Mass and isospin breaking effects in the Witten-Sakai-Sugimoto model

比亚科教授

河南大学数学和统计学院

合作Bartolini、Bolognesi和Rainaldi[PRD110, 026017 (2024)]

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Outline

- 1 Holographic QCD
 - Quark and pion masses
 - Isospin symmetry breaking
 - The Skyrme model

2 Conclusions

The Sakai-Sugimoto (SS) model [Prog.Theor.Phys.113, 843 (2005)] builds on the work of Witten [Adv.Theor.Math.Phys.2, 253 (1998)] where N_f D8- and $\overline{\mbox{D8}}$ -branes are intersecting N_c D4-branes

- The SS model: type IIA string theory
- SUSY-breaking by anti-periodic boundary conditions on the S^1 for the fermions on D4
- Chiral symmetry is explicit by the two 8-branes, when they stretch
- Chiral symmetry breaking is string geometric as the 8-branes touch and merge – the low-energy supergravity geometry is that of a cigar-shaped space, which is AdS₅-like

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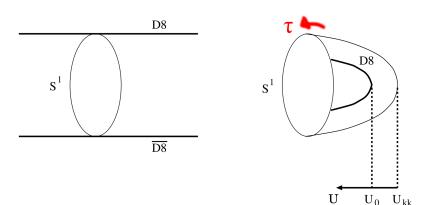
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Sakai-Sugimoto – chiral symmetry breaking



[Sakai-Sugimoto, Prog.Theor.Phys.113, 843 (2005)]

ullet Notice that the confined geometry ends at $U_{
m KK}$

• The 't Hooft limit is considered $N_c \gg N_f$, so that the 8-branes can be considered in the probe branes embedded in the D4-background (color d.o.f.)

$$egin{aligned} \mathrm{d}s^2 &= \left(rac{u}{R}
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- The flavor d.o.f. are described by the DBI action and the Chern-Simons term at level N_c both scale as N_c
- The leading order approximation to the DBI action is the 5-dimensional Yang-Mills term, hence the Sakai-Sugimoto model:

$$S = -\kappa \operatorname{Tr} \int_{\mathrm{AdS}_5} \mathcal{F} \wedge *\mathcal{F} + N_c \int_{\mathrm{AdS}_5} \omega_5,$$

with YM coefficient $\kappa=rac{\lambda N_c}{216\pi^3}$, and 't Hooft coupling $\lambda=g_{
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$$g = h(z)k(z)dx^{\mu}dx_{\mu} + h^{2}(z)dz^{2}, \qquad k(z) = h^{-3}(z) = 1 + z^{2}.$$

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The flavor fields are expanded as

$$\mathcal{A}_{\mu} = \sum_{n} v_{\mu}^{2n-1}(x) \psi_{2n-1}(z) + \sum_{n} a_{\mu}^{2n}(x) \psi_{2n}(z),$$
 $\mathcal{A}_{z} = \Pi(x) \phi_{0}(z) + \sum_{n} H^{n}(x) \phi_{n}(z),$

with profile functions

$$\begin{array}{ll} \text{pions}: \phi_0(z) = \frac{1}{\sqrt{\pi\kappa}} \frac{1}{k(z)}, \qquad \text{vectors}: -h^{-1}(z) \partial_z(k(z) \partial_z \psi_n) = \lambda_n \psi_n, \\ \\ \phi_n(z) = \frac{1}{\sqrt{\lambda_n}} \partial_z \psi_n(z), \end{array}$$

with vector meson masses $M_n \sim \sqrt{\lambda_n}$

Fitting the pion decay constant and the rho meson mass, one obtains

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Sakai-Sugimoto – The Skyrme model

If we truncate to the pions, one gets the Skyrme model

$$S = \widetilde{T} (2\pilpha')^2 \int \mathrm{d}^4 x \; \operatorname{Tr} \left[A L_\mu^2 + B [L_\mu, L_
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with the left-invariant chiral current $L_\mu=U^{-1}\partial_\mu U$, and the constants determined by string theory

$$A=rac{9u_{
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The Skyrme coupling is determined by the model

$$e^2 = rac{27\pi^7}{2b} rac{1}{\lambda N_c} \sim (7.32\cdots)^2$$

which can be compared to [Adkins-Nappi-Witten, NPB228, 552 (1983)], where they find e=5.45 by fitting to the masses of the nucleon and Delta resonance

