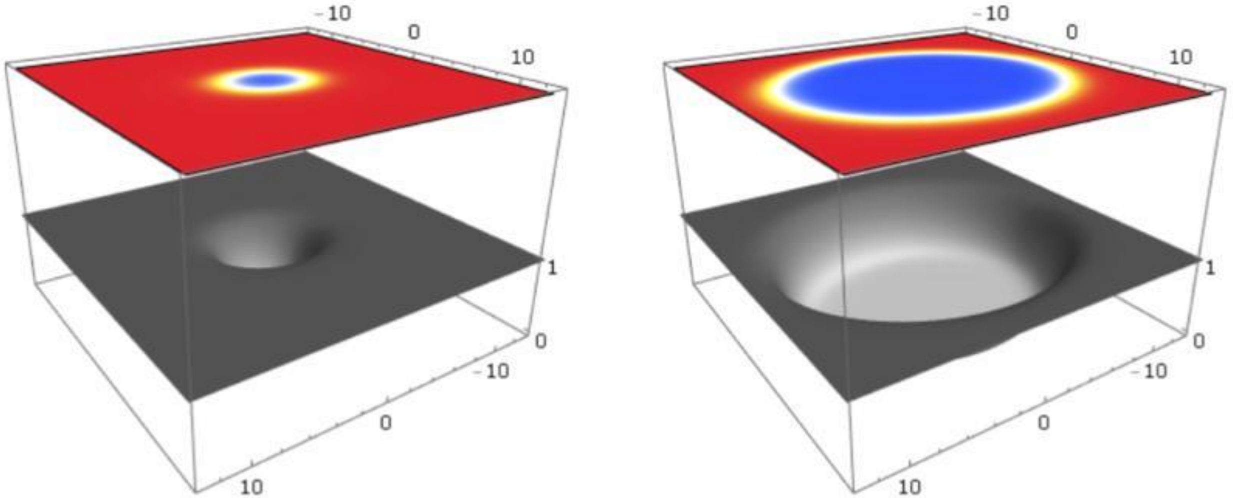
HOLOGRAPHIC BUBBLE NUCLEATION IN ASTROPHYSICS OR THE EARLY UNIVERSE

PHASE TRANSITIONS IN A COOLING ENVIRONMENT



With Javier Subils and David Mateos

Reference: 2508.xxxxx

Wilke van der Schee

Holographic applications:
from Quantum Realms to the Big Bang
11 – 19 July 2025, UCAS Beijing

OUTLINE

First order phase transitions

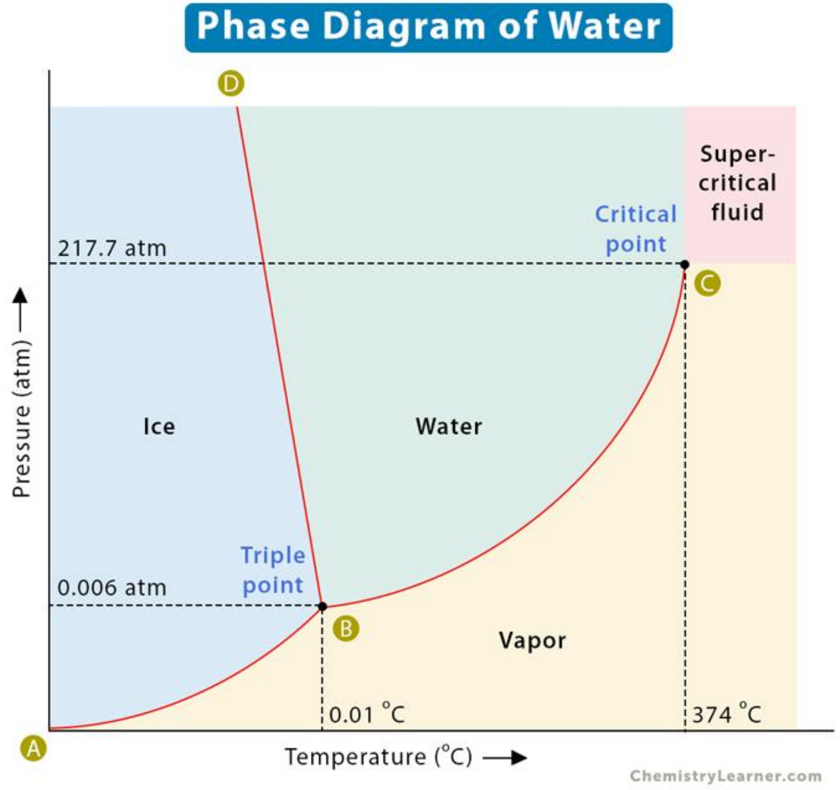
- Sources: early universe, astrophysics
- Signature: gravitational waves (mHz, MHz)
- A window into the Planck scale?

