Effective theories for strongly-coupled matter with gapped excitations

EFTs are tools for making predictions on the low-energy dynamics of interacting phases of matter, with some extent of independence on the microscopic details of the system and initial conditions.

Two examples I wish to discuss here

* kinetic theory

Based on Boltzmann equation for 1-particle distribution f:
$f(t, x^i, p^i) \equiv$ probability to find a particle at time t, position x^i and w/ momentum p^i

$$\frac{d}{dt} f \equiv \partial_t f + v^i \partial_i f + \dot{p}^i \partial_{p^i} f = \underbrace{C[f]}_{\text{collision kernel}}$$

→ assumes an underlying particle description
→ must specify the collision kernel
→ must specify the equilibrium distribution of particles
eg Maxwell-Boltzmann $f_0 = e^{-\beta(p \cdot u + \mu)}$ rel part

$$f_0 = \frac{1}{e^{\beta(\epsilon_p - \mu)} + 1} \quad \text{Fermi liquids}$$

→ Then solve for $f(t, x^i, p^i)$ out of eq

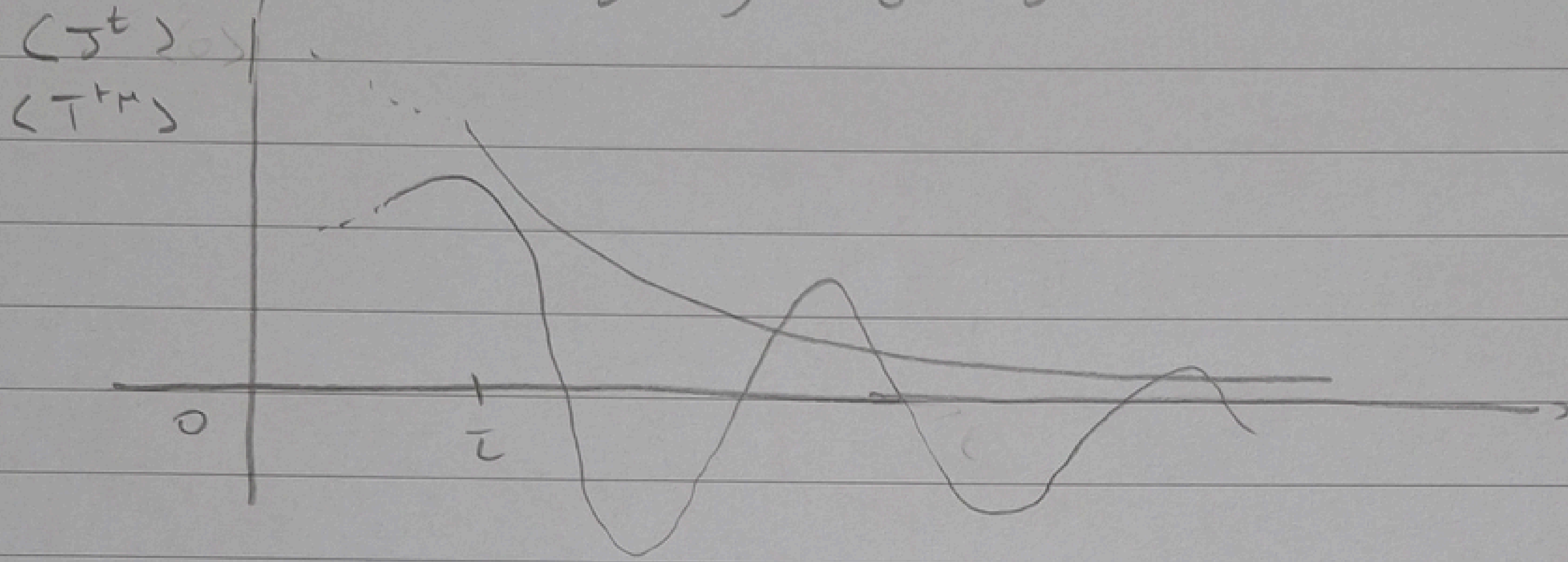
* Hydrodynamics:

Start by enumerating conservation laws from global symmetries

eg $\nabla_\mu T^{\mu\nu} = 0 \qquad \nabla_\mu J^\mu = 0$

Assume local equilibrium on sufficiently coarse-grained scales $(t, x^i) \gtrsim (\tau, \ell)$ → thermal ensemble $\langle O \rangle = \frac{1}{Z} \mathrm{Tr}[O e^{-\beta H}]$

