Far From Equilibrium Physics in Finite Size System — Thermalization

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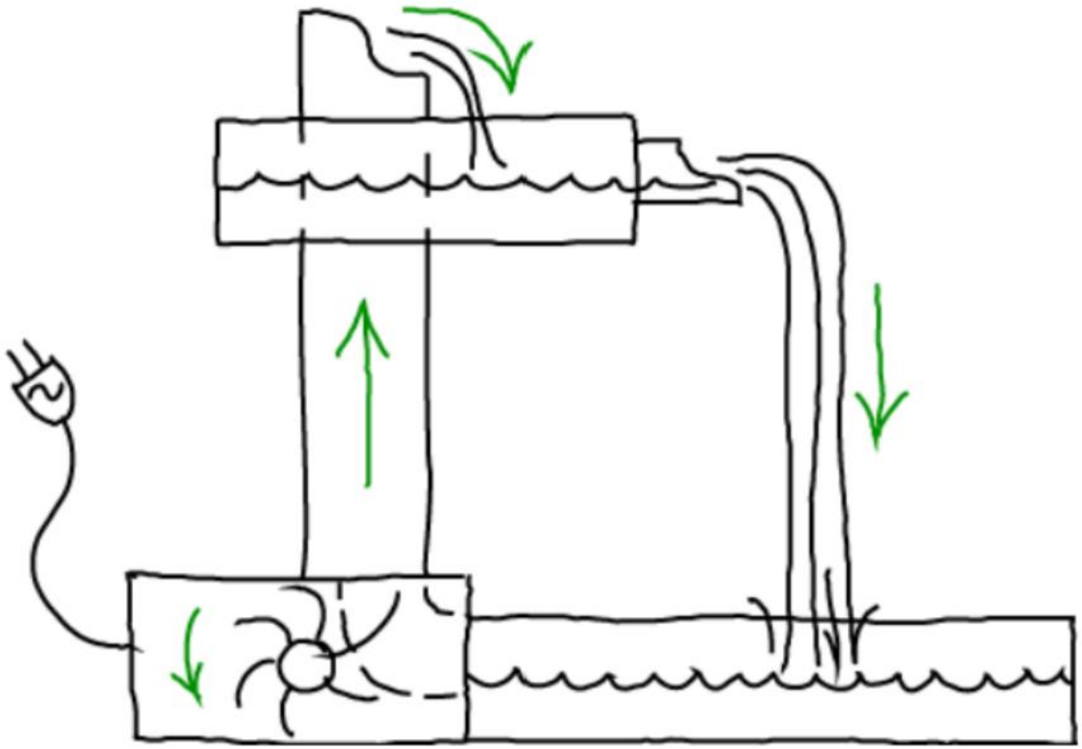
1. Non-equilibrium Steady State
2. Holographic Finite Size System and BCFT
3. NESS decay, shock wave and dissipation

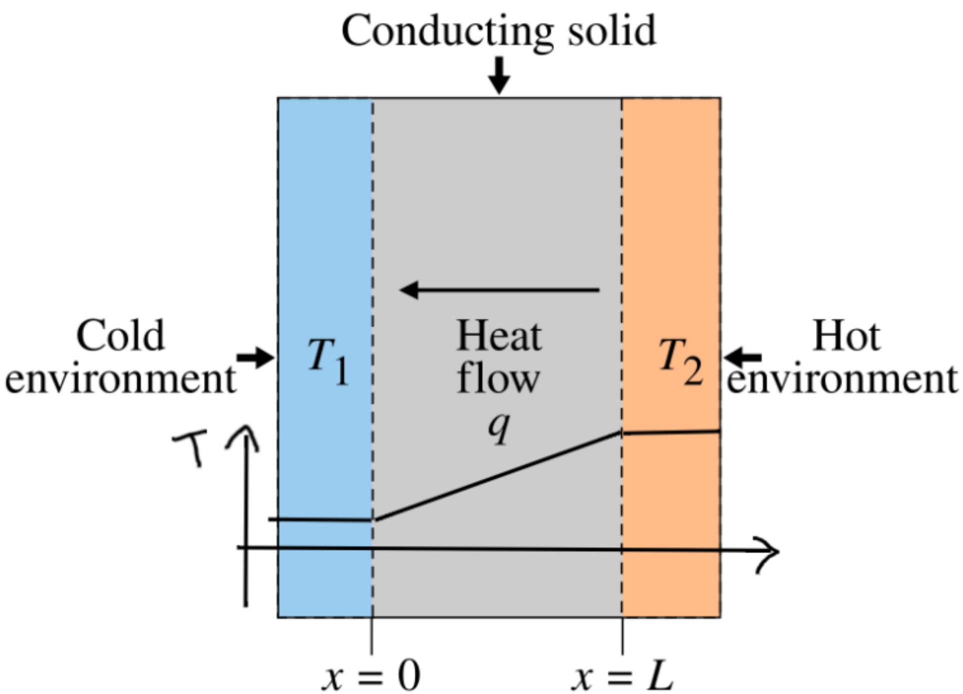
Non-equilibrium Steady State

Macroscopic stationary, Dynamic Balance, External driven.



图片来自：https://www.physicallensonthecell.org/chemical-physics/non-equilibrium-steady-states

Energy flow between two systems

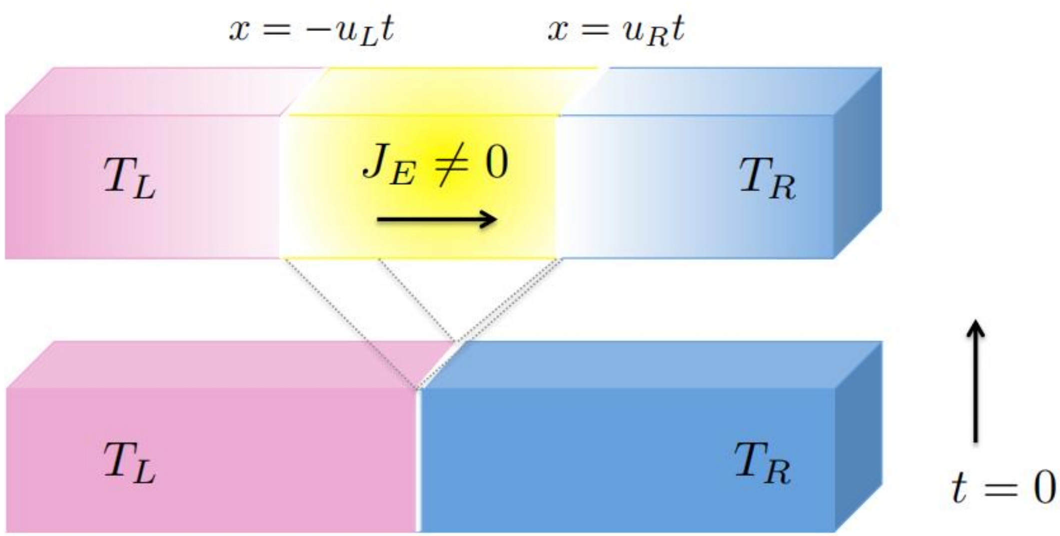


Fourier's law

$$q \propto -\frac{dT}{dx}$$

图片来自：https://en.wikipedia.org/wiki/Thermal_conductivity_and_resistivity

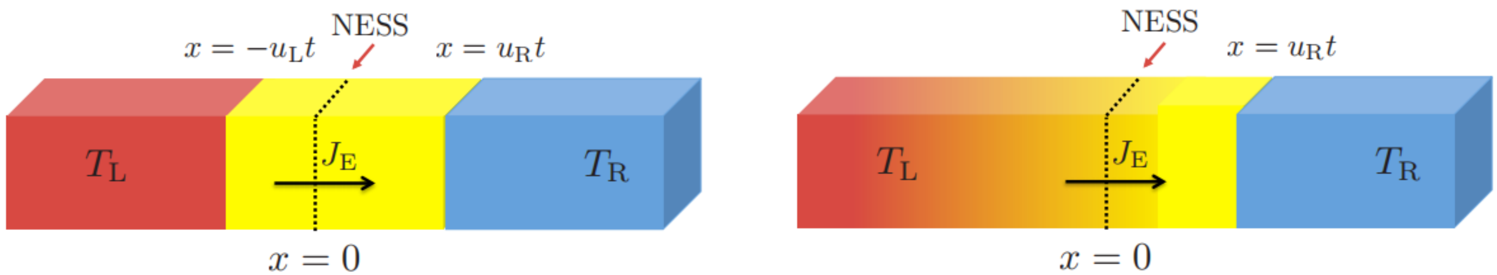
Energy flow in Quantum Critical Systems



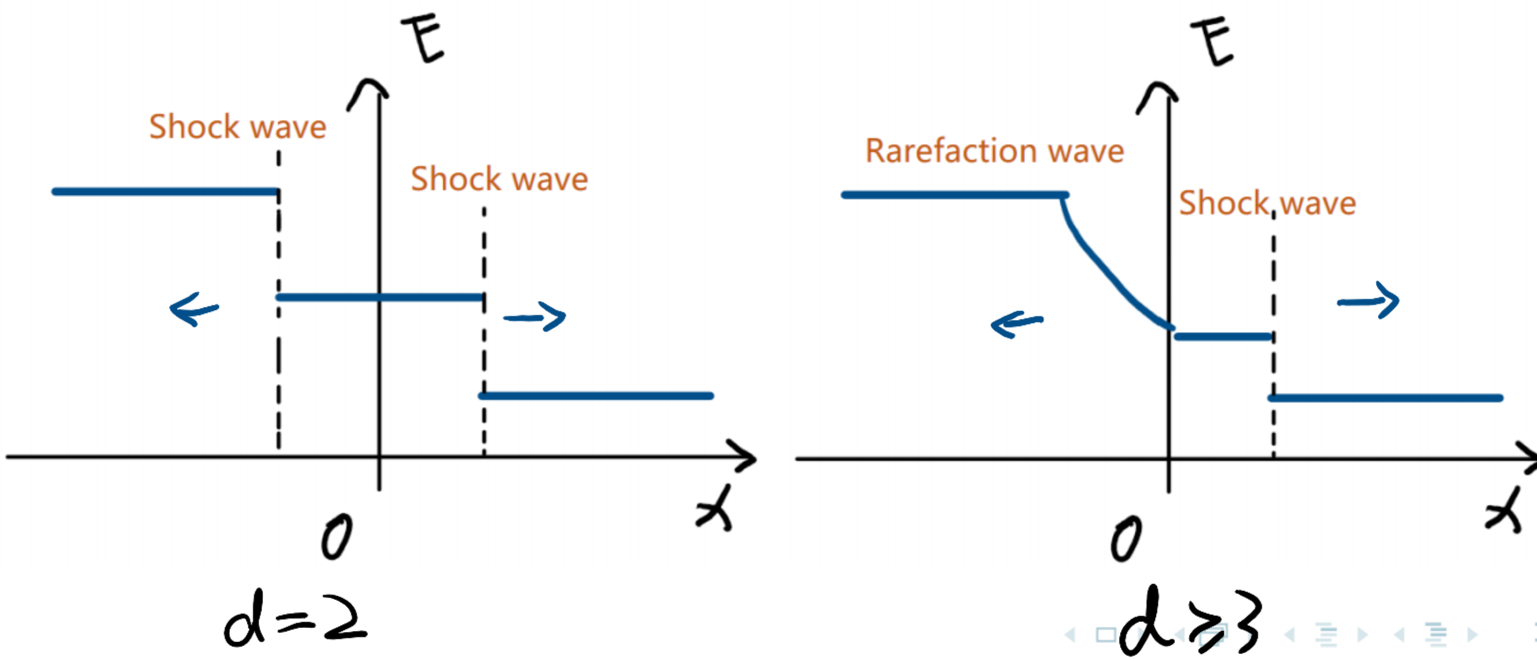
M. J. Bhaseen, B. Doyon, A. Lucas, and K. Schalm, Nature Phys **11**, 509 (2015).

