Holographic applications: from Quantum Realms to the Big Bang

Machine Learning Holographic CMT

2025.07.15

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光州

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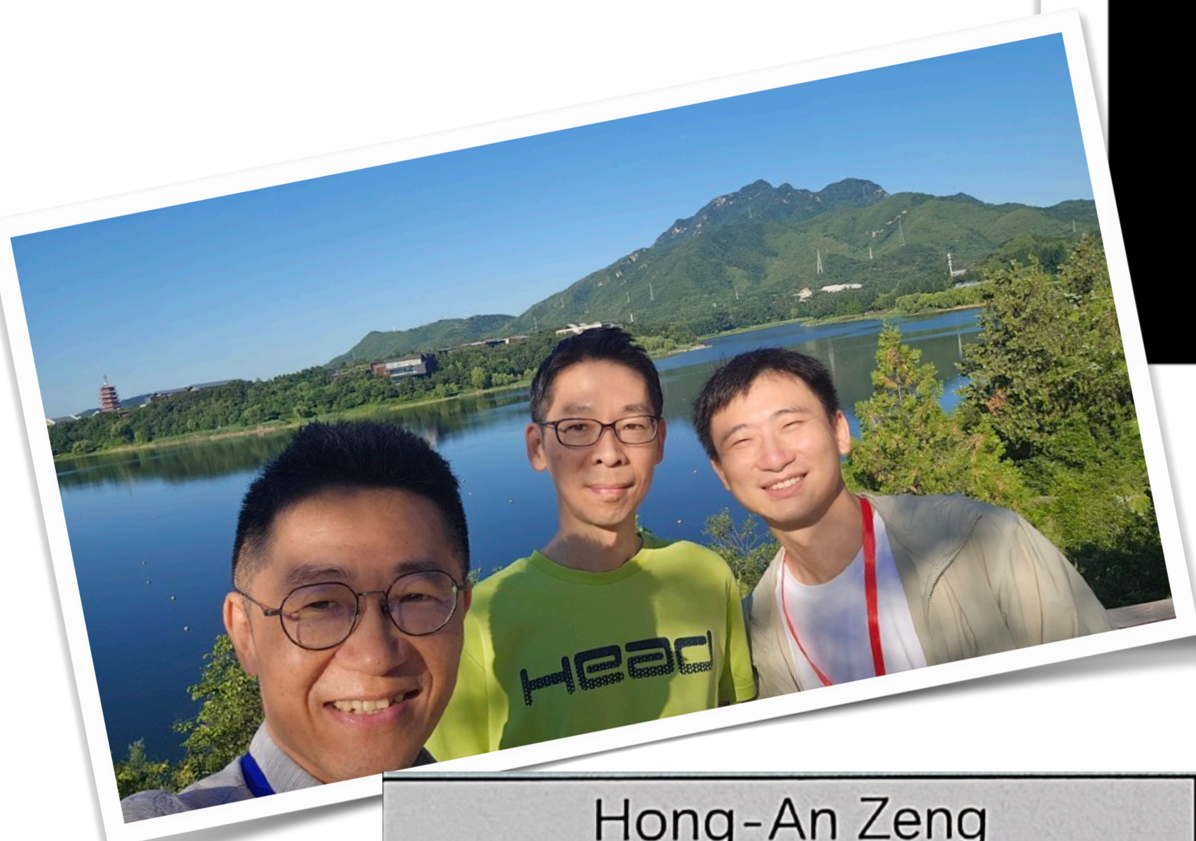
Invited lectures at "Holographic applications: from Quantum Realms to the Big Bang", 12 Jul. 2025

Machine Learning Holographic QCD

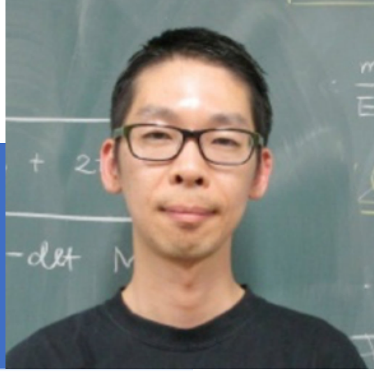
Koji Hashimoto (Kyoto U, MLPhys)



Hong-An Zeng Jilin University	Neural Network for Holographic QCD
Liqiang Zhu Central China Normal Uni.	Bayesian Inference of the Critical Endpoint in 2+1-Flavor System from Holographic QCD
Yiping Si UCAS	Holographic Calculation of Thermal Photon Emission Rate and Electric Conductivity

Machine Learning Holographic CMT

Based on selected review of Koji's lecture

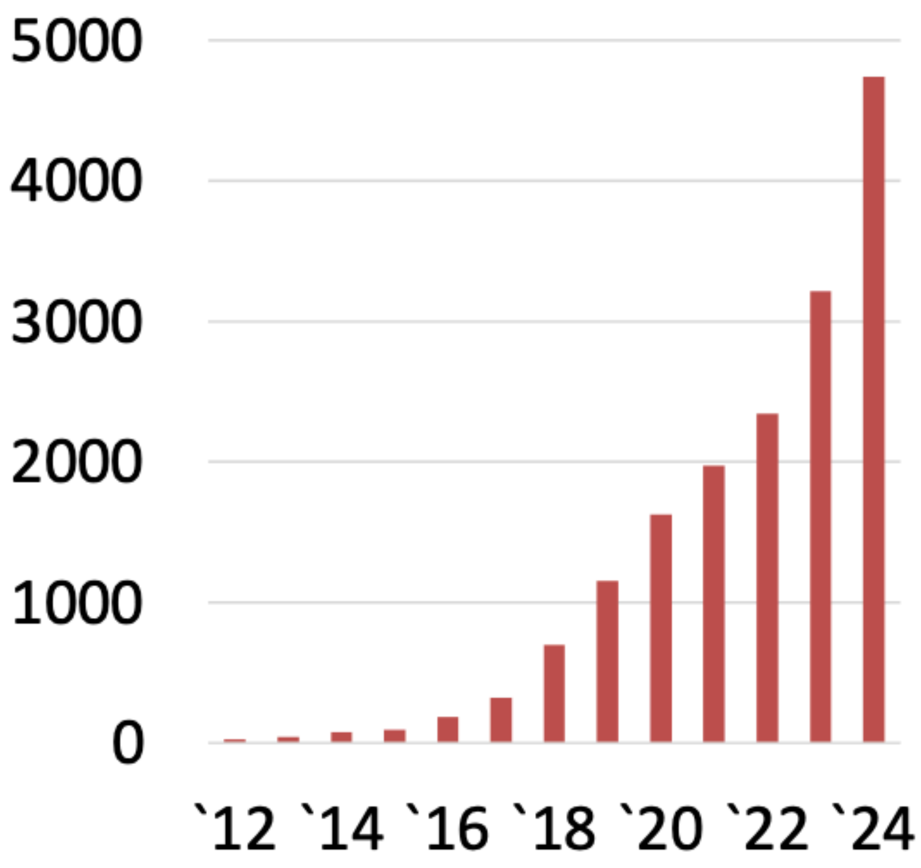


Expanding research field in the world

arXiv papers of ML+Phys

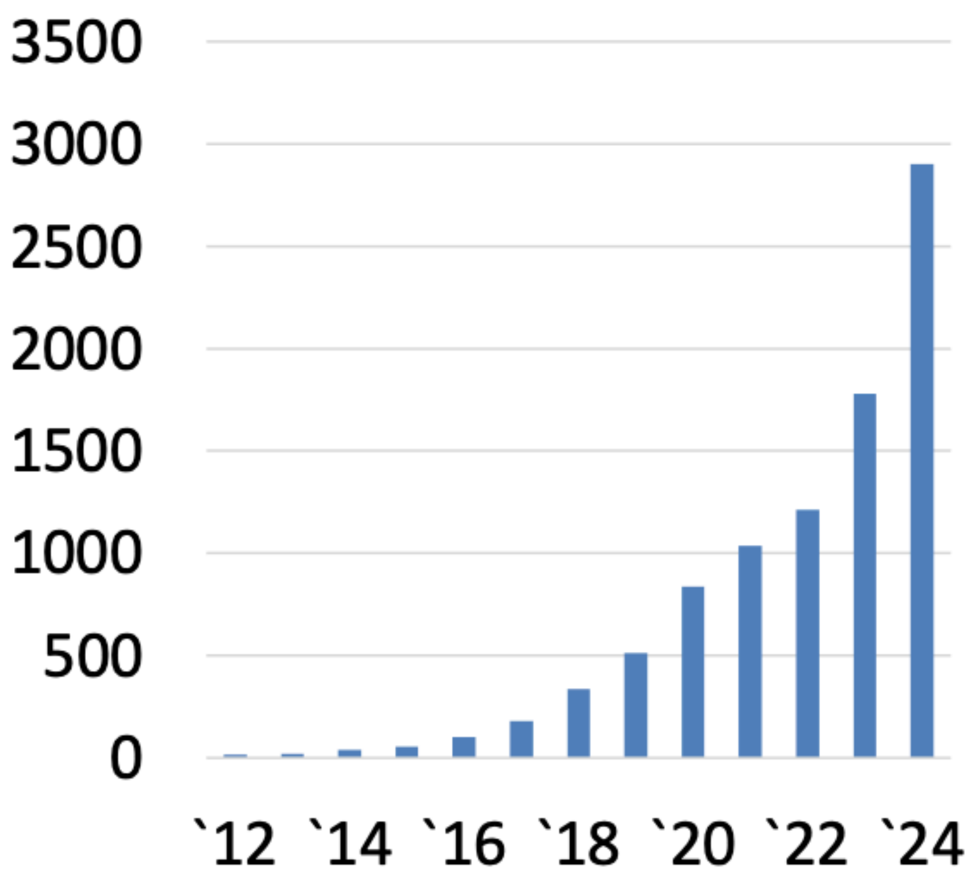
Phys category

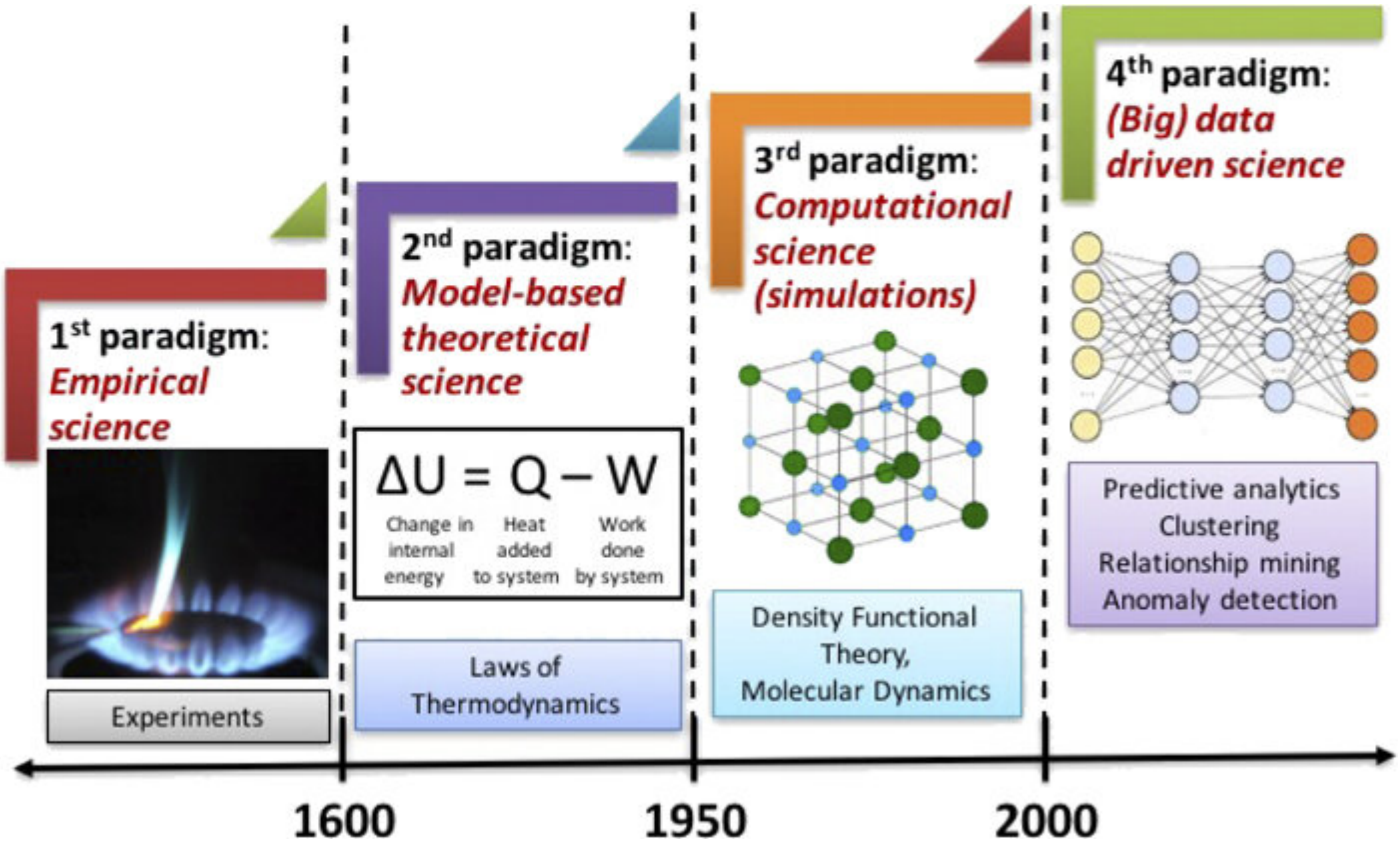
(abstract includes "machine (deep) learning")



CS category

(abstract includes "physics" and "learning")




1st paradigm: Empirical science
Experiments
2nd paradigm: Model-based theoretical science
ΔU = Q – W
Change in internal energy
Heat added to system
Work done by system
Laws of Thermodynamics
3rd paradigm: Computational science (simulations)
Density Functional Theory, Molecular Dynamics
4th paradigm: (Big) data driven science
Predictive analytics
Clustering
Relationship mining
Anomaly detection
1600
1950
2000

MLPhYs 学術変革領域研究(A) 学習物理学の創成
Foundation of "Machine Learning Physics"

Past conferences

- String Data 2023
- String Data 2022
- String Data 2021
- String Data 2020
- String Data 2019
- String Data 2018
- String Data 2017

Physics Reports 839 (2020) 1–117

Contents lists available at ScienceDirect

Physics Reports

journal homepage: www.elsevier.com/locate/physrep

Data science applications to string theory

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Rudolf Peierls Centre for Theoretical Physics, University of Oxford, Department of Physics, Clarendon Laboratory, Parks Road, Oxford OX1 3PU, UK

Machine learning and the physical sciences

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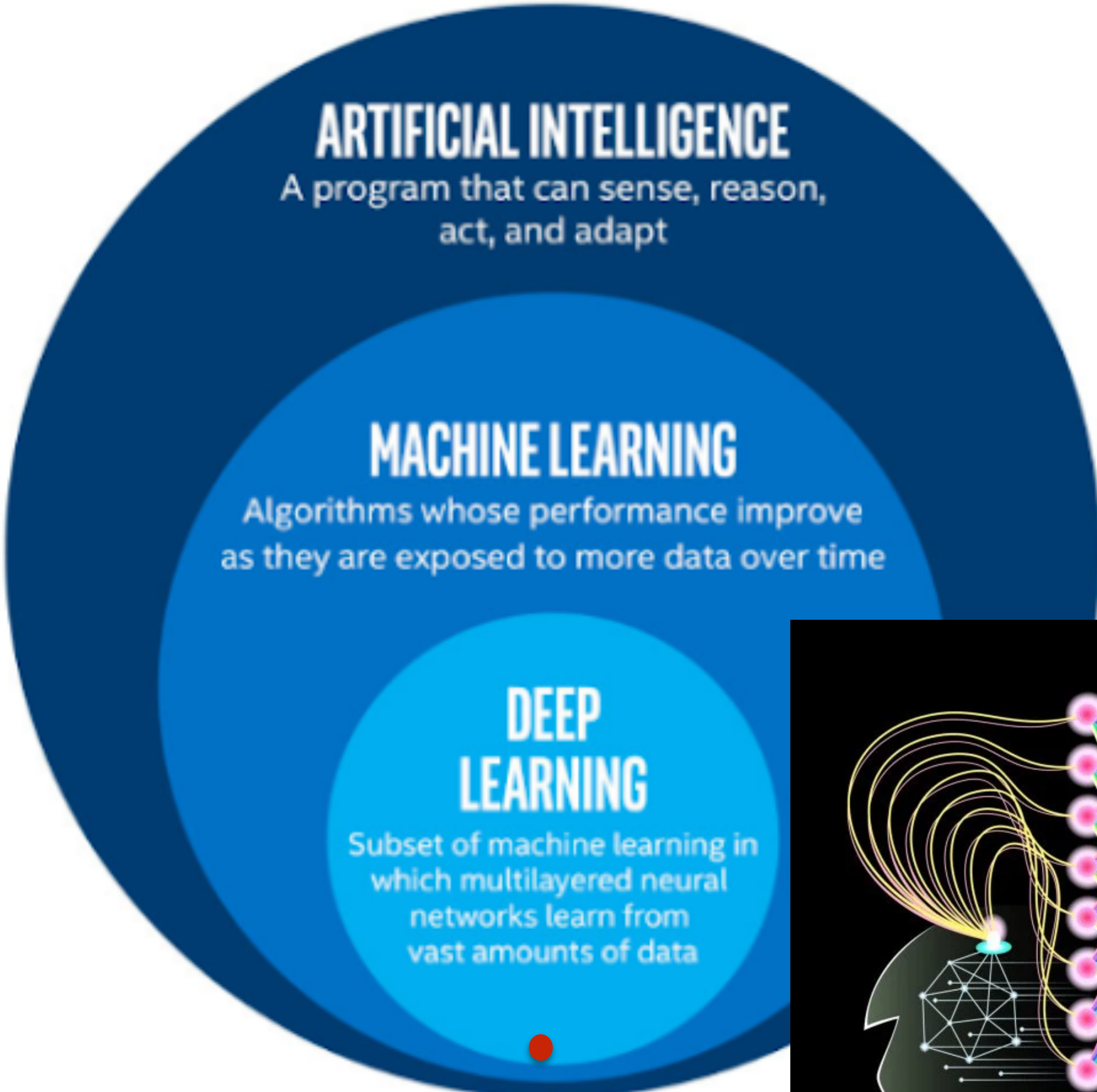
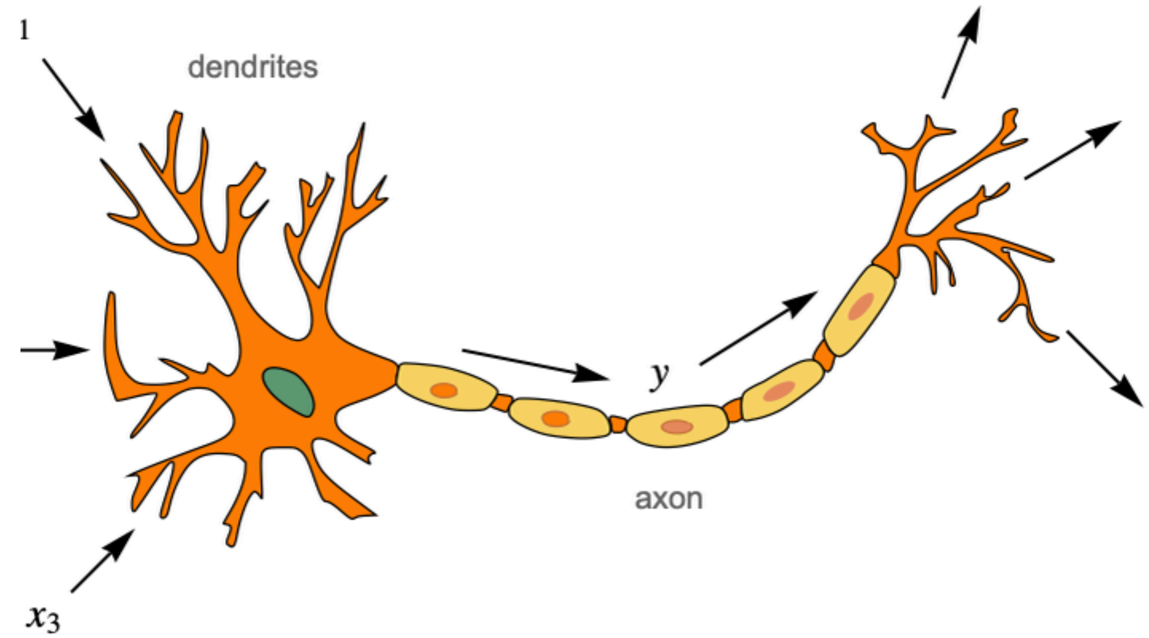
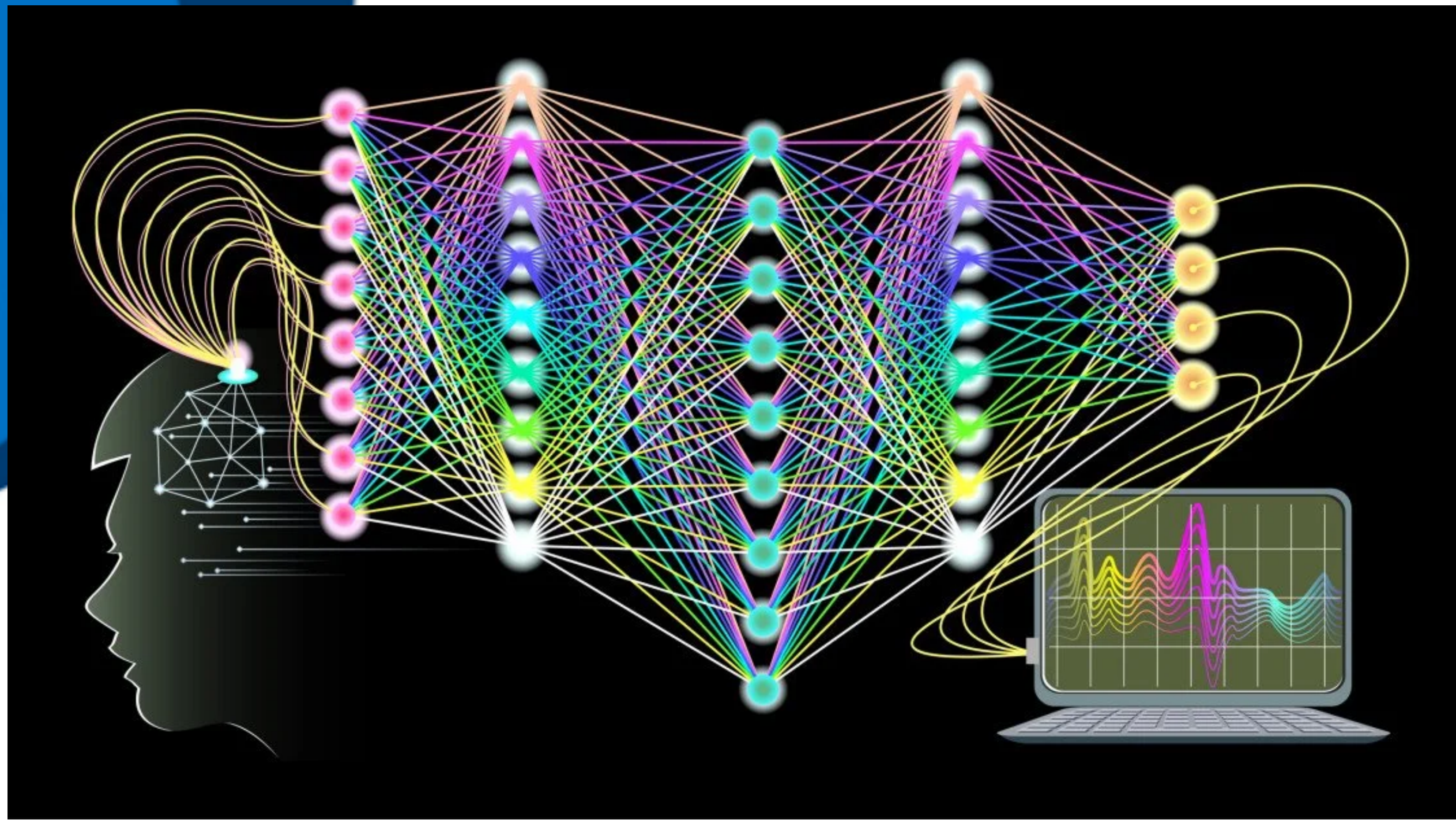
CONTENTS