On such scales, conserved currents $T^{t\mu}$, J^t do not decay, transported away by hydrodynamic modes



Non-conserved operators expanded order by order in gradients of hydrodynamic fields : dissipation by thermal bath

$$\underbrace{\langle J^i \rangle = \sum_{n\geq 0} D^{(n)i}_{A} \langle O^A \rangle}_{\text{constitutive relations}} \qquad \text{where } O^A = \{J^t, T^{t\mu}\}$$

$D^{(n)}$ diff operator of order n.

Allows to close the system and solve.

Hydrodynamics : - emerges from kinetic theory on $t \gtrsim \tau$, the inverse of the smallest eigenval of $C[f]$ see (3)

- but does not need to assume particles, in fact emerges also from holography at strong coupling / SYK models.

Make hydro appealing away from weak coupling / many-body. But when is it applicable?

Where have kinetic theory / hydro been applied?

- electronic flows in Graphene.
- strange metallic transport in overdoped cuprates high T_c SC.
- dynamics of QGP formed in heavy ion collisions.

(and obviously holography)

One concrete example: charge diffusion from kinetic theory

Assume a global $U(1)$. Hydro says. Thermo

$$\partial_t n + \partial_i j^i = 0 \qquad j^i = -\sigma_0(\partial_i\mu - E^i) \qquad \delta n = \chi_{nn}\delta\mu$$

FT: $(-i\omega + k^2\sigma_0\chi_{nn})\delta\tilde{n} + ik\tilde{E}_x = 0 \qquad \tilde{E}_x = i\omega\delta\tilde{A}_x + ik\delta\tilde{A}_t$

$$\delta\tilde{n} = \frac{-ik}{-i\omega + k^2\sigma_0\chi_{nn}}(i\omega\delta\tilde{A}_x + ik\delta\tilde{A}_t)$$

$\Rightarrow$ diffusion mode $\omega = ik^2 D_n$, $D_n = \chi_{nn}\sigma_0$

Kinetic theory: (relativistic) Romatschke 1512.02641

$$p^\nu\partial_\nu f + F^{\alpha\beta}p_\beta\nabla_{p^\alpha}f = \underbrace{\frac{p^\alpha u_\alpha}{\tau}(f - f_{eq})}_{RTA} \qquad f_{eq} = e^{\frac{p^\alpha u_\alpha + \mu(t,x^i)}{T}}$$

local eq distr. fn

$$J^\mu = \int \underbrace{\frac{d^3p}{(2\pi)^3}\frac{p^\mu}{p^0}}_{\text{Lorentz invt measure}} f$$

See also Bajec et al 2403.17769: $\mu\neq 0$

Belitz-Kirkpatrick 2112.14246 FLs

$\tilde{f} = f_0 + \delta\tilde{f}$

Linearize and solve: $\delta f_{eq} = f_0\frac{\delta\mu}{T}$, $\delta\tilde{f} = \frac{f_0}{T_0}\frac{\delta\tilde{\mu} + \tau\vec{v}\cdot\vec{E}}{1+\tau(-i\omega + \vec{k}\cdot\vec{v})}$ $\quad \vec{v} = \frac{\vec{p}}{p^0}$

