Quantum-Gravity-Corrected Wilson Loop Correlators and Super-Yang-Mills Theory at Infrared

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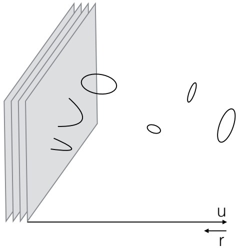
Holographic Applications:
From Quantum Realms to the Big Bang
Beijing, July 14, 2025

Based on arXiv:2412.11107 and work in progress with
Xiao-Long Liu, Cong-Yuan Yue, Wenni Zheng, Mei Huang

AdS/CFT Correspondence

- Maldacena (1997):

$$\text{4D } U(N)\ \mathcal{N}=4 \text{ SYM} \quad \Leftrightarrow \quad \text{IIB string theory on } AdS_5 \times S^5$$



- Gubser, Klebanov, Polyakov (1998); Witten (1998):

$$Z_{CFT} = Z_{AdS}\,, \quad Z_{AdS} \approx Z_{SUGRA} \text{ when } \lambda \gg 1 \text{ and } N \gg 1$$

$$N^2 = \frac{\pi}{2}\frac{L^3}{G_5}\,, \quad \lambda \equiv g_{YM}^2 N = \left(\frac{L}{\ell_s}\right)^4$$

- Rey, Yee (1998); Maldacena (1998):
  The AdS/CFT cannot produce the linear term in $V_{q\bar{q}}$ at $T=0$.

Quantum Gravity (QG)

- How to compute quantum gravity corrections?
  - UV quantum gravity:
    Introduce higher-curvature terms in (super-)gravity action, suppressed by $\frac{1}{M_p}$.
  - IR quantum gravity:

    Quantum fluctuations vs Thermal fluctuations

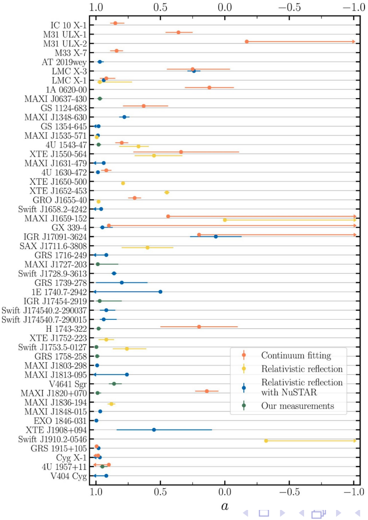
    The quantum fluctuations become manifest as $T \to 0$. All the physical observables should be averaged within an appropriate IR quantum gravity path integral.
- How to possibly observe quantum gravity effects?

  gravitational waves & black hole image (photon ring)

Do real black holes have low temperatures?

- Black holes from astrophysical measurements:

('23 Draghis et al.)



Higher-D Black Hole's Near-Horizon Region

- Schwarzschild black holes:

  no near-extremal limit, no near-horizon AdS throat

- (Near-)extremal Reissner-Nordström black holes:
  ('14 Almhieiri, Polchinski; '16 Maldacena, Stanford, Yang)

  near-horizon AdS_2 throat $\Rightarrow$ AdS_2 JT/Schwarzian

- (Near-)extremal D1-D5 black holes: ('18 Cotler, Jensen)

  near-horizon BTZ region $\Rightarrow$ AdS_3 gravity/Cotler-Jensen

- (Near-)extremal Kerr black holes:
  ('08 Guica, Hartman, Song, Strominger)

  near-horizon NHEK geometry = warped $\text{AdS}_3 \ltimes [0, \pi]$

  $\Rightarrow$ warped AdS_3 gravity / new 2d dual theory ('25 **JN**, Tian)

Quark Confinement

- Why we never observe a free quark directly?

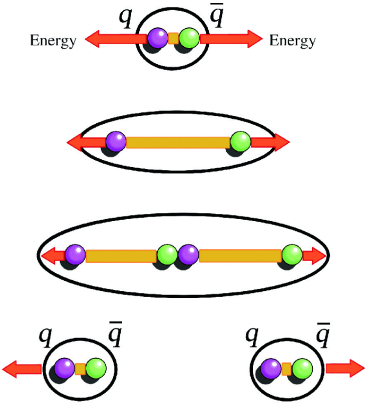
Answer: quark (color) confinement

Related problem: Yang-Mills mass gap problem

(one of the Millennium Problems in mathematics)

Qualitative Explanation

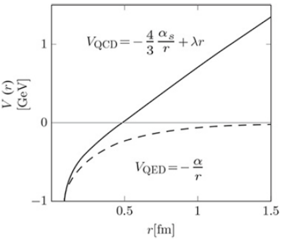
- How do we qualitatively explain confinement?

