Near-extremal holographic charge correlators

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Tuesday, July 15, 2025

Holographic applications: from Quantum Realms to the Big Bang,

UCAS International Conference Center, China

References

- *Near-extremal holographic charge correlators*, with David Ramirez, Mikel Sanchez-Garitaonandia and Clément Supiot, [arXiv:2506.11974].
- Ongoing work with the same collaborators.
- *Hydrodynamic diffusion and its breakdown near AdS_2 fixed points*, with D. Areán, R. Davison and K. Suzuki, [arXiv:2011.12301].

$AdS_2 \times M^d$ near-extremal spacetimes

- Recent evidence for duality between near-AdS_2 spacetimes and SYK models [Kitaev'15, Sachdev'15, Maldacena & Stanford'16, Maldacena, Stanford & Yang'16, Jensen'16, Engelsoy, Mertens & Verlinde '16].

- Large families of holographic theories have black hole solutions with an $AdS_2 \times S^d$ near-extremal geometry.

- Some thermodynamic properties of these black holes can be deduced by KK-reducing to AdS_2 [Almheiri & Polchinski'14, Nayak et al'18, Iliesiu & Turiaci'20].

- To what extent are their dynamical properties at low temperature controlled by the AdS_2 geometry/the Schwarzian effective action?

$AdS_2 \times R^d$ near-extremal spacetimes

- Charged planar black holes extensively investigated in applied AdS/CFT, eg

$$\mathcal{L} = R - \frac{1}{2}\partial\phi^2 - V(\phi) - \frac{Z(\phi)}{4}F^2 - \frac{Y(\phi)}{2}\sum_{I=1}^{d}\partial\psi_I^2\,, \quad \psi_I = m\delta_{Ii}x^i$$

- They are dual to charged fluids in a flat geometry, possibly subject to various instabilities at low temperature.

- In the normal phase, they often display an emergent $AdS_2 \times R^d$ near-extremal geometry

$$ds^2 = \frac{\ell_2^2}{\zeta^2}\left[-\left(1-\frac{\zeta^2}{\zeta_h^2}\right)dt^2 + \frac{d\zeta^2}{1-\frac{\zeta^2}{\zeta_h^2}}\right] + \frac{d\vec{x}_{(d)}^2}{r_e^2}$$

$AdS_2\times R^d$ near-extremal spacetimes: EFT at finite T

- At frequencies $\omega, k \lesssim T$, the low-energy dynamics is controlled by hydrodynamics, whether $Tr_e \gtrsim 1$ [Erdmenger et al'08, Banerjee et al'08] or $Tr_e \lesssim 1$ [Edalati et al '09,'10], eg for charged fluids:

$$\nabla_\mu T^{\mu\nu} = 0 \qquad \nabla_\mu J^\mu = 0$$

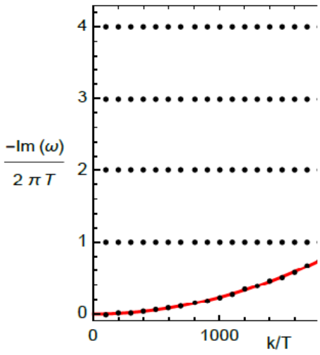
- Solving for linear perturbations, retarded Green's functions display gapless hydrodynamic poles

longitudinal sound: $\omega(k, T) = \pm c_s(T)k - i\Gamma(T)k^2 + O(k^3)$

longitudinal thermal diffusion: $\omega(k, T) = -iD_{th}(T)k^2 + O(k^4)$

transverse momentum diffusion: $\omega(k, T) = -iD_\eta(T)k^2 + O(k^4)$

$AdS_2\times R^d$ near-extremal spacetimes: Low-T spectrum



[Arean et al, 2011.12301]

- For $Tr_e \lesssim 1$, the hydrodynamic modes coexist with a tower of gapped modes controlled by the SL(2,R) symmetry of the $AdS_2\times R^2$ geometry:

$$\omega_n = -2i\pi T\,[\Delta(k)+n]$$

Coalesce into a branch cut $\omega^{2\Delta(k)-1}$ at $T=0$ [Faulkner et al'09, Edalati et al'09,'10].

