Mass and isospin breaking effects in the Witten-Sakai-Sugimoto model

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Outline

Sakai-Sugimoto – AdS/QCD

The Sakai-Sugimoto (SS) model [Prog.Theor.Phys.113, 843 (2005)] builds on the work of Witten [Adv.Theor.Math.Phys.2, 253 (1998)] where N_f D8- and $\overline{\text{D8}}$-branes are intersecting N_c D4-branes

	0	1	2	3	(4)	5	6	7	8	9
D4	○	○	○	○	○					
D8-$\overline{\text{D8}}$	○	○	○	○		○	○	○	○	○

- The SS model: type IIA string theory
- SUSY-breaking by anti-periodic boundary conditions on the S^1 for the fermions on D4
- Chiral symmetry is explicit by the two 8-branes, when they stretch
- Chiral symmetry breaking is string geometric as the 8-branes touch and merge – the low-energy supergravity geometry is that of a cigar-shaped space, which is AdS_5-like

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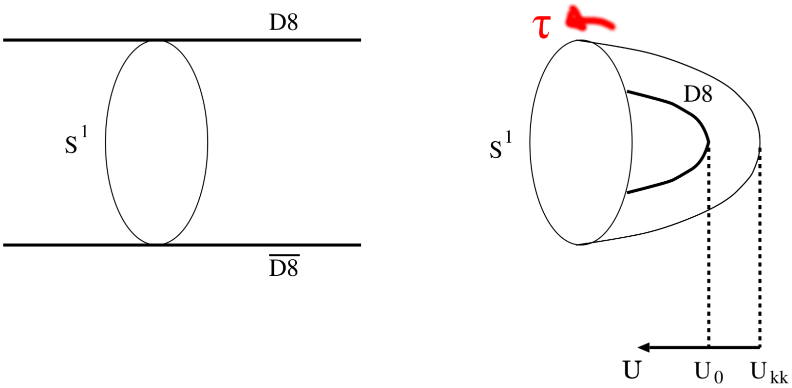
Sakai-Sugimoto – chiral symmetry breaking



[Sakai-Sugimoto, Prog.Theor.Phys.113, 843 (2005)]

- Notice that the confined geometry ends at $U_{\rm KK}$

Sakai-Sugimoto – AdS/QCD

- The 't Hooft limit is considered $N_c \gg N_f$, so that the 8-branes can be considered in the probe branes embedded in the D4-background (color d.o.f.)

$$ds^2 = \left(\frac{u}{R}\right)^{3/2} (\eta_{\mu\nu} dx^\mu dx^\nu + f(u) d\tau^2) + \left(\frac{R}{u}\right)^{3/2} \left(\frac{du^2}{f(u)} + u^2 d\Omega_4^2\right)$$

$$e^\phi = g_s \left(\frac{u}{R}\right)^{3/4}, \qquad F_4 = dC_3 = \frac{2\pi N_c}{V_4}\epsilon_4, \qquad f(u) = 1 - \frac{u_{\rm KK}^3}{u^3}.$$

- The flavor d.o.f. are described by the DBI action and the Chern-Simons term at level N_c – both scale as N_c
- The leading order approximation to the DBI action is the 5-dimensional Yang-Mills term, hence the Sakai-Sugimoto model:

$$S = -\kappa \,\mathrm{Tr} \int_{\mathrm{AdS}_5} \mathcal{F} \wedge *\mathcal{F} + N_c \int_{\mathrm{AdS}_5} \omega_5,$$

with YM coefficient $\kappa = \frac{\lambda N_c}{216\pi^3}$, and 't Hooft coupling $\lambda = g_{\rm YM}^2 N_c$ (fixed),

$$g = h(z)k(z) dx^\mu dx_\mu + h^2(z) dz^2, \qquad k(z) = h^{-3}(z) = 1 + z^2.$$

- The U(2) gauge fields is split as $\mathcal{A}_\alpha = A_\alpha^a T^a + \widehat{A}_\alpha \frac{\mathbb{1}}{2}$

