Spin polarization in off-equilibrium medium at weak and strong coupling

Shu Lin

Sun Yat-Sen University

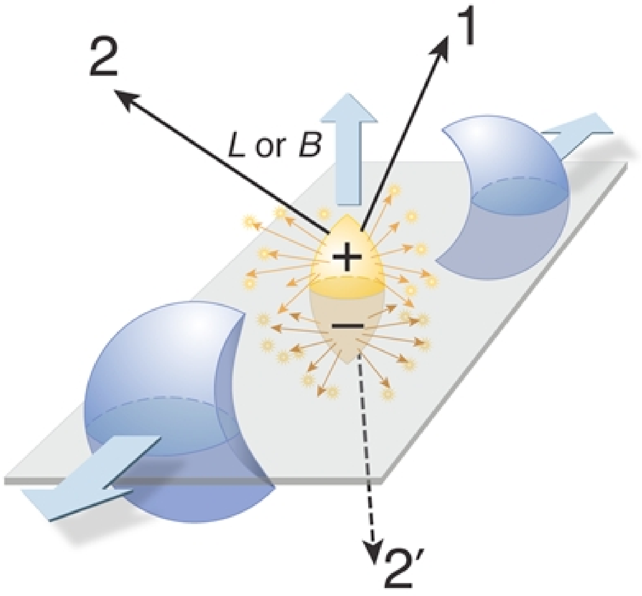
Holographic applications: from Quantum Realms to the Big Bang, Beijing, July 11-19, 2025

Li, SL, 2505.21883
SL, Tian, 2410.22935

Outline

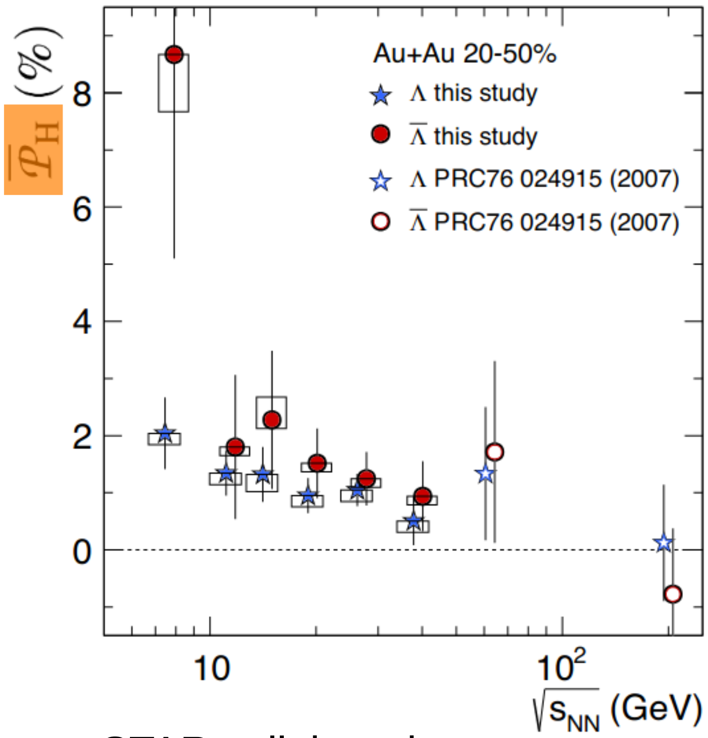
- Rich spin phenomena in heavy ion collisions
- What insights we wish to gain from holography
- Spectral function of particle in off-equilbrium state
- Polarized spectral function at weak coupling
- Polarized spectral function at strong coupling
- Summary

Global polarization of Λ (S=1/2)



$$L_{ini} \sim 10^5\hbar \rightarrow S_{final}$$

Liang, Wang, PRL 2005, PLB 2005

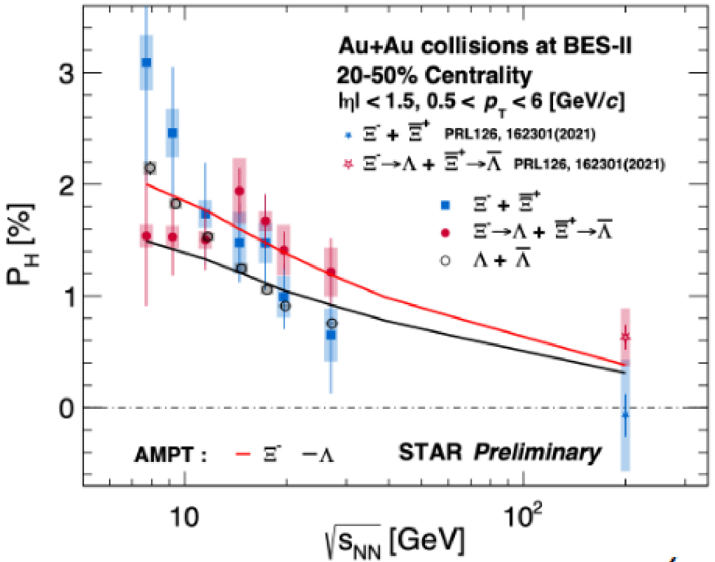


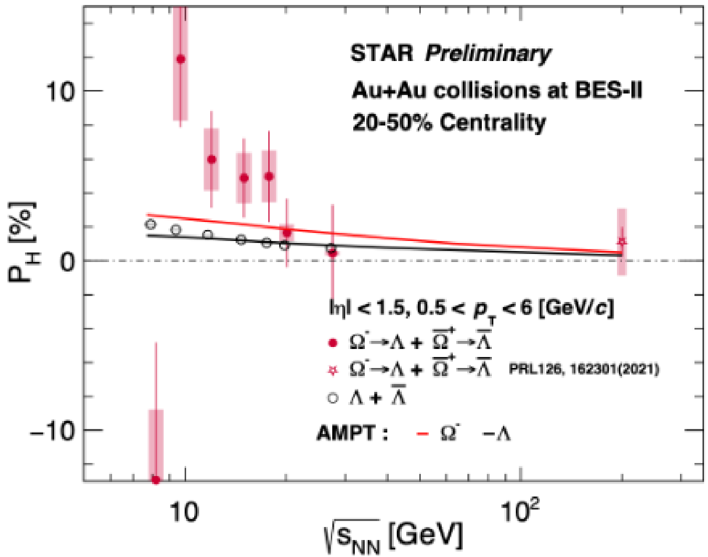
STAR collaboration, Nature 2017

$$e^{-\beta(H_0 - \mathbf{S}\cdot\boldsymbol{\omega})}$$

Becattini et al, PRC 2017

Global polarization of Ξ, Ω (S=1/2)