CFT: diffusion equation $\rightarrowtail$ wave equation

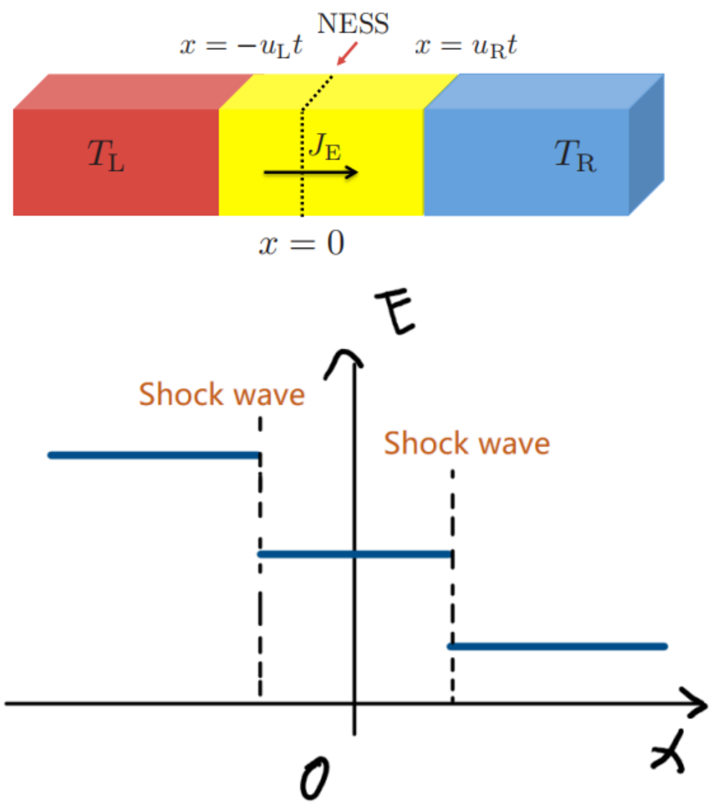
Universal Non-equilibrium Steady State



A. Lucas, K. Schalm, B. Doyon and M. J. Bhaseen, Phys. Rev. D94(2), 025004 (2016)



Shock Wave Velocity



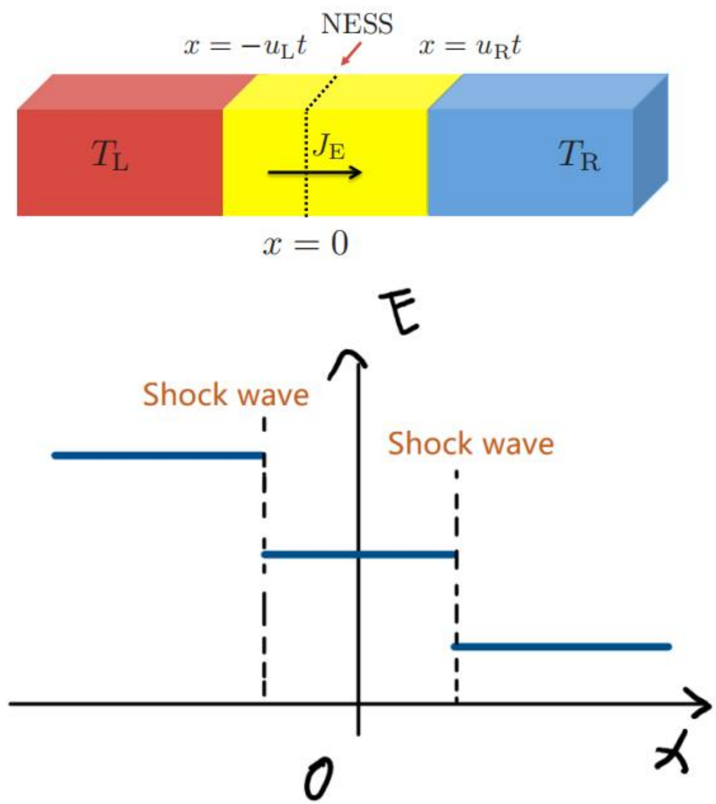
Rankine-Hugoniot jump conditions

$$v_L(T_L^{tt}-T_E^{tt})=T_L^{tx}-T_E^{tx}$$
$$v_R(T_R^{tt}-T_E^{tt})=T_R^{tx}-T_E^{tx}$$
$$v_L(T_L^{tx}-T_E^{tx})=T_L^{xx}-T_E^{xx}$$
$$v_R(T_L^{tx}-T_E^{tx})=T_L^{xx}-T_E^{xx}$$

with stress tensor

$$T_{L,R}^{\mu\nu}=\begin{bmatrix} T_{L,R}^{tt} & 0 \\ 0 & c_s^2 T_{L,R}^{tt}\end{bmatrix}$$
$$T_E^{\mu\nu}=c_s^2\mathcal{E}_E\left(\eta^{\mu\nu}+(1+c_s^{-2})u^\mu u^\nu\right)$$

Shock Wave Velocity



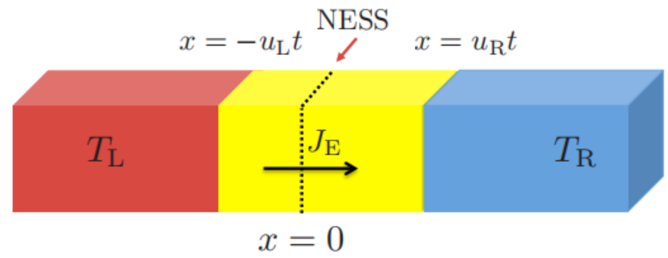
Shock wave velocity $v_{L,R}$ becomes

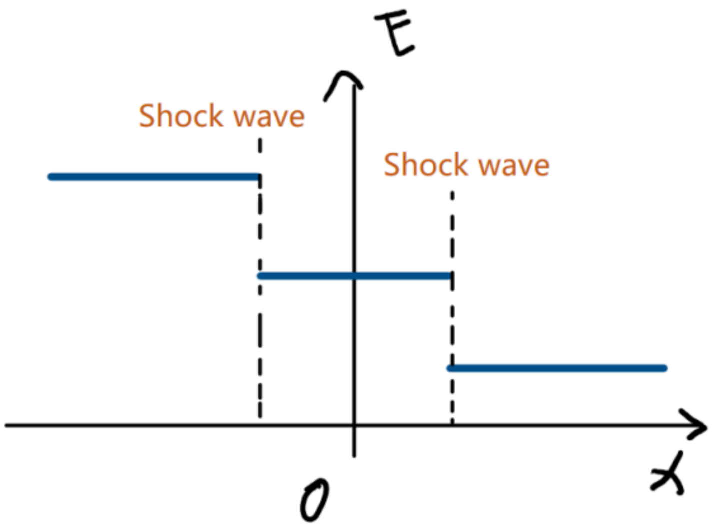
$$v_L = -c_s\sqrt{\frac{1+c_s^2\chi}{c_s^2+\chi}}$$

