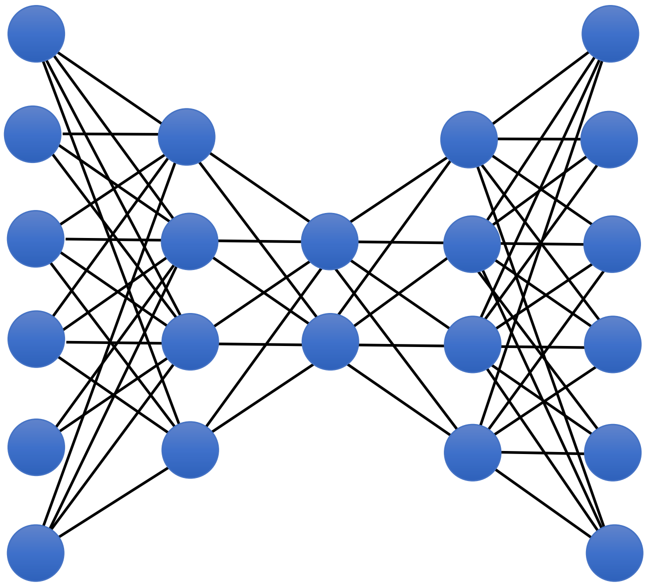
||

Quantum mechanics in d-dim.

Similarity!?



Wormholes in Penrose diagram
of maximally extended eternal
AdS Schwarzschild black hole
[Iizuka, Sugishita, KH `17]

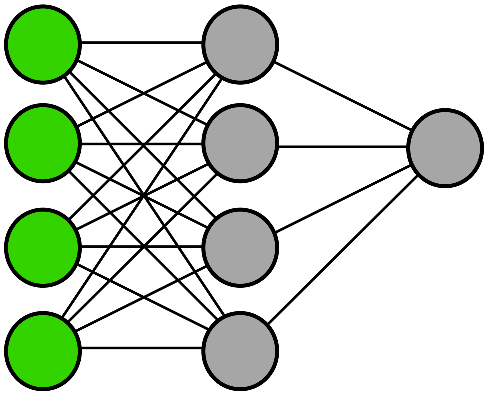


Deep Autoencoder

③ When is NN a spacetime?

General NN is not a space

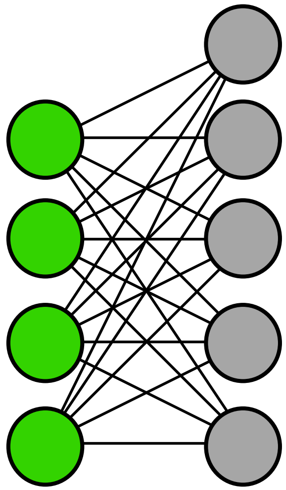
No notion of which unit is close to which



Perceptron model

[Rosenblatt 1958]

[Rumelhart, McClelland 1986]



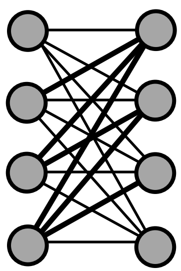
Boltzmann machine

[Ackley, Hinton, Sejnowski 1985]

3. When is NN a spacetime?

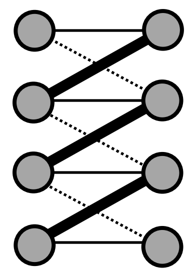
Sparsity + weight sharing, for NN to be a space

No locality



Fully connected

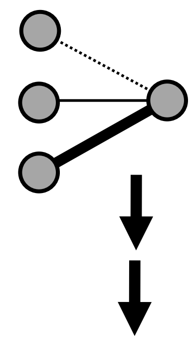
Locality imposed



Convolutional layer

[Fukushima `80]

$=$



Parallelly translated

Input: $\phi(n\Delta x)$

Output:

$$a\phi(n\Delta x) + b\partial_x\phi(n\Delta x) + c\partial_x^2\phi(n\Delta x) + \cdots$$

3. When is NN a spacetime?

NN depth as time

Dynamical system

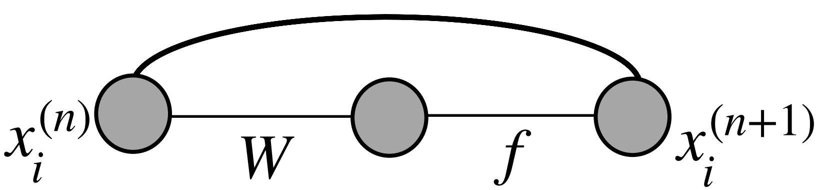
$$\dot{x}_i = f_i(x(t)) \quad \Longrightarrow \quad x_i(t_{n+1}) = \underline{x_i(t_n) +} \, \Delta t \cdot f_i(x(t_n))$$

$$t_{n+1} = t_n + \Delta t$$

Discretized time

ResNET (Residual network) : easily trained deep model

[K.He et al.,1512.03385]



$$x_i^{(n+1)} = f(W_{ij} x_j^{(n)}) \, \underline{+ \, x_i^{(n)}}$$

Skip connection

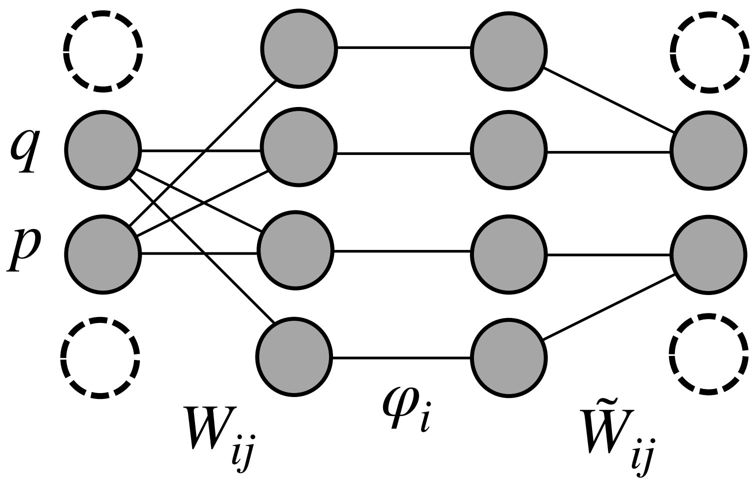
3. When is NN a spacetime?

Hamilton dynamics is a NN

1802.08313

$$\dot{q} = \frac{\partial H}{\partial p}, \qquad \dot{p} = -\frac{\partial H}{\partial q}$$

Time-dependent Hamiltonian = weights/activation



$$W = \begin{pmatrix} 0 & 0 & v & 0 \\ 0 & 1+\Delta t w_{11} & \Delta t w_{12} & 0 \\ 0 & \Delta t w_{21} & 1+\Delta t w_{12} & 0 \\ 0 & u & 0 & 0 \end{pmatrix}$$

$$\varphi_i = \begin{pmatrix} \Delta t f(x) \\ 1 \\ 1 \\ \Delta t g(x) \end{pmatrix} \qquad \tilde{W} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ \lambda_1 & 1 & 0 & 0 \\ 0 & 0 & 1 & \lambda_2 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$H = w_{11} pq + \frac{1}{2} w_{12} p^2 - \frac{1}{2} w_{21} q^2 + \frac{\lambda_1}{v} F(vp) - \frac{\lambda_2}{u} G(uq)$$

$$(F' = f, \quad G' = g)$$

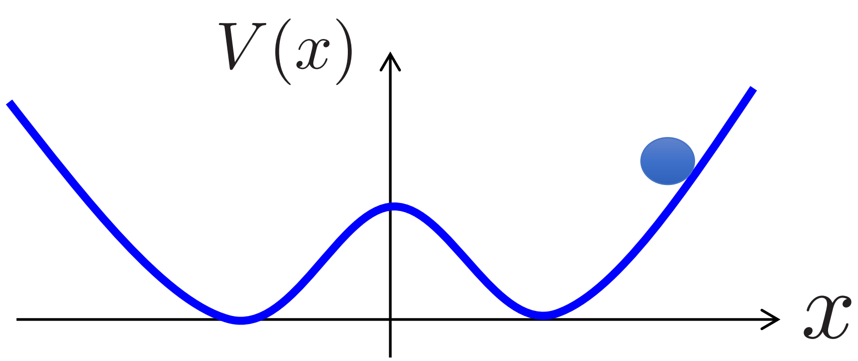
③ When is NN a spacetime?

Q. Find a Hamiltonian

Consider a particle motion $x(t)$ in a given potential $V(x)$ in 1 dimension, with unknown time-dependent friction force $h(t)\dot{x}$.
One tried many initial conditions $(x(t=0), \dot{x}(t=0))$ and collected those which stop at $t=10$.

Q. From given data of the initial conditions, find $h(t)$.

$$m\ddot{x} = h(t)\dot{x} + \frac{\partial V(x)}{\partial x}$$



Roadmap

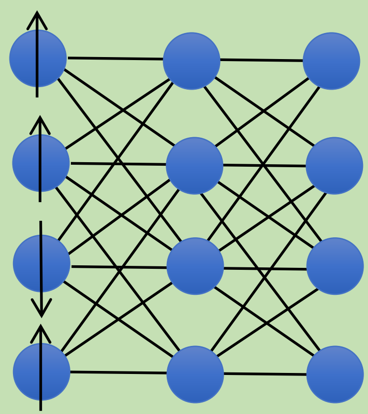
Quantum
gravity
in (d+1)-dim.

'tHooft `93
Susskind `94
Maldacena `97

Quantum
mechanics
in d-dim.

3. General spacetime

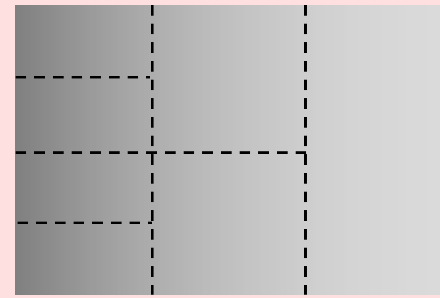
?



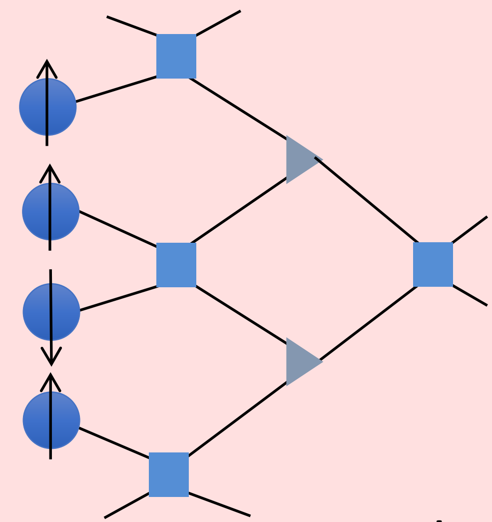
Neural network

Carleo, Troyer `17

Anti de Sitter spacetime



Swingle `10



Tensor network

Machine Learning Holographic QCD

1. Quantum gravity — 4 pages
2. Neural network quantum states — 6 pages
3. When is NN a spacetime? — 5 pages
4. Spacetime emergent from data — 15 pages

Discussion: Quantum gravity $\subset$ ML ?

