Schwinger-Keldysh holography: hydrodynamics and beyond

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Based on 2012.08362, 2506.00926, 2503.03374

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SJTU (上海交大): Y. Ahn, M. Baggioli, M. Matsumoto (now at Chuo U.)

HIT (哈工大): X. Sun, Z. Yang

Outline

- ☐ Background & Motivation: hydrodynamics and AdS/CFT
- ☐ SK holography: from charge diffusion to Maxwell-Cattaneo
- ☐ SK holography: dissipative neutral fluid
- ☐ Summary & Discussion

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Hydro is applied in many fields: astrophysics, biology, engineering, …

The past two decades witnessed entry of hydro in new century physics

---Hydro flows observed in exotic quantum many-body systems (quark-gluon plasma, unitary fermi gas, graphene)

---Theoretical foundations of hydro pushed forward: anomalous hydro, higher gradient extension and resurgence, hydro as an EFT, …

"AdS/CFT": an important role in understanding aspects of hydro [large N limit: quantum dynamics on the boundary is captured by "classical" dynamics of AdS gravity]

Basically, two approaches for "studying hydro via AdS/CFT"

From AdS / CFT correspondence to hydrodynamics

Giuseppe Policastro (Pisa, Scuola Normale Superiore), Dam T. Son (Washington U., Seattle), Andrei O. Starinets (Washington U., Seattle) (May, 2002)

Published in: *JHEP* 09 (2002) 043 • e-Print: hep-th/0205052 [hep-th]

Viscosity in strongly interacting quantum field theories from black hole physics #

P. Kovtun (Washington U., Seattle), Dan T. Son (Washington U., Seattle), Andrei O. Starinets (Washington U., Seattle) (Mar, 2004)

Published in: *Phys.Rev.Lett.* 94 (2005) 111601 • e-Print: hep-th/0405231 [hep-th]

Assume long-time long wavelength limit of boundary system is governed by the following viscous hydro:

$$\nabla_\mu T^{\mu\nu}_{\text{hydro}} = 0 \qquad\qquad T^{\text{hydro}}_{\mu\nu} = (\epsilon + P)u_\mu u_\nu + P g_{\mu\nu} + \eta_0 \nabla_{\langle\mu} u_{\nu\rangle} + \cdots$$

Derive the Kubo formulas and calculate them via AdS/CFT

solve classical Einstein equations in AdS $\quad \langle T_{\mu\nu} T_{\rho\sigma} \rangle \to \eta_0$

$$\boxed{\frac{\eta_0}{s} = \frac{1}{4\pi}\frac{\hbar}{k_B}}$$

A more systematic approach: fluid-gravity correspondence

Nonlinear Fluid Dynamics from Gravity #1

Sayantani Bhattacharyya (Tata Inst.), Veronika E Hubeny (Durham U. and Durham U., Dept. of Math.), Shiraz Minwalla (Tata Inst.), Mukund Rangamani (Durham U. and Durham U., Dept. of Math.) (Dec, 2007)

Published in: *JHEP* 02 (2008) 045 • e-Print: 0712.2456 [hep-th]

"Derive" constitutive relations and hydro equations from AdS gravity

Dynamics of AdS gravity $\Longrightarrow$

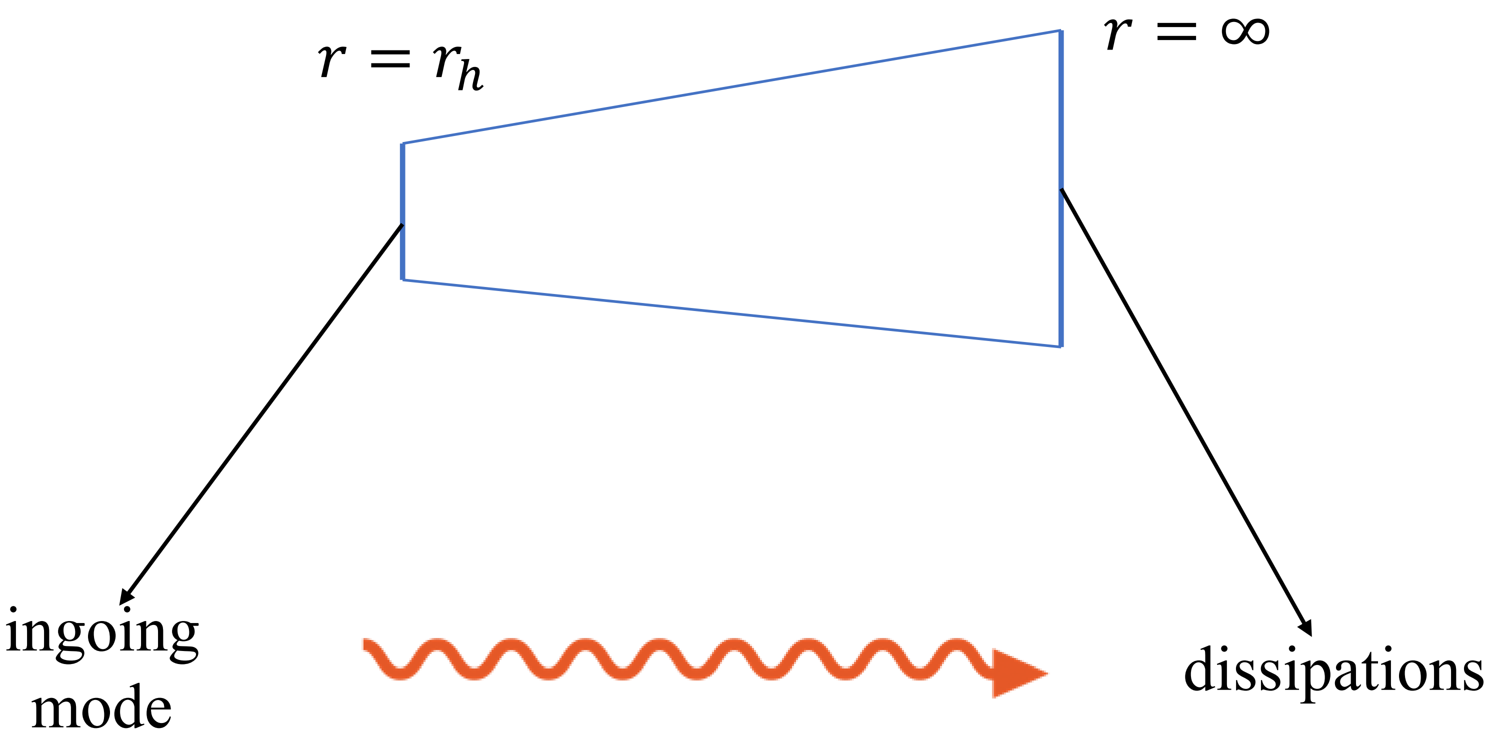
$$\nabla_\mu T^{\mu\nu}_{\text{hydro}} = 0$$