$$v_R = c_s\sqrt{\frac{c_s^2+\chi}{1+c_s^2\chi}}$$

where $\chi = \sqrt{T_R^{tt}/T_L^{tt}}$

Boost velocity

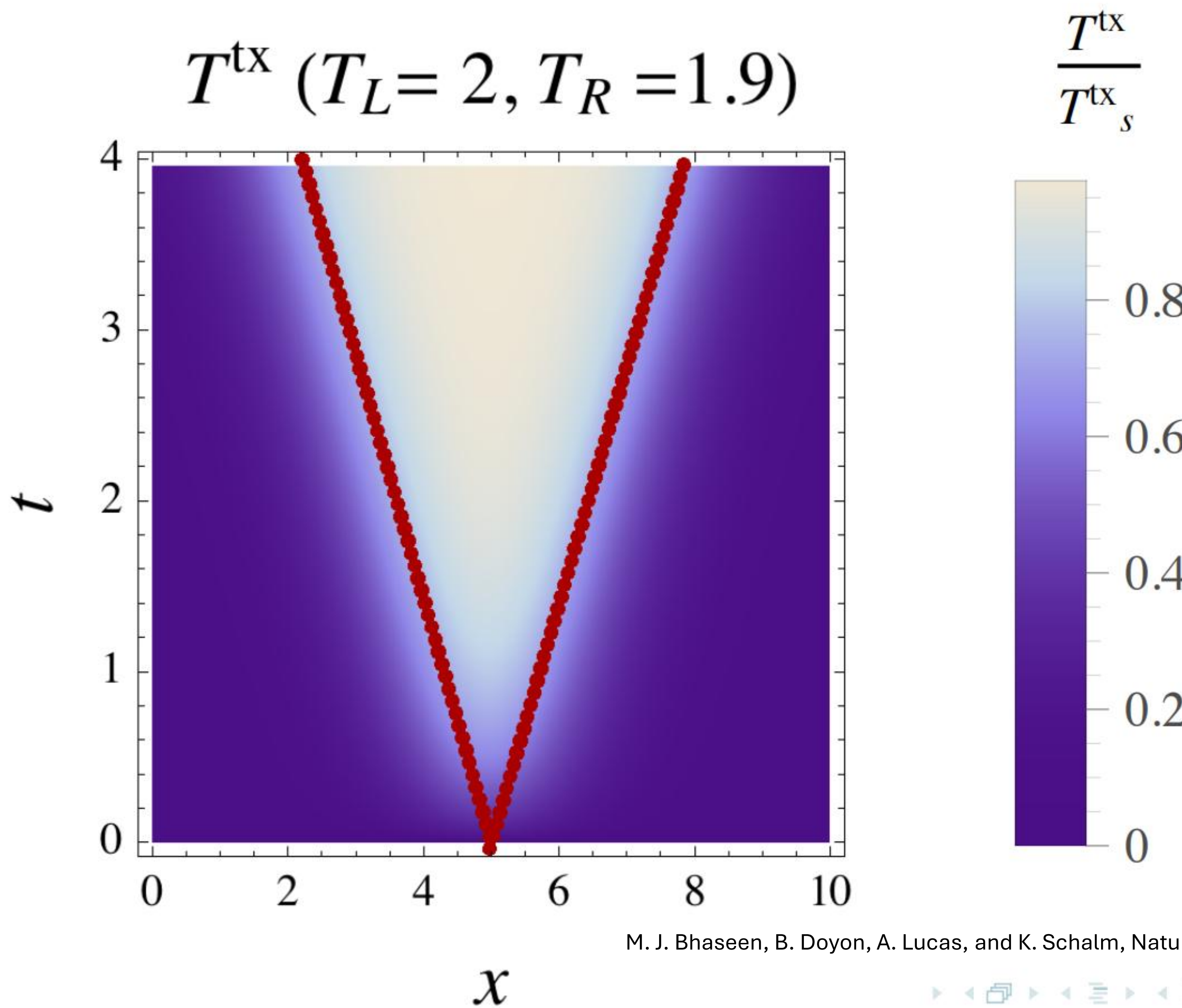




Boost velocity $u^\mu = \gamma(1, v_E)$ becomes

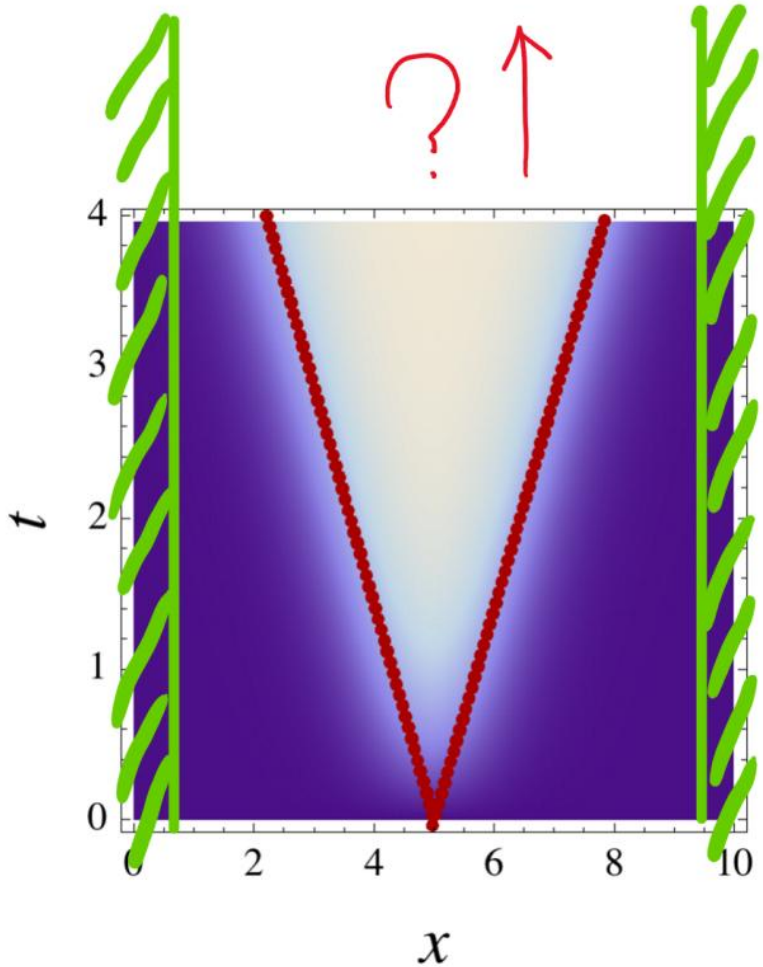
$$v_E = \frac{c_s(1-\chi)}{\sqrt{(1+c_s^2\chi)(c_s^2+\chi)}}$$

NESS $\rightarrowtail$ boosted black brane



M. J. Bhaseen, B. Doyon, A. Lucas, and K. Schalm, Nature Phys **11**, 509 (2015).

Finite Size System and Thermalization



Infinite system ⟶ Finite size system

NESS ⟶ Thermalization

1 Non-equilibrium Steady State

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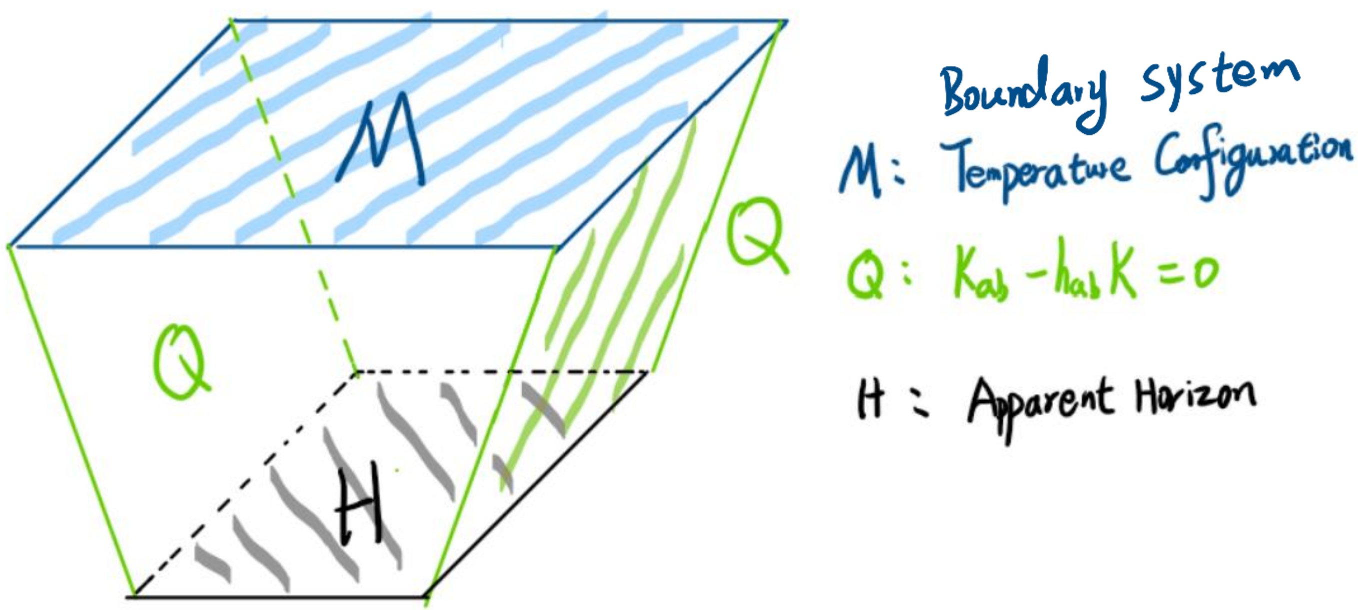
3 NESS decay, shock wave and dissipation

AdS/BCFT and Boundary Condition



T. Takayanagi, Holographic Dual of BCFT, Phys. Rev. Lett. **107**, 101602 (2011).

Holographic Set Up

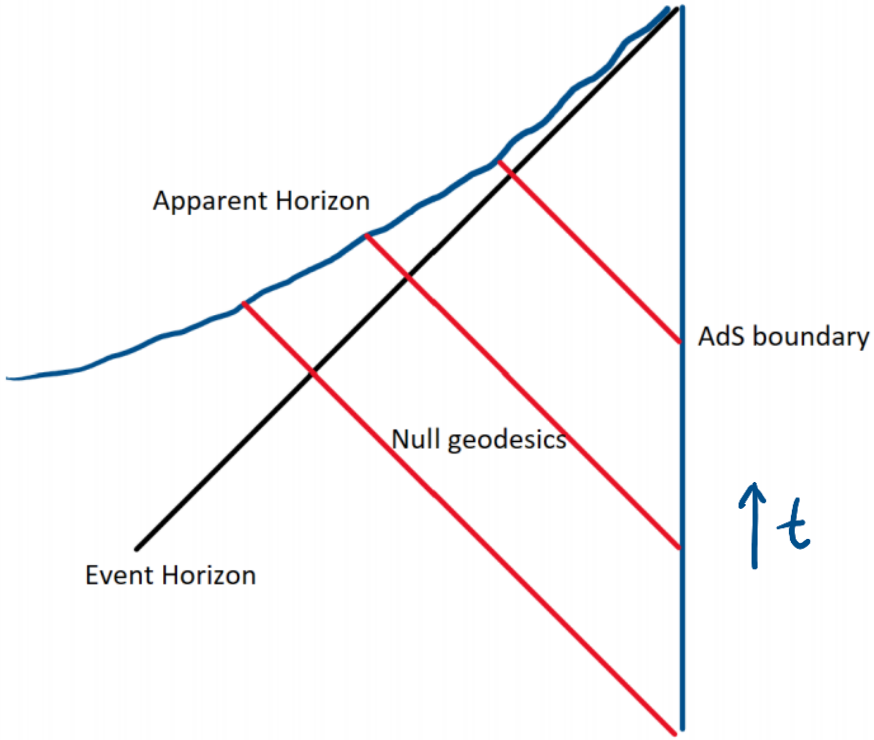
The action is

$$S = \int d^4x \sqrt{-g}\, [R - 2\Lambda]\,, \tag{1}$$

and the line element becomes

$$\begin{aligned} ds^2 = &- A(u,t,x)\, dt^2 + 2dtdr - 2F_i(u,t,x)\, dtdx^i \\ &+ \Sigma^2(u,t,x)\left[e^{B(u,t,x)} dx^2 + e^{-B(u,t,x)} dy^2\right]. \end{aligned} \tag{2}$$