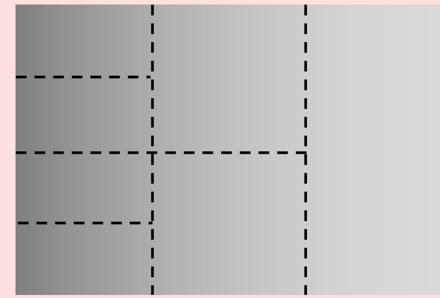
Roadmap

4.

Quantum gravity in (d+1)-dim.

General spacetime

Anti de Sitter spacetime



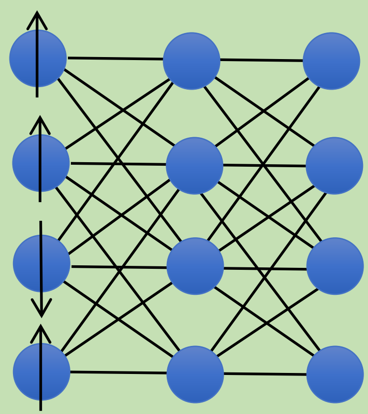
‘tHooft `93
Susskind `94
Maldacena `97

||

|| ?

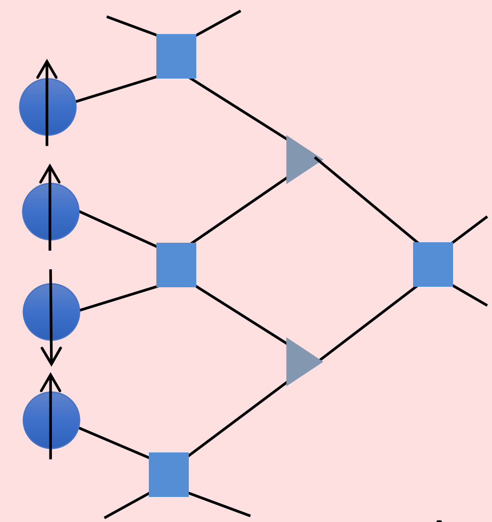
|| Swingle `10

Quantum mechanics in d-dim.

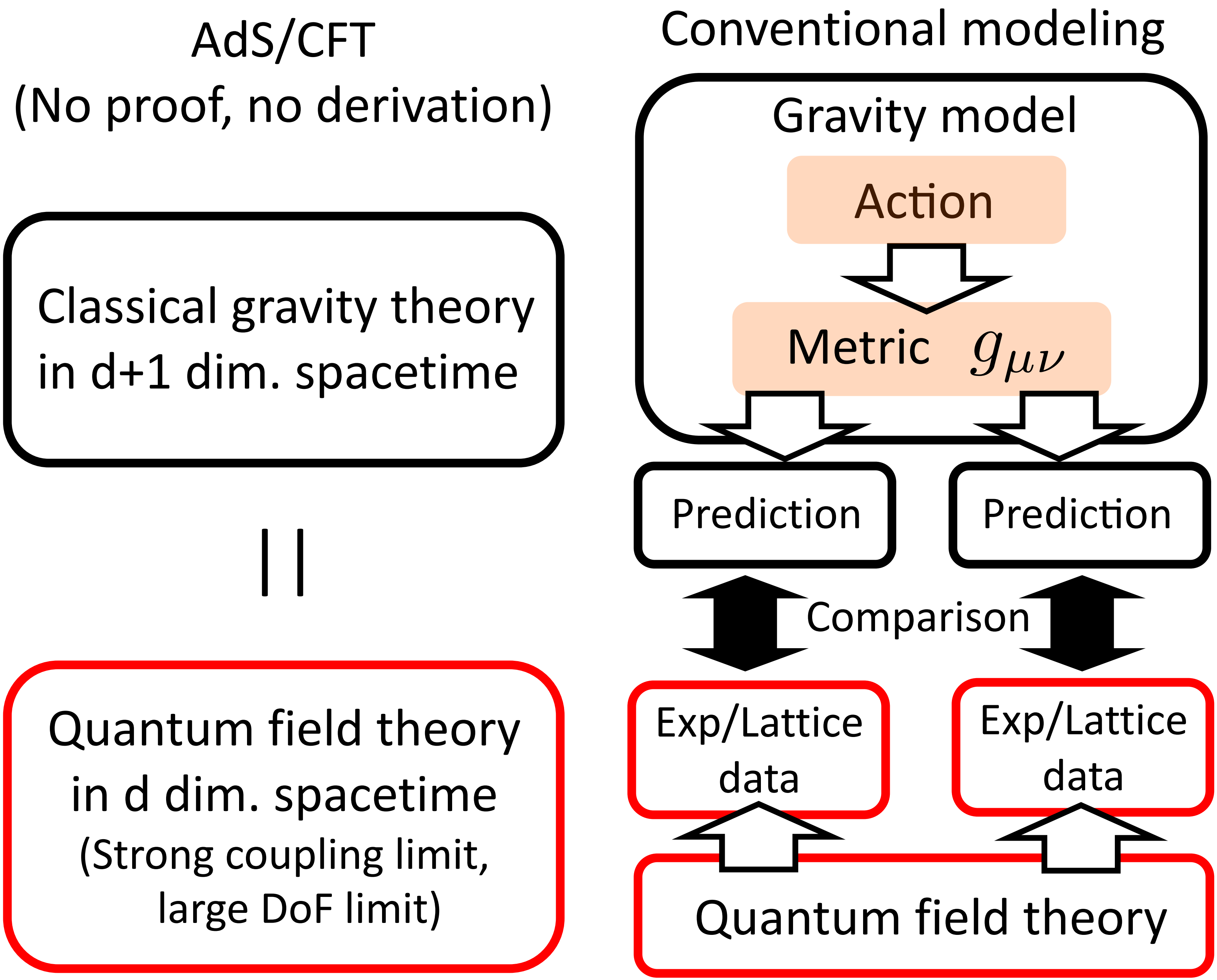


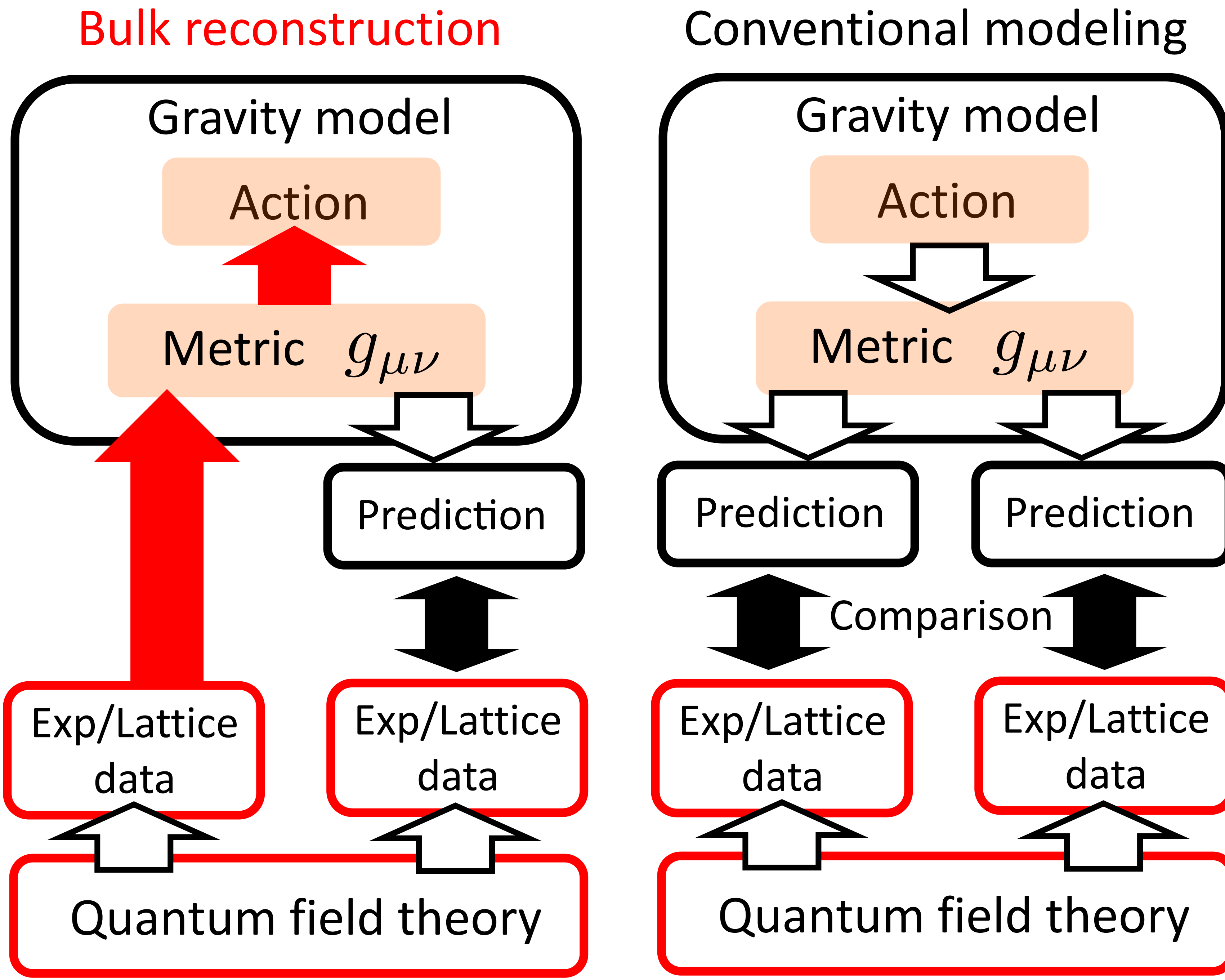
Neural network

Carleo, Troyer `17



Tensor network

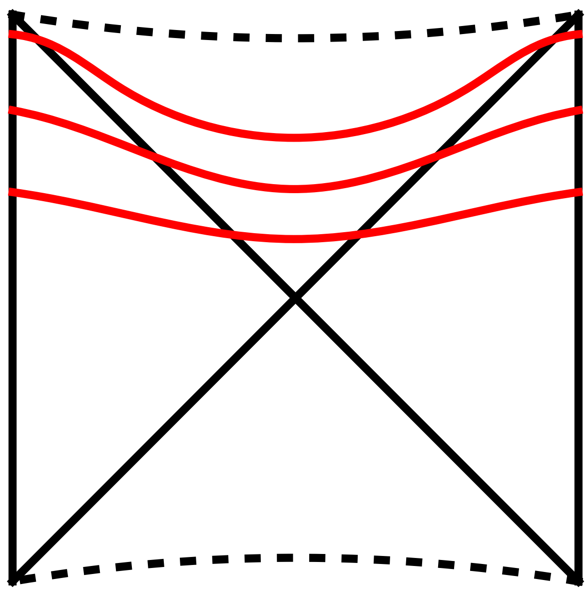

AdS/CFT
(No proof, no derivation)
Classical gravity theory in d+1 dim. spacetime
||
Quantum field theory in d dim. spacetime
(Strong coupling limit, large DoF limit)
Conventional modeling
Gravity model
Action
Metric $g_{\mu\nu}$
Prediction
Prediction
Comparison
Exp/Lattice data
Exp/Lattice data
Quantum field theory


