Holography, Kinetic Theory and Hidden Symmetry

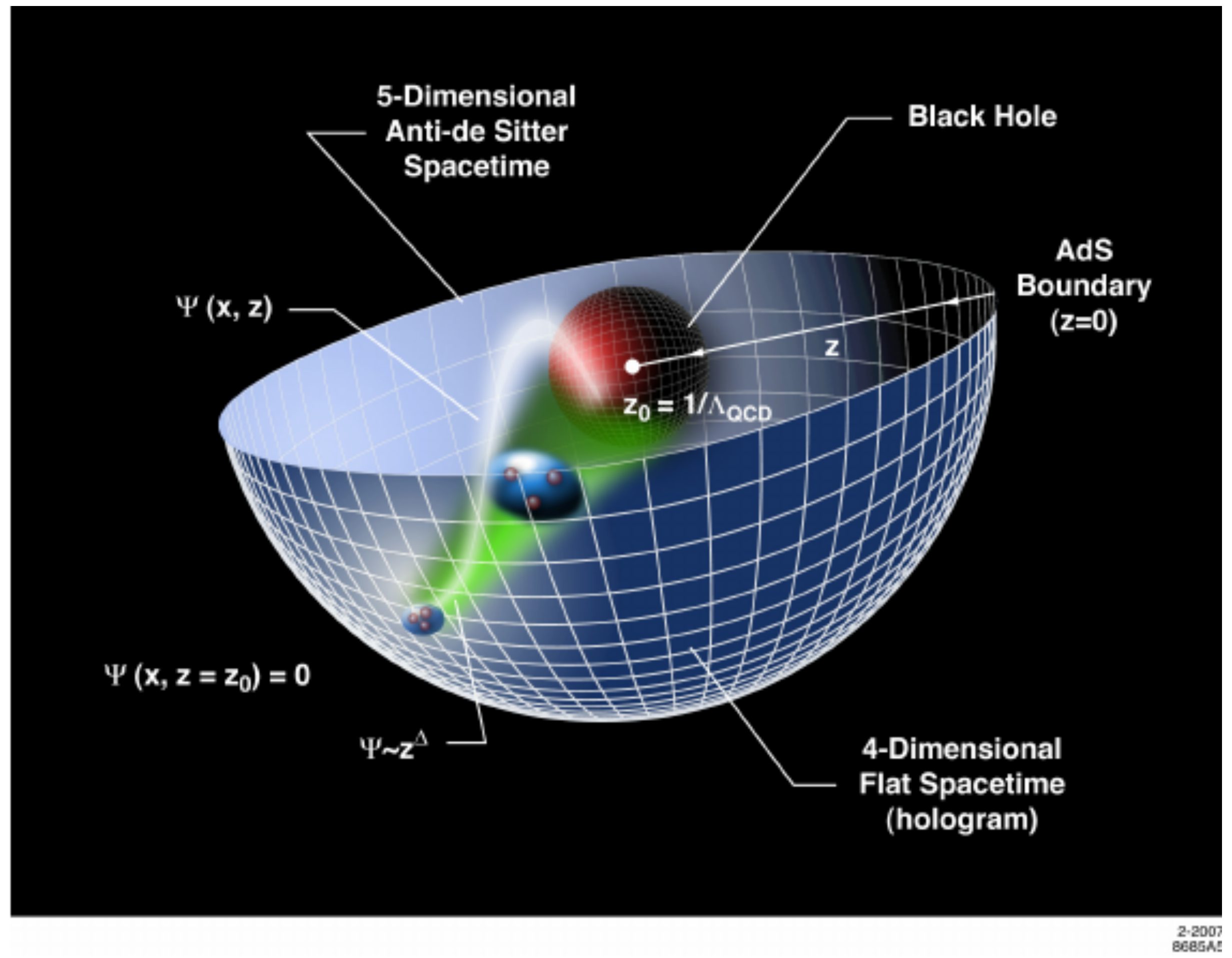
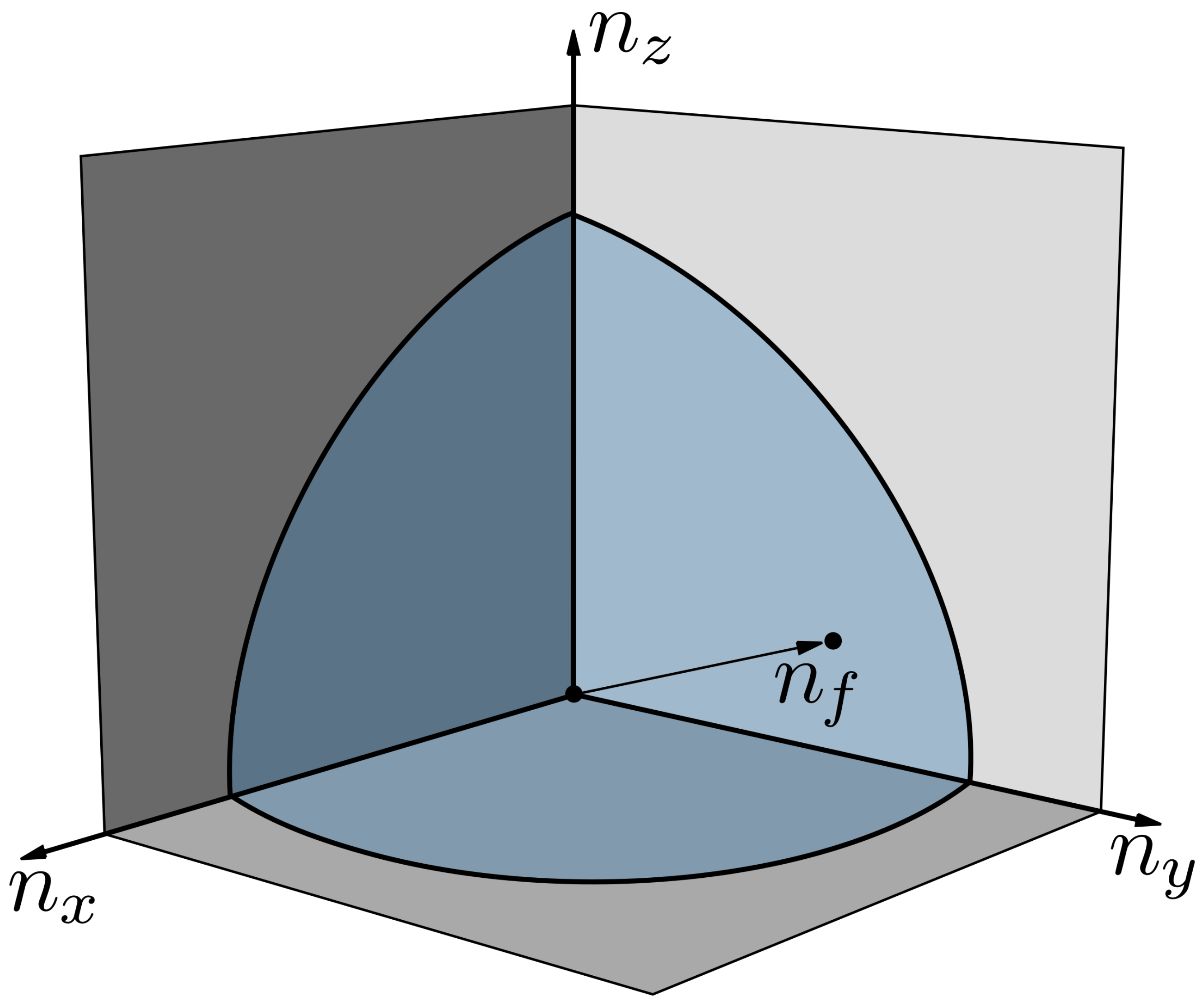


Fig. from 0802.0514



CUHK-SZ

Yi Yin

An-Brants-Heller-YY, in preparation

Holography workshop @ UCAS July. 15th 2025

Hydrodynamics

- Equations for energy/momentum and charge density for many-particle systems near equilibrium
- Successful in describing quark matter, cold atom, neutron star merge
- Modern view: effective field theory organized by symmetry principle

Motivation for Non-hydro.

