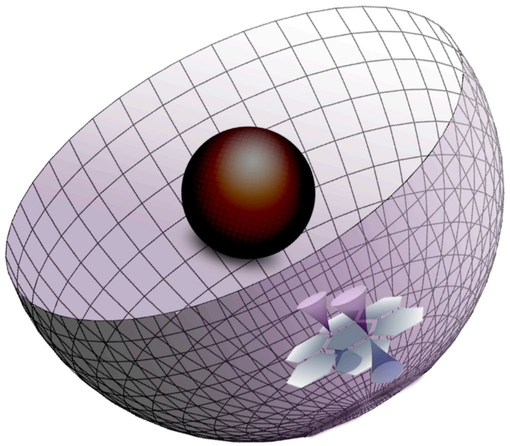
ER=EPR, and Strange metal from Entanglement

A parallelism between Y-SYK and quantum gravity



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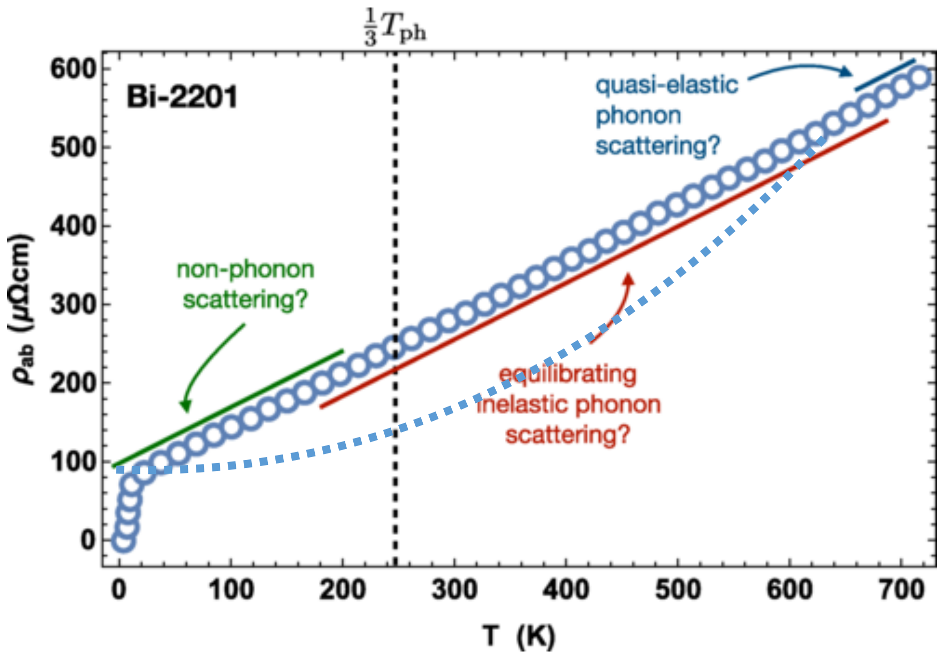
Strange Metal.

1. $$\rho \sim T$$

2. $$\sigma_{xx}/\sigma_{xy} \sim T^2$$



Essence of High Tc is in the Normal state!

References

- P.W. Anderson, Phys. Rev. Lett. 67 (1991) 2092.
- P.W. Anderson, Physics Today 66 (2013) 9.