- Surprisingly, gapless modes persist at $T=0$ besides the branch cut [Edalati et al'09,'10; Davison & Parnachev'13, Arean et al'20].

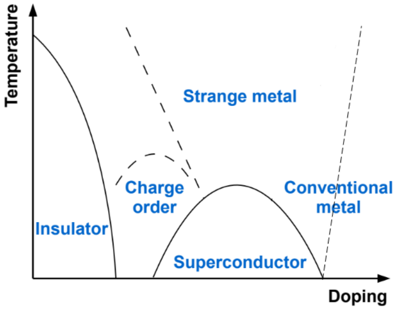
$$\omega = -iD_{th}(T=0)k^2+\dots$$

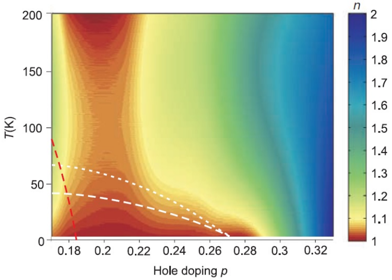
- This usually signals an emergent global symmetry (eg Fermi liquids, superfluids)

$AdS_2 \times R^d$ near-extremal spacetimes

- Can we derive an EFT which captures both the hydrodynamic modes and the gapped critical modes?
- What is the emergent symmetry and what causes it?
- Relation to the Schwarzian and AdS_2?
- This could also be relevant for the strange metallic phase of cuprate high T_c superconductors.

Strange metals: Drude or quantum critical?





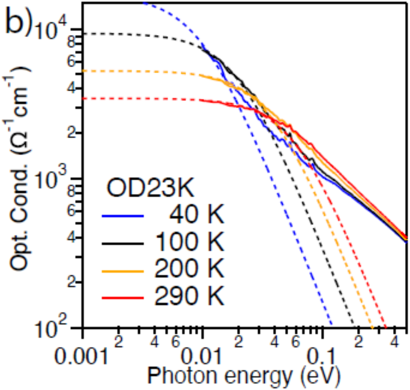
[Cooper et al, Science'09]

- $\rho \sim T$ over most of the OD phase diagram. From the Drude formula [Legros et al, 1805.02512]:

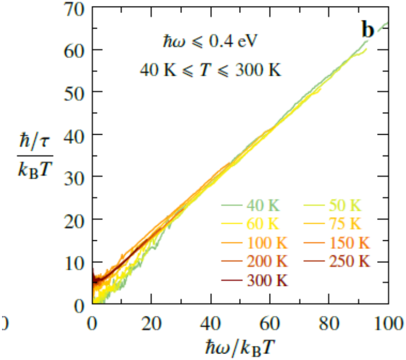
$$\rho_{Drude} = \frac{m}{ne^2}\tau_{tr} \quad \Rightarrow \quad \tau_{tr} = \mathcal{O}(1)\frac{\hbar}{k_B T} \simeq \tau_{Planck}$$

- Also reminiscent of quantum criticality [Sachdev, Zaanen], where T is the only scale.

Strange metals: Drude or quantum critical?

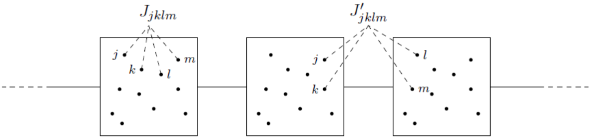


[Van Heumen et al, 2205.00899]

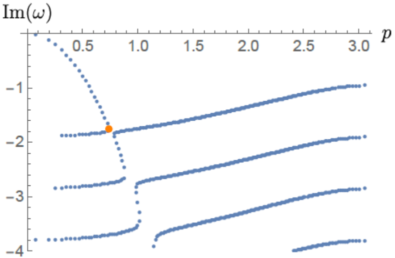


[Michon et al, 2205.04030]

- AC conductivity displays a Drude peak and extra mid-IR spectral weight. Emergent branch cut [Van der Marel, cond-mat/0309172; Van Heumen et al, 2205.00899]?
- Data also displays ω/T scaling: quantum criticality?