- Beyond "vanilla" hydro. : chaos, spin, critical dynamics, neuron network
- Studying the evolution of matter (e.g. QCD matter) with varying scale
- Thermalization and hydrodynamization
- This talk: behavior outside hydro. Regime
- (Not covered: including additional slow modes in hydro. regime)

e.g. Stephanov-Yin, 2018

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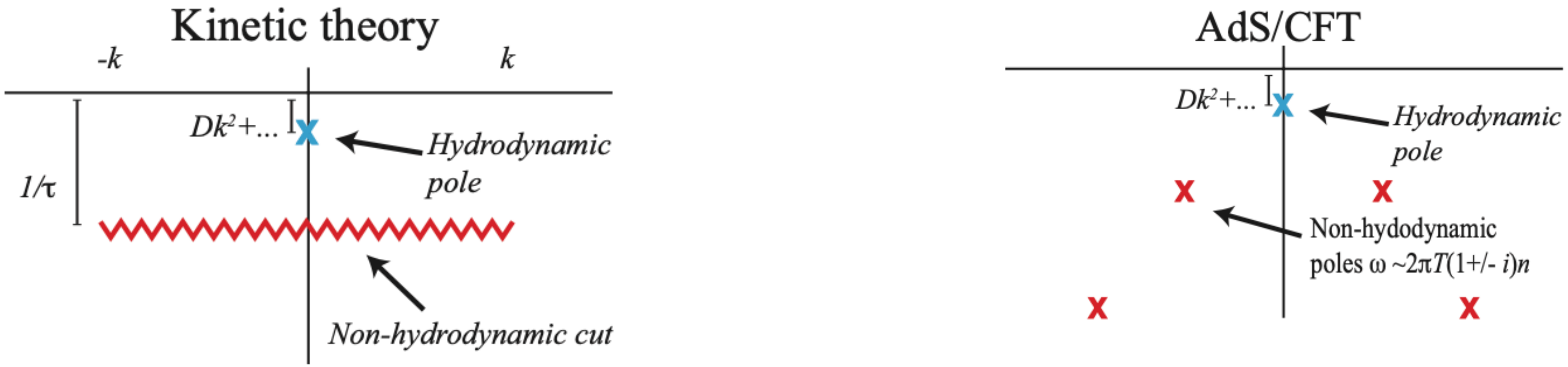
Non-Hydro. Excitations



The analytic structure of retarded Green function

Fig. from Kurkela-Wiedemann-Wu, EPJC 19'

- Non-hydro. behaviors are diverse and complicated; no simple guiding principle to describe them

"Happy families are all alike; every unhappy family is unhappy in its own way"

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Hidden Symmetry

An-Brants-Heller-YY, in preparation

- We propose hidden symmetries as a new guiding principle for describing a class of non-hydro behavior
  - Applicable for holography, kinetic theory and many
- Towards classification of non-hydro behaviors from hidden symmetry
  - Holo.: spontaneously broken
  - (Collisionless) kinetic theory: unbroken

Warm-up: Mueller-Israel-Stewart (MIS) Theory

Symmetry implies gapless excitations but non-hydro. modes are gapped (e.g. quasi-normal modes), how can they be described by hidden symmetry?

MIS Theory for Charge Density (Maxwell-Cattaneo)

- Conservation

$$\partial_t n + \vec{\partial} \cdot \vec{j} = 0$$

- Relaxation for non-hydro. variable $\vec{j}$:

$$\partial_t \vec{j} = -\frac{1}{\tau_J}(\vec{j} - D\vec{\partial}n)$$

Analogous to MIS eqn. for T^{ij}

- No obvious connection to symmetry
- but it does

Nickel-Son Theory

Nickel-Son, 2010'

- Motivation: describing the emergence of hydro. in holo. liquid

Charge $U(1)$ (V_μ) $\times$ Hidden $U(1)_H$ (A_μ) $\longrightarrow$ $U(1)_{\text{dia}}$ $(\delta A = \delta V)$

Spontaneous Breaking

- D.o.f.s: Goldstone ϕ+massive gauge field A_μ

$$\mathscr{L} = \mathscr{L}_\phi[\phi; A, V] + \mathscr{L}_A[A]$$

$$\mathscr{L}_\phi = f^2 \left(\partial_t\phi - V_t + A_t\right)^2 - g^2(\vec{\partial}\phi - \vec{V} + \vec{A})^2$$

$f \sim$ decay constant g ~ Higgs mass

- Dissipation: $\mathscr{L}_A[\vec{A}] \rightarrow$ dissipative crruent $\sim \lambda(\partial_t\vec{A} - \vec{\partial}A_t)$

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Duality between NS and MIS

- Goldstone+hidden gauge field $A_\mu \sim$ n+$\vec{j}$
- Damping of $A \sim$ Relaxation of $\vec{j}$
- The saddle point of NS action $\leftrightarrow$ MIS eqn. with

$$f^2 = \chi \,, \qquad \frac{\lambda}{g^2} = \tau_J \,, \qquad (\frac{g}{f})^2 = \frac{D}{\tau_J} \leq 1$$

Transparency in causality constraint

- Lesson learned: some non-hydro. modes can be described by hidden but breaking symmetry

Hidden Symmetry Model

An-Brants-Heller-YY, in preparation

Nickel-Son aims at understanding the emergence of hydro. We now reverse the logic, and generalize NS theory to describe non-hydro.

Construction

- Introducing finite (even infinite) no. of hidden gauge symmetries K that are spontaneously broken
- Field content: A^n(gauge) + ϕ^n(Goldstone), $n = 1, \ldots, K$
- Nearest neighboring: A^n couples to $A^{n\pm1}$ through ϕ^n, ϕ^{n-1}
- Organizing action by symmetry

$$V \overset{\phi^K}{\text{--------}} (A)^K \quad \ldots \quad (A)^n \overset{\phi^n}{\text{--------}} (A)^{n-1} \quad \text{------} \; (A)^1$$

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Action

$$\mathscr{L} = \sum \mathscr{L}^n_\phi[\phi^n; A^n, A^{n+1}] + \mathscr{L}^n_A[A^n], \qquad n = 1, \ldots, K$$

$$\mathscr{L}^n_\phi = (f^n)^2(\partial_t \phi^n - A^{n+1}_t + A^n_t)^2 - (g^n)^2(\vec{\partial}\phi^n - \vec{A}^{n+1} + \vec{A}^n)^2$$

$$\mathscr{L}^n_A[A^n] = \frac{1}{2}\left[\, \epsilon_n (F^n_{0i})^2 - (\mu^n)^{-2}(F^n_{ij})^2\right] + \mathcal{O}(\partial^4)\,,$$

