Holographic applications: from Quantum Realms to the Big Bang

Quantum critical phenomenon by holography

——the science with eight decimal places

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07/14/2025，HuaiRou，UCAS

Outlines

I. Introduction: the quantum critical phenomenon

II. Diagnosing the quantum phase transition by holography

III. Diagnosing the quantum phase transition by UV physics

Acknowledgement to collaborators：

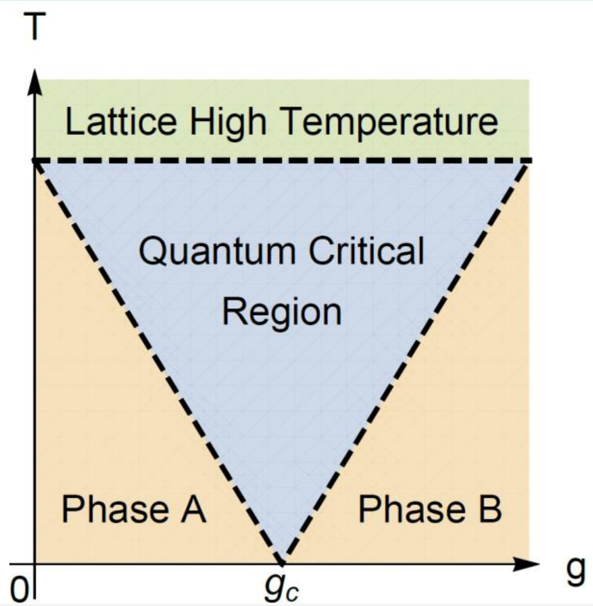
Peng Liu, Chao Niu, Jianpin Wu, Zhuoyu Xian, Zhenhua Zhou, Fang-jing Cheng, Zhe Yang, Zhou-jian Cao

Quantum critical phenomenon

Quantum critical phenomenon refers to the unusual physical properties observed in a material over a wide range of temperatures and tuning parameters near QCP.

- The (second-order) quantum phase transition occurs at T=0, which is driven by quantum fluctuations controlled by system parameters.

- The quantum critical point (QCP) is the specific value of the tuning parameter. It is a point of non-thermal instability in the ground states.



A diagram of quantum phase transition

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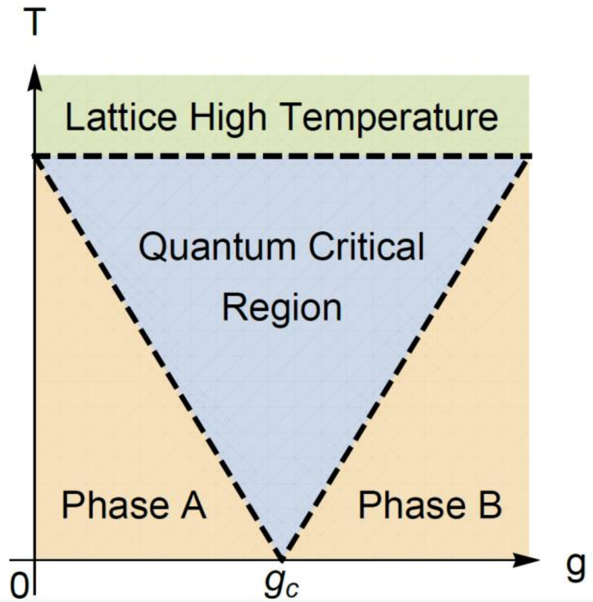

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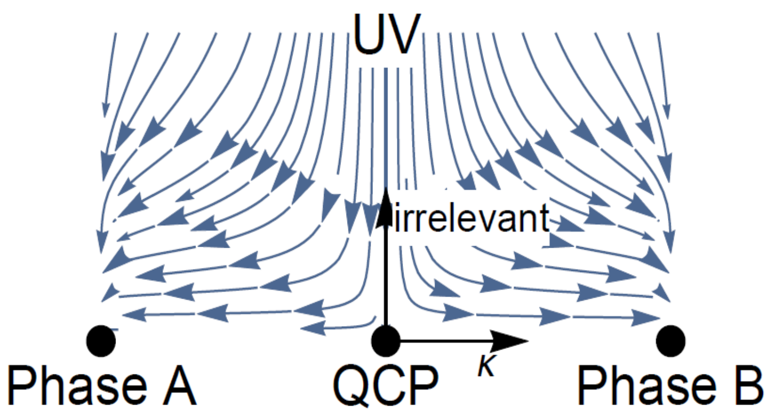
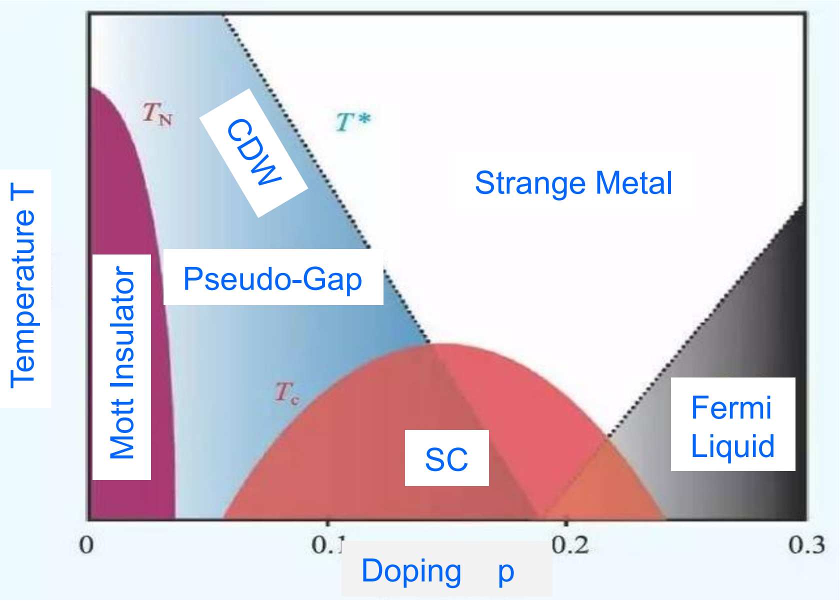


RG flow of QPT

Why is quantum critical phenomenon important

Quantum criticality is a major frontier in condensed matter physics and beyond

- Unconventional superconductivity
- Strange metals and Non-fermi liquid behavior
- Novel emergent phenomenon (emergent particles and excitations)
- Topological phases and other states