Evolution



And the boundary condition on $Q \cap M$ becomes

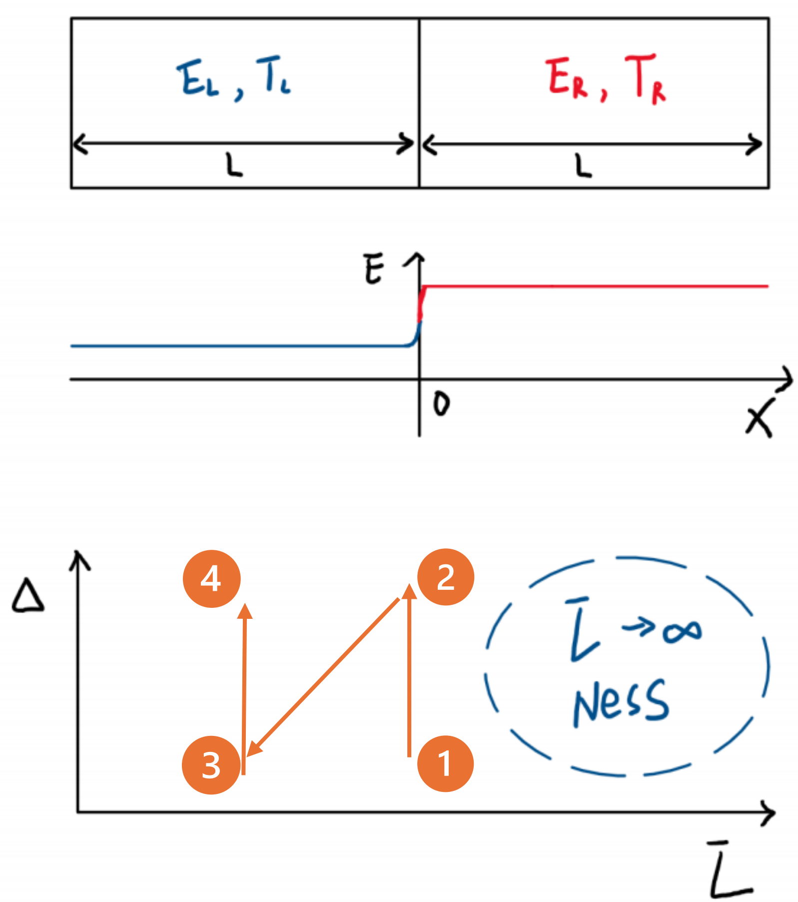
$$\partial_x T^{tt}|_{Q\cap M} = 0 \quad (3)$$

$$T^{tx}|_{Q\cap M} = 0 \quad (4)$$

Isolated finite size system.

P. M. Chesler and L. G. Yaffe, Numerical solution of gravitational dynamics in asymptotically anti-de Sitter spacetimes, J. High Energ. Phys. **2014**, 86 (2014).

Dimensionless Parameters



Scaling Symmetry

$$\{1/r, t, x, y\} \to \lambda\{1/r, t, x, y\}$$

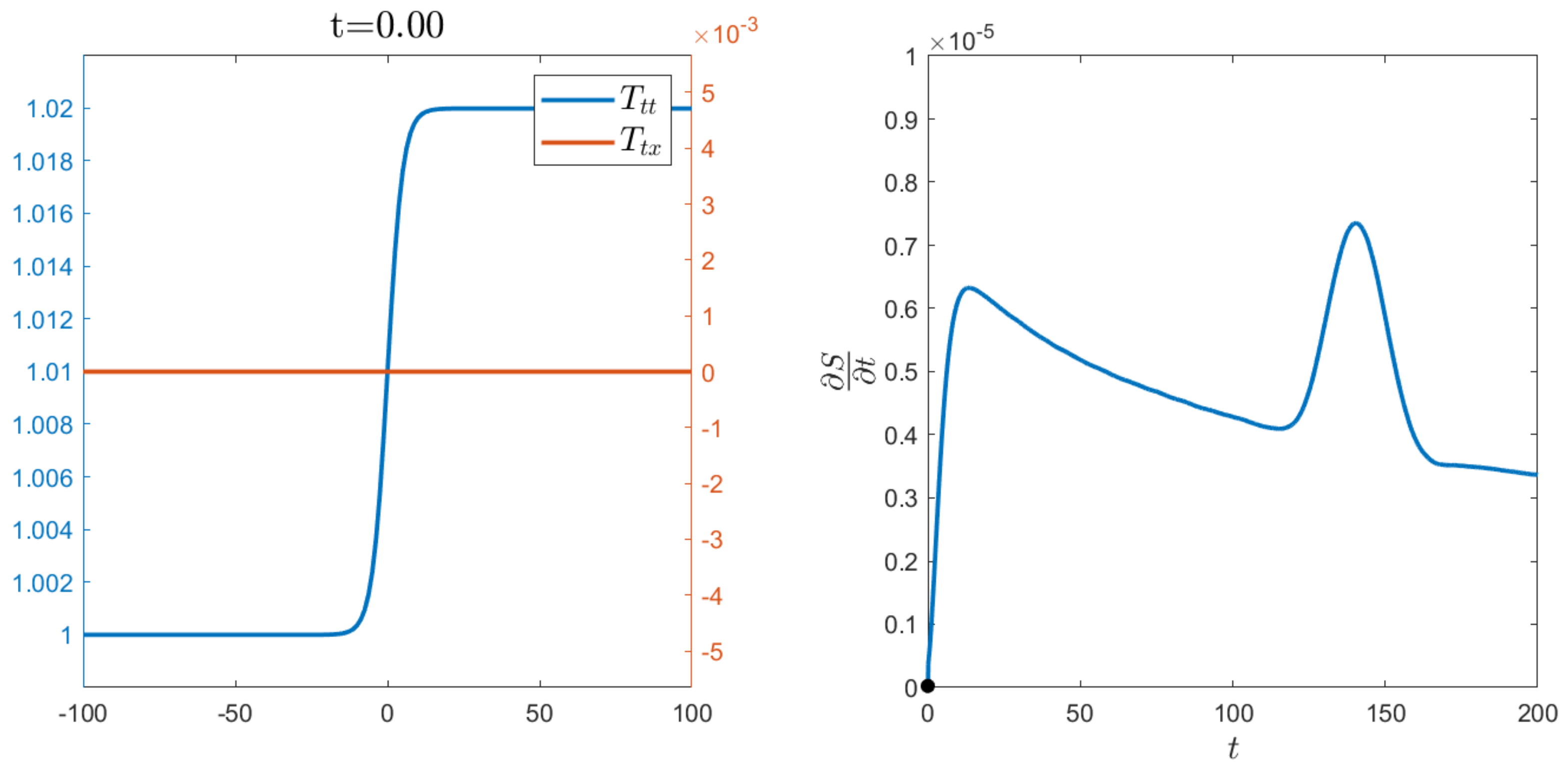
$$E \to \frac{1}{\lambda^3}$$

Dimensionless Parameters

$$\Delta = \frac{E_R - E_L}{E_L}$$

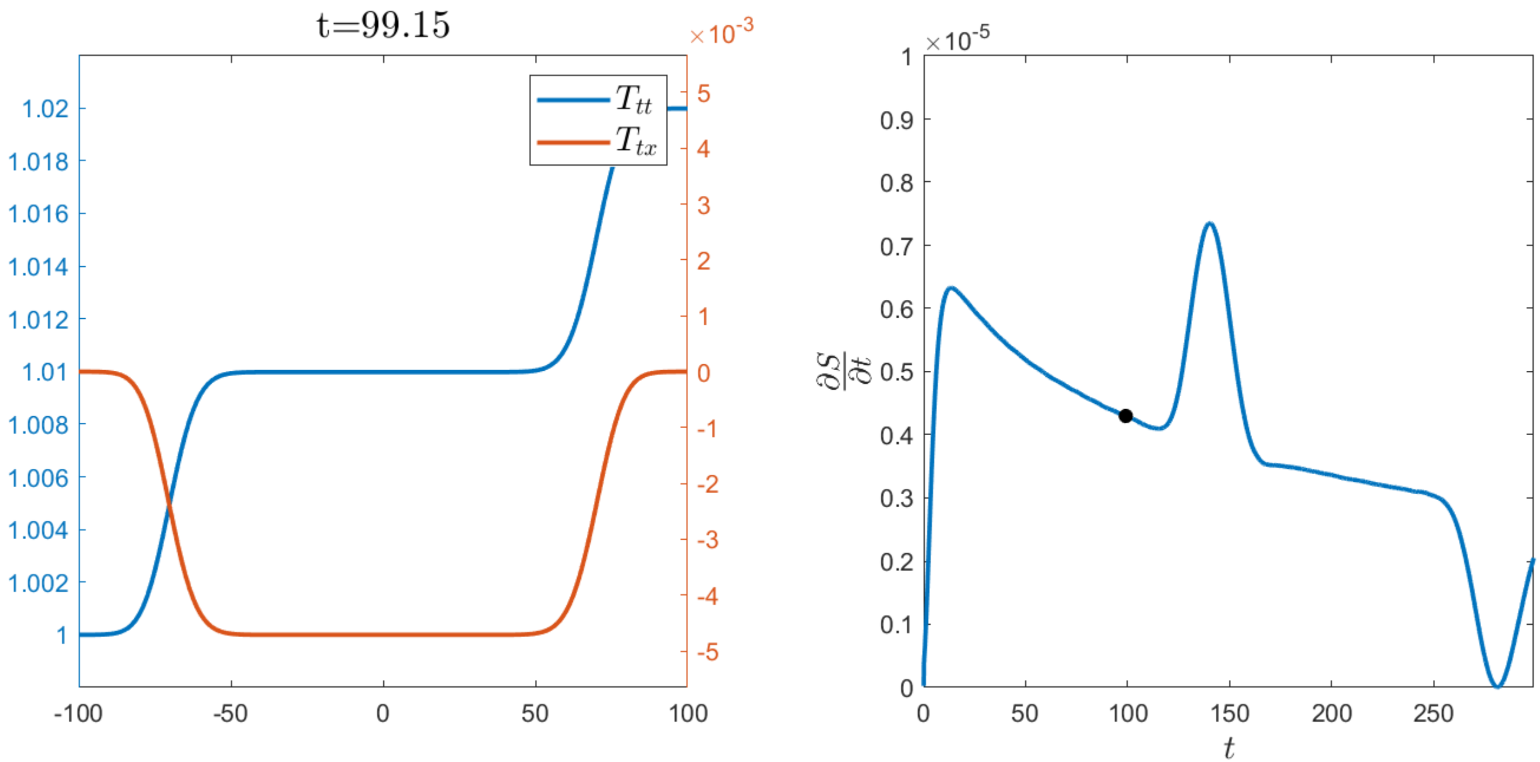
$$\bar{L} = L(E_R + E_L)^{1/3} 2^{-1/3}$$