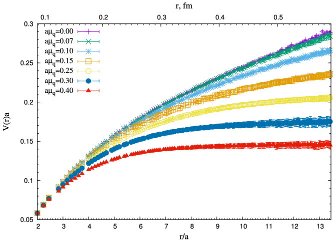


String theory was originally invented to solve this problem!

Quantitative Expectation

- How do we quantitatively describe confinement?
  The potential between a quark and an anti-quark pair:





- Can we theoretically compute this potential?
  Wilson loop $\langle W \rangle$ + quantum-corrected AdS/CFT

Wilson Loop

- Given a closed loop $\mathcal{C}$ in spacetime.
- A Wilson loop in Yang-Mills theory is defined as

$$W(\mathcal{C}) \equiv \frac{1}{N} \mathrm{Tr} \left(P\, e^{i \int_{\mathcal{C}} dx^\mu A_\mu} \right) .$$

- $W(\mathcal{C})$ is a gauge-invariant quantity, and captures many important features of Yang-Mills theory.
- All the QCD observables can be reformulated in terms of $\langle W \rangle$, $\langle WW \rangle$, $\cdots$

Outline

Quantum Gravity

In a quantum gravity (QG) theory, we should be able to evaluate:

- The partition function:

$$Z = \int [Dg_{\mu\nu}]\, [D\phi]\, e^{-S_{grav}[g_{\mu\nu}] - S_{matter}[\phi,\, g_{\mu\nu}]}$$

- The correlation functions, e.g., for $g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$

$$\langle h_{\lambda\xi} h_{\rho\sigma} \rangle = \int [Dh_{\mu\nu}]\, h_{\lambda\xi} h_{\rho\sigma}\, e^{-S_{grav}[\bar{g}_{\mu\nu} + h_{\mu\nu}]}$$

$$\text{where} \qquad S_{grav} = \int d^D x\, \sqrt{g}\, (R + \Lambda)$$

  We also want to compute the matter correlators $\langle \phi\phi \rangle$.

- There are still many open issues (e.g., renormalization and black hole singularity).

JT Gravity to Schwarzian

- Start from the JT gravity:

  ('14 Almhieiri, Polchinski; '16 Maldacena, Stanford, Yang)

$$I_{JT} = -\frac{1}{16\pi G}\left[\int d^2x\,\sqrt{g}\,\phi\,(R+2) + 2\int_{bdy}\phi_b\,K\right]$$

- Plugging the bulk equations of motion back into the action:

$$I = -\frac{1}{8\pi G}\int dt\,\phi_r(t)\,Sch(f,t)\,,\quad Sch(f,t) \equiv -\frac{1}{2}\frac{f''^2}{f'^2} + \left(\frac{f''}{f'}\right)'\,.$$

  where f is reparametrization zero modes of boundary time u:

$$f(t) = t + \varepsilon(t)\,,$$

  and the boundary value of dilaton is

$$\phi_b = \phi|_{bdy} = \frac{\phi_r(t)}{\epsilon}\,.$$

Schwarzian as Toy Model QG

- We assume $\phi_r(u)$ to be a constant denoted by $\bar{\phi}_r$.
- Defining $C \equiv \bar{\phi}_r/(8\pi G)$, we obtain the Schwarzian action:

$$I_{Schw}[f] = -C \int dt\, Sch(f, t)$$

- The 2-point correlation functions can be computed:
  ('16 Maldacena, Stanford, Yang; '19 Qi, Sin, Joon)

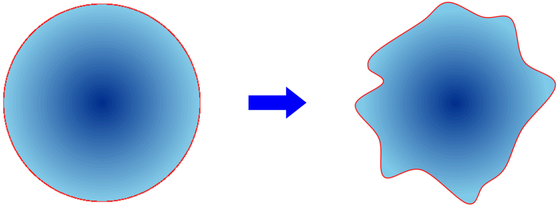
$$\langle \varepsilon(u)\, \varepsilon(0) \rangle = \frac{1}{2\pi C} \left[-\frac{(|u| - \pi)^2}{2} + (|u| - \pi) \sin |u| + a + b \cos u \right]$$

  which plays the role of $\langle h_{\lambda\xi} h_{\rho\sigma} \rangle$ in this toy model QG.

- For a matter field χ on the fluctuating AdS$_2$ spacetime background, its 2-point function $\langle \chi(u)\, \chi(0) \rangle$ can also be computed, which plays the role of $\langle \phi\phi \rangle$.

