15 July 2025

Beijing

The Unique SYK-rised Model to Strange Metals

王依力（Yi-Li Wang）

Hanyang University

- Sang-Jin Sin, Y.-L. W.,   arXiv: 2507.09442
- Y.-L. W., Young-Kwon Han, Xian-Hui Ge, Sang-Jin Sin, Phys. Rev. B 112 (2025) 3, 035121
- Y.-L. W., Xian-Hui Ge, Sang-Jin Sin, Phys.Rev.B 111 (2025) 11, 115135

Content

Part I

Introduction

Background and Motivation

Strange Metal

Violates various features of Landau Fermi liquids.

- Normal phase of high-temperature superconductivity.
- Metal phase of strongly interacting systems.

No theoretical framework

1. Introduction

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ARTICLE

Universal theory of strange metals from spatially random interactions

View article page

Aavishkar A. Patel, Haoyu Guo, Ilya Esterlis, and Subir Sachdev

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SYK-rised Coupling

$$g_{ijl}(r)\psi_i^\dagger(\tau,r)\psi_j(\tau,r)\phi_l(\tau,r)$$

$$\rho = \rho_0 + AT$$

$$\begin{cases} \langle g_{ijl}(\mathbf{r})\rangle = 0 \\ \langle g^*_{ijl}(\mathbf{r})g_{i'j'l'}(\mathbf{r}')\rangle = g^2\delta_{ii'jj'll'}\delta(\mathbf{r}-\mathbf{r}') \end{cases}$$

Reference

- A. A. Patel, H. Guo, I. Esterlis, and S. Sachdev, Science381,790-793(2023).

1. Introduction

"UNIVERSAL" theory

Can we generalise the theory to find other approaches to linear resistivity?

Random couplings between Fermi surface and higher-rank vectors or tensors

Random couplings among multiple fermions and critical bosons

1. Introduction

Vector Coupling

$$S = \int d\tau d^2 r \Big[\sum_{i=1}^{N} \psi_i^\dagger(r,\tau) \left(\partial_\tau - \frac{\nabla^2}{2m} - \mu \right) \psi_i(r,\tau)$$

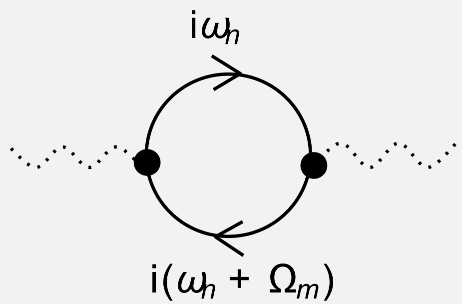
$$+ \sum_{i,j=1}^{N} v_{ij}(r) \psi^\dagger(r,\tau)\psi_i(r,\tau) - \frac{1}{2K^2} \sum_{l=1}^{N} g_{ab} a_l^a \left(-\partial_\tau^2 + \mathbf{q}^2\right) a_l^b$$

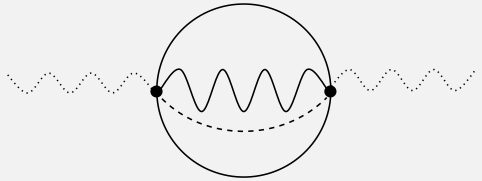
$$+ \sum_{i,j,l=1}^{N} \frac{K_{ijl}(r)}{KN} \frac{\mathrm{i}}{m} \psi_i^\dagger \nabla_a \psi_j \mathbf{a}_l^a + \frac{1}{K^2 N^{3/2}} \sum_{i,j,s,t=1}^{N} \frac{\tilde{K}_{ijst}(r)}{2m} \mathbf{a}_s \cdot \mathbf{a}_t \psi_i^\dagger \psi_j \Big]$$

$$\langle K_{ijl}(r) \rangle = 0, \quad \langle K_{ijl}^*(r) K_{i'j'l'}(r') \rangle = K^2 \delta_{ii'jj'll'} \delta(r - r')$$

$$\langle \tilde{K}_{ijl}(r) \rangle = 0, \quad \langle \tilde{K}_{ijl}^*(r) \tilde{K}_{i'j'l'}(r') \rangle = \tilde{K}^2 \delta_{ii'jj'll'} \delta(r - r')$$

$$\langle v_{ij}(r) \rangle = 0, \quad \langle v_{ij}(r) v_{i'j'}(r') \rangle = \delta_{ii',jj'} \delta(r - r')$$