Deep Learning 101



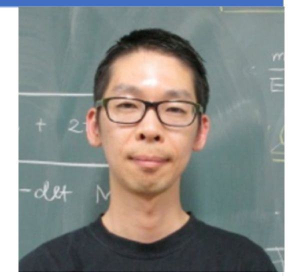
My talk



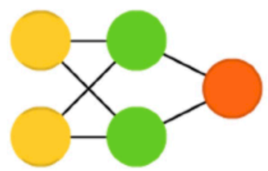


Machine learning = function approximator

Input: a vector $(v_1, v_2, v_3, \ldots)$
Output: a value $f(v_1, v_2, v_3, \ldots)$

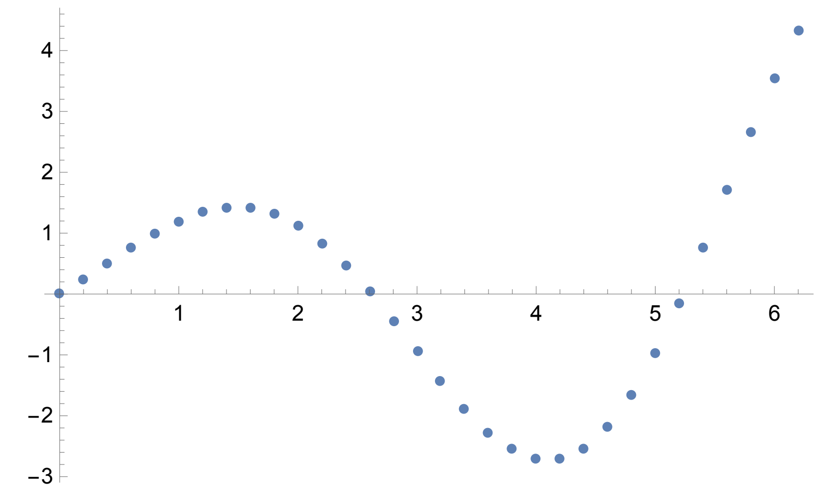


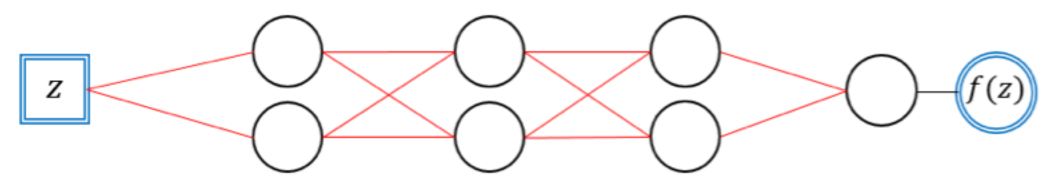
Network architecture = Function ansatz



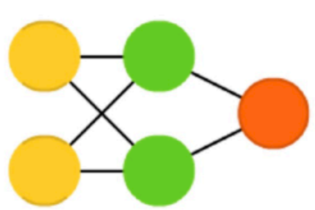
Perceptron model
[Rosenblatt 1958]
[Rumelhart, McClelland 1986]

Universal approximation theorem :
Any function can be approximated with more hidden units [Cybenko 1989] [Roux, Bengio 2008]





Perceptron model



$$f = W_i^{(2)} \varphi\left(W_{ij}^{(1)} x_j\right)$$

“Unit” (circles) : Vector components

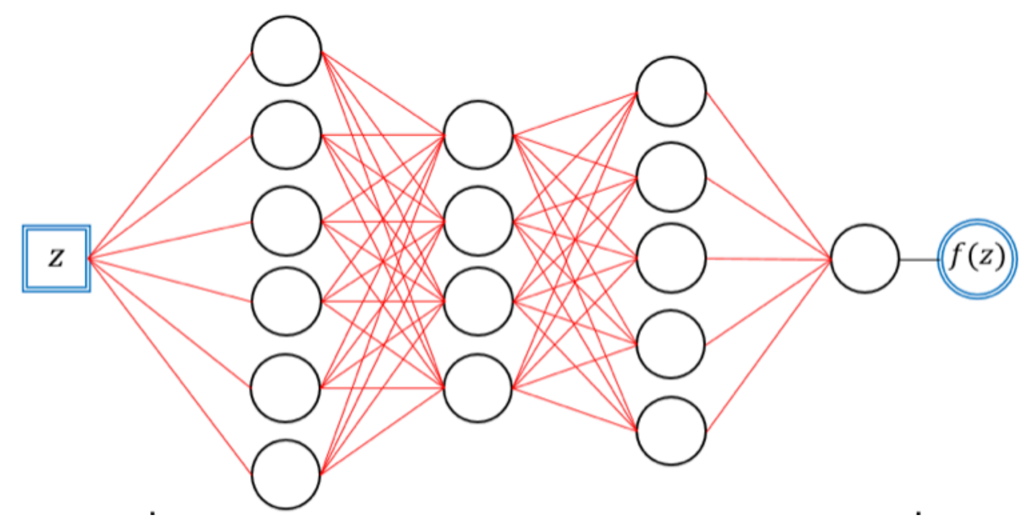
“Weight” (lines) : Linear transformation to be optimized

“Activation function” (hidden line-end) : Nonlinear component-wise transf.

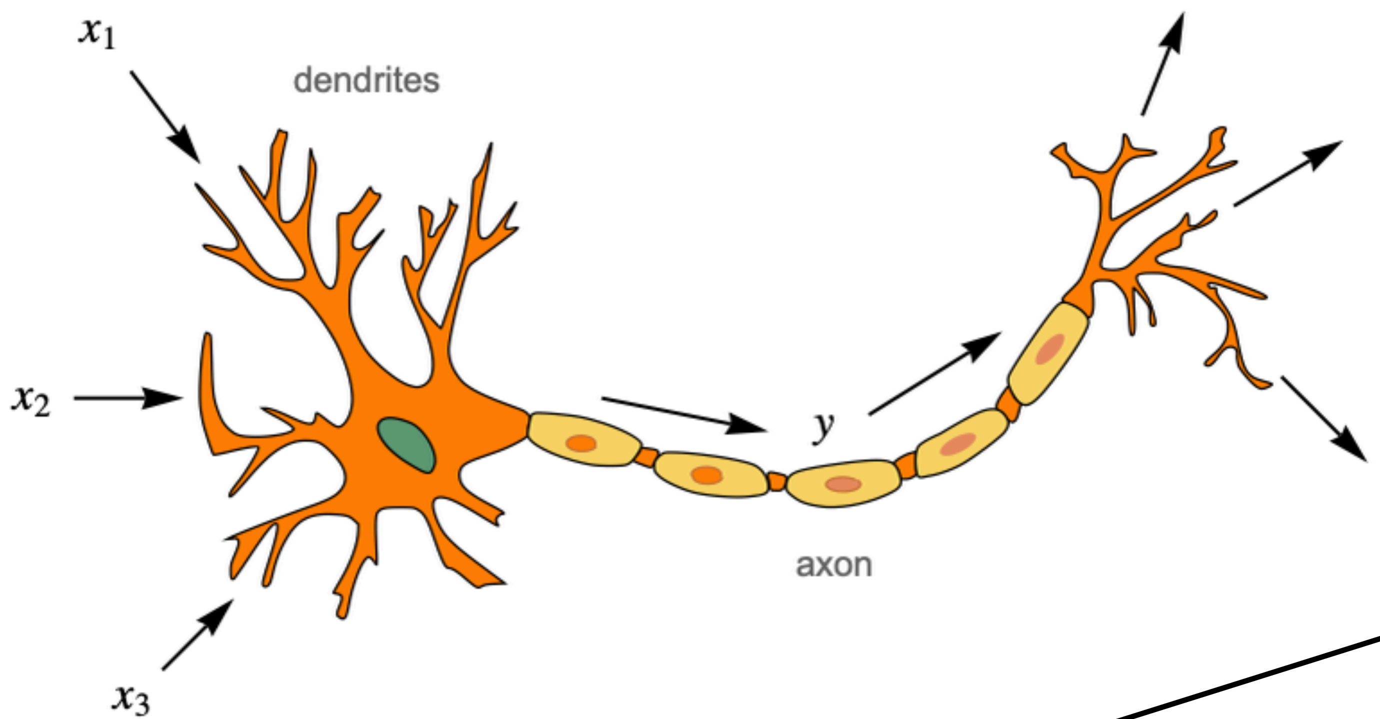
$$\varphi(x) \equiv \frac{1}{1+e^{-x}}$$

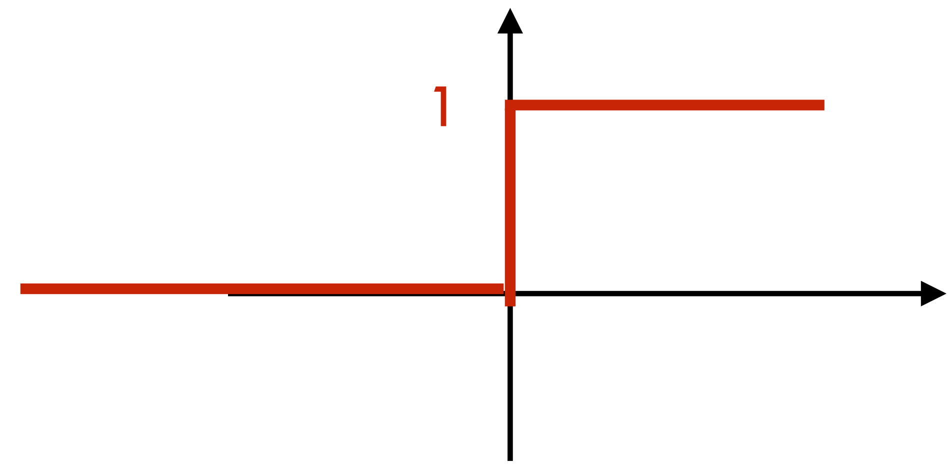
- Training protocol :

1) Prepare many sets $\{(x_j, f)\}$: input + output
2) Train the network (adjust W) by lowering “Loss function” $E \equiv \sum_{\text{data}} \left| f - W_i^{(2)} \varphi\left(W_{ij}^{(1)} x_j\right) \right|$



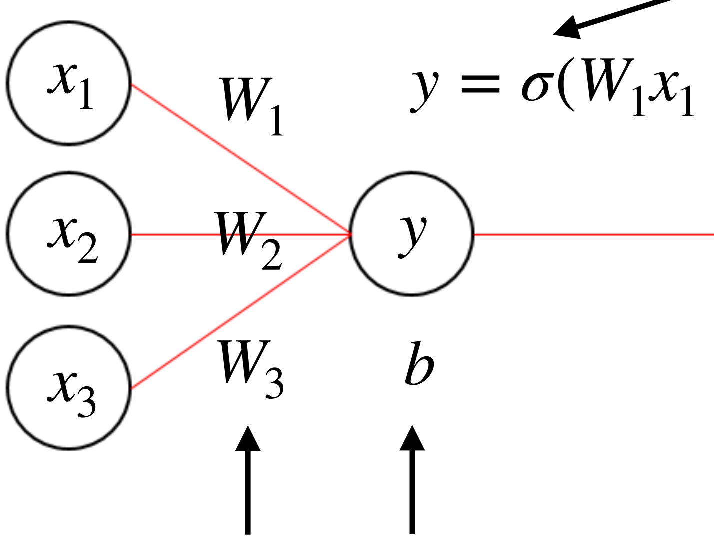
Deep Learning 101



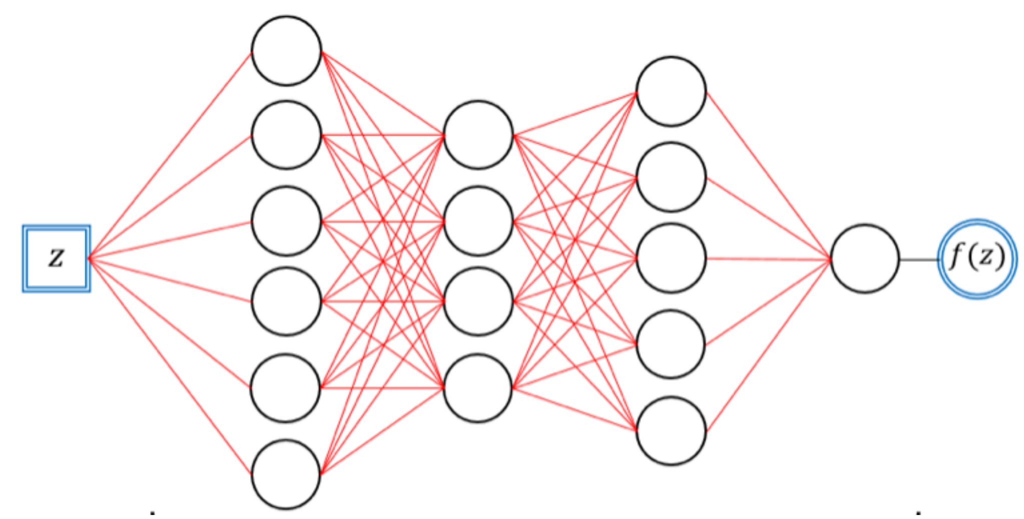


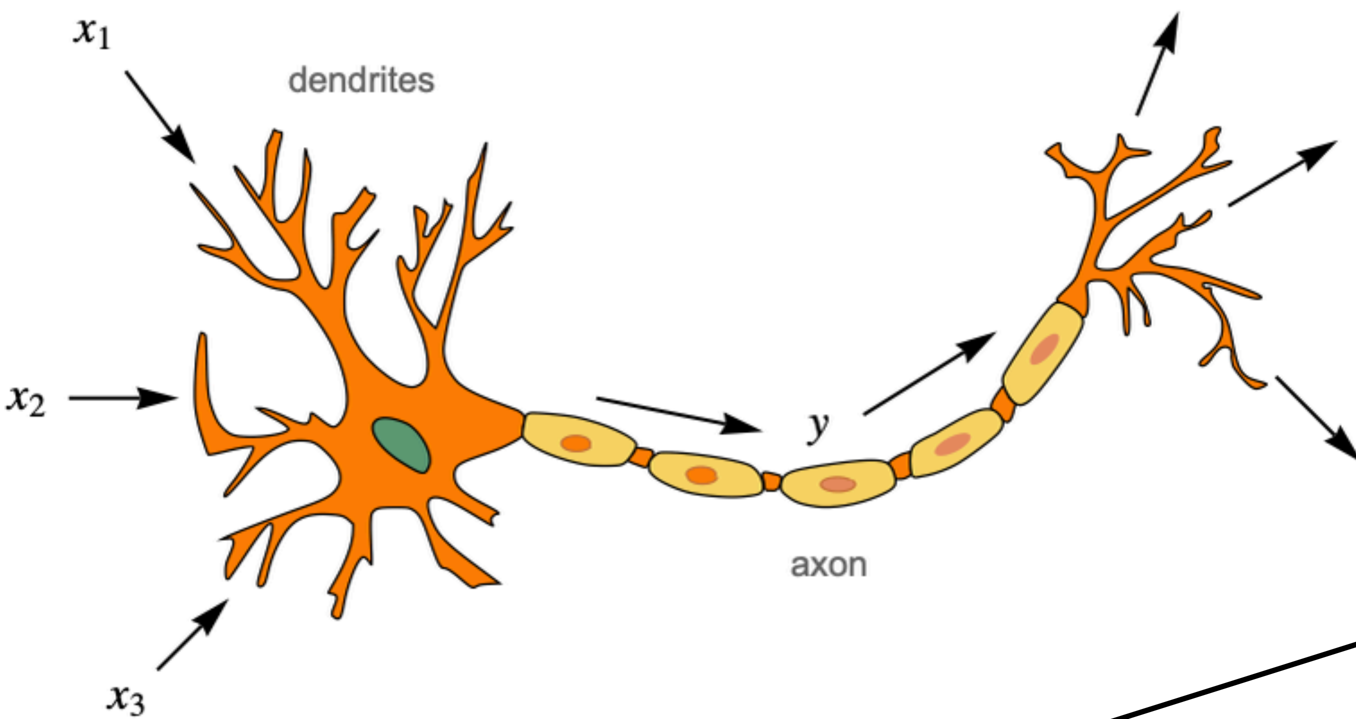
Activation function

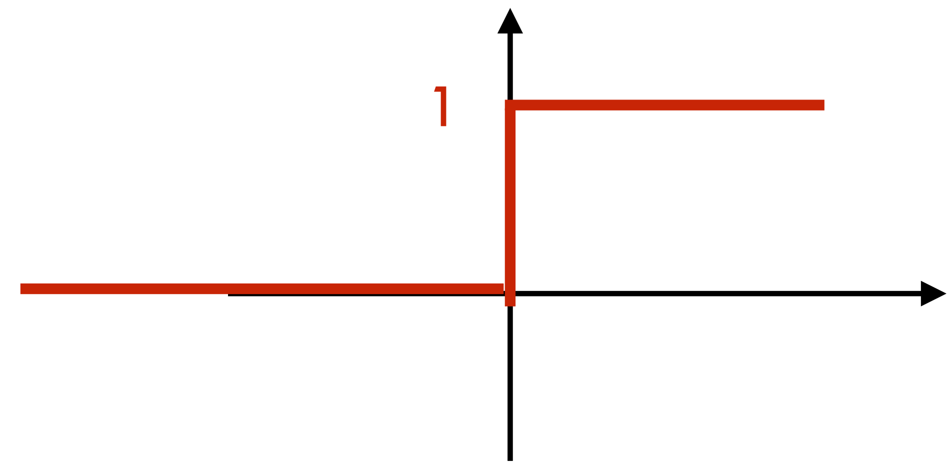
$$y = \sigma(W_1x_1 + W_2x_2 + W_3x_3 - b)$$