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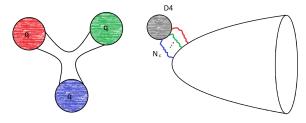
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• the coupling of N_c strings from the D4-branes to the 8-branes

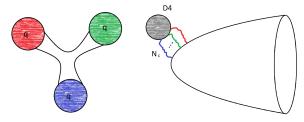


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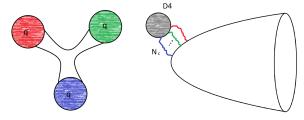
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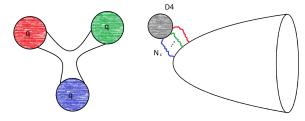
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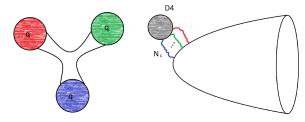
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[Hata-Sakai-Sugimoto-Yamato, Prog.Theor.Phys.117, 1157 (2007)]

$$A_M = -if(\xi)g\partial_M g^{-1}, \qquad f(\xi) = rac{\xi^2}{\xi^2 +
ho^2}, \ \xi^2 = (x - X)^2 + (z - Z)^2, \qquad g(x) = rac{(z - Z) - i(\mathbf{x} - \mathbf{X}) \cdot au}{\xi}.$$

• The Abelian electric field is new, this field acts as a size stabilization against gravitational collapse

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Rotation of the instanton gives rise to spin and isospin quantum numbers

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$$m_q(\psi_L^{\dagger}\psi_R + \psi_R^{\dagger}\psi_L),$$

which must be nonlocal in the bulk!

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Outline

- Holographic QCD
 - Quark and pion masses
 - Isospin symmetry breaking
 - The Skyrme model

2 Conclusions

Chiral symmetry breaking induced by the Yukawas

 We will introduce the chiral symmetry breaking along the lines of the Standard Model: i.e. by the mass term (Yukawa interactions):

$$egin{aligned} S_{ ext{AK}} &= c \int ext{d}^4 x \; \operatorname{Tr}\left[M\left(e^{ ext{i}arphi} + e^{- ext{i}arphi} - 2\mathbb{1}
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Quantum baryons

• Turning on isospin moduli:

$$\widehat{A}_z = -rac{N_c}{16\pi^2\kappa}rac{
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- Protons: $|p\uparrow\rangle\propto(a_1+\mathrm{i}a_2)$ and $|p\downarrow\rangle\propto(a_4-\mathrm{i}a_3)$
- Neutrons: $|n\uparrow\rangle \propto (a_4 + \mathrm{i}a_3)$ and $|n\downarrow\rangle \propto (a_1 \mathrm{i}a_2)$
- Angular velocity: $\chi^a = -i \operatorname{Tr}(A^{\dagger} \dot{A} \tau^a)$ with $A = \mathbb{1}_2 a_4 + i a_i \tau^i$
- Matching linearized EOM with BPST core [Hashimoto-Sakai-Sugimoto Prog.Theor.Phys.120, 1093 (2008)]

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• Linearized EOMs:

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• Diagonalize into the mass basis (π and η)

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Solutions:

$$A_{\eta} = b \chi^{j} \partial_{X^{j}} H_{+} - \tilde{b} \operatorname{Tr} \left(\mathsf{A} \tau^{j} \mathsf{A}^{\dagger} \tau^{3} \right) \partial_{X^{j}} H_{+}, \quad A_{\pi} = d \chi^{j} \partial_{X^{j}} H_{-} - \tilde{d} \operatorname{Tr} \left(\mathsf{A} \tau^{j} \mathsf{A}^{\dagger} \tau^{3} \right) \partial_{X^{j}} H_{-}.$$

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Modified Green's functions

$$H_\pm \equiv -\kappa \sum_{n=0} \phi_n(z) \phi_n(Z) rac{1}{4\pi} rac{e^{-\sqrt{\lambda_n}r}}{r}, \qquad \lambda_0 = m_\pm^2, \qquad m_\pm^2 = m^2 (1\pm\epsilon).$$