Dissipation: $\lim_{\omega \to 0} \epsilon_n(\omega) = \frac{i\lambda_n}{\omega}$

$V \overset{\phi^K}{\text{--------}} (A)^K \;\; \ldots \;\; (A)^n \overset{\phi^n}{\text{--------}} (A)^{n-1} \;\; \text{------} \;\; (A)^1$

- Boundary condition $\partial_t A^1_\mu$: fixed by hydro. limit

Describing K "hidden massive photons" propagating in medium

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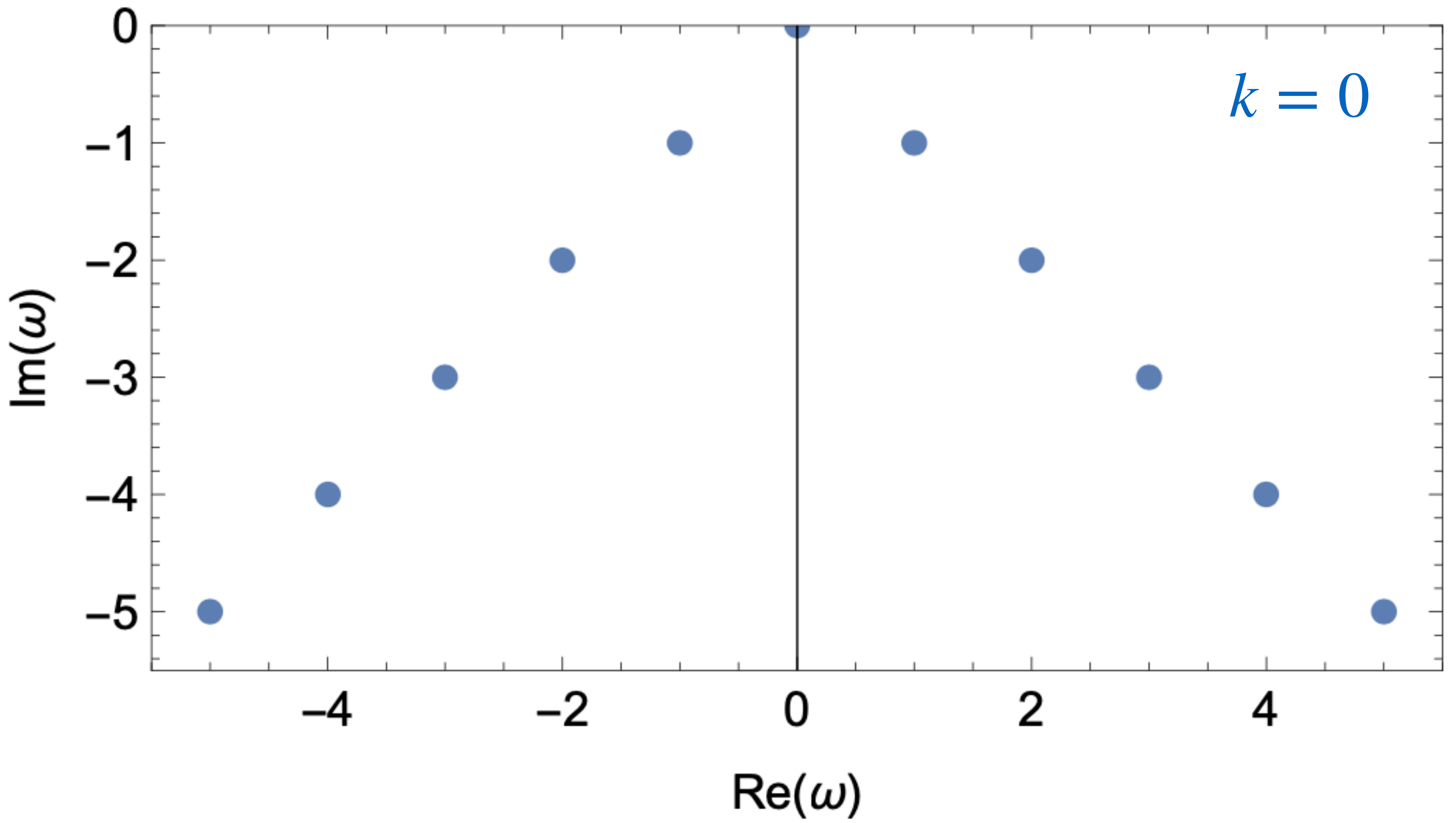

- Holographic-like (Christmas tree) modes by tuning f_n, g_n, ϵ_n

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Continuum limit $K \to \infty$

- 4+1d gauge theory in curved spacetime: "n" labels fifth coord. $\rho_n = na$

$$\mathscr{L} \sim \sqrt{-h}h^{MP}h^{NQ}F_{MN}F_{PQ}$$

- $\left(A_\mu(\rho_n), A_\rho(\rho_n)\right) = \left(A^n_\mu, \phi^n/a\right)$

- $(f(\rho), g(\rho), \epsilon(\rho), \kappa(\rho)) \to$ metric

e.g. $g_n^2(\vec{\partial}\phi_n - \overrightarrow{A}_{n+1} + \overrightarrow{A}_n)^2 \to g^2(\rho)F_{\rho i}F_{\rho i} \sim h^{\rho\rho}h^{ii}F_{\rho i}F_{\rho i}$