High-temperature superconductivity

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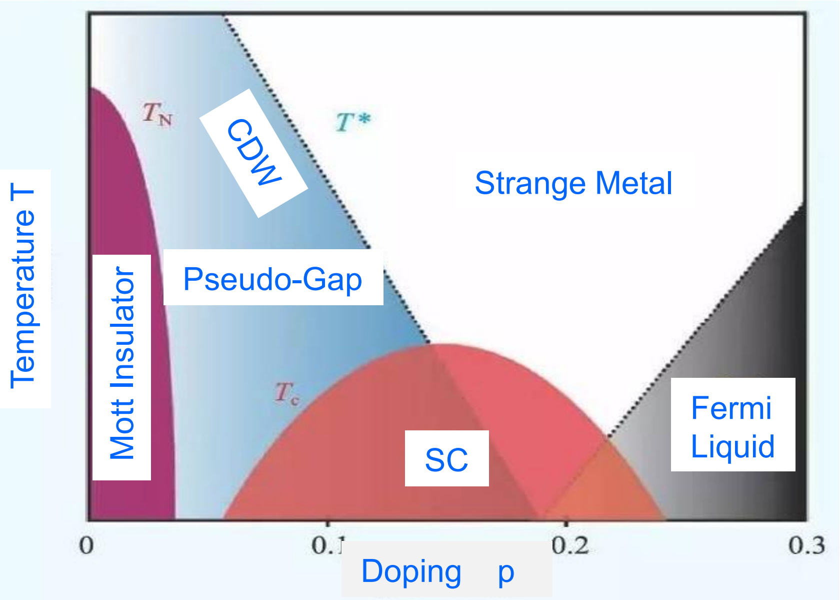
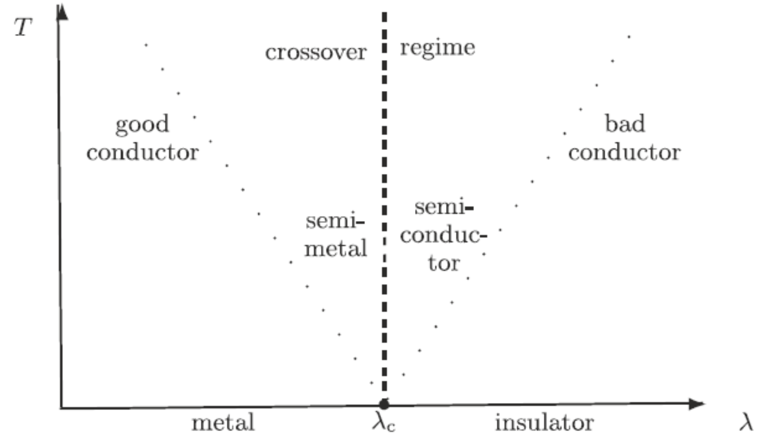


High-temperature superconductivity

$\lambda_c \propto U / J$



MIT in Mott-Hubbard model

Significant challenges and Key problems in QCP

- Inaccessibility of absolute zero temperature
- Competition between quantum and thermal fluctuations
- Theoretical complexity (Non-fermi liquid behavior)

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Quantum criticality presents profound theoretical challenges

- Strong coupling and non-perturbative physics
- Highly entangled states
- Dimensionality and dynamics
- Fractionalization and emergence
- Competition with other orders
- Numerical intractability

Holographic duality as a revolutionary tool for QCP

Quantum-critical systems → Classical gravitational theories

- Universal scaling from geometry
- Geometrical description of the entanglement entropy
- Novel phases and instabilities

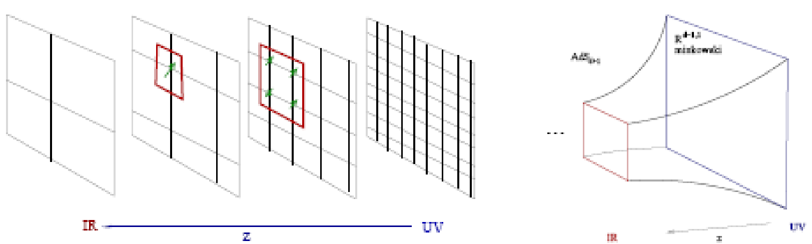

Figure 1: The extra ('radial') dimension of the bulk is the resolution scale of the field theory.

McGreevy, Adv.High Energy Phys. 2010 (2010) 723105

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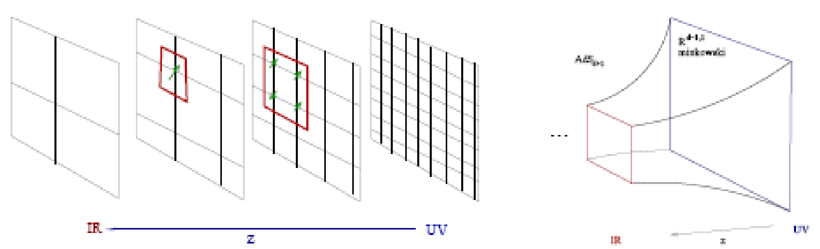

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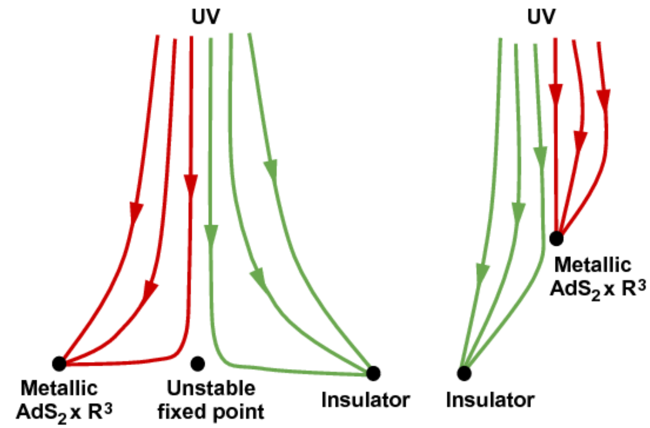

McGreevy, Adv.High Energy Phys. 2010 (2010) 723105

Donos and Hartnoll, Nature Phys.9, 649 (2013)

Diagnosing the quantum phase transition By Holography (traditional IR approach)

Ling, Liu, Niu, Wu and Xian, JHEP04, 114 (2016).
Ling, Liu, Wu. PRD 93 (2016) 12, 126004.
Ling, Liu, Wu and Zhou, PLB766, 41 (2017).
Ling, Liu, and Wu, JHEP 10 (2017) 025.
Ling, Liu, and Wu, PLB 768 (2017) 288.
Ling, and Xian, JHEP 09 (2017) 003.

How to diagnose the occurrence of QPT

- QPTs are often very difficult to analyze as they naturally occur in strongly correlated systems.

- Many QPTs do not involve symmetry breaking or traditional order parameters.

- Entanglement entropy (EE) is also notoriously hard to calculate.