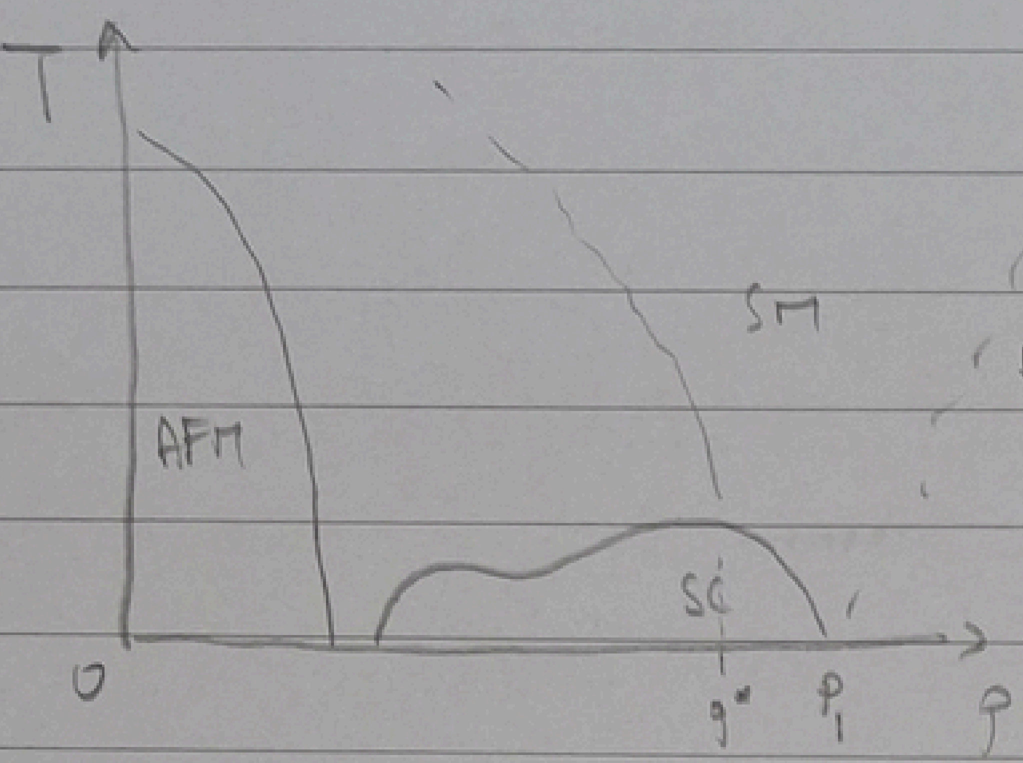
$$\Rightarrow \tilde{G}_{nn} = -\chi_{nn}\frac{1 + (i\omega - \frac{1}{\tau})\frac{L}{2ik}}{1 - \frac{L}{2ik\tau}}, \qquad L = \underbrace{\ln\left(\frac{\omega - k + i/\tau}{\omega + k + i/\tau}\right)}_{\text{ln branch cut arising from angular integral}}$$

Exact pole $\omega = -\frac{i}{\tau} + \frac{ik}{\tan(k\tau)}$ $\quad k\tau < \frac{\pi}{2}$: principal Riemann sheet

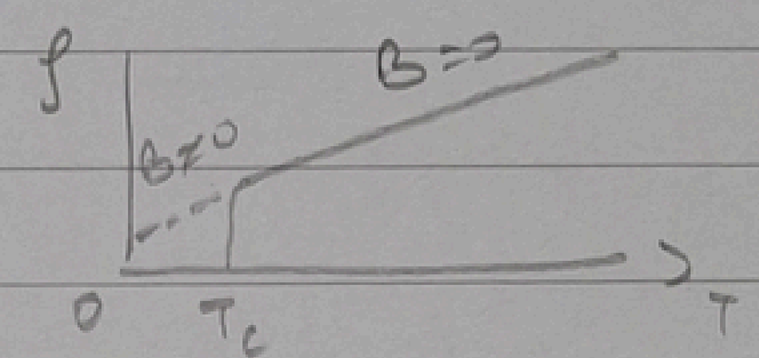
At $k = \frac{\pi}{2\tau}$, pole goes through ln b.c. bkdwn of hydro at $\tau k = \pi$ pole at ∞

At $k \ll \frac{1}{\tau}$, $\omega = -i\frac{\chi}{3}\tau k^2 + O(k^4) \to$ diffusion pole

Cuprate high T_c superconductors



At p^* $\quad \rho(T) \sim T$



Legros et al 1805.02512 studied the low T regime w/ $B \neq 0$ at p^*
Cooper et al Science 323 2009

→ SC suppressed

Show that $\rho(T) \sim T$ compatible w/ a "Planckian" relaxation rate
$\rho(T) = \frac{m^*}{ne^2}\frac{1}{\tau_{tr}}$ through the Drude formula, $\tau_{tr} = O(1)\frac{\hbar}{k_B T}$.

This is a "transport" timescale, which cannot immediately be related to a microscopic collision rate between particles.