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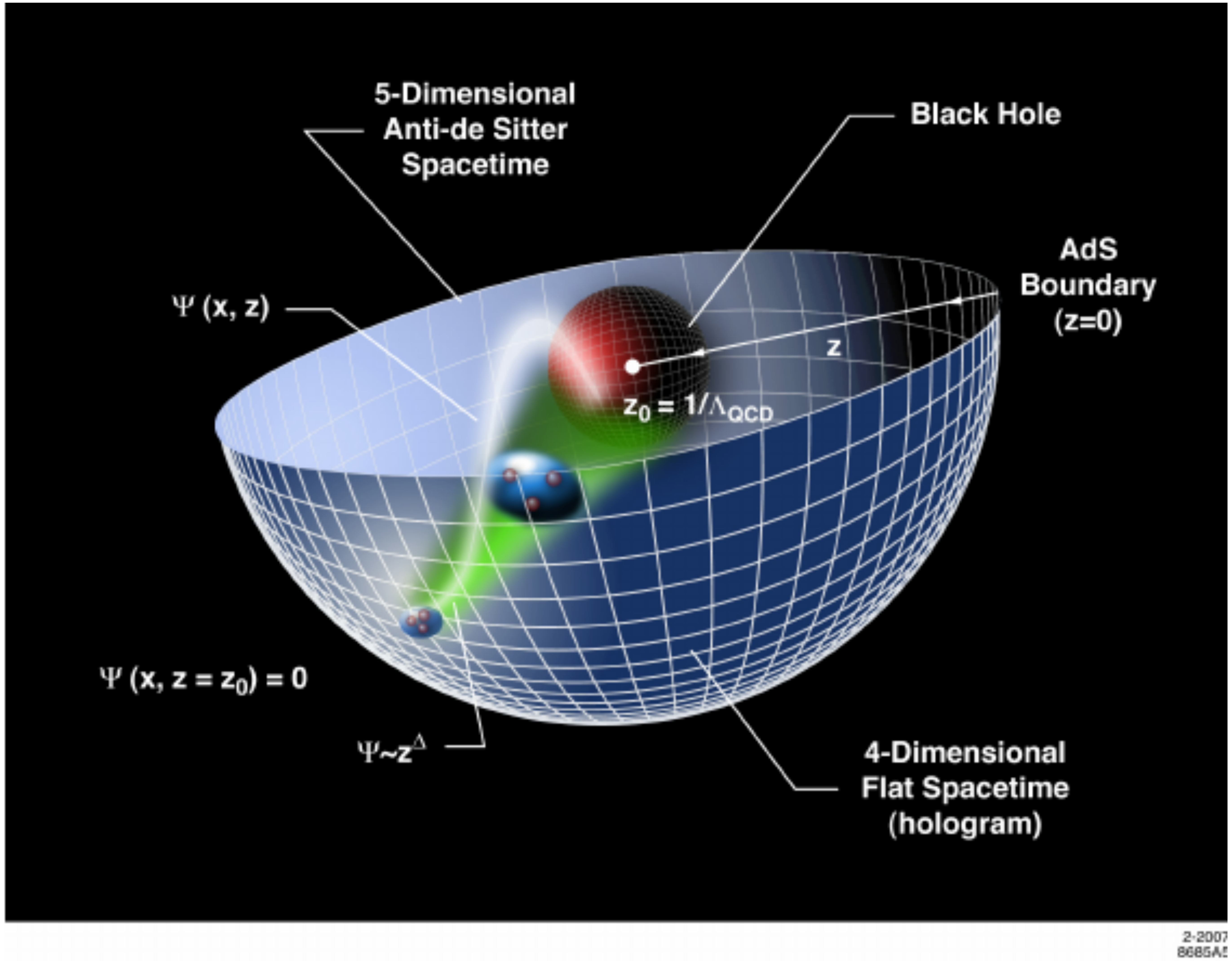
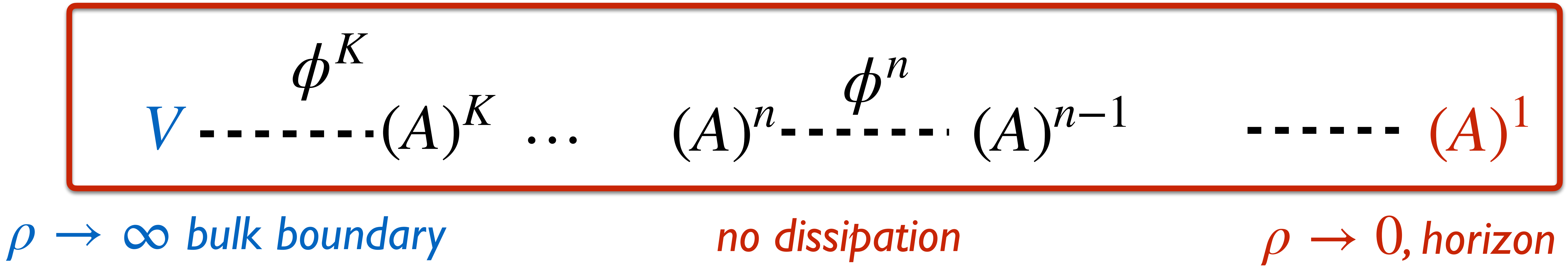

5-Dimensional
Anti-de Sitter
Spacetime
Black Hole
AdS
Boundary
(z=0)
Ψ (x, z)
z
$z_0 = 1/\Lambda_{QCD}$
Ψ (x, z = z_0) = 0
Ψ~z^Δ
4-Dimensional
Flat Spacetime
(hologram)
2-2007
8685A5

Relation to Holography Model



- Subclass of hidden symmetry model in continuum limit
  - Only $\mathscr{L}_A^n(A^n), n = 1$ contains dissipation
  - Quasi-normal modes as hidden gauge bosons

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Discussion

- Bottom-up holography ~ setting f, g phenomenologically
- Modelling strongly coupled systems with a few hidden gauge fields
- Renormalization group analysis: $K \to K/2 \to K/4$

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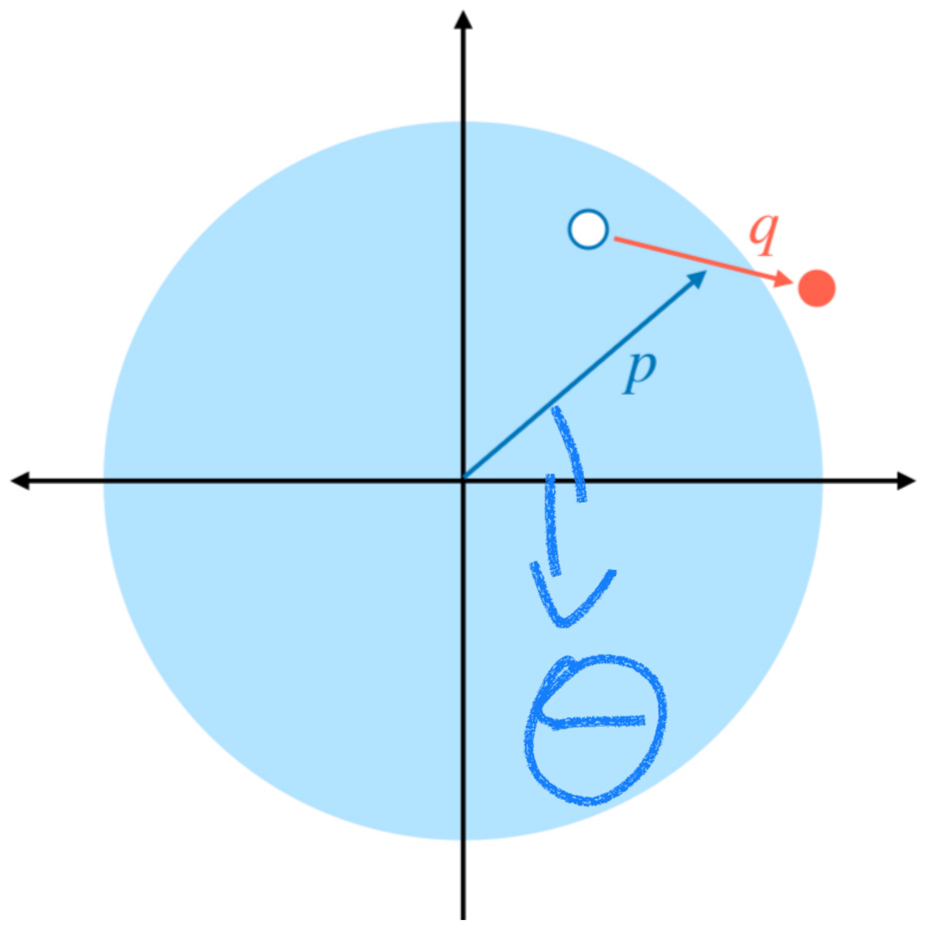
Kinetic Theory

Hidden Symmetry in Kinetic Theory

Else et al 2402.14066;

Also Delacretaz et al, ;



- Kinetic theory: describing the evolution of distribution function $n(x, \vec{p})$
- Liouville theorem
  - phase space volume is preserved without collision
  - symmetry: quasi-particles near Fermi surface (F.S.) with velocity $\vec{v}(\theta) = v_F(\cos\theta, \sin\theta)$ can be assigned with independent $U_\theta(1)$ phase
- Formulating collisionless kinetic theory with infinite $U_\theta(1)$