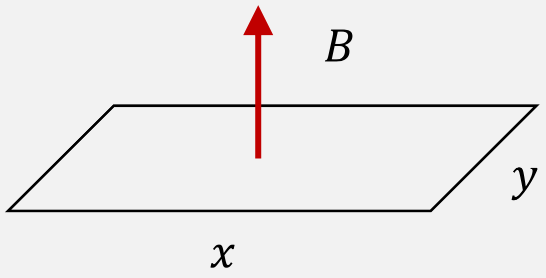
Linear Resistivity

Reference

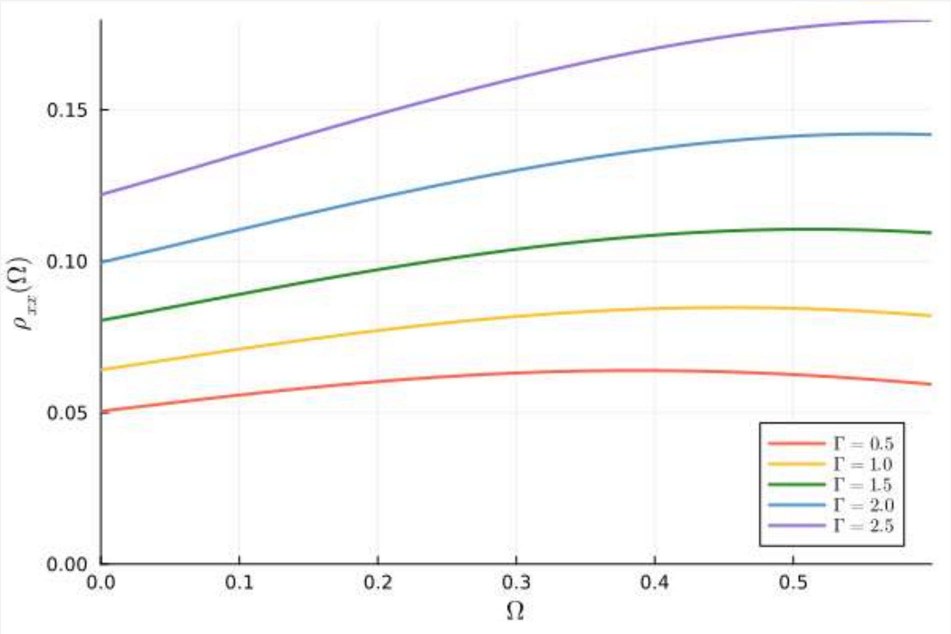
- Y.-L. W., Xian-Hui Ge, Sang-Jin Sin, Phys.Rev.B 111 (2025) 11, 115135

Vector Coupling

Linearity is preserved in the presence of a magnetic field.



In (2+1)D, both scalar and vector couplings lead to resistivity linear in temperature.



References

- Y.-L. W., Young-Kwon Han, Xian-Hui Ge, Sang-Jin Sin, Phys. Rev. B 112 (2025) 3, 035121

1. Introduction

"UNIVERSAL" theory

Can we generalise the theory to find other approaches to linear resistivity?

√

Random couplings between Fermi surface and higher-rank vectors or tensors

?