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Inserting the solutions into the Yang-Mills:

$$T = rac{\pi^2
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$$\mathcal{I} = I_0 \mathbb{1}_3 + \delta egin{pmatrix} 1 & 0 & 0 \ 0 & 1 & 0 \ 0 & 0 & -2 \end{pmatrix}, \qquad \delta = rac{\pi^2
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 \bullet The AK term contains $\mathcal{O}(\epsilon)$ modification of the angular momentum ($\mathcal{J}_2\approx 1.054)$:

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Quantized energy

$$E_{\mathrm{full}} = \underbrace{\frac{1}{2I_A}j(j+1)}_{\mathrm{ANW}} + \underbrace{\frac{1}{2}\left(\frac{1}{I_C} - \frac{1}{I_A}\right)i_3^2}_{\mathrm{PDCP}} - \underbrace{\frac{1}{I_0}K_\zeta i_3}_{\mathrm{Bigazzi-Niro}} + \underbrace{\frac{1}{2I_0}K_\zeta^2 + M_0}_{\mathrm{Rigazzi-Niro}},$$

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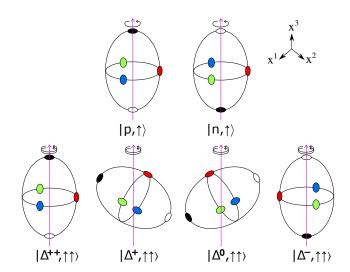
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ight)i_{3}^{2}}_{\mathrm{BRGR}} - \underbrace{\frac{1}{I_{0}}K_{\zeta}i_{3}}_{\mathrm{Bigazzi-Niro}} + rac{1}{2I_{0}}K_{\zeta}^{2} + M_{0},$$

WSS中同位能对称破缺

比亚科



Proton-neutron mass splitting [Bigazzi-Niro, PRD98 (2018), 046004]

$$M_p - M_n = rac{\epsilon m^2 N_c}{48\pi^3 \kappa}
ho \mathcal{J}_2 \simeq -4.74 \, \mathrm{MeV} \,.$$

PDG value

$$\left(M_p-M_n\right)^{\mathrm{PDG}}=-1.293\,\mathrm{MeV}$$

- Baryon observables always overestimated by meson-fitted parameters in large-N_c theories.
- It's better to compare ratios:

$$2\frac{M_p - M_n}{M_p + M_n} \simeq -2.5 \times 10^{-3},$$

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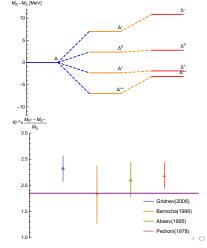
Isospin breaking: application to the Δs

• Splitting of Δs

$$\frac{1}{2} (M_{\Delta^{++}} - M_{\Delta^{+}} - M_{\Delta^{0}} + M_{\Delta^{-}}) = \frac{3\delta}{(I+\delta)(I-2\delta)} \simeq \frac{27\pi}{8} \frac{m\epsilon^{2}}{N_{c}\lambda} \approx 3.41\,\mathrm{MeV}\,.$$

Numerical values of the splitting

$$\begin{split} M_{\Delta^{++}} - M_{\Delta^{+}} &\simeq \frac{3\delta}{I_{0}^{2}} + \frac{\epsilon m^{2}N_{c}}{48\pi^{3}\kappa}\rho\mathcal{J}_{2} \\ &\approx -1.25\,\mathrm{MeV}, \\ M_{\Delta^{+}} - M_{\Delta^{0}} &\simeq \frac{\epsilon m^{2}N_{c}}{48\pi^{3}\kappa}\rho\mathcal{J}_{2} \\ &\approx -4.67\,\mathrm{MeV}, \\ M_{\Delta^{0}} - M_{\Delta^{-}} &\simeq -\frac{3\delta}{I_{0}^{2}} + \frac{\epsilon m^{2}N_{c}}{48\pi^{3}\kappa}\rho\mathcal{J}_{2} \\ &\approx -8.08\,\mathrm{MeV} \,. \end{split}$$



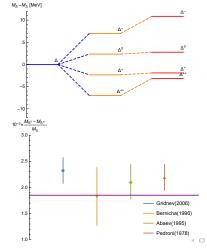
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- 1 Holographic QCD
 - Quark and pion masses
 - Isospin symmetry breaking
 - The Skyrme model