NESS Recurrence and Entropy Increasing



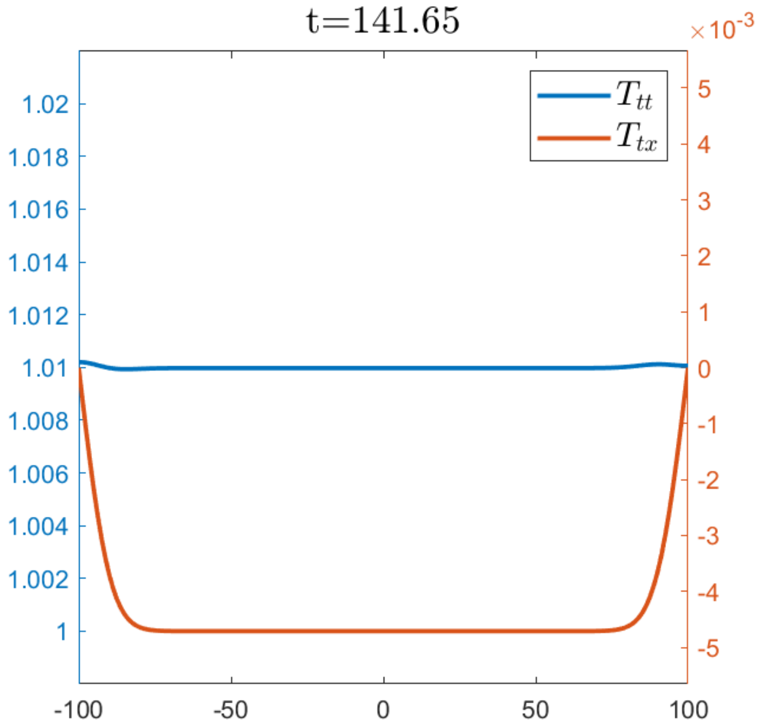
$\Delta = 0.02$, $L = 100$

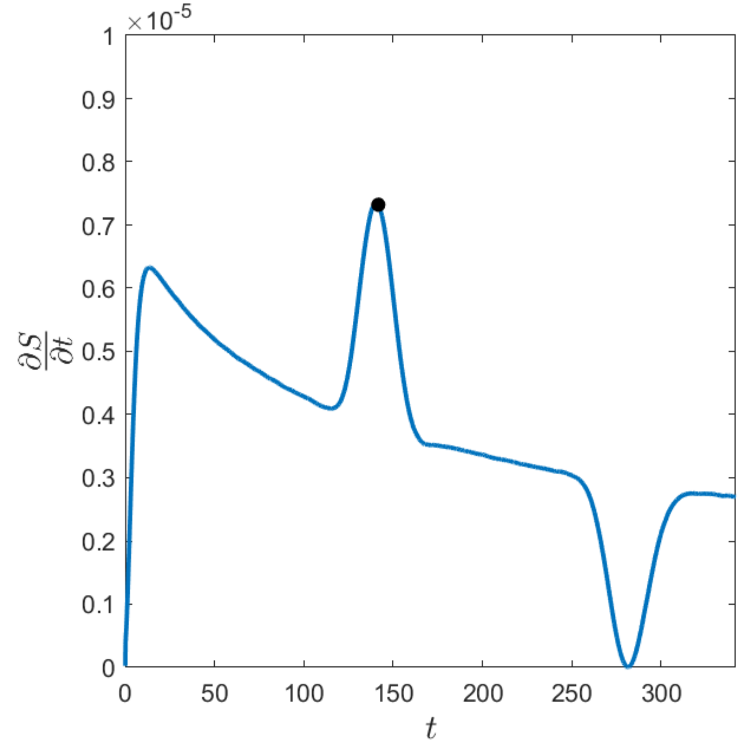
NESS Recurrence and Entropy Increasing



$\Delta = 0.02$, $\tau = 100$

NESS Recurrence and Entropy Increasing





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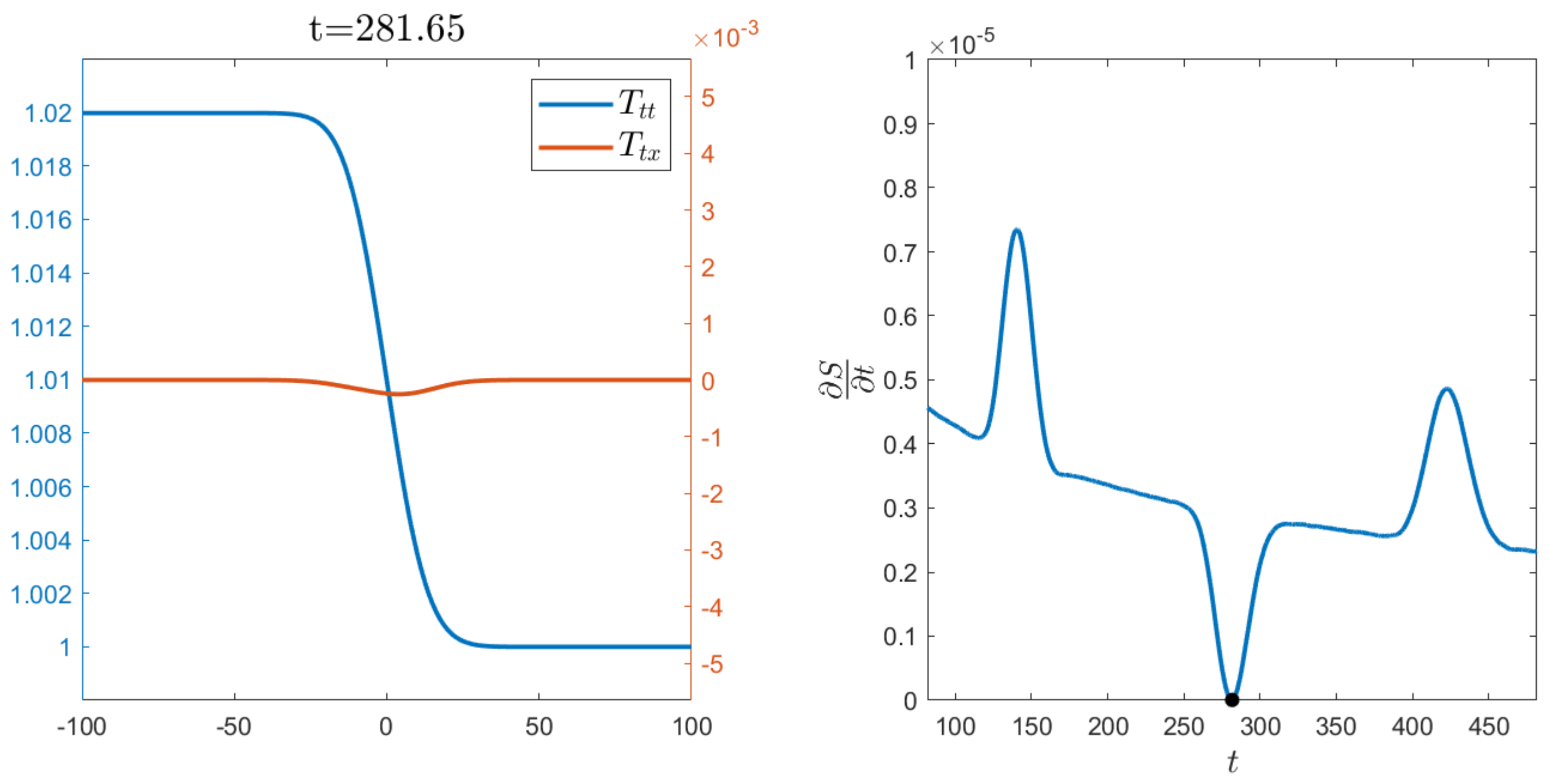
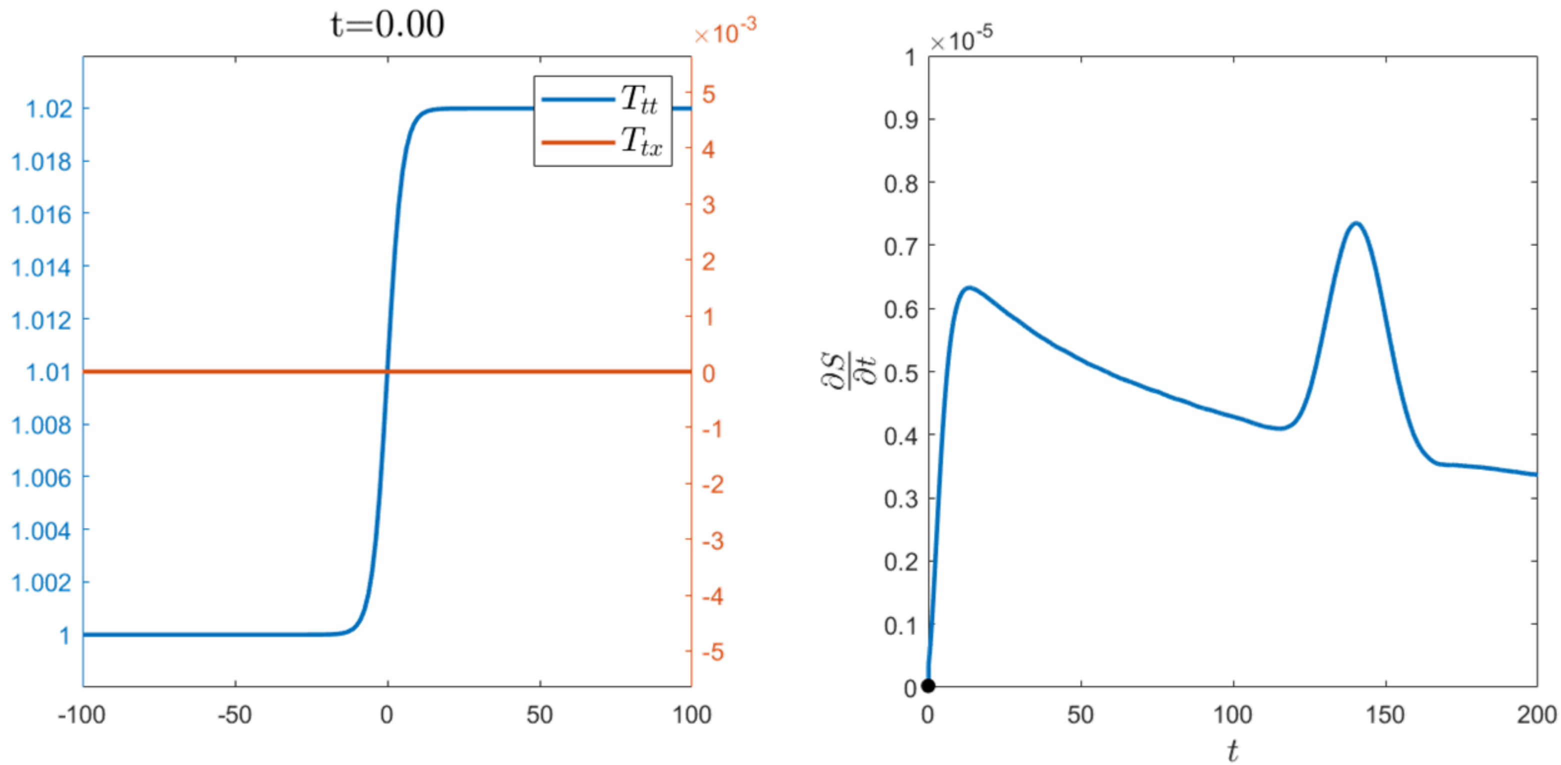
NESS Recurrence and Entropy Increasing