Random couplings among multiple fermions and critical bosons

Reference

- Sang-Jin Sin, Y.-L. W., arXiv: 2507.09442

Part II

Generalised Model

Large-N Critical Theory

2. Generalised Model

Mutiple-Field Coupling in any Dimensions

$$S_g = \frac{g_{\{i\}\{j\}\{l\}}(r)}{N^{(2n+m-1)/2}} \int d\tau d^d r \sum_{\{i\},\{j\},\{l\}} \psi^\dagger_{i_1,k}(r,\tau)\ldots\psi^\dagger_{i_n,k}(r,\tau)$$
$$\times\ \psi_{j_1,k+q}(r,\tau)\ldots\psi_{j_n,k+q}(r,\tau)\phi_{l_1,q}(r,\tau)\ldots\phi_{l_m,q}(r,\tau)$$

$$\langle g_{\{i\}\{j\}\{l\}}(r)\rangle = 0,$$
$$\langle g_{\{i\}\{j\}\{l\}} g^*_{\{i'\}\{j'\}\{l'\}}(r')\rangle = g^2\delta(r-r')\delta_{\{i\}\{i'\},\{j\}\{j'\},\{l\}\{l'\}},$$
$$\delta_{\{i\}\{i'\}} \equiv \delta_{i_1 i'_1}\delta_{i_2 i'_2}\ldots\delta_{i_n i'_n}.$$

Find n,m,d

that result in

linear

resistivity

2. Generalised Model

G-Sigma formalism

$$\bar{\delta} = \delta(r - r')$$

$$\begin{aligned}\frac{S[G,\Sigma;D,\Pi]}{N} = & -\ln\det(\partial_\tau + \varepsilon(k)\delta(x-x') - \mu + \Sigma) \\ & + \frac{1}{2}\ln\det\left((-\partial_\tau^2 + q^2 + m_b^2)\delta(x-x') - \Pi\right) \\ & - \mathrm{Tr}\,(\Sigma \cdot G) + \frac{1}{2}\mathrm{Tr}\,(\Pi \cdot D) + \frac{v^2}{2}\mathrm{Tr}\left((G\bar{\delta}) \cdot G\right) \\ & - \frac{g^2}{2}\mathrm{Tr}\left((G^n D^m \bar{\delta}) \cdot (-G)^n\right)\end{aligned}$$

Saddle point

$$\begin{aligned}G_*[\Sigma](x_1,x_2) &= (-\partial_\tau - \varepsilon(k) + \mu - \Sigma)^{-1}(x_1,x_2) \\ \Sigma_*[G](x_1,x_2) &= ng^2 G^{2n-1}(x_1,x_2) D^m(x_1,x_2)\bar{\delta} + v^2 G(x_1,x_2)\bar{\delta} \\ D_*[\Pi](x_1,x_2) &= (-\partial_\tau^2 + q^2 + m_b^2 - \Pi)^{-1}(x_1,x_2) \\ \Pi_*[D](x_1,x_2) &= mg^2 G^{2n}(x_1,x_2) D^{m-1}(x_1,x_2)\bar{\delta}\end{aligned}$$

2. Generalised Model

Critical point

$$m_b^2 - \Pi(0,0) = 0$$
$$\Pi(i\Omega_m) - \Pi(0) \sim -c_B|\Omega_m|^{\eta'}$$

- $\eta' < 2$ $\quad D(i\Omega_m, q) = \frac{1}{\Omega_m^2 + q^2 + c_B|\Omega_m|^{\eta'}} \simeq \frac{1}{q^2 + c_B|\Omega_m|^{\eta'}}$
- $\eta' \geq 2$ $\quad D(i\Omega_m, q) = \frac{1}{\Omega_m^2 + q^2 + c_B|\Omega_m|^{\eta'}} \simeq \frac{1}{q^2 + c'_B|\Omega_m|^{2}}$

$$\Rightarrow \quad D(i\Omega_m, q) \simeq \frac{1}{q^2 + c_B|\Omega_m|^{\eta}}, \quad \eta \leq 2$$

2. Generalised Model

Potential Disorder

$$\Sigma_v(\Omega_m) = v^2 \int \frac{d^d k}{(2\pi)^2} G(i\Omega_m, k) = -i\frac{\Gamma}{2}\text{sgn}(\Omega_m),$$