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Gauge/Gravity Duality

- Long wavelength limit

$$\omega \to 0 \qquad\qquad \frac{\eta}{s},\ \ \sigma_D,\ \ D,\ \ v_B$$

Infrared (IR) physics ⟷ Horizon formula

(Scaling symmetry)

$$AdS_4,\quad AdS_2,\quad HL...$$

Metal-Insulator Transition as a QPT

Donos and Gauntlett, JHEP 1404, 040 (2014).

- The action of Q-lattice model

$$L = R + 6 - \frac{1}{4}F_{ab}F^{ab} - |\partial\Phi|^2 - m^2|\Phi|^2$$

$$ds^2 = \frac{1}{z^2}\left(-fSdt^2 + \frac{dz^2}{fS} + \hat{V}_x dx^2 + \hat{V}_y dy^2\right)$$

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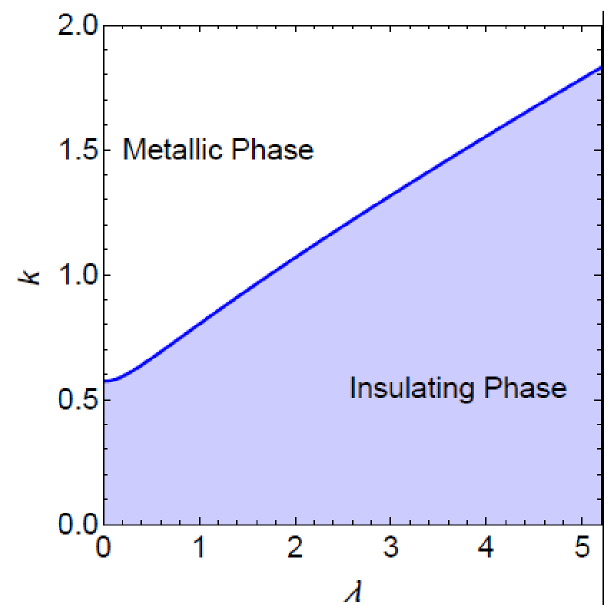
- DC conductivity

$$\sigma_{DC} = \left.\left(\sqrt{\frac{\hat{V}_y}{\hat{V}_x}} + \frac{\mu^2 a^2 \sqrt{\hat{V}_x \hat{V}_y}}{k^2 \phi^2}\right)\right|_{z=1}$$

Critical point: $\partial_T \sigma_{DC}(k, \lambda) = 0$

Question:
What role can the holographic entanglement entropy play in quantum phase transition?



$T / \mu = 0.001$

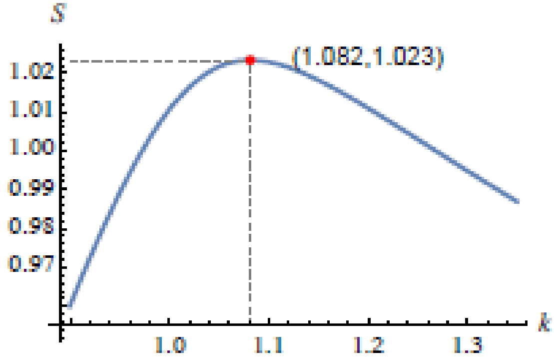
Ling, Liu, Niu, Wu and Xian, JHEP04, 114 (2016). Ling, Liu, Wu. Phys.Rev.D 93 (2016) 12, 126004.

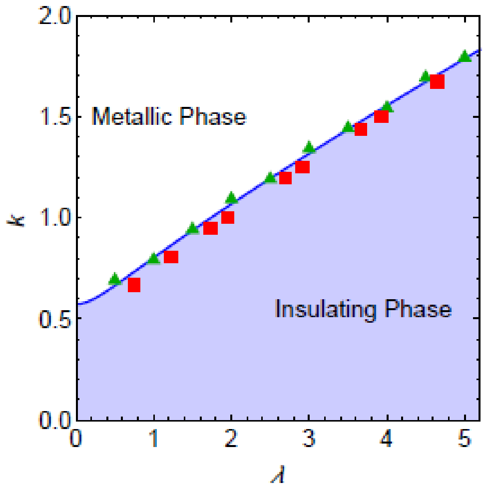
Diagnosing QPT by holographic entanglement entropy

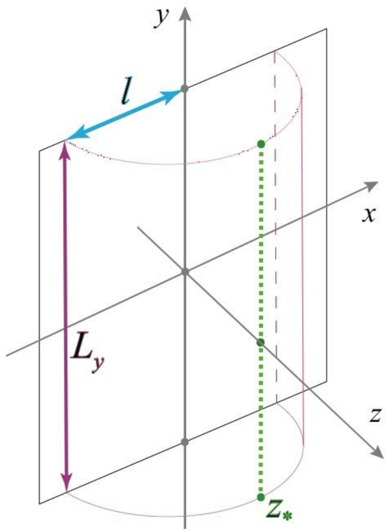
Ryu and Takayanagi, PRL96, 181602(2006)

Ling, Liu, Niu, Wu and Xian, JHEP04, 114 (2016)

- HEE close to QPTs :