[Gu et al, 1609.07832]



[Choi et al, 2010.08558]

- 1d spatial extensions of SYK models also display a similar spectrum at low temperatures.
- 2d Yukawa-SYK models reproduce aspects of strange metal thermodynamics and transport [Patel et al, 2203.04990; Guo et al, 2308.01956].

This talk

- Present an analytic expression for the probe G^R_{JJ} in a planar black hole spacetime with a near-extremal $AdS_2 \times R^2$ geometry.

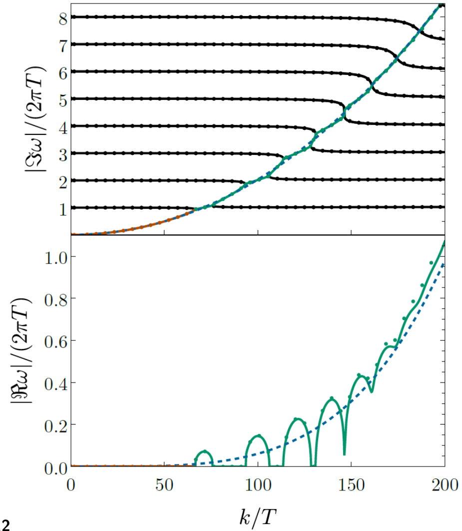
- The calculation is a careful extension beyond leading order of matching calculations done in previous literature [Davison & Parnachev, 1303.6334], assuming
$$\omega r_e \sim T r_e \sim k^2 r_e^2 \ll 1$$

- Keeping track of irrelevant deformations away from $AdS_2 \times R^2$ was essential.

- Captures all the dynamics of the hydro pole, the gapped poles, and the emergent $T = 0$ spectrum: strongly suggests that the EFT can be constructed a la fluids/gravity (see also [Moitra et al, 2005.00016; C. Yue's talk]).

Key result: Analytic computation of G^R_{JJ}

$$G^R_{JJ} = \frac{\omega^2\left(1 - \mathbf{k}^2\mathcal{G}/3\right)}{i\omega + \frac{\mathbf{k}^2}{6}\left(4\pi\mathbf{T} + 2(\mathbf{k}^2 + i\omega)\mathcal{G} + 3\mathbf{k}^2\log 3 - 6\right)}, \quad (\boldsymbol{\omega}, \mathbf{k}, \mathbf{T}) = r_e(\omega, k, T)$$

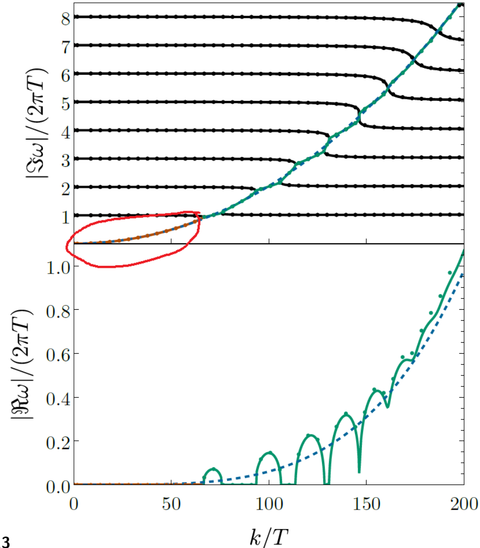


$$\mathcal{G} = \pi\cot\left(\frac{i\boldsymbol{\omega}}{2\mathbf{T}}\right) + \gamma + \psi\left(\frac{i\boldsymbol{\omega}}{2\pi\mathbf{T}}\right) - \log\left(\frac{9}{4\pi\mathbf{T}}\right),$$

Result valid at at $0 \ll r_e T \ll 1$

Charge diffusion pole

$$G^R_{JJ} = \frac{\omega^2\left(1 - \boldsymbol{k}^2\mathcal{G}/3\right)}{i\omega + \frac{\boldsymbol{k}^2}{6}\left(4\pi \boldsymbol{T} + 2(\boldsymbol{k}^2 + i\omega)\mathcal{G} + 3\boldsymbol{k}^2\log 3 - 6\right)},$$



