I. Hydro w/ approximate spatial translations

local thermodynamic equilibrium : $x^\mu \gg x^\mu_{UV}$

At x^μ_{UV}, microscopic dofs become important

Set of densities of (almost) conserved operators :

$$\langle O^A \rangle \; , \quad A = 1 \ldots N \qquad \langle . \rangle = \frac{1}{Z} \mathrm{Tr}\left(. \, e^{\beta H} \right)$$

$$\overset{\text{eg}}{\parallel}$$

$$\{ \varepsilon, g^i, n, \ldots \}$$

All fields fns of x^μ

$$\boxed{\partial_t O^A + \underbrace{\partial_i j^{iA}}_{\text{spatial fluxes}} = \underbrace{f^A}_{\text{symmetry-breaking current.}}}$$

1st law : $d\left(\frac{P}{T}\right) = O^A \underbrace{ds_A}$
conjugate thermodynamic sources

eg $\left\{ -\frac{1}{T}, \frac{v^i}{T}, \frac{\mu}{T}, \ldots \right\}$

where $\frac{P}{T} = s + O^A s_A \qquad ds = - s_A dO_A$

Write down the local second law :

$$\boxed{\Delta = T \partial_t s + T \partial_i s^i \geq 0}$$

Use the first law and rearrange as ;

$$\boxed{\Delta = - \tilde{j}^{iA} \, \delta \partial_i s_A - \tilde{f}^i \delta v_i}$$

and $j^{Ai} = \bar{j}^{Ai} + \tilde{j}^{Ai}$ $\qquad \boxed{\bar{j}^{Ai} = \frac{v^i}{T} \frac{\delta P}{\delta s_A} + P \delta^{Ai}}$

$$f^i = \bar{f}^i + \tilde{f}^i$$

- At ideal level and in absence of expl. breaking, recovers usual fluid hydro.

- $\delta \partial_i s_A$ means non-hydrostatic derivative terms, ie vanishing on hydrostatic equilibrium configurations $O^A = \bar{O}^A$, $s_A = \bar{s}_A$

- $\exists$ non-ideal terms which do not vanish on eq config $\bar{\Delta} = 0$
  → corrections away from homogeneous eq
  → can be repackaged in redef. of the O^A's $\to \bar{O}^A$

- there is a new non-hydrostatic source caused by expl. bkg
  $\delta v^i = v^i - \bar{v}^i$

Now the task is to write non-hydrostatic terms:

$$\begin{pmatrix} \tilde{j}^{Ai} \\ \tilde{f}^i \end{pmatrix} = \begin{pmatrix} \alpha^{AiBj} & \beta^{Aij} \\ \gamma^{iBj} & \kappa^{ij} \end{pmatrix} \begin{pmatrix} \delta \partial_j s_B \\ \delta v_j \end{pmatrix}$$

and impose $\begin{cases} \Delta \geq 0 & \text{(positive quadratic form)} \\ \text{Onsager reciprocity relations} \end{cases}$

- $\tilde{f}^i$ is expanded as any other current

- $\alpha, \beta, \gamma, \kappa$ differential operators

- symmetric terms contribute to Δ: dissipative
  anti-symmetric terms do not contribute to Δ: non-dissipative
  still, non-equilibrium

Recovers all the usual terms from fluid hydro

→ terms due to expl. bkg

Leading term contributes to κ^{ij}:

$$\kappa^{ij} = -\chi_{gg}\,\gamma\,\delta^{ij} - \gamma_h\,\epsilon^{ij} \quad \Rightarrow \Delta \supset \gamma(\delta v)^2 \Rightarrow \gamma \geq 0$$