Weight Bias

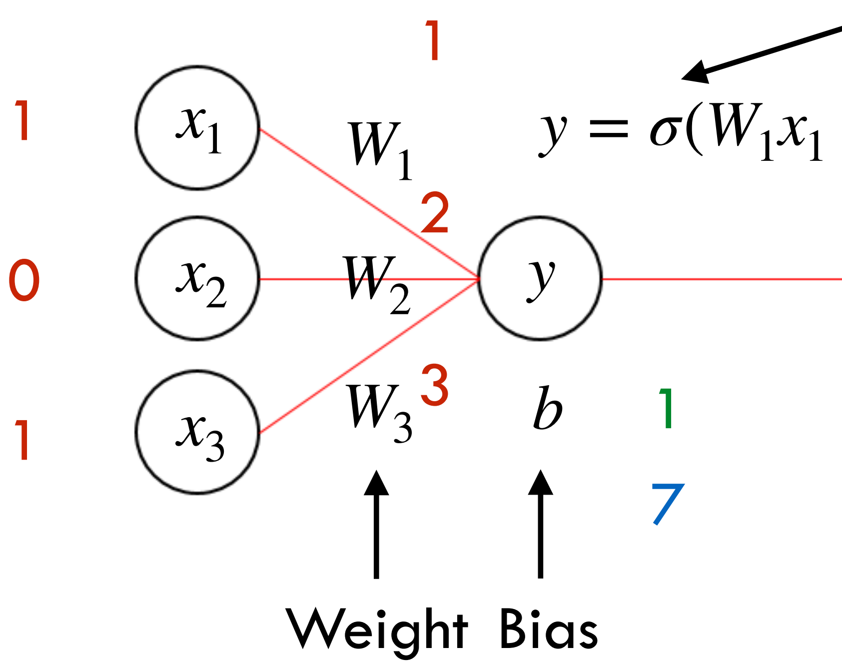






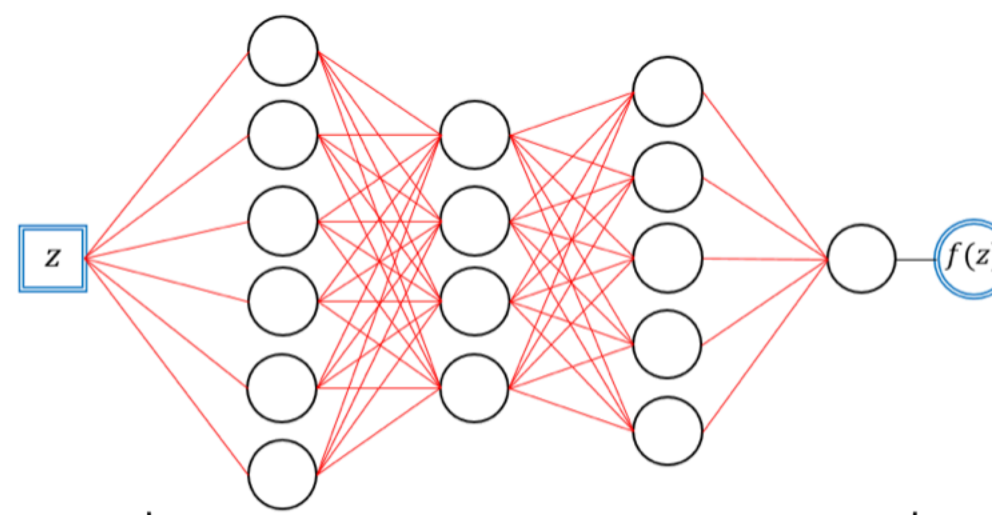
Activation function

$$y = \sigma(W_1x_1 + W_2x_2 + W_3x_3 - b)$$

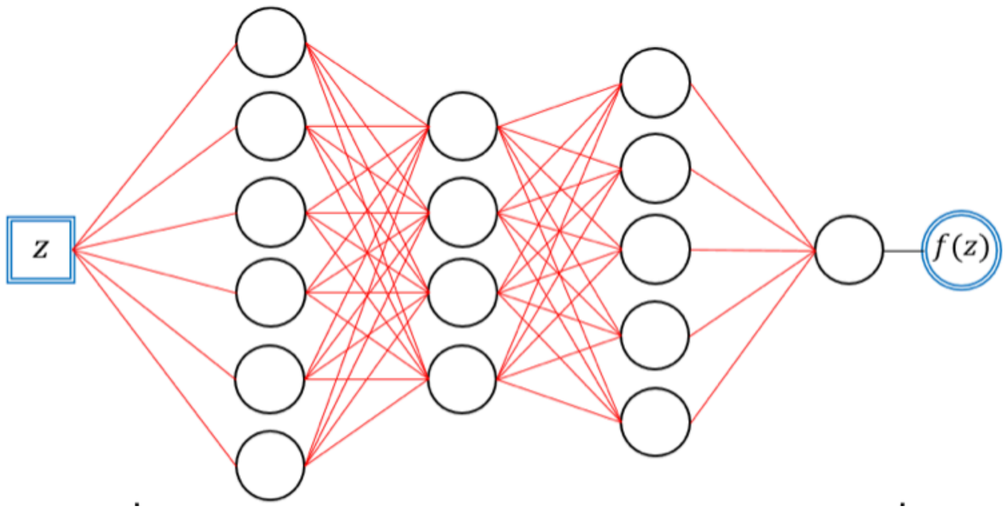


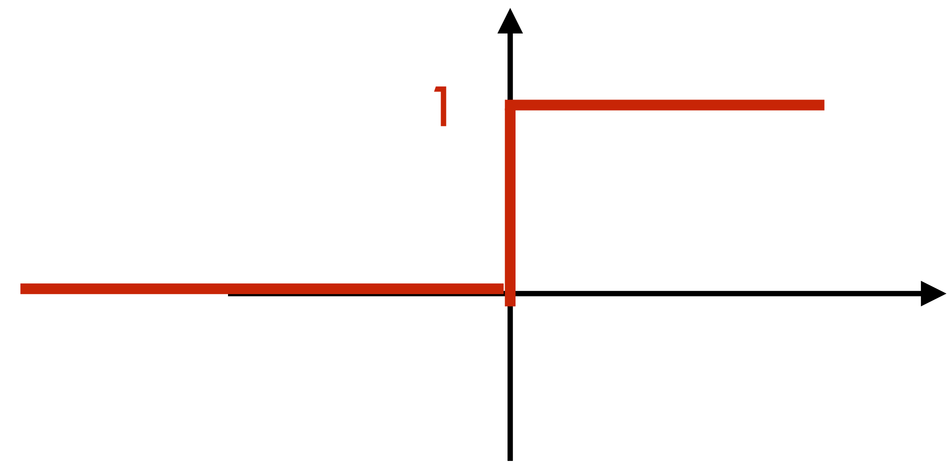
4 - 1 >0

4 - 7 <0



Deep Learning 101





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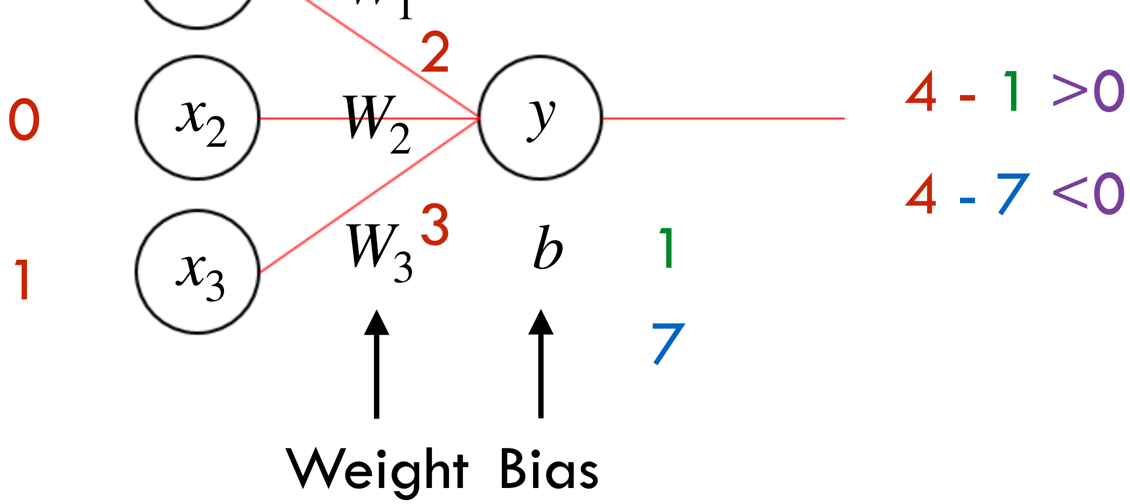
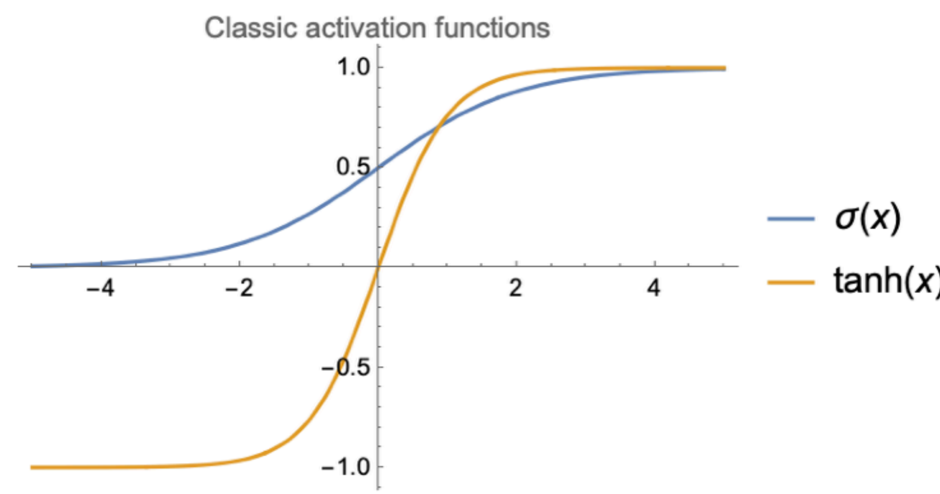
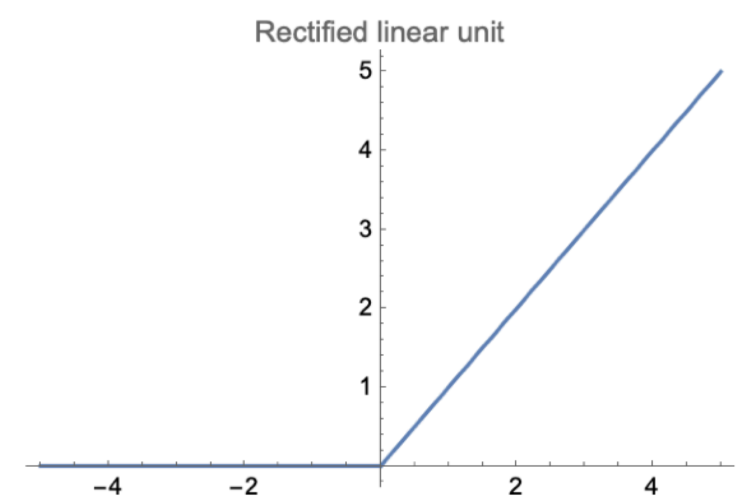
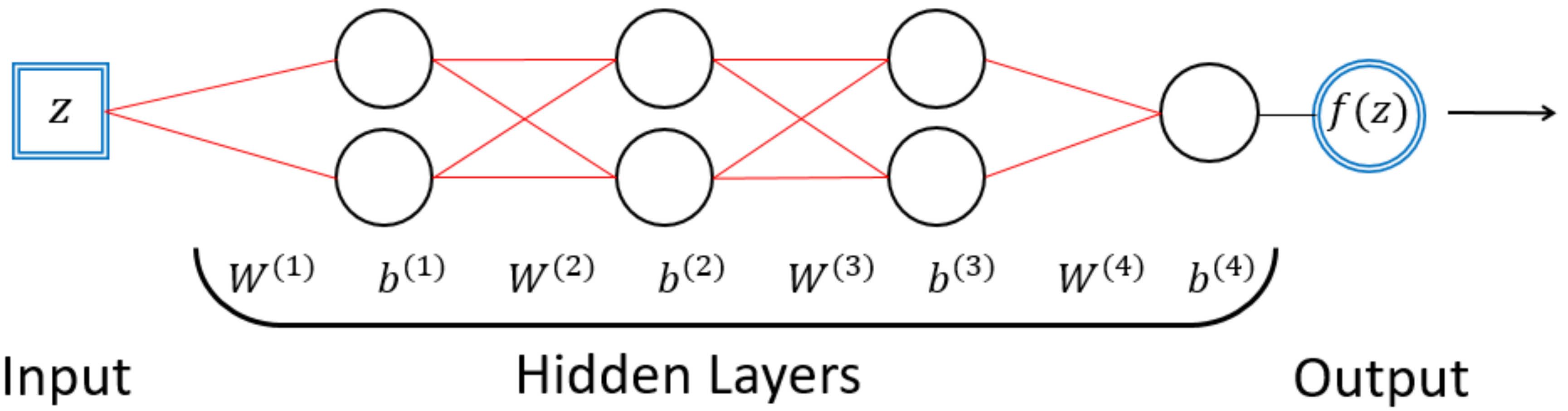
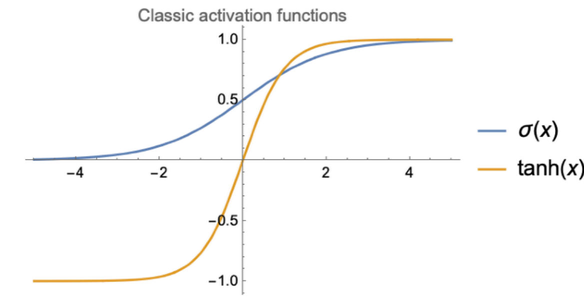






Standard Deep Learning

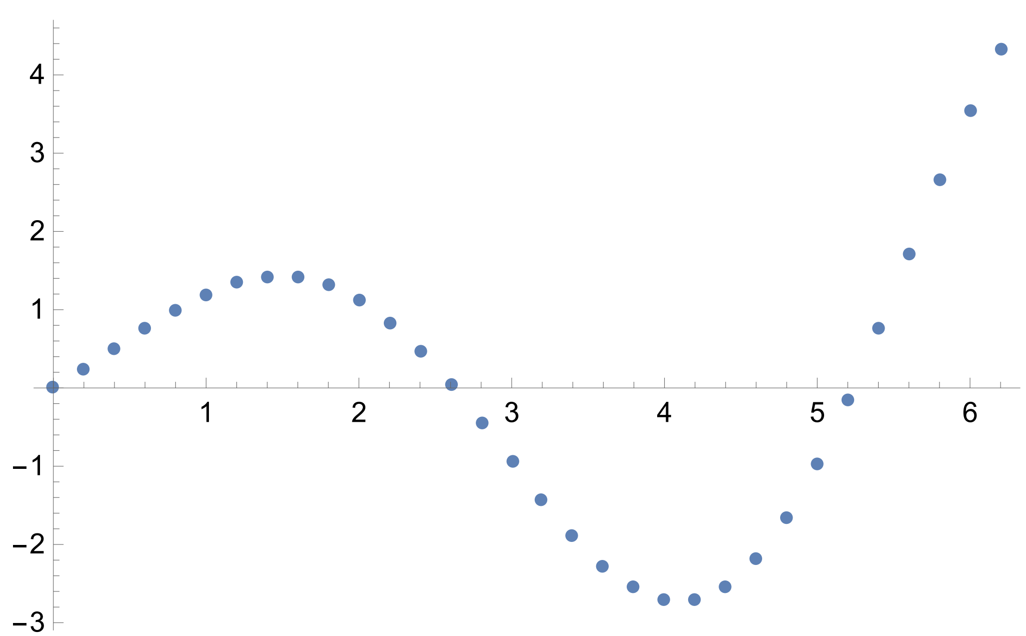


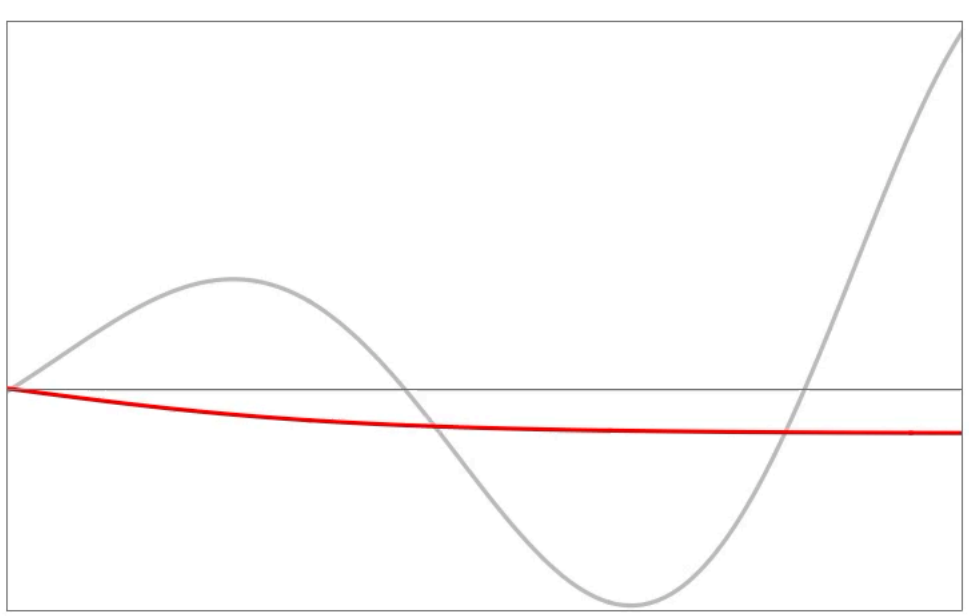


$$z_j^{(i+1)} = \sigma(W_{jk}^{(i)} \cdot z_k^{(i)} + b_j^{(i)})$$

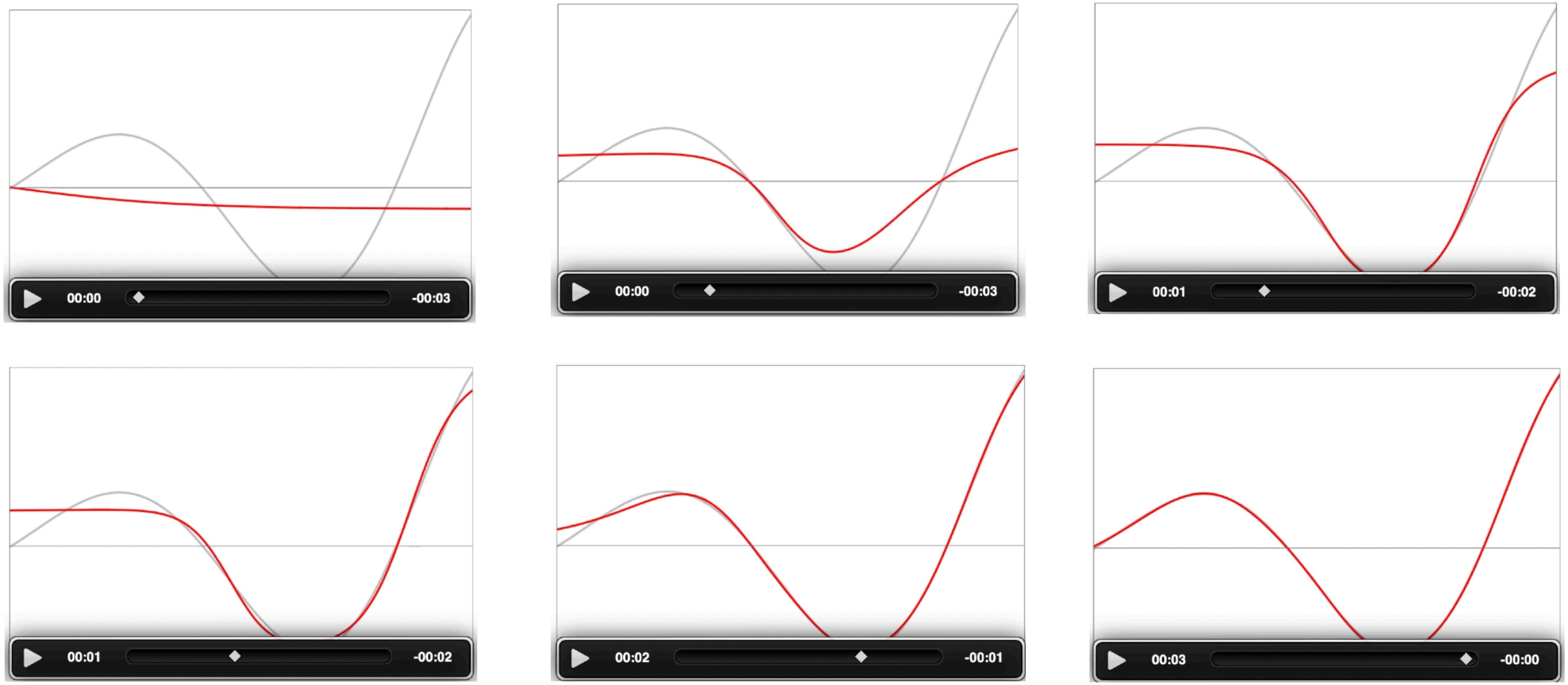
~2000 epoch 19 parameters

$$\mathrm{tanh}\left[\begin{pmatrix}\square & \square\end{pmatrix} \mathrm{tanh}\left[\begin{pmatrix}\square & \square \\ \square & \square\end{pmatrix} \left(\mathrm{tanh}\left[\begin{pmatrix}\square & \square \\ \square & \square\end{pmatrix} \left(\mathrm{tanh}\left[\begin{pmatrix}\square \\ \square\end{pmatrix} z + \begin{pmatrix}\square \\ \square\end{pmatrix}\right]\right) + \begin{pmatrix}\square \\ \square\end{pmatrix}\right] + \begin{pmatrix}\square \\ \square\end{pmatrix}\right)\right] + \begin{pmatrix}\square\end{pmatrix}\right]$$

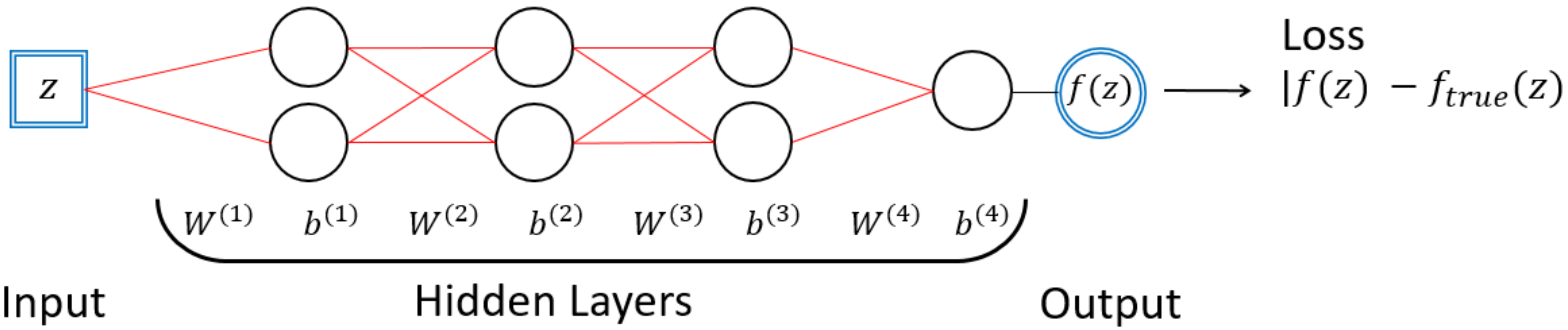




Standard Deep Learning



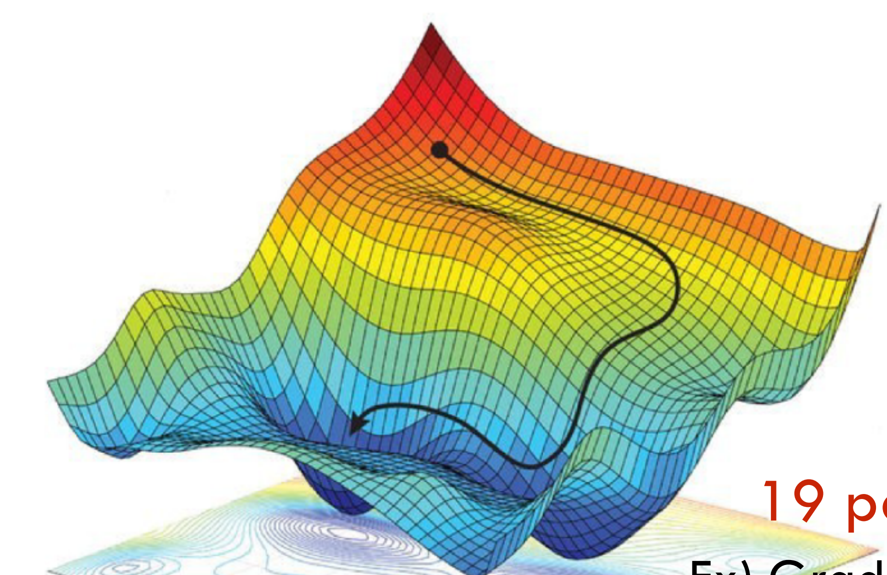
Standard Deep Learning



$$z_j^{(i+1)} = \sigma(W_{jk}^{(i)} \cdot z_k^{(i)} + b_j^{(i)})$$ ~2000 epoch **19 parameters**

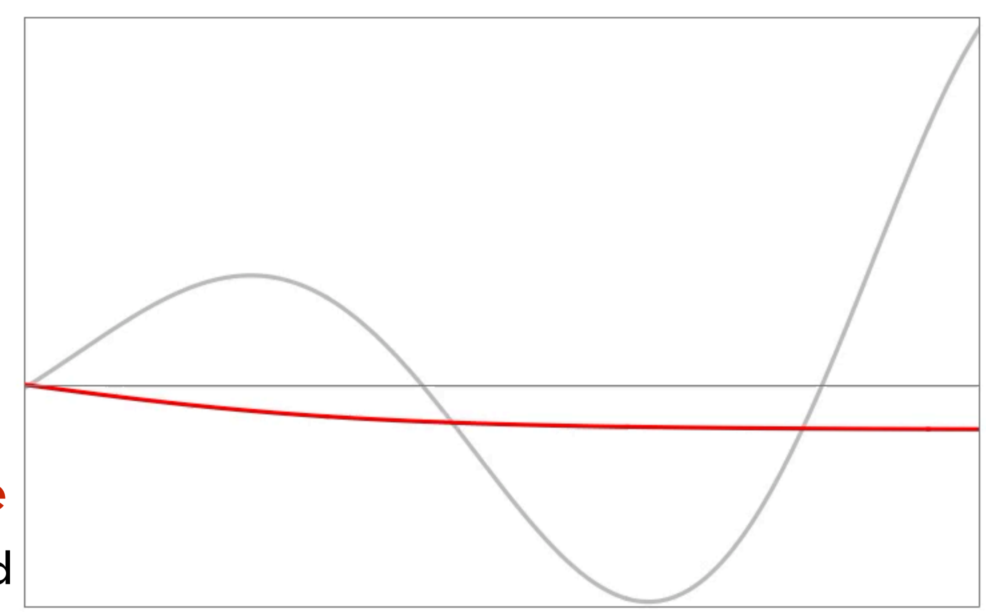
$$\tanh\left[\begin{pmatrix}\square & \square\end{pmatrix} \tanh\left[\begin{pmatrix}\square & \square \\ \square & \square\end{pmatrix} \left(\tanh\left[\begin{pmatrix}\square & \square \\ \square & \square\end{pmatrix} \left(\tanh\left[\begin{pmatrix}\square \\ \square\end{pmatrix} z + \begin{pmatrix}\square \\ \square\end{pmatrix}\right]\right) + \begin{pmatrix}\square \\ \square\end{pmatrix}\right] + \begin{pmatrix}\square \\ \square\end{pmatrix}\right)\right] + \begin{pmatrix}\square\end{pmatrix}\right]\Big|$$

$$\begin{pmatrix}0.878421 & 6.21182\end{pmatrix} \begin{pmatrix}1.55725 & -2.14132 \\ -2.49736 & -1.0517\end{pmatrix} \begin{pmatrix}-1.21558 & -0.693247 \\ 2.86862 & 3.01908\end{pmatrix} \begin{pmatrix}0.317991 \\ 0.521212\end{pmatrix} \begin{pmatrix}-1.96854 \\ -0.020086\end{pmatrix} \begin{pmatrix}0.320358 \\ -0.614063\end{pmatrix} \begin{pmatrix}-0.442947 \\ 0.854547\end{pmatrix} \begin{pmatrix}1.33224\end{pmatrix}$$