Dynamics

- Bubble nucleation and false vacua
    - Bubble velocities and shapes

Critical bubbles

- Determines the minimal size for relaxation to true vacuum
    - Technical: de Turck trick at constant temperature
    - Action, tunneling and the nucleation rate
    - Comparison with effective field theory



SUPERCOOLED

Fun fact

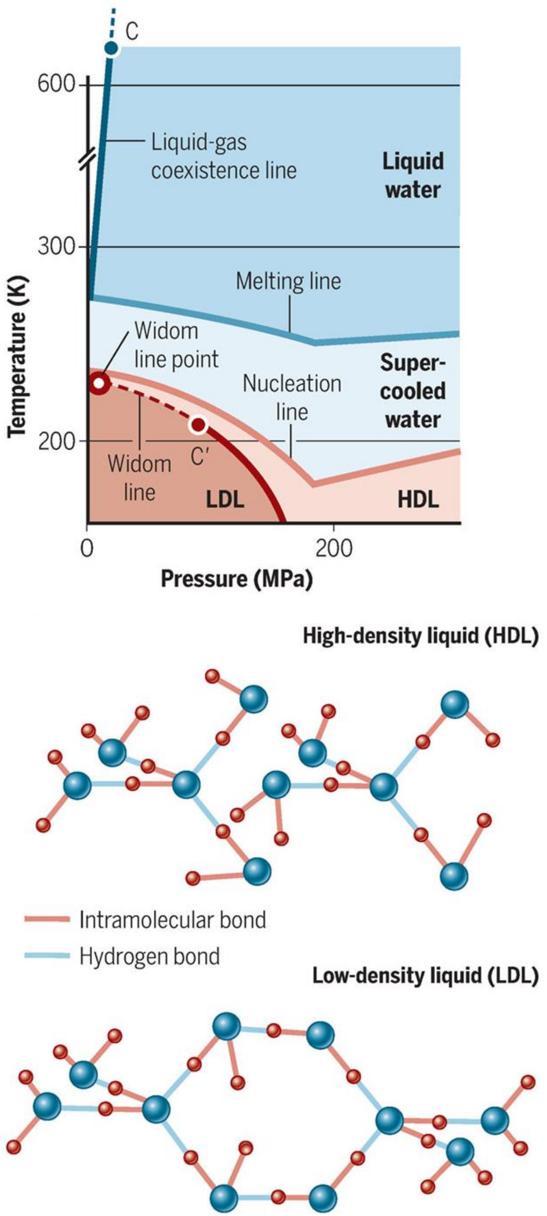
- Supercooled water at –44 degrees, one micrometer
- X-ray femtoscopy identifies high and low density liquids
- A second critical point in the supercooled region ☺

More seriously

- If the Standard Model* has a first order phase transition (FOPT):
  - Universe cools down fast, but extremely homogeneous
  - Universe will supercool beyond critical temperature
  - Bubbles will nucleate, but at what rate? How?
  - *technically this is then ***beyond*** standard model (quite some examples)
    - Really unique opportunity to probe extremely high energy scales

- If QCD has a critical point:
  - FOPT potentially present during neutron star mergers

Experimental confirmation
Kim *et al.* provide experimental evidence for the Widom line and, hence, of a second-order critical point in the supercooled region of the water phase diagram.



Paola Gallo and H. Eugene Stanley, Supercooled water reveals its secrets (2017)

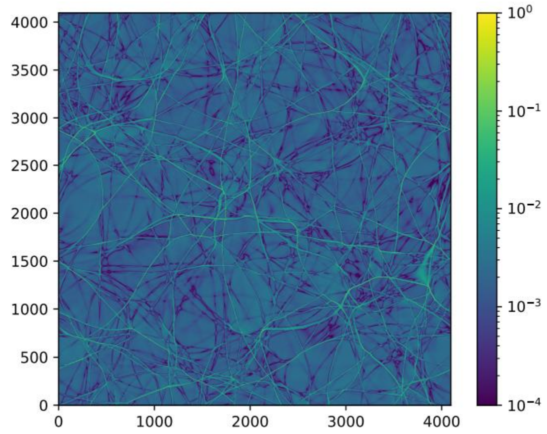
BUBBLE COLLISIONS

Bubble collisions in the Early Universe or in Astrophysics

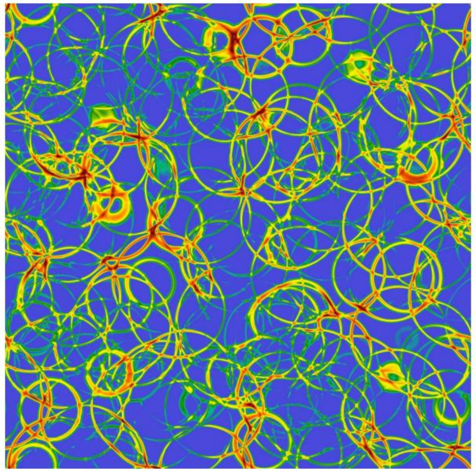
- If the Standard Model has a first order phase transition (FOPT):
  - Universe cools down fast, but extremely homogeneous
  - Universe will supercool beyond critical temperature
  - Bubbles will nucleate, but at what rate? How?

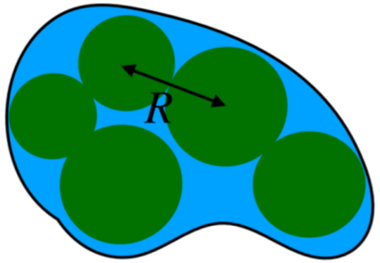
- FOPT potentially present during neutron star mergers
  - Merger could cause both superheating and supercooling (subcooling?)