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- The flavor fields are expanded as

$$\mathcal{A}_\mu = \sum_n v_\mu^{2n-1}(x)\psi_{2n-1}(z) + \sum_n a_\mu^{2n}(x)\psi_{2n}(z),$$
$$\mathcal{A}_z = \Pi(x)\phi_0(z) + \sum_n H^n(x)\phi_n(z),$$

with profile functions

$$\text{pions} : \phi_0(z) = \frac{1}{\sqrt{\pi\kappa}}\frac{1}{k(z)}, \qquad \text{vectors} : -h^{-1}(z)\partial_z(k(z)\partial_z\psi_n) = \lambda_n\psi_n,$$
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with vector meson masses $M_n \sim \sqrt{\lambda_n}$

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Sakai-Sugimoto – The Skyrme model

- If we truncate to the pions, one gets the Skyrme model

$$S = \widetilde{T}(2\pi\alpha')^2 \int \mathrm{d}^4x \ \mathrm{Tr}\left[AL_\mu^2 + B[L_\mu, L_\nu]^2\right],$$

with the left-invariant chiral current $L_\mu = U^{-1}\partial_\mu U$, and the constants determined by string theory

$$A = \frac{9u_{\mathrm{KK}}}{4\pi}, \qquad B = \frac{R^3 b}{2\pi^4}, \qquad b \sim 15.25$$

- The Skyrme coupling is determined by the model

$$e^2 = \frac{27\pi^7}{2b}\frac{1}{\lambda N_c} \sim (7.32\cdots)^2$$

which can be compared to [Adkins-Nappi-Witten, NPB228, 552 (1983)], where they find $e = 5.45$ by fitting to the masses of the nucleon and Delta resonance

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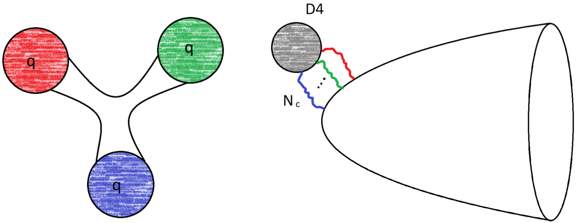
The baryon is:

- the coupling of N_c strings from the D4-branes to the 8-branes



- the instanton in an (x^1, x^2, x^2, z) slice of the AdS$_5$-like geometry
- the Skyrmion in the pion effective theory, which is a soliton of 1 dimension less than the instanton, but same S^3 target space

$$\pi_3(S^3) = \mathbb{Z} \ni k = B$$

- The 3rd homotopy group is due to the mappings being from $\sim \partial\mathbb{R}^4 \simeq S^3$ in the instanton case, and from $\mathbb{R}^3 \cup \{\infty\} \simeq S^3$ in the Skyrmion case
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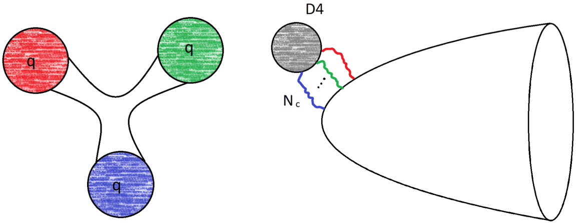

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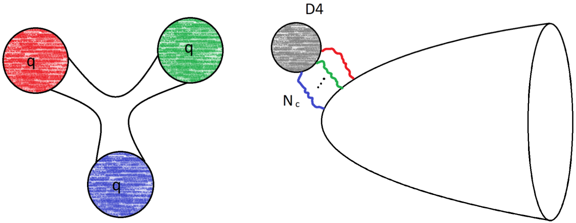

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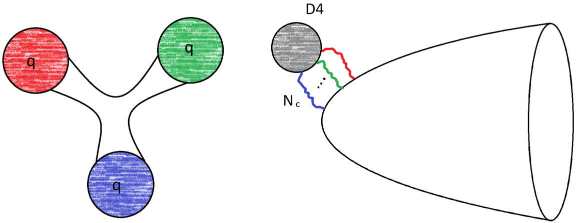