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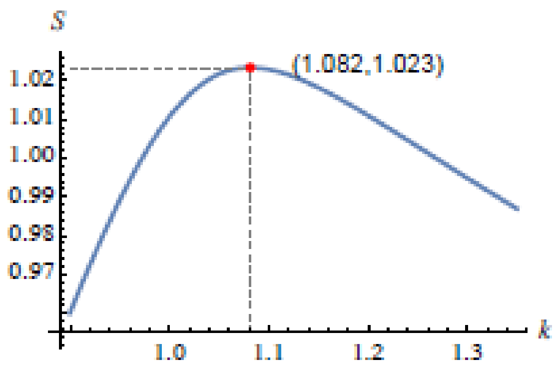
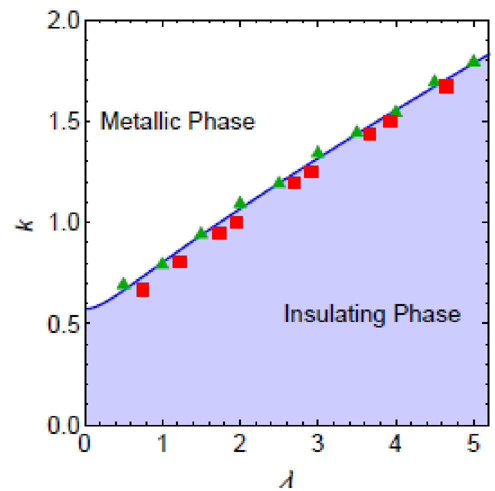
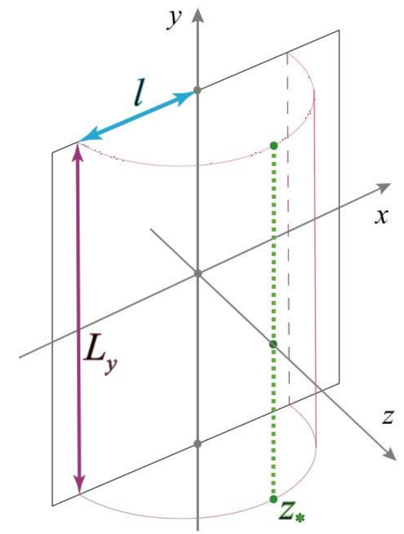
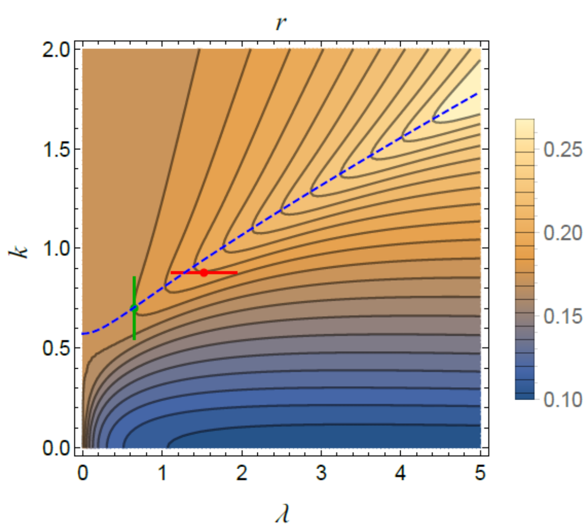






- Near horizon analysis:

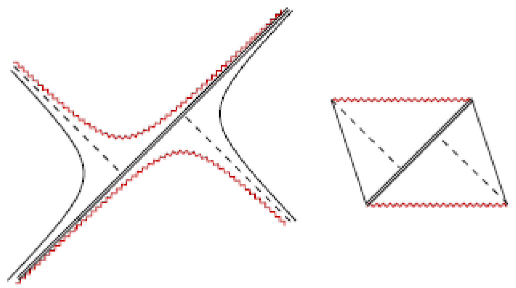
$$r = \lim_{z_* \to 1} \frac{S}{l} = \left. \frac{\sqrt{V_x V_y}}{\mu^2} \right|_{z=1}$$



Diagnosing QPT by holographic butterfly effect

- Holographic butterfly velocity

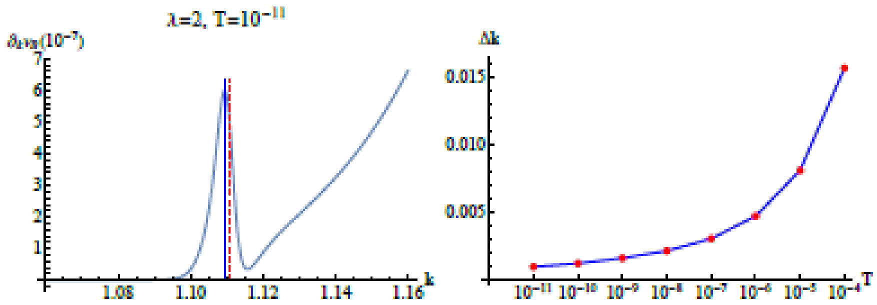
$$v_B = \sqrt{\frac{\pi T V_{\tilde{y}}(r_0)}{V_x^{'}(r_0)V_{\tilde{y}}(r_0) + V_x(r_0)V_{\tilde{y}}^{'}(r_0)}}$$



Shenker and Stanford, JHEP 03 (2014) 067

- Critical behavior of HBE

Ling, Liu, and Wu, JHEP 10 (2017) 025



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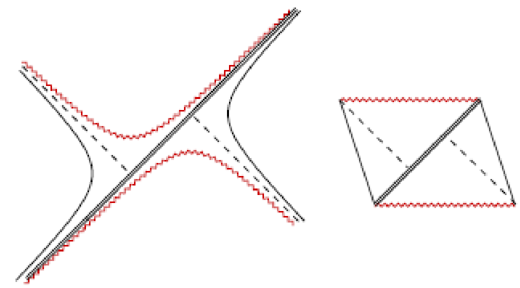

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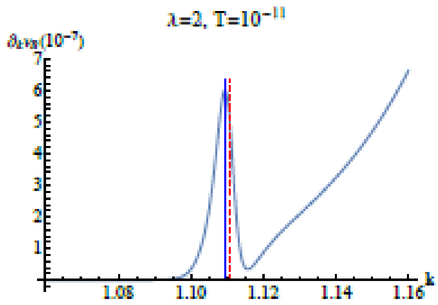
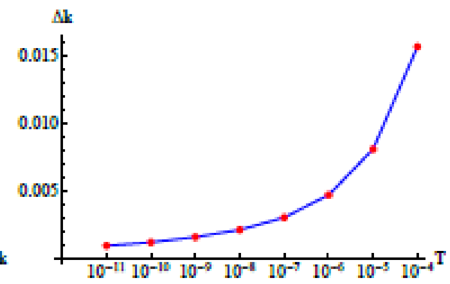
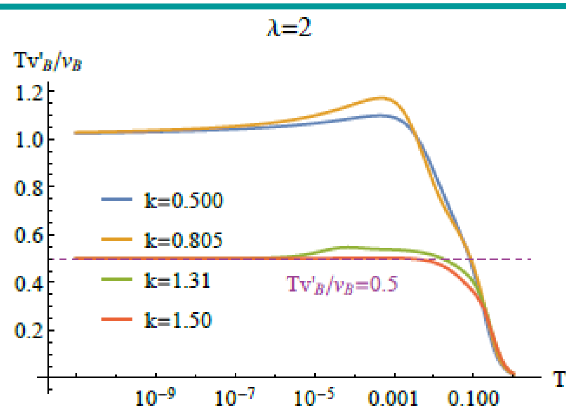




- Scaling behavior of HBE



Comments on diagnosing QPT at finite temperature

Advantage of IR:

- IR physics primarily probe low-energy excitations and capture long-range correlations. It is related to the macroscopic structure of the system and thus usually the signature of QPT (which is scale independent) can be easily captured.

Disadvantage of IR:

- IR physics often mix quantum and classical (thermal) correlations, thus sensitive to the change of temperature.

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Disadvantage of IR:

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Advantage of UV:

- UV physics primarily probe quantum correlations because they capture short-range scales where quantum fluctuations dominate over thermal effects.