- Collisions of bubbles: unique signature in gravitational waves (!)
  - LISA can potentially measure these (mHz)



(c) $|\nabla \cdot \mathbf{v}|(x, y, z = 0)$





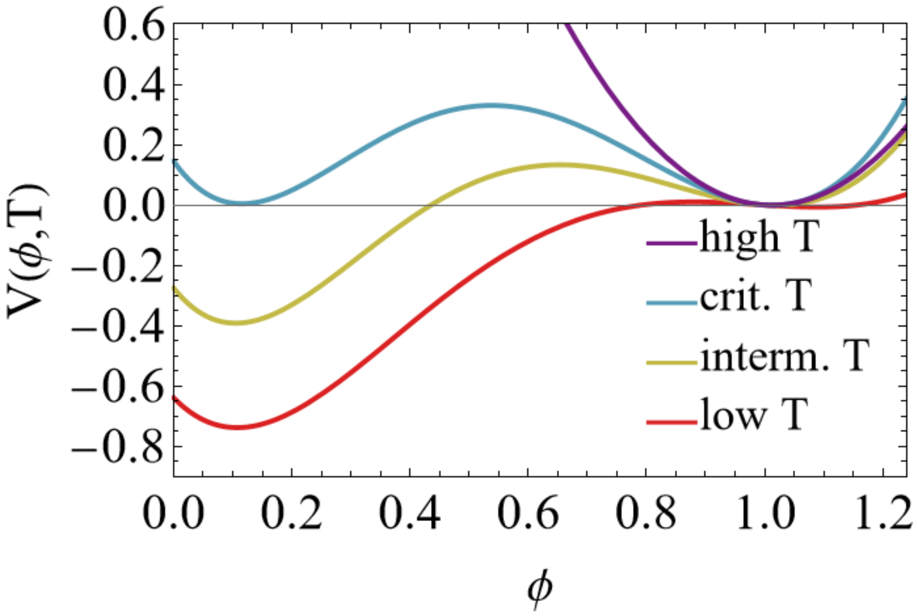
Andreas Ekstedt, Oliver Gould, Joonas Hirvonen, Benoit Laurentd, Lauri Niemic, Philipp Schichoe and Jorinde van de Vis,
How fast does the WallGo? A package for computing wall velocities in first-order phase transitions (2024)
Mark Hindmarsh, Stephan J. Huber, Kari Rummukainen and David J. Weir, Numerical simulations of acoustically generated gravitational waves at a first order phase transition (2015)
Jorge Casalderrey-Solana, David Mateos, Mikel Sanchez-Garitaonandia, Mega-Hertz Gravitational Waves from Neutron Star Mergers (2022)

NUCLEATION IN EFFECTIVE FIELD THEORY

An effective potential at constant temperature:

$$S(T) = \int_0^\beta \mathrm{d}\tau \int \mathrm{d}^3x \left(\frac{1}{2}Z(\varphi,T)((\partial_\tau\varphi)^2 + (\nabla\varphi)^2) + V_{\mathrm{eff}}(\varphi,T)\right)$$

$$V_{\mathrm{eff}}(\varphi,T) = \frac{1}{2}\gamma(T^2 - T_0^2)\varphi^2 - \frac{1}{3}\alpha T\varphi^3 + \frac{1}{4}\lambda\varphi^4$$

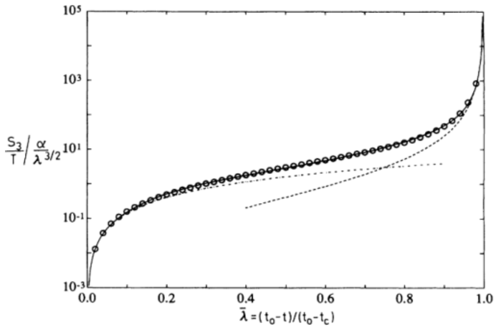


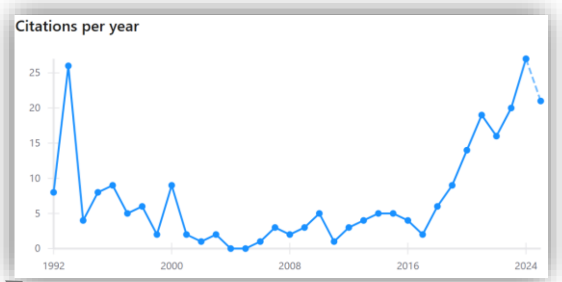
Three regimes

- High temperature: one stable hot phase
- Intermediate ($T_0 < T < T_c$): two (meta)stable phases
- Low temperature: one stable cold phase

Cooling the system:

- Below T_c and above T_0 the system can tunnel to the true minimum (supercooled)



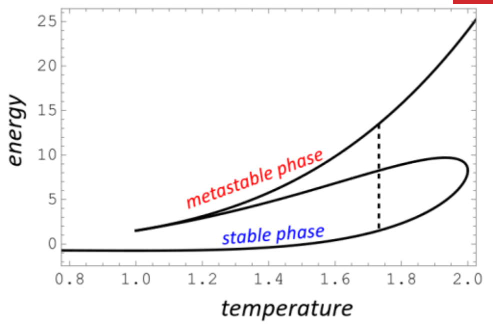


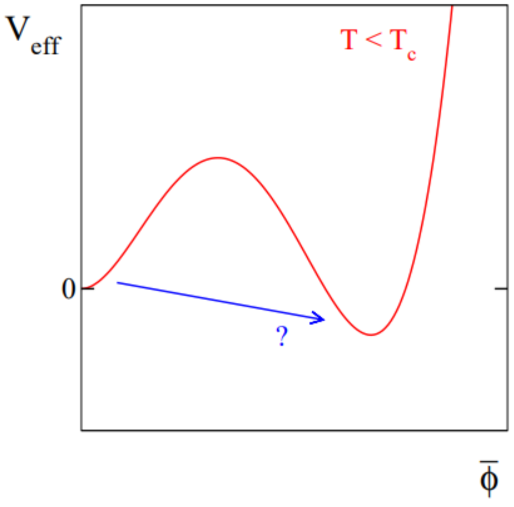
K. Enqvist, J. Ignatius, K. Kajantie and K. Rummukainen, Nucleation and bubble growth in a first-order cosmological electroweak phase transition (1992)