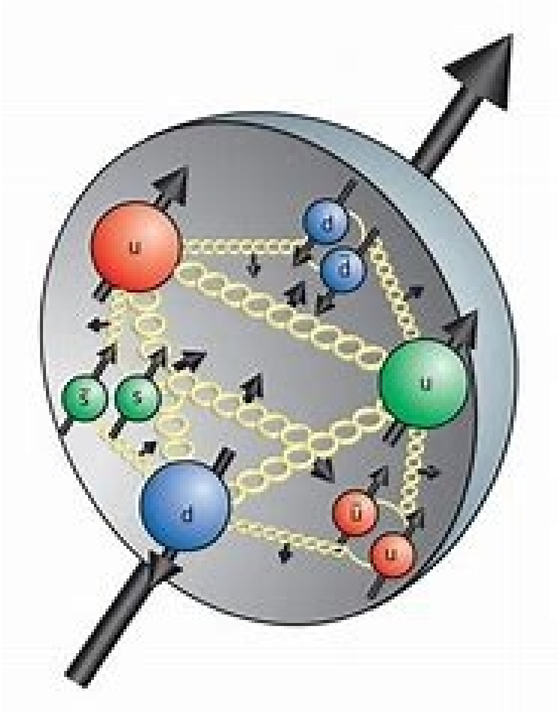


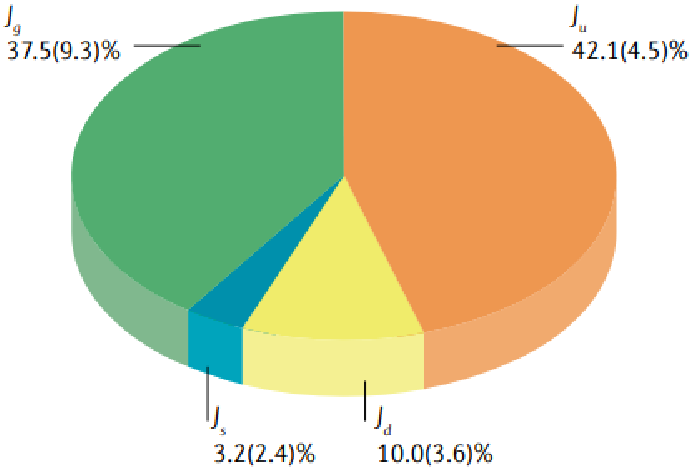
point particle $P_H \simeq \frac{(s+1)}{3}\frac{\omega}{T}$ Becattini et al, PRC 2017

quark model $P_\Lambda = P_s$ $P_\Xi \simeq \frac{4P_s - P_d}{3}$ $P_\Omega \simeq \frac{5P_s}{3}$ Liang, Wang, PRL 2005

→ $P_\Omega \simeq \frac{5P_\Lambda}{3}$ $P_\Xi \simeq P_\Lambda$ same prediction from non-interacting nature of models disfavored by data?