Problem

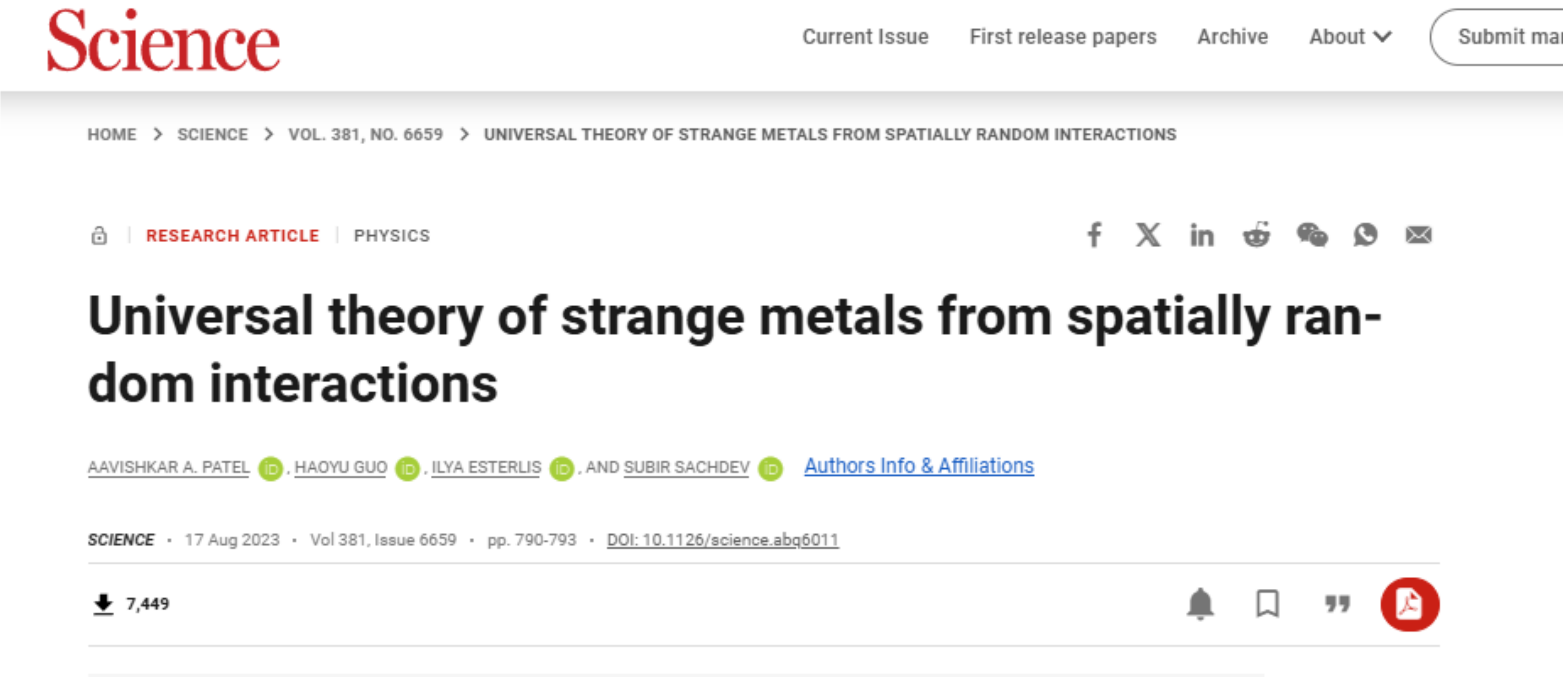
- Exp: Universal behavior of strongly correlated metal,
  Theory: Difficult to write a (solvable QFT) model for 35 years.

- Even the lowest dim. H_{int} produces $\rho \sim T^{\alpha}, \quad \alpha > 1,$
  ==> No microscopic understanding for mechanism

- Phenomenology: momentum is dissipated Strangely Fast : $1/\tau \sim T$
  You will see why!

A model for Linear Resistivity

2023.Aug. Patel and Sachdev et.al, wrote 2+1 model based on SYK model.



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RESEARCH ARTICLE | PHYSICS

Universal theory of strange metals from spatially random interactions

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7,449

Based on SYK : all to all with Random coupling

$$L = \frac{1}{2}\sum_{i=1}^{N} \chi_i \partial_\tau \chi_i - \frac{1}{4!}\sum_{i,j,k,l=1}^{N} J_{ijkl}\chi_i\chi_j\chi_k\chi_l,$$

$$\langle J_{ijkl} J_{i'j'k'l'} \rangle = J^2 \delta_{ii'} \delta_{jj'} \delta_{kk'} \delta_{ll'}$$

1. Solvable.
2. $\Gamma \sim T$
3. Maximal Chaos

****

4. 0-dim => No transport!

Yukawa-SYK in 2+1 dim.

$$\mathscr{H} = \mathscr{H}_\psi + \mathscr{H}_\phi + \sum_{ijk} g_{ijk}(x)\phi_i\psi_j\psi_k(t,x) + \sum_{ij} v_{ij}(x)\psi_i\psi_j$$

$$\begin{cases} \langle g_{ijl}(\mathbf{r})\rangle = 0 \\ \langle g^*_{ijl}(\mathbf{r}) g_{i'j'l'}(\mathbf{r}')\rangle = g^2\delta_{ii'jj'll'}\delta(\mathbf{r}-\mathbf{r}') \end{cases}$$

$$\rho = \rho_0 + AT$$

Resistivity linear in T at Low tem.

References

- A.A. Patel, H. Guo, I. Esterlis and S. Sachdev, Science 381 (2023) abq6011
- I. Esterlis, H. Guo, A.A. Patel and S. Sachdev, Phys. Rev. B 103 (2021) 235129
- H. Guo, A.A. Patel, I. Esterlis and S. Sachdev, Phys. Rev. B 106 (2022) 115151

Yili Wang, XianHui Ge, YK Han + SJS

1. Yili Wang, Xian-Hui Ge, and S. Sin, arXiv: 2406.11170
2. Yili Wang, Young-Kwon Han, Xian-Hui Ge, and S. Sin, arXiv: 2501.07792
3. ER=EPR and strange metal from quantum entanglement.
4. Most general strange metal model. TBA

1. How universal is the (spatially random disorder) mechanism?

   Yukawa-SYK, & Vector-SYK, Model. $\phi\psi\psi$, $A_\mu^{ext}\psi\partial_\mu\psi$

2. Does it work in 3+1 ? No.

   => layered structure is important. (CuO_2 plane)