Disadvantage of UV:

- UV physics primarily probe high-energy excitations and it is related to the microscopic structure of the system and thus usually hard to explore. The signature of QPT is weak.

Diagnosing the quantum phase transition By UV physics

Fang-Jing Cheng (程芳景）, Zhe Yang（杨哲）, Yi Ling, Jian-Pin Wu, Zhou-Jian Cao and Peng Liu.
e-Print: 2507.07899

The complex UV behavior of the strongly correlated theory on the boundary

↓

The controlled deformations of the asymptotic AdS geometry in the bulk

[Donos and Gauntlett, JHEP 06, 007 (2014)]

The holographic setup

- The action of EMDA model

$$S = \frac{1}{16\pi G}\int d^4 x\sqrt{-g}\left\{R + 6cosh\psi - \frac{3}{2}[(\partial\psi)^2 + 4sinh^2\psi(\partial\chi)^2] - \frac{1}{4}cosh^{\gamma/3}(3\psi)F_{\mu\nu}F^{\mu\nu}\right\}$$

- The ansatz of the background

$$ds^2 = \frac{1}{z^2}[-P(z)dt^2 + \frac{1}{P(z)}dz^2 + V_1(z)dx^2 + V_2(z)dy^2]$$

$$A = \mu(1-z)adt, \;\; \psi = z\phi(z), \;\; \chi = kx,$$

$$P(z) = U(z)(1-z)\left(1 + z + z^2 - \frac{\mu^2 z^3}{4}\right)$$

- Three-parameter black brane solutions

T/μ, λ/μ, k/μ

Temperature — Lattice amplitude — Wave number

$$\gamma = -\frac{1}{6}$$

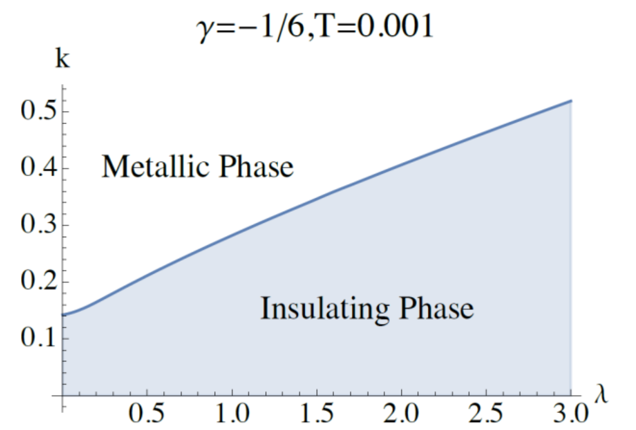
$$\lambda/\mu = 2$$

[Fu, Wang, Liu, Zhang, Kuang and Wu, JHEP 04, 148 (2022)]

The transport behavior

- The Metal-Insulator Transition (MIT)

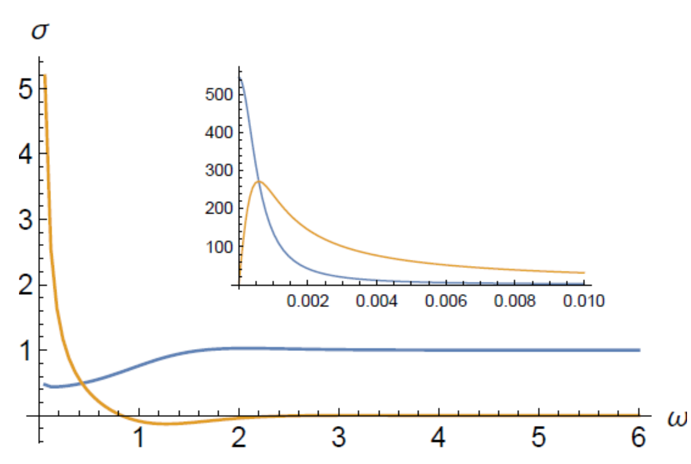
Critical line: $\partial_T \sigma_{DC}(k,\lambda) = 0$



- The optical conductivity by linear perturbations

$$g_{tx} = e^{-i\omega t}\delta h_{tx}(z), \quad A_x = e^{-i\omega t}\delta a_x(z), \quad \chi = e^{-i\omega t}\delta\chi(z),$$

$$\sigma(\omega) = \left.\frac{\delta a'_x(z)}{i\omega\delta a_x(z)}\right|_{z\to 0}.$$



[Cheng, Yang, Ling, Wu, Cao and Liu. e-Print: 2507.07899]

The science with eight decimal places

- The high frequency conductivity

$\omega \to \infty$

$$\mathrm{Re}[\sigma_{AC}] = 1 + \frac{C_\sigma}{\omega^2} + \frac{P_\sigma}{\omega^4} + O(\omega^{-6})$$

[Cheng, Yang, Ling, Wu, Cao and Liu. e-Print: 2507.07899]

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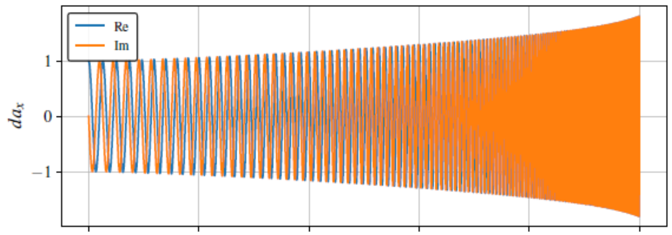
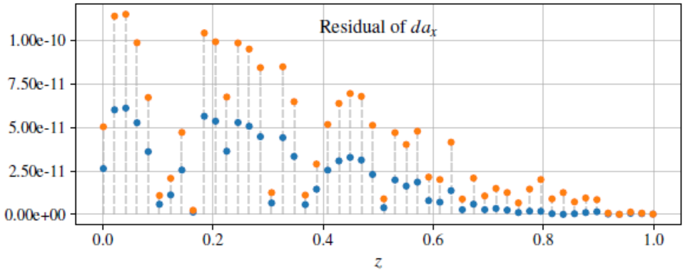
Two numerical challenges at low temperatures and high frequencies