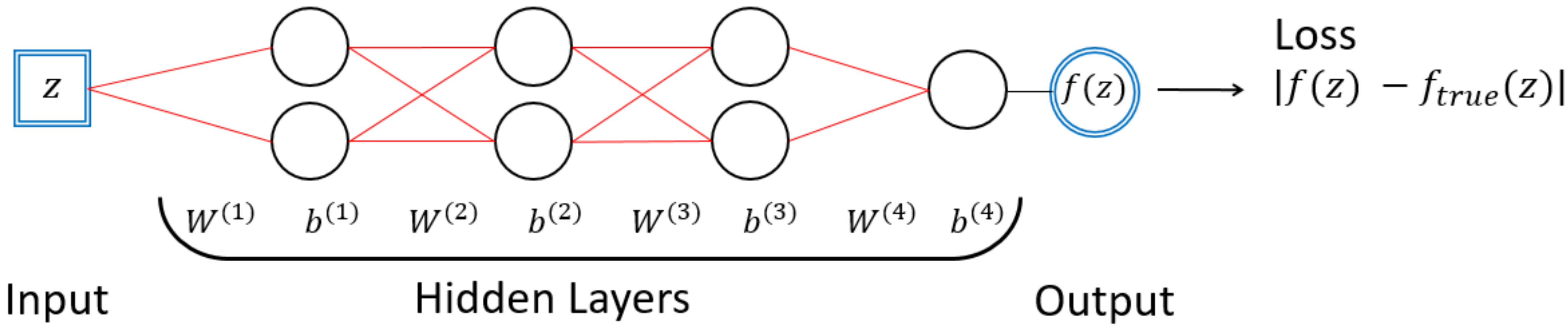


19 parameter space

Ex) Gradient decent method



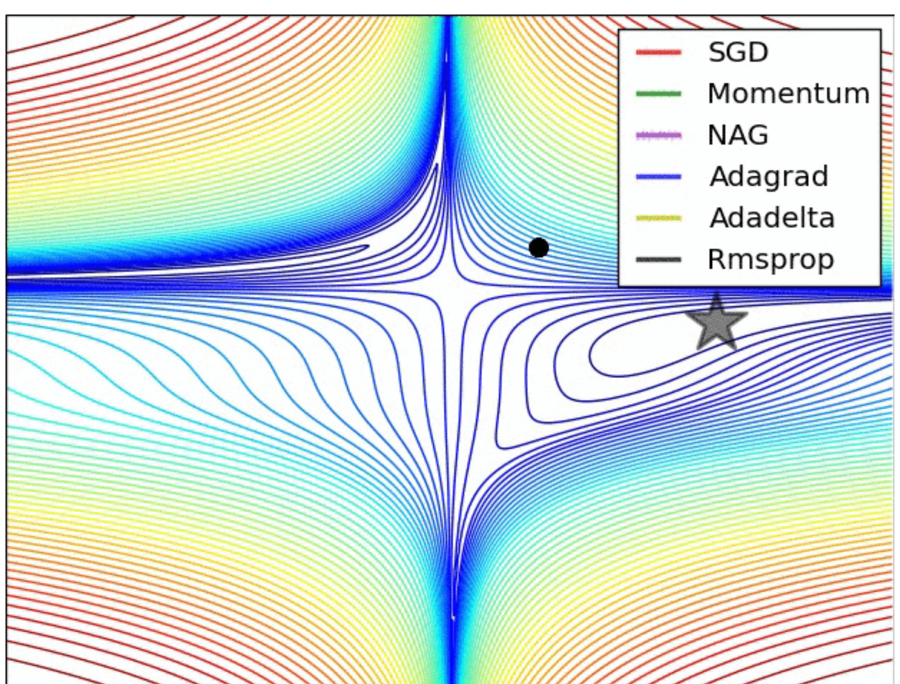
Standard Deep Learning

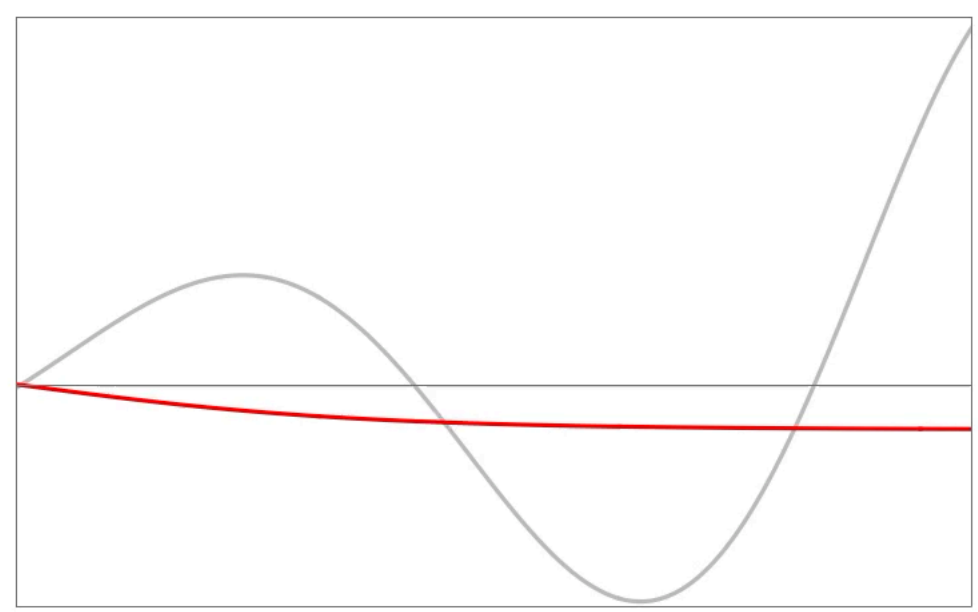


$z_j^{(i+1)} = \sigma(W_{jk}^{(i)} \cdot z_k^{(i)} + b_j^{(i)})$ ~2000 epoch 19 parameters

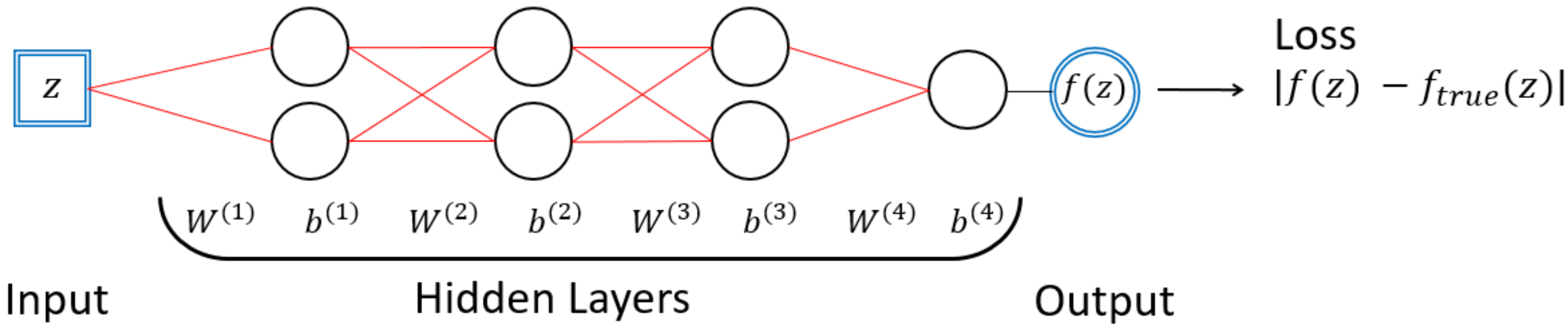
$$\tanh\left[\begin{pmatrix} \square & \square \end{pmatrix} \tanh\left[\begin{pmatrix} \square & \square \\ \square & \square \end{pmatrix} \left(\tanh\left[\begin{pmatrix} \square & \square \\ \square & \square \end{pmatrix} \left(\tanh\left[\begin{pmatrix} \square \\ \square \end{pmatrix} z + \begin{pmatrix} \square \\ \square \end{pmatrix}\right]\right) + \begin{pmatrix} \square \\ \square \end{pmatrix}\right] + \begin{pmatrix} \square \\ \square \end{pmatrix}\right)\right] + \begin{pmatrix} \square \end{pmatrix}\right]$$

$$\begin{pmatrix} 0.878421 & 6.21182 \end{pmatrix} \begin{pmatrix} 1.55725 & -2.14132 \\ -2.49736 & -1.0517 \end{pmatrix} \begin{pmatrix} -1.21558 & -0.693247 \\ 2.86862 & 3.01908 \end{pmatrix} \begin{pmatrix} 0.317991 \\ 0.521212 \end{pmatrix} \begin{pmatrix} -1.96854 \\ -0.020086 \end{pmatrix} \begin{pmatrix} 0.320358 \\ -0.614063 \end{pmatrix} \begin{pmatrix} -0.442947 \\ 0.854547 \end{pmatrix} \begin{pmatrix} 1.33224 \end{pmatrix}$$



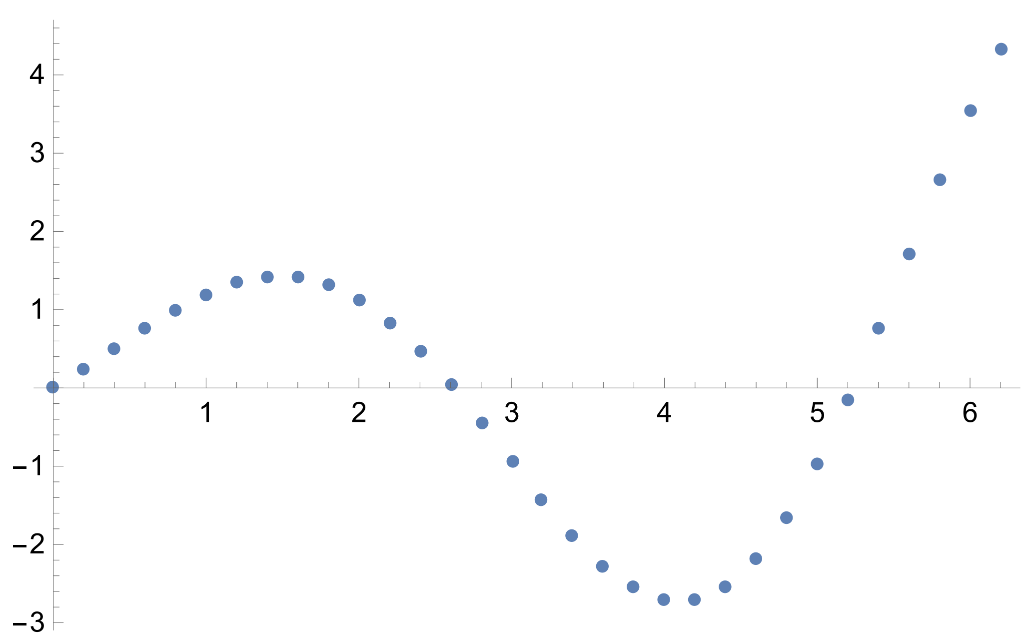


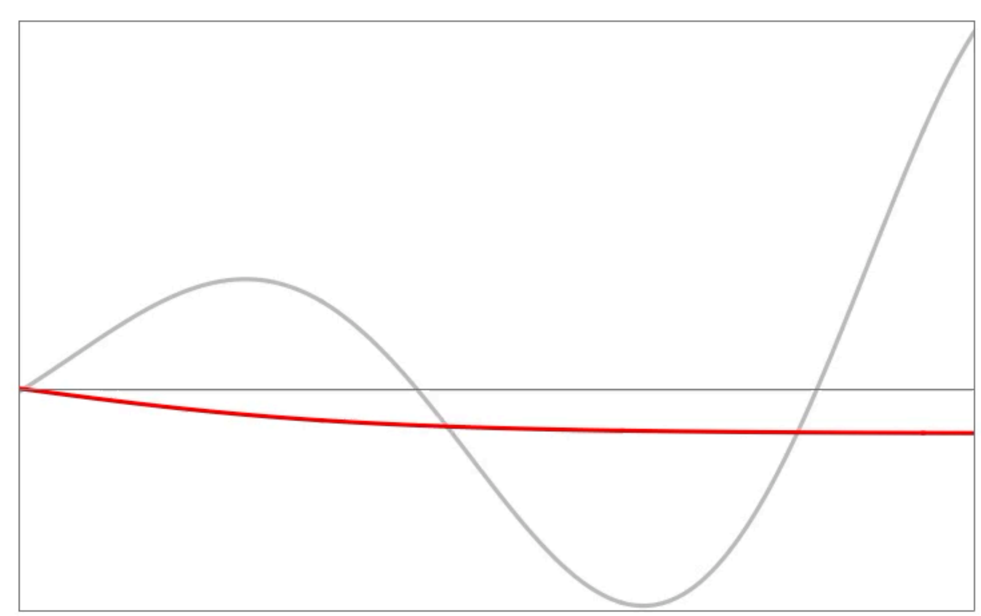
Standard Deep Learning



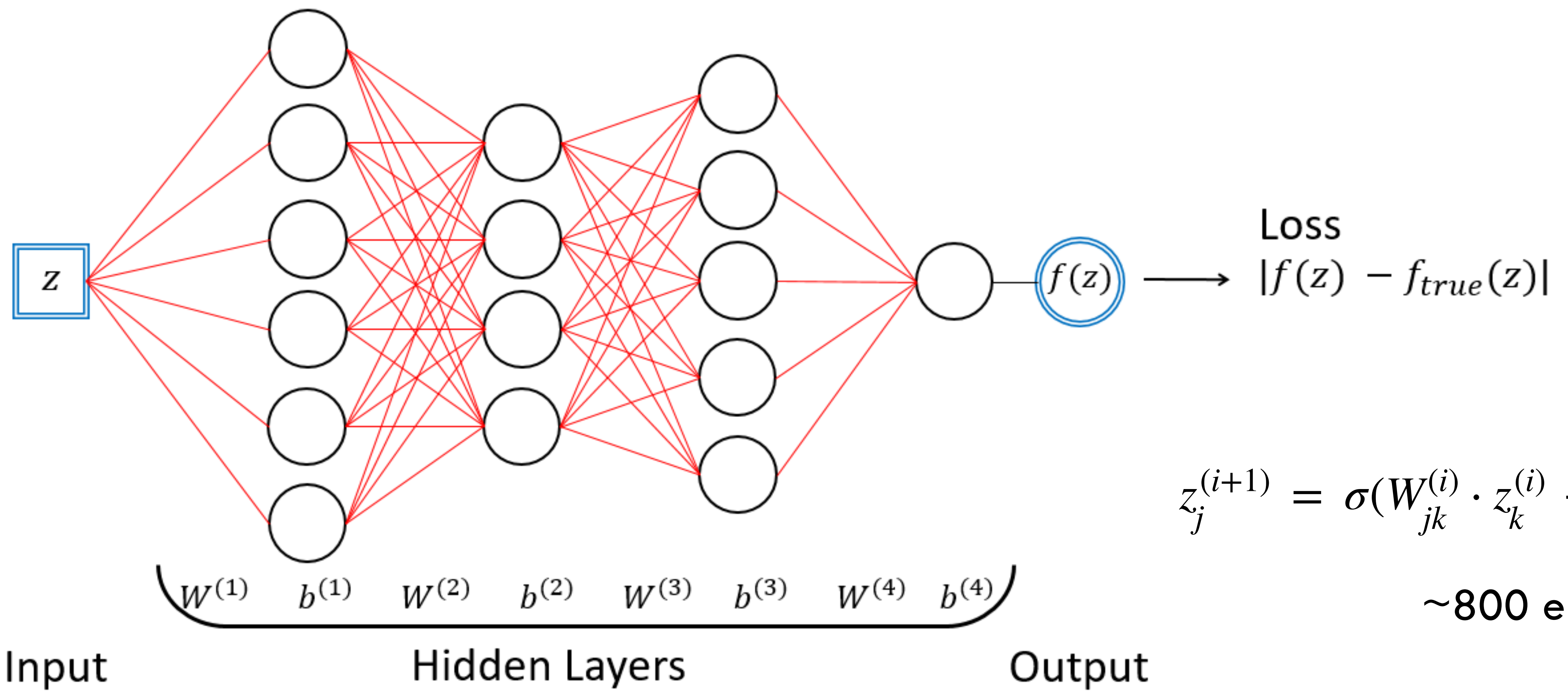
$$z_j^{(i+1)} = \sigma(W_{jk}^{(i)} \cdot z_k^{(i)} + b_j^{(i)}) \qquad \sim 2000 \text{ epoch}$$

```
net = NetChain[{LinearLayer[2], Tanh, LinearLayer[2], Tanh, LinearLayer[2], Tanh, LinearLayer[]}]
      네트워크 체인  선형층                 쌍곡…  선형층                 쌍곡…  선형층                 쌍곡…  선형층
solutions = NetTrain[net, data, <|"Property" → Function[#Net[Range[0, 2 π, 0.2]]], "Form" → "List",
            네트워크 훈련                           함수
   "Interval" → Quantity[30, "Rounds"]|>, (*LearningRate→0.001,*) TimeGoal → 600];
                수량                                              작업 시간 지정
```



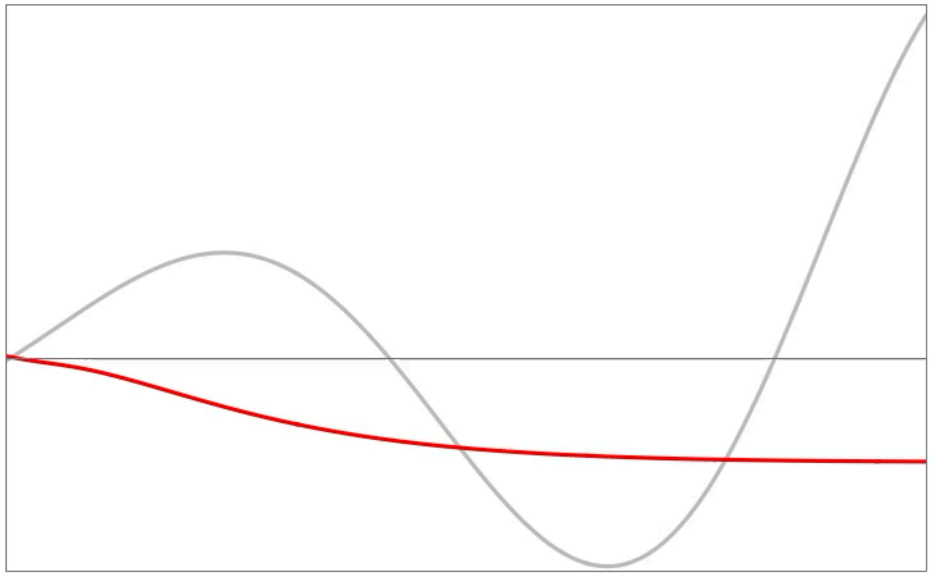


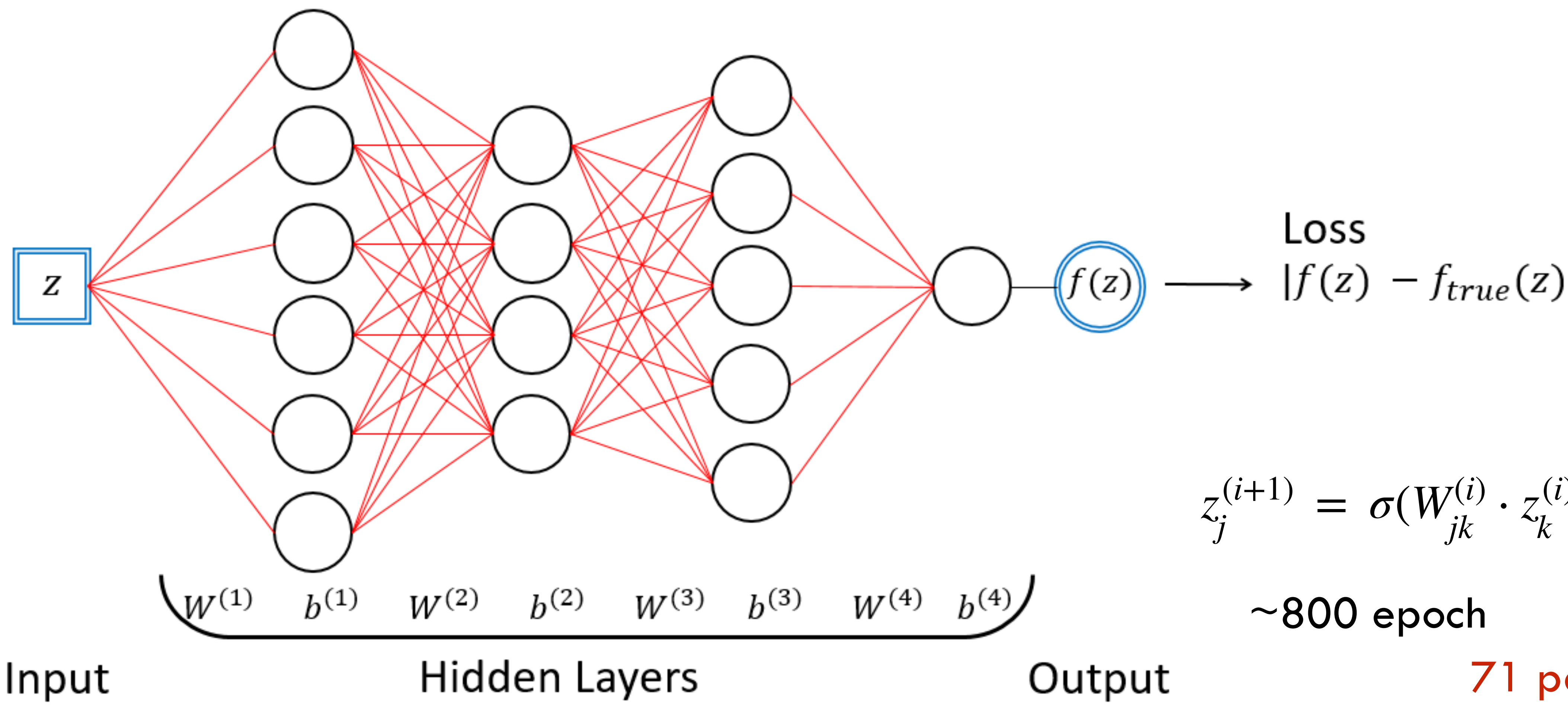
Standard Deep Learning



$$z_j^{(i+1)} = \sigma(W_{jk}^{(i)} \cdot z_k^{(i)} + b_j^{(i)})$$

~800 epoch





$$z_j^{(i+1)} = \sigma(W_{jk}^{(i)} \cdot z_k^{(i)} + b_j^{(i)})$$

~800 epoch

71 parameters

$$\text{Show}\left[\left\{\begin{pmatrix}0.155467\\0.503477\\-0.19021\\-0.573821\\0.238013\\0.93589\end{pmatrix},\begin{pmatrix}-0.421564\\-0.664526\\0.768171\\1.02467\\-1.01427\\-0.94677\end{pmatrix},\begin{pmatrix}0.584066 & 0.331801 & -0.29404 & -0.586447 & 0.820878 & 1.04534\\0.208755 & 0.465765 & -1.21082 & -0.63667 & 1.73886 & -0.53486\\0.918991 & 0.328462 & -1.08441 & 0.474161 & -0.0382366 & -0.502777\\0.600695 & -0.32061 & -0.914458 & 0.73871 & 0.124301 & 0.402687\end{pmatrix},\right.\right.$$

$$\begin{pmatrix}0.172725\\-0.0946543\\0.0526899\\0.201293\end{pmatrix},\begin{pmatrix}0.308126 & -1.0333 & 1.21593 & 1.62945\\-0.217696 & 0.696293 & -0.338222 & -2.68779\\0.0759481 & -0.664867 & 0.86521 & 1.7725\\0.445185 & 0.0762033 & -1.15522 & -2.01414\\-0.595479 & 0.916218 & -0.400813 & -2.12668\end{pmatrix},\begin{pmatrix}0.621626\\-0.192633\\0.402367\\-0.14795\\-0.331163\end{pmatrix},$$