$$T^{\mu\nu}_{\text{hydro}} = T^{\mu\nu}_{\text{hydro}}[u_\mu, T; \eta_0, \cdots]$$

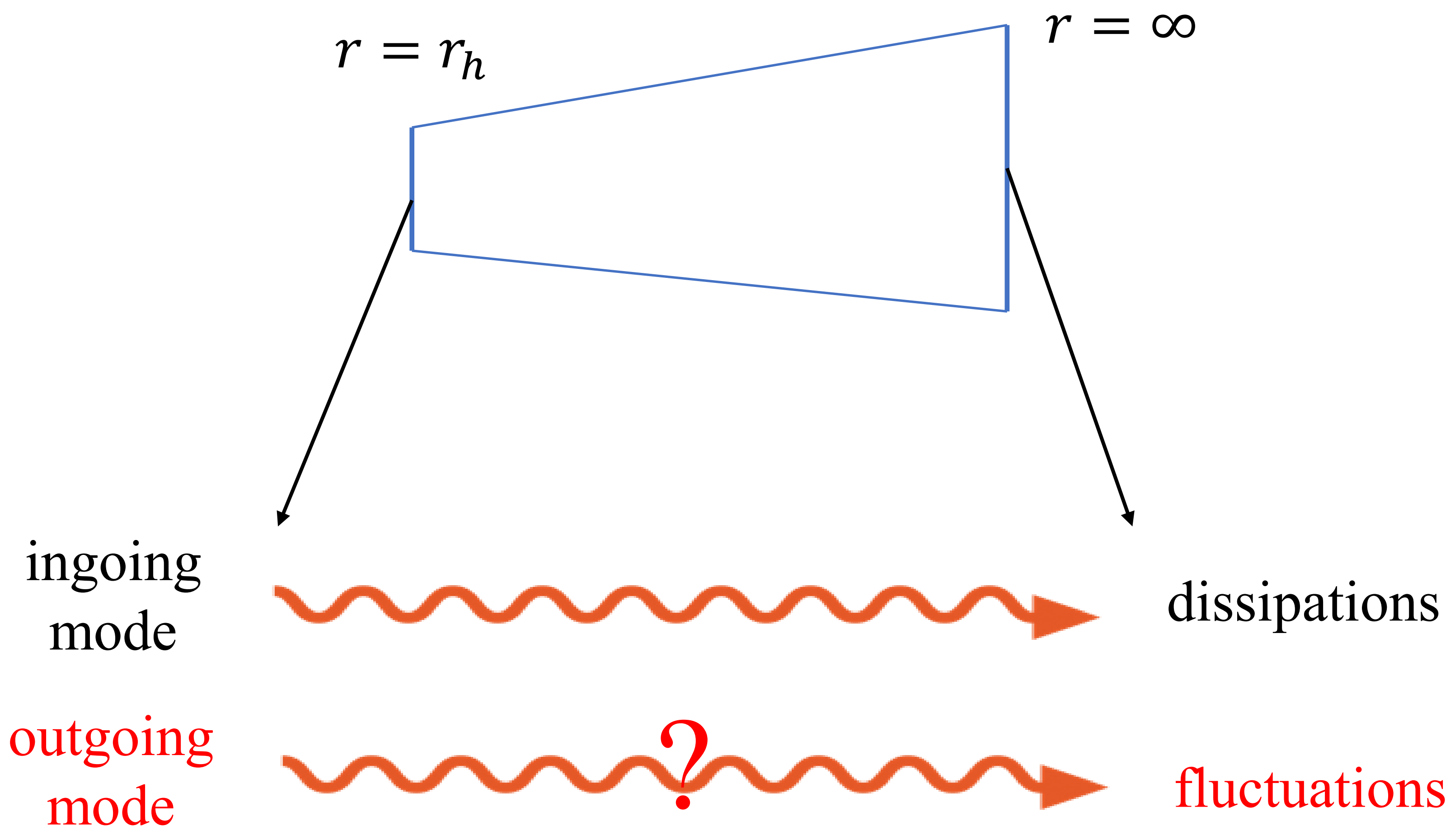
This provides a systematic way of getting higher derivative terms

Essentially, in this approach one perturbs a system in local equilibrium (the first approach starts with a global equilibrium state, and perturb it)

A crucial issue for horizon behavior of bulk field:



Classical dynamics in asymptotic AdS $\Longrightarrow$ $\nabla_\mu T^{\mu\nu}_{\mathrm{hydro}} = 0, \qquad T^{\mu\nu}_{\mathrm{hydro}} = \cdots$



we expect "stochastic hydro" on the boundary $\nabla_\mu T^{\mu\nu}_{\text{hydro}} = \zeta^\nu$

Can we do better ???