1. Rapid oscillations: $T/\mu = 0.005, \omega/\mu = 20$

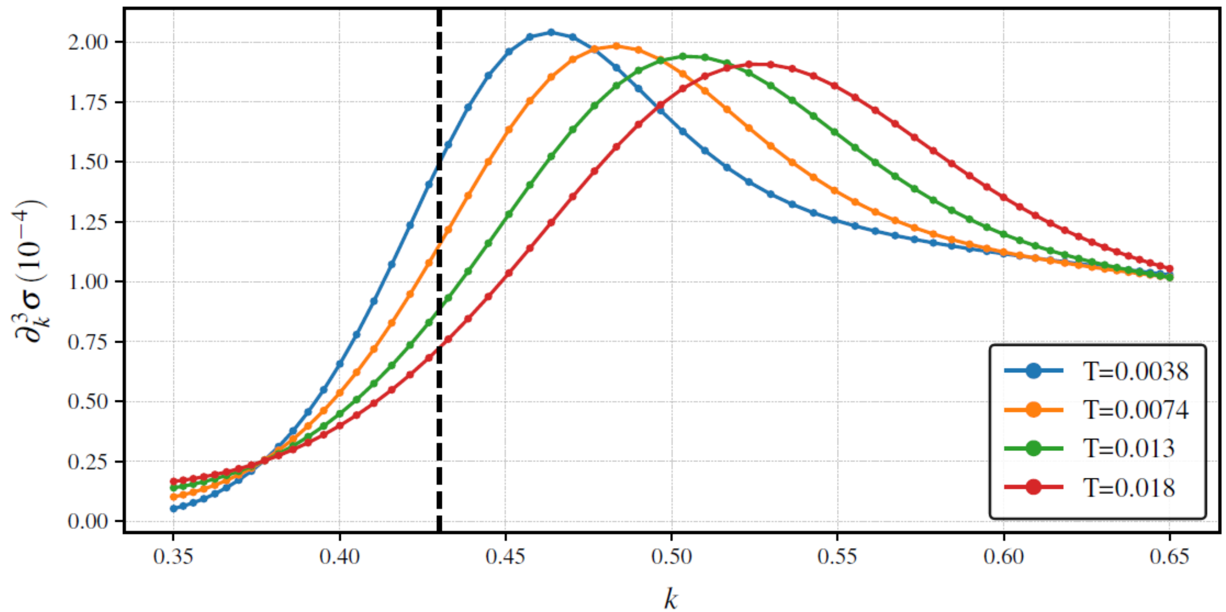




2. Small deviations of the signal:

$(T,\ \omega/\mu)$	k				
	0.33	0.38	0.43	0.48	0.53
(0.01, 20)	1.00126150	1.00125966	1.00125760	1.00125533	1.00125267
(0.002, 20)	1.00126148	1.00125963	1.00125756	1.00125528	1.00125262
(0.01, 50)	1.00020052	1.00020050	1.00020047	1.00020045	1.00020041

The science with eight decimal places

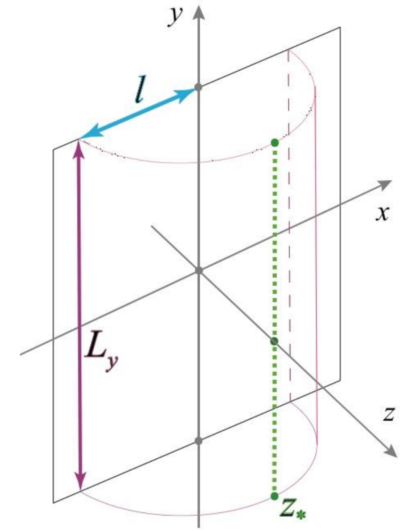


The third derivative of UV conductivity w. r. t. k exhibits a pronounced extremum near the QCP.

The behavior of entanglement measures

- The holographic entanglement entropy(HEE)

$$S_A = \frac{\text{Area}(\gamma_A)}{4G_N},$$



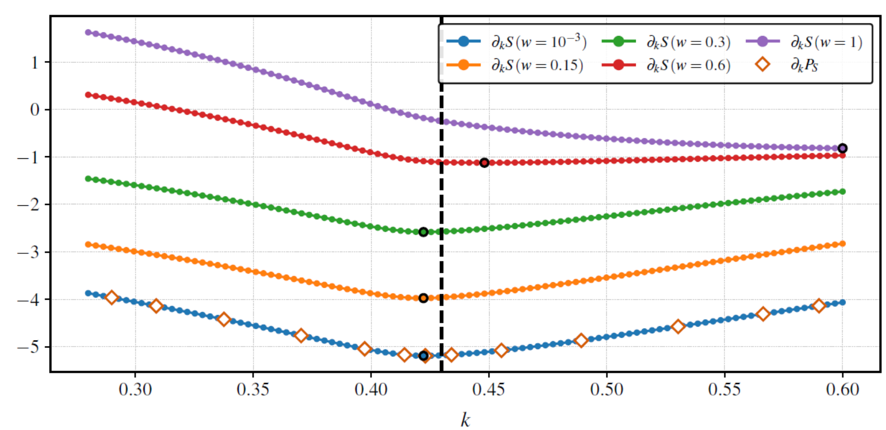
The width of the strip: $$w = 2\mu \int_0^{z_s} dz\, z^2 \sqrt{\frac{V_1(z_s)V_2(z_s)}{P(z)V_1(z)W(z_s,z)}},$$

$$S = \frac{L_y}{2\mu G_N}\left[-\frac{1}{z_s} + \int_0^{z_s} \frac{dz}{z^2}\left(\frac{z_s^2 V_1(z)V_2(z)}{\sqrt{P(z)V_1(z)W(z_s,z)}} - 1\right)\right]$$

$$W(z_s,z) \equiv z_s^4 V_1(z)V_2(z) - z^4 V_1(z_s)V_2(z_s)$$

- The UV behavior of HEE

$$w \to 0 \qquad S(w) = \frac{L_y}{4\mu G_N}\left[-\frac{C_{-1}}{w} + P_S + O(w)\right]$$



The first derivative of HEE

The behavior of entanglement measures

- The holographic mutual information (HMI)

Region A and C are separated by B:

The width of the strip A,B,C: a, b, c

$$I(A;C) = S(A) + S(C) - S(A \cup C)$$

$$S(A \cup C) = \min\{S(A) + S(C),\, S(B) + S(A+B+C)\}.$$

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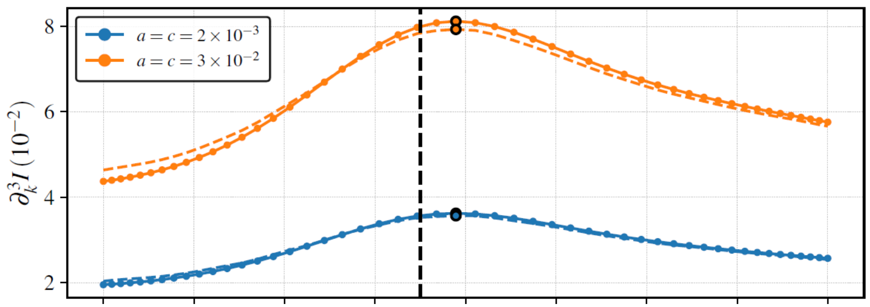
- UV behavior of HMI