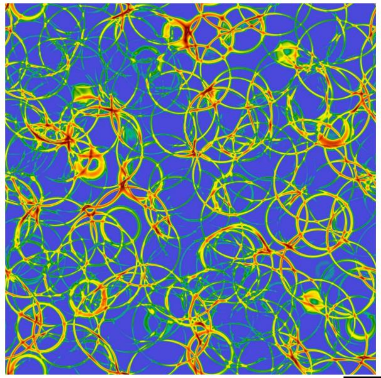
See also talk by Zhenhan Jin, Tue 18:20

See also talk by Hao-Tian Sun, Mon 17:55

BUBBLE NUCLEATION AND GROWTH







Vacuum-like decay

- Wall velocity can accelerate to the speed of light

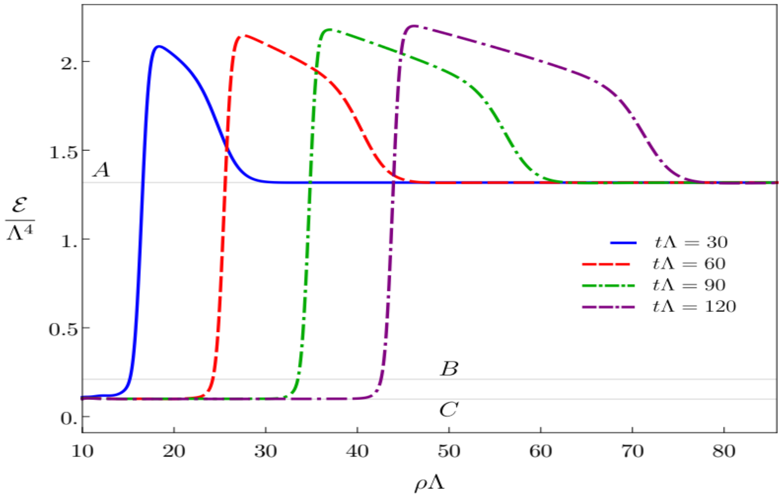
Thermal-like decay

A conjectured result for the nucleation rate is thus

$$\frac{\Gamma}{V}\bigg|_{\text{high }T} \simeq \left(\frac{\lambda_{-}}{2\pi}\right)\left(\frac{\hat{S}_{3d}}{2\pi T}\right)^{\frac{3}{2}}\left|\frac{\det'[-\nabla^2+V''(\hat{\phi})]}{\det[-\nabla^2+V''(0)]}\right|^{-\frac{1}{2}} e^{-\beta\hat{S}_{3d}}\,. \tag{9.80}$$

It should be stressed, however, that the simplistic approach based on the negative eigenmode λ_{-} does not really give a theoretically consistent answer [9.22, 9.23]; rather, we should understand the above analysis in the sense that a rate exists, and the formulae as giving its order of magnitude.

- Deflagration: subsonic detonation
- Friction from the remaining plasma
- Energy density (typically) needs to be balanced
- This means a wall that becomes wider with time:



Mikko Laine and Aleksi Vuorinen, Basics of Thermal Field Theory, A Tutorial on Perturbative Computations (2017)
Yago Bea, Jorge Casalderrey-Solana, Thanasis Giannakopoulos, Aron Jansen, David Mateos, Mikel Sanchez-Garitaonandia and Miguel Zilhão, Holographic Bubbles with Jecco: Expanding, Collapsing and Critical (2022)