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hydro diffusion pole $\omega, k \lesssim T$

$$i\omega = \boldsymbol{k}^2\left(1 - \frac{2\pi\boldsymbol{T}}{3}\right) + \frac{\boldsymbol{k}^4}{6}\log\left[\frac{243}{16}\left(\frac{1}{2\pi\boldsymbol{T}}\right)^4\right] + \dots$$

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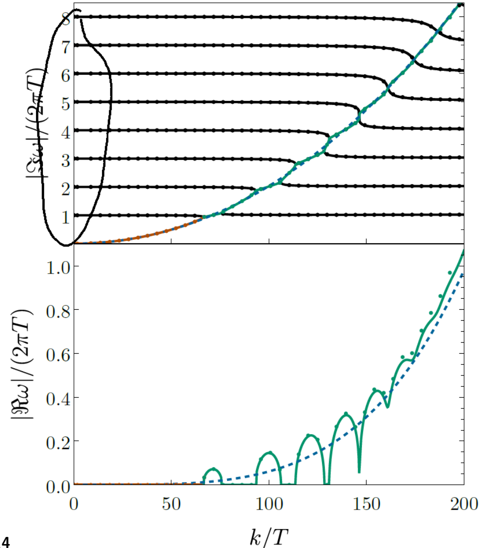


Gapped IR poles

$$\omega_n = -2i\pi T\left[\Delta(k=0) + n\right]$$

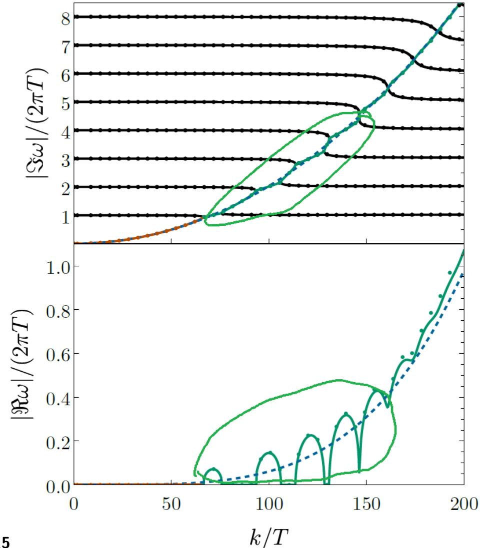
$$\Delta(\boldsymbol{k}) = \frac{1}{2} + \frac{1}{2}\sqrt{1 + \frac{4\boldsymbol{k}^2}{3}}$$

Originate from [Faulkner et al, 0907.2694]

$$G^R_{IR} \sim T^{2\Delta-1}\frac{\Gamma(\Delta - i\frac{\omega}{2\pi T})}{\Gamma(1 - \Delta - i\frac{\omega}{2\pi T})}$$

Pole-snatching

$$G^R_{JJ} = \frac{\omega^2\left(1 - \boldsymbol{k}^2\mathcal{G}/3\right)}{i\omega + \frac{\boldsymbol{k}^2}{6}\left(4\pi\boldsymbol{T} + 2(\boldsymbol{k}^2 + i\omega)\mathcal{G} + 3\boldsymbol{k}^2\log 3 - 6\right)},$$



Parity: poles with a real part come in pairs [Kaminski et al, 0911.3610].

Hydro pole snatches one of the poles from the tower of critical poles and leaves the imaginary axis.