UV behavior: tiny a, b, c

$$b = 10^{-4}$$



The third derivative of HMI

$T/\mu = 10^{-3}$

The behavior of entanglement measures

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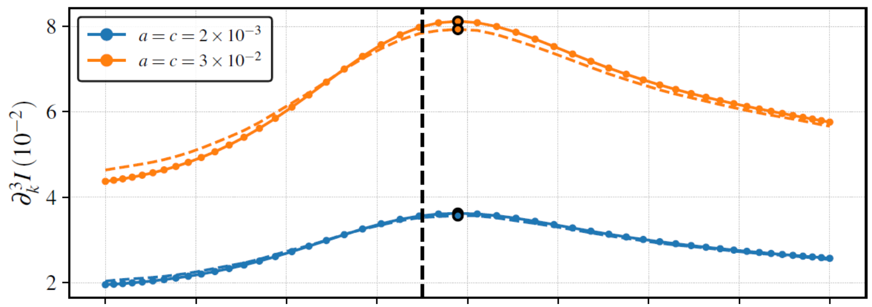


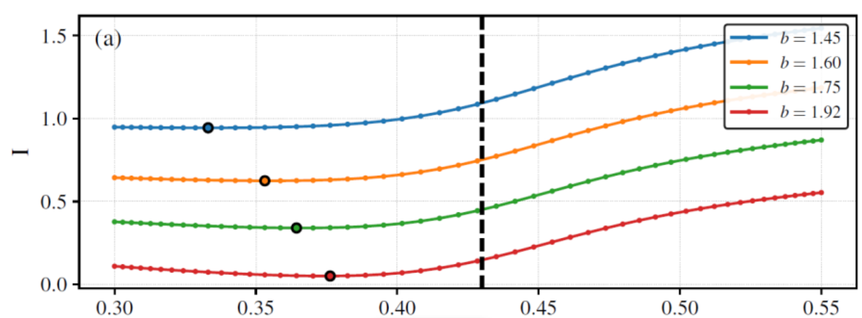
The third derivative of HMI

$T/\mu = 10^{-3}$

- IR behavior of HMI

  IR behavior: very large a, c

$$a = c = 40$$



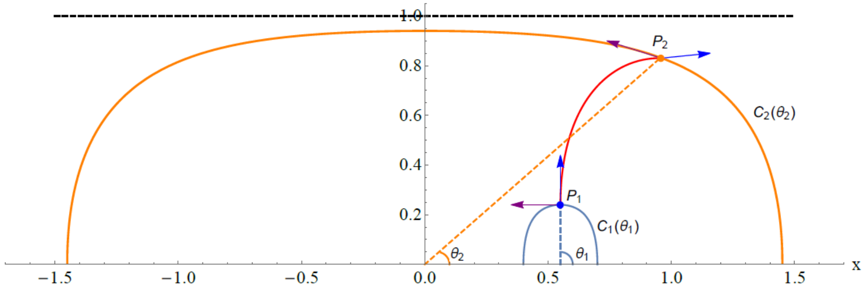
HMI

The behavior of entanglement measures

- The holographic entanglement wedge of cross section (EWCS)

$$E_W = \frac{L_y}{4\mu G_N}\int_{C_{p_1,p_2}} \sqrt{g_{xx}g_{yy}x'(z)^2 + g_{zz}g_{yy}}dz$$

$$x'(z)^3\left(\frac{g_{xx}g'_{yy}}{2g_{yy}g_{zz}} + \frac{g'_{xx}}{2g_{zz}}\right) + x'(z)\left(\frac{g'_{xx}}{g_{xx}} + \frac{g'_{yy}}{2g_{yy}} - \frac{g'_{zz}}{2g_{zz}}\right) + x''(z) = 0$$



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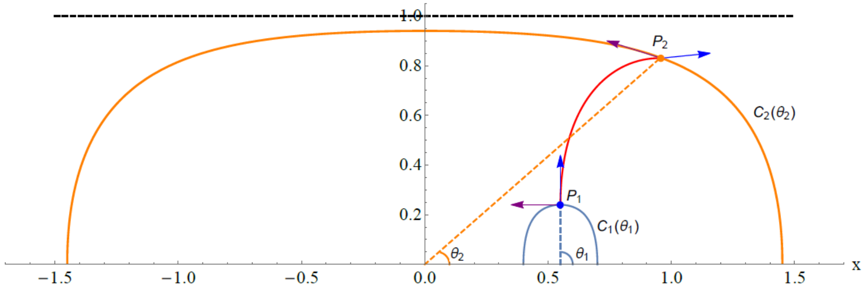
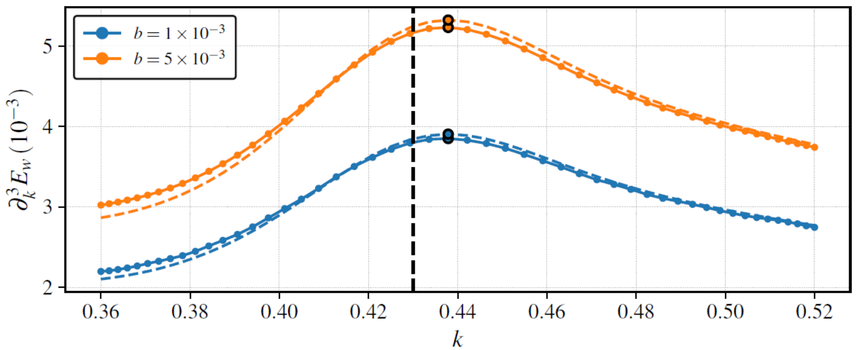
$$x'(z)^3\left(\frac{g_{xx}g'_{yy}}{2g_{yy}g_{zz}} + \frac{g'_{xx}}{2g_{zz}}\right) + x'(z)\left(\frac{g'_{xx}}{g_{xx}} + \frac{g'_{yy}}{2g_{yy}} - \frac{g'_{zz}}{2g_{zz}}\right) + x''(z) = 0$$