Interaction effect significant



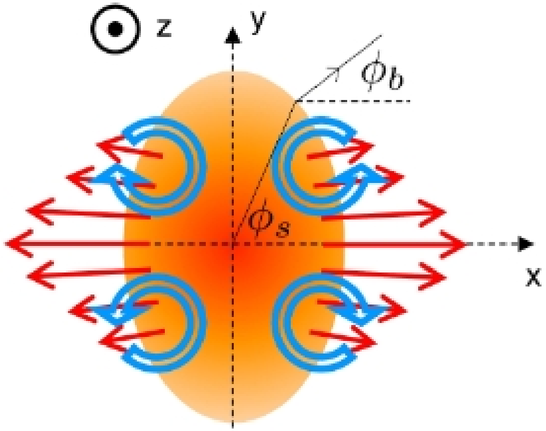


$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + L_q + \Delta G + L_g$$

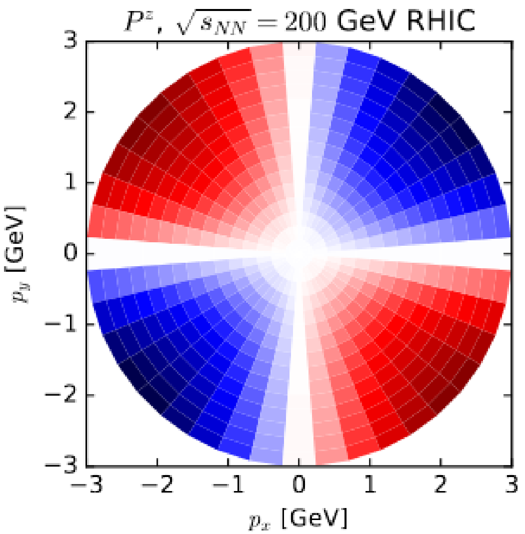
~40%

Alexandrou et al, PRD 2020

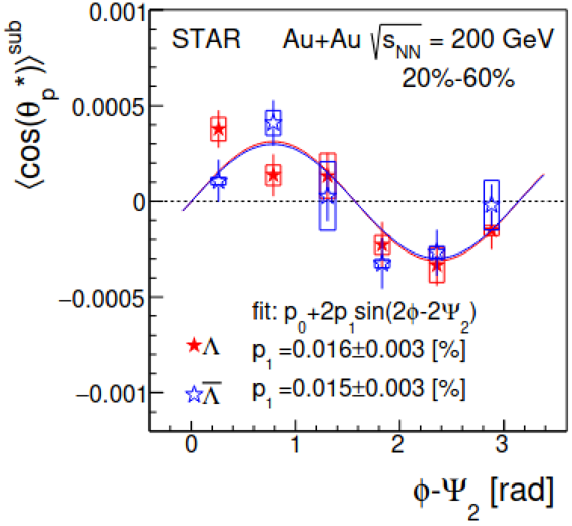
Local polarization of Λ: vorticity only



$$S^i \sim \omega^i$$



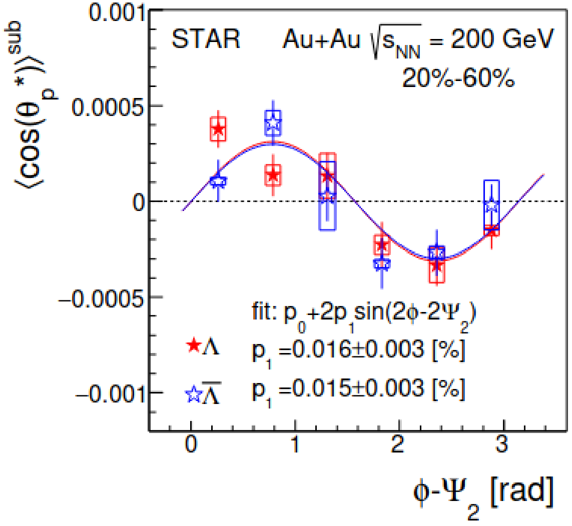
Becattini, Karpenko, PRL 2018
Wei, Deng, Huang, PRC 2019
Wu, Pang, Huang, Wang, PRR 2019
Fu, Xu, Huang, Song, PRC 2021



STAR collaboration, PRL 2019

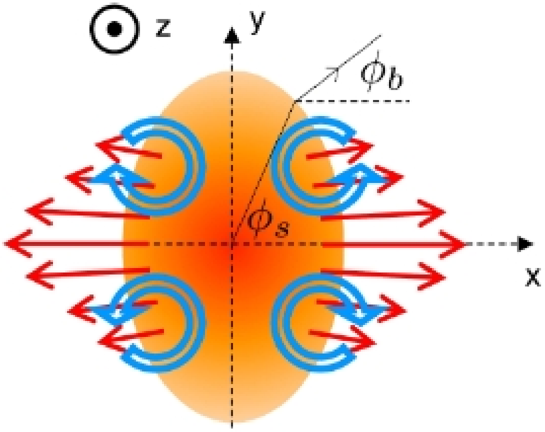
wrong sign

Local polarization of Λ: vorticity + shear



STAR collaboration, PRL 2019

vorticity



$$S^i \sim \omega^i$$

wrong sign

shear

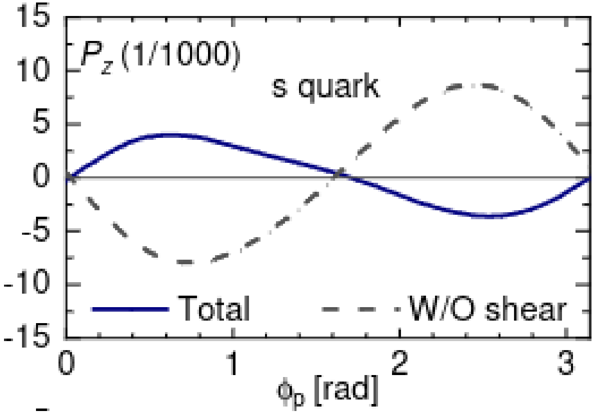
$$S^i \sim \epsilon^{ijk}\hat{p}_k\hat{p}_l\sigma_{jl}$$

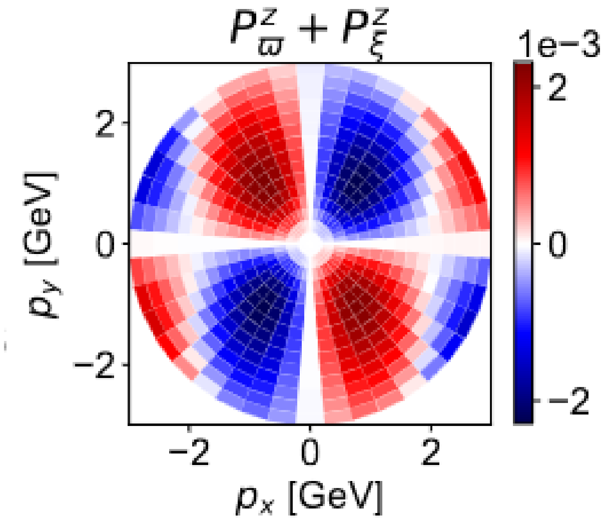
$$S^z \sim \left(\langle p_y^2\rangle - \langle p_x^2\rangle\right)\partial_y u_x$$