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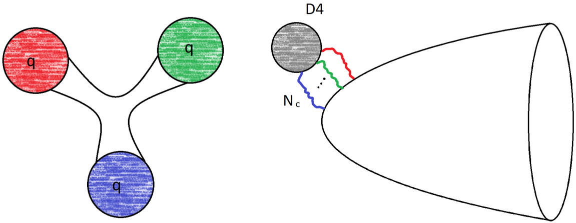

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The instanton in SS

- In the large-λ limit, the curved-space instanton is well approximated by the flat-space BPST instanton solution in the non-Abelian fields [Hata-Sakai-Sugimoto-Yamato, Prog.Theor.Phys.117, 1157 (2007)]

$$A_M = -\mathrm{i} f(\xi) g \partial_M g^{-1}, \qquad f(\xi) = \frac{\xi^2}{\xi^2 + \rho^2},$$

$$\xi^2 = (x - X)^2 + (z - Z)^2, \qquad g(x) = \frac{(z - Z) - \mathrm{i}(\mathbf{x} - \mathbf{X}) \cdot \boldsymbol{\tau}}{\xi}.$$

- The Abelian electric field is new, this field acts as a size stabilization against gravitational collapse

$$\widehat{A}_0 = \frac{N_c}{8\pi^2\kappa} \frac{1}{\xi^2} \left(1 - \frac{\rho^4}{(\xi^2 + \rho^2)^2}\right).$$

- Minimization of the pseudo-moduli Z, ρ gives

$$Z = 0, \qquad \rho^2 = \frac{N_c}{8\pi^2\kappa}\sqrt{\frac{6}{5}}.$$

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$$m_q(\psi_L^\dagger \psi_R + \psi_R^\dagger \psi_L),$$

which must be nonlocal in the bulk!

- [Aharony-Kutasov, PRD78, 026005 (2008)] solved this problem with a Wilson line

$$S_{\rm AK} = c \int \mathrm{d}^4x \ \mathrm{Tr}\, P \left[M\left(e^{\mathrm{i}\varphi} + e^{-\mathrm{i}\varphi} - 2\mathbb{1}\right)\right], \qquad \varphi = -\int \mathrm{d}z\, A_z,$$

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Outline

1. Holographic QCD
   - Quark and pion masses
   - Isospin symmetry breaking
   - The Skyrme model
2. Conclusions

Chiral symmetry breaking induced by the Yukawas

- We will introduce the chiral symmetry breaking along the lines of the Standard Model: i.e. by the mass term (Yukawa interactions):

$$S_{\rm AK} = c \int \mathrm{d}^4 x \ \mathrm{Tr}\left[M\left(e^{\mathrm{i}\varphi} + e^{-\mathrm{i}\varphi} - 2\mathbb{1}\right)\right], \qquad e^{\mathrm{i}\varphi} = \mathcal{P}\exp\left(\frac{\mathrm{i}}{2}\int \mathrm{d}z \, A_z\right),$$

$$M = m_q \mathbb{1}_2 + \epsilon m_q \tau^3, \qquad m_q = \frac{1}{2}(m_u + m_d), \qquad \epsilon = \frac{m_u - m_d}{m_u + m_d}.$$

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Quantum baryons

- Turning on isospin moduli:

$$\widehat{A}_z = -\frac{N_c}{16\pi^2\kappa}\frac{\rho^2}{(\xi^2+\rho^2)^2}\boldsymbol{\chi}\cdot\mathbf{x}, \qquad \widehat{A}_i = -\frac{N_c}{16\pi^2\kappa}\frac{\rho^2}{(\xi^2+\rho^2)^2}\epsilon^{iab}\chi^a x^b.$$

- Protons: $|p\uparrow\rangle \propto (a_1 + \mathrm{i}a_2)$ and $|p\downarrow\rangle \propto (a_4 - \mathrm{i}a_3)$
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Isospin breaking: modified tails of the soliton

- Linearized EOMs:

$$\kappa k(z)\left(\partial_i^2\widehat{A}_z - \partial_i\partial_z\widehat{A}_i\right) \;=\; m_q c\left[\int_{-\infty}^{+\infty}\mathrm{d}z\,\widehat{A}_z + \epsilon\int_{-\infty}^{+\infty}\mathrm{d}z\,A_z^{a=3}\right],$$