This feature persists across the entire overdoped SM phase

$$p^* < p < p_1$$

Some success been achieved in reproducing this data, as well as more sophisticated probes (ADMR) NdLSO, LSCO
Grissonnanche et al 2011.13054
Ataei et al 2203.05035

Boltzmann eqn + magnetic field: $\frac{dp^i}{dt} = -e(E^i + \epsilon^{ijk} v_j B_k)$

→ exact solution in the dc limit Chambers formula

Main assumption: p-dependent scattering rate $\frac{1}{\tau(p)} = \frac{1}{\tau_{iso}} + \frac{1}{\tau_{aniso}(p)}$

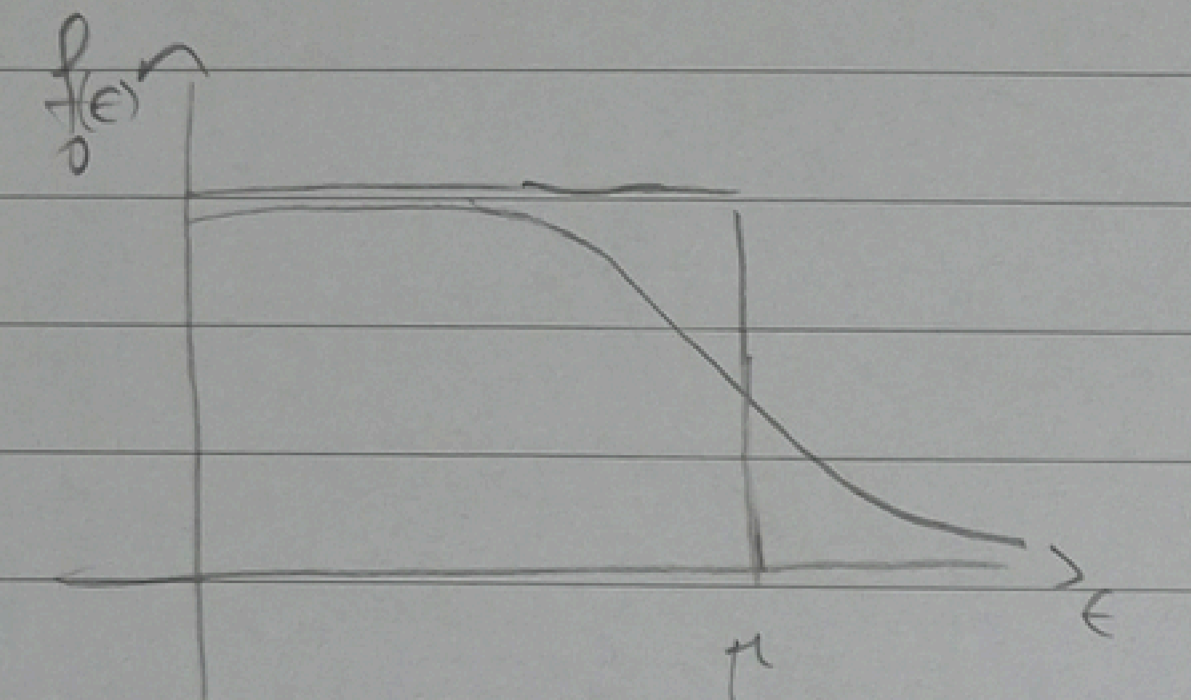
w/ $\frac{1}{\tau_{iso}} = \frac{1}{\tau_0} + \frac{a k_B T}{\hbar}$

Results seem material dependent as Abdel-Jawad et al cond-mat/0609763 Tl2201 make the opposite T-assumption.