Bulk reconstruction
Conventional modeling
Gravity model
Action
Metric $g_{\mu\nu}$
Prediction
Exp/Lattice data
Quantum field theory
Gravity model
Action
Metric $g_{\mu\nu}$
Prediction
Prediction
Comparison
Exp/Lattice data
Exp/Lattice data
Quantum field theory

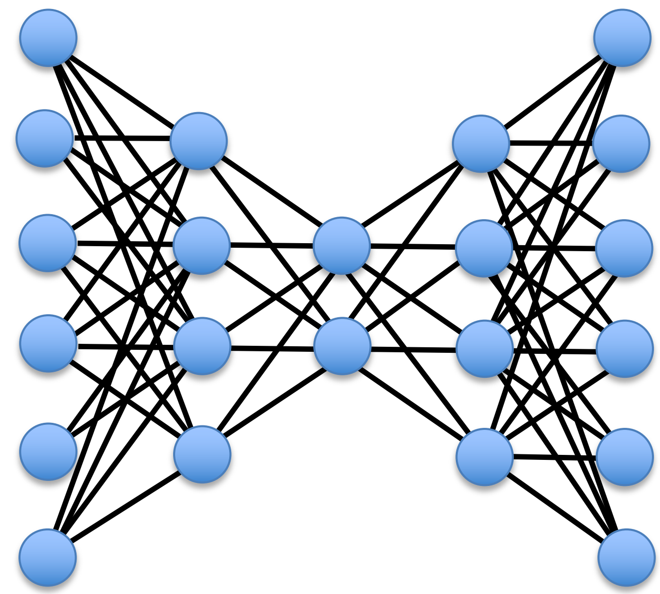
Comparison of solvers

Reconstruction method	No use of Einstein eq	Lattice input
Holographic renormalization [deHaro Solodukhin Skenderis 00]		✓
Entanglement, Complexity [Hammersley 07] [Bilson 08]… [KH Watanabe 21]	✓	
Correlators [Hammersley 06] [Hubeny Liu Rangamani 06]	✓	
AdS/DL [KH Tanaka Tomiya Sugishita 18]	✓	✓
Wilson loop [KH 20]	✓	✓

Similarity!?



Wormholes in Penrose diagram of maximally extended eternal AdS Schwarzschild black hole [Iizuka, Sugishita, KH `17]



Deep Autoencoder

④ Spacetime emergent from data

Gravity side

Classical scalar field theory in unknown 5-dim. spacetime

1802.08313
1809.10536

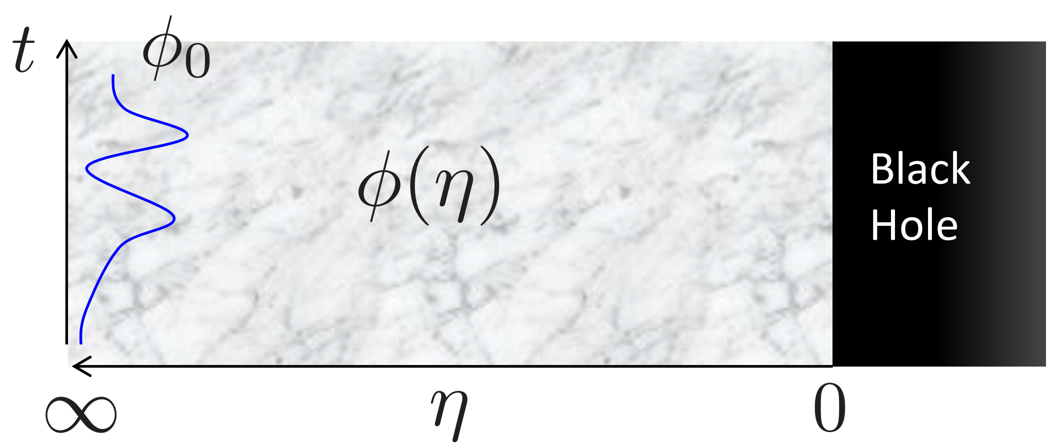
$$S = \int d\eta d^4x \sqrt{\det g}\,\left[(\partial_\eta \phi)^2 - V(\phi)\right]$$

$$\begin{cases} ds^2 = -f(\eta)dt^2 + d\eta^2 + g(\eta)(dx_1^2 + \cdots + dx_{d-1}^2) \\ V[\phi] = -\dfrac{3}{L^2}\phi^2 + \dfrac{\lambda}{4}\phi^4 \end{cases}$$

Data: $(\phi_0, Z[\phi_0])$

$\Updownarrow$

$(\phi|_{\eta=\infty}, \partial_\eta \phi|_{\eta=\infty}, \partial_\eta \phi|_{\eta=0})$



④ Spacetime emergent from data

Equation of motion as a feedforward NN

Eq. of motion $\partial_\eta^2 \phi + \underline{\underline{h(\eta)}} \partial_\eta \phi - \dfrac{\delta V[\phi]}{\delta \phi} = 0$

metric $h(\eta) \equiv \partial_\eta \left[\log \sqrt{f(\eta) g(\eta)^{d-1}} \right]$

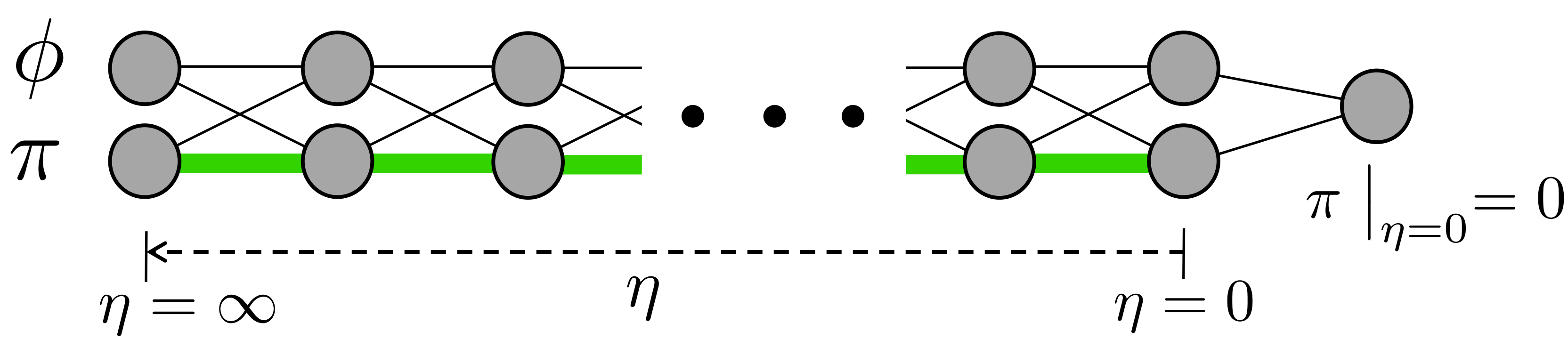
⇩

Discretization
Hamilton form

$$\begin{cases} \phi(\eta + \Delta\eta) = \phi(\eta) + \Delta\eta\, \pi(\eta) \\ \pi(\eta + \Delta\eta) = \pi(\eta) + \Delta\eta \left(h(\eta) \pi(\eta) - \dfrac{\delta V(\phi(\eta))}{\delta \phi(\eta)} \right) \end{cases}$$