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$$A_\eta = \frac{1}{\sqrt{2}}(A_z^{a=3} + \widehat{A}_z), \qquad A_\pi = \frac{1}{\sqrt{2}}(A_z^{a=3} - \widehat{A}_z).$$

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$$A_\eta = b\chi^j\partial_{X^j}H_+ - \tilde{b}\,\mathrm{Tr}\left(\mathsf{A}\tau^j\mathsf{A}^\dagger\tau^3\right)\partial_{X^j}H_+,\;\; A_\pi = d\chi^j\partial_{X^j}H_- - \tilde{d}\,\mathrm{Tr}\left(\mathsf{A}\tau^j\mathsf{A}^\dagger\tau^3\right)\partial_{X^j}H_-.$$

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$$H_\pm \equiv -\kappa\sum_{n=0}\phi_n(z)\phi_n(Z)\frac{1}{4\pi}\frac{e^{-\sqrt{\lambda_n}r}}{r}, \qquad \lambda_0 = m_\pm^2, \qquad m_\pm^2 = m^2(1\pm\epsilon).$$

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Isospin breaking: modified inertia

- Inserting the solutions into the Yang-Mills:

$$T = \frac{\pi^2\rho^4\kappa}{2} m \left[\left(\frac{8}{3mR} - \left(2 + \sqrt{1+\epsilon} + \sqrt{1-\epsilon}\right)\right)\chi^2 - \left(2 - \sqrt{1+\epsilon} - \sqrt{1-\epsilon}\right)\chi_\zeta^2 \right].$$

- Inertia tensor:

$$\mathcal{I} = I_0 \mathbb{1}_3 + \delta \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}, \qquad \delta = \frac{\pi^2\rho^4\kappa}{12} m\epsilon^2 + \mathcal{O}(m^2, \epsilon^4).$$

- I_0 requires numerical computation!
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Isospin breaking: new quantization

- The AK term contains $\mathcal{O}(\epsilon)$ modification of the angular momentum ($\mathcal{J}_2 \approx 1.054$):

$$L_{\rm AK} = c\,\mathrm{Tr}\int \mathrm{d}^3x \left[M e^{\mathrm{i}\varphi} \mathsf{A} U \mathsf{A}^\dagger + e^{-\mathrm{i}\varphi} \mathsf{A} U^\dagger \mathsf{A}^\dagger M - 2\mathbb{1}_2 \right]$$
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$$J_i = \mathcal{I}_{ij}\chi_j \qquad \longrightarrow \qquad J_i = \mathcal{I}_{ij}\chi_j - K_i,$$

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- Hamiltonian

$$H_{\rm full} = \frac{1}{2I_A}\left(J_\xi^2 + J_\eta^2\right) + \frac{1}{2I_C}\left(J_\zeta^2 + K_\zeta^2 + 2J_\zeta K_\zeta\right) + M_0.$$

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$$E_{\rm full} = \underbrace{\frac{1}{2I_A} j(j+1)}_{\text{ANW}} + \underbrace{\frac{1}{2}\left(\frac{1}{I_C} - \frac{1}{I_A}\right) i_3^2}_{\text{BBGR}} - \underbrace{\frac{1}{I_0} K_\zeta i_3}_{\text{Bigazzi-Niro}} + \frac{1}{2I_0} K_\zeta^2 + M_0,$$

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$$L_{\rm AK} = c\,{\rm Tr}\int {\rm d}^3x\left[Me^{{\rm i}\varphi}\mathsf{A}U\mathsf{A}^\dagger + e^{-{\rm i}\varphi}\mathsf{A}U^\dagger\mathsf{A}^\dagger M - 2\mathbb{1}_2\right] = \frac{m_q c\epsilon N_c}{12\kappa}\rho^3\mathcal{J}_2\chi^i R_{3i},$$

- The usual angular momentum gets modified:

$$J_i = \mathcal{I}_{ij}\chi_j \qquad \longrightarrow \qquad J_i = \mathcal{I}_{ij}\chi_j - K_i,$$