Fermionic Propagator

$$G(i\omega, k) \simeq \frac{1}{i\text{sgn}(\omega)\Gamma/2 - k^2/(2m) + \mu}$$

Critical Point

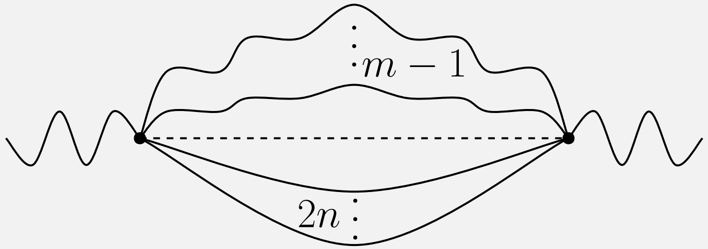
$$m_b^2 - \Pi(0,0) = 0$$

Bosonic Propagator

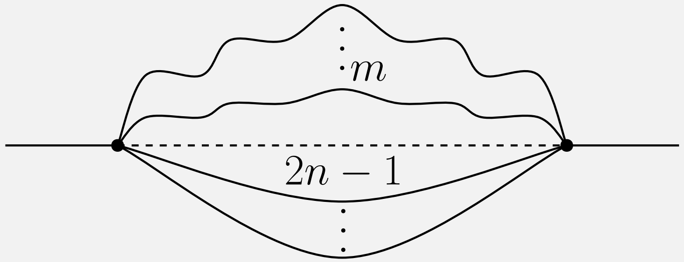
$$D(i\Omega_m, q) \simeq \frac{1}{q^2 + c_B|\Omega_m|^\eta}, \quad \eta \leq 2$$

2. Generalised Model

bosonic self-energy



fermionic self-energy



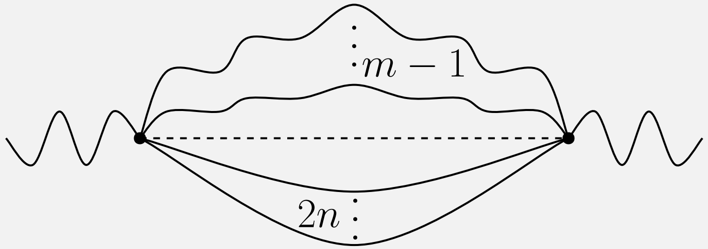
$$\mathcal{I}^d_{\alpha,\beta}(ix) \equiv \int_{-\infty}^{\infty}\left(\prod_{i=1}^{\alpha}\frac{d\omega_i}{2\pi}d\xi_{k_i}\frac{1}{i\mathrm{sgn}(\omega_i)\Gamma/2-\xi_{k_i}}\right)$$

$$\times\left(\int_{-\infty}^{\infty}\prod_{j=1}^{\beta-1}\frac{d\Omega_j}{2\pi}\int_0^{\infty}\frac{d^dq_j}{(2\pi)^2}\frac{1}{c_B|\Omega_j|^\eta+q_j^2}\right)\int_0^{\infty}\frac{d^dq_\beta}{(2\pi)^2}\frac{1}{c_B|x+\sum_{j=1}^{\beta-1}\Omega_j+\sum_{i=1}^{2n}\omega_i|^\eta+q_\beta^2}$$

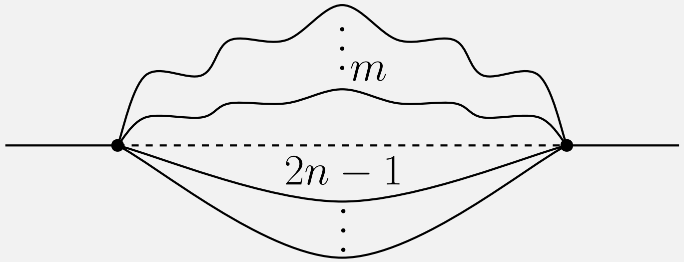
$$\sim(-i)^\alpha\int_{-\infty}^{\infty}\left(\prod_{j=1}^{\beta-1}d\Omega_j|\Omega_j|^{\frac{\eta(d-2)}{2}}\right)\left|x+\sum_{j=1}^{\beta-1}\Omega_j\right|^{\frac{\eta(d-2)}{2}+\alpha}$$