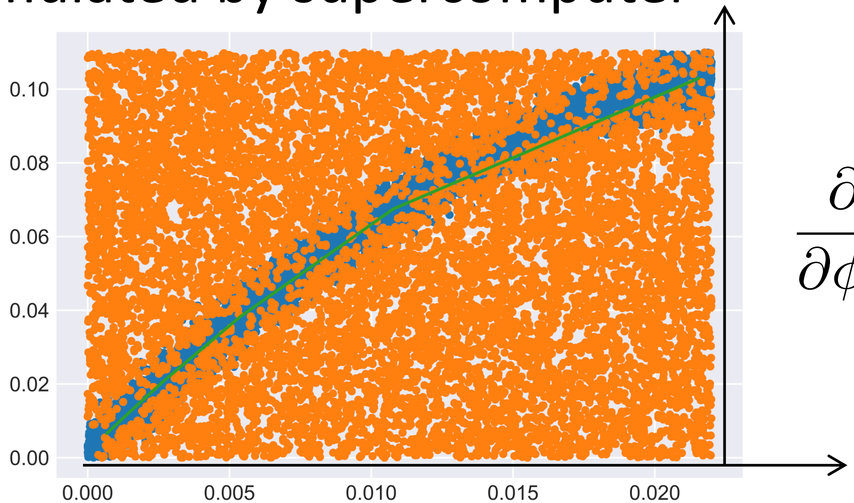
⇩

Feedfoward neural network for classification



Training with data of quark condensate

Data of quantum chromodynamics
simulated by supercomputer



$$\frac{\partial}{\partial \phi_0} Z[\phi_0]$$: Quark condensate

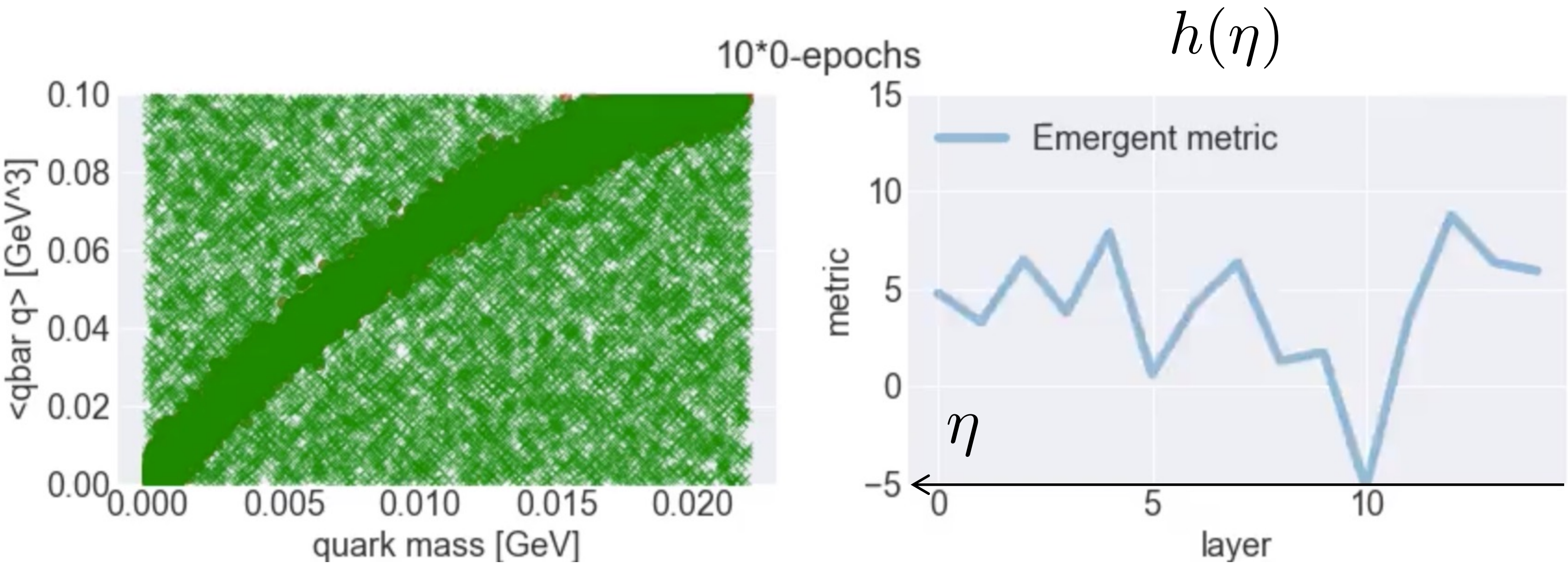
ϕ_0 : Quark mass

T=205(7) [MeV]

[W. Unger, "The Chiral Phase Transition of QCD with 2+1 Flavors : A lattice study on Goldstone modes and universal scaling," Ph.D. thesis, der Universitat Bielefeld, 2010]

④ Spacetime emergent from data

Spacetime metric emergent as NN



Trained values of potential :

1/L = 237(3)[MeV] , λ/L = 0.0127(6)

④ Spacetime emergent from data

AdS QCD model for meson spectra

[Karch, Kaz, Son, Stephanov `06]

Classical gauge theory in 5-d dilaton gravity background

$$S = \int d^4x dz \, e^{-\Phi} \sqrt{-g} \left(F_{MN}\right)^2$$

Dilaton $\Phi(z)$, metric $ds^2 = e^{2A(z)} \left(dz^2 + \eta_{\mu\nu} dx^\mu dx^\nu \right)$

AdS boundary ($z \sim 0$) : $B(z) \equiv \Phi(z) - A(z) \sim \log z$

Solve EoM for gauge field $A_\mu(z, x^\mu) = v_n(z) \rho_\mu(x^\mu)$

$$\frac{\partial}{\partial z}\left(e^{-B} \frac{\partial}{\partial z} v_n \right) + \omega^2 e^{-B} v_n = 0$$

When frequency takes a proper discrete value $\omega^2 \sim m_n^2$, gauge field is normalizable : vector meson spectra.

4. Spacetime emergent from data

Bring the bulk EoM to neural network

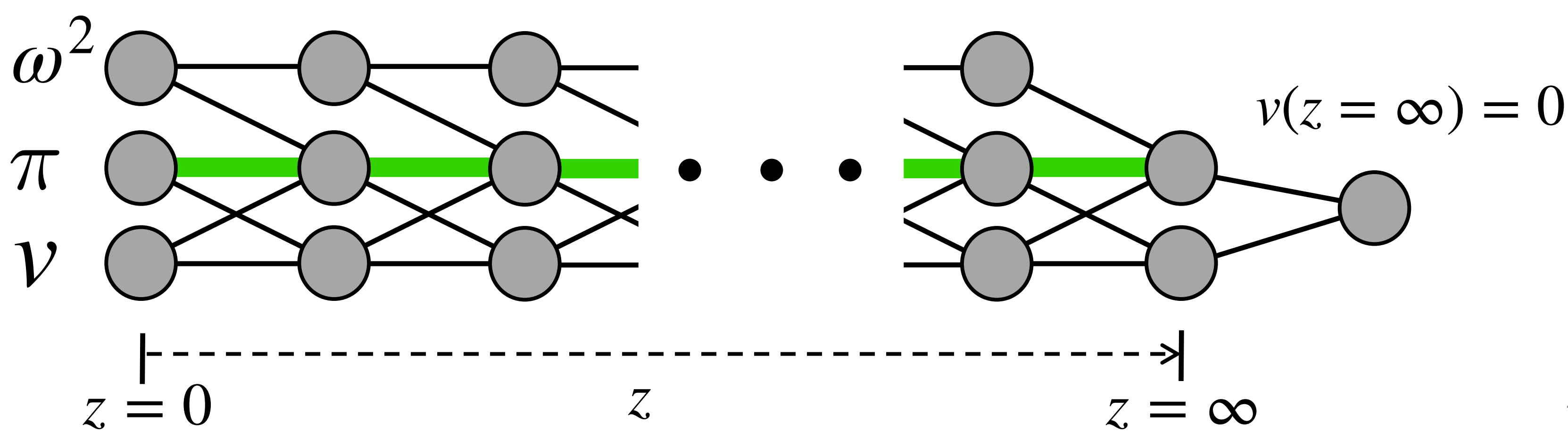
2005.02636

Bulk EoM
$$\frac{\partial}{\partial z}\left(e^{-B}\frac{\partial}{\partial z}v_n\right)+\omega^2 e^{-B}v_n=0$$

Discretization

Hamilton form
$$\begin{cases} v_n(z+\Delta z)=v_n(z)+\Delta z\,\pi_n(z) \\ \pi_n(z+\Delta z)=\pi_n(z)+\Delta z\left(B'(z)\pi_n(z)-\omega^2 v_n(z)\right) \end{cases}$$