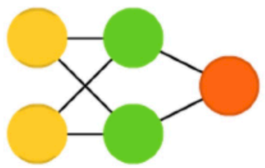
$$\left.\left.\begin{pmatrix}0.903621 & -2.09077 & 1.97153 & -1.85892 & -1.05424\end{pmatrix},\begin{pmatrix}0.319326\end{pmatrix}\right\}\right]$$

Recap

Machine learning = function approximator

Input: a vector $(v_1, v_2, v_3, \ldots)$
Output: a value $f(v_1, v_2, v_3, \ldots)$

Network architecture = Function ansatz



Perceptron model
[Rosenblatt 1958]
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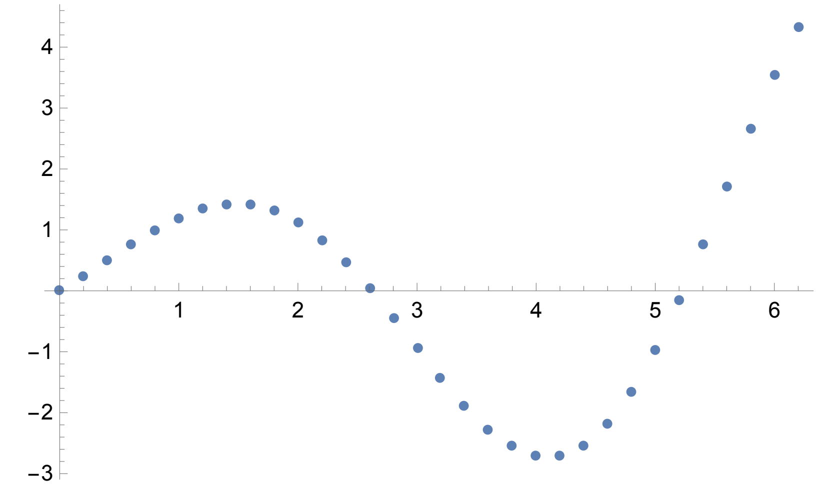
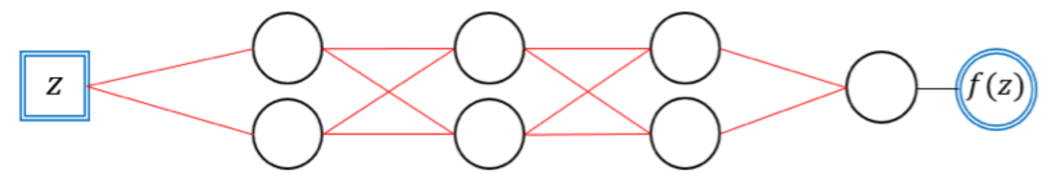
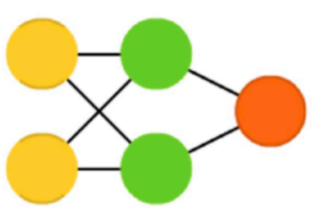
Universal approximation theorem :
Any function can be approximated with more hidden units [Cybenko 1989] [Roux, Bengio 2008]





Perceptron model



$$f = W_i^{(2)} \varphi\left(W_{ij}^{(1)} x_j\right)$$

"Unit" (circles) : Vector components

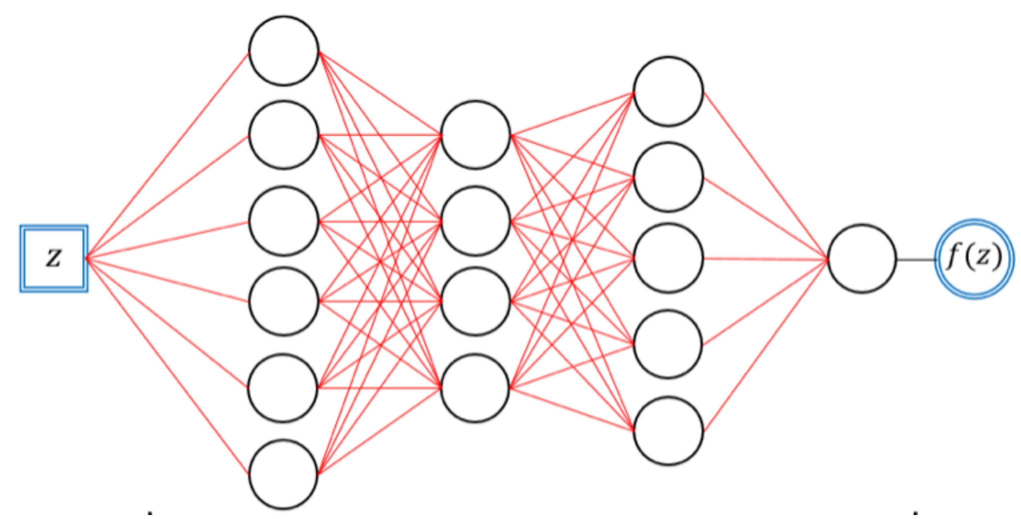
"Weight" (lines) : Linear transformation to be optimized

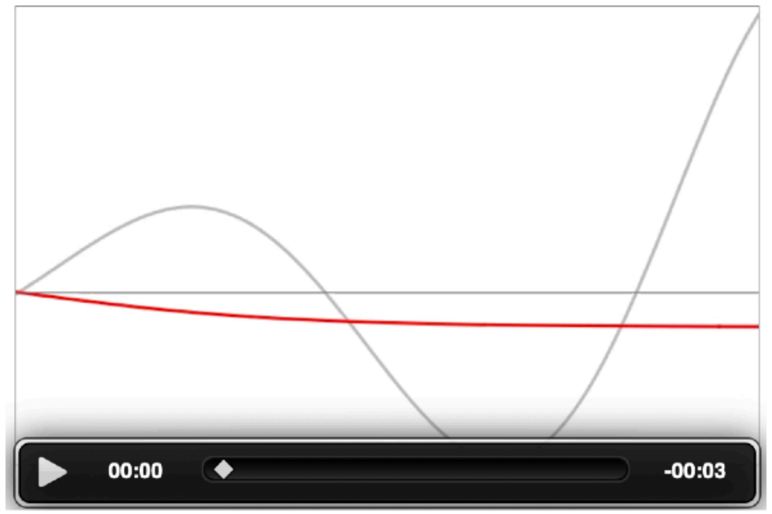
"Activation function" (hidden line-end) : Nonlinear component-wise transf.

$$\varphi(x) \equiv \frac{1}{1+e^{-x}}$$

- Training protocol :

1) Prepare many sets $\{(x_j, f)\}$: input + output
2) Train the network (adjust W) by lowering "Loss function" $E \equiv \sum_{\text{data}} \left| f - W_i^{(2)} \varphi\left(W_{ij}^{(1)} x_j\right) \right|$

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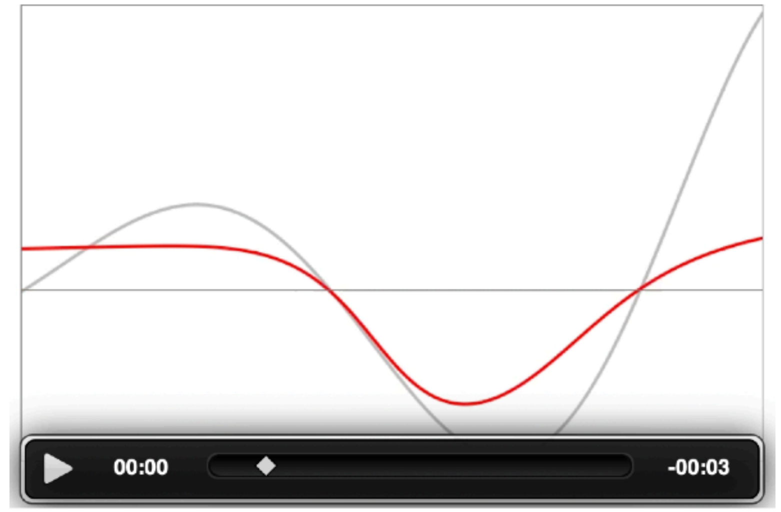
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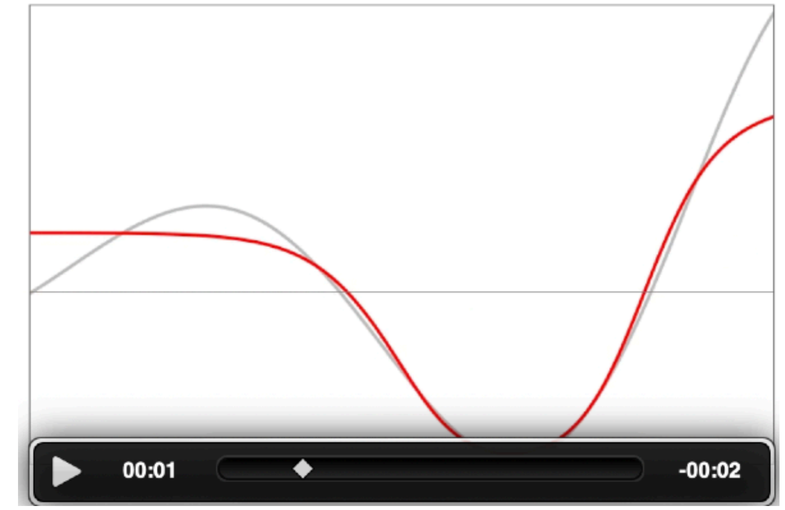
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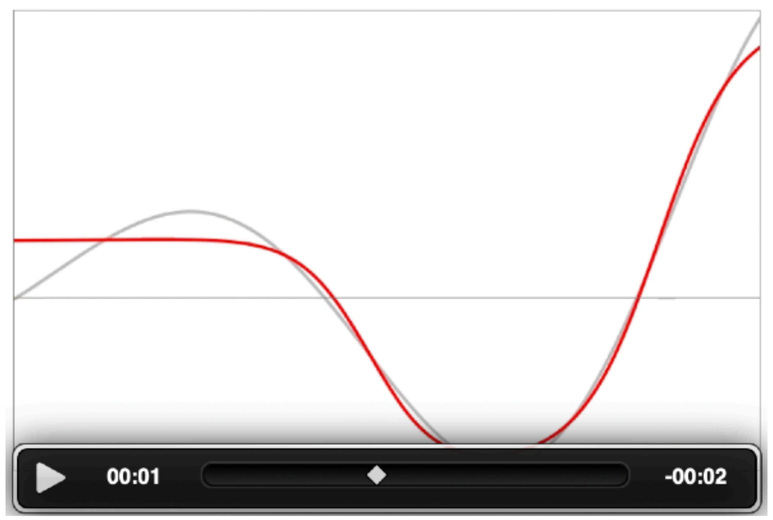
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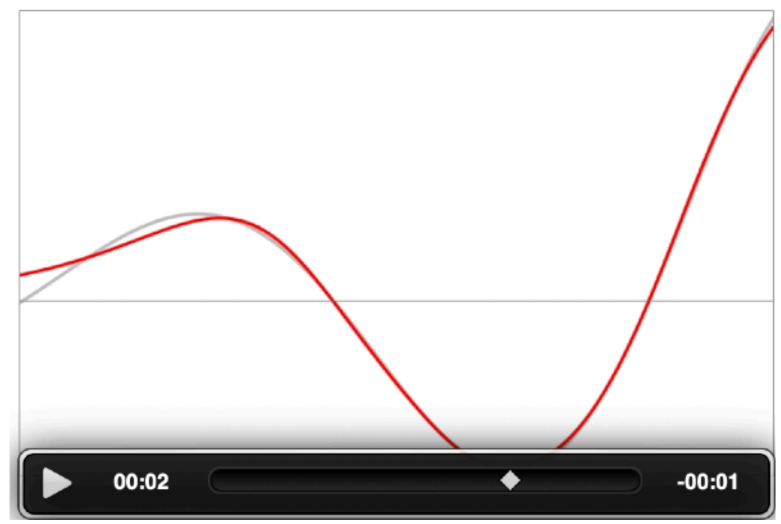
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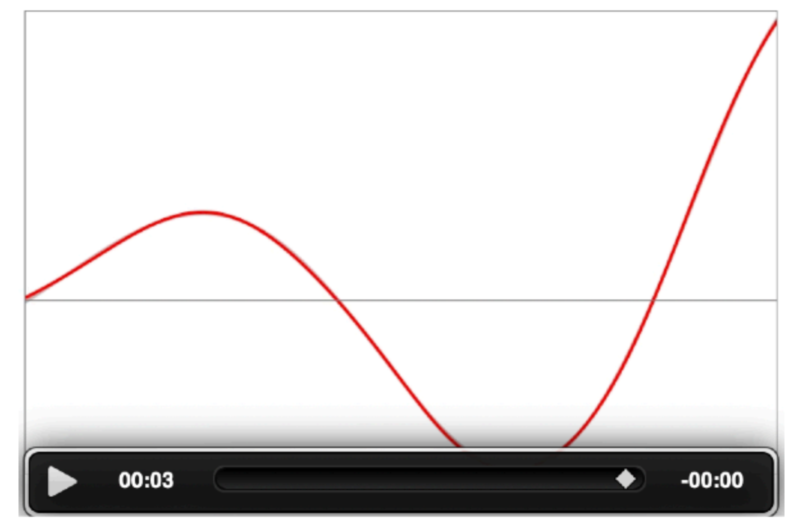
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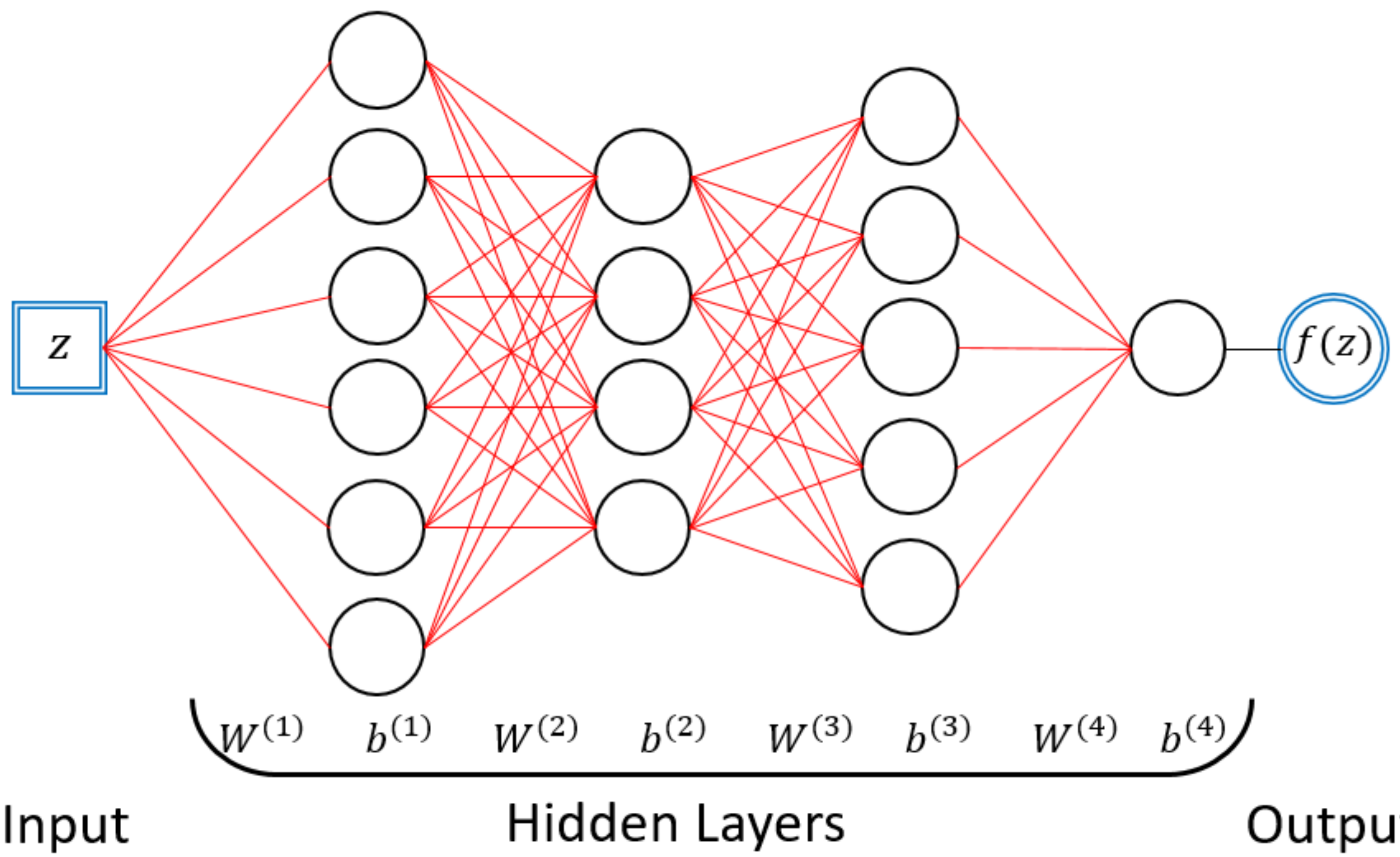
Standard Deep Learning



Loss
$|f(z) - f_{true}(z)|$

$$z_j^{(i+1)} = \sigma(W_{jk}^{(i)} \cdot z_k^{(i)} + b_j^{(i)})$$

~800 epoch

71 parameters

Show $\left[\left\{ \begin{pmatrix} 0.155467 \\ 0.503477 \\ -0.19021 \\ -0.573821 \\ 0.238013 \\ 0.93589 \end{pmatrix}, \begin{pmatrix} -0.421564 \\ -0.664526 \\ 0.768171 \\ 1.02467 \\ -1.01427 \\ -0.94677 \end{pmatrix}, \begin{pmatrix} 0.584066 & 0.331801 & -0.29404 & -0.586447 & 0.820878 & 1.04534 \\ 0.208755 & 0.465765 & -1.21082 & -0.63667 & 1.73886 & -0.53486 \\ 0.918991 & 0.328462 & -1.08441 & 0.474161 & -0.0382366 & -0.502777 \\ 0.600695 & -0.32061 & -0.914458 & 0.73871 & 0.124301 & 0.402687 \end{pmatrix}, \right.\right.$

$\begin{pmatrix} 0.172725 \\ -0.0946543 \\ 0.0526899 \\ 0.201293 \end{pmatrix}, \begin{pmatrix} 0.308126 & -1.0333 & 1.21593 & 1.62945 \\ -0.217696 & 0.696293 & -0.338222 & -2.68779 \\ 0.0759481 & -0.664867 & 0.86521 & 1.7725 \\ 0.445185 & 0.0762033 & -1.15522 & -2.01414 \\ -0.595479 & 0.916218 & -0.400813 & -2.12668 \end{pmatrix}, \begin{pmatrix} 0.621626 \\ -0.192633 \\ 0.402367 \\ -0.14795 \\ -0.331163 \end{pmatrix},$

$\left.\left. \begin{pmatrix} 0.903621 & -2.09077 & 1.97153 & -1.85892 & -1.05424 \end{pmatrix}, \begin{pmatrix} 0.319326 \end{pmatrix} \right\}\right]$

Simple physic problem?

Reference I

Chinese Physics C Vol. 45, No. 7 (2021) 073111

AdS/Deep-Learning made easy: simple examples*

Mugeon Song[1,#†] Maverick S. H. Oh[1,2,#‡] Yongjun Ahn[1§] Keun-Young Kima[1♮]

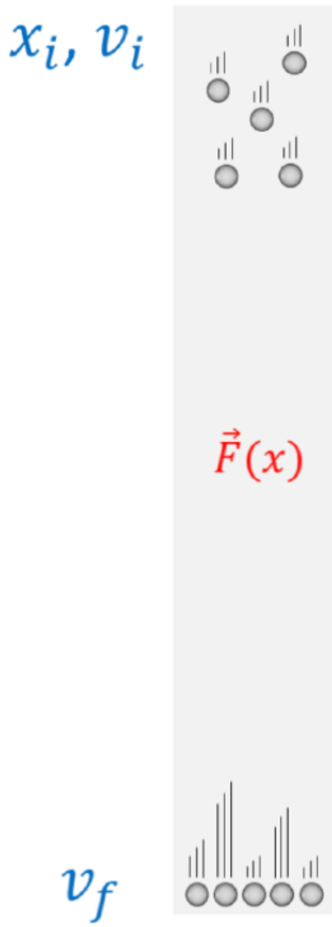
[1]Gwangju Institute of Science and Technology (GIST), Department of Physics and Photon Science, Gwangju, South Korea
[2]University of California–Merced, Department of Physics, Merced, CA, USA

Abstract: Deep learning has been widely and actively used in various research areas. Recently, in gauge/gravity duality, a new deep learning technique called AdS/DL (Deep Learning) has been proposed. The goal of this paper is to explain the essence of AdS/DL in the simplest possible setups, without resorting to knowledge of gauge/gravity duality. This perspective will be useful for various physics problems: from the emergent spacetime as a neural network to classical mechanics problems. For prototypical examples, we choose simple classical mechanics problems. This method is slightly different from standard deep learning techniques in the sense that we not only have the right final answers but also obtain physical understanding of learning parameters.