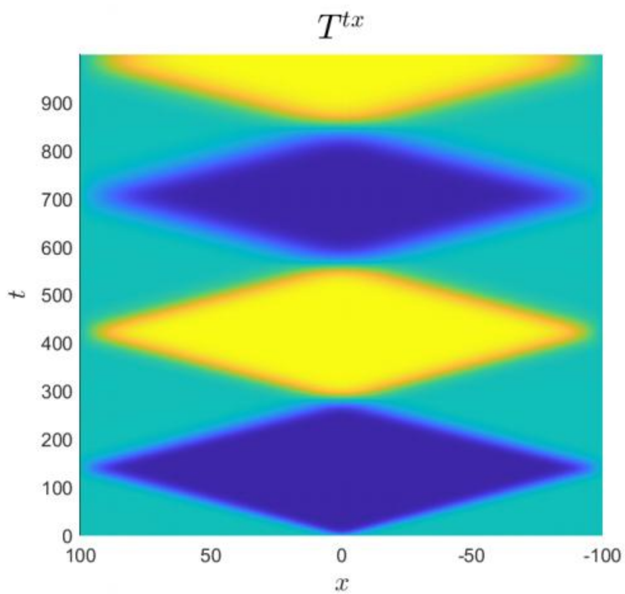
$\Delta = 0.02$, $L = 100$

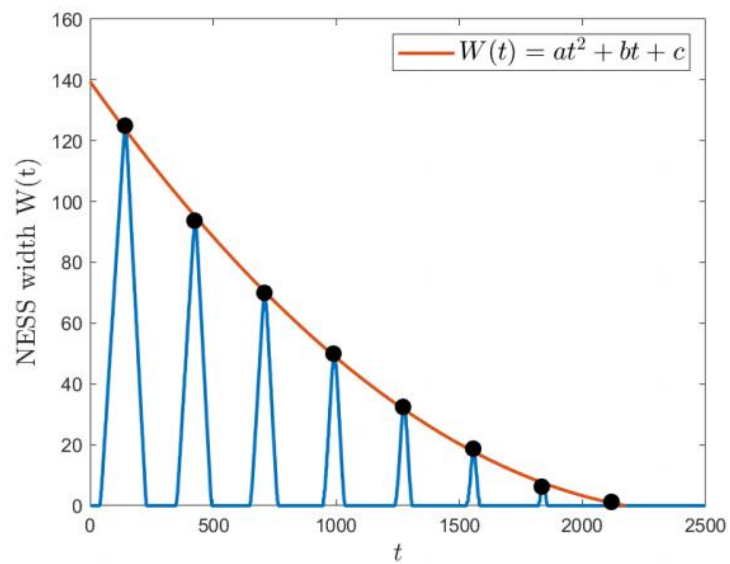
NESS Recurrence and Entropy Increasing



$\Delta = 0.02$, $\tau = 100$

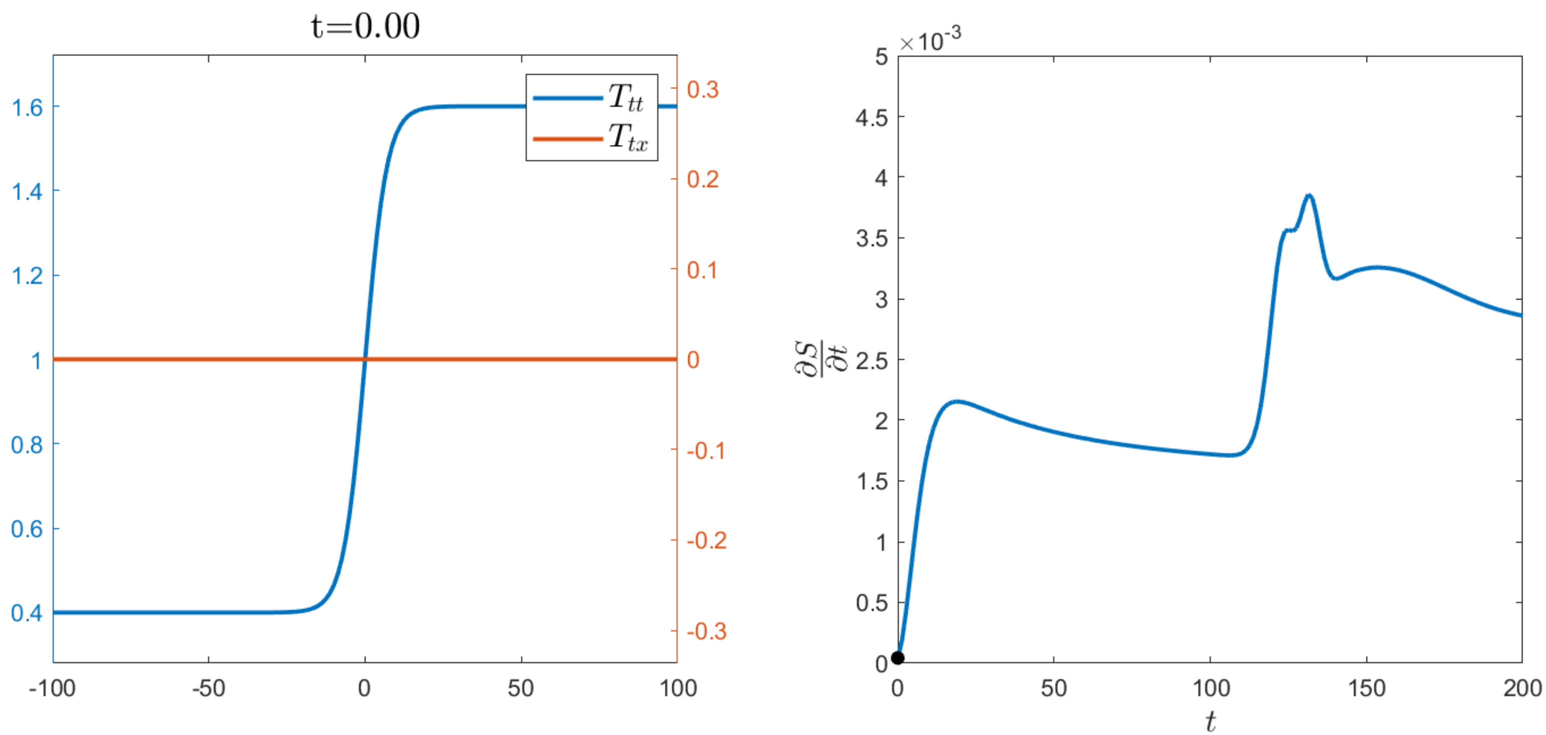
NESS recurrence and decay





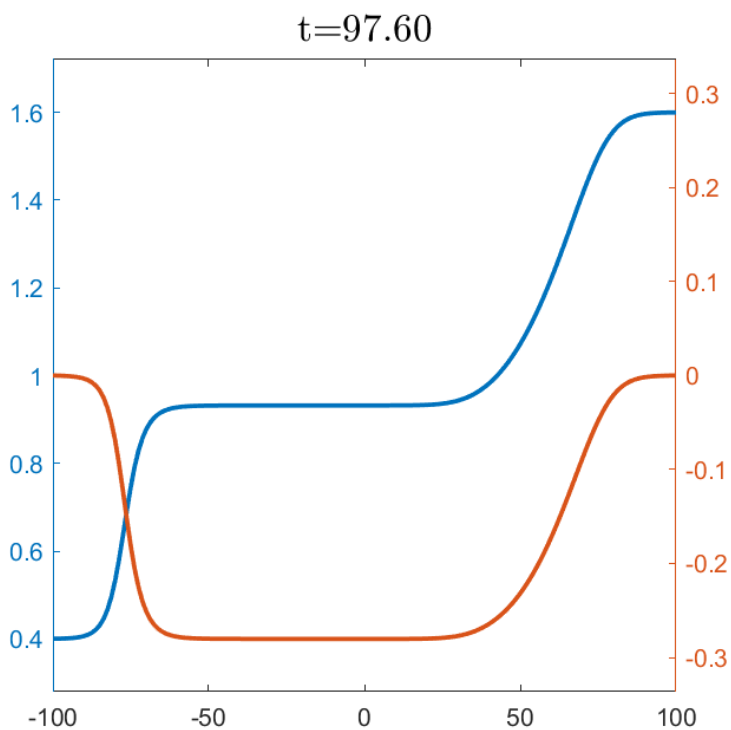
NESS recurrence $\rightarrowtail$ NESS width decay $\rightarrowtail$ quasinormal modes

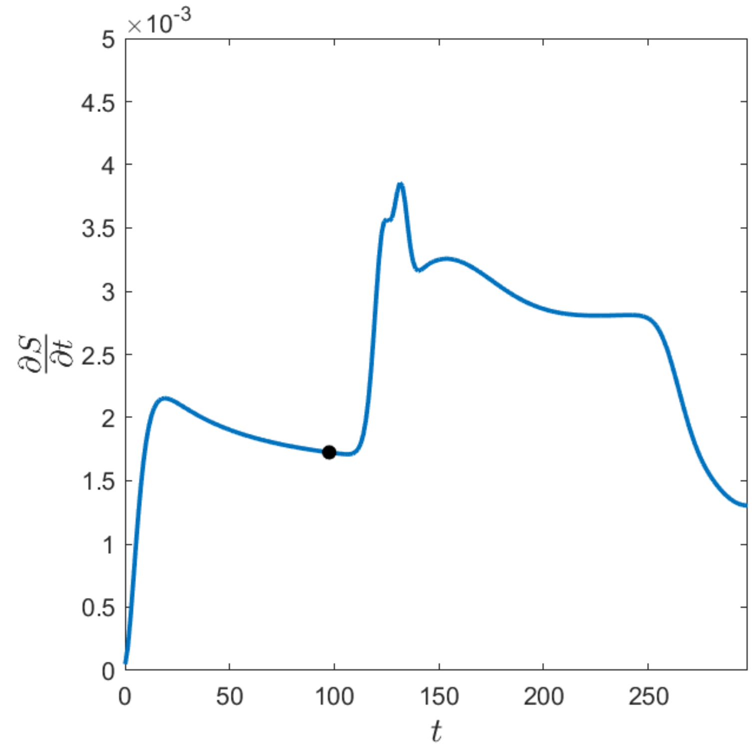
Shock wave propagation and Entropy Increasing