2. Generalised Model

bosonic self-energy



fermionic self-energy



$$D(i\Omega_m, q) \simeq \frac{1}{q^2 + c_B|\Omega_m|^\eta}, \quad \eta \leq 2$$

$$\Pi(i\Omega) - \Pi(0) \sim -c_B g^2 \Omega^{2n+m-2+\frac{\eta(d-2)(m-1)}{2}}$$

$$\Sigma(i\omega) \sim -ic_F g^2 \omega^{2n+m-2+\frac{\eta(d-2)m}{2}}$$

2. Generalised Model

$$\Pi(i\Omega) - \Pi(0) \sim \; -c_B g^2 \Omega^{2n+m-2+\frac{\eta(d-2)(m-1)}{2}}$$

$$\Sigma(i\omega) \sim \; -ic_F g^2 \omega^{2n+m-2+\frac{\eta(d-2)m}{2}}$$

- $\eta < 2$ $\quad$ $$2n + m - 2 + \frac{\eta(d-2)}{2} + \frac{\eta(d-2)(m-2)}{2} = \eta$$

$$\Rightarrow \quad \eta = -\frac{2(m+2n-2)}{dm - d - 2m}$$

$$\Rightarrow \quad \Pi(i\Omega) - \Pi(0) \sim \; -c_B g^2 \Omega^{-\frac{2(m+2n-2)}{d(m-1)-2m}}$$

$$\Sigma(i\omega) \sim \; -ic_F g^2 \omega^{-\frac{d(m+2n-2)}{d(m-1)-2m}}$$

2. Generalised Model

$$\Pi(i\Omega) - \Pi(0) \sim -c_B g^2 \Omega^{2n+m-2+\frac{\eta(d-2)(m-1)}{2}}$$
$$\Sigma(i\omega) \sim -ic_F g^2 \omega^{2n+m-2+\frac{\eta(d-2)m}{2}}$$

- $\eta = 2$

$$\Pi(i\Omega) - \Pi(0) \sim -c_B g^2 \Omega^{2n+m-2+(d-2)(m-1)}$$
$$\Sigma(i\omega) \sim -ic_F g^2 \omega^{2n+m-2+(d-2)m}$$

2. Generalised Model

- $\eta < 2$

$$\Pi(i\Omega) - \Pi(0) \sim \; - c_B g^2 \Omega^{-\frac{2(m+2n-2)}{d(m-1)-2m}}$$
$$\Sigma(i\omega) \sim \; - i c_F g^2 \omega^{-\frac{d(m+2n-2)}{d(m-1)-2m}}$$

- $\eta = 2$

$$\Pi(i\Omega) - \Pi(0) \sim \; - c_B g^2 \Omega^{2n+m-2+(d-2)(m-1)}$$
$$\Sigma(i\omega) \sim \; - i c_F g^2 \omega^{2n+m-2+(d-2)m}$$