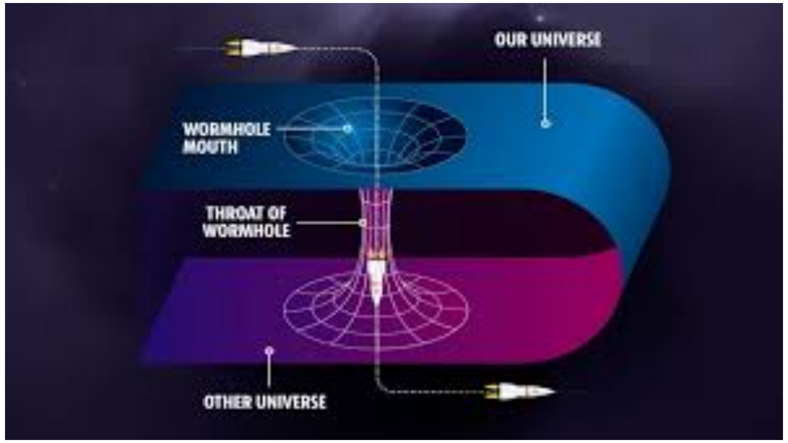
3. T^2 inverse Hall angle behavior? No!

4. The most general QFT model?

   $\phi\psi\psi$, $A_\mu^{ext}\psi\partial_\mu\psi$, dim 5/2 => all others are irrelevant

Our Q: why it works as a Mechanism for the Planckian dissipation

ANS: randomness brings us Wormhole, a short cut in space



Claim:

<—> $\langle g^*_{ijl}(\mathbf{r}) g_{i'j'l'}(\mathbf{r}')\rangle = g^2 \delta_{ii'jj'll'}\delta(\mathbf{r}-\mathbf{r}')$

- Gravitational Wormhole
- Field theory wormhole

Quenched vs Annealed

$$W_q = \int D[g]P[g]\left[\ln\left(\int D[\Psi]e^{-S_{\rm tot}[g,\psi,\phi]}\right)\right] = \langle \log Z\rangle_{dis}$$

$$W_a = \ln\left(\int D[\Psi]\int D[g]P[g]e^{-S_{\rm tot}[g,\psi,\phi]}\right) \quad = \log\langle Z\rangle_{dis}$$

Quenched case: replica trick. Replica off diagonal case is 1/N suppressed.

For annealed case, $H_{int}(x) = g(x)^2 + g(x)\int dt\mathcal{O}(x,t) = \left[g(x) + \int dt\mathcal{O}(x,t)\right]^2 - \iint dt_1 dt_2 \mathcal{O}(x,t_1)\mathcal{O}(x,t_2)$

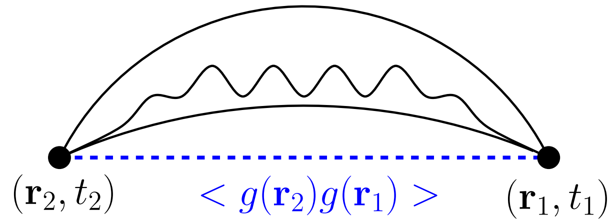
$$S_{\rm int} \equiv -\frac{g^2}{2N^2}\int d\tau_1 d\tau_2 d^2\boldsymbol{r}\sum_{a,b,c=1}^{N}\psi_a^\dagger\psi_b(\boldsymbol{r},\tau_1)\phi_c(\boldsymbol{r},\tau_1)$$
$$\times\psi_b^\dagger\psi_a(\boldsymbol{r},\tau_2)\phi_c(\boldsymbol{r},\tau_2). \quad (8)$$

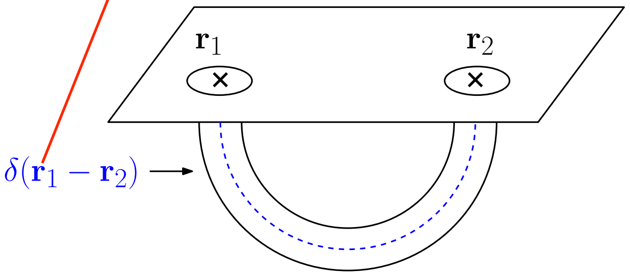
Quenched => wormhole

$$H_{ijk} = \phi_i \psi_j \psi_k (t, x)$$

$$-F_q = \langle \log Z \rangle_{dis} = \sum_{n=0}^{\infty} \int_{\mathbf{x}_1 \ldots \mathbf{x}_n} \left\langle \langle \prod_{a=1}^{n} g_{ijk}(\mathbf{x}_a) H_{ijk}(\mathbf{x}_a) \rangle_{\Phi,c} \right\rangle_{dis},$$

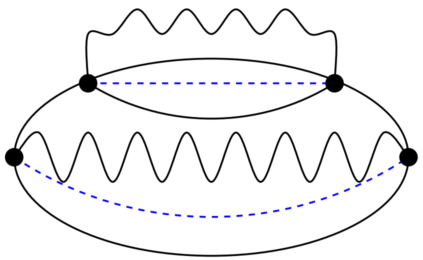
n=2 : <g(x)g(y)> $\langle \phi\psi\psi(x)\phi\psi\psi(y) \rangle$