Schwinger-Keldysh holography

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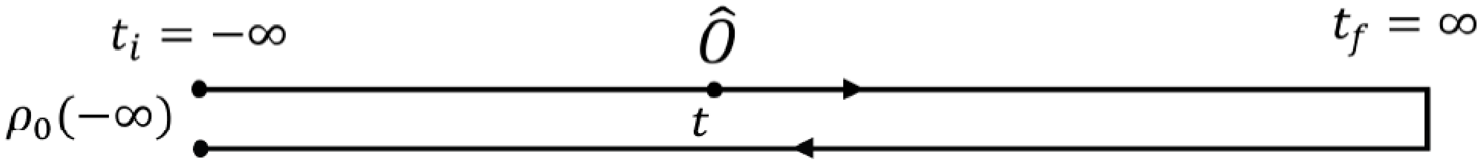
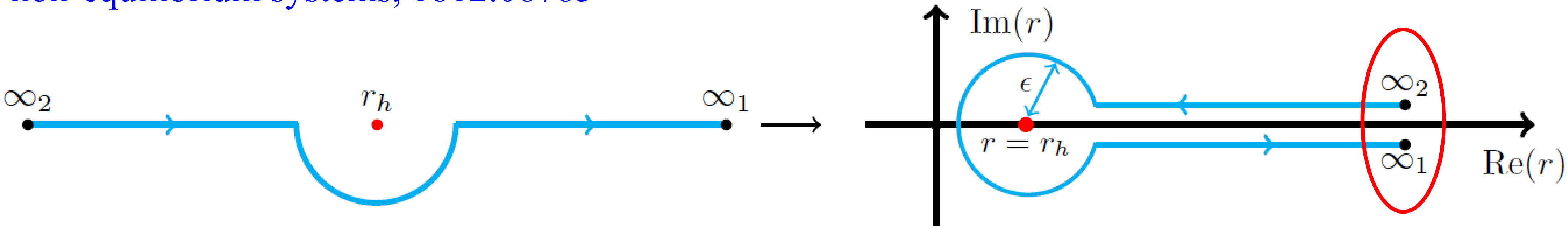
Holographic SK contour

to address both fluctuations and dissipations, we need Schwinger-Keldysh (SK) closed time path



Gravity dual of SK closed time path

P. Glorioso, M. Crossley and H. Liu, A prescription for holographic Schwinger-Keldysh contour in non-equilibrium systems, 1812.08785



Basically, we shall study double Dirichlet problem in the bulk

(alternative approaches: Herzog-Son 2002, van Rees-Skenderis 2008/9)

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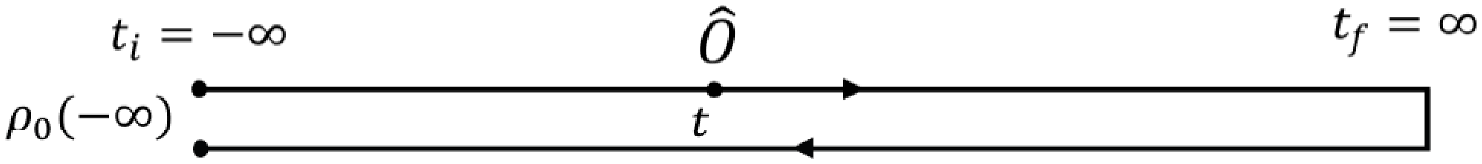

thermal density matrix ρ_0

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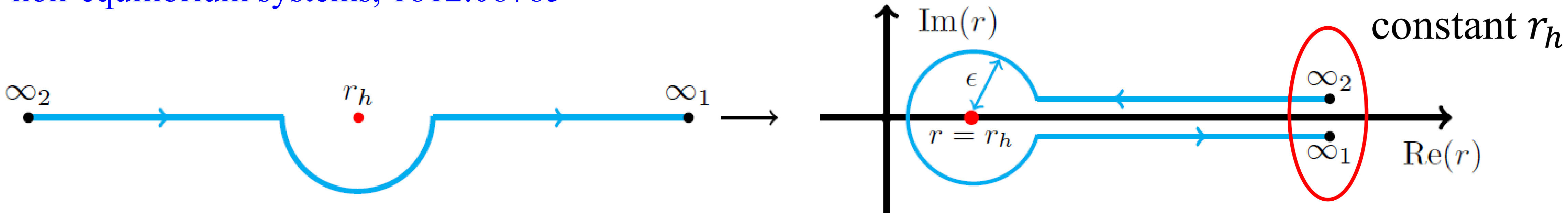

Basically, we shall study double Dirichlet problem in the bulk

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Basic idea: holographic renormalization group (RG)

$$\int [D\Phi] e^{iS_{\text{bulk}}[\Phi]} = \boxed{Z_{\text{AdS}} = Z_{\text{CFT}}} = \int [D\psi] e^{iS_{micro}[\psi]}$$

Holographic RG

gravity duals of heavy modes and hydro fields ϕ ???

resolved by exploring bulk gauge & diffeo. symmetry

partially on-shell AdS/CFT in the saddle approx.: un-integrate out bulk components dual to ϕ

integrate out heavy modes

$$= \int [D\phi] e^{iS_{eff}[\phi]}$$

$S_{eff}[\phi] \supset$ (hydro eom., constitutive relations, constraints, …)