$$\Sigma(i\omega) \equiv -i c_F g^2 \omega^{\varsigma}$$

Part III

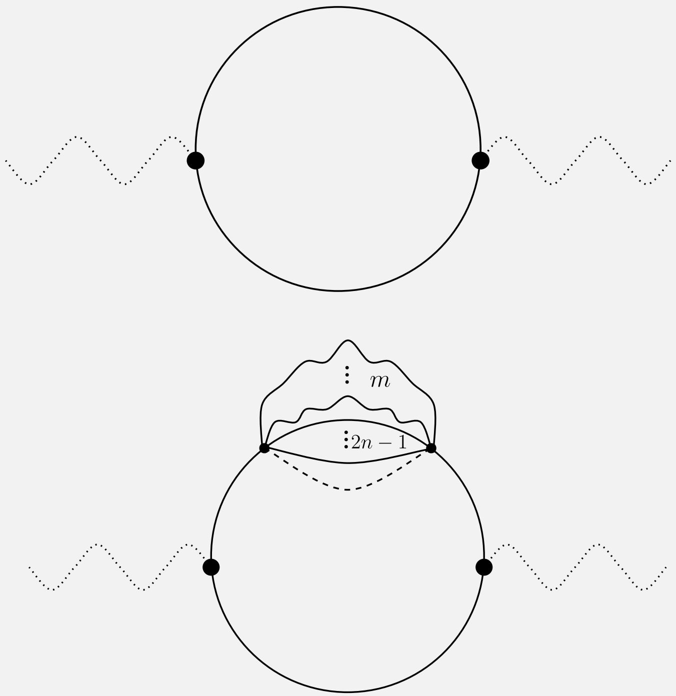
Conductivity

Emergence of Linearity

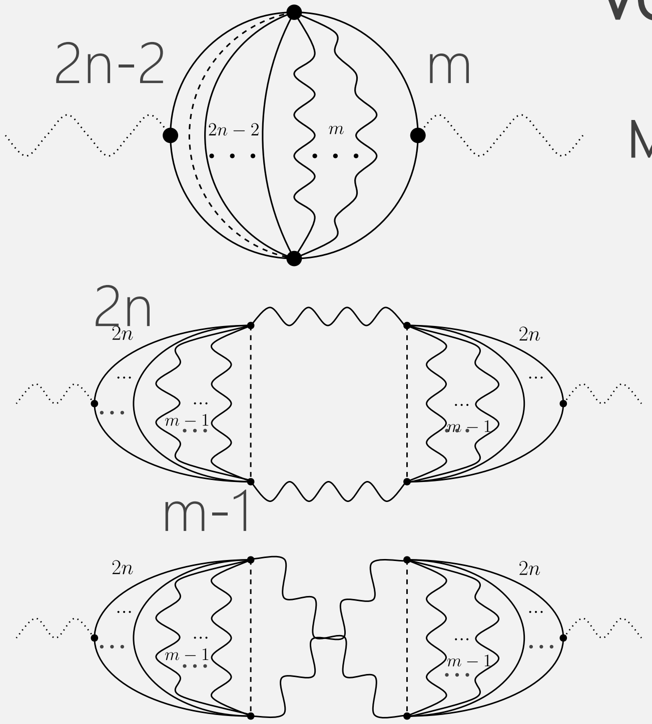
3. Conductivity

Kubo Formula

$$\sigma^{\mu\nu}(i\Omega_m) = -\frac{1}{\Omega_m}\left[\tilde{\Pi}^{\mu\nu}(\Omega)\right]_{\Omega=0}^{\Omega=i\Omega_m}$$