2 Conclusions

It's hard to calculate I_0 in the WSS (3-dimensional PDE)



we'll do it in the Skyrme model

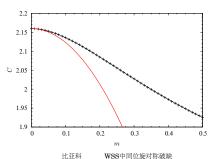
Detour – the Skyrme model: small pion mass expansion

• The linear tail:

$$f'' + rac{2f'}{r} - rac{2f}{r^2} - m^2 f = 0, \qquad f(r) = C(m) rac{mr+1}{r^2} e^{-mr}, \qquad r \gg 1.$$

$$g_{NN\pi}=rac{8\pi M_{
m Sk}}{f_\pi e^2}C(m).$$

$$C(m) = C(0) - 2m^2C(0)^2\left(\beta - \frac{1}{4}\right) + \mathcal{O}(m^3), \qquad C(0) = R(0)^2.$$



WSS中同位能对称破缺

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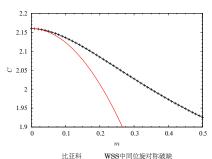
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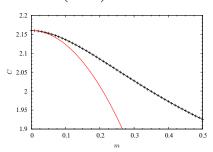
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• C(m)'s m-dependence:

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比亚科

• Mass matrix for π and η :

$$M^2 = m^2 egin{pmatrix} 1 & 0 & 0 & 0 \ 0 & 1 & 0 & 0 \ 0 & 0 & 1 & \epsilon \ 0 & 0 & \epsilon & 1 \end{pmatrix} \sim m^2 \operatorname{diag}ig(1, 1, 1 + \epsilon, 1 - \epsilonig).$$

Tails

$$\begin{split} \pi^1 &= C \frac{mr+1}{r^2} e^{-mr} \hat{x}^1, \qquad \pi^2 = C \frac{mr+1}{r^2} e^{-mr} \hat{x}^2, \\ \pi^3 &= \frac{D}{2} \frac{m\sqrt{1+\epsilon}\, r+1}{r^2} e^{-m\sqrt{1+\epsilon}\, r} \hat{x}^3 + \frac{E}{2} \frac{m\sqrt{1-\epsilon}\, r+1}{r^2} e^{-m\sqrt{1-\epsilon}\, r} \hat{x}^3, \\ \eta &= \frac{D}{2} \frac{m\sqrt{1+\epsilon}\, r+1}{r^2} e^{-m\sqrt{1+\epsilon}\, r} \hat{x}^3 - \frac{E}{2} \frac{m\sqrt{1-\epsilon}\, r+1}{r^2} e^{-m\sqrt{1-\epsilon}\, r} \hat{x}^3, \end{split}$$

Moments of inertial

$$I = rac{4\pi}{9} rac{D^2 + E^2 + 4C^2}{R_{
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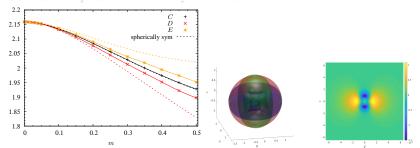
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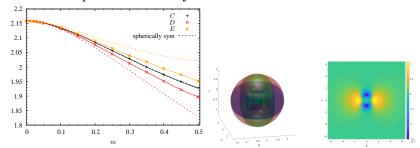
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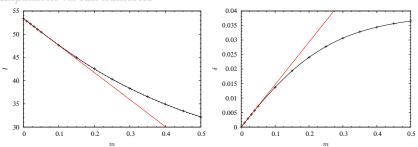
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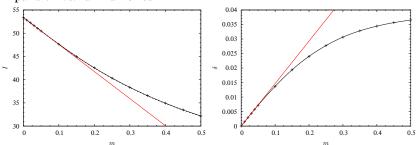
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Expansion vs. full numerics



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2 Conclusions

- The mechanism of mass splitting the Yukawa way is complicated and requires 4 conditions:
 - Mass-splitting in the quark/pion mass term
 - 2 Including the η (extending SU(2) to U(2))
 - Chern-Simons (or in 4D Wess-Zumino) term
 - 4 Vectors
- Easy to calculate:
 - In the Skyrme model: the zeroth-order moment of inertia (I_0)
 - In holography (WSS): the leading-in- ϵ splitting of the moment of inertial
- Myriads of possibilities for improvement
 - \circ Full-PDE computation of the baryon with core and tail $\Rightarrow I_0$
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谢谢大家!!

有问题吗?