$\Delta = 3$, $\bar{L} = 100$

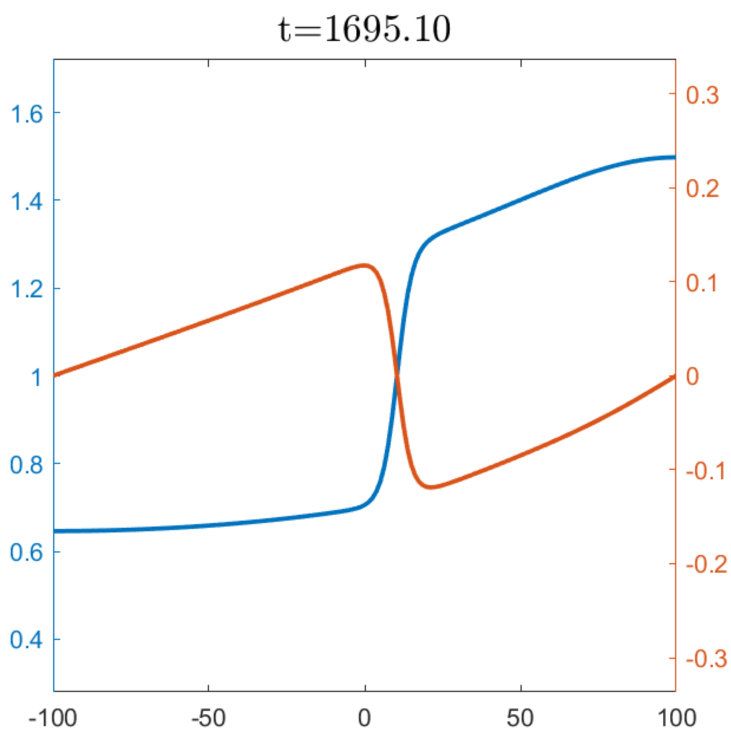
Shock wave propagation and Entropy Increasing

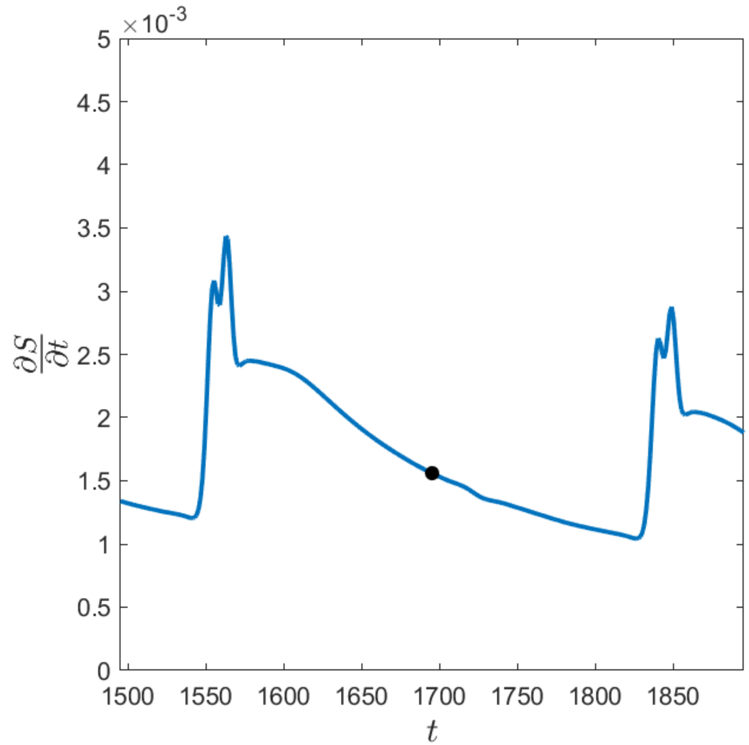




$\Delta = 3, \quad \bar{L} = 100$

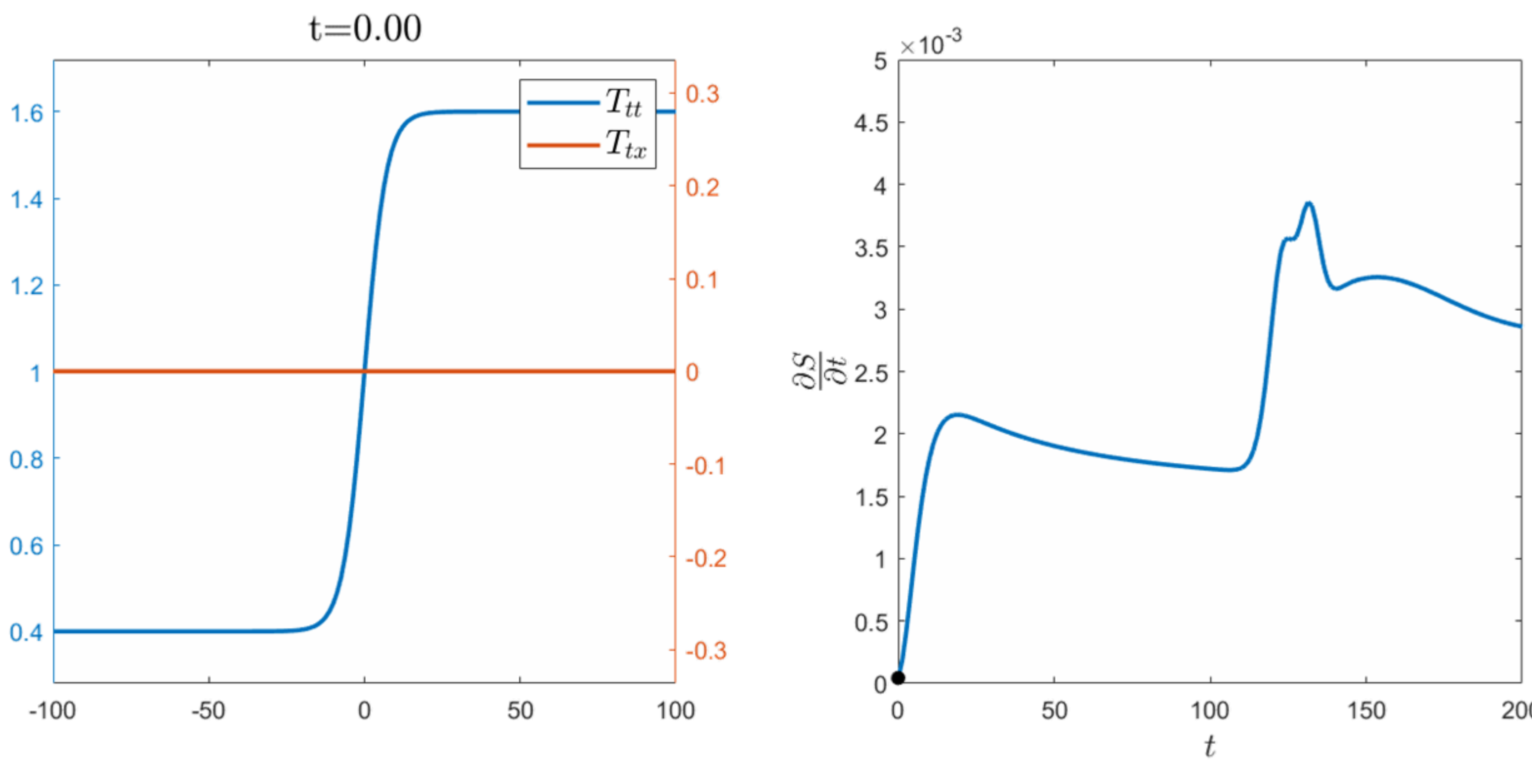
Shock wave propagation and Entropy Increasing