Vertex Corrections



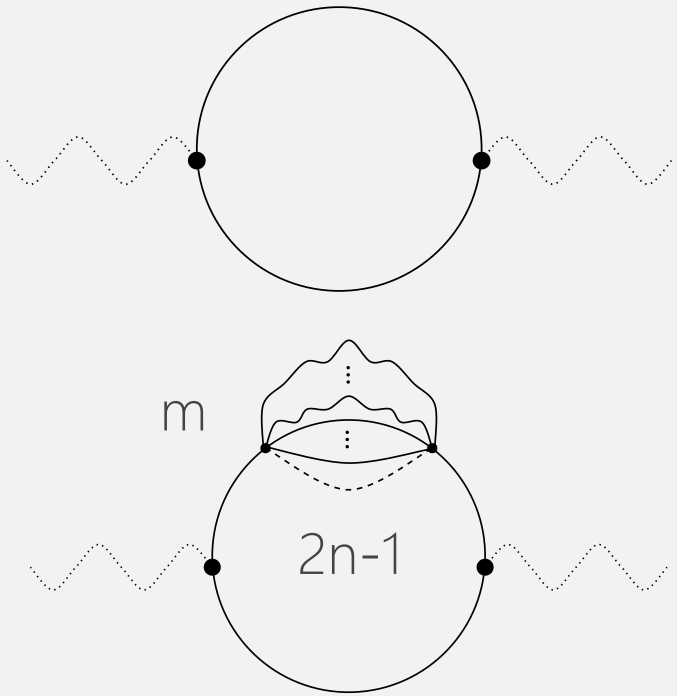
Maki-Thompson diagram

Aslamazov-Larkin diagram

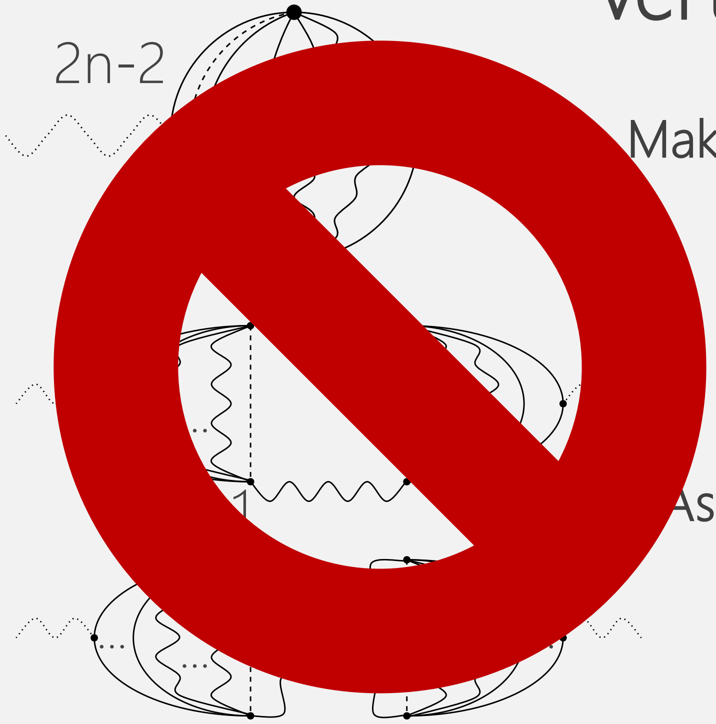
3. Conductivity

Kubo Formula

$$\sigma^{\mu\nu}(i\Omega_m) = -\frac{1}{\Omega_m}\left[\tilde{\Pi}^{\mu\nu}(\Omega)\right]_{\Omega=0}^{\Omega=i\Omega_m}$$



Vertex Corrections



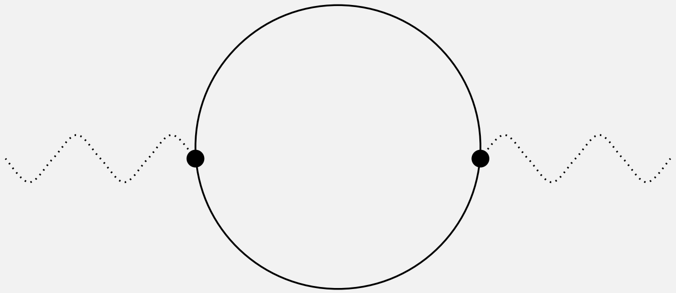
Maki-Thompson diagram

Aslamazov-Larkin diagram

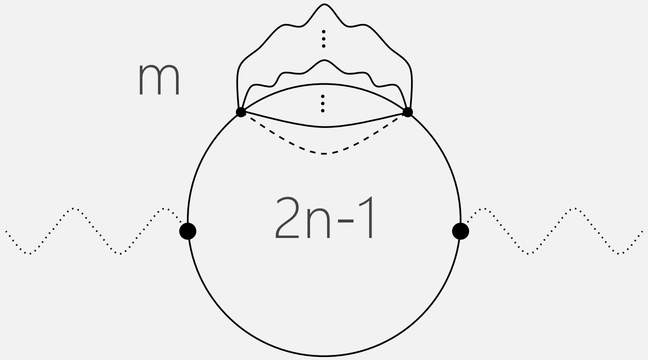
3. Conductivity

Kubo Formula

$$\sigma^{\mu\nu}(i\Omega_m) = -\frac{1}{\Omega_m}\left[\tilde{\Pi}^{\mu\nu}(\Omega)\right]_{\Omega=0}^{\Omega=i\Omega_m}$$



$$\tilde{\Pi}_0^{\mu\nu}(i\Omega_m) \simeq -v_F^2\mathcal{N}\delta^{\mu\nu}\frac{\Omega_m}{\Omega_m + \mathrm{sgn}(\Omega_m)\Gamma},$$



$$\Sigma(i\omega) \equiv -ic_F g^2\omega^{\varsigma}$$

$$\tilde{\Pi}_g^{\mu\nu}(i\Omega) = -\frac{2}{m^2}\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}\int_{-\infty}^{\infty}\frac{d^2k}{(2\pi)^2}k^\mu k^\nu G(i\omega_n)\Sigma(i\omega)G(i\omega)G(i(\omega+\Omega))$$

$$\sim g^2\delta^{\mu\nu}\Omega^{\varsigma+1}$$

3. Conductivity

Kubo Formula

$$\sigma^{\mu\nu}(i\Omega_m) = -\frac{1}{\Omega_m}\left[\tilde{\Pi}^{\mu\nu}(\Omega)\right]_{\Omega=0}^{\Omega=i\Omega_m}$$