- The AK shift

$$K_i = -\frac{\epsilon m^2 N_c}{12\pi}\rho^3 R_{3i}\mathcal{J}_2.$$

- Hamiltonian

$$H_{\rm full} = \frac{1}{2I_A}\left(J_\xi^2 + J_\eta^2\right) + \frac{1}{2I_C}\left(J_\zeta^2 + K_\zeta^2 + 2J_\zeta K_\zeta\right) + M_0.$$

- Quantized energy

$$E_{\rm full} = \underbrace{\frac{1}{2I_A}j(j+1)}_{\rm ANW} + \underbrace{\frac{1}{2}\left(\frac{1}{I_C} - \frac{1}{I_A}\right)i_3^2}_{\rm BBGR} - \underbrace{\frac{1}{I_0}K_\zeta i_3}_{\text{Bigazzi-Niro}} + \frac{1}{2I_0}K_\zeta^2 + M_0,$$

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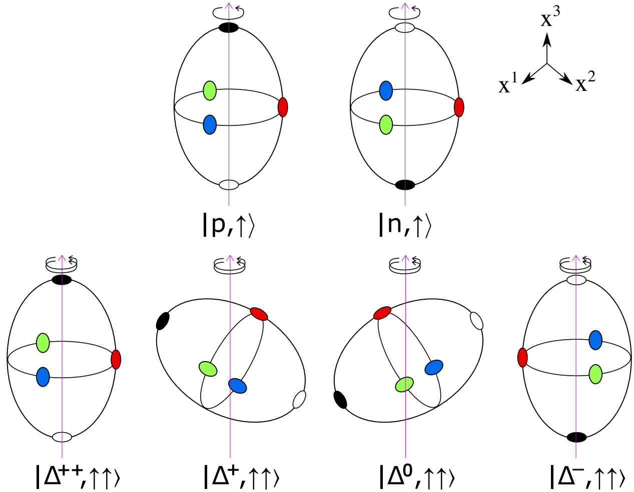
Isospin breaking: new quantization



Isospin breaking: proton-neutron splitting

- Proton-neutron mass splitting [Bigazzi-Niro, PRD98 (2018), 046004]

$$M_p - M_n = \frac{\epsilon m^2 N_c}{48\pi^3 \kappa} \rho \mathcal{J}_2 \simeq -4.74\,\text{MeV}\,.$$

- PDG value

$$(M_p - M_n)^{\text{PDG}} = -1.293\,\text{MeV}$$

- Baryon observables always overestimated by meson-fitted parameters in large-N_c theories.
- It's better to compare ratios:

$$2\frac{M_p - M_n}{M_p + M_n} \simeq -2.5 \times 10^{-3},$$

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Isospin breaking: application to the Δs

- Splitting of Δs

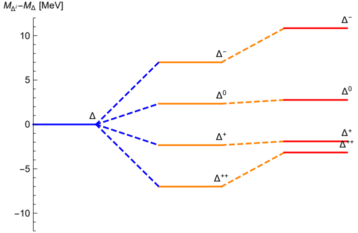
$$\frac{1}{2}(M_{\Delta^{++}} - M_{\Delta^{+}} - M_{\Delta^{0}} + M_{\Delta^{-}}) = \frac{3\delta}{(I+\delta)(I-2\delta)} \simeq \frac{27\pi}{8}\frac{m\epsilon^2}{N_c\lambda} \approx 3.41\,\text{MeV}\,.$$

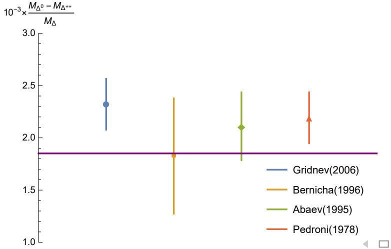
- Numerical values of the splitting

$$M_{\Delta^{++}} - M_{\Delta^{+}} \simeq \frac{3\delta}{I_0^2} + \frac{\epsilon m^2 N_c}{48\pi^3\kappa}\rho\mathcal{J}_2 \approx -1.25\,\text{MeV}\,,$$

$$M_{\Delta^{+}} - M_{\Delta^{0}} \simeq \frac{\epsilon m^2 N_c}{48\pi^3\kappa}\rho\mathcal{J}_2 \approx -4.67\,\text{MeV}\,,$$

$$M_{\Delta^{0}} - M_{\Delta^{-}} \simeq -\frac{3\delta}{I_0^2} + \frac{\epsilon m^2 N_c}{48\pi^3\kappa}\rho\mathcal{J}_2 \approx -8.08\,\text{MeV}\,.$$