- UV behavior of EWCS

UV behavior: tiny a, b, c

$$b = 10^{-4}$$



The third derivative of EWCS

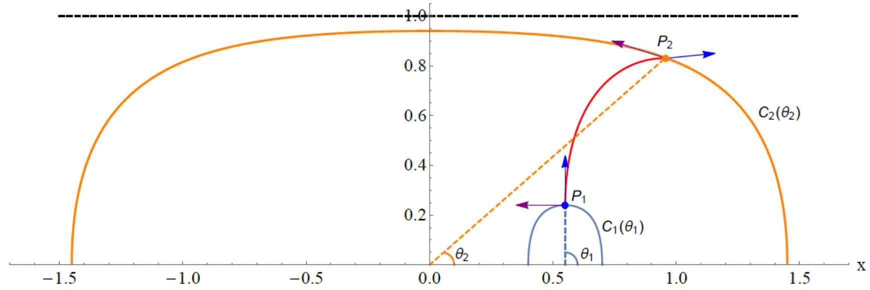
$T/\mu = 10^{-3}$

The behavior of entanglement measures

- The holographic entanglement wedge of cross section (EWCS)

$$E_W = \frac{L_y}{4\mu G_N}\int_{C_{p1,p2}} \sqrt{g_{xx}g_{yy}x'(z)^2 + g_{zz}g_{yy}}dz$$

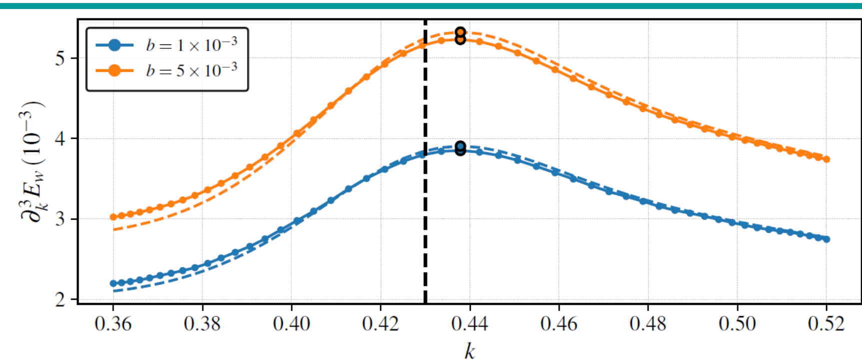
$$x'(z)^3\left(\frac{g_{xx}g'_{yy}}{2g_{yy}g_{zz}} + \frac{g'_{xx}}{2g_{zz}}\right) + x'(z)\left(\frac{g'_{xx}}{g_{xx}} + \frac{g'_{yy}}{2g_{yy}} - \frac{g'_{zz}}{2g_{zz}}\right) + x''(z) = 0$$



- UV behavior of EWCS

UV behavior: tiny a, b, c

$b = 10^{-4}$



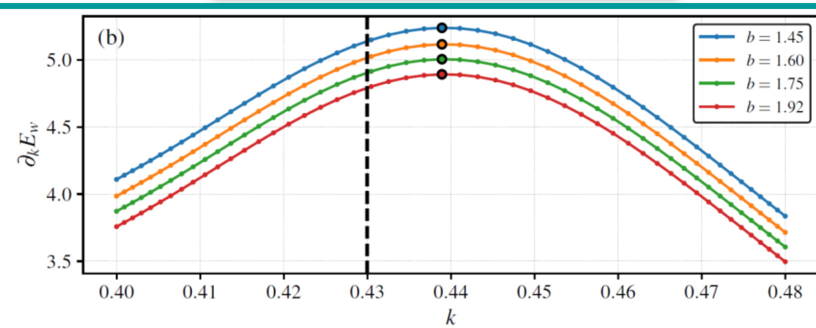
The third derivative of EWCS

$T/\mu = 10^{-3}$

- IR behavior of EWCS

IR behavior: very large a, c

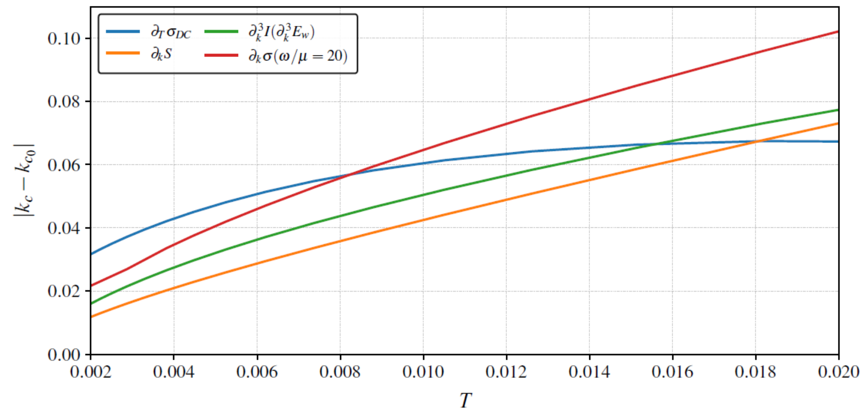
$a = c = 40$



The first derivative of EWCS

The robustness of UV signatures

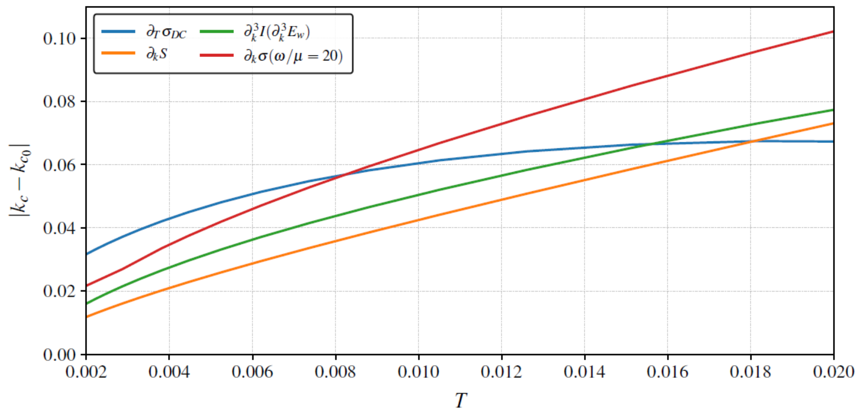
- The behavior of UV observables at finite temperature



The UV observables exhibit smaller deviations from the zero-temperature critical point than the DC conductivity

The robustness of UV signatures

- The behavior of UV observables at finite temperature



The UV observables exhibit smaller deviations from the zero-temperature critical point than the DC conductivity

- The comparison of UV observables with IR

TABLE I. Comparison between IR and UV observables.

IR observable	**UV observable**
Probe low-energy excitations	**Probe high-energy degrees of freedom**
Capture long-range correlations	**Capture short-range correlations**
Limit: Smearing of the thermal signal	**Advantage: Distinction of the critical signal**

Summary

- The gauge/gravity duality provides important tools for understanding the quantum critical phenomenon.
- We reveal for the first time that ultraviolet (UV) observables can diagnose quantum phase transitions (QPTs).
- These UV diagnostics show enhanced robustness to thermal fluctuations compared to typical infrared (IR) diagnostics.
- This work opens a new window for exploring quantum critical phenomena via UV physics in the laboratory.