Keywords: gauge/gravity duality, holographic principle, machine learning

DOI: 10.1088/1674-1137/abfc36

Deep Learning for ODE: classical mechanics



$$m\dot{v} = mg - F(x)$$
$$\dot{x} = v$$

Machine learning = function approximator

Input: a vector $(v_1, v_2, v_3, \ldots)$
Output: a value $f(v_1, v_2, v_3, \ldots)$

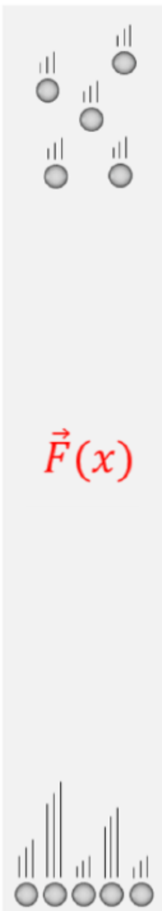
Network architecture = Function ansatz

Perceptron model
[Rosenblatt 1958]
[Rumelhart, McClelland 1986]

Universal approximation theorem :
Any function can be approximated with more hidden units [Cybenko 1989] [Roux, Bengio 2008]

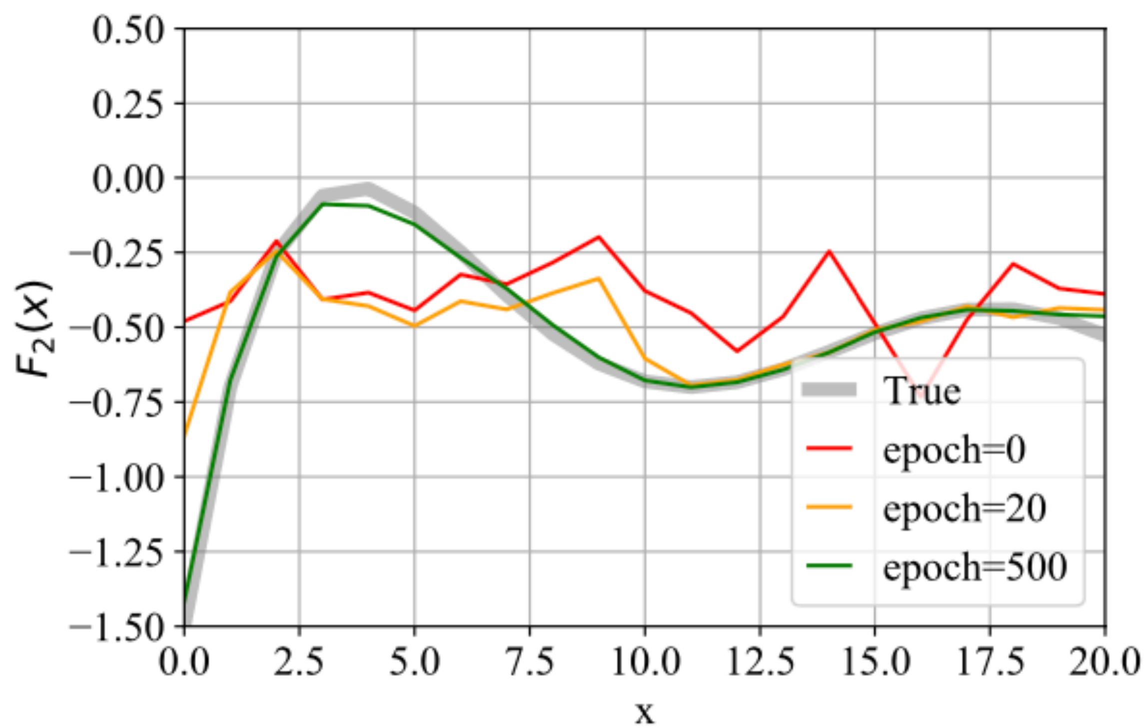
22

x_i, v_i



v_f

$$m\dot{v} = mg - F(x)$$
$$\dot{x} = v$$



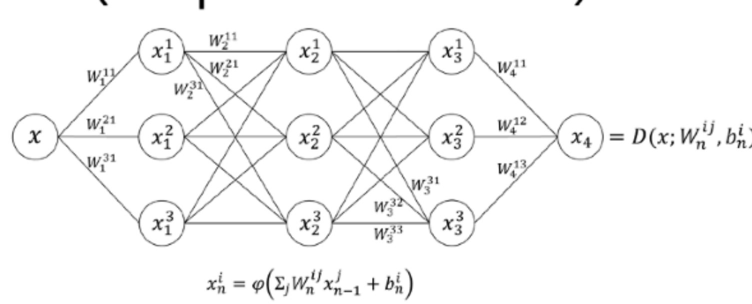
1. (Data) 3. (Numerical ODE solving) 5. (Loss) 4. (Solution)

$$(v_i, x_i) \longrightarrow \begin{array}{c} m\dot{v} = mg - F(x) \\ \dot{x} = v \end{array} \longrightarrow L = \mathrm{avg}(\,|v_f - V_f|\,) \longleftarrow V_f$$

$$\frac{\partial L}{\partial W_n}, \frac{\partial L}{\partial b_n}$$

$$F(x) = D(x; W_n, b_n)$$

2. (Deep neural network) 6. (Parameters optimizing)



Generalization

AdS/Deep learning: optical conductivity

$$m\ddot{x} = F$$

$$A(z)f''(z)+B(z)f'(z)+C(z)f(z) = F(z)$$

$$\boxed{(fA_x')' + \frac{w^2}{f}A_x = \frac{4\mu^2}{\gamma^2 r_h^2} r^2 A_x\,.}$$

$$\sigma(\omega) = \frac{1}{e^2}\frac{A_x'}{i\omega A_x}\bigg|_{r=0}\,.$$

$$A(z)f''(z)+B(z)f'(z)+C(z)f(z) = D(z)g(z)$$

$$E(z)g''(z)+F(z)g'(z)+G(z)g(z) = H(z)f(z)$$

$$\boxed{\begin{aligned}
&\partial_z^2 A_x = \zeta\,\partial_z A_x + \left(\frac{z^2\mu^2}{f} - \xi\right) A_x + \frac{iz\mu}{f}\Phi\,,\\
&\partial_z^2 \Phi = \zeta\,\partial_z \Phi + \left(\frac{\alpha^2}{f} + \frac{f'}{zf} - \xi\right) \Phi - \frac{iz\alpha^2\mu}{f} A_x\,,\\
&\zeta := \frac{2i\omega}{(1-z)f'(1)} - \frac{f'(z)}{f(z)}\,, \qquad \xi := \frac{\omega^2}{f(z)^2} + \frac{i\omega}{(1-z)f'(1)}\left(\frac{i\omega}{(1-z)f'(1)} - \frac{1}{1-z} - \frac{f'(z)}{f(z)}\right)
\end{aligned}}$$

Linear response

One-point function

$$\langle O\rangle_s = \frac{\delta \underline{S}[\phi^{(0)}]}{\delta\phi^{(0)}}$$

GKP-Witten Relation

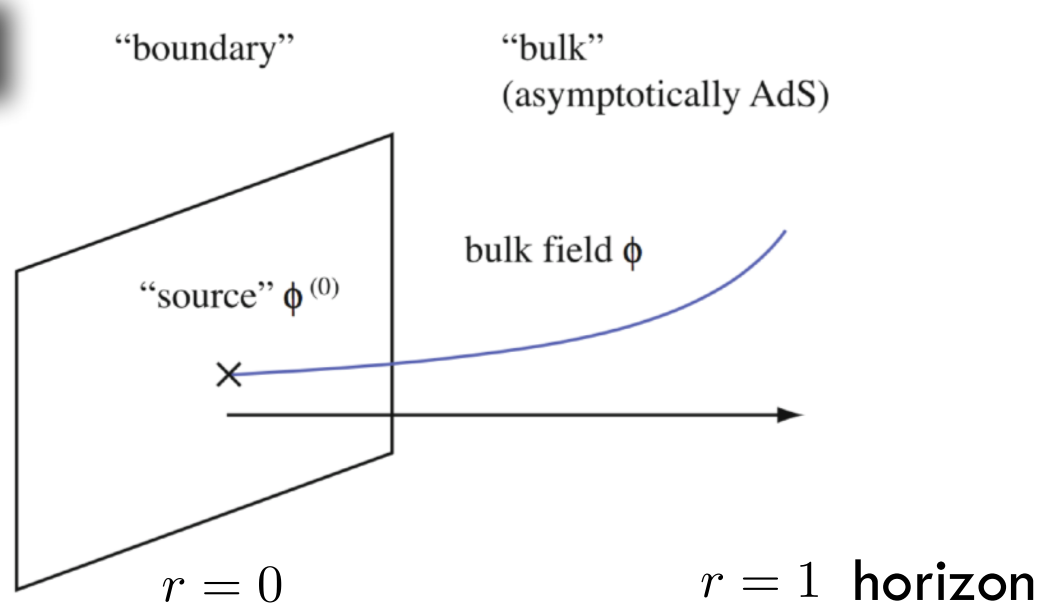
$$Z_{\text{gauge}} = Z_{\text{AdS}}$$

$$\boxed{\left\langle \exp\left(i\int \phi^{(0)} O\right)\right\rangle = e^{i\underline{S}[\phi|_{u=0}=\phi^{(0)}]}.}$$

On-shell action ~ partition function

Variation of the On-shell action
~ One point function

Corollary



Linear response theory: 2nd order ODE

$$\delta A_x(r,\omega) = \frac{E}{i\omega} + J_x(\omega) r + \cdots$$

Source　Expectation value

HOLOGRAPHIC CONDUCTIVITY*

DAVID TONG

$$S_{\text{bulk}} = \int d^4x\sqrt{-g}\left[\frac{1}{2\kappa^2}\left(R+\frac{6}{L^2}\right)-\frac{1}{4e^2}F_{AB}F^{AB}\right] .$$

$$ds^2 = \frac{L^2}{r^2}\left(-f(r)dt^2+\frac{dr^2}{f(r)}+\eta_{\mu\nu}dx^\mu dx^\nu\right) \qquad A_0 = \mu\left(1-\frac{r}{r_{\text{h}}}\right) .$$

$$f(r) = 1-\left(1+\frac{r_{\text{h}}^2\mu^2}{\gamma^2}\right)\left(\frac{r}{r_{\text{h}}}\right)^3+\frac{r_{\text{h}}^2\mu^2}{\gamma^2}\left(\frac{r}{r_{\text{h}}}\right)^4 .$$

$$(fA_x')'+\frac{w^2}{f}A_x = \frac{4\mu^2}{\gamma^2 r_{\text{h}}^2}r^2A_x .$$

$$A_x = \frac{E}{i\omega}e^{i\omega t}+\langle J_x\rangle r+\ldots$$

$$\sigma(\omega) = \frac{1}{e^2}\frac{A_x'}{i\omega A_x}\bigg|_{r=0} .$$

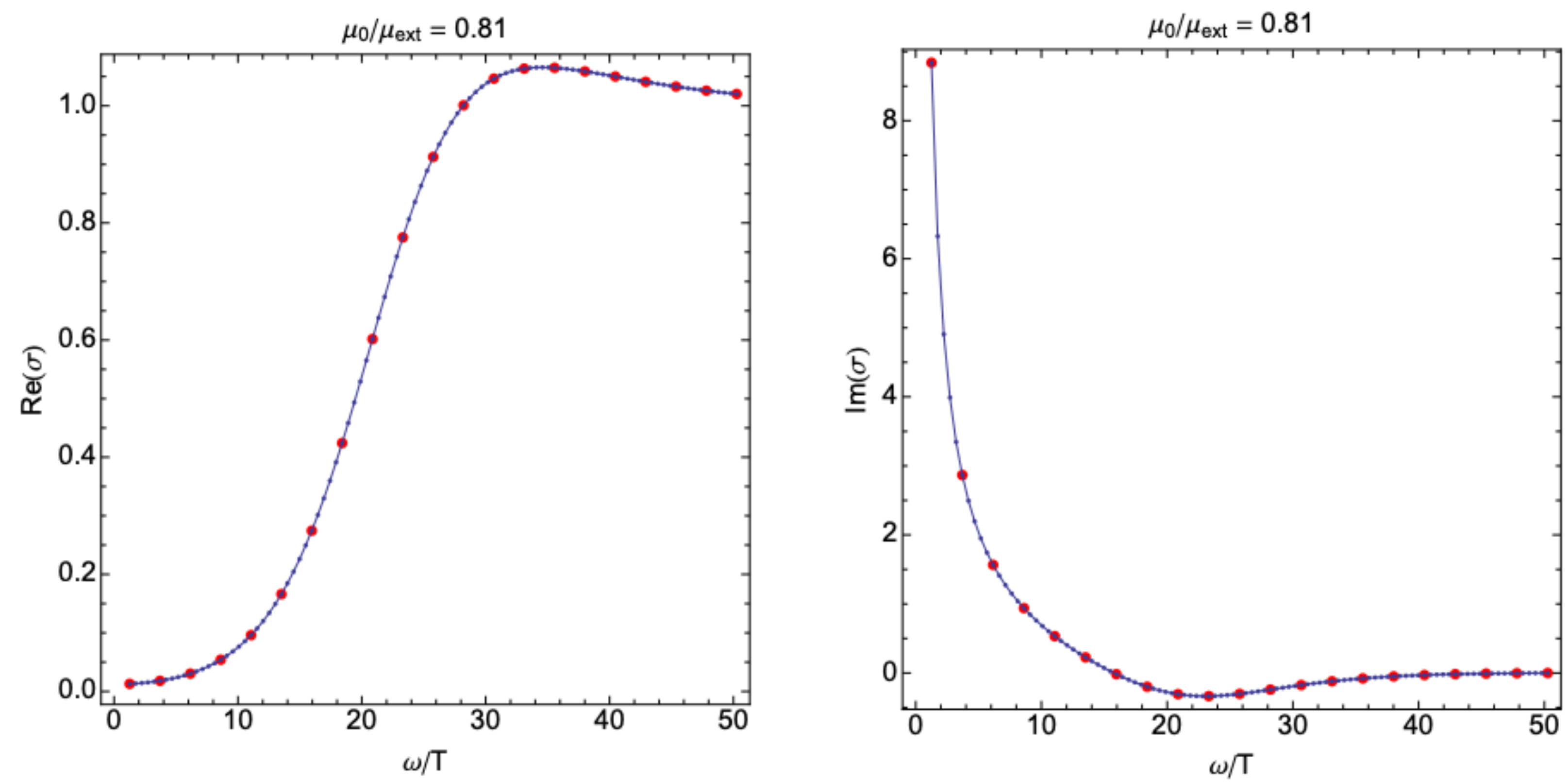


Fig. 5. Holographic optical conductivity.

Reference I

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Deep learning bulk spacetime from boundary optical conductivity

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Action

$$\mathcal{S} = \int \mathrm{d}^4x\sqrt{-g}\left(R+6-\frac{1}{4}F_{ab}F^{ab}-\frac{1}{2}\sum_{I=1}^{2}(\partial X_I)^2\right)$$

EOM

$$R_{ab}-\frac{1}{2}g_{ab}\left(R+6-\frac{1}{4}F_{ab}F^{ab}-\frac{1}{2}\sum_{I=1}^{2}(\partial X_I)^2\right)-F_{ac}F_b^{\,c}-\frac{1}{2}\sum_{I=1}^{2}\partial_a X_I\partial_b X_I=0\,,$$

$$\nabla^a F_{ab}=0\,,\qquad \nabla_a\nabla^a X_I=0\,,$$

Background

$$\mathrm{d}s^2=\frac{1}{z^2}\left[-f(z)\mathrm{d}t^2+\frac{\mathrm{d}z^2}{f(z)}+\mathrm{d}x^2+\mathrm{d}y^2\right],\qquad f(z)=1-\frac{\alpha^2}{2}z^2-\left(1-\frac{\alpha^2}{2}+\frac{\mu^2}{4}\right)z^3+\frac{\mu^2}{4}z^4$$

$$A=\mu\,(1-z)\,\mathrm{d}t\,,\qquad X_1=\alpha\,x\,,\quad X_2=\alpha\,y$$

Flucutation EOM I

$$\delta g_{tx}=e^{-i\omega t}\frac{h_{tx}(z)}{z^2}\,,\qquad \delta A_x=e^{-i\omega t}a_x(z)\,,\qquad \delta X_1=e^{-i\omega t}\frac{\psi_x(z)}{\alpha}$$

$$a_x''(z)+\frac{f'(z)}{f(z)}a_x'(z)+\left(\frac{\omega^2}{f(z)^2}-\frac{\mu^2z^2}{f(z)}\right)a_x(z)-\frac{i\mu z}{f(z)}\phi(z)=0\,,\qquad \phi(z):=-\frac{f(z)\psi_x'(z)}{\omega\,z}$$

$$\phi''(z)+\frac{f'(z)}{f(z)}\phi'(z)+\left(\frac{\omega^2}{f(z)^2}-\frac{\alpha^2}{f(z)}-\frac{f'(z)}{zf(z)}\right)\phi(z)+\frac{i\alpha^2\mu z}{f(z)}a_x(z)=0\,,$$

$$\sigma(\omega)=\frac{1}{i\omega}G^R_{j^xj^x}(\omega)=\frac{1}{i\omega}\frac{a_x^{(R)}}{a_x^{(S)}}=\frac{A_x'(z_{\mathrm{fin}})}{i\omega A_x(z_{\mathrm{fin}})}-\frac{1}{f'(1)}$$

Flucutation EOM II

$$A_x(z):=(1-z)^{-\frac{i\omega}{f'(1)}}a_x(z)\,,\qquad \Phi(z):=(1-z)^{-\frac{i\omega}{f'(1)}}\phi(z)\,,$$

$$\partial_z^2A_x=\zeta\,\partial_zA_x+\left(\frac{z^2\mu^2}{f}-\xi\right)A_x+\frac{iz\mu}{f}\Phi\,,$$

$$\partial_z^2\Phi=\zeta\,\partial_z\Phi+\left(\frac{\alpha^2}{f}+\frac{f'}{zf}-\xi\right)\Phi-\frac{iz\alpha^2\mu}{f}A_x\,,$$

$$\zeta:=\frac{2i\omega}{(1-z)f'(1)}-\frac{f'(z)}{f(z)}\,,\qquad \xi:=\frac{\omega^2}{f(z)^2}+\frac{i\omega}{(1-z)f'(1)}\left(\frac{i\omega}{(1-z)f'(1)}-\frac{1}{1-z}-\frac{f'(z)}{f(z)}\right)$$

AdS/Deep learning: optical conductivity

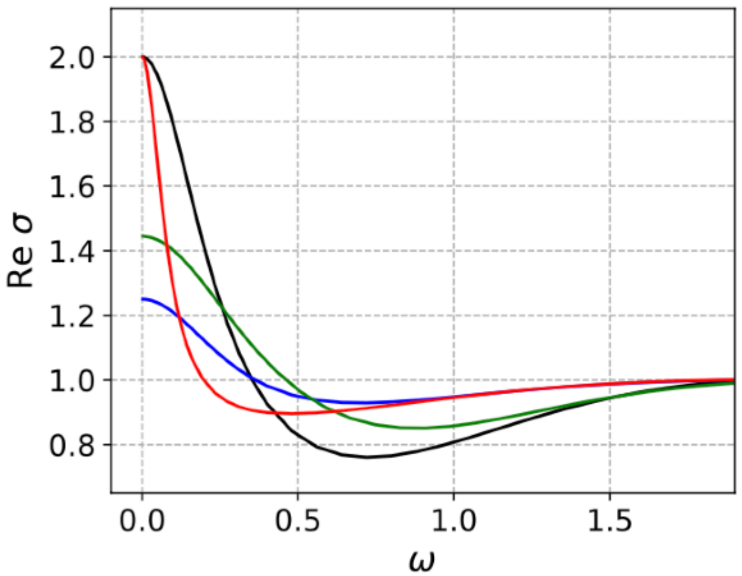
Setup

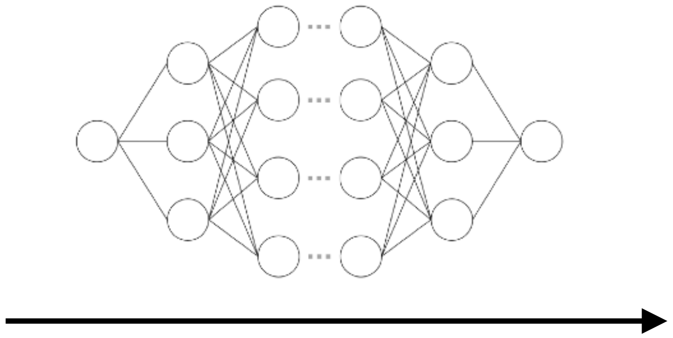
$$\mathcal{S} = \int \mathrm{d}^4x\sqrt{-g}\left(R + 6 - \frac{1}{4}F_{ab}F^{ab} - \frac{1}{2}\sum_{I=1}^{2}(\partial X_I)^2\right)$$

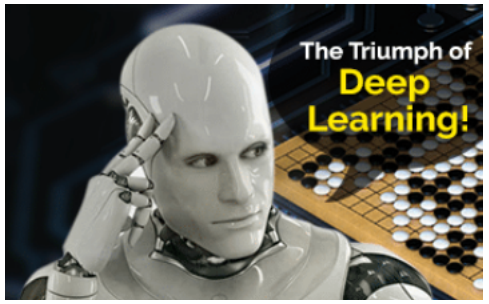
$$\mathrm{d}s^2 = \frac{1}{z^2}\left[-f(z)\mathrm{d}t^2 + \frac{\mathrm{d}z^2}{f(z)} + \mathrm{d}x^2 + \mathrm{d}y^2\right],$$

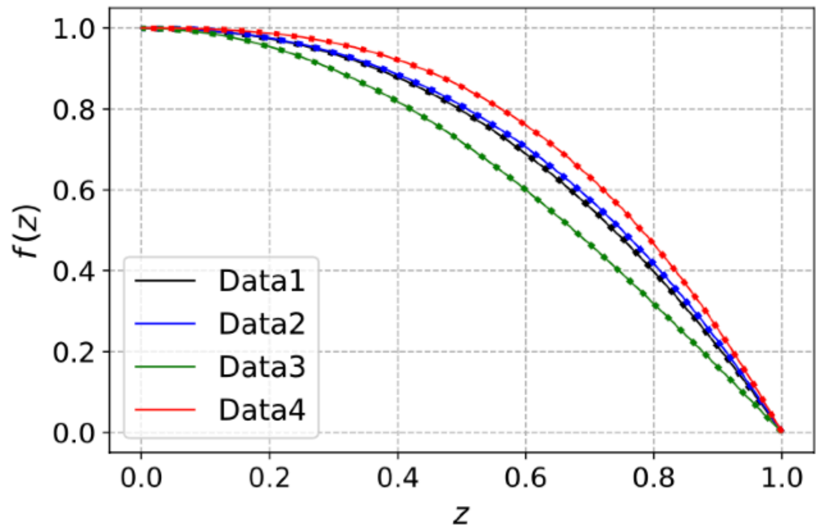
$$A = \mu\,(1-z)\,\mathrm{d}t\,, \qquad X_1 = \alpha\, x\,, \quad X_2 = \alpha\, y$$

What is the bulk metric giving the conductivity at boundary









Harder problem

arXiv > hep-th > arXiv:2502.10245

High Energy Physics – Theory

[Submitted on 14 Feb 2025]