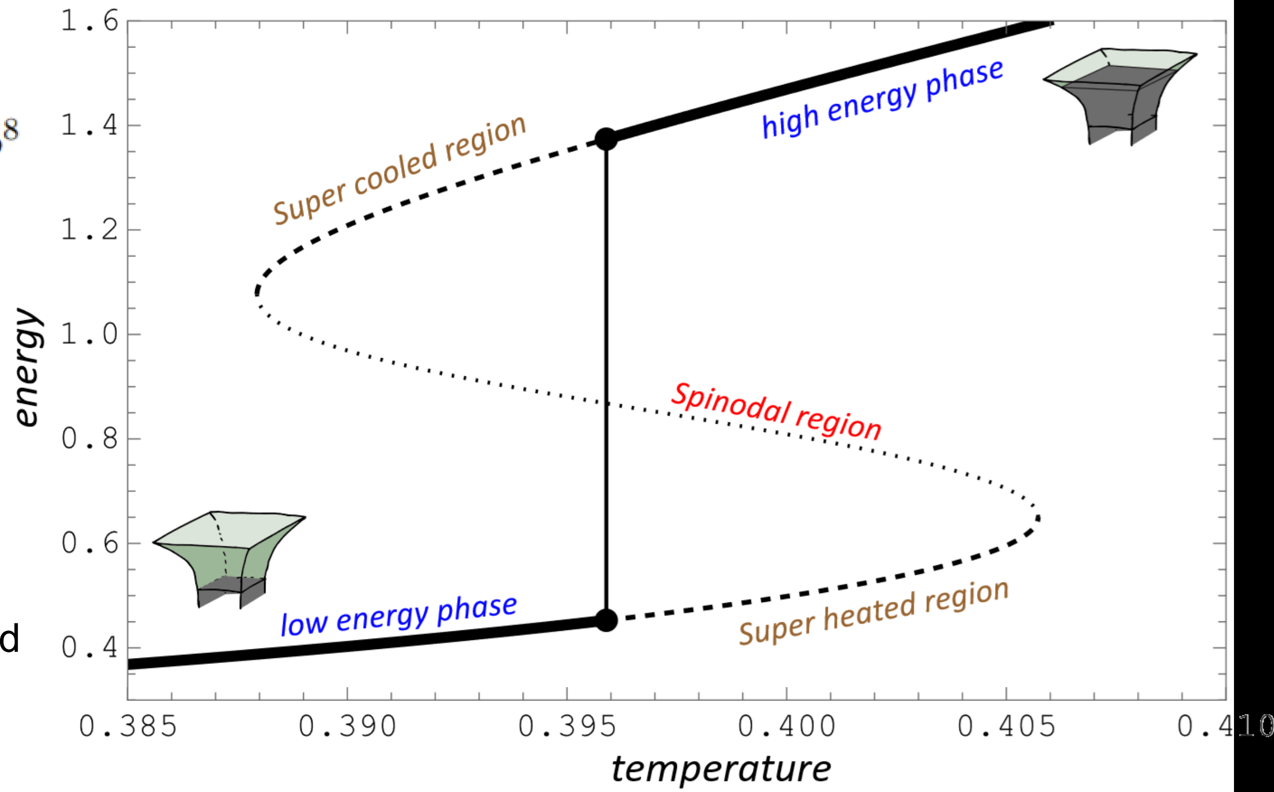
SUPERCOOL HOLOGRAPHY

A non-conformal holographic toy model

$$S = \frac{2}{8\pi G}\int_{\mathcal{M}} d^5x\sqrt{-g}\left(\frac{1}{4}R[g] - \frac{1}{2}(\partial\phi)^2 - V(\phi)\right)$$

$$L^2 V(\phi) = -3 - \frac{3}{2}\phi^2 - \frac{1}{3}\phi^4 + \left(\frac{1}{3\phi_M^2} + \frac{1}{2\phi_M^4}\right)\phi^6 - \frac{1}{12\phi_M^4}\phi^8$$

- Source for scalar provides extra scale (j = 1)



The universe cools; what happens?

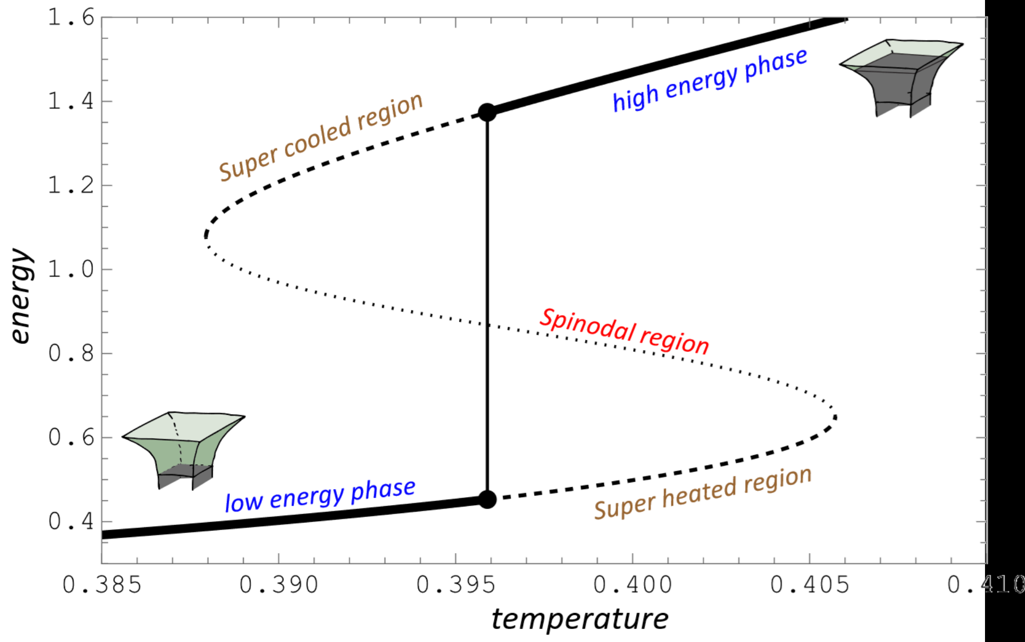
- First it becomes supercooled (metastable)
- If no bubbles form: it enters spinodal branch
  - Negative specific heat, imaginary speed of sound
  - Gubser-Mitra: unstable (Gregory-Laflamme)
  - System phase separates
- If bubbles form:
  - They keep on colliding (gravitational waves!)
  - If given enough time only solution is phase separated at critical temperature (unlike water)
- Finally enters low energy phase

Yago Bea and David Mateos, Heating up Exotic RG Flows with Holography

SUPERCOOLED AND CRITICAL HOLOGRAPHY

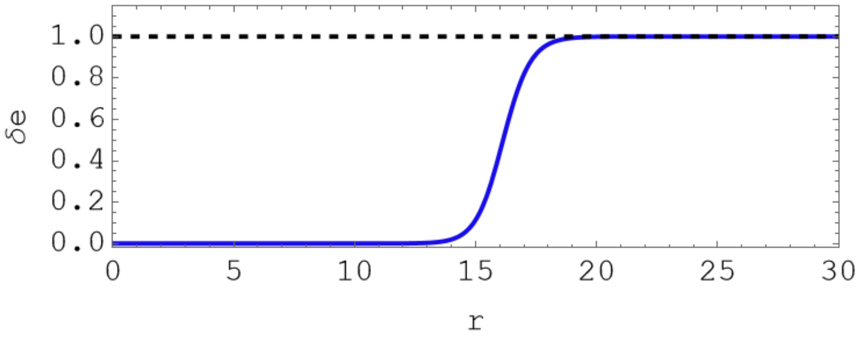
Bubble nucleation

- Small bubbles disappear (surface tension is too expensive)
- Large bubbles can expand
  - Somewhat non-trivial, but Gubser-Mitra implies that the end state is the phase separated state at the critical temperature
  - Interesting question if this is possible in an expanding universe (but for now we're in a box ☺)