Hydro EFT machinery

P. Glorioso, H. Liu, PoS TASI 2017 (2018) 008

Diffusion EFT: Maxwell in double AdS

P. Glorioso, M. Crossley, H. Liu, 1812.08785

J. de Boer, M. P. Heller, N. Pinzani-Fokeeva, 1812.06093

YB, T. Demircik, M. Lublinsky, 2012.08362

[J. K. Ghosh, et al., 2012.03999]
Similar topic but different formalism

bulk model: $$S_0 = -\frac{1}{4}\int d^5x\sqrt{-g}\,F_{MN}F^{MN}\,, \qquad S_{\rm ct} = \cdots$$

$$F_{MN} = \nabla_M C_N - \nabla_N C_M$$ C_M: bulk Maxwell field

Dirichlet boundary conditions

$$C_\mu(r=\infty_{\rm s}, x^\alpha) = \underline{B_{{\rm s}\mu}(x^\alpha)}$$ C_r fixed by gauge choice

boundary gauge potential + diffusive field

This is different from the treatment in usual “on-shell” AdS/CFT, in which $B_{s\mu}$ would have been external gauge potential for boundary theory

diffusive field & boundary gauge symmetry

Crossley-Glorioso-Liu, 1511.03646

couple the boundary current J^μ to an external gauge potential $\boldsymbol{\mathcal{A}}_\mu$

$$Z[\mathcal{A}_\mu] = \int_{\rho_0} [D\psi] e^{i\int d^4x \mathcal{L}[\psi, \mathcal{A}_\mu]}$$

current conservation guaranteed by background gauge symmetry

$$Z[\mathcal{A}'_\mu] = Z[\mathcal{A}_\mu], \quad \mathcal{A}'_\mu = \mathcal{A}_\mu + \partial_\mu \lambda \implies \partial_\mu J^\mu = 0$$

non-locality of Z: shear mode is integrated out

imagine an "integrate-in" procedure $Z[\mathcal{A}_\mu] = \int [D\underline{\varphi}] e^{iS_{eff}[\varphi, \mathcal{A}_\mu]}$

φ emerges from promoting gauge-transformation parameter λ into dynamical ones

$\lambda \to \varphi(x)$ Stuekelberg-like fields

$$C_\mu(r = \infty_s, x^\alpha) = \underline{B_{s\mu}(x^\alpha) \equiv \mathcal{A}_{s\mu} + \partial_\mu \varphi_s}$$

Dynamical components of Maxwell equations: $C_\mu = C_\mu[r;\, B_{\mathrm{s}\mu}(x^\alpha)]$

Effective action for diffusion $S_0 + S_{\mathrm{ct}}\big|_{C_\mu \to C_\mu[r;\, B_{\mathrm{s}\mu}]} = S_{eff}[\mathcal{A}_{\mathrm{s}\mu},\, \varphi_{\mathrm{s}}]$

To first order in derivative of $B_{\mathrm{s}\mu} \equiv \mathcal{A}_{\mathrm{s}\mu} + \partial_\mu \varphi_{\mathrm{s}}$

$$S_{eff} = \int d^4x \left(2r_h^2 B_{\mathrm{a}0} B_{\mathrm{r}0} - r_h B_{\mathrm{a}i} \partial_0 B_{\mathrm{r}i} + \frac{\mathrm{i} r_h^2}{\pi} B_{\mathrm{a}i}^2 + \cdots \right)$$

$$\boxed{\begin{aligned} B_{\mathrm{r}\mu} &\equiv (B_{1\mu} + B_{2\mu})/2, \\ B_{\mathrm{a}\mu} &\equiv B_{1\mu} - B_{2\mu} \end{aligned}}$$

[recover diffusive hydro]

$$J_{\mathrm{r}}^\mu \equiv \delta S_{eff}/\delta \mathcal{A}_{\mathrm{a}\mu}, \qquad \delta S_{eff}/\delta \varphi_{\mathrm{a}} = 0 \Longrightarrow \partial_\mu J_{\mathrm{r}}^\mu = 0$$

$$J_{\mathrm{r}}^0 = \underbrace{2r_h^2 B_{\mathrm{r}0}}, \qquad J_{\mathrm{r}}^i = \underbrace{-r_h \partial_i B_{\mathrm{r}0} + r_h(\partial_i B_{\mathrm{r}0} - \partial_0 B_{\mathrm{r}i})} + \underbrace{\frac{\mathrm{i}}{\pi} 2r_h^2 B_{\mathrm{a}i}}$$

hydro current (mean field part) noise current

$$\partial_\mu J_{\mathrm{r}}^\mu = 0 \Longrightarrow \partial_\mu J_{hydro}^\mu = \xi$$

Causal diffusion EFT: nonlinear Maxwell in double AdS

Y. Ahn, M. Baggioli, YB, M. Matsumoto, X. Sun, 2506.00926

(See also Y. Liu, Y.-W. Sun and X.-M. Wu, 2411.16306)

$$S = \int d^5x\sqrt{-g}\left[-\frac{1}{4}F_{MN}F^{MN} + \alpha\left(\frac{1}{4}F_{MN}F^{MN}\right)^2\right]$$

Consider a bulk ansatz $A_M(r, x^\mu) = \underbrace{\delta_0^M \bar{A}_M(r)}_{\text{background charge density } \rho} + \underbrace{\delta A_M(r, x^\mu)}_{\text{dynamics of density fluctuation } n}$