Deep learning-based holography for T-linear resistivity

Byoungjoon Ahn, Hyun-Sik Jeong, Chang-Woo Ji, Keun-Young Kim, Kwan Yun

Science

Current Issue

HOME > SCIENCE > VOL. 377, NO. 6602 > STRANGER THAN METALS

REVIEW | PHYSICS

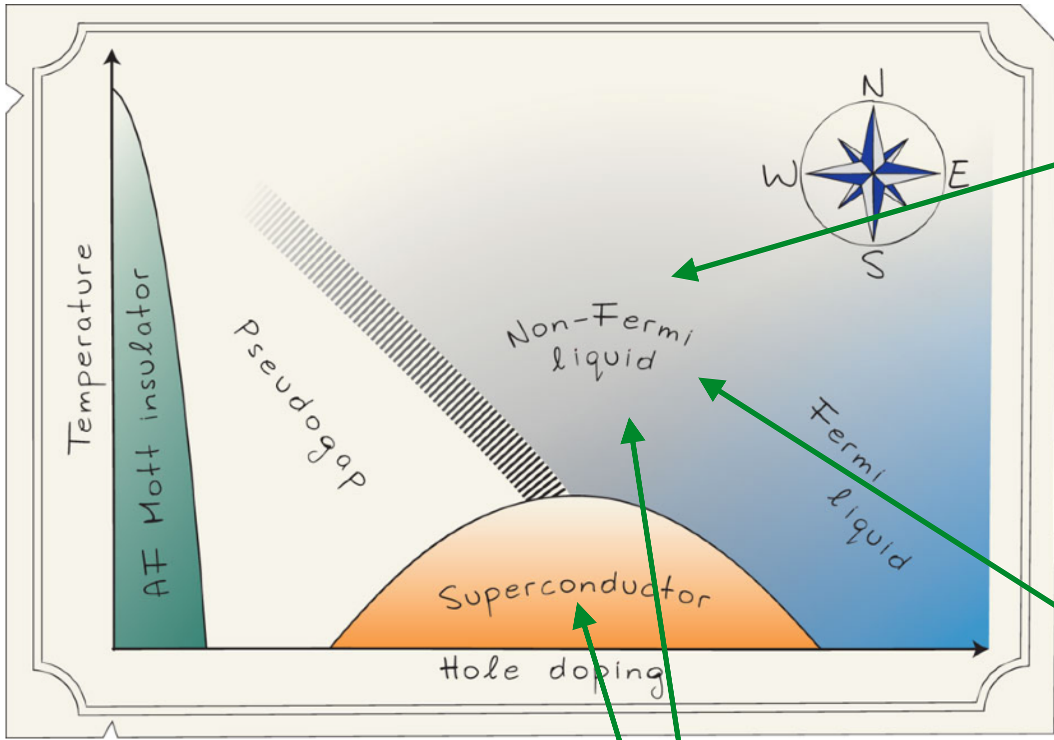
Stranger than metals

PHILIP W. PHILLIPS, NIGEL E. HUSSEY, AND PETER ABBAMONTE Authors Info & Affiliations

SCIENCE • 8 Jul 2022 • Vol 377, Issue 6602 • DOI: 10.1126/science.abh4273

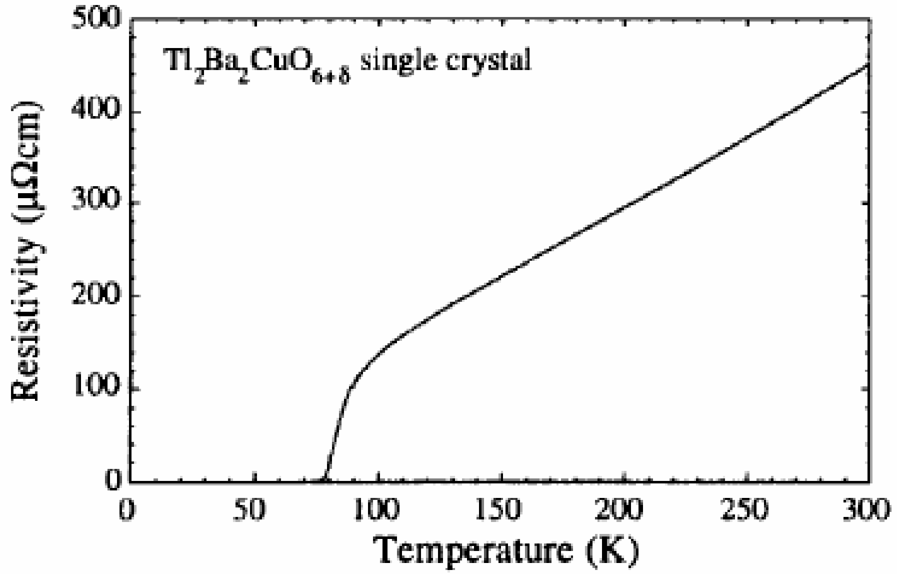
Some universal properties in CMT

Cuprate phase diagram

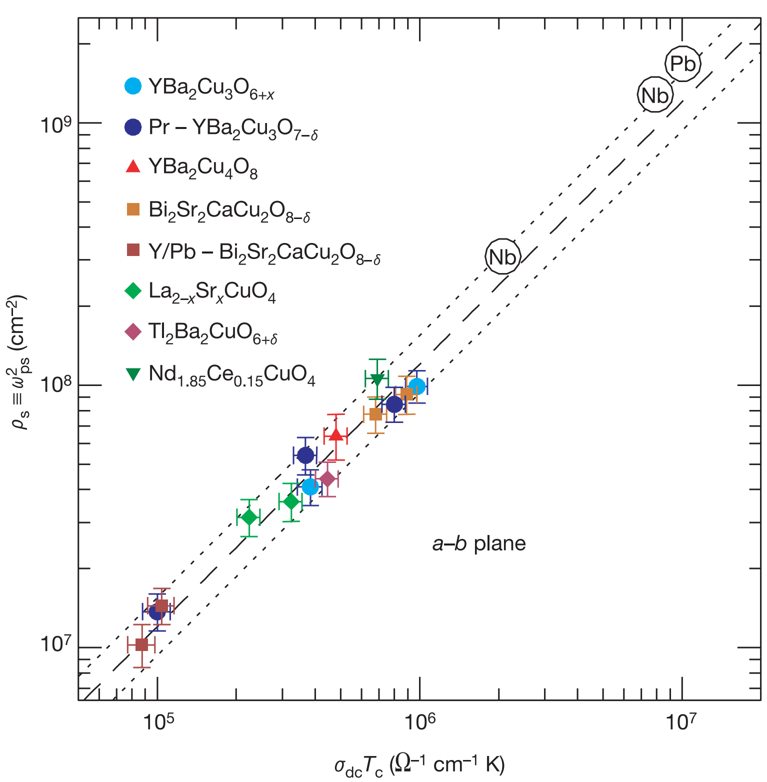


[Peter Wahl: 2012, Nature Physics]

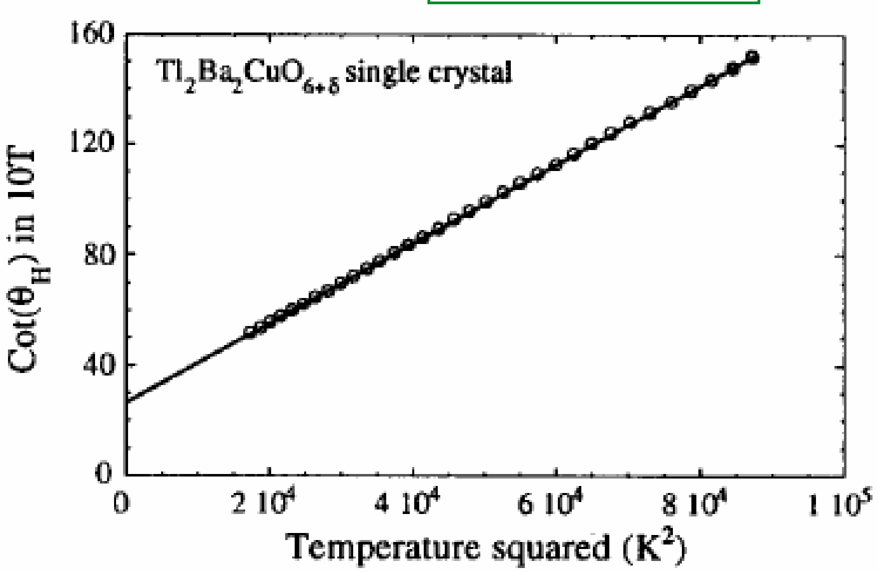
- DC resistivity $\boxed{\rho \sim T}$



- Homes law $\boxed{\rho_s(T=0) = C\sigma_{DC}(T_c)T_c}$



- Hall angle $\boxed{\sigma_{xx}/\sigma_{xy} \sim T^2}$



Mackenzie, 1997

	$\rho \propto T$ as $T \to \infty$	$\rho \propto T$ as $T \to 0$	Extended criticality	$\cot\Theta_H \propto T^2$ (at low H)	Modified Kohler's (at low H)	H-linear MR (at high H)	Quadrature MR
UD p-cuprates	$\checkmark$ [6]	$\times$ [20]	$\times$ [21]	$\checkmark$ [22]	$\checkmark$ [23]	-	-
OP p-cuprates	$\checkmark$ [4]	-	-	$\checkmark$ [24]	$\checkmark$ [25]	$\checkmark$ [26]	$\times$ [27]
OD p-cuprates	$\checkmark$ [6]	$\checkmark$ [8]	$\checkmark$ [8]	$\checkmark$ [28]	$\times$ [29]	$\checkmark$ [29]	$\checkmark$ [29]
$La_{2-x}Ce_xCuO_4$	$\times$ [30]	$\checkmark$ [31]	$\checkmark$ [31]	$\times$ [32]	$\times$ [33]	$\checkmark$ [34]	$\times$ [34]
Sr_2RuO_4	$\checkmark$ [35]	$\times$ [36]	$\times$ [37]	$\times$ [38]	$\times$ [36]	$\times$ [36]	$\times$ [36]
$Sr_3Ru_2O_7$	$\checkmark$ [10]	$\checkmark$ [10]	$\times$ [10]	$\times$	-	-	-
$FeSe_{1-x}S_x$	$\times$ [39]	$\checkmark$ [40]	$\times$ [40]	$\checkmark$ [41]	$\checkmark$ [41]	$\checkmark$* [42]	$\checkmark$* [42]
$BaFe_2(As_{1-x}P_x)_2$	$\times$ [43]	$\checkmark$ [44]	$\times$ [44]	-	$\checkmark$ [45]	$\checkmark$ [46]	$\checkmark$ [46]
$Ba(Fe_{1/3}Co_{1/3}Ni_{1/3})_2As_2$	-	$\checkmark$ [47]	$\times$ [47]	-	-	$\checkmark$ [47]	$\checkmark$ [47]
$YbRh_2Si_2$	$\times$ [48]	$\checkmark$ [49]	$\checkmark$ [50]	$\checkmark$ [51]	-	-	-
$YbBAl_4$	$\times$ [52]	$\checkmark$** [52]	$\checkmark$** [52]	-	-	-	-
$CeCoIn_5$	$\times$ [53]	$\checkmark$ [54]	$\times$ [54]	$\checkmark$ [53]	$\checkmark$ [53]	-	-
$CeRh_6Ge_4$	$\times$ [55]	$\checkmark$ [55]	$\times$ [55]	-	-	-	-
$(TMTSF)_2PF_6$	-	$\checkmark$ [56]	$\checkmark$ [56]	-	-	-	-
MATBG	$\checkmark$ [57]	$\checkmark$ [58]	$\checkmark$ [58]	$\checkmark$ [59]	-	-	-

a

	$\rho \propto T$ as $T \to 0$	$\rho \propto T$ as $T \to \infty$	$\sigma \propto \omega^{-2/3}$	Quadrature MR	Extended criticality	Experimental Prediction
Phenomenological						
MFL	$\checkmark$ [65]	$\times$[65]	$\times$	$\times$	$\times$	loop currents [104]
EFL	- [b]	-	-	$\times$	$\times$	loop currents [105]
Numerical						
ECFL	$\times$	$\checkmark$[106]	-	-	$\times$	$\times$
HM (QMC/ED/CA)	- [107]	$\checkmark$ [107–111]	$\times$	-	-	-
DMFT/EDMFT	$\checkmark$ [112]	$\checkmark$[113, 114]	$\times$	-	$\checkmark$ [114]	-
QCP	$\checkmark$[115]	-	-	-	$\times$	-
Gravity-based						
SYK	$\checkmark$ [116, 117]	$\checkmark$[c] [117]	$\times$	$\checkmark$[d] [118]	-	$\times$
AdS/CFT	$\checkmark$ [119]	$\checkmark$ [119]	$\checkmark$[e] [88, 123]	$\times$	$\times$	$\times$
AD/EMD	$\checkmark$ [124–126]	$\checkmark$ [88, 123, 124, 126, 127]	$\checkmark$ [88, 123, 127]	$\times$	$\checkmark$[123]	Fractional A-B [126]

No concrete holography model of "T-linear resistivity + T^2 -Hall angle together" yet, even though there are many interesting holography models partly successful?

ML Example: Holographic model

EMD(Einstein Maxwell Dilaton) model [ArXiv:1005.4690][hep-th], [ArXiv:1401.5436][hep-th]

$$S = \int \mathrm{d}^{p+1}x\,\sqrt{-g}\left[R - \frac{1}{2}\partial\phi^2 - \frac{1}{4}Z(\phi)F^2 + V(\phi) - \frac{1}{2}Y(\phi)\sum_{i=1}^{p-1}\partial\psi_i^2\right].$$

$\longrightarrow$ Many variations

$$Z(\phi) \sim e^{\gamma\phi}, \qquad V(\phi) \sim V_0 e^{-\delta\phi}, \qquad Y(\phi) \sim e^{\lambda\phi}$$

$$\mathrm{d}s^2 = r^{\frac{2\theta}{p-1}}\left[-f(r)\frac{\mathrm{d}t^2}{r^{2z}} + \frac{L^2\mathrm{d}r^2}{r^2 f(r)} + \frac{\mathrm{d}\vec{x}^2}{r^2}\right], \quad A = Qr^{\zeta-z}\mathrm{d}t\,, \quad \phi = \kappa\ln r$$

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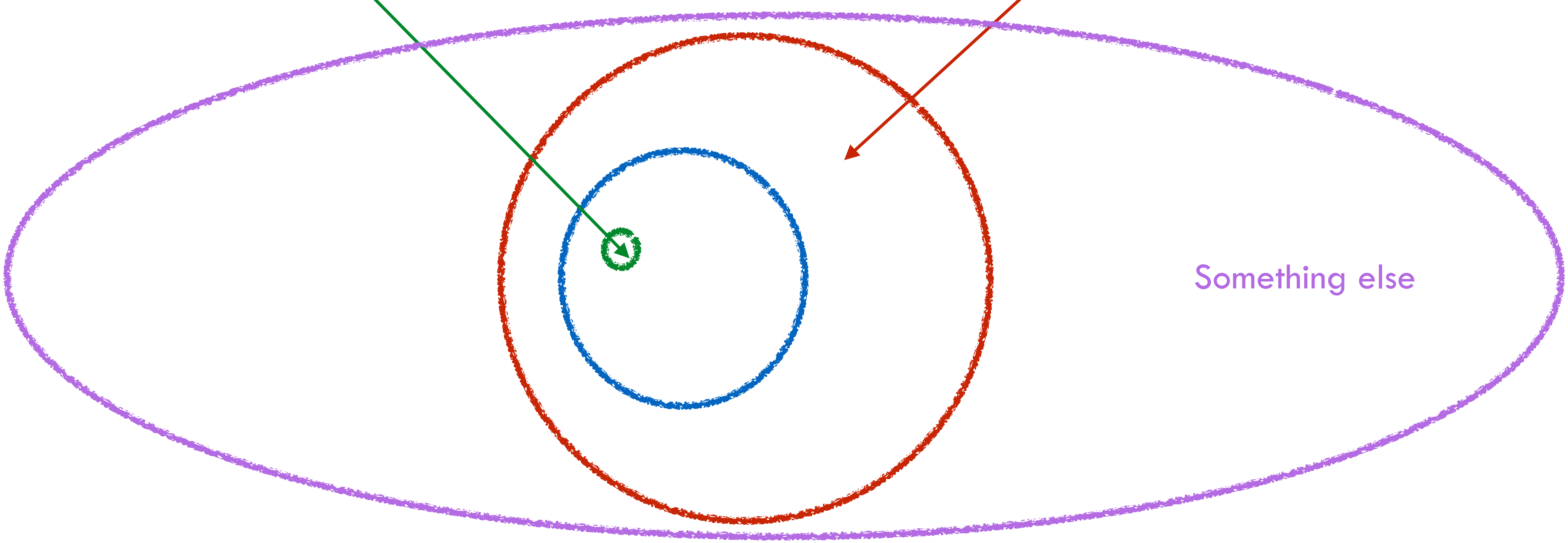

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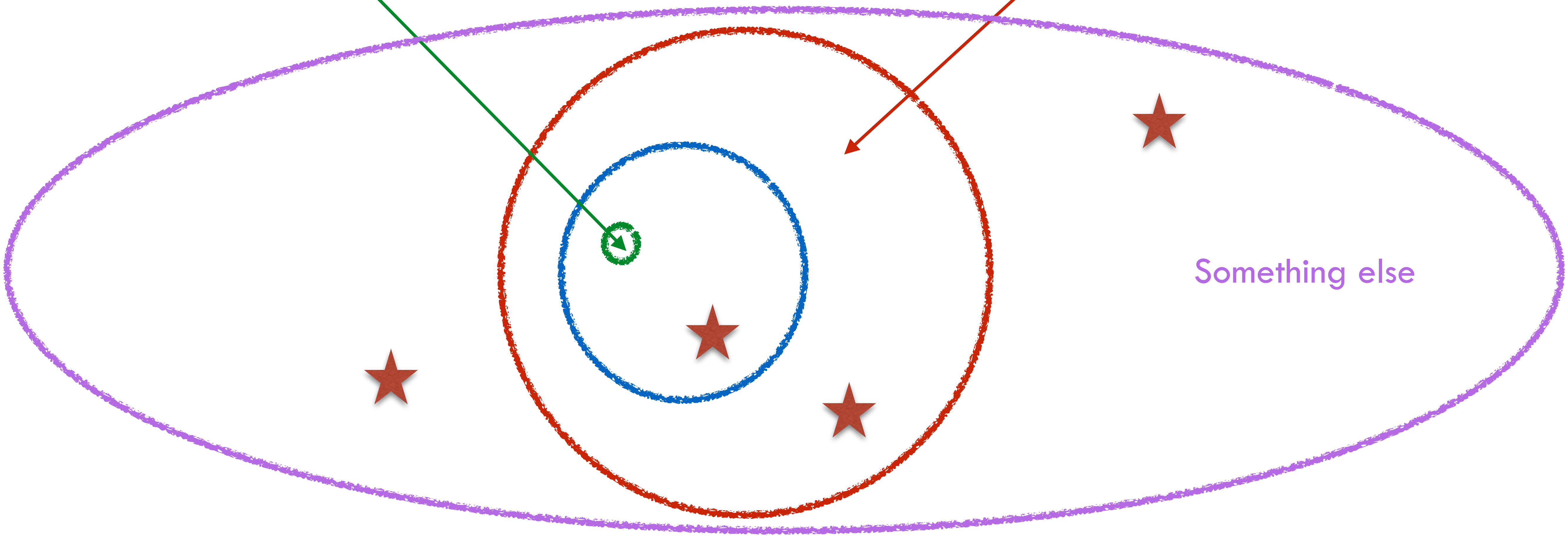

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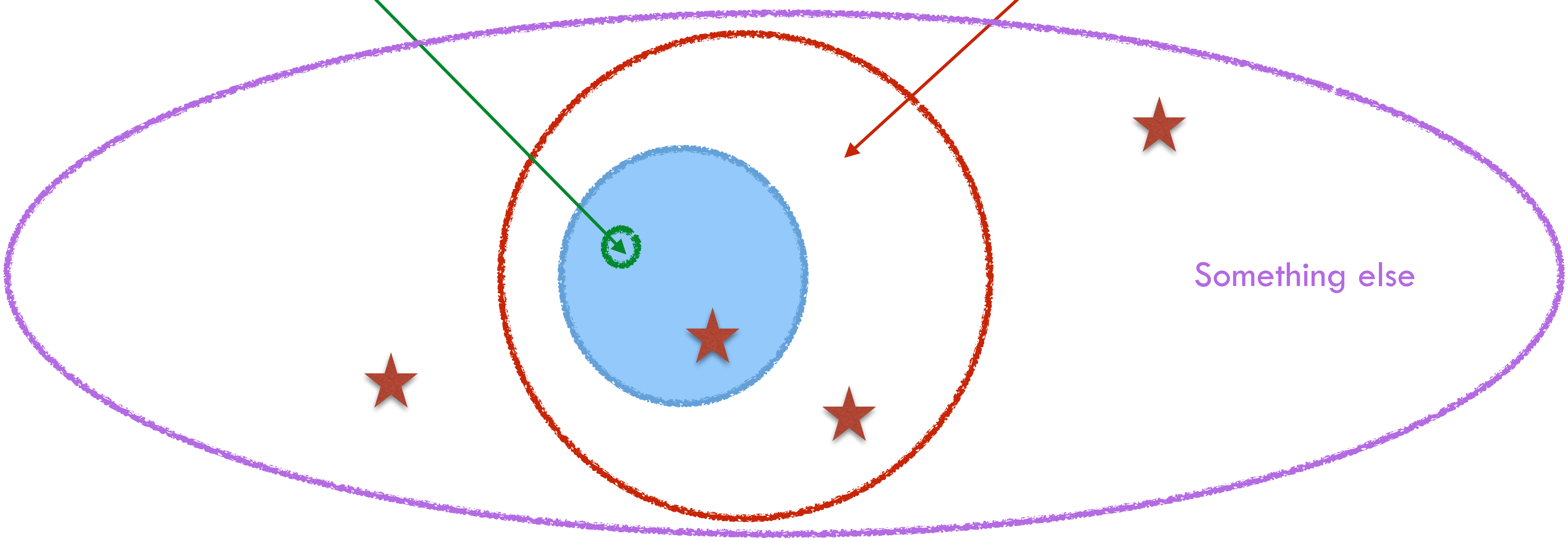