SL(2,R) invariance:

$$n(k \ll T) \mapsto n(T \ll k) - 1$$

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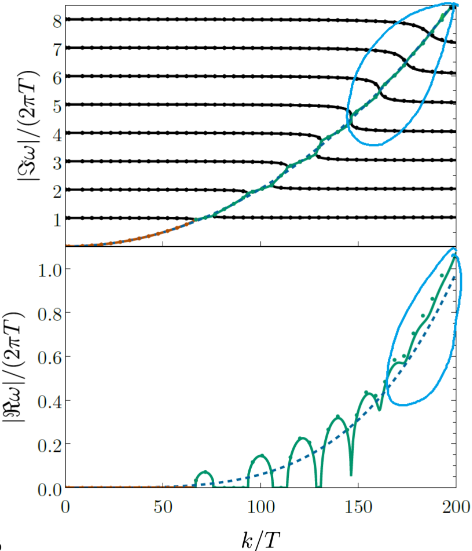

At $T = 0$

$$\mathcal{G} = \pm i\pi + \gamma + \log\left(\frac{2i\boldsymbol{\omega}}{9}\right)$$

Gapless pole survives

$$\boldsymbol{\omega} = -i\boldsymbol{k}^2 + i\frac{4}{3}\boldsymbol{k}^4\log(\boldsymbol{k})$$

$$+i\frac{2}{3}\boldsymbol{k}^4\left(\gamma + \log 2 - \frac{5}{4}\log 3 \mp i\pi\right)$$

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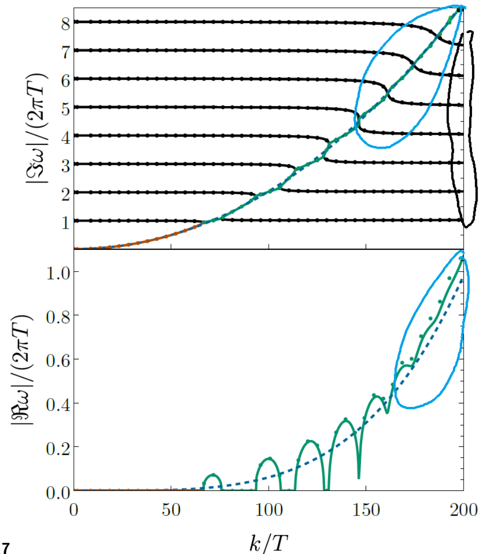

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$$+ i\frac{2}{3}\boldsymbol{k}^4\left(\gamma + \log 2 - \frac{5}{4}\log 3 \mp i\pi\right)$$

and log branch cut forms

Holographic model

- Background spacetime

$$S_{\text{background}} = \int \mathrm{d}^4x\sqrt{-g}\left[R - 2\Lambda - \frac{1}{2}\sum_{I=1}^{2}(\partial\psi_I)^2\right]$$

$$\mathrm{d}s^2 = \frac{\ell^2}{r^2}\left(-f(r)\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{f(r)} + \mathrm{d}\vec{x}^2\right),$$

$$f(r) = 1 - \frac{m^2r^2}{2} - \left(1 - \frac{m^2r_h^2}{2}\right)\frac{r^3}{r_h^3}, \quad \psi_I = m\delta_{Ii}x^i$$

- Emergent $AdS_2 \times R^2$ geometry

$$r = r_e - \epsilon\frac{r_e^2}{3\zeta}, \quad r_h = r_e - \epsilon\frac{r_e^2}{3\zeta_h}, \quad t = \epsilon^{-1}\tau\,, \quad r_e = \frac{\sqrt{6}}{m}$$

- Analogous results for other spacetimes, eg Reissner-Nordström.

Gauge field dynamics

- Probe gauge field

$$S = -\int \mathrm{d}^4 x \sqrt{-g}\, \frac{1}{4} F^2$$

- Gauge field perturbation

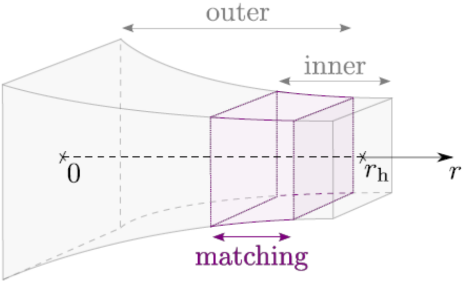
$$\delta A_\mu = \mathrm{e}^{-i(\omega t - kx)} a_\mu(r)\,, \qquad E_x = \omega a_x(r) + k a_t(r)$$