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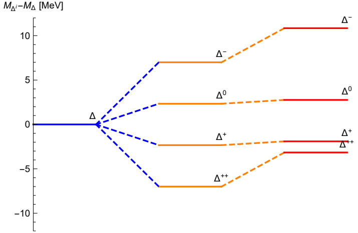
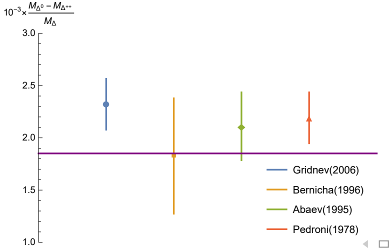




Outline

It's hard to calculate I_0 in the WSS (3-dimensional PDE)

$$\Downarrow$$

we'll do it in the Skyrme model

Detour – the Skyrme model: small pion mass expansion

- The linear tail:

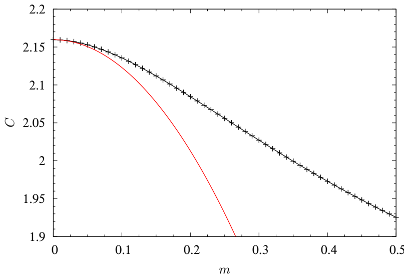
$$f'' + \frac{2f'}{r} - \frac{2f}{r^2} - m^2 f = 0, \qquad f(r) = C(m)\frac{mr+1}{r^2}e^{-mr}, \qquad r \gg 1.$$

- Related to $g_{NN\pi}$:

$$g_{NN\pi} = \frac{8\pi M_{\rm Sk}}{f_\pi e^2} C(m).$$

- $C(m)$'s m-dependence:

$$C(m) = C(0) - 2m^2 C(0)^2\left(\beta - \frac{1}{4}\right) + \mathcal{O}(m^3), \qquad C(0) = R(0)^2.$$



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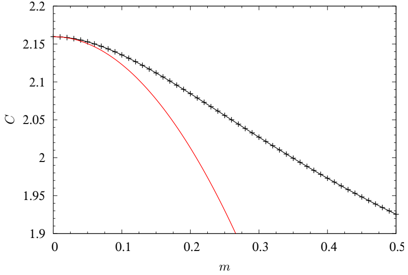

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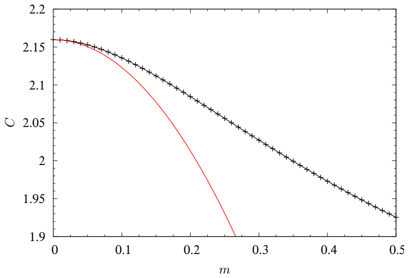

Detour – the Skyrme model: splitting

- Mass matrix for $\boldsymbol{\pi}$ and η:

$$M^2 = m^2 \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \epsilon \\ 0 & 0 & \epsilon & 1 \end{pmatrix} \sim m^2 \,\mathrm{diag}\left(1, 1, 1+\epsilon, 1-\epsilon\right).$$

- Tails:

$$\pi^1 = C\frac{mr+1}{r^2}e^{-mr}\hat{x}^1, \qquad \pi^2 = C\frac{mr+1}{r^2}e^{-mr}\hat{x}^2,$$

$$\pi^3 = \frac{D}{2}\frac{m\sqrt{1+\epsilon}r+1}{r^2}e^{-m\sqrt{1+\epsilon}r}\hat{x}^3 + \frac{E}{2}\frac{m\sqrt{1-\epsilon}r+1}{r^2}e^{-m\sqrt{1-\epsilon}r}\hat{x}^3,$$

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- Moments of inertia:

$$I = \frac{4\pi}{9}\frac{D^2+E^2+4C^2}{R_{\mathrm{UV}}} - \frac{2\pi}{3}\left(\sqrt{1+\epsilon}D^2 + \sqrt{1-\epsilon}E^2 + 4C^2\right)m + \mathcal{O}(m^2, m^2\epsilon),$$