Neural-Network representation



④ Spacetime emergent from data

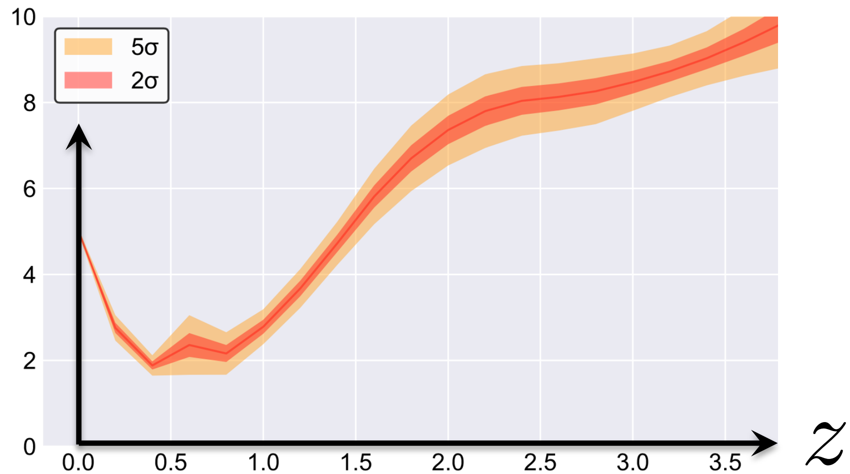
Training with QCD data: hadron spectra

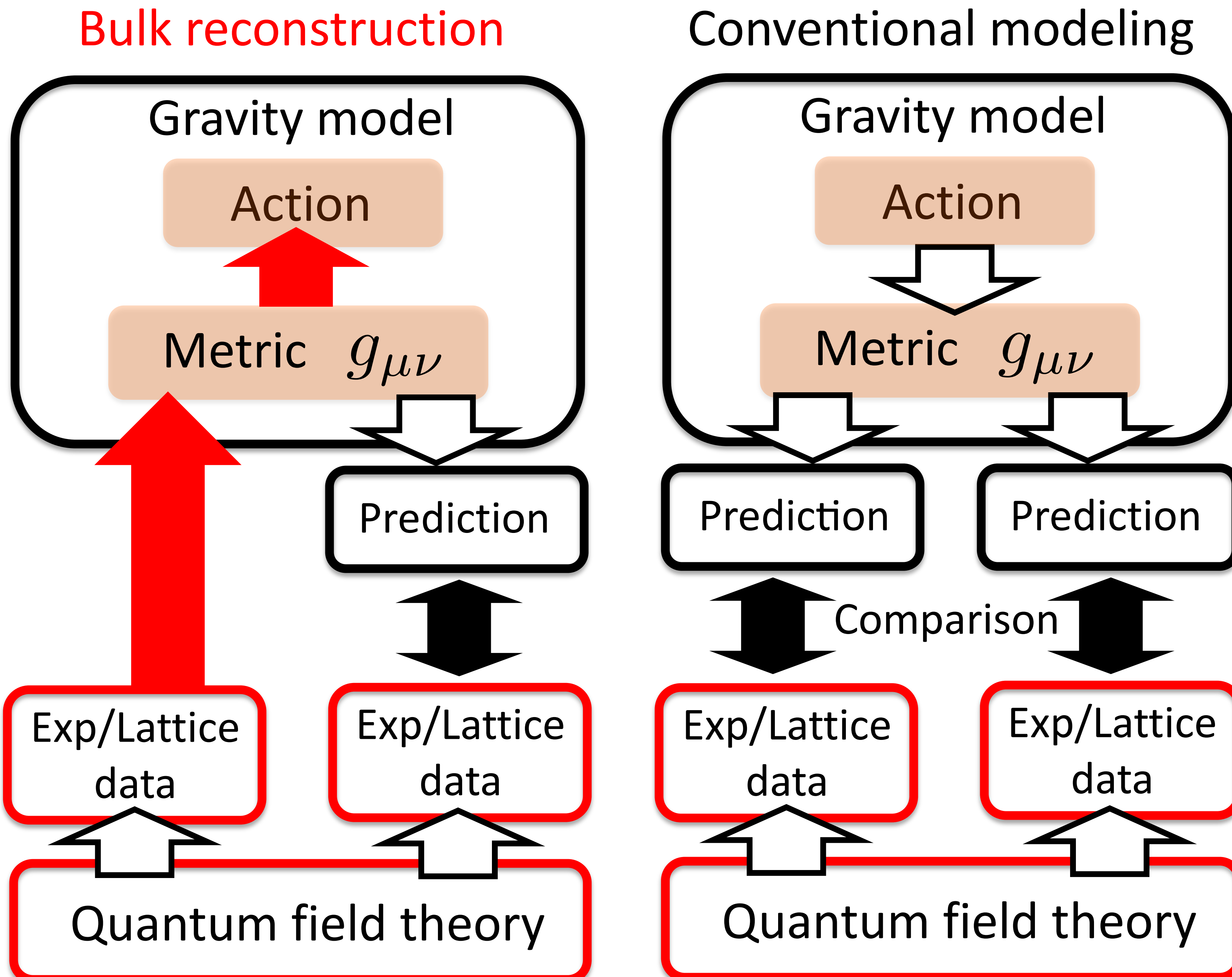
2005.02636

Input : PDG data for rho meson mass

$$m_\rho^{(1)} = 0.77 \text{ GeV}, \; m_\rho^{(2)} = 1.45 \text{ GeV}$$

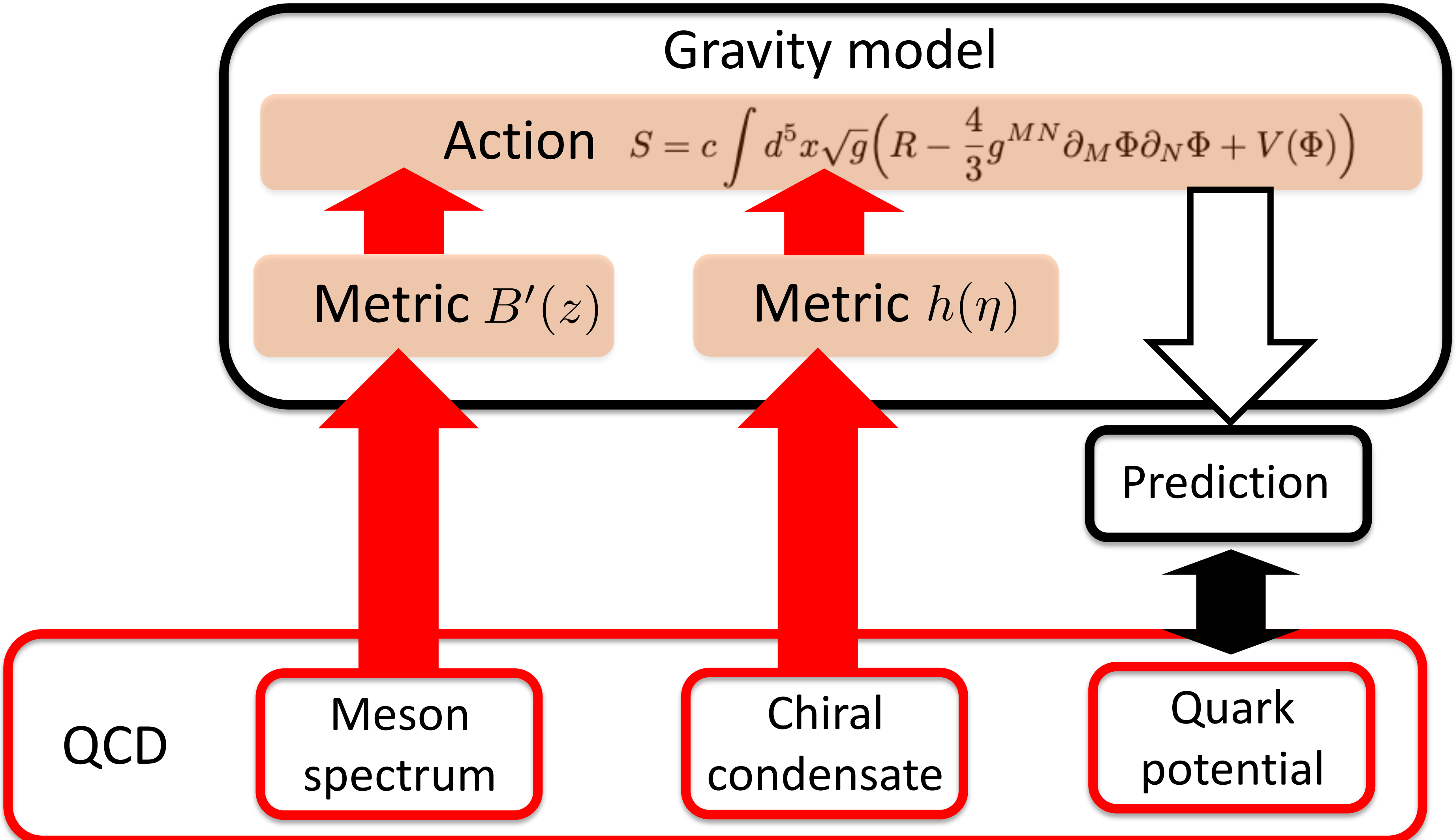
Result: Emergent metric $B'(z) = \Phi'(z) - A'(z)$




Bulk reconstruction
Conventional modeling
Gravity model
Action
Metric $g_{\mu\nu}$
Prediction
Comparison
Exp/Lattice data
Quantum field theory

4. Spacetime emergent from data

Two independent information of metric



④ Spacetime emergent from data

Two independent information of metric

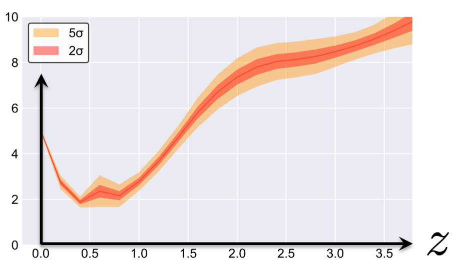
Gravity model

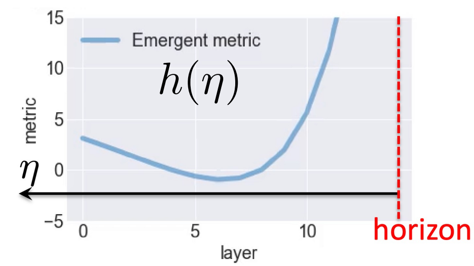
Action $S = c\int d^5x\sqrt{g}\left(R - \frac{4}{3}g^{MN}\partial_M\Phi\partial_N\Phi + V(\Phi)\right)$

Metric $B'(z)$

Metric $h(\eta)$

$B'(z) = \Phi'(z) - A'(z)$





Meson spectrum

Chiral condensate

④ Spacetime emergent from data

Deriving the dilaton potential (T=0)

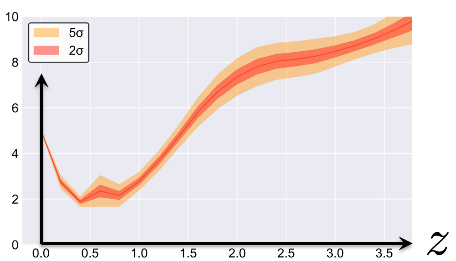
Gravity model

Action $S = c\int d^5x\sqrt{g}\left(R - \frac{4}{3}g^{MN}\partial_M\Phi\partial_N\Phi + V(\Phi)\right)$

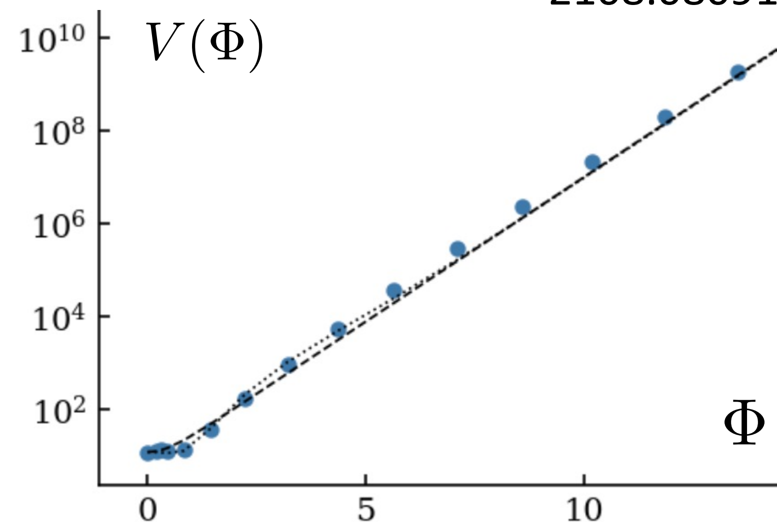
Metric $B'(z)$

Meson spectrum

$B'(z) = \Phi'(z) - A'(z)$



2108.08091



$V = 12\cosh(1.433\Phi)$

$V = 12\cosh(1.430\Phi) - 16.778\Phi^2 + 5.943\Phi^4$

Cf. [Gubser, Nellore, 0804.0434]

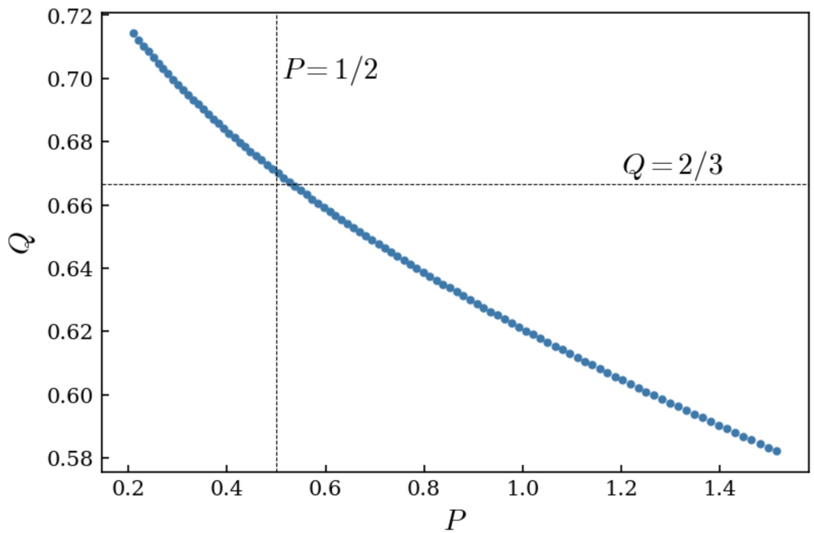
4. Spacetime emergent from data

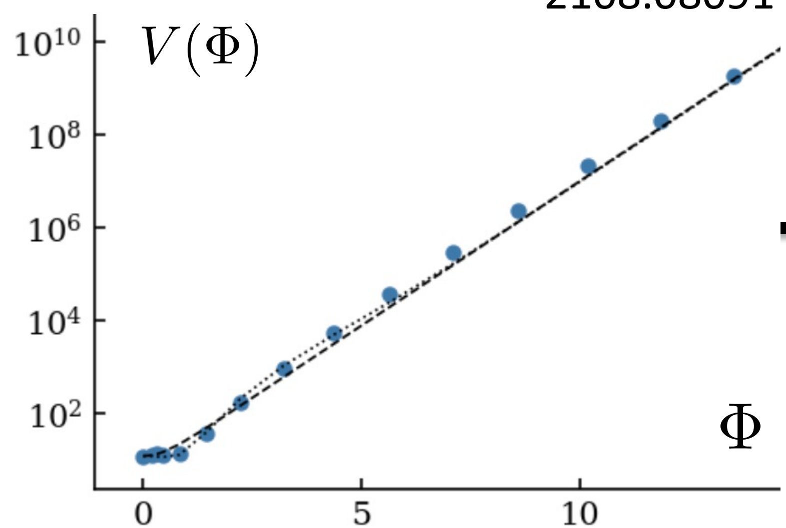
It's a nice dilaton potential !