- Perturbation equation

$$\left[\frac{f(r) E_x'}{\omega^2 - k^2 f(r)}\right]' + \frac{1}{f(r)} E_x = 0$$

Matching calculation

$$\left[\frac{f(r)E_x'}{\omega^2 - k^2 f(r)}\right]' + \frac{1}{f(r)}E_x = 0$$



[CREDIT: C. SUPIOT]

- Outer region

$$r^2\omega^2/f^2, r^2k^2/f \ll 1$$

- Inner region

$$\zeta/r_e, \zeta_h/r_\mathrm{e} \gtrsim \epsilon$$

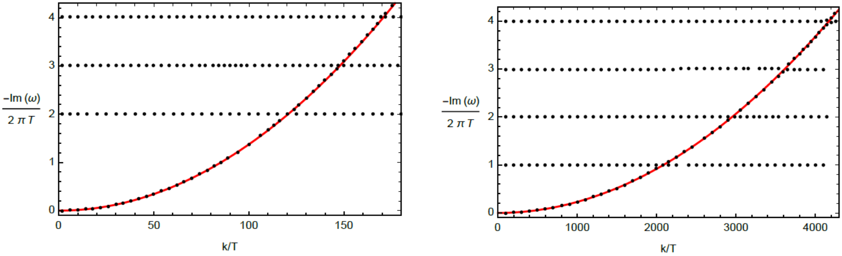
- Expand perturbation equation in both regions.

- Solve in each region: inner solution, outer solution.

- Expand ingoing wave condition order by order in ϵ:

$$\delta A_\mu \sim e^{-i\omega(t+r^\star)}, \qquad r^\star(r) = \int_0^r \frac{\mathrm{d}r_1}{f(r_1)}$$

Backreaction?



[Arean et al'20]

- Leading order result from matching [Davison & Parnachev'13]

$$\omega_\perp(k, T = 0) = -iD_\eta(T = 0)k^2 + O(k^4)$$

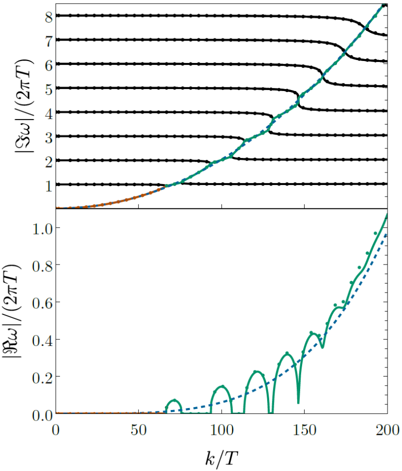
- Next-order correction, small temperature correction: WIP.

- Non-analytic contributions? [Moitra et al, 2005.00016; C. Yue's talk]

- (Yukawa-)SYK models?

Summary and outlook

$$G^R_{JJ} = \frac{\boldsymbol{\omega}^2\left(1-\boldsymbol{k}^2\mathcal{G}/3\right)}{i\boldsymbol{\omega}+\frac{\boldsymbol{k}^2}{6}\left(4\pi\boldsymbol{T}+2(\boldsymbol{k}^2+i\boldsymbol{\omega})\mathcal{G}+3\boldsymbol{k}^2\log 3-6\right)}, \quad (\boldsymbol{\omega},\boldsymbol{k},\boldsymbol{T}) = r_e(\omega,k,T)$$



$$\mathcal{G} = \pi\cot\left(\frac{i\boldsymbol{\omega}}{2\boldsymbol{T}}\right)+\gamma+\psi\left(\frac{i\boldsymbol{\omega}}{2\pi\boldsymbol{T}}\right) - \log\left(\frac{9}{4\pi\boldsymbol{T}}\right),$$

- Result valid at at $0 \ll r_e T \ll 1$: EFT for scales all the way to $1/r_e$?
- Construct holographic effective action [Nickel & Son, 1009.3094; Davison et al, 2210.14802]: Identification of emergent symmetry.
- Backreaction? Impact of Schwarzian action on $T=0$ gapless modes?