We confirmed n obeys Maxwell-Cattaneo model (for large enough ρ)

$$\tau\partial_0^2 n + \partial_0 n = D\vec{\nabla}^2 n$$

[Inspired by Grozdanov-Lucas-Poovuttikul 1810.10016, Ke-Yin 2208.01046, Jain-Kovtun 2309.00511, …]

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UV completion

We confirmed n obeys Maxwell-Cattaneo model (for large enough ρ)

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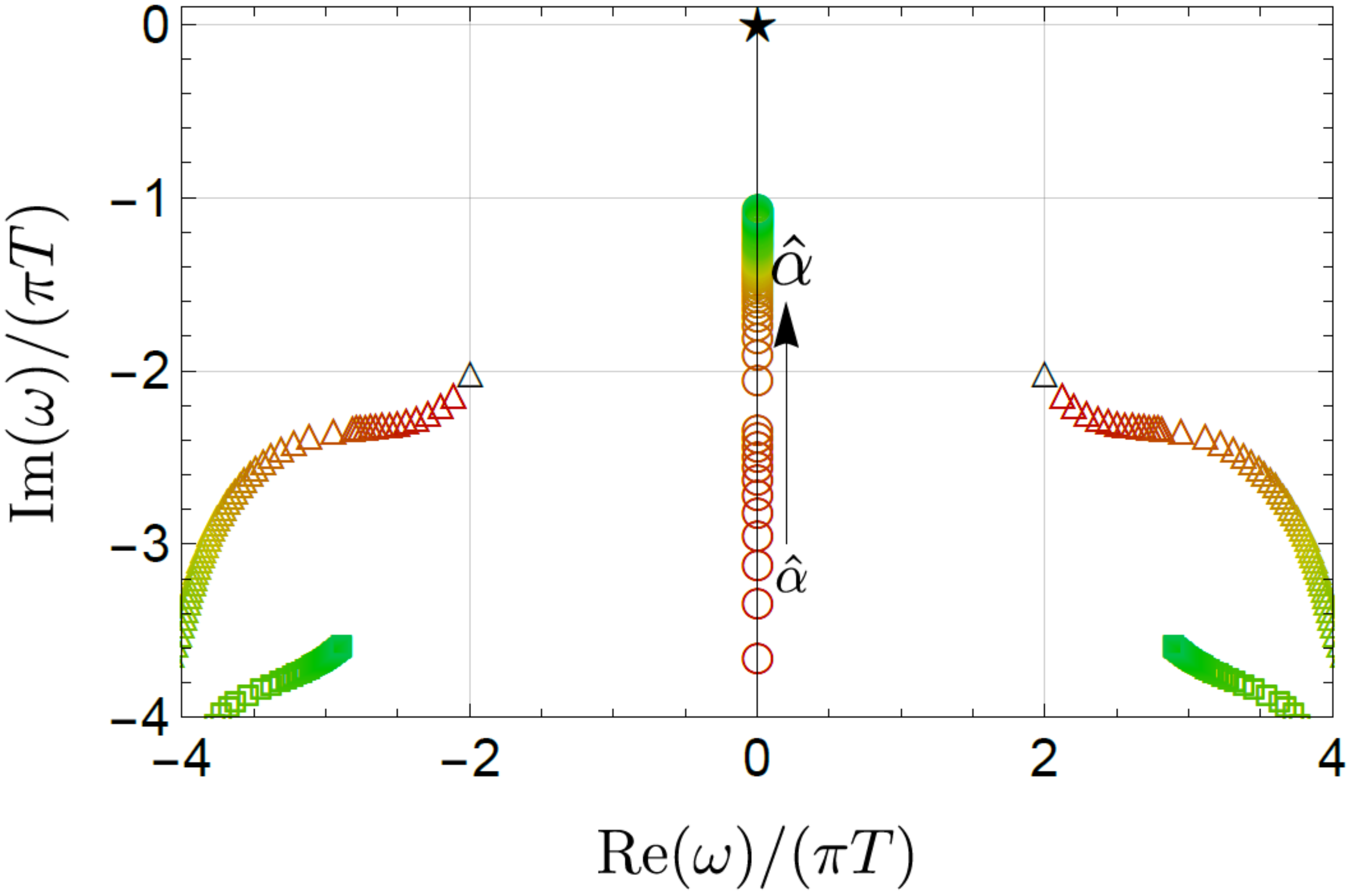
[Inspired by Grozdanov-Lucas-Poovuttikul 1810.10016, Ke-Yin 2208.01046, Jain-Kovtun 2309.00511, …]

QNMs as increasing $\hat{\alpha} = \alpha\rho^2$ (at $k = 0$)