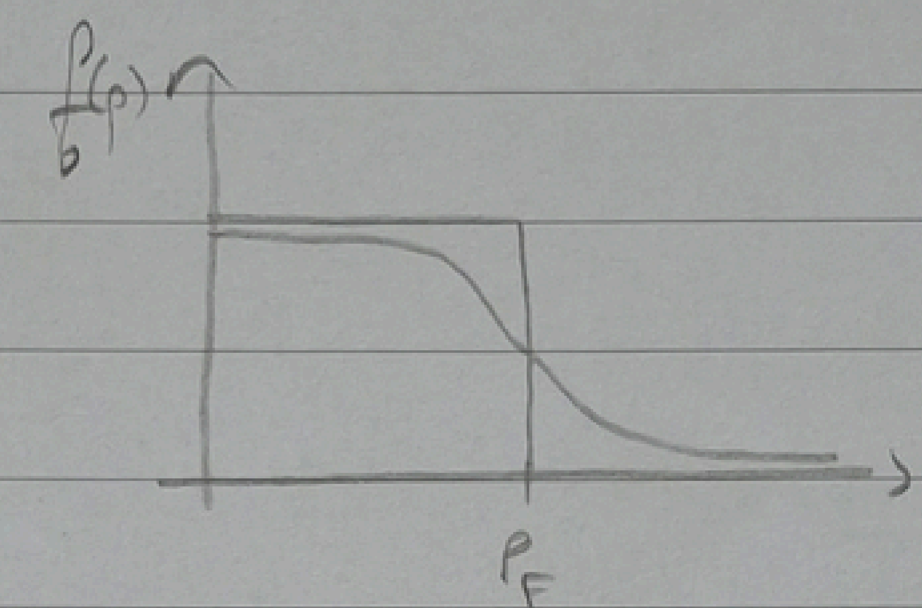
In this procedure, fit for the p-dependence of $\tau(p)$, but also for the Fermi velocity $\vec{v}_F(\vec{p}) \equiv \nabla_{\vec{p}}\, \epsilon(\vec{p})$

$\rightarrow$ Compare to ARPES measurements and verify they match.

For a simple spherical Fermi surface $\quad f_0(\epsilon) = \dfrac{1}{e^{\frac{(\epsilon-\mu)}{T}}+1}$



or

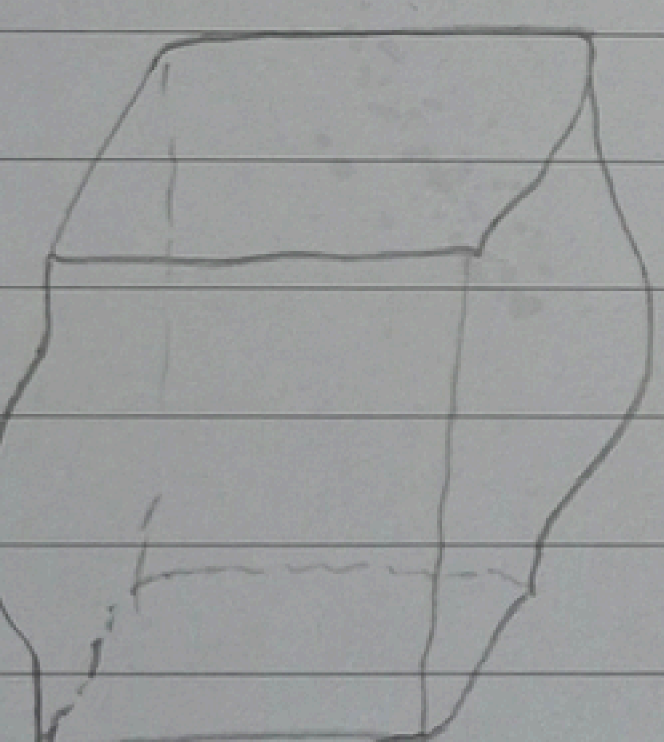


$\epsilon = \mu + v_F(p - p_F)$

$v_F = \dfrac{p_F}{m_*}$

$\Rightarrow v_F$ angle independent

Cuprates do not have a spherical FS $\Rightarrow v_F$ changes around FS



cylindrical-like FS

Chambers formula:

$$\sigma_{ij} = \frac{e^2}{4\pi^3} \int d^3p \left(-\frac{\partial f_0}{\partial \epsilon}\right) v_i[\vec{p}(t=0)] \int_{-\infty}^{0} v_j[\vec{p}(t)]\, e^{t/\tau(p)}$$

Using this formula gives reasonable agreement w/ data.

On the other hand, Ayres et al 2012.01208 fail to reproduce all aspects of magneto-transport data on Tl2201 using BTE.

Several points of caution regarding the use of BTE:

* OD cuprates are not necessarily weakly-coupled

eg $\rho(T) \sim T$ { seems incompatible w/ umklapp / disorder scattering; ARPES measures $\tau_{qp} \sim \frac{1}{T} \ll \frac{1}{T^2}$ as in FLs.

Is $\tau_{qp} \sim \tau_{tr}$ consistent?

* Effect of spatial disorder $\rightarrow$ spatially inhomogeneous τ_{qp}?

* RTA too simplistic?