Gravity model

Action $S = c \int d^5x \sqrt{g} \left(R - \frac{4}{3} g^{MN} \partial_M \Phi \partial_N \Phi + V(\Phi) \right)$

2108.08091





Fit the asymptotic part by $V(\Phi) \sim e^{2Q\Phi}\Phi^P$ for different values of dilaton initial cond.

Cf. [Gursoy, Kiritsis, 0707.1324]
[Gursoy, Kiritsis, Nitti, 0707.1349]

------ $V = 12\cosh(1.433\Phi)$

$V = 12\cosh(1.430\Phi) - 16.778\Phi^2 + 5.943\Phi^4$

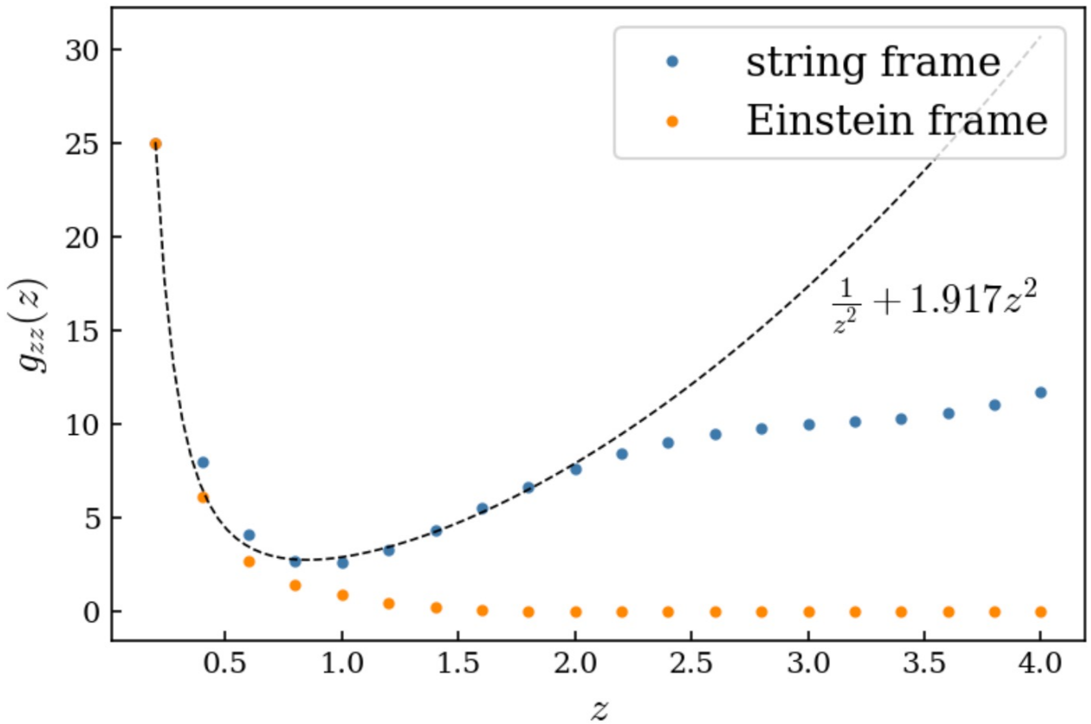
Cf. [Gubser, Nellore, 0804.0434]

④ Spacetime emergent from data

String frame metric has a bottom

Gravity model

Action $S = c\int d^5x\sqrt{g}\left(R - \frac{4}{3}g^{MN}\partial_M\Phi\partial_N\Phi + V(\Phi)\right)$



Prediction

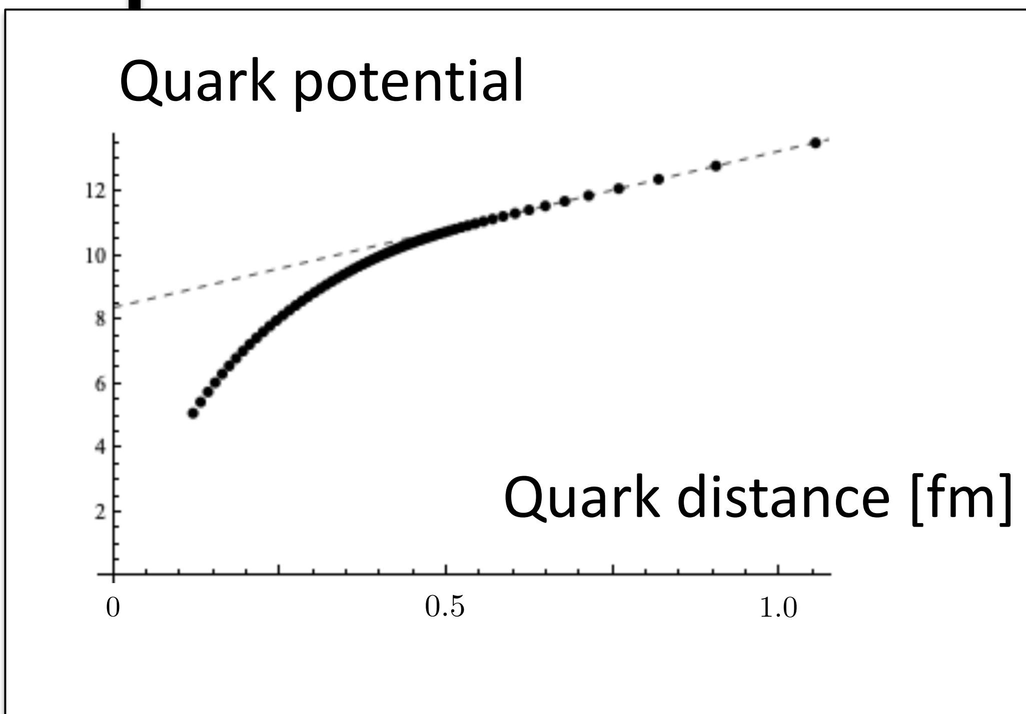
Quark potential

④ Spacetime emergent from data

Prediction of string breaking (T=0)

Gravity model

Action $S = c\int d^5x\sqrt{g}\left(R - \frac{4}{3}g^{MN}\partial_M\Phi\partial_N\Phi + V(\Phi)\right)$



Prediction

Quark potential

④ Spacetime emergent from data

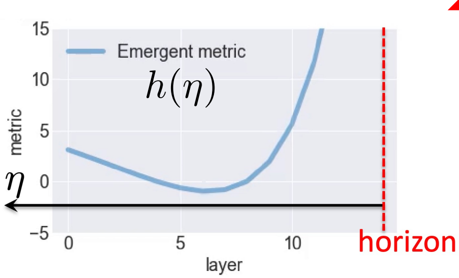
Deriving the dilaton potential (finite T)

Gravity model

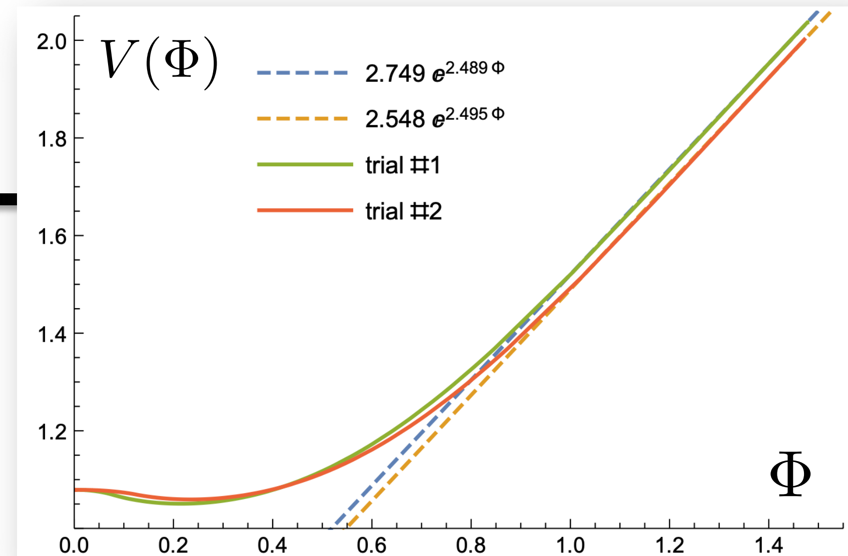
Action $S = c\int d^5x\sqrt{g}\left(R - \frac{4}{3}g^{MN}\partial_M\Phi\partial_N\Phi + V(\Phi)\right)$

2209.04638

Metric $h(\eta)$



Chiral condensate



④ Spacetime emergent from data

Prediction of string breaking (finite T)

Gravity model

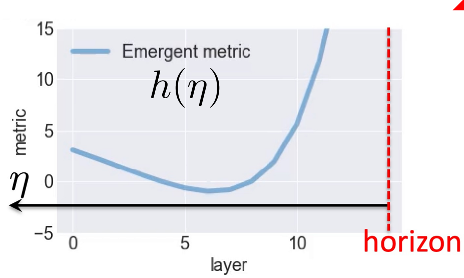
Action $S = c\int d^5x\sqrt{g}\left(R - \frac{4}{3}g^{MN}\partial_M\Phi\partial_N\Phi + V(\Phi)\right)$