γ leads to momentum relaxation

$$\partial_t g^i + \partial_j \tau^{ji} = -\gamma\,\chi_{gg}\,\delta v^i - \gamma_h\,\epsilon^{ij}\delta v^j \quad \Rightarrow g^i \sim e^{-\gamma t} \quad \text{or} \quad G^R_{g^i g^j} = \delta^{ij}\frac{\chi_{gg}}{\gamma - i\omega}$$

$$\Rightarrow \omega = -i\frac{\gamma}{\chi_{gg}} \quad \text{Drude pole}$$

γ_h is an anti-sym term allowed when parity is broken

$\rightarrow$ magnetic fields. Eg then $f^i = j_k F^{ik} = nBv^k\epsilon^{ik} + \dots$

Then, "microscopic" formula for $\gamma_h = nB$ and cyclotron resonance.

pole at $\omega = \pm\omega_h - i\gamma$, $\omega_h = \frac{\gamma_h}{\chi_{gg}}$

The point is no need to assume B in order to predict such a pole $\rightarrow$ breaking of translations and parity.

At the next order:

$$\text{from } \gamma^{iBi},\quad \tilde{f}^i = -\gamma\,\delta v^i - \gamma_h\,\epsilon^{ij}\delta v^j - \lambda_n\,\partial^i\mu - \lambda_s\,\partial^i T - \lambda_g\,\partial_t\,\delta v^i$$

$$\text{while}\quad \tilde{j}^i = \lambda_n\,\delta v^i + \dots$$

$$\tilde{s}^{\delta v^i} = \lambda_s\,\delta v^i + \dots$$

see also 2112.14373 Armas & Jain

$\lambda_n, \lambda_s, \lambda_g$ are all <u>non-dissipative</u> <u>out-of-eq</u> <u>terms</u>.

Lead to <u>shifts</u> <u>in cyclotron</u> <u>resonance</u> and <u>weight</u> <u>of</u> <u>Drude pole</u>

$$\boxed{\sigma_{ii} = \frac{i}{\omega}G^R_{j^i j^i} = \frac{(n+\lambda_n)^2}{\chi_{gg}+\lambda_g}\,\frac{1}{\gamma - i\omega}} \qquad \omega_c = \frac{n + 2\lambda_n}{\chi_{gg}+\lambda_g}B$$

So far, no reference to any specific mechanism for broken $k^{\wedge}$.

Break translations through inhomogeneous chemical potential:

$\boxed{\mu_{eq}(x) = \mu_0 + \varepsilon \bar{\mu}(x)}$ $\varepsilon \ll 1$ to be in hydro $\sim |\partial_x \mu|/\mu_0$

Option 1: $\bar{\mu}(x)$ could have a random distribution → relevant for eg Graphene
case analyzed in the dc limit by
Andreev Kivelson Spivak 1011.3068
Lucas 1506.02662

$\boxed{\text{Kinetic thy in RTA: } \text{absent? } 2403.17769}$

Advantage: use usual hydro eqn $\boxed{\begin{aligned} \nabla_\mu T^{\mu\nu} &= J_\lambda F^{\nu\lambda} \\ \nabla_\mu J^\mu &= 0 \end{aligned}}$

Derive expression for γ order by order in ε : $\boxed{\gamma \sim O(\eta, \sigma_0)}$

Technically non-trivial to go beyond leading order.

Option 2: take $\bar{\mu}(x)$ periodic "lattice"

Chagnet & Schalm 2303.17685, 2409.11035 (et al)
integrate over $\bar{\mu}(x)$ and they could obtain $\Delta_{n,s,g}$ and match to the previous approach.

Holography: Option 2 matched by constructing corresponding inhomogeneous spatially periodic BHs, extracting their dc transport conductivities and longest-lived QNMs

Option 3: $\boxed{\mathcal{L} = R - 2\Lambda - \frac{1}{2}\sum_I (\partial \psi_I)^2 - \frac{1}{4} F^2 , \quad \psi_I = m\, \delta_{Ii}\, x^i}$

Andrade and Withers 1311.5157
homogeneous model of momentum relaxation
Transport properties at $\omega, k \neq 0$ further studied in
Davison & Goutéraux 1411.1062, 1505.05092) GR
Blake 1505.06992, 1507.04870) Fluids gravity

In 2309.04033 w/ Shukla, we demonstrate complete matching w/ these results.