DC transport data $\to$ $\omega \to 0$ limit of out-of-eq retarded Green's fn

eg $\sigma_{ij} = \lim_{\omega \to 0} \frac{i}{\omega} G^R_{J_i J_j}(\omega, k=0)$

$\to$ within hydro regime $\omega \lesssim \frac{1}{\tau_{qp}}$

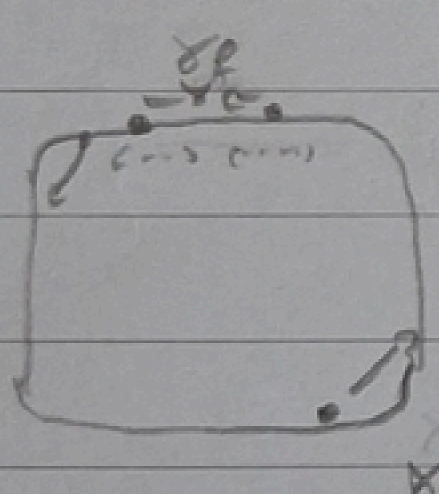
In fact, derive hydro from kinetic theory (many works in QCD litterature). Less familiar approach in traditional cond mat, since no hierarchy of scales $\tau_{qp,el} \ll \tau_{imp}, \tau_{umklapp}$ in ordinary metals.

With Graphene, electron hydro come back to the forefront.

{ RTA
{ Simple spherical FS $\to$ Galilean hydro w/ "microscopic" expressions for transport coefficients:

$$\kappa = c_p \frac{1}{3} (v_F^*)^2 \tau \quad , \quad \eta = \frac{nm}{5} (v_F^*)^2 \left(1 + \frac{f_1}{3}\right)$$

[Belitz and Kirkpatrick 2112.14246]



Square FS w/ rounded edges: Cook & Lucas 1503.05652

Two rates. Breaking of Galilean symmetry (and isotropy)

* Back to OD cuprates, it is unclear which hydro theory emerges from the $\tau(p)$ and geometry of FS $\to$ impact on whether Drude ρ correct
* Further, not obvious as well whether the match to data is a true property of the underlying hydro description or whether the emergent hydro suffices.

One major difference between kinetic theory and hydrodynamics:

hydro only has gapless modes, while kinetic theory also includes gapped excitations (usually branch cuts)
$\to$ makes the $\omega \ll \frac{1}{\tau}$ limit non-trivial.

To better understand which hydro emerges, necessary to incorporate momentum relaxation (metals)

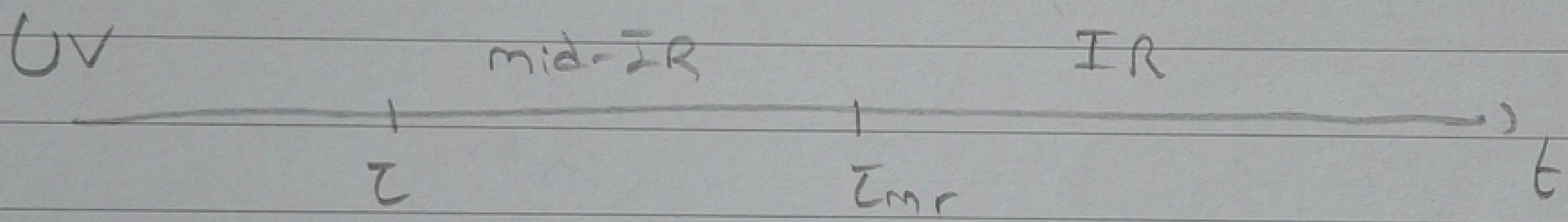
$$\partial_t g^i + \partial_j \tau^{ji} = f^i$$

Logic: expand f^i as any other hydro current.

Why does this make sense? f^i in general not a conserved current $\rightarrow$ decays to local eq on scales of $O(\tau, \ell)$

$\Rightarrow$ its ev $\langle f^i \rangle$ can be expanded in the usual way (with a few bells and whistles, see below).

Thus we have something like this



The goal of the momentum-relaxed theory is to describe both IR and mid-IR regimes.

IR: true hydro theory, w/ only conserved currents
$\rightarrow$ charge, energy

mid-IR: hydro th$^{=}$ w/ approximate symmetry
$\rightarrow$ leading to gapless + slow modes (momentum)
on these scales, momentum is almost conserved.