Wormhole

n=4

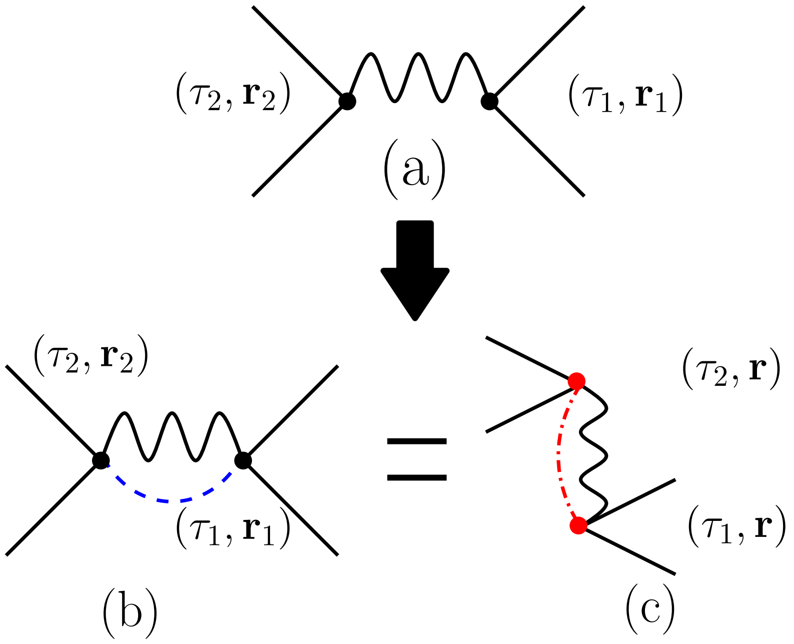


Many Wormholes

The role of the QFT-WH:

Without it, distance dependent suppression

With => no such suppression



$$\frac{1}{r}e^{-mx}$$

Causal suppression!

$$\Downarrow$$

$$1$$

In many body:

m -> 1/L, L ~MFP

Is SM a weak coupling phenomena?

Even with small coupling, achieve strongly correlated system.

Disorder => wormhole: decoherence?

1. Use equivalence of Quenched = Annealed In large N limit.

   We calculated ρ(T) and show it is the same as that of Patel. et.al

2. Ask what we mean by spatial randomness!

Annealed picture: vertex gives EPR

$$H_{int}(x) = g(x)^2 + g(x)\int dt\mathcal{O}(x,t) = \left[g(x) + \int dt\mathcal{O}(x,t)\right]^2 - \iint dt_1 dt_2 \mathcal{O}(x,t_1)\mathcal{O}(x,t_2)$$

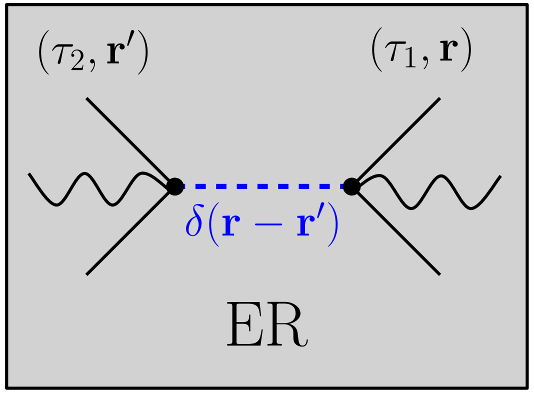
$$S_{\rm int} = -\frac{g^2}{2N^2}\int d\tau_1 d\tau_2 d^2\boldsymbol{r} \sum_{a,b,c=1}^{N} \psi_a^\dagger\psi_b(\boldsymbol{r},\tau_1)\phi_c(\boldsymbol{r},\tau_1) \times\psi_b^\dagger\psi_a(\boldsymbol{r},\tau_2)\phi_c(\boldsymbol{r},\tau_2). \quad (8)$$

This vertex gives Entanglement!

Only one p conservation for two vertex->

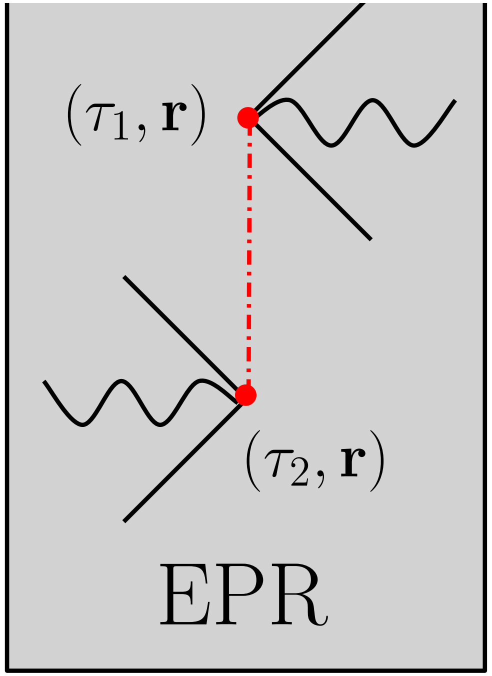
Long range momentum correlation

$$|\Psi\rangle = \int d\boldsymbol{p}\sqrt{P(\boldsymbol{p})}\,|\boldsymbol{p},-\boldsymbol{p}\rangle, \text{ with } \int d\boldsymbol{p}P(\boldsymbol{p}) = 1.$$