Nonlinear Bosonization of Fermi Surfaces: The Method of Coadjoint Orbits

Luca V. Delacrétaz,[1,2] Yi-Hsien Du,[1] Umang Mehta,[1,3] and Dam Thanh Son[1,2,4]

[1] *Kadanoff Center for Theoretical Physics,*
University of Chicago, Chicago, Illinois 60637, USA

Collisionless dynamics of general non-Fermi liquids from hydrodynamics of emergent conserved quantities

Dominic V. Else
Perimeter Institute for Theoretical Physics, Waterloo, ON N2L 2Y5, Canada

Action for Kinetic Theory

- Unbroken $U_\theta(1)$: scalar $\varphi(x;\theta)$, no hidden gauge fields

$$\varphi(t,\vec{x};\theta) \to \varphi(t,\vec{x};\theta) + a(\vec{x};\theta)$$

shift symmetry

$$\mathscr{L} = \int_\theta f^2(\partial_t\varphi_\theta(x))^2 + \tilde{f}^2\,\vec{v}\cdot\vec{\partial}\varphi_\theta(x)(\partial_t\varphi_\theta(x))$$

$$(\vec{\partial}\varphi_\theta(x))^2$$

Forbidden

$$\partial_t\delta n_\theta(x) + \vec{v}\cdot\vec{\partial}\delta n_\theta(x) = 0$$

E.o.M.: Boltzmann equation with $\delta n_\theta(x) = f^2\partial_t\phi_\theta(x)$ and $(f = \tilde{f})$

- Can be extended to Landau Fermi liquid, non-Fermi liquid

Holography v.s. Kinetic Theory

	Holo	Kinetic
Symmetry	Broken	Unbroken
Field	$A + \phi$	φ
Label	Extra d	Phase space

- Hidden symmetry: describing and classifying different non-hydro.

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Generalization

[Submitted on 25 Sep 1996]

Phonons as Goldstone Bosons

H. Leutwyler (University of Bern and CERN)

- Energy-momentum sector: hidden diffeomorphism
  - understanding "mysterious" high-frequency sound
  - constraining non-linear coupling among non-hydro. modes

Weiyao Ke and YY, PRL 23, JHEP 24;

- Spin kinetic theory (SU(2))

Zonglin Mo, YY in preparation

- Fluctuations/dissipation (see paper)

An-Brants-Heller-YY, in preparation

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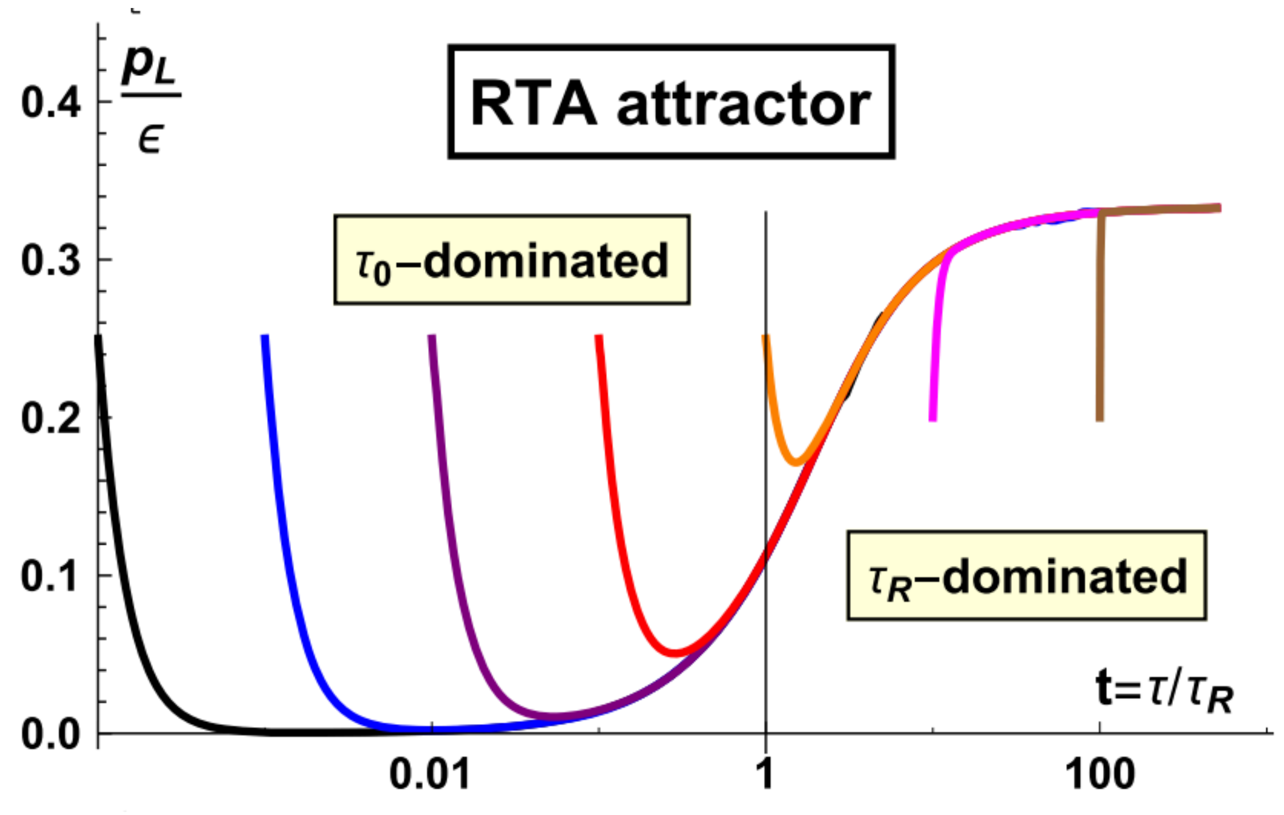
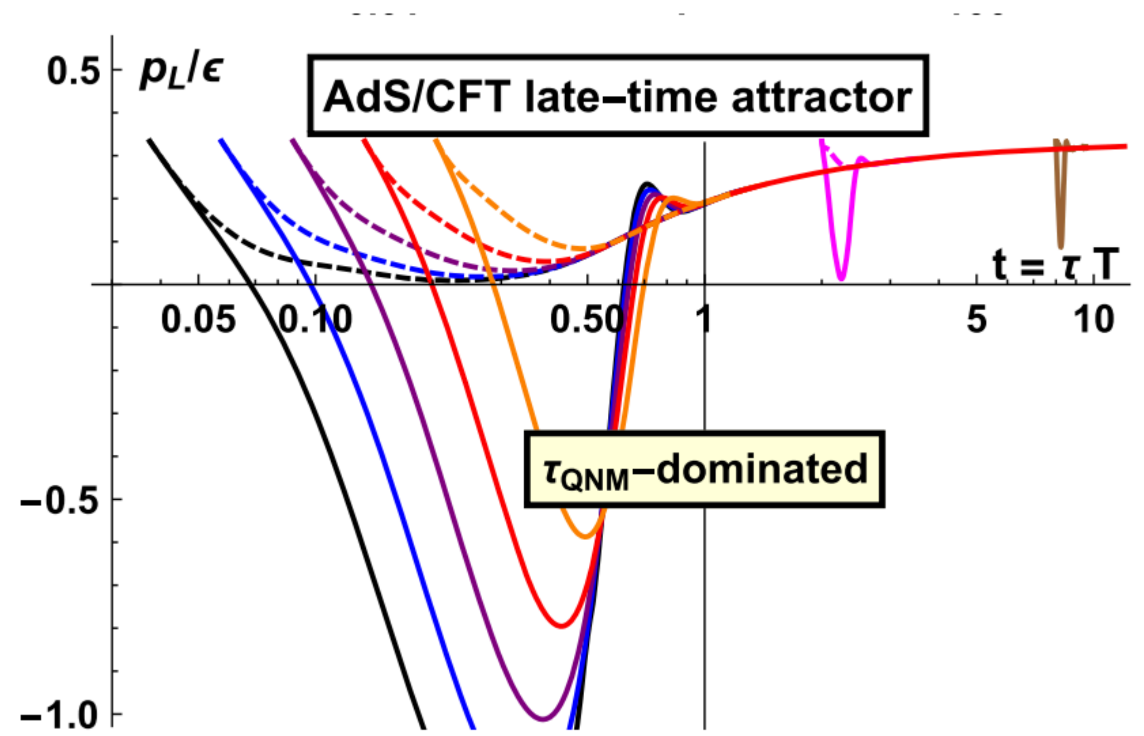
Symmetry and Thermalization