- $\hat{\alpha} = 0$

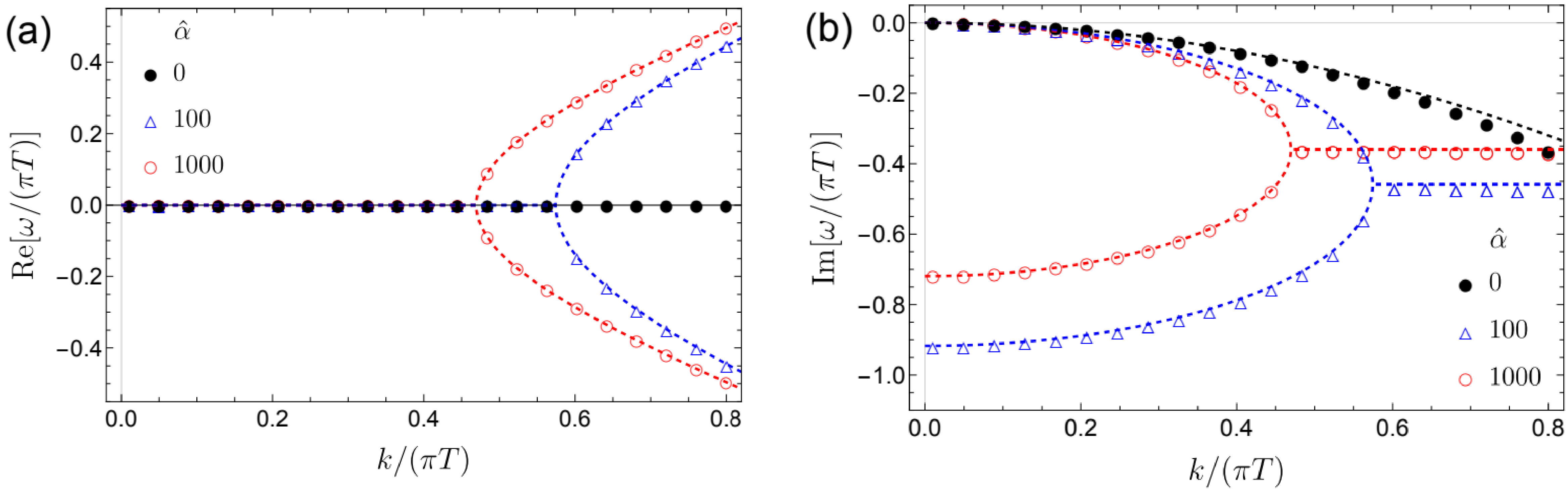
  $\omega_\star = 0, \qquad \omega_{\boldsymbol{\Delta}} = -(i \pm 1)2\pi T,$

- $\hat{\alpha}$ increased (towards green color)

the pair $\omega_{\boldsymbol{\Delta}}$ moves down into complex plane; a purely imaginary mode $\omega_1 = -i/\tau$ climbs along the imaginary axes

Quasi-hydro regime:
for large enough $\hat{\alpha}$, ω_1 gets closer to ω_* but well separated from all the rest non-hydro modes

QNMs at $k \neq 0$ for several representative $\hat{\alpha}$



For large enough $\hat{\alpha}$, the first two QNMs are well fitted by telegrapher equation

$$\omega^2 + \mathrm{i}\omega/\tau = v_0^2 k^2, \qquad v_0^2 = D/\tau,$$

Derivation of boundary EFT in the quasi-hydro regime

Sakai-Sugimoto
hep-th/0412141

$$\delta A_\mu(r, x^\alpha) = \underbrace{\delta A^{\rm h}_\mu(r, x^\alpha)}_{\text{hydro mode}} + \underbrace{\delta A^{\rm nh}_\mu(r, x^\alpha)}_{\text{non-hydro mode}} + \cdots$$

$$\delta A^{\rm h}_\mu(r \to \infty_s, x^\alpha) = B_{s\mu}(x^\alpha) + \cdots + \frac{\mathfrak{J}^{\rm h}_{s\mu}(x^\alpha)}{r^2} + \cdots \qquad \mathfrak{J}^{\rm h}_{s\mu} = \mathfrak{J}^{\rm h}_{s\mu}[B_{s\mu}(x)]$$

$$\delta A^{\rm nh}_\mu(r \to \infty_s, x^\alpha) = \mathcal{V}_{s\mu}(x^\alpha) + \cdots + \frac{\mathfrak{J}^{\rm nh}_{s\mu}(x^\alpha)}{r^2} + \cdots \qquad \mathcal{V}_{s\mu} = \mathcal{V}_{s\mu}[\mathfrak{J}^{\rm nh}_{s\mu}(x)]$$

We solve equations for $\delta A^{\rm h}_\mu$ in the hydro regime, i.e., assume $\partial_\mu \sim 0$

We solve equations for $\delta A^{\rm nh}_\mu$ in the non-hydro regime, i.e., assume $\partial_\mu \to (\omega \sim \omega_1 \equiv -i/\tau, k \sim 0)$

$$S_{\rm eff}^{\rm h} = \int d^4x \left(\chi B_{a0} B_{r0} - \sigma_0 B_{ai} \partial_0 B_{ri} + {\rm i}\beta^{-1} \sigma B_{ai}^2\right),$$

$$S_{\rm eff}^{\rm nh} = \int d^4x \Big\{ -\frac{a_1}{2}\frac{\sigma_0}{\tau} \left(B_{ai}\partial_0 \mathfrak{J}_{ri}^{\rm nh} + \mathfrak{J}_{ai}^{\rm nh}\partial_0 B_{ri} - 2{\rm i}\beta^{-1} B_{ai}\mathfrak{J}_{ai}^{\rm nh}\right) + 2a_1 \mathfrak{J}_{ai}^{\rm nh}(\partial_0 + \tau^{-1})\mathfrak{J}_{ri}^{\rm nh}$$
$$+ {\rm i}a_1(\partial_0+\tau^{-1})\left[\cot\left(\frac{\beta}{2\tau}\right) + (\partial_0+\tau^{-1})\frac{\beta}{2\tau}\sin^{-2}\left(\frac{\beta}{2\tau}\right)\right] \underline{\mathfrak{J}_{ai}^{\rm nh}\mathfrak{J}_{ai}^{\rm nh}} \Big\},$$

$$\chi = \left(\int_{r_h}^{\infty} \frac{dy}{y^3[1+3\mathcal{C}(y)]}\right)^{-1}, \qquad \sigma_0 = r_h\left[1+\mathcal{C}(r_h)\right], \qquad \underline{\tau = -\frac{\rm i}{\omega_1}, \qquad a_1 = {\rm i}a'(\omega_1).}$$

extracted from ingoing solution at $k = 0$

$$\mathcal{C}(r) \equiv \alpha(\bar{A}_0'(r))^2$$

ra-terms: MC equation

$$\partial_0 n + \vec{\nabla}\cdot\vec{j} = 0$$
$$\vec{j} + \tau\partial_0\vec{j} = -D\vec{\nabla} n$$

$$\vec{j} \sim \mathfrak{J}_{ri}^{\rm nh}$$

aa-terms: thermal fluctuations, non-hydro mode obeys a more generic KMS symmetry

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neutral fluid EFT: Einstein in double AdS