In quenched ==> In annealed

ER ==> EPR



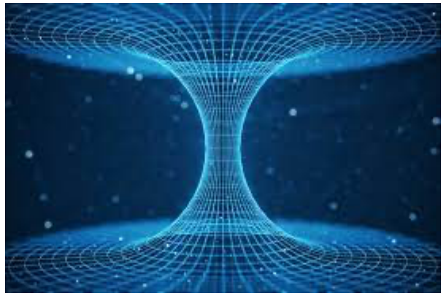
A Parallelism in information loss prob.

Quantum gravity vs Yukawa-SYK

Wormhole in QG and information loss

Hawking (1987)

deCoherence by the Wormhole :



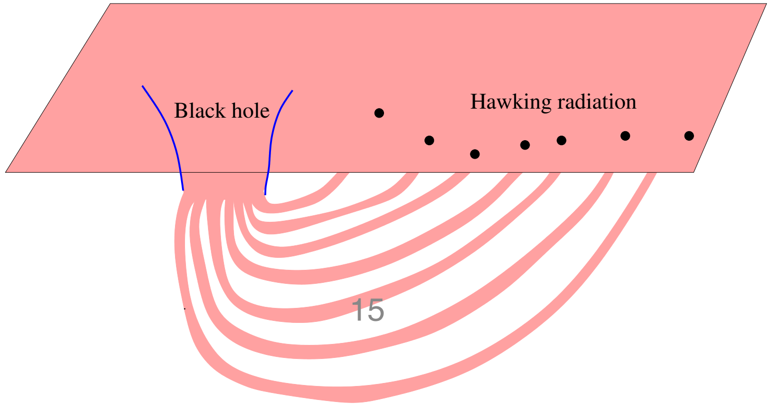
Info. can leak away by WormHole

VS

- Maldacena+Susskind : arXiv:1306.0533
  Any entangled pair is connected by a microscopic wormhole

ER=EPR

- Hawking Radiation is entangled by WH
- Not thermal, => info not lost.



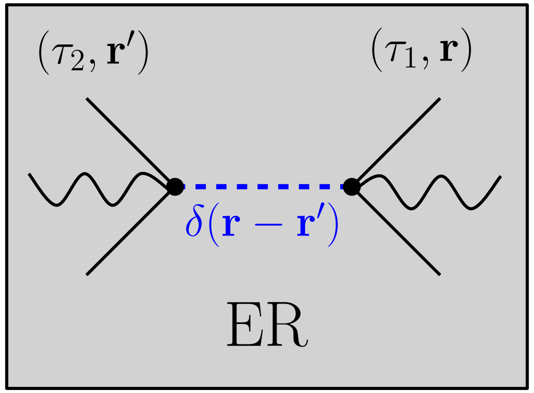
Parallelism in Yukawa-SYK and information loss

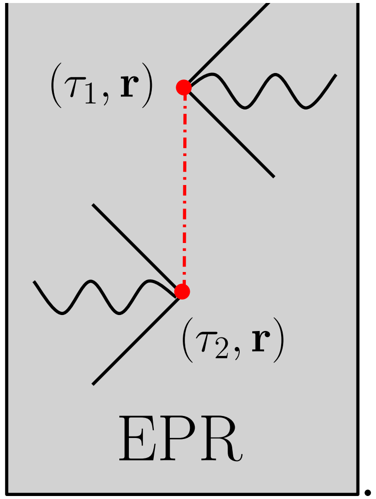
Usual SYK is disorder averaged hence describe a Mixed state !

the coupling correlation is the way to deliver the information from one vertex to the other.

Consistent with Hawking's view :

=> SM is mixed state property not really a qm property.