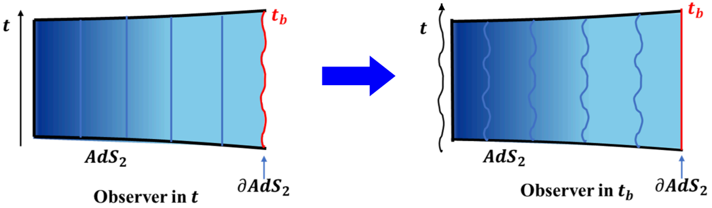
Bulk vs Boundary in Schwarzian Theory

- ('14 Almhieiri, Polchinski; '16 Maldacena, Stanford, Yang)
  The only physical degree of freedom is the boundary's reparametrization $f(t)$.



Embedding of AdS_2 into $\mathrm{AdS}_{D\geq 4}$

- ('20 Iliesiu, Turiaci; '24 Liu, Yue, **JN**, Zheng)



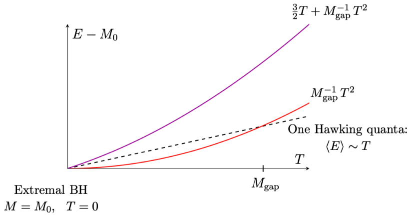
- By requiring continuity and smoothness conditions, we can embed AdS_2 with quantum fluctuations into $\mathrm{AdS}_{D\geq 4}$.

Resolving Radiation Paradox

- The quantum corrections to S and M can be computed:
  ('20 Iliesiu, Turiaci; '19 Larsen, **JN**, Zeng; '20 David, **JN**)

$$S(\beta, Q) = S_0 + \frac{4\pi^2 \phi_{b,Q}}{\beta} - \frac{3}{2} \log \frac{\beta}{e\, \phi_{b,Q}}\,, \quad M(\beta, Q) = M_0 + \frac{2\pi^2 \phi_{b,Q}}{\beta^2} + \frac{3}{2\beta}\,.$$

- Consequently, a paradox of Hawking radiation is resolved:
  ('91 Preskill, Schwarz, Shapere, Trivedi, Wilczek; '20 Iliesiu, Turiaci)



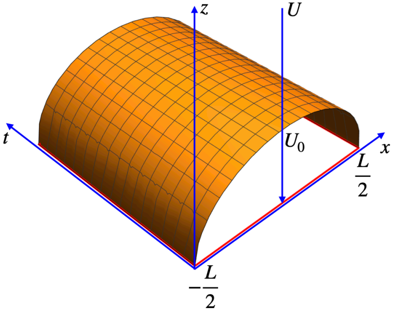
- Quantum corrected greybody factor of asymptotically flat RN black holes ('24 Brown, Iliesiu, Penington, Usatyuk; '25 Lin, Iliesiu, Usatyuk)
- Quantum corrected absorption cross section of black holes: (2501.17470 Emparan)

Applications

- Quantum corrected holographic strange metal:
  (2410.11487 Liu, **JN**, Pando Zayas)
- Quantum corrected Wilson loop and SYM confinement:
  (2412.11107 Liu, **JN**, Yue, Zheng)
- Quantum corrected quasi-normal modes:
  (2506.22945 Jiang, **JN**, Cai, Tian, Zhang)
- Quantum corrected Wilson loop and SYM mass gap:
  ('25 Liu, **JN**, Yue, Zheng, Huang)
- Quantum corrected fluid/gravity correspondence:
  ('25 **JN**, Pando Zayas, Yue) (See Cong-Yuan Yue's talk)
- Quantum corrected the ratio of shear viscosity and entropy:
  ('25 Cremonini, Liu, **JN**, Li)

Holographic $\langle W \rangle$

- ('98 Rey, Yee; '98 Maldacena)



$S_{NG} \propto$ (the minimal surface in AdS anchored to the Wilson loop).

$$e^{-T \cdot V_{q\bar{q}}(L)} \sim \langle W \rangle \sim e^{-S_{NG}} \quad \Rightarrow \quad V_{q\bar{q}} = -\frac{4\pi^2 \left(2 g_{YM}^2 N\right)^{1/2}}{\Gamma\left(\frac{1}{4}\right)^4 L}$$

- ('98 Witten; '98 Rey, Theisen, Yee; '98 Brandhuber, Itzhaki, Sonnenschein, Yankielowicz; '99 Greensite, Olesen) At finite temperature, holographic Wilson loop $\langle W \rangle$ can produce confining $V_{q\bar{q}}$.