YB, X. Sun, Holographic derivation of effective action for a dissipative neutral fluid, 2503.03374

(early attempts: Crossley-Glorioso-Liu 1504.07611, de Boer-Heller-Pinzani-Fokeeva, 1504.07616)

Einstein gravity in 5D AdS space $\quad S = S_0 + S_{\rm GH} + S_{\rm ct}$

Bulk metric ansatz (dual to boundary "thermal equilibrium + perturbations")

$$\begin{aligned} ds^2 &= \bar{G}_{MN} d\sigma^M d\sigma^N + H_{MN} d\sigma^M d\sigma^N \\ &= \underbrace{2 dr d\sigma^0 - r^2 f(r)(d\sigma^0)^2 + r^2 \delta_{ij} d\sigma^i d\sigma^j}_{\text{thermal equilibrium state}} + \underbrace{H_{MN} d\sigma^M d\sigma^N}_{\text{dissipations \& fluctuations}} \end{aligned}$$

Dirichlet-type boundary conditions

$$H_{ab}(r \to \infty_{\rm s}, \sigma^a) = r^2 \left[B_{\text{s}ab}(\sigma) + \mathcal{O}(r^{-1}) \right] \qquad H_{Mr} \text{ gauge-fixed}$$

???

boundary metric perturbations + hydro fields

hydrodynamic fields & boundary diffeomorphism

turn on a background curved metric $g_{\mu\nu}$ on the boundary

Crossley-Glorioso-Liu, 1511.03646

$$Z[g_{\mu\nu}(x)] = \int_{\rho_0} [D\psi] e^{i\int d^4x\, \mathcal{L}[\psi, g_{\mu\nu}]}$$

$$Z[\bar{g}_{\alpha\beta}(\bar{x})] = Z[g_{\mu\nu}(x)] \quad \Longrightarrow \quad \nabla_\mu T^{\mu\nu} = 0$$

non-locality of Z: hydro modes are integrated out

imagine an "integrate-in" procedure $Z[g_{\mu\nu}] = \int [DX^\mu] e^{iS_{eff}[X^\mu; g_{\mu\nu}]}$

X^μ emerges from promoting diffeo-transformation parameters x^μ into dynamical ones

$$x^\mu \to X^\mu(\sigma^a) \qquad \text{Stuekelberg-like fields}$$

This is in analogy with "diffusion EFT"

Accordingly, we have the following promotion:

$$dl_0^2 = g_{\mu\nu}(x)dx^\mu dx^\nu \xrightarrow{\boxed{x^\mu \to X^\mu(\sigma^a)}} dl^2 = g_{\mu\nu}(X)dX^\mu dX^\nu$$

$$= g_{\mu\nu}(X^\mu(\sigma))\frac{\partial X^\mu}{\partial \sigma^a}\frac{\partial X^\nu}{\partial \sigma^b}d\sigma^a d\sigma^b$$

$$\equiv \underbrace{\mathcal{G}_{ab}(X^\mu(\sigma))}_{\text{AdS boundary data}} d\sigma^a d\sigma^b \simeq ds^2|_{\Sigma_{\mathrm{UV}}}$$

$$X_{\mathrm{s}}^\mu(\sigma) = \delta_a^\mu \sigma^a + \pi_{\mathrm{s}}^\mu(\sigma), \qquad g_{\mathrm{s}\mu\nu}(x) = \eta_{\mu\nu} + A_{\mathrm{s}\mu\nu}(x)$$

$$B_{\mathrm{s}ab}(\sigma) = A_{\mathrm{s}ab}(\sigma) + A_{\mathrm{s}a\nu}(\sigma)\partial_b \pi_{\mathrm{s}}^\nu(\sigma) + A_{\mathrm{s}\mu b}(\sigma)\partial_a \pi_{\mathrm{s}}^\mu(\sigma) + \pi_{\mathrm{s}}^\alpha(\sigma)\partial_\alpha A_{\mathrm{s}ab}(\sigma)$$
$$+ \partial_a \pi_{\mathrm{s}b}(\sigma) + \partial_b \pi_{\mathrm{s}a}(\sigma) + \partial_a \pi_{\mathrm{s}}^\mu(\sigma)\partial_b \pi_{\mathrm{s}\mu}(\sigma) + \cdots$$

$$
\begin{aligned}
ds^2 &= \bar{G}_{MN} d\sigma^M d\sigma^N + H_{MN} d\sigma^M d\sigma^N \\
&= 2 dr d\sigma^0 - r^2 f(r) (d\sigma^0)^2 + r^2 \delta_{ij} d\sigma^i d\sigma^j + H_{MN} d\sigma^M d\sigma^N
\end{aligned}
$$

solve bulk Einstein eqns: $H_{MN} = H_{MN}[r, B_{\mathrm{s}ab}(\sigma)]$

$$
S = S_0 + S_{\mathrm{GH}} + S_{\mathrm{ct}} \Longrightarrow S_{eff} = S_{eff}[B_{\mathrm{s}ab}(\sigma)]
$$