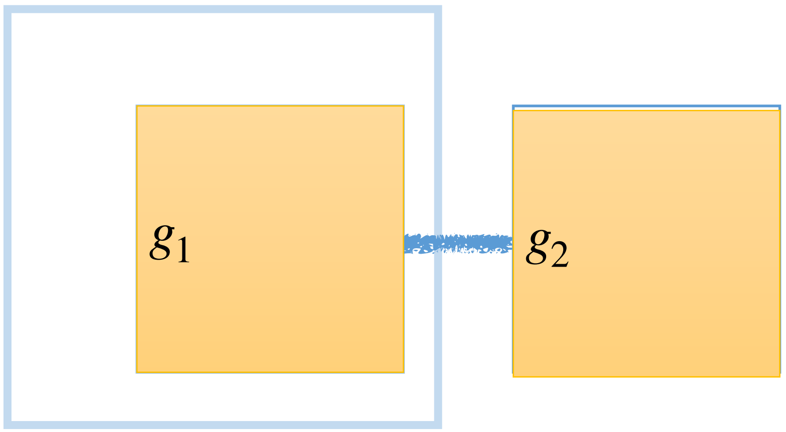




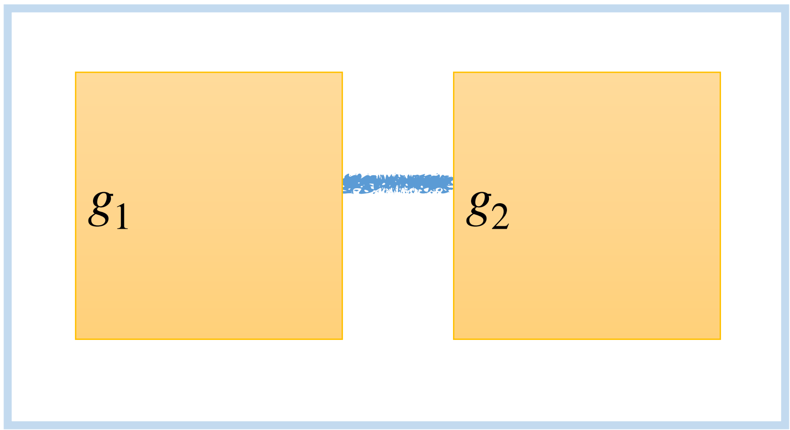
- 1. Quenched <=> Anealed
  <=> ER=EPR

- 2.This is the origin of the Planckian Dissipation.
  (Because this is mechanism of long range p delivery.)

wormhole and information loss: g vs g(x) ?



System



System include both

Open system: information leaks

Closed system: No information loss

sample = Each box ,

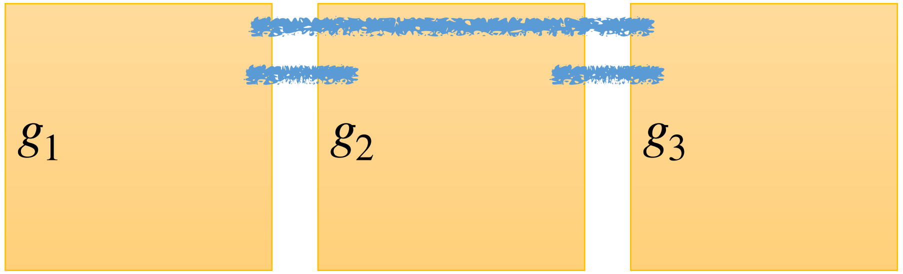
Mixed: sum of independent many ,

What do we mean by spatially Random $g(x)$

- 1. Spatial inhomogeneity is important.

$$Sys = \sum_i (subSys)_i \,, \qquad \text{NOT} \qquad Sys = \langle Sys_i \rangle$$

- <=> system is sum of many small pieces of subregions



Glue all to all fashion

Interacting <=> Entangled

1. 2. sample= sum of the boxes

qm: the whole is 1 system by connection.

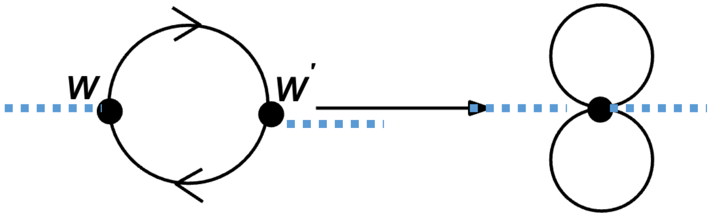
Parallel to the Raamsdonk's idea! Entanglement is a glue of spacetime.

Strange metalicity is more than Entanglement

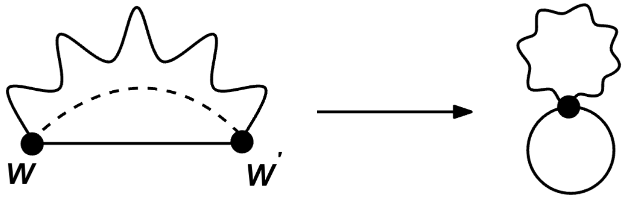
Need all 3 conditions

1. 2+1 dim!
2. Spatial dependence of g !
3. Inclusion of minimal dim operator.

$$\langle g^*_{ijl}(\mathbf{r}) g_{i'j'l'}(\mathbf{r}')\rangle = g^2 \delta_{ii'jj'll'}\delta(\mathbf{r}-\mathbf{r}')$$



Boson self-energy



Electron self-energy

$$\Sigma_v(\mathrm{i}\omega_n) \equiv -\mathrm{i}\frac{\Gamma}{2} sgn(\omega_n)$$

Potential disorder

$$\Sigma_K(\mathrm{i}\omega_n) \equiv -\mathrm{i}c_K\omega_n \ln\left(\frac{c_0}{\omega_n}\right)$$

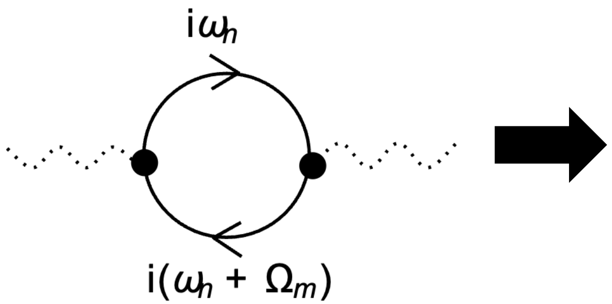
Electron-boson coupling

Momentum conservation at each vertex is lost!