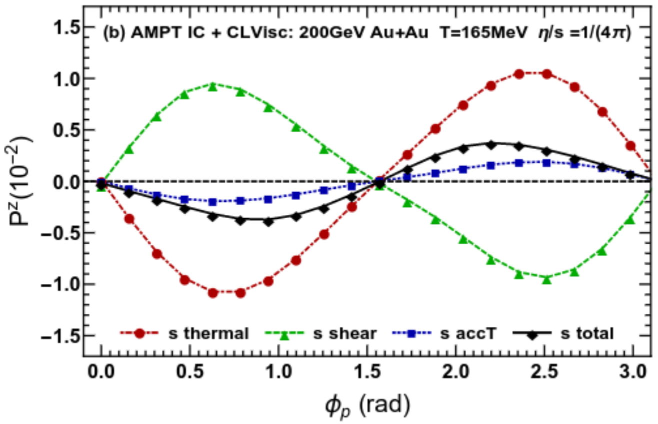
right sign

Hidaka, Pu, Yang, PRD 2018
Liu, Yin, JHEP 2021
Becattini, et al, PLB 2021

Sensitivity on interaction: shear







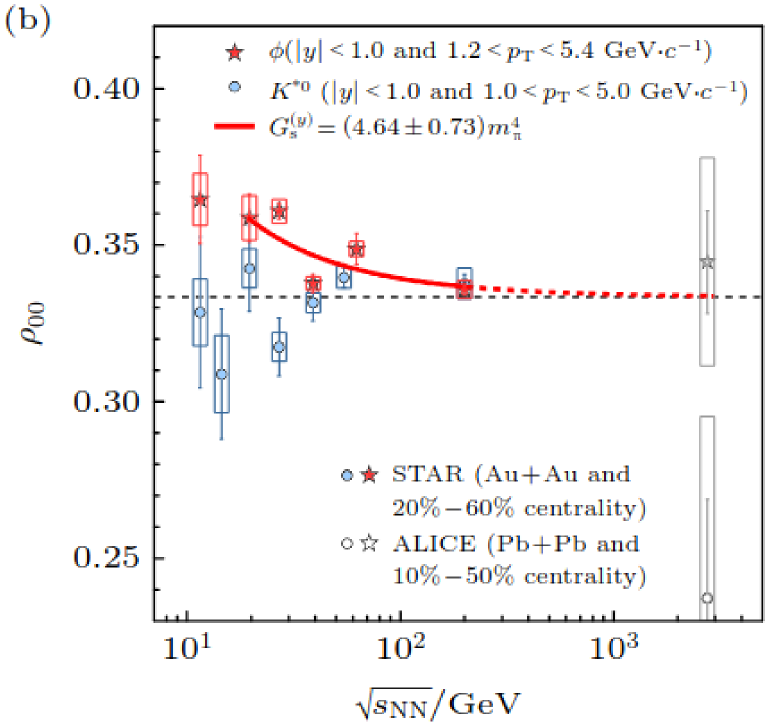
Fu, Liu, Pang, Song, Yin, PRL 2021
Becattini, et al, PRL 2021
Yi, Pu, Yang, PRC 2021

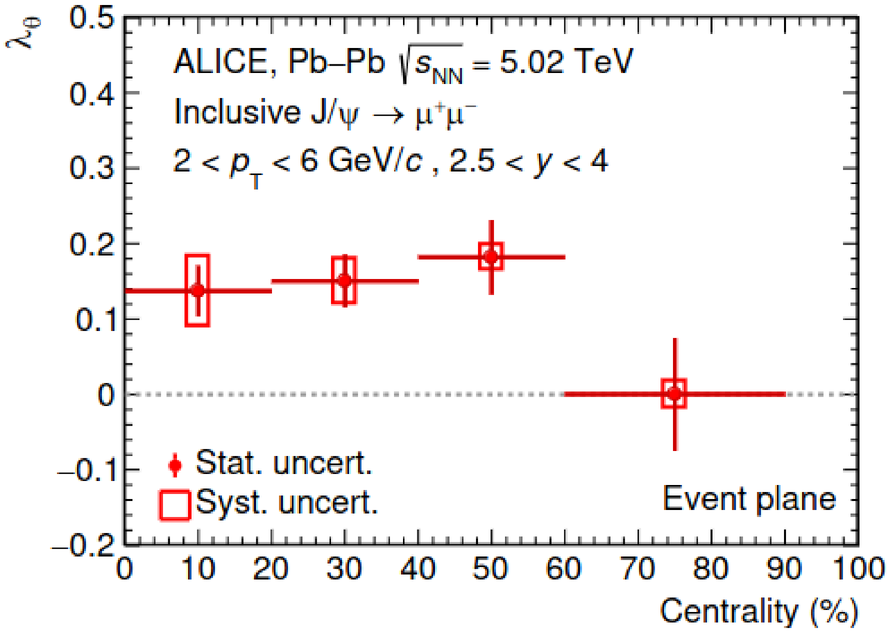
two scenarios
Λ: point particle
Λ: quark model

$$S^\mu(p^\alpha) = \frac{1}{4m} \frac{\int d\Sigma \cdot p\, n_0(1-n_0)\mathcal{A}^\mu}{\int d\Sigma \cdot p\, n_0}$$

Interaction effect?

Spin alignment of ϕ, J/Ψ (S=1)





splitting of longitudinal/transverse polarizations

Holograpic studies: Hou et al PRD 2024, JHEP 2024, Huang et al PRD 2025

Interaction effect in medium: weak coupling picture

free theory $$S^i \sim \left(1\,\omega^i + 1\,\epsilon^{ijk}\,\hat{p}_j\,\hat{p}_l\,\sigma_{kl} + 1\,\epsilon^{ijk}\,\hat{p}_j\,\frac{\partial_k \beta}{\beta}\right)\delta(p^2 - m^2)$$

- distribution function correction

$$(\partial_t + \hat{p}\cdot\nabla_x)\, f(t, x, p) = -C[f] \qquad \delta f \propto \frac{\nabla_x f}{g^4}$$

deviation of steady state from local equilibrium

$$S^i \sim \left((1 + O(1))\epsilon^{ijk}\hat{p}_j\hat{p}_l\sigma_{kl}\right)\delta(P^2 - m^2)$$

SL, Wang, JHEP 2022, PRD 2025
Fang, Pu, Yang, PRD 2024
Fang, Pu, PRD 2025

- spectral function correction in off-equilibrium state

focus of this talk

SL, Tian, 2410.22935
Fang, Pu, Yang, 2503.13320

Spectral function of probe "particle" in medium

$$\rho_{\alpha\beta}(\omega,\vec{p}) = \int d^4x e^{ip\cdot x} \left\langle \Psi_\alpha(x)\Psi_\beta^\dagger(0) + \Psi_\beta^\dagger(0)\Psi_\alpha(x) \right\rangle$$

$\omega, p \gg \partial_X$ particle localized in fluid element

$\langle\cdots\rangle = \mathrm{Tr}[D\cdots]$ D: density matrix

probe particle in off-equilibrium fluid $D = D_{\mathrm{probe}} \otimes D_{\mathrm{fluid}}.$

$D^{(0)}{}_{\mathrm{probe}} \otimes D^{(1)}{}_{\mathrm{fluid}}$ $D^{(1)}{}_{\mathrm{probe}} \otimes D^{(0)}{}_{\mathrm{fluid}}$