We successfully obtained the action up to cubic order in B_{sab}

$$
S^{(1)}_{eff} = \int d^4\sigma \, \underline{\frac{r_h^4}{2} \left[3B_{\mathrm{a}00} + B_{\mathrm{a}ii} \right]}
$$

ideal part

$$
\begin{aligned}
S^{(2)}_{eff} = \int d^4\sigma \, \Big\{ & \underline{\frac{3r_h^4}{4} B_{\mathrm{a}00} B_{\mathrm{r}00} + r_h^4 B_{\mathrm{a}0i} B_{\mathrm{r}0i} - \frac{r_h^4}{4} B_{\mathrm{a}ii} B_{\mathrm{r}00} + \frac{3r_h^4}{4} B_{\mathrm{a}00} B_{\mathrm{r}ii}} \\
& \underline{+ \frac{r_h^4}{4} B_{\mathrm{a}ii} B_{\mathrm{r}jj} - \frac{r_h^4}{2} B_{\mathrm{a}ij} B_{\mathrm{r}ij}} + \frac{\mathrm{i} r_h^4}{6\pi} \left(3B_{\mathrm{a}ij} B_{\mathrm{a}ij} - B_{\mathrm{a}ii} B_{\mathrm{a}jj} \right) \\
& \underline{+ \frac{r_h^3}{6} \left(B_{\mathrm{a}ii} \partial_0 B_{\mathrm{r}jj} - 3B_{\mathrm{a}ij} \partial_0 B_{\mathrm{r}ij} \right)} - \frac{\mathrm{i}\pi r_h^3}{8} B_{\mathrm{a}00} \partial_0 B_{\mathrm{a}ii} \Big\} .
\end{aligned}
$$

dissipative part

Recover classical hydrodynamics

$$S_{eff} = S_{eff}[B_{sab}(\underline{\sigma})]$$

From fluid spacetime to physical spacetime: $\sigma^a \to x^\mu$

$$X_s^\mu(\sigma) = \delta_a^\mu \sigma^a + \pi_s^\mu(\sigma), \qquad g_{s\mu\nu}(x) = \eta_{\mu\nu} + A_{s\mu\nu}(x)$$

$$\delta_a^\mu \sigma^a = x^\mu - \pi_r^\mu(x) + \pi_r^\nu(x)\partial_\nu \pi_r^\mu(x) + \cdots$$

$$S_{eff} = S_{eff}[\pi_s^\mu(x^\alpha), A_{s\mu\nu}(x^\alpha)]$$

$$\frac{\delta S_{eff}}{\delta \pi_{a\mu}} = 0 \quad \Longrightarrow \quad \begin{array}{ll} \text{shear mode}: & \tilde{\omega} = -\frac{1}{2}\mathrm{i}\tilde{k}^2 + \cdots, \\ \text{sound mode}: & \tilde{\omega} = \pm\frac{1}{\sqrt{3}}\tilde{k} - \frac{2}{3}\mathrm{i}\tilde{k}^2 + \cdots \end{array}$$

Stochastic hydrodynamics

$$T_{\rm r}^{\mu\nu}(x) = \frac{2}{\sqrt{-g_{\rm r}}}\frac{\delta S_{eff}}{\delta g_{{\rm a}\mu\nu}(x)}$$

function of hydro field X^μ

We shall go from hydro fields X^μ to fluid velocity u^μ

$$u_{\rm s}^\mu \equiv \frac{1}{b_{\rm s}}\frac{\partial X_{\rm s}^\mu}{\partial\sigma^0}, \qquad \text{with} \quad b_{\rm s} \equiv \sqrt{-g_{{\rm s}\mu\nu}(X_{\rm s}^\mu)\frac{\partial X_{\rm s}^\mu}{\partial\sigma^0}\frac{\partial X_{\rm s}^\nu}{\partial\sigma^0}}$$

$$T_{\rm r}^{\mu\nu} = T_{\rm hydro}^{\mu\nu} + \underline{T_{\rm stoc}^{\mu\nu}} \qquad \partial_\mu T_{\rm r}^{\mu\nu} = 0 \Longrightarrow \partial_\mu T_{\rm hydro}^{\mu\nu} = \underline{\zeta^\nu}$$

$$T_{\rm hydro}^{\mu\nu} = \epsilon u^\mu u^\nu + P(g_{\rm r}^{\mu\nu} + u^\mu u^\nu) - \eta_0\sigma^{\mu\nu} \qquad \eta_0 = r_h^3 = (\pi T)^3$$

$$\langle\zeta^\mu(x)\rangle = 0, \qquad \langle\zeta^\mu(x)\zeta^\nu(x')\rangle = \mathcal{M}^{\mu\nu}\delta^{(4)}(x-x')$$

$$\mathcal{M}^{ij} = -\frac{2r_h^4}{3\pi}\partial_i\partial_j - \frac{2r_h^4}{\pi}\delta_{ij}\vec{\partial}^2$$

Summary & Discussion

With the SK holography, we derived the low energy EFT for hydro modes associated with conserved charge density or energy/momentum; achieved by solving Maxwell or Einstein equations in the double AdS black brane

In presence of a (large enough) background charge density, the nonlinear Maxwell in AdS is dual to Maxwell-Cattaneo model; With SK holography, we derived the boundary EFT action in the quasi-hydro regime

It would be interesting to start with a local equilibrium state (as in fluid-gravity), and derive more complete EFT for a holographic dissipative fluid

It would be interesting to understand aspects of SK EFT beyond hydro regime (e.g., a bulk Maxwell indeed contains infinitely many modes). This is supposed to be of relevance for studying far-from-equilibrium dynamics.

Thanks for listening!