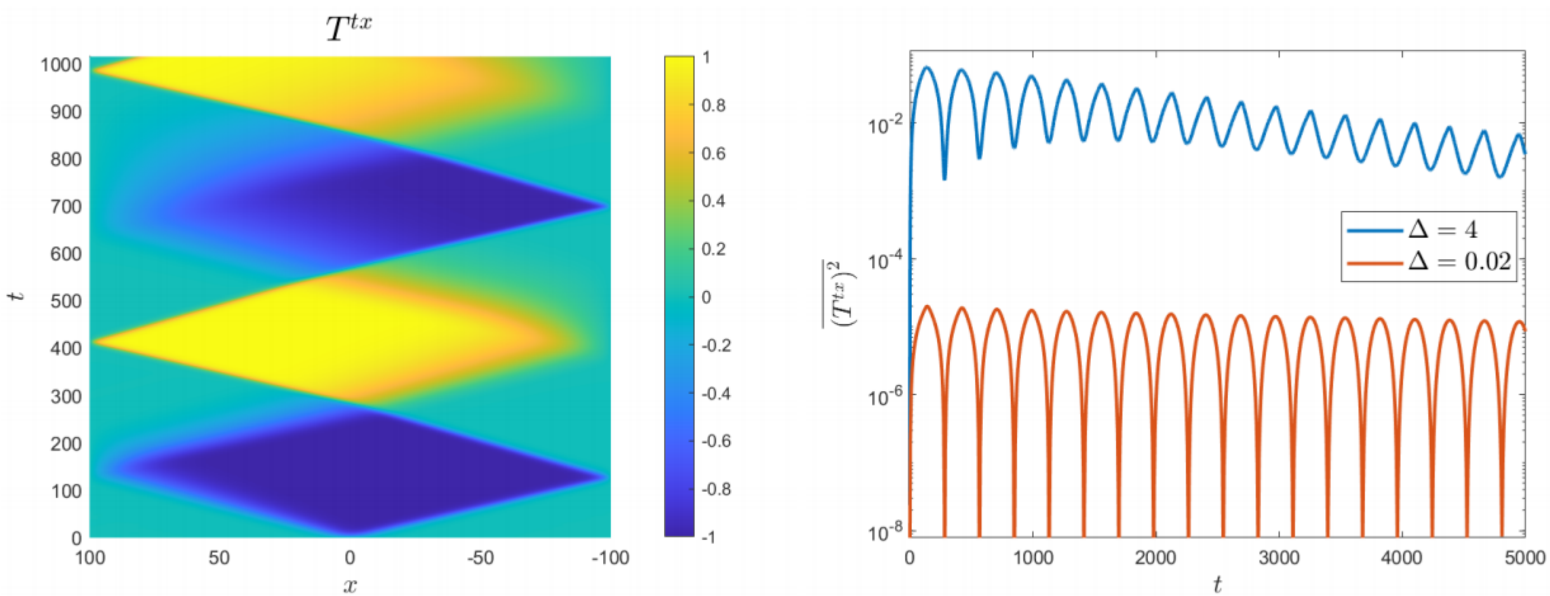
$\Delta = 3, \quad \bar{L} = 100$

Shock wave propagation and Entropy Increasing



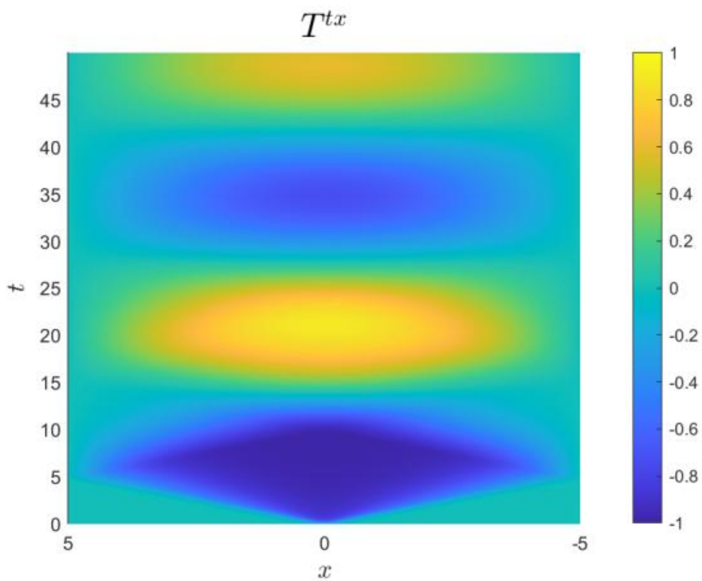
$\Delta = 3$, $\overline{L} = 100$

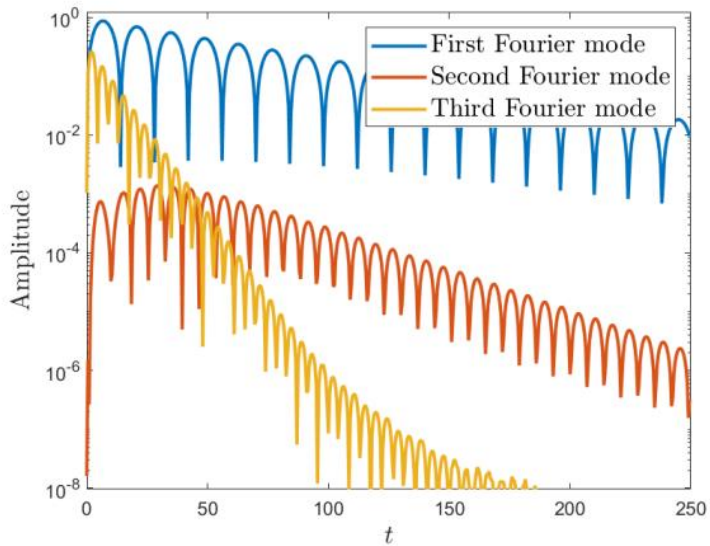
Shock wave oscillation and decay



Rarefaction wave superposition $\longrightarrow$ Shock wave propagation $\longrightarrow$ quasinormal modes

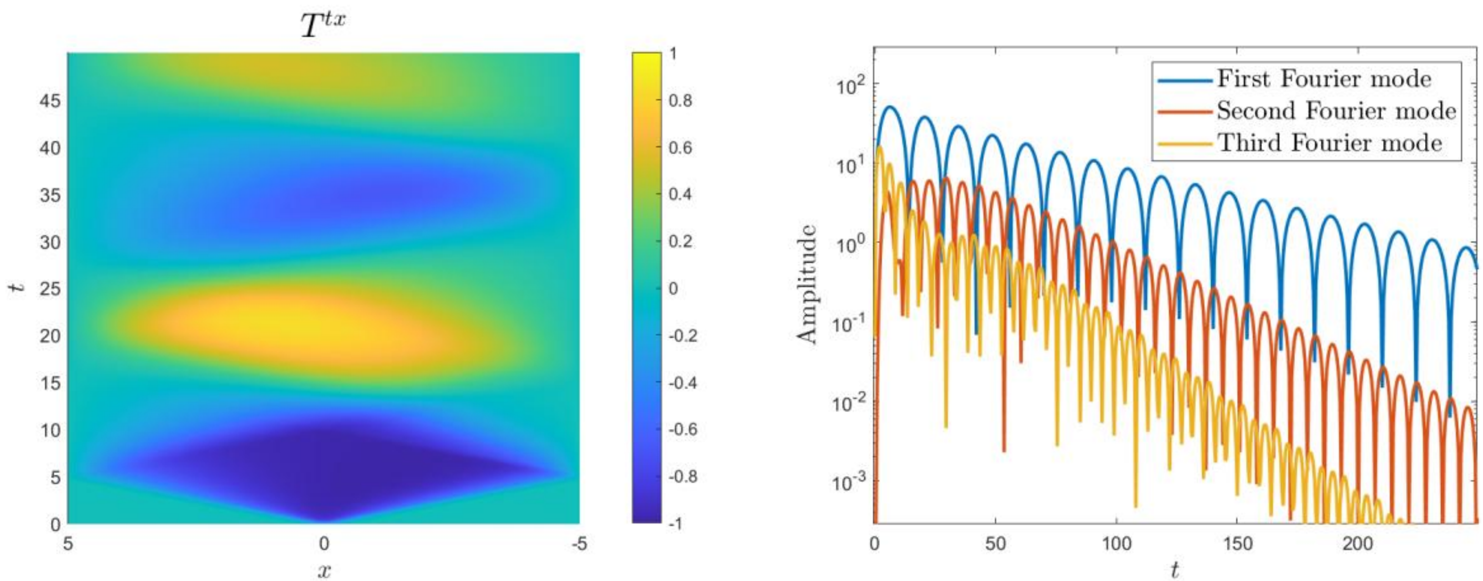
quasinormal modes decay





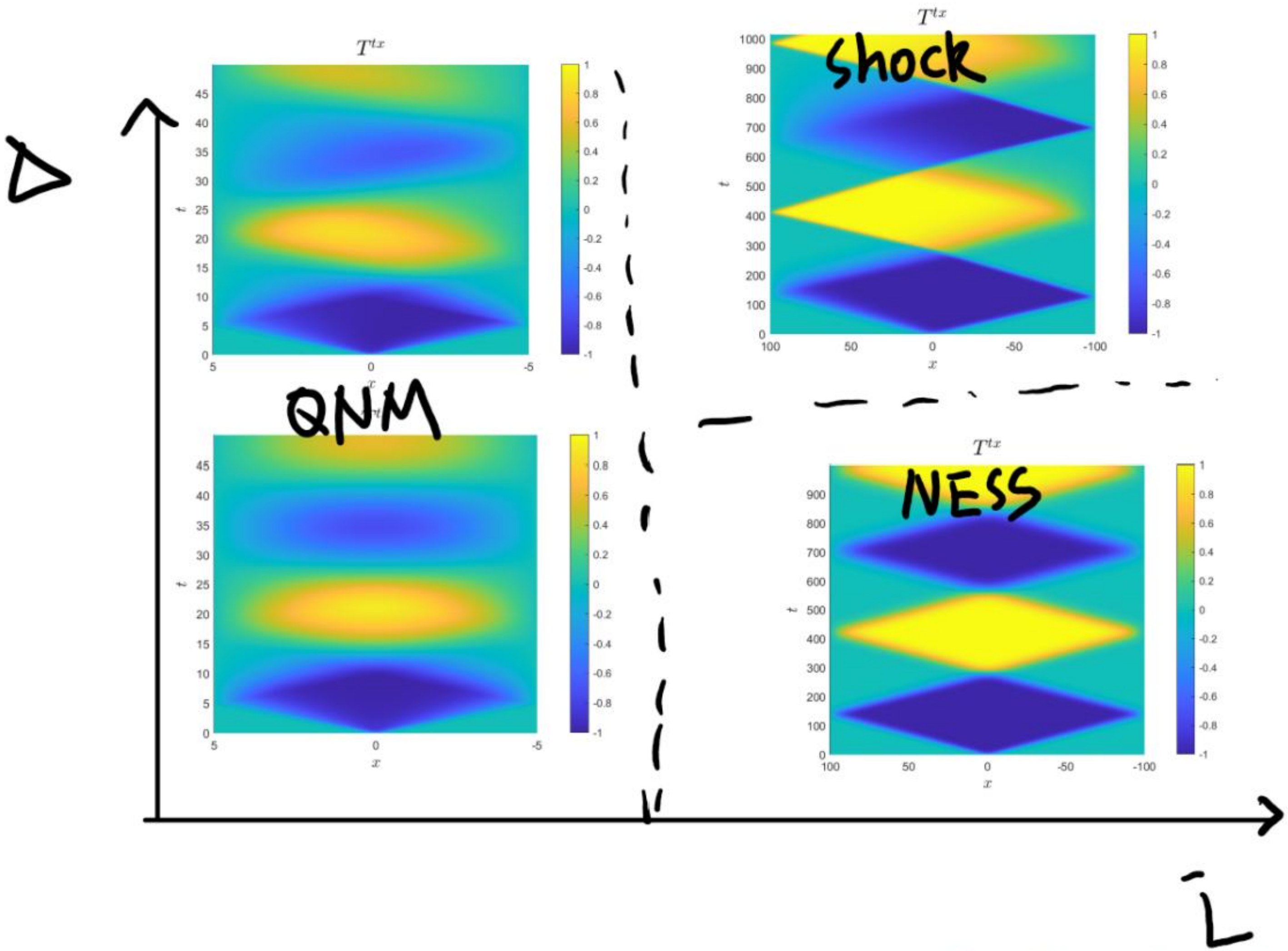
$\Delta = 0.02$, $L = 5$

quasinormal modes decay



$\Delta = 198, \quad \bar{L} = 5$

Different dynamical behavior of thermalization



Thank you!