$\rho = 2\,\mathrm{Im}\,G^R$ → $S^i \sim \mathrm{tr}[\rho\,\sigma^i]$

Off-equilibrium spectral function at weak coupling

- Probe quark

Spectral function from self-energy

$$\frac{i}{2}\not{\partial} S_R(X,P) + \not{P} S_R(X,P) - \left(\Sigma_R(X,P) S_R(X,P) + \frac{i}{2}\{\Sigma_R(X,P), S_R(X,P)\}_{\rm PB}\right) = -1$$

$$S_R = S_R^{(0)} + S_R^{(1)} + \cdots ,$$

$$\{A,B\}_{\rm PB} = \partial_P A \cdot \partial_X B - \partial_X A \cdot \partial_P B$$

$$S_R^{(0)} = -\frac{1}{\not{P}} - \frac{1}{\not{P}}\Sigma_R \frac{1}{\not{P}}$$

$$S_R^{(1)} = -\frac{1}{\not{P}}\delta\Sigma_R \frac{1}{\not{P}} + \gamma^5\gamma^\beta P^\nu T^{\mu\lambda}\epsilon_{\beta\lambda\mu\nu}\frac{-1}{(P^2)^2}$$

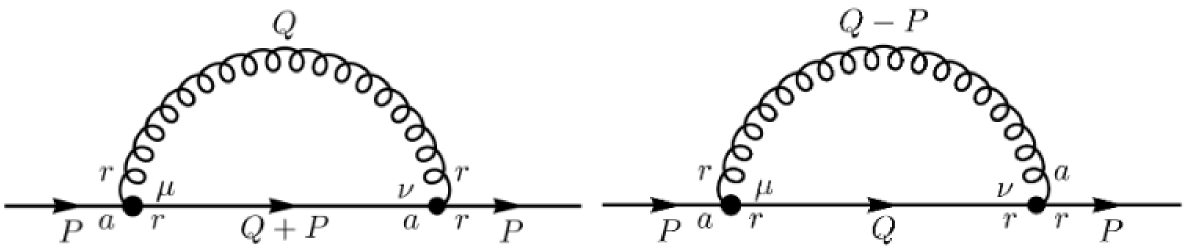
$$T_{\mu\lambda} = \partial_{[\mu}\Sigma^R_{\lambda]}$$

off-equilibrium self-energy

gradient of equilibrium self-energy

SL, Tian, 2410.22935

Equilibrium self-energy



$P \gg T$, $P^2 \ll p/\beta$,

energetic quark
nearly on-shell

$$\frac{\Sigma_{ar}}{g^2 C_F} = 2i\not{P}(A+B) + 4ip_0\gamma^0 A.$$

$$p_0 = p(1 + 8A)$$

$$A = \frac{1}{2(2\pi)^2}\frac{-i\pi}{2p\beta}.$$

modified dispersion:

finite damping

$$S_R^{(1)} = \gamma^5\gamma_\beta P_\nu \partial_\mu \Sigma_\lambda^R \frac{-1}{(P^2)^2}\epsilon^{\beta\lambda\mu\nu} \sim O(T\partial)$$

$D^{(0)}{}_{\text{probe}} \otimes D^{(1)}{}_{\text{fluid}}$

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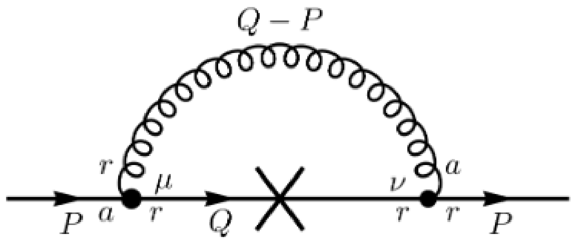
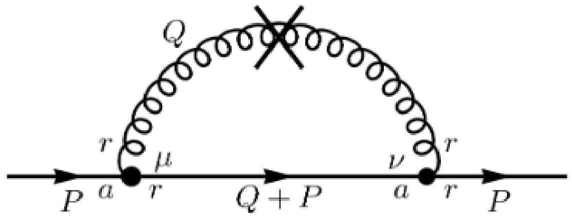
Off-equilibrium self-energy





off-equilibrium propagators

Hidaka, Pu, Yang 2017

Huang et al 2020

Hattori et al 2020

$$\frac{\delta\Sigma_{ar}}{g^2 C_F} = \gamma^5\gamma^\mu \mathcal{A}_\mu,$$

$$\mathcal{A}^0 = i\omega^i p_i \beta(-4\delta A - 2\delta B),$$

$$\mathcal{A}^k = i\omega_\parallel^k \beta p(-4\delta A - 2\delta B) + \omega_\perp^k \beta p(-4\delta A - \delta B) + \epsilon^{ijk}\hat{p}_i\hat{p}_l\sigma_{jl}\beta p(-\delta B) + \epsilon^{ijk} p_i \partial_j \beta(-\delta C)$$

$$\delta S_R^{(0)} = -\frac{1}{\not{P}}\delta\Sigma_R \frac{1}{\not{P}} \sim O(p\partial)$$

dominant contribution $\quad P \gg T$

$D^{(1)}_{\text{probe}} \otimes D^{(0)}_{\text{fluid}}$

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Weak coupling summary

free $$\gamma^5\gamma_i \frac{2\pi}{2}\left(\omega^i + \epsilon^{ijk}\hat{p}_k \frac{\partial_j \beta}{\beta} + \epsilon^{ijk}\hat{p}_k \hat{p}_l \sigma_{lj}\right) \beta \tilde{f}(p).$$

interaction $$\int dp_0 \delta\rho(P) \tilde{f}(p_0)$$

$O(\lambda)$ $$= \frac{g^2 C_F}{2(2\pi)^2} \frac{\pi}{2p} \gamma^5 \gamma_i [0.95\omega^i_{\parallel} + 1.48\omega^i_{\perp} - 0.52\epsilon^{ijk}\hat{p}_j \hat{p}_l \sigma_{kl} - 0.02\epsilon^{ijk}\hat{p}_j \frac{\partial_k \beta}{\beta}] \tilde{f}(p)$$