⬇

resistivity

$$\Re\rho = \Re\frac{1}{\sigma} \sim \frac{2\Gamma}{\mathcal{N}v_F^2} + g^2 c_g' \Omega^\varsigma$$

notes

$$\Pi(i\Omega_m) - \Pi(0) \sim -c_B|\Omega_m|^{\eta'}$$

$$D(i\Omega_m, q) \simeq \frac{1}{q^2 + c_B|\Omega_m|^\eta}, \quad \eta \leq 2$$

$$\eta = 2 \begin{cases} \varsigma = 2n + m - 2 + (d-2)m = 1 \\ \eta' = 2n + m - 2 + (d-2)(m-1) > 2 \end{cases}$$

➡ no solution

3. Conductivity

Kubo Formula

$$\sigma^{\mu\nu}(i\Omega_m) = -\frac{1}{\Omega_m}\left[\tilde{\Pi}^{\mu\nu}(\Omega)\right]_{\Omega=0}^{\Omega=i\Omega_m}$$

↓

resistivity

$$\Re\rho = \Re\frac{1}{\sigma} \sim \frac{2\Gamma}{\mathcal{N}v_F^2} + g^2 c_g' \Omega^{\varsigma}$$

notes

$$\Pi(i\Omega_m) - \Pi(0) \sim -c_B|\Omega_m|^{\eta'}$$

$$D(i\Omega_m, q) \simeq \frac{1}{q^2 + c_B|\Omega_m|^{\eta}}, \quad \eta \leq 2$$

- $\eta < 2$

$$\begin{cases} \varsigma = -\dfrac{d(m+2n-2)}{d(m-1)-2m} = 1 \\ \eta' = -\dfrac{2(m+2n-2)}{d(m-1)-2m} < 2 \end{cases}$$

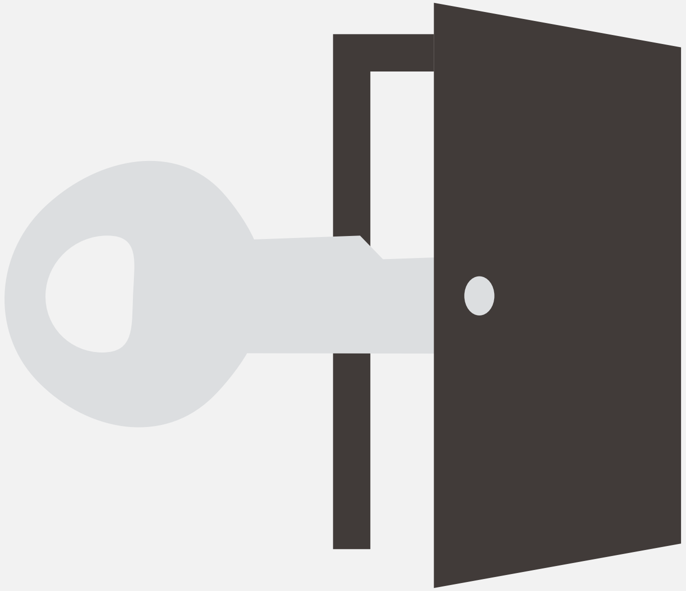
→ $n = m = 1, \quad d = 2$

Part IV

Conclusion

Discussion and Outlooks

4. Conclusion



- Among all interactions considered here, Yukawa-type couplings in 2d is the only one leading to linear resistivity.
- Yukawa-type and QED-type interactions are the minimal building blocks for linear-T resistivity.
- More transport properties to be explored, e.g. magnetoresistance.

Thank You

王依力（Yi-Li Wang）
Hanyang University