A critical bubble

- An unstable time-independent solution of the EOM
- At T_c the phase separated phase satisfies this:
  - The limit of a large bubble



- For smaller bubbles this would not be stable: the bubble would collapse

Yago Bea and David Mateos, Heating up Exotic RG Flows with Holography

Some technicalities: DeTurck

Solve a slightly modified problem

- Take a reference metric (the homogeneous solution, $\overline{G}^{MN}$) and define $\xi^P = G^{MN}(\Gamma^P_{MN} - \overline{\Gamma}^P_{MN})$
- Solve the modified Einstein equations:

$$R_{MN} - \frac{R}{2}G_{MN} - \left(\nabla_{(M}\xi_{N)} - \frac{1}{2}\nabla_A\xi^A G_{MN}\right) = \kappa_5^2 T_{MN}$$

- Check that:

$$\xi_A\xi^A = 0$$

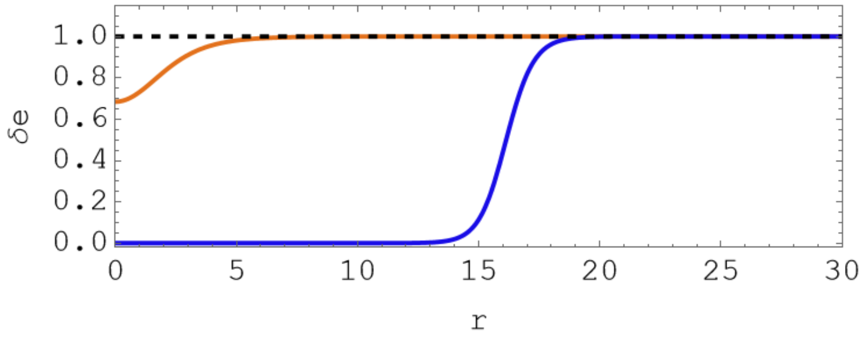
- This helps imposing boundary conditions and to make the PDEs elliptic
  - Since we know the solution exists there should not be any \`soliton' with ξ non-zero
- Technically we work at large but finite N_c, otherwise the nucleation rate vanishes
  - Spherical symmetry
  - Fully backreacted AdS_5 solutions though ☺

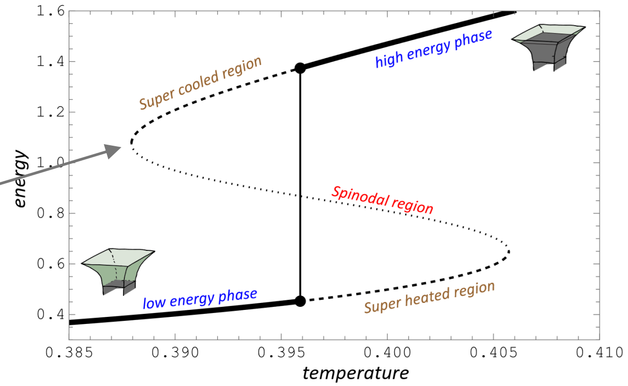
Óscar J. C. Dias, Jorge E. Santos and Benson Way, Numerical Methods for Finding Stationary Gravitational Solutions (2015)

SUPERCOOLED AND CRITICAL HOLOGRAPHY

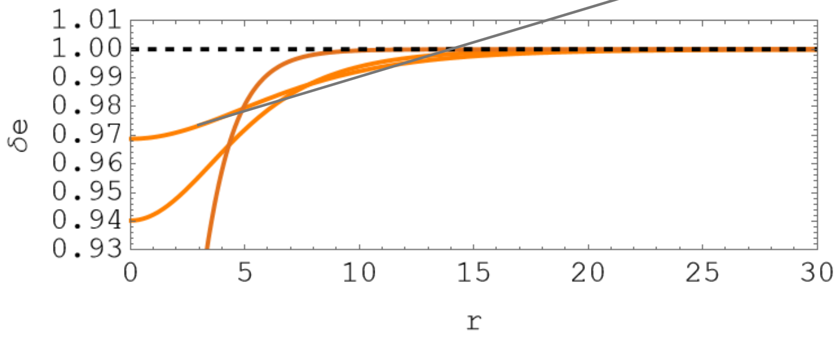
A critical bubble

- An unstable time-independent solution of the EOM
- For smaller bubbles this would not be stable: the bubble would collapse
  - Stable if wall is less steep
  - Stable if more pressure at the centre of the bubble