Quantum-Corrected AdS_2

- The AdS_2 metric can be rewritten in the light-cone coordinates:

$$ds^2 = -L^2 \frac{4\pi^2}{\beta^2} \frac{du\, dv}{\sin^2 \frac{\pi}{\beta}(u-v)}\,,$$

where $u \equiv t + z$, $v \equiv t - z$.

- Let u and v have quantum fluctuations:

$$u \to f(u)\,, \quad v \to f(v)\,,$$

the metric becomes

$$ds^2 = -L^2 \frac{4\pi^2}{\beta^2} \frac{f'(u)\, f'(v)}{\sin^2 \frac{\pi}{\beta}\,[f(u) - f(v)]}\, du\, dv \equiv -4\, L^2 \mathcal{O}(u,\, v)\, du\, dv\,,$$

where $\mathcal{O}$ is called a bilocal operator. ('16 Maldacena, Stanford, Yang)

- The quantum-corrected metric is ('18 Blommaert, Mertens, Verschelde)

$$\langle ds^2 \rangle = -4\, L^2 \langle \mathcal{O}(u,\, v) \rangle\, du\, dv = \langle G_{\partial\partial}(2z) \rangle\, (dt^2 + dz^2)$$

with

$$\langle G_{\partial\partial}(2z) \rangle = \frac{1}{2C} \int_0^\infty d\omega\, \sinh(2\pi\sqrt{2C\omega})\, e^{-\omega z}\, \Gamma(1 \pm i\sqrt{2C\omega})^2\,.$$

Quantum Corrected $\langle W \rangle$ and Super-Yang-Mills Confinement

('24 Liu, Yue, **JN**, Zheng)

- Near-extremal AdS_5 Reissner-Nordström black brane:

$$ds^2 = \frac{u^2}{L_{\text{AdS}}^2} f(u)\, dt_E^2 + \frac{L_{\text{AdS}}^2}{u^2} \frac{du^2}{f(u)} + \frac{u^2}{L_{\text{AdS}}^2} d\vec{x}^{\,2}$$

- After some changes of coordinates and introducing quantum fluctuations in near-horizon AdS_2, we obtain

$$ds^2 = \alpha' \left[\frac{U^2}{R^2} \left(f(U)\, h(U)\, dt_E^2 + d\vec{x}^{\,2} \right) + R^2 f^{-1}(U)\, h(U)\, \frac{dU^2}{U^2} \right]$$

with $\quad R^2 \equiv L_{AdS}^2/\alpha\,, \quad$ a constant C with length dimension,

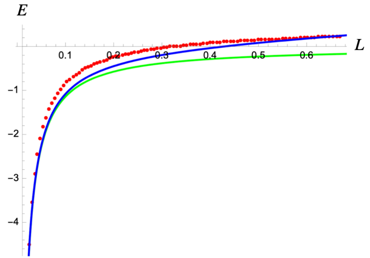
$$h(U) = 1 + \frac{R^4}{96\,\pi^4 C^2 (U - U_T)^2} - \frac{5 R^6}{1152\, C^3\, (\pi^6 (U - U_T)^3)} + \frac{35 R^8}{18432\, \pi^8 C^4 (U - U_T)^4} + \cdots$$

Quantum Corrected $\langle W \rangle$ and Super-Yang-Mills Confinement

('24 Liu, Yue, **JN**, Zheng)

- Using this quantum-corrected AdS_5 metric, we can compute $\langle W \rangle$ and then extract the quark anti-quark potential:

$$V_{q\bar{q}} = -\frac{4\pi^2}{\Gamma\left(\frac{1}{4}\right)^4}\frac{R^2}{L} + \frac{5\,\Gamma\left(\frac{1}{4}\right)^2}{4608\,\pi^5\,\Gamma\left(\frac{3}{4}\right)^2}\frac{R^2}{C^2}L + \mathcal{O}\left(C^{-3}, \frac{U_T}{U_0}\right)$$



Green: Coulomb potential

Blue: analytic expansion

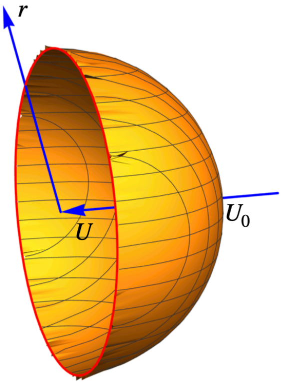
Red: numerical result

- Within AdS/CFT, confinement comes from quantum gravity!