Polarized spectral function

SL, Tian, 2410.22935

Off-equilibrium spectral function at strong coupling

- Probe baryon

Holographic model for baryon

$$S = i \int d^{D+1}x \sqrt{-g}\bar{\psi}\left(\Gamma^M \nabla_M - m\right)\psi,$$

Iqbal, Liu 2009

5D Dirac fermion ⟷ 4D Weyl fermion

$$\psi = \begin{pmatrix} \psi_R \\ \psi_L \end{pmatrix}$$

“lump of quark/gluon”

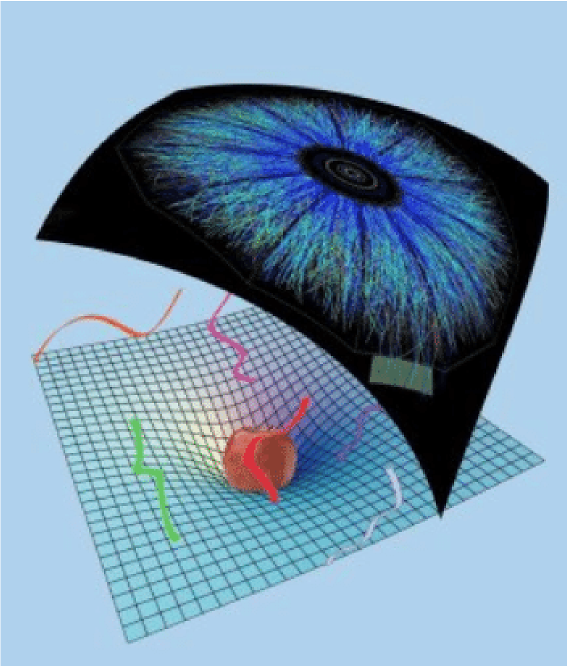
One baryon = R + L Weyl fermions

focus on spectral function in off-equilibrium QGP

Holographic model for QGP

$$ds^2 = -2u_\mu(x)dx^\mu dr - r^2 f(b(x)r)u_\mu u_\nu dx^\mu dx^\nu + r^2 P_{\mu\nu}dx^\mu dx^\nu$$ local equilibrium

$$+ 2r^2 bF(br)\sigma_{\mu\nu}dx^\mu dx^\nu + \frac{2}{3} r u_\mu u_\nu \partial_\lambda u^\lambda dx^\mu dx^\nu - r u^\lambda \partial_\lambda(u_\mu u_\nu)dx^\mu dx^\nu$$ steady state



Bhattacharyya et al, JHEP 2008

$O(\partial^0)$ $\quad$ $O(\partial)$

$$T^{\mu\nu} = (\pi T)^4 (\eta^{\mu\nu} + 4u^\mu u^\nu) - 2(\pi T)^3 \sigma^{\mu\nu}$$

local equilibrium $\quad$ η

steady state

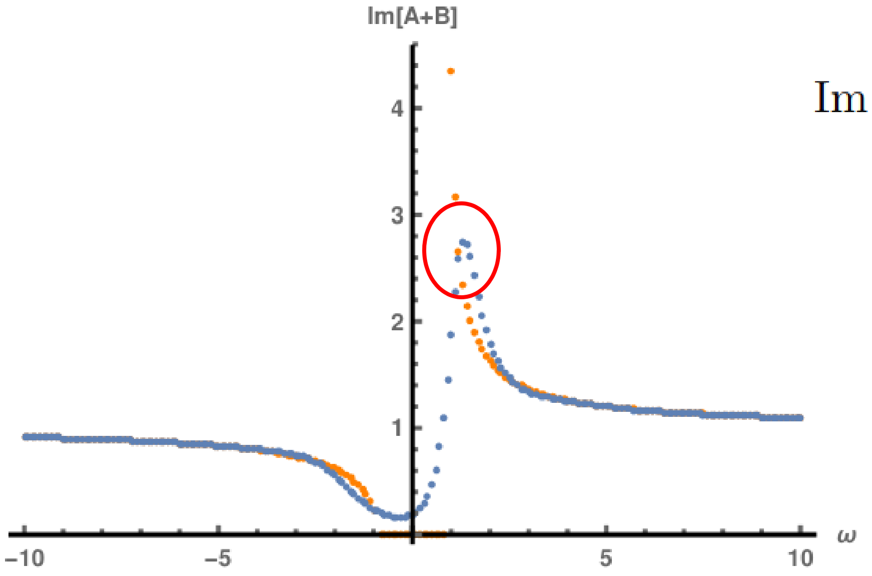
σ_{ij} $\quad$ ~~$\partial_0 b = \frac{1}{3}\partial_i u_i$~~ $\quad$ $\partial_i b = \partial_0 u_i$

T-grad = acceleration

Equilibrium spectral function

$$G_R = A + B\hat{p}\cdot\vec{\sigma}$$

$$\rho = 2\mathrm{Im}G_R = \mathrm{Im}[A+B]\,(1-\hat{p}\cdot\vec{\sigma}) + \mathrm{Im}[A-B]\,(1+\hat{p}\cdot\vec{\sigma})$$



orange: vacuum

Iqbal, Liu 2009

$$\mathrm{Im}[A+B] = \mathrm{sgn}(\omega)\theta(\omega^2-p^2)\left(\frac{\omega+p}{\omega-p}\right)^{1/2}$$