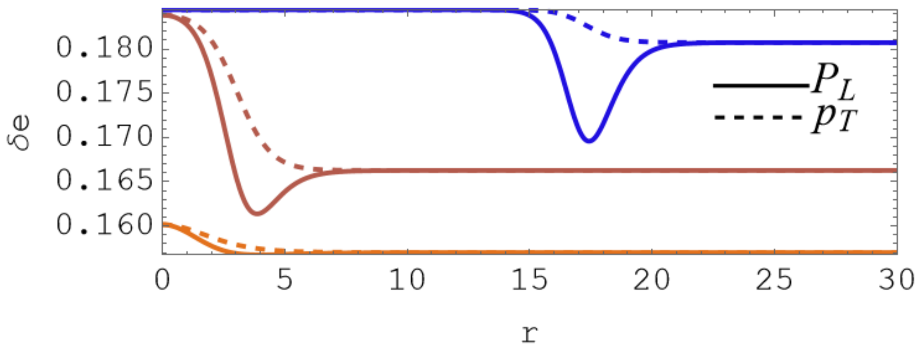


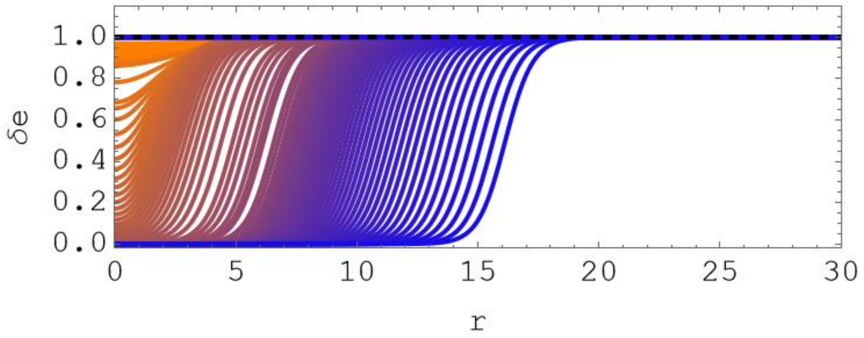
- Even smaller bubbles (regarding slope, closer to T_0):
  - The size starts growing again (!):

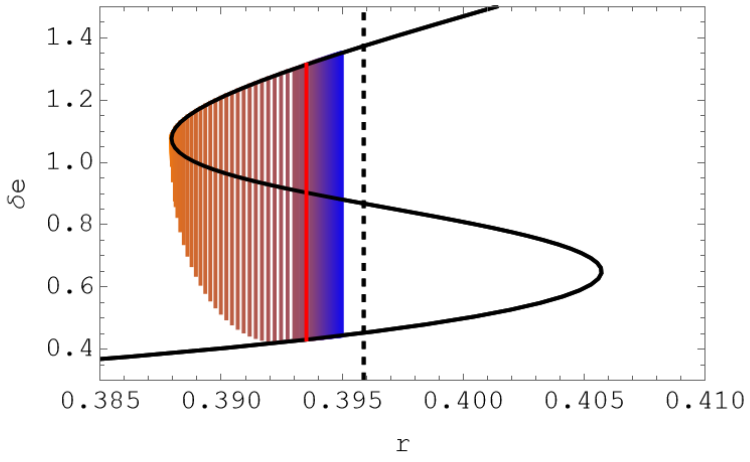


SUPERCOOLED AND CRITICAL HOLOGRAPHY

A few more details: pressures







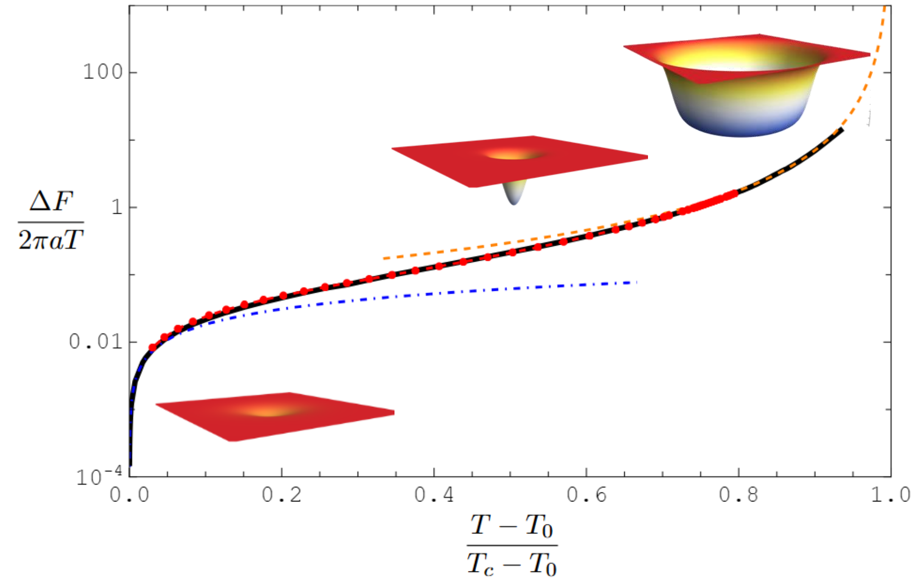
- Pressure inside is higher than outside (but energy density is lower)
  - Remember: at T_c pressures of both phases are equal
- Transverse pressure is always smaller than longitudinal pressure
- Transverse pressure can be non-monotonic (always?)
- Longitudinal pressure is always monotonic