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- Leading order (with $C = D = E$):

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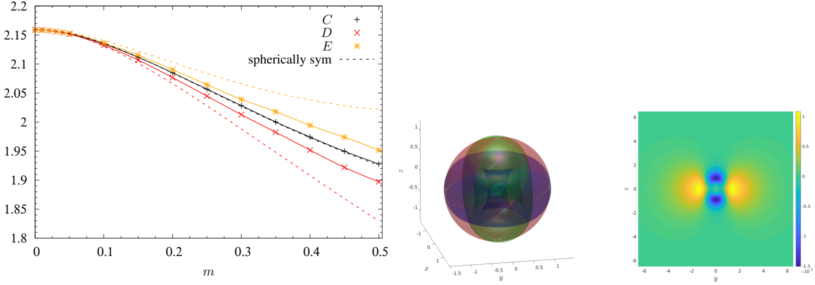

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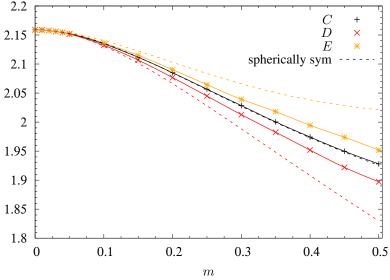

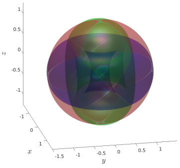

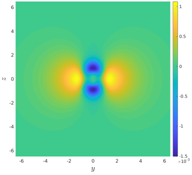

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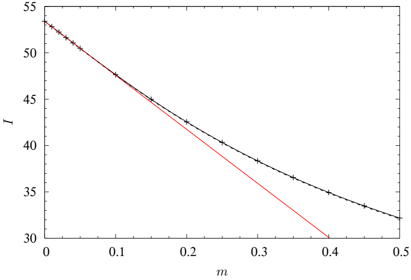

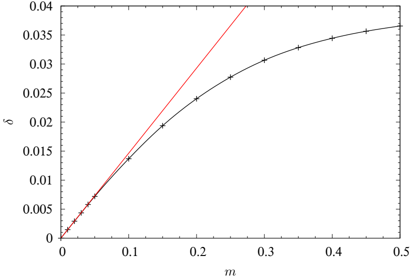

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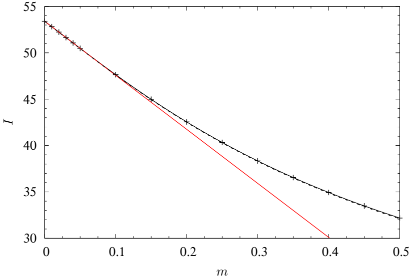

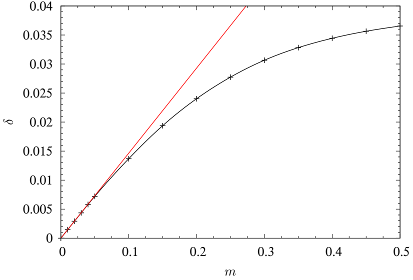

Outline

Conclusions

- The mechanism of mass splitting the Yukawa way is complicated and requires 4 conditions:
  1. Mass-splitting in the quark/pion mass term
  2. Including the η (extending $\mathrm{SU}(2)$ to $\mathrm{U}(2)$)
  3. Chern-Simons (or in 4D Wess-Zumino) term
  4. Vectors
- Easy to calculate:
  - In the Skyrme model: the zeroth-order moment of inertia (I_0)
  - In holography (WSS): the leading-in-ϵ splitting of the moment of inertia
- Myriads of possibilities for improvement:
  - Full-PDE computation of the baryon with core and tail $\Rightarrow I_0$
  - Higher-order $\epsilon, N_c^{-1}, \lambda^{-1}, \ldots$ corrections
  - 3 flavors ($\mathrm{U}(3)$)
  - Non-antipodal branes?
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谢谢大家！！

有问题吗?