no spacelike spectral

blue: equilibrium QGP
soften the singularity
develop spacelike spectral

focus on timelike spectral for baryon

Gradient corrections

$$(\Gamma^M \nabla_M - m)\psi = 0$$

gradient correction to Dirac field → gradient correction to baryon

$$D^{(0)}_{\text{fluid}} \otimes D^{(1)}_{\text{baryon}}$$

$$(\Gamma^M \nabla_M - m)\psi = 0$$

gradient correction to Dirac operator → gradient correction to medium

$$D^{(1)}_{\text{fluid}} \otimes D^{(0)}_{\text{baryon}}$$

$D^{(1)\,\text{le}}_{\text{fluid}}$ $D^{(1)\,\text{ss}}_{\text{fluid}}$

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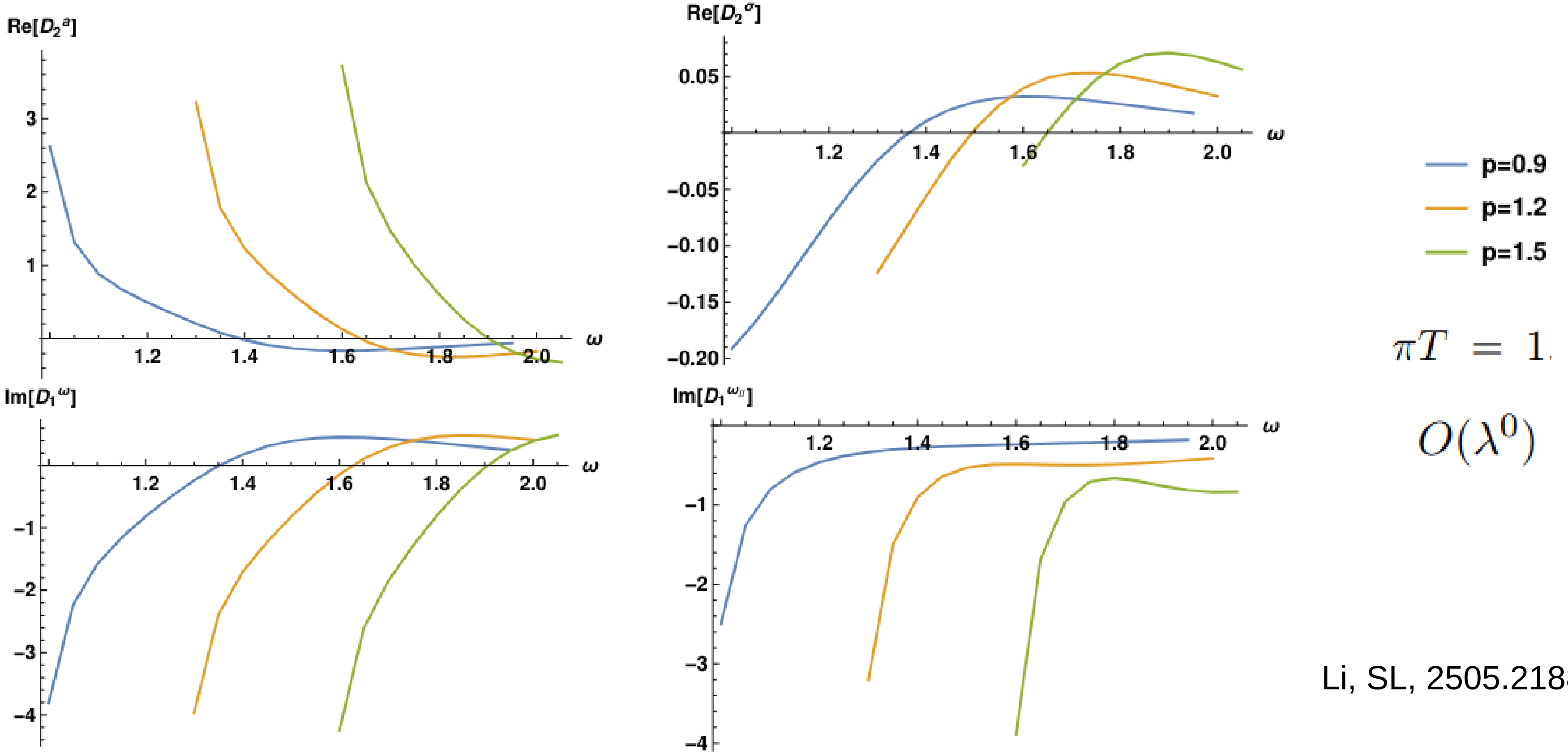
$$\frac{1}{2}\mathrm{tr}\left[(\delta\rho^L+\delta\rho^R)\sigma_k\right]=4\left(-\mathrm{Re}[D_2^a]\epsilon^{ijk}\hat{p}_j\partial_0 u_i-\mathrm{Re}[D_2^\sigma]\epsilon^{ijk}\hat{p}_j\hat{p}_l\sigma_{il}+\mathrm{Im}[D_1^\omega]\omega_k+\mathrm{Im}[D_1^{\omega^\parallel}]\omega_k^\parallel\right)$$