- Hidden symmetry modes dominate early-time (high frequency) attractor behavior
  - Holography: gauge bosons
  - Kinetic theory: scalar fields
- Attractor is related to symmetry of theory





Kurkela-Schee-Wiedemann-Wu, PRL 19'

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Outlook and Summary

Summary

- Introducing hidden symmetry to describe diverse non-hydro. behavior in one and the same conceptual framework
- Connecting several recent developments (but seemingly unrelated)
  - Holography, early-time attractor, EFT for kinetic theory
- Towards understanding of non-hydro. behaviors at deeper level
- Model for generative AI?

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Back-up

Hidden Symmetry and Canonical Transformation

- Distribution at different time are related by canonical transformation in collisionless regime

$$f(t_0, \vec{x}, \vec{p}) = f(t, \vec{x} - \nabla^p \varphi(t, \vec{x}, \vec{p}), \vec{p} + \nabla \varphi(t, \vec{x}, \vec{p}))$$

- Kinetic equation can be equivalently formulated with infinite number of scalar fields $\varphi(t, x, \hat{p})$

- Linearized C.T. corresponds to infinite $U(1)$ (independent phase choice in phase space)

$\varphi(t, \vec{x}, \hat{p}) \leftrightarrow$ Hidden Goldstones $\phi^n(t, \vec{x})$ $(\hat{p} \sim n)$

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Lesson Learned

- Non-hydro can be described by hidden but breaking symmetry
  - Massive hidden gauge field ~ non-hydro. (gapped)
  - Unbroken U(1)~ hydro. mode
- Analogy: massive meson (ρ) as the vector boson of breaking hidden local $SU(2)$

Bando et al, 1980s; Son-Stephanov, 2004 (precessor of ADS/QCD)

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Duality between NS and MIS

Goldstone $n = f^2 \partial_t \phi$ $A_t = 0$ *gauge*

Hidden Gauge field $\vec{j} = -g^2(\nabla\phi - \vec{A})$

Hidden Ohm current Relaxation

- The saddle point of NS action is equivalent to MIS eqn. with

$$f^2 = \chi\,, \qquad \frac{\lambda}{g^2} = \tau_J\,, \qquad (\frac{g}{f})^2 = \frac{D}{\tau_J} \leq 1$$

Transparency in causality constraint

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Sum Rule for Transport Coefficients

$$\sigma = \sum_n \lambda_n$$ c.f. parallel circuit

$$\chi^{-1} = \sum_n (f_n)^{-2}$$

$$\kappa = \sum_n \mu_n^{-1}$$ $G_R = \ldots + \kappa k^2$

$$\frac{\tau_J}{\sigma} = \sum_n \left(\frac{\lambda_n}{\sigma}\frac{1}{g_n^2} - \epsilon_n\right)$$

- Second-order transport coeff. are sensitive to non-hydro excitations
  - Without "hidden photon", $\kappa = 0$
  - $g_n/f_n, (\mu_n \epsilon_n)^{-1} \leq 1$ (causality) constraints $\tau_J/D, \kappa$
- NB: AdS/CFT: $\tau_J/D = \pi/2$; kinetic theory: $\tau_J/D \geq 3, \kappa = 0$