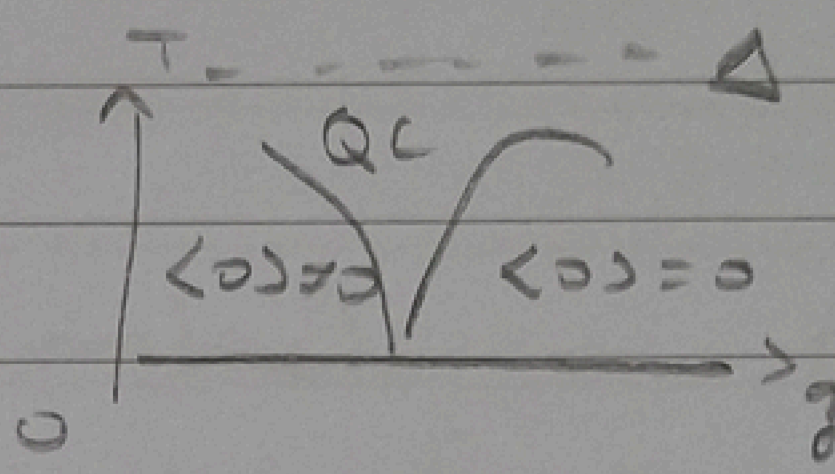
Experimental relevance : Strange metallic phase of cuprates

Measure ac conductivity, $\sigma(\omega) \equiv \frac{i}{\omega} G^R_{JJ}(\omega)$

- weak-momentum relaxation : $\sigma(\omega) = \frac{n^2}{\chi_{JJ}} \frac{1}{\Gamma - i\omega}$ Drude

or w/ B field : $\sigma_\pm(\omega) = \sigma_{xx} \pm i\sigma_{xy} = \frac{n^2}{\chi_{JJ}} \frac{1}{\Gamma - i(\omega \mp \omega_c)}$

- quantum critical (QPT)