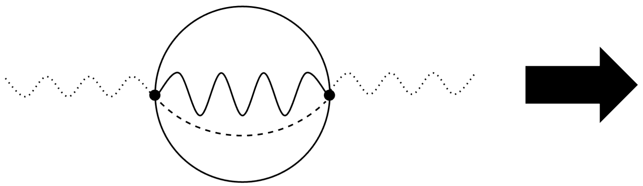
together with two vertex=> conserved!

$$\rho = \rho_0 + T^2 \rightarrow \rho_0 + T^1$$

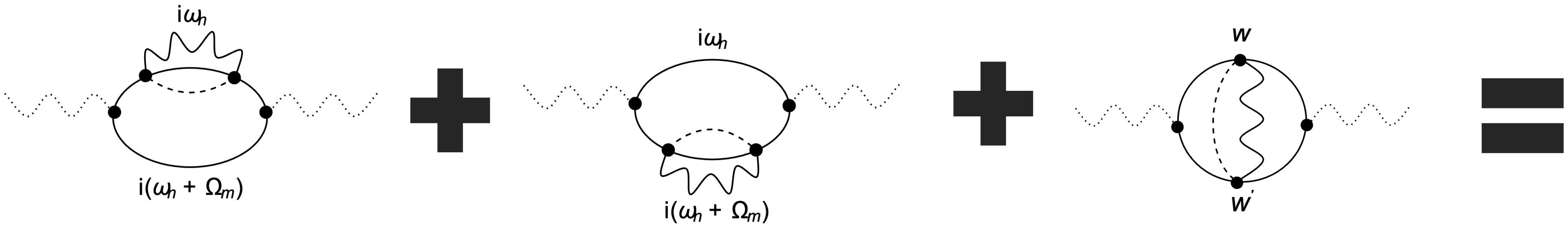
spatial randomness: changing the scaling law



residual resistivity



linearity



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Summary and future work

- Spatial random coupling => Wormhole
  => EPR+ Strange transport (long range momentum delivery).

- (quenched <—> annealed) => (ER = EPR)

- Remaining Q: In what system Y-SYK is realized and
  How Non-disordered system can be SM?

Epilogue from the movie "Dr. Strange"

- Dr. Strange came through the wormhole and shouted,

  운동량 배달 왔어요!

- Momentum delivery!   运动量快递

Thank you

- Interacting pair <=> Entangled pair

- Strange Metalicity needs more quantitative dynamical answer.

- Claim: Random correlation of the coupling => wormhole
  => Plankian Transport

$$\langle g_i(\vec{x}) g_j(\vec{x}') \rangle = \delta_{ij} \delta(\vec{x} - \vec{x}')$$

III. Equivalence of the Quenched and Annealed disorder.

$$W_q = \int D[g]P[g]\left[\ln\left(\int D[\Psi]e^{-S_{\rm tot}[g,\psi,\phi]}\right)\right],$$

$$W_a = \ln\left(\int D[\Psi]\int D[g]P[g]e^{-S_{\rm tot}[g,\psi,\phi]}\right)$$

$$W_q = W_a.$$

$$g^2 + g(x)\mathcal{O}(x) = (g + \mathcal{O}(x))^2 - \mathcal{O}(x)\mathcal{O}(x)$$

For annealed case,

one can integrate out the disorder variable $g_{ijk}(\boldsymbol{r})$ to get an effective theory, which can be described by replacing the interaction (1) with

$$S_{\rm int} \equiv -\frac{g^2}{2N^2}\int d\tau_1 d\tau_2 d^2\boldsymbol{r}\sum_{a,b,c=1}^{N}\psi_a^\dagger\psi_b(\boldsymbol{r},\tau_1)\phi_c(\boldsymbol{r},\tau_1) \times\psi_b^\dagger\psi_a(\boldsymbol{r},\tau_2)\phi_c(\boldsymbol{r},\tau_2). \quad (8)$$

Quenched case: replica trick.
Replica off diagonal case is 1/N suppressed.

- For $g_{ijk}(x)\phi_i\psi_j\psi_k(t,x)$, I=1,..., N, with large N,
  the replica index a from $g_{ijk}(x)\phi_i^a\psi_j^a\psi_k^a(t,x)$
  can be dropped after $R \to 0$.
- The equivalence can be shown case by case.
  We have done at the level of conductivity to discuss SM.