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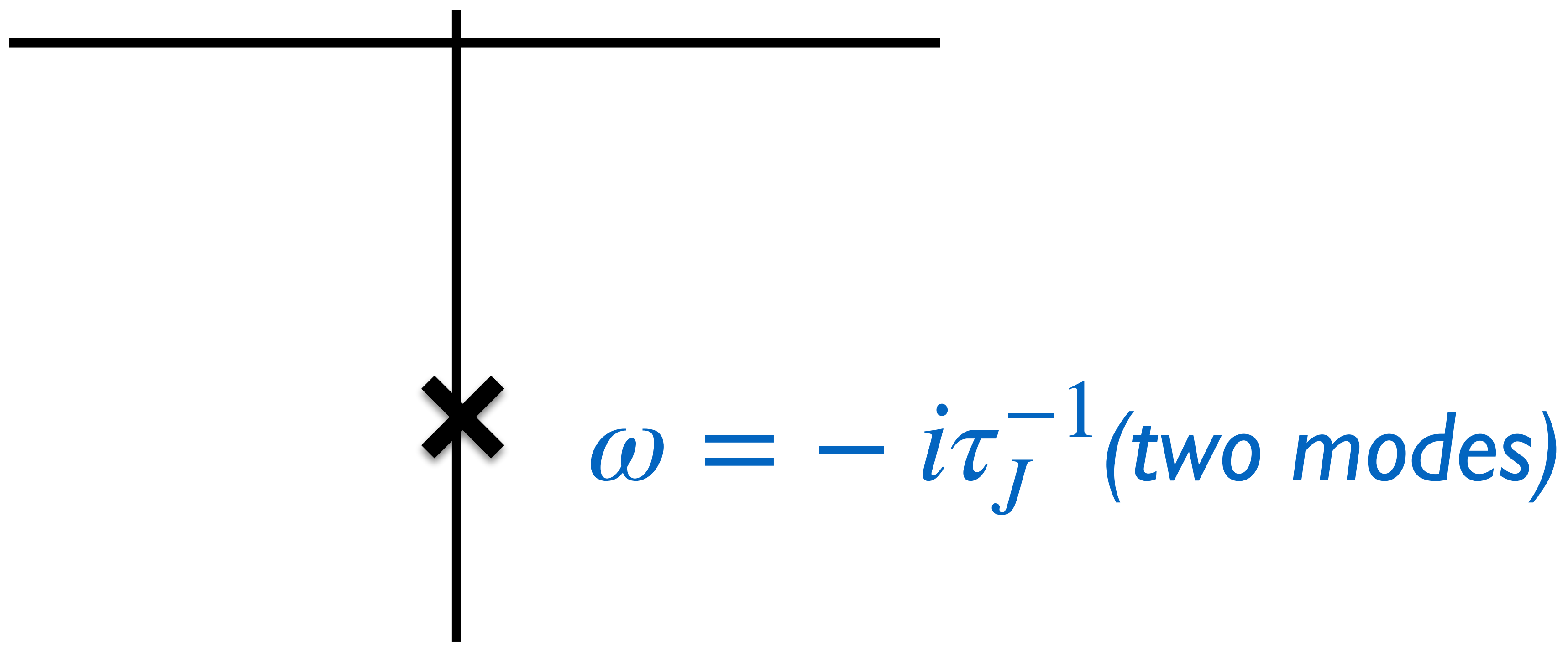
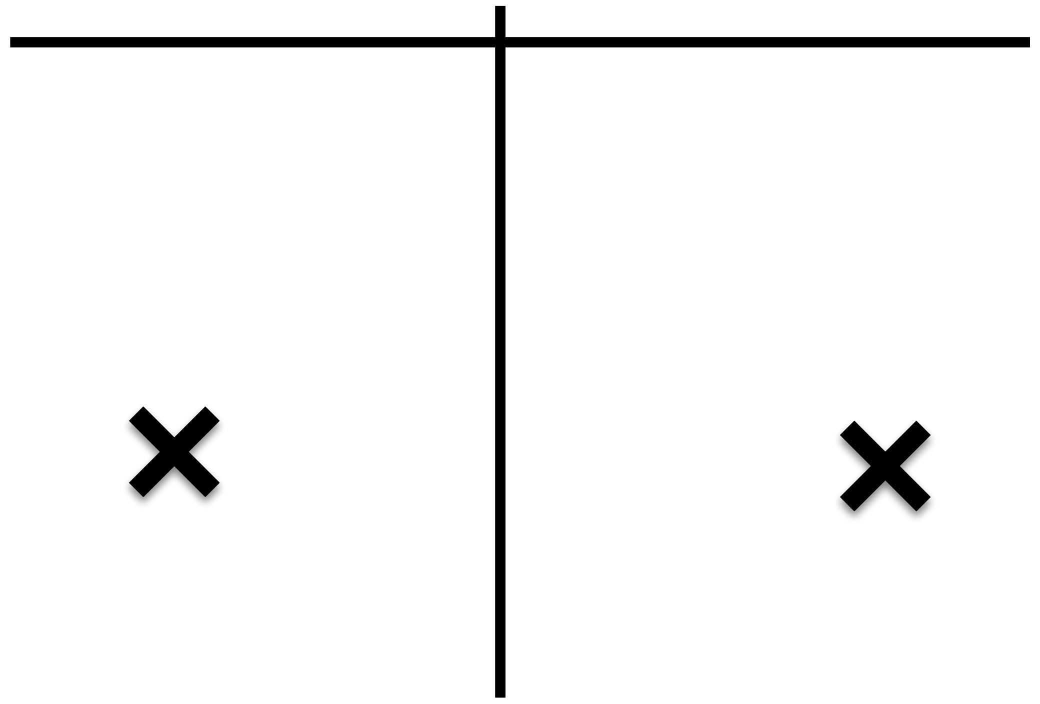
Quasi-normal Modes as Massive Photon in Plasma

- One combination of Goldstones corresponds to hydro. mode
- Other Hidden Goldstones mixed with hidden plasmon
- Transverse damping modes $(\epsilon_n, \mu_n^{-1} = 0)$ become propagating $(\epsilon_n, \mu_n^{-1} \neq 0)$



Nickel-Son at k=0



with kinetic term $(\epsilon_1 \neq 0)$

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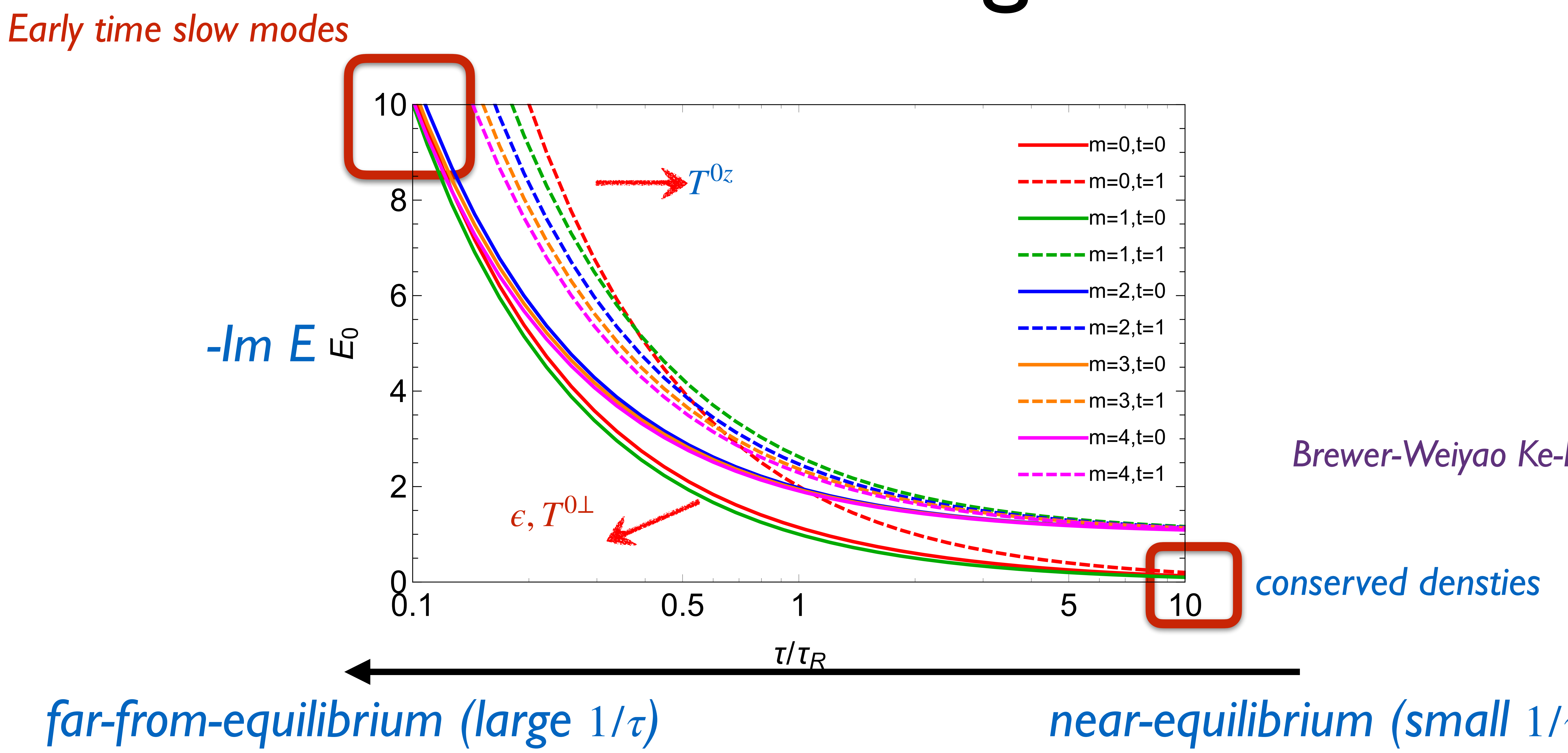
Hydrodynamization as Symmetry breaking



Brewer-Weiyao Ke-Li Yan-YY, PRD Letter 2024

- Far-from-equilibrium: hidden symmetry approximately restored
- Hydro. modes stand out when hidden symmetry is breaking