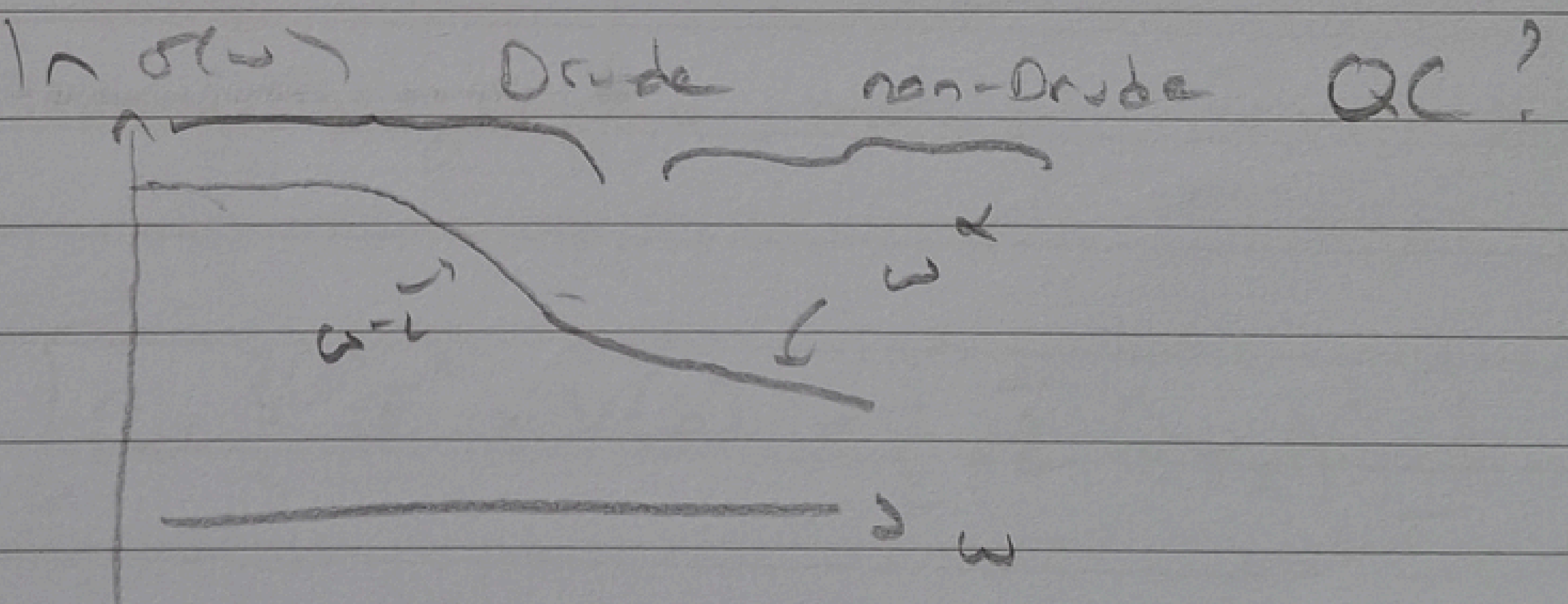


=> emergent scale invariance CFT_{IR}

$\sigma(\omega, T) = \Sigma(\omega/T)$ → expect poles at $\omega \sim T$

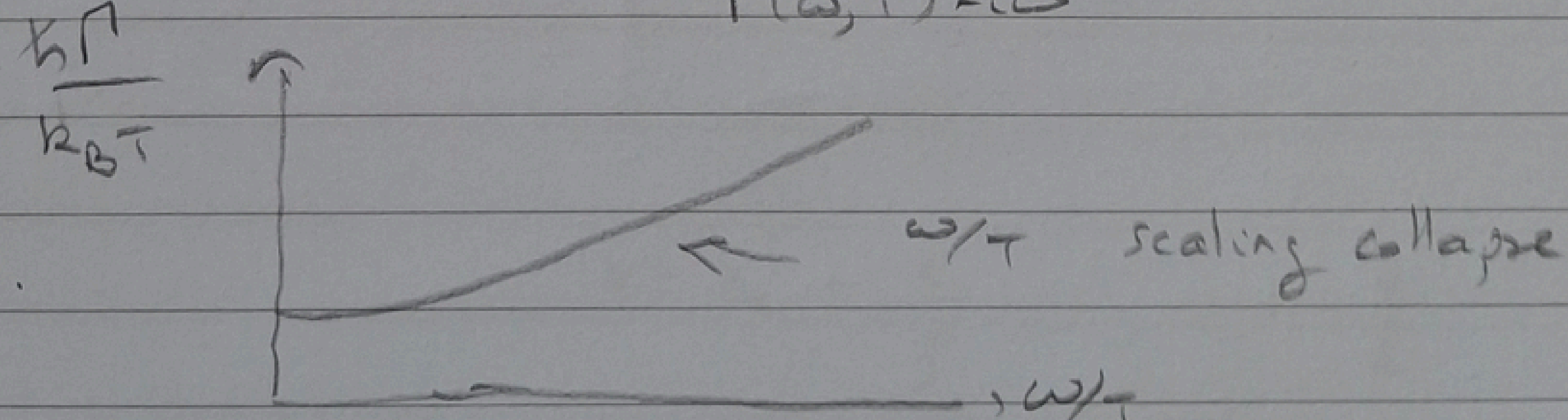
for $\omega \lesssim \Delta$ ← UV scale

In cuprates
cond-mat/0309172
2205.00899

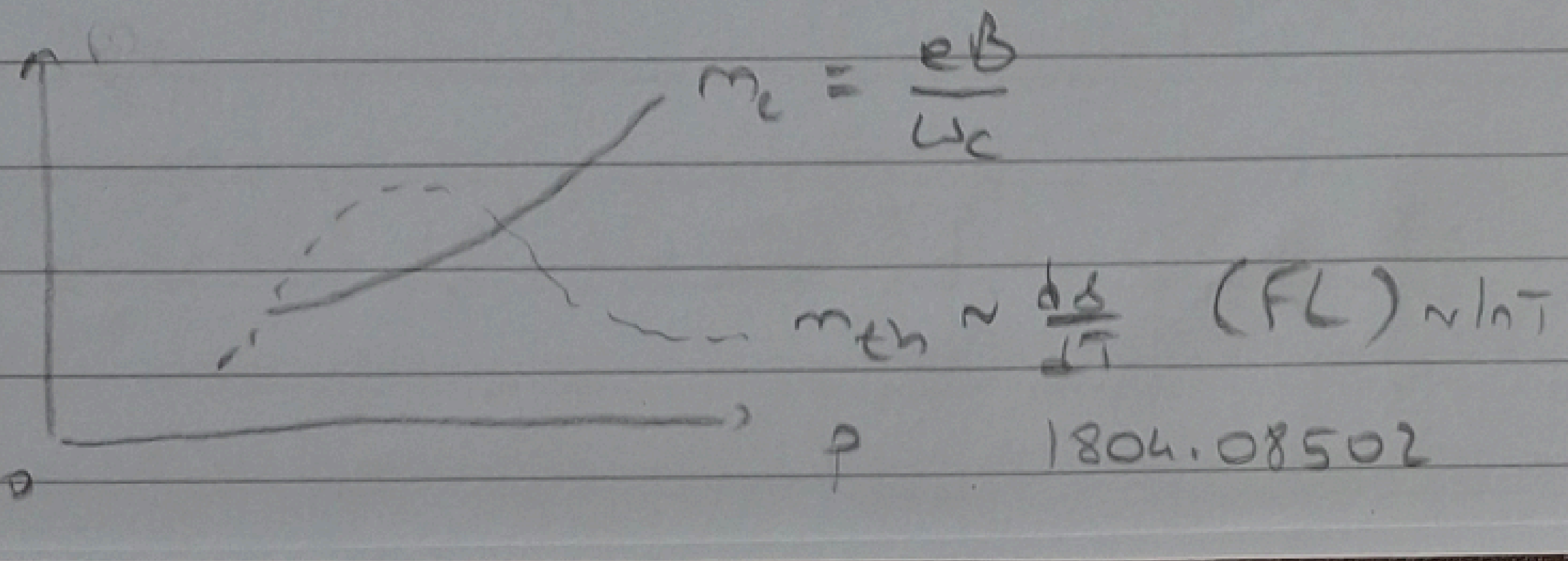


Further, define $\sigma(\omega, T) = \frac{K}{\Gamma(\omega, T) - i\omega}$ and extract Γ

2205.04030



2205.12444
Magnetoconductivity



1804.08502

$\underline{m_{th}}$: $\underbrace{\text{thermodynamic measurement}}$ → insensitive to out-of-eq
correlations $\lambda_{g,\eta,s}$

$\underline{m_e}$: $\underbrace{\text{spectroscopic}}$ $\underbrace{\text{measurement}}$ → linear response, out of eq
→ sensitive to $\lambda_{g,\eta,s}$

Calculation of $\lambda_{g,\eta,s}$ microscopic with disorder?

→ $\underline{\text{Y-SYK models}}$ 2308.01956
found deviations between m_{th} and m_e but no
connection made to EFT.

$\underline{\text{Cuprate}}$ $\underbrace{\text{strange}}$ metals → motivate us to consider
interplay between slow-mode and tower of critical
modes.

Let's investigate this in the context of spacetimes w/
infrared near-extremal geometry $AdS_2 \times \mathbb{R}^2$.