SYK-rised Vector Model

$$S = \int d\tau d^2 r \Big[\sum_{i=1}^{N} \psi_i^\dagger(r,\tau) \left(\partial_\tau - \frac{\nabla^2}{2m} - \mu \right) \psi_i(r,\tau)$$

$$+ \sum_{i,j=1}^{N} v_{ij}(r) \psi^\dagger(r,\tau)\psi_i(r,\tau) - \frac{1}{2K^2} \sum_{l=1}^{N} g_{ab} a_l^a \left(-\partial_\tau^2 + \mathbf{q}^2\right) a_l^b$$

$$+ \sum_{i,j,l=1}^{N} \frac{K_{ijl}(r)}{KN} \frac{\mathrm{i}}{m} \psi_i^\dagger \nabla_a \psi_j \mathbf{a}_l^a + \frac{1}{K^2 N^{3/2}} \sum_{i,j,s,t=1}^{N} \frac{\tilde{K}_{ijst}(r)}{2m} \mathbf{a}_s \cdot \mathbf{a}_t \psi_i^\dagger \psi_j \Big]$$

vector field

potential disorder

SYK-rised random coupling

$$\langle K_{ijl}(r) \rangle = 0, \quad \langle K_{ijl}^*(r) K_{i'j'l'}(r') \rangle = K^2 \delta_{ii'jj'll'} \delta(r - r')$$

$$\langle \tilde{K}_{ijl}(r) \rangle = 0, \quad \langle \tilde{K}_{ijl}^*(r) \tilde{K}_{i'j'l'}(r') \rangle = \tilde{K}^2 \delta_{ii'jj'll'} \delta(r - r')$$

$$\langle v_{ij}(r) \rangle = 0, \quad \langle v_{ij}(r) v_{i'j'}(r') \rangle = \delta_{ii',jj'} \delta(r - r')$$

02

SYK-rised Vector Model

$$G(x_1,x_2) \equiv -\frac{1}{N}\sum_i \langle \mathcal{T}\left(\psi_i(x_1)\psi_i^\dagger(x_2)\right)\rangle$$

$$D^{\mu\nu}(x_1,x_2) \equiv \frac{1}{N}\sum_l \langle \mathcal{T}\left(a_l^\mu(x_1)a_l^\nu(x_2)\right)\rangle$$

Vector model

↓

G-Σ action

Saddle Point Equation

↓

Dyson's Equations

$$\begin{aligned} 0 &= \frac{\delta S}{N} \\ &= \mathrm{Tr}(\delta\Sigma(G_*[\Sigma]-G)+\delta G(\Sigma_*[G]-\Sigma) \\ &\quad + \frac{1}{2}\delta\Pi_{ab}(D^{ab}-D_*{}^{ab}[\Pi^{ab}])+\delta D_{ab}(\Pi^{ab}-\Pi_*^{ab}[D^{ab}])) \end{aligned}$$

$$G = G_* = (-\partial_\tau - \varepsilon_k + \mu - \Sigma)^{-1}$$

$$\Sigma = \Sigma_* = v^2 G(\tau, r=0)\delta^3(r) + \frac{(k_1+k_2)^a(k_1+k_2)^b}{4m^2}D_{ab}G\bar{\delta} + \frac{1}{4m^2}D_{ab}D^{ab}G\bar{\delta}$$

$$D_{ab} = D_{*ab} = K^2(-g^{ab}(-\partial_\tau^2+\mathbf{q}^2) - K^2\Pi^{ab})^{-1}$$

$$\Pi_{ab} = \Pi_{*ab} = -\frac{(k_1+k_2)_a(k_1+k_2)_b}{4m^2}G\bar{\delta}\cdot G - \frac{1}{4m^2}GD_{ab}G\bar{\delta}$$

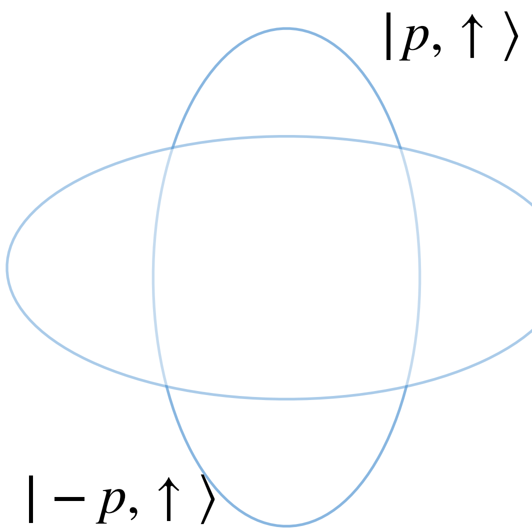
III. Equivalence of the Quenched and Annealed disorder.

- Quenched: $H_{int} = J\mathcal{O}$, should use replica trick.

$$< O >_Q = -\frac{\delta}{\delta J} < lnZ[J] >_{dis} = -\lim_{R\to 0}\frac{1}{R}\frac{\delta Z^R[J]}{\delta J}$$

- Annealed :

$$< O >_A = -\frac{\delta}{\delta J}\ln < Z[J] >_{dis}$$



So the pair is coming from $|p, \uparrow\rangle \otimes |-p, \uparrow\rangle$ which is a triplet. So triplet superconductor is possible. While the singlet is possible only for the discrete 4 points. It is similar to the superconductor out of spineless fermions. The point of the chiral superconductor.