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Hidden Gauge Symmetry and Massive Vector Mesons

- Massive meson like ρ, a_1 can be treated as the vector boson of spontaneously breaking hidden local $SU(2)$

Bando et al, 1980s

- Generalization to finite (infinite) number of HLS (predecessor of AdS/QCD)

Son-Stephanov, 2004

Reproducing a host of hadronic phenomena with acceptable precision

Pion ~ hydro. modes

Massive meson ~ non-hydro. modes

Message

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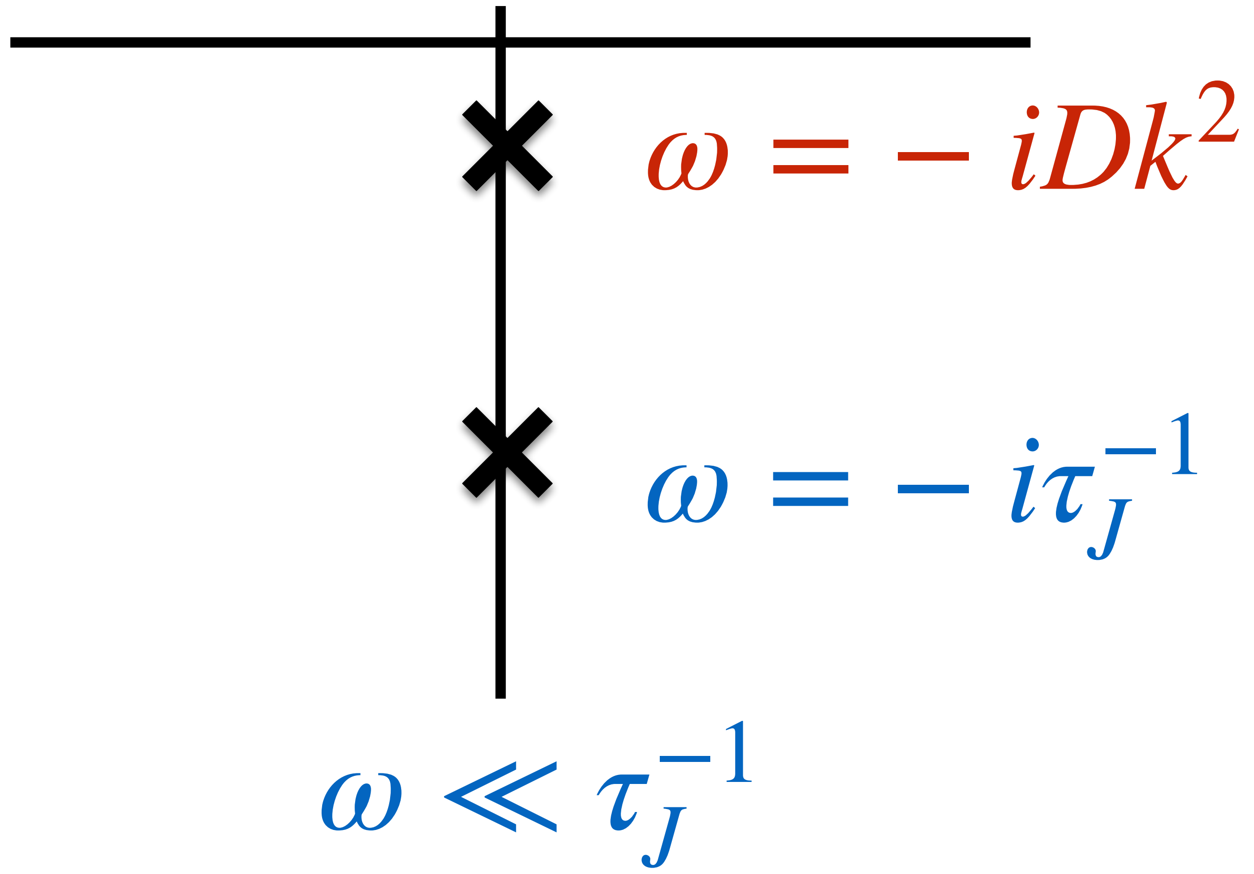
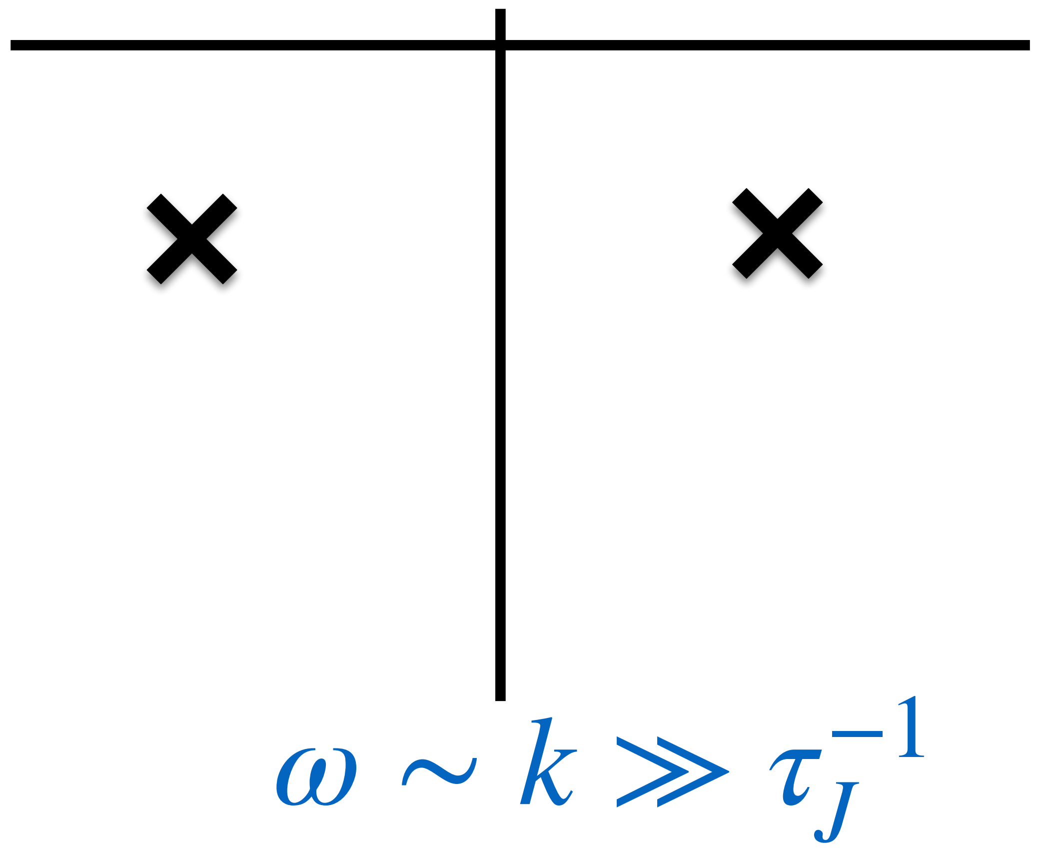
Dispersion

$$\tau_J \omega^2 - \omega - iDk^2 = 0$$

Longitudinal Channel





- High-frequency regime: propagating waves
  - Describing phonon-like excitation in some materials *Baggioli*
- Transverse (independent of k): damping modes only

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