$$D^{(0)}_{\mathrm{fluid}}\otimes D^{(1)}_{\mathrm{baryon}}$$



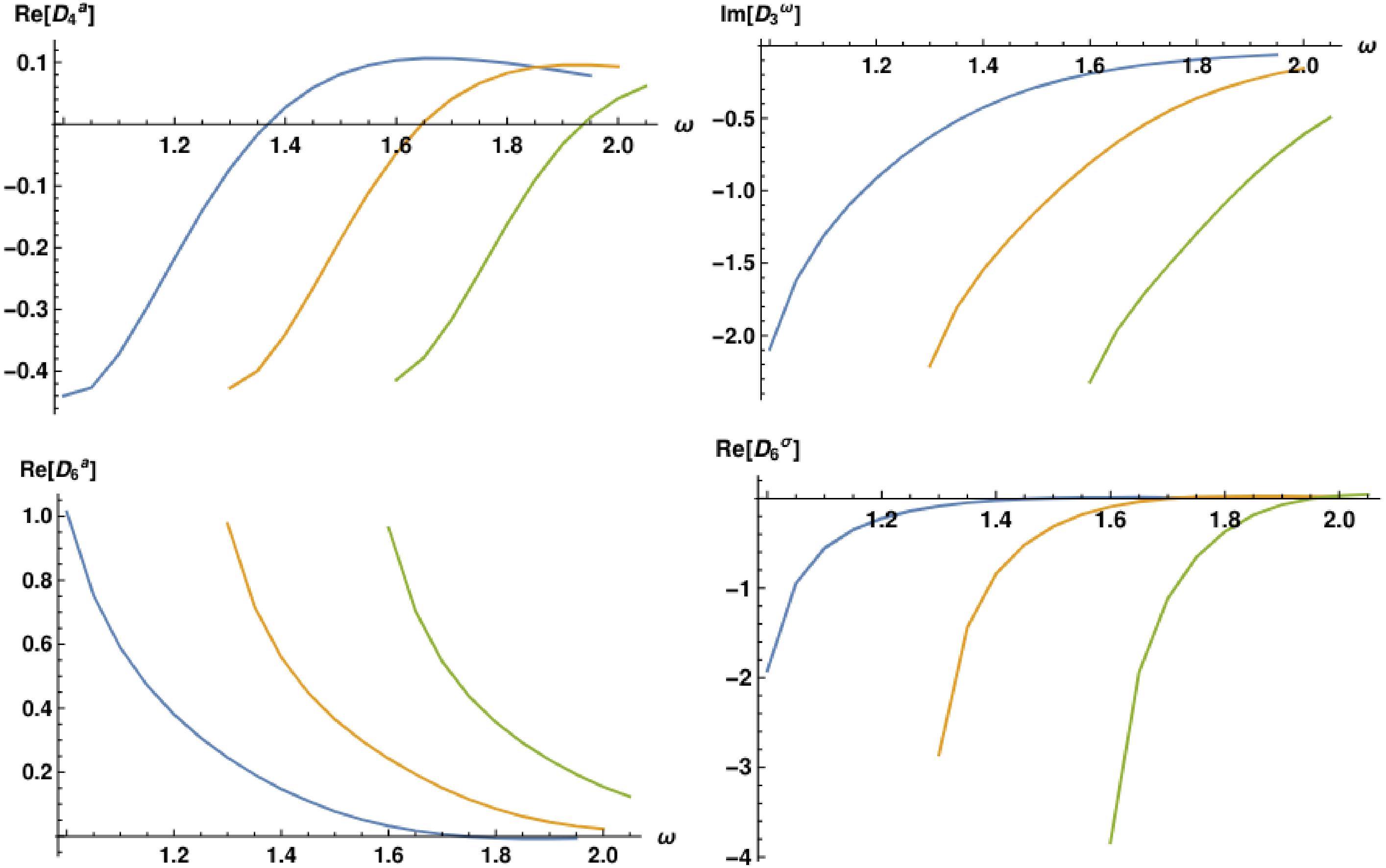
$\pi T = 1.$

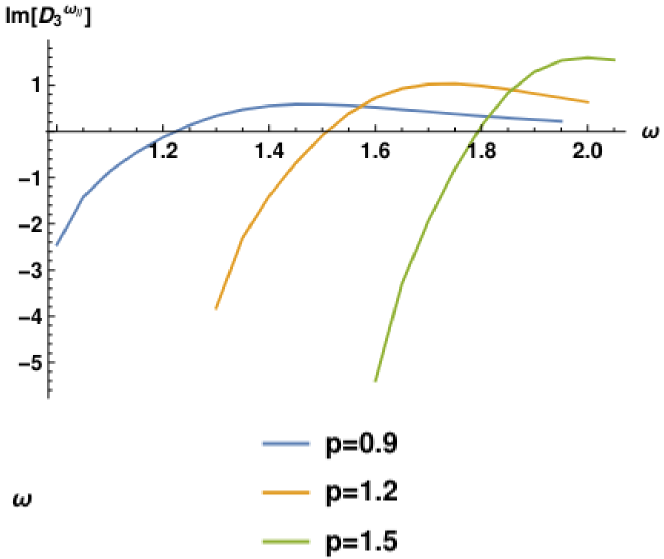
$O(\lambda^0)$

Li, SL, 2505.21883

$$\frac{1}{2}\mathrm{tr}[(\delta\rho^L+\delta\rho^R)\sigma_k] = 4\Big[-\mathrm{Re}[D_4^a + D_6^a]\epsilon^{ijk}\hat{p}_j\partial_0 u_i - \mathrm{Re}[D_6^\sigma]\epsilon^{ijk}\hat{p}_j\hat{p}_l\sigma_{il} + \mathrm{Im}[D_3^\omega]\omega_k + \mathrm{Im}[D_3^{\omega\parallel}]\omega_k^\parallel\Big]$$

$D^{(1)\,le}_{fluid}$ $D^{(1)\,ss}_{fluid}$





$O(\lambda^0)$ $\pi T = 1$

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Strong coupling summary

$\mathrm{tr}[(\delta\rho^L + \delta\rho^R)\sigma_k]$	$\epsilon^{ijk}\hat{p}_j\partial_0 u_i$	$\epsilon^{ijk}\hat{p}_j\hat{p}_l\sigma_{il}$	$\omega_k^\perp$	$\omega_k^\parallel$
$D^{(1)}_{\mathrm{probe}}$	$-4\mathrm{Re}[D_2^a]$	$-4\mathrm{Re}[D_2^\sigma]$	$4\mathrm{Im}[D_1^\omega]$	$4\mathrm{Im}[D_1^\omega + D_1^{\omega^\parallel}]$
$D^{(1)\mathrm{le}}_{\mathrm{fluid}}$	$-4\mathrm{Re}[D_4^a]$		$4\mathrm{Im}[D_3^\omega]$	$4\mathrm{Im}[D_3^\omega + D_3^{\omega^\parallel}]$
$D^{(1)\mathrm{ss}}_{\mathrm{fluid}}$	$-4\mathrm{Re}[D_6^a]$	$-4\mathrm{Re}[D_6^\sigma]$		

Polarized spectral function

Li, SL, 2505.21883

Weak (quark) vs Strong (baryon)

Common

- Polarized spectral function in off-equilibrium state

  $D^{(1)}{}_{probe}$ $D^{(1)\,le}{}_{fluid}$ $D^{(1)\,ss}{}_{fluid}$

- Correction suppressed for energetic particles

Difference

- Weak $O(\lambda)$ vs strong $O(\lambda^0)$

Summary

- Sensitiveness of polarization to interaction
- Spectral function in off-equilibrium state
- vorticity, shear, T-grad generically lead to (interaction dependent) polarized spectral function

Outlook

- Mass dependence of polarized spectral function
- Direct access to polarization: Schwinger-Keldysh extended holography