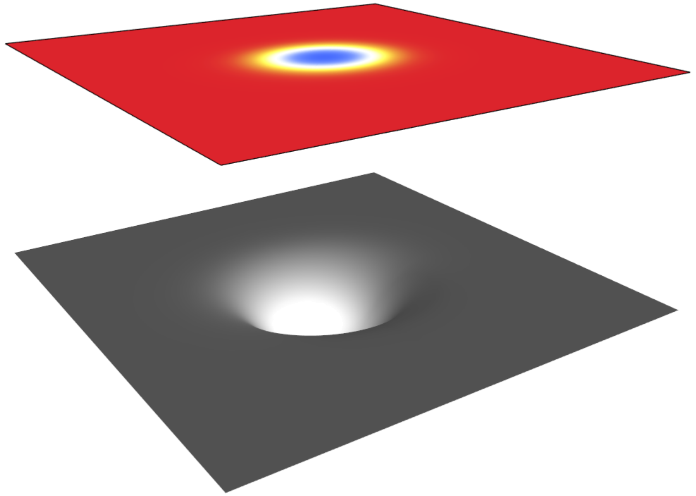
$$\frac{\Gamma}{V}\Big|_{\text{high }T} \simeq \left(\frac{\lambda_-}{2\pi}\right)\left(\frac{\hat{S}_{3d}}{2\pi T}\right)^{\frac{3}{2}}\left|\frac{\det'[-\nabla^2+V''(\hat{\phi})]}{\det[-\nabla^2+V''(0)]}\right|^{-\frac{1}{2}} e^{-\beta \hat{S}_{3d}}$$

Wilke van der Schee, CERN/Utrecht

WHAT KIND OF BUBBLES WILL NUCLEATE?

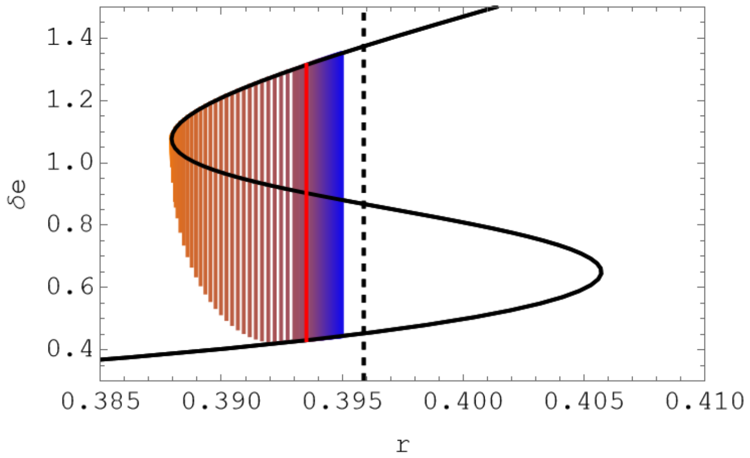
We compute the action:





- Larger bubbles are (much) harder to nucleate
- No final answer, but quiet some supercooling can be expected

- Effective field theory approach can work well (red, after fitting (!))
- Thin wall works for large bubbles (orange, $(T\text{-}T_c)^{-2}$)
- Small bubble effective description not as good (blue, $(T\text{-}T_0)^{4/3}$)



See also talk by Zhenhan Jin, Tue 18:20

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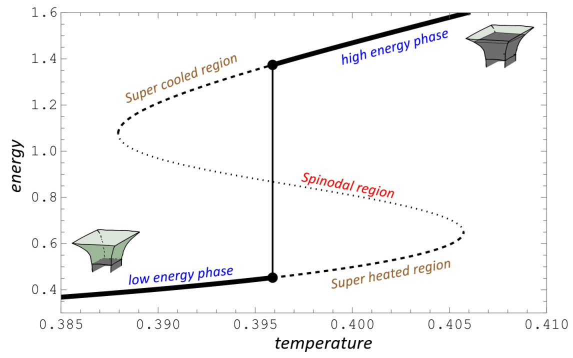
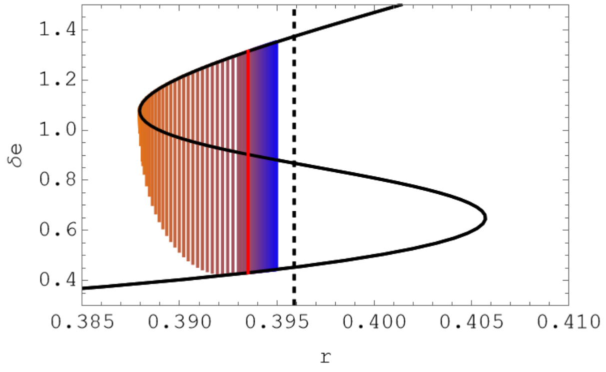
DISCUSSION



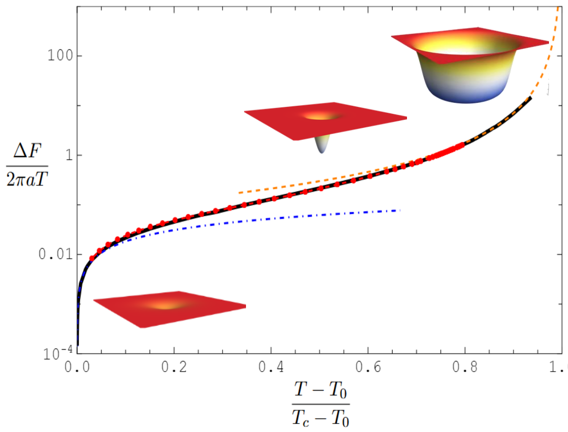
Bubble nucleation in holography

- Full microscopic construction of critical bubbles
  - Backreacted AdS_5, limiting behaviour fits effective potential



- No final answer, but quiet some supercooling can be expected

- Applications to neutron stars and early universe
  - Really not much certain both for nucleation and dynamics
  - Great help to have (toy!) models where microscopics is under full control



- Dynamics is clearly the next step
  - Velocities are interesting, wall profiles
  - Competition with cooling due to expansion