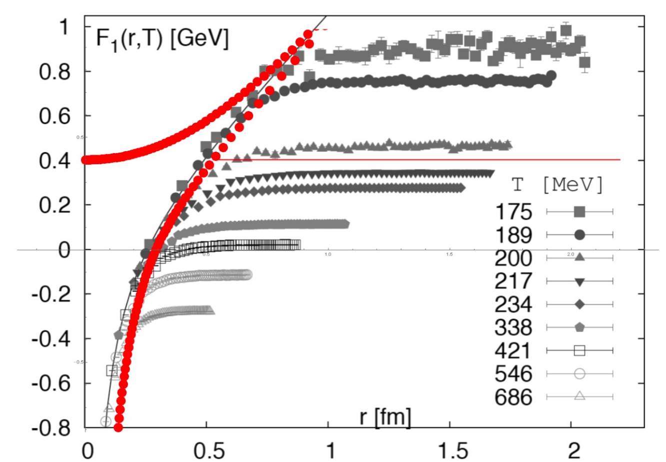
Metric $h(\eta)$

Prediction

Chiral condensate

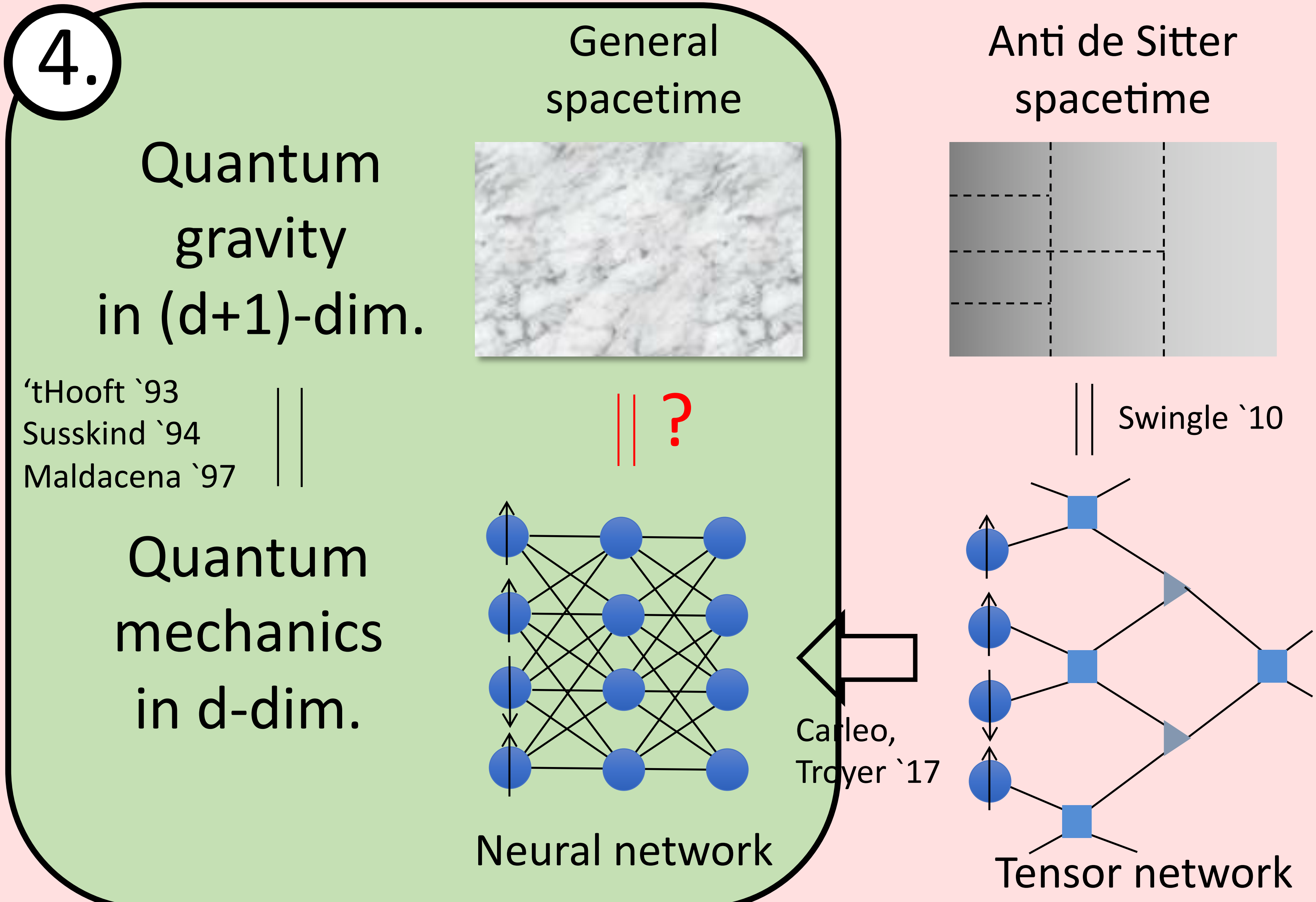
Quark potential





Lattice data (grey) : Petreczky, J.Phys.G37(2010)094009

Roadmap


4.
General spacetime
Anti de Sitter spacetime
Quantum gravity in (d+1)-dim.
‘tHooft `93
Susskind `94
Maldacena `97
?
Swingle `10
Quantum mechanics in d-dim.
Carleo, Troyer `17
Neural network
Tensor network

Machine Learning Holographic QCD

1. Quantum gravity — 4 pages
2. Neural network quantum states — 6 pages
3. When is NN a spacetime? — 5 pages
4. Spacetime emergent from data — 15 pages

Discussion: Quantum gravity $\subset$ ML ?

Discussion: Quantum gravity ⊂ ML ?

3 steps for quantum gravity

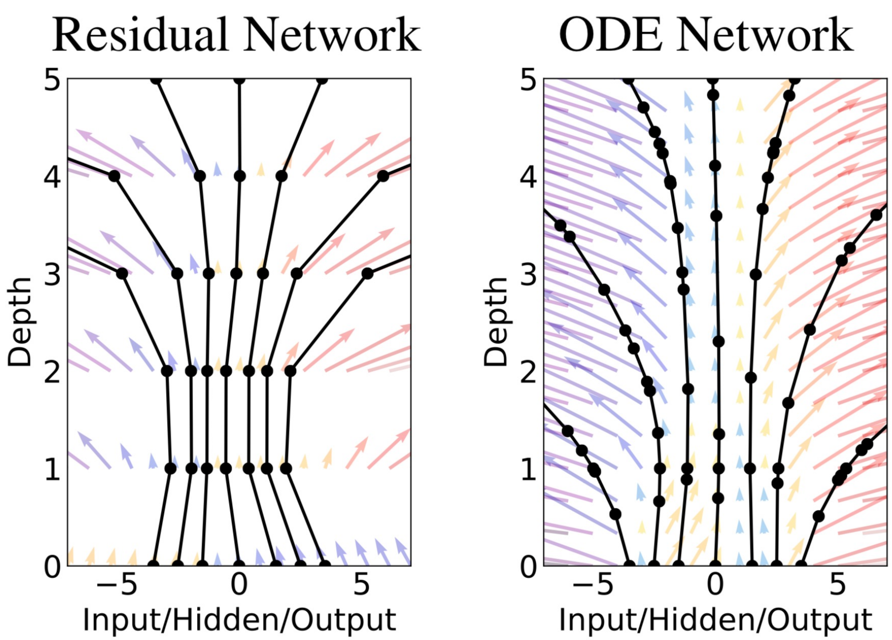
Quantum Mechanics side	Gravity side		NN architecture
	metric $g_{\mu\nu}$	field ϕ	
Large DoF limit	Classical	Classical	Feedforward NN
Large DoF expansion	Classical	Quantum	Deep Boltzmann
Finite DoF	Quantum	Quantum	?

- Neural ODE : free from discretization
- Quantum AdS/CFT ⊂ Deep Boltzmann machine
- Which part of geometry is the neurons?

Discussion: Quantum gravity ⊂ ML ?

Neural ODE : free from discretization

2006.00712



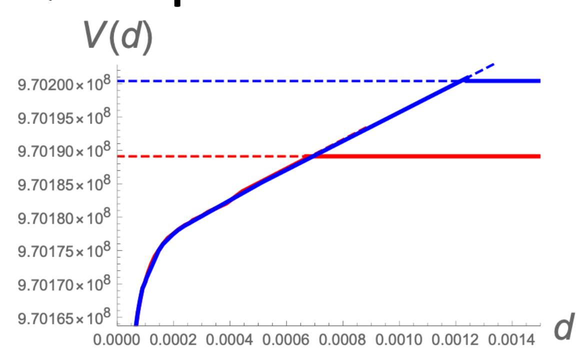
Neural ODE [R.T.Q.Chen, Y.Rubanova, J.Bettencourt, D.Duvenaud 1806.07366]

$$\frac{d\phi(\eta)}{d\eta} = f(\phi(\eta), \eta, h(\eta))$$

Emergent metric

$$\begin{aligned} h(\eta) = & \, 8.2351\tilde{\eta}^8 + 8.0108\tilde{\eta}^7 + 7.6071\tilde{\eta}^6 \\ & + 6.9468\tilde{\eta}^5 + 150.8853\tilde{\eta}^4 - 130.8117\tilde{\eta}^3 \\ & + 55.5384\tilde{\eta}^2 - 2.22235\tilde{\eta}^1 + 3.7719. \\ & \tilde{\eta} = 1 - \eta \end{aligned}$$

Q Qbar potential

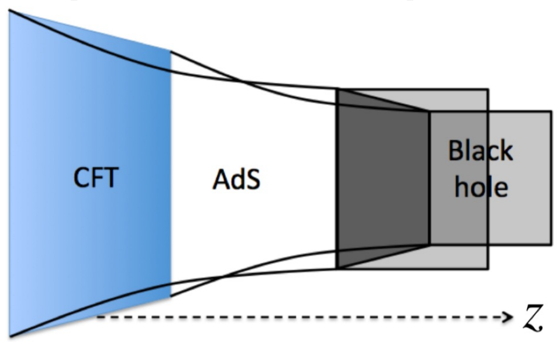


Discussion: Quantum gravity ⊂ ML ?