Circular Spatial $\langle W \rangle$ and Super-Yang-Mills Confinement

('24 Liu, Yue, **JN**, Zheng)

- We can also consider a circular spatial $\langle W \rangle$:



Circular Spatial $\langle W \rangle$ and Super-Yang-Mills Confinement

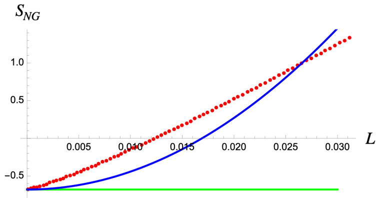
('24 Liu, Yue, **JN**, Zheng)

- From $\langle W \rangle$, we can extract the quark anti-quark potential:

$$S_{\text{NG}} = \frac{\pi R^2}{12}\left(-{}_3F_2\left(\frac{1}{2},\frac{1}{2},\frac{3}{4};1,\frac{7}{4};1\right) - \frac{24\pi^2}{\Gamma\left(\frac{1}{4}\right)^4}\right) + \frac{BR^2\,\Gamma\left(\frac{1}{4}\right)^4}{2\pi^3}\frac{L^2}{C^2} + \mathcal{O}\left(C^{-3},\, U_T/U_0\right)\,,$$

which obeys the area law!

- Numerical results:



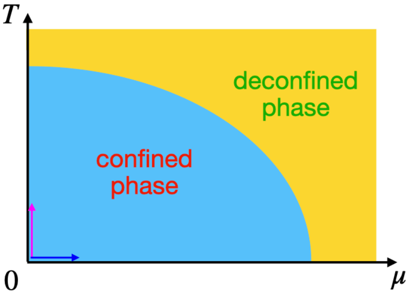
Green: constant (classical)

Blue: analytic expansion

Red: numerical result

How to Understand The Result?

- ('98 Witten)
  At finite temperature, the confinement can be obtained, i.e., the conformal symmetry is broken.
- ('24 Liu, Yue, **JN**, Zheng)
  Our works corresponds to finite chemical potential at zero temperature.
- SYM phase diagram: ('06 Yamada, Yaffe)



Quantum Phase Transition?

- An extremal RN black hole has

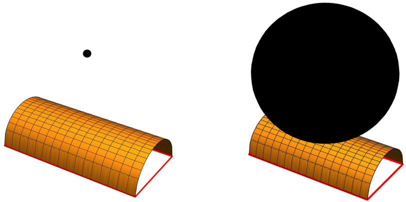
$$M = Q \sim \mu \sim r_+$$

- Recall that $V_{q\bar{q}}$ at $T = 0$ has no μ-dependence:

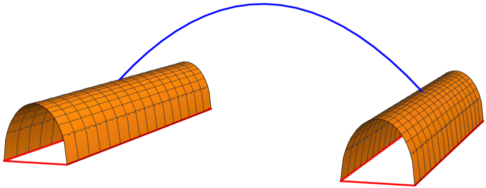
$$V_{q\bar{q}} = -\frac{4\pi^2}{\Gamma\left(\frac{1}{4}\right)^4}\frac{R^2}{L} + \frac{5\,\Gamma\left(\frac{1}{4}\right)^2}{4608\,\pi^5\,\Gamma\left(\frac{3}{4}\right)^2}\frac{R^2}{C^2}L + \mathcal{O}\left(C^{-3}, \frac{U_T}{U_0}\right)$$

$\Rightarrow$ μ is necessary, but quantum fluctuations are more essential.

- Quantum phase transition at $T = 0$?



$\langle WW \rangle$ and SYM's Mass Gap



- ('98 Berenstein, Corrado, Fischler, Maldacena)

$$\frac{\langle W(L)\, W(0) \rangle}{\langle W(L) \rangle \, \langle W(0) \rangle} \sim \frac{1}{L^{2\Delta}}\,.$$

- ('25 Liu, Yue, **JN**, Zheng, Huang)

$$\frac{\langle W(L)\, W(0) \rangle}{\langle W(L) \rangle \, \langle W(0) \rangle} \sim e^{-ML}\,, \quad M \propto C^{-1}\,.$$

Summary and Prospect

A few applications of near-horizon AdS_2 quantum gravity include:

- Resolution of the Hawking radiation paradox
- Quantum-gravity-corrected greybody factor of Hawking radiation
- Quantum-gravity-corrected black hole's absorption cross section
- Holographic strange metal, low-T fluid/gravity correspondence, $\cdots$
- **Holographically, super-Yang-Mills confinement originates from quantum gravity fluctuations in the near-horizon region.**

In the near future:

- Can quantum gravity fluctuations generate mass gap of SYM? We expect $\langle W(L)\, W(0) \rangle \sim e^{-ML}$ with the correct Regge behavior. ('25 Liu, Yue, **JN**, Zheng, Huang)

Thank you!