Deep Learning for ODE: classical mechanics

physics informed neural networks

1. (Data)

$$(v_i,\ V_f) \longrightarrow$$

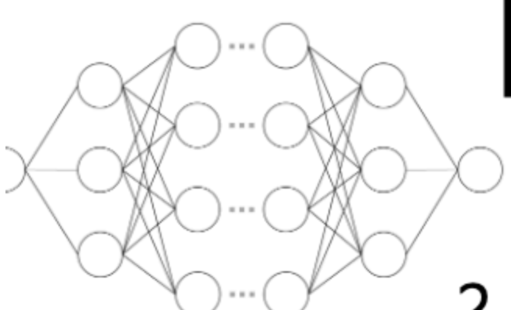
3. (Loss)

$$L = \text{avg}_t(\ |eom(t)|\ ) + |v(t_i) - v_i| + |v(t_f) - v_f| \\ + |x'(t) - v(t)| + |x(t_i) - x_i| + |x_f|$$

$$eom(t) = mv'(t) - \big(mg - F\big(x(t)\big)\big) = 0$$

$$\frac{\partial L}{\partial W_n}, \frac{\partial L}{\partial b_n}$$

$$F(x) = D_3(x; W_n, b_n)$$



$$x(t) = D_1(t; W_n, b_n)$$

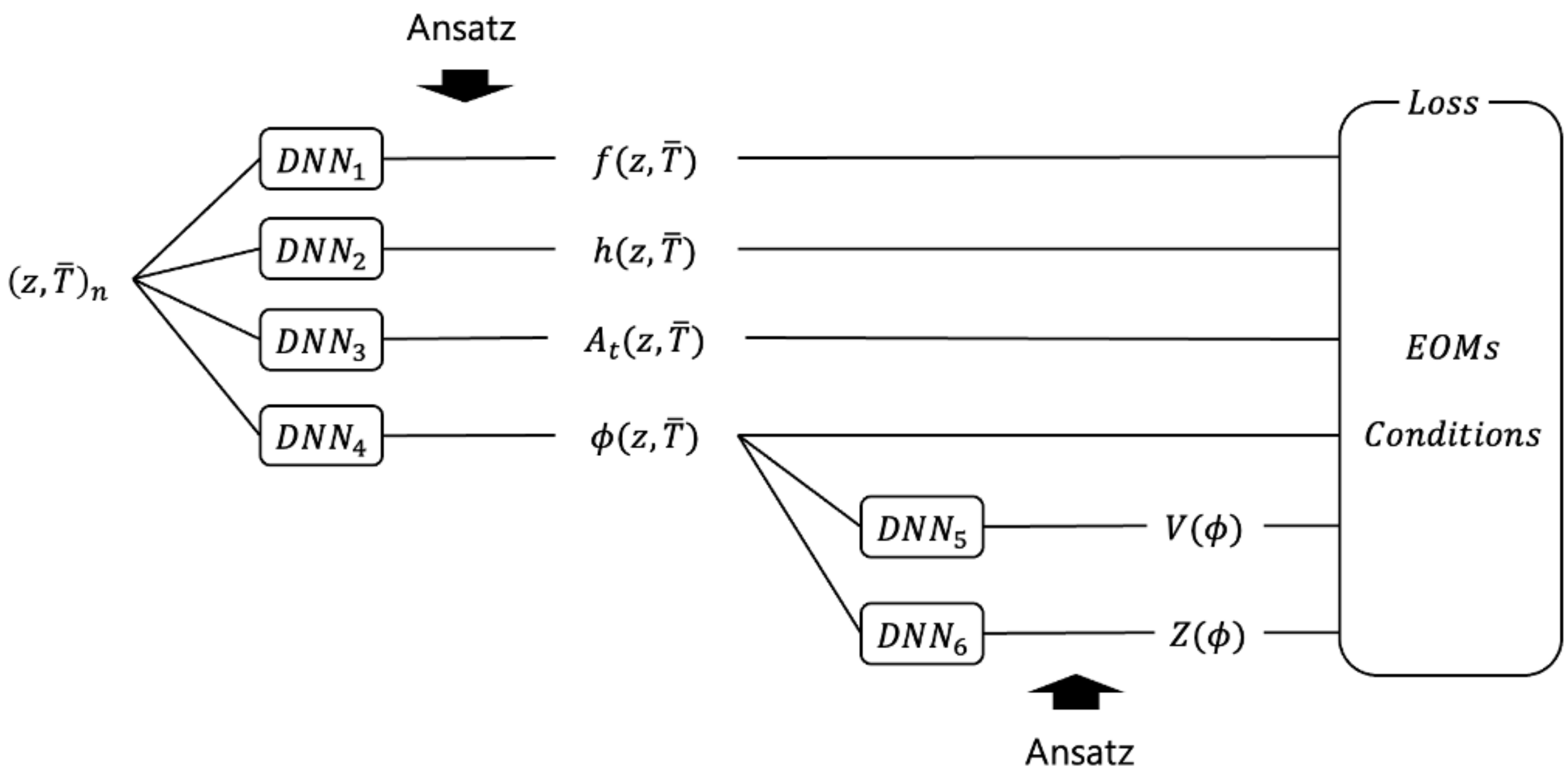
$$v(t) = D_2(t; W_n, b_n)$$

2. (Deep neural networks)

4. (Parameters optimizing)

$$\frac{\partial L}{\partial W_n}, \frac{\partial L}{\partial b_n}, \frac{\partial L}{\partial W_n}, \frac{\partial L}{\partial b_n}$$

Towards holographic strange model



Ansatz

$f_b = 1$

$f_h = 0$

$f'_h = -4\pi T$

$h_b = 1$

$h_h = \frac{s}{4\pi}$

$A_{t_b} = \mu$

$A_{t_h} = 0$

$\phi_b = 0$

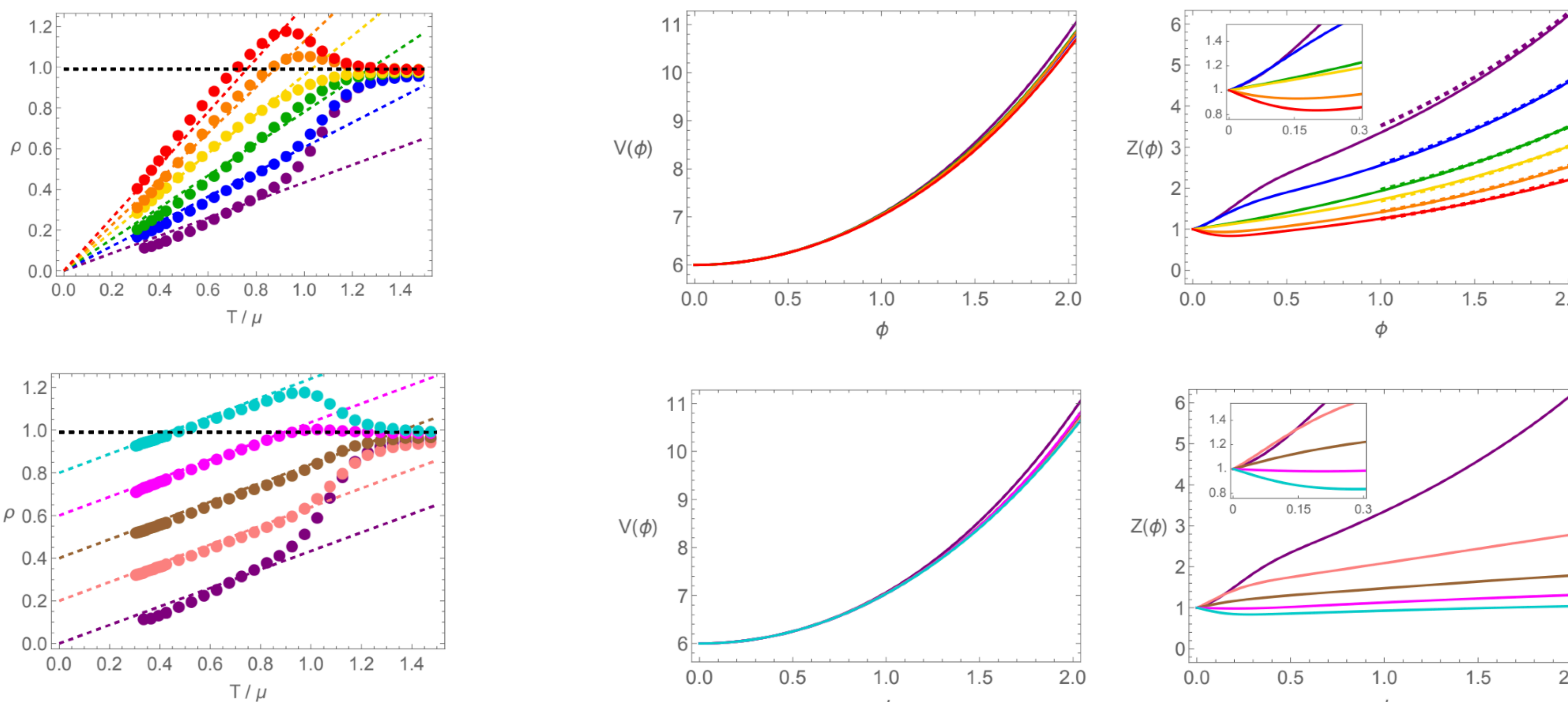
$V_b = -6$

$Z_b = 1$

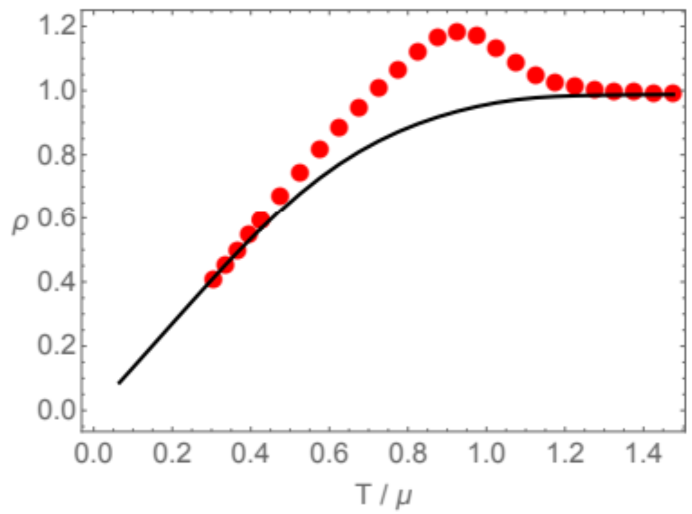
$$Loss = \frac{1}{N}\sum_{n=1}^{N}\left(\sum_{i=1}^{4} \left| EOM_i((z,\bar{T})_n) \right| + \sum_j \left| Condition_j((z,\bar{T})_n) \right| \right) + L_{etc}$$

$$S=\int \mathrm{d}^{p+1}x\,\sqrt{-g}\left[R-\frac{1}{2}\partial\phi^2-\frac{1}{4}Z(\phi)F^2+V(\phi)-\frac{1}{2}Y(\phi)\sum_{i=1}^{p-1}\partial\psi_i^2\right].$$

Gubser Rocha model $V(\phi)=6\cosh\frac{\phi}{\sqrt{3}}$, $Z(\phi)=e^{\frac{\phi}{\sqrt{3}}}$ $Y(\phi)=1$

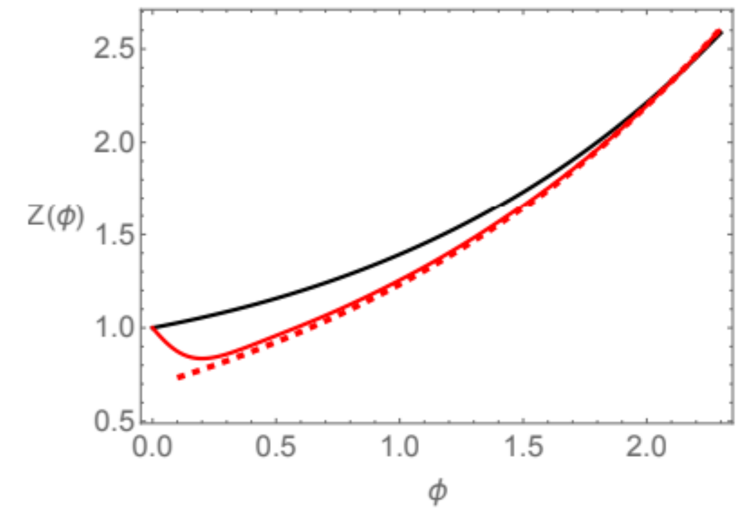


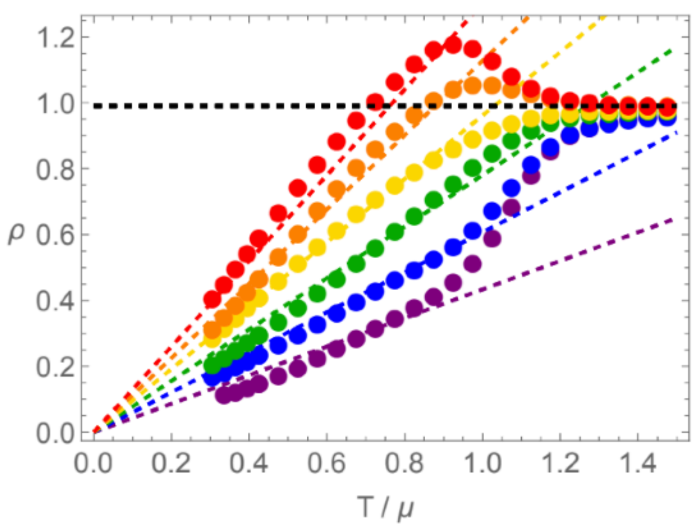
Towards holographic strange model

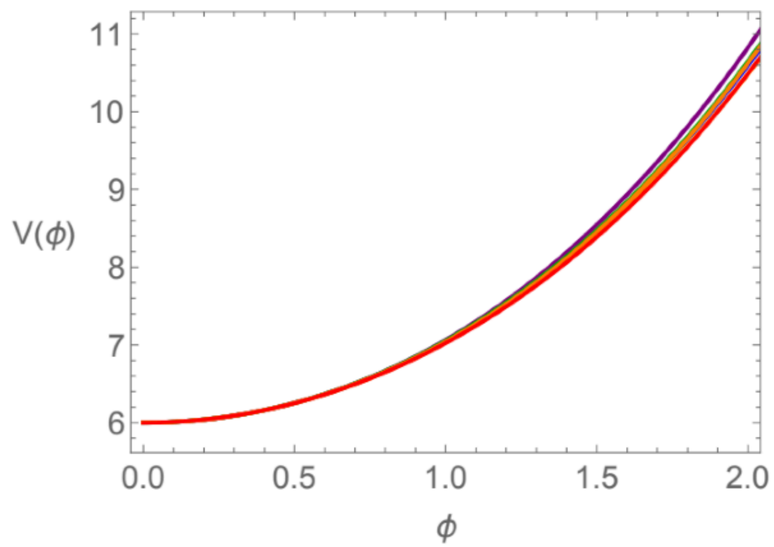


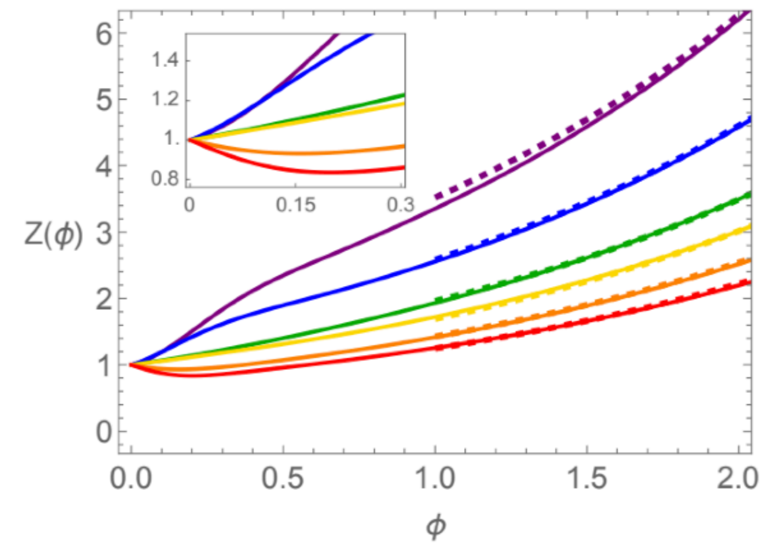
$$= \int \mathrm{d}^{p+1}x\,\sqrt{-g}\,\Big[R - \frac{1}{2}\partial\phi^2 - \frac{1}{4}Z(\phi)F^2 + V(\phi) - \frac{1}{2}Y(\phi)\sum_{i=1}^{p-1}$$

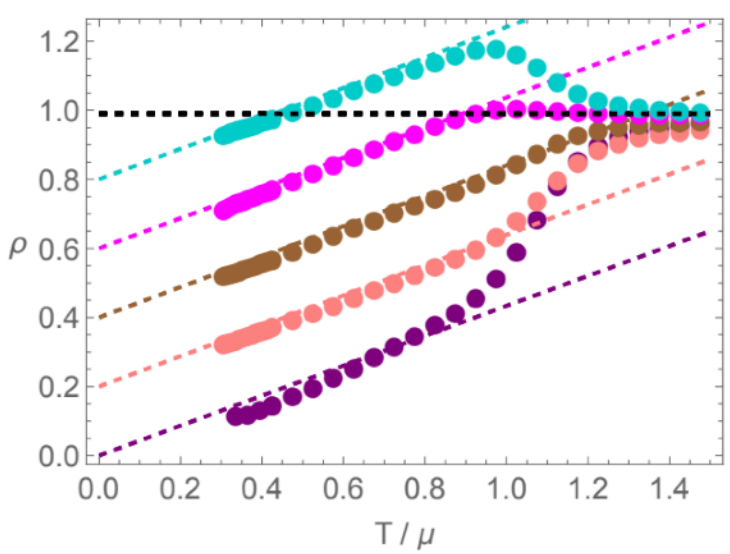
$$V(\phi) = 6\cosh\frac{\phi}{\sqrt{3}}\,, \qquad Z(\phi) = e^{\frac{\phi}{\sqrt{3}}}$$

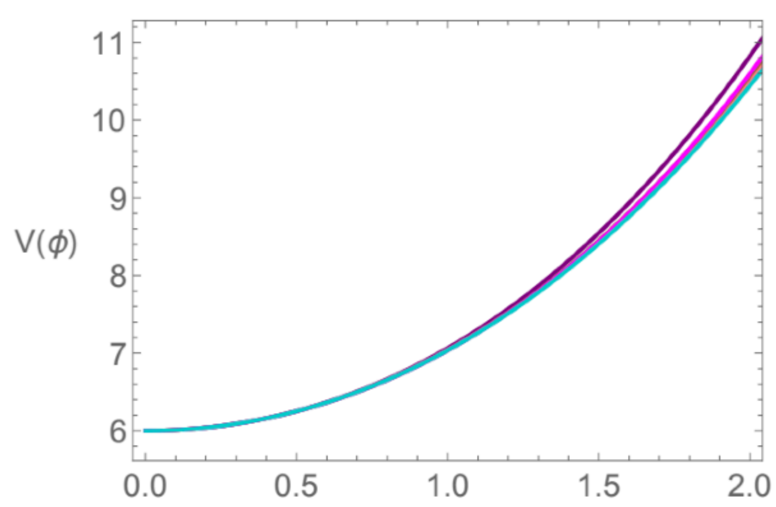


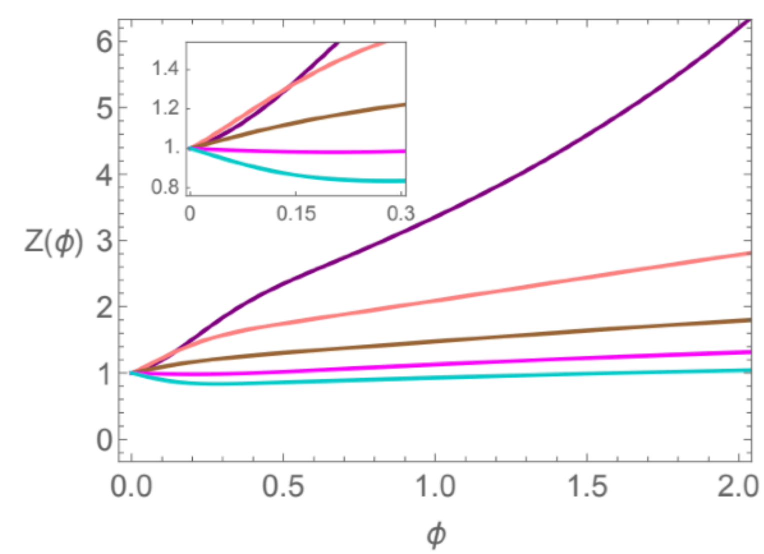












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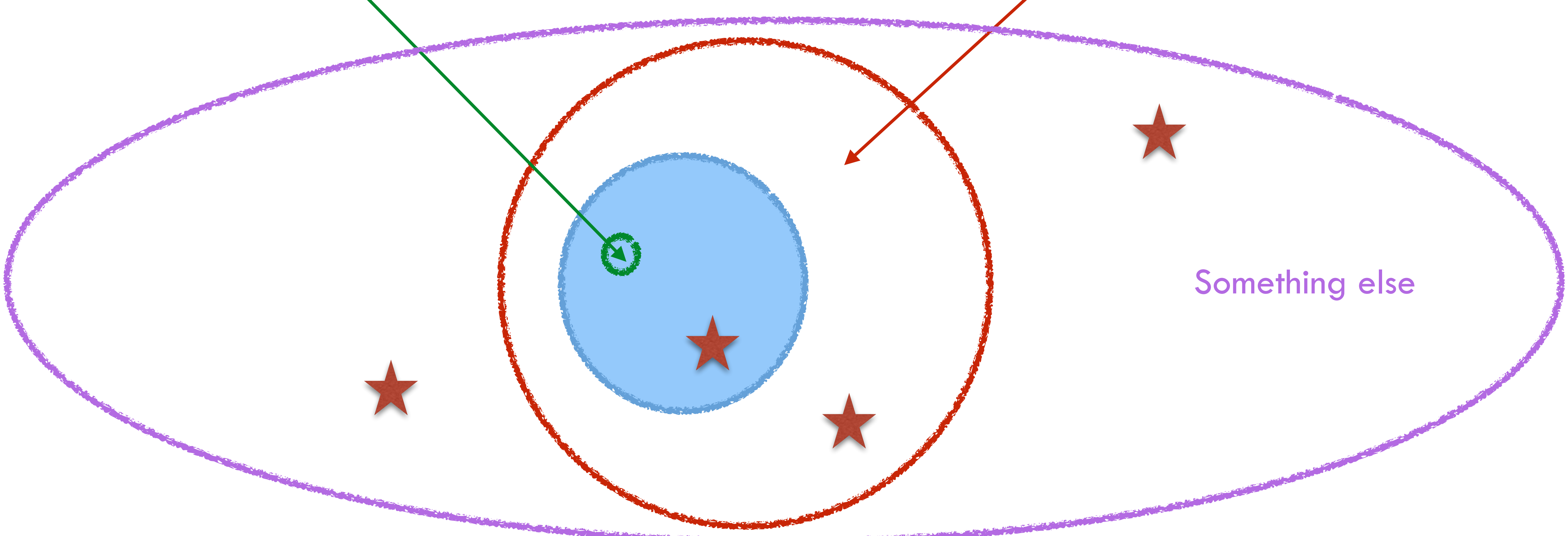

Ongoing and future work

- Methodology development

  - ResNet,

  - Neural ODE, Neural integral

  - PINN (Physics Informed Neural Network)

  - PDE
- Other physical quantities

  - ARPES: Fermionic spectral function

  - Quantum info: complexity, entanglement entropy, etc

  - Applications to other physics problems (including ODE, PDE, Integral)
- Figuring out action itself for a specific problem

  - so far, the form of the action is fixed

  - Linear T resistivity + T^2 Hall angle together

Job opportunity

asia pacific center for theoretical physics

Prof. Hyun-Sik Jeong
Junior Research Group Leader

Looking for a postdoc who will work on physics (holography) by machine learning
From Sep. 2025

apctp.hepth@gmail.com

Thank you