Quantum AdS/CFT ⊂ Deep Boltzmann

AdS/CFT

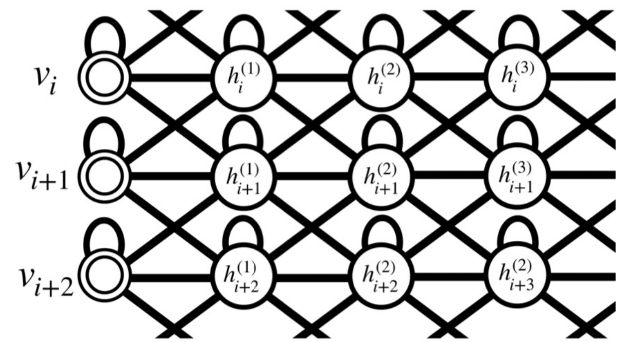
[Maldacena 1997]



$$Z_{\mathrm{QFT}}[J] = \int_{\phi(z=0)=J} \mathcal{D}\phi \, \exp\left(-S_{\mathrm{gravity}}[\phi]\right)$$

Deep Boltzman machine

[Salakhutdinov, Hinton 2009]



$$P(v_i) = \sum_{h_i \in \{0,1\}} \exp\left[-\mathcal{E}(v_i, h_i)\right]$$

[KH `19] [You,Yang,Qi `18] (See also [Gan,Shu `17][Howard `18])

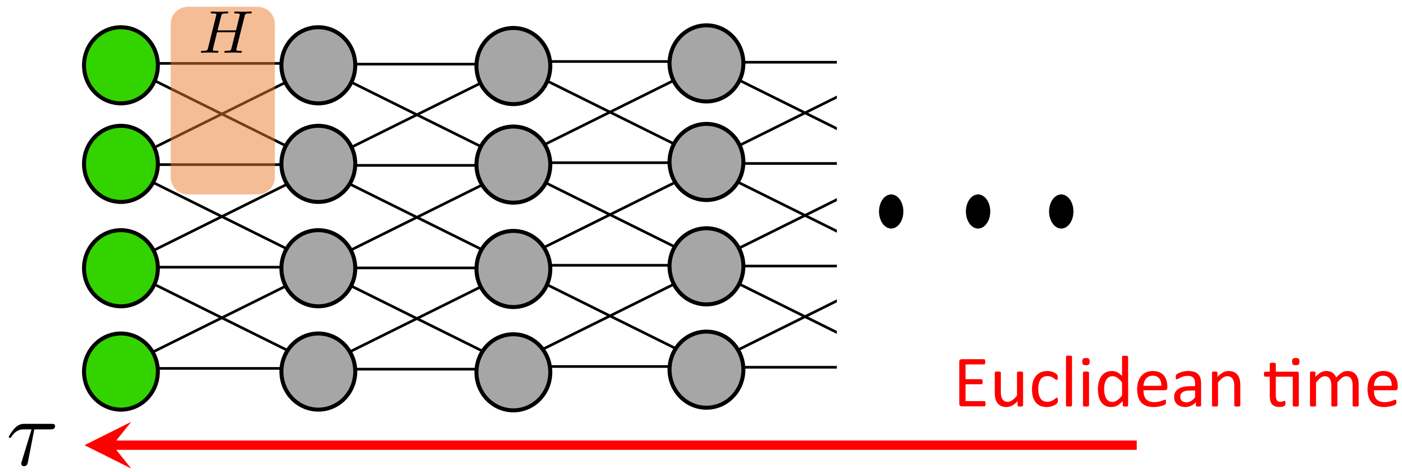
Discussion: Quantum gravity ⊂ ML ?

Physical picture of Deep Boltzmann

Ground state wave function for given Hamiltonian
is identified as a deep Boltzmann machine

[Carleo, Nomura, Imada `18], ..

$$|\psi\rangle = \lim_{\tau\to\infty} e^{-\tau H}|\text{any}\rangle = e^{-\Delta\tau H}e^{-\Delta\tau H}\cdots|\text{any}\rangle$$



$$\psi(x_i) = \sum_{h_j^{(n)}\in\{0,1\}} \exp\left[-\sum_{ij} w_{ij}^{(0)} x_i h_j - \sum_n \sum_{ij} w_{ij}^{(n)} h_i^{(n)} h_j^{(n+1)}\right]$$

Discussion: Quantum gravity ⊂ ML ?

A transformer has the AdS isometry

2402.02362

$$h_i = \sum_{j=1}^{n} \mathrm{ReLU}\left((xw^{(q)})_i (xw^{(k)})_j^{\mathrm{T}} \right) (xw^{(v)})_j$$

$x \in \mathbf{R}^{n\times d}$: set of data $x_i \in \mathbf{R}^d (i = 1, 2, \cdots, n)$

$w^{(q),(k),(v)} \in \mathbf{R}^{d\times d}$: query, key and value weights

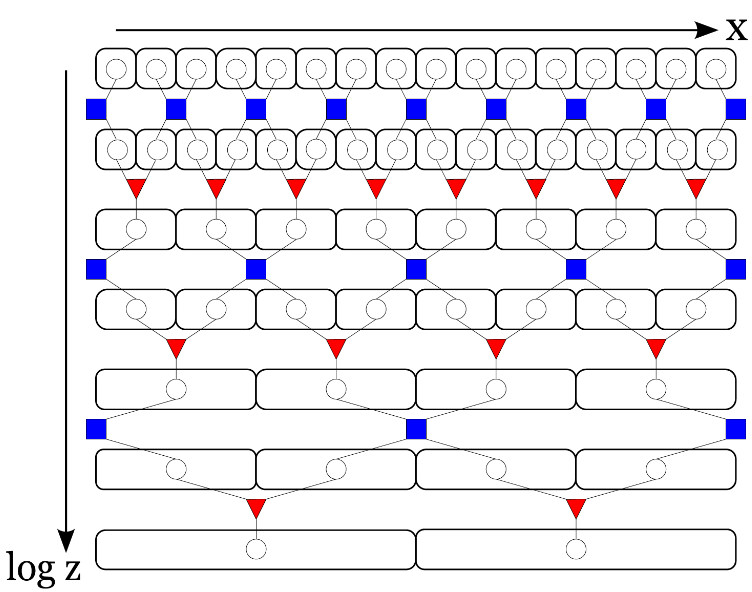
<u>NN sym</u> : rescaling $\quad w^{(a)} \mapsto \alpha^{(a)} w^{(a)}, \quad w^{(q)} w^{(k)} w^{(v)} = 1$

Discussion: Quantum gravity ⊂ ML ?

AdS/CFT discretized the bulk, but fixed

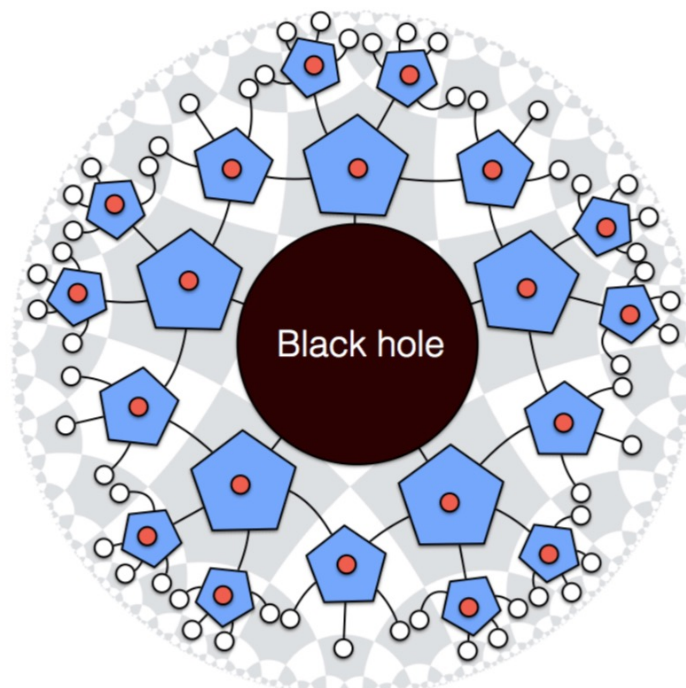
AdS/MERA

[Swingle `09]



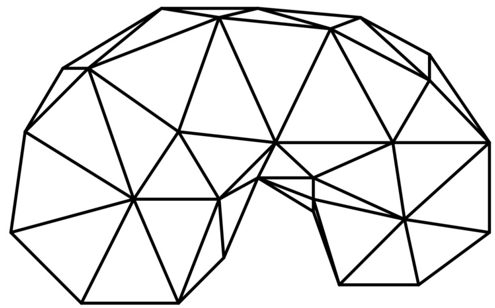
Quantum codes for holography

[Pastawski, Yoshida, Harlow, Preskill `15]



Discussion: Quantum gravity ⊂ ML ?

Quantum spacetime? Regge vs Matrix

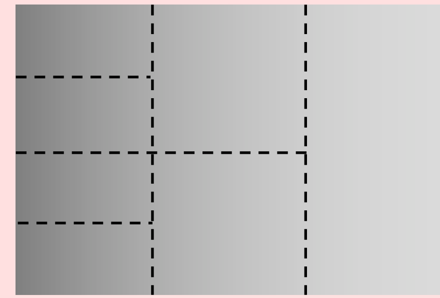


Regge calculus [Regge `61]	Dynamical triangulation [Ambjorn, Loll `98]
Fixed lattice architecture, variable lengths	Randomly generated lattice architecture, fixed lengths
⇩	⇩
Suits conventional NN	Novel “QG NN”

Roadmap

General spacetime

Anti de Sitter spacetime



Quantum gravity in (d+1)-dim.

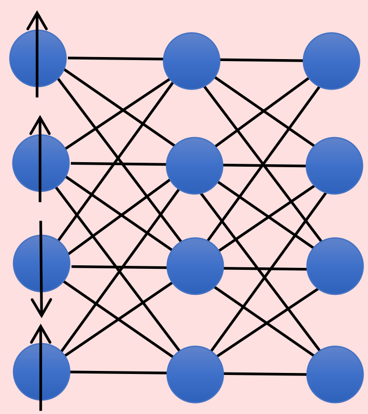
‘tHooft `93
Susskind `94
Maldacena `97

||

|| ?

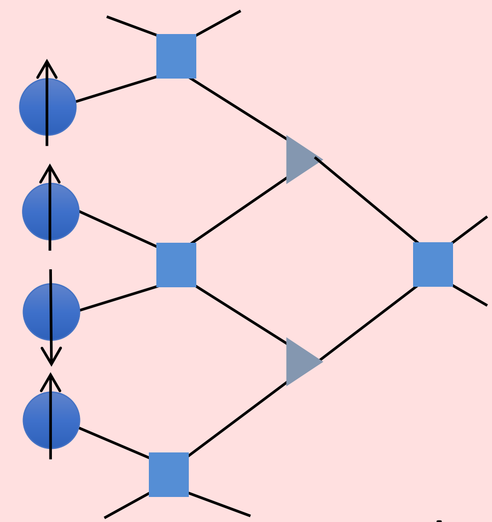
|| Swingle `10

Quantum mechanics in d-dim.



Neural network

Carleo, Troyer `17



Tensor network