Phase Structure of 2+1-Flavor Holographic QCD

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Based on a just finished work
with Jin-Yang Shen, Xin-Yi Liu, Jin-Rui Wu, Yue-Liang Wu

Holographic applications: from Quantum Realms to the Big Bang @ UCAS

Outline

- Introduction & Motivation (QCD phase transition/diagram)
- (Bottom-up) Holographic QCD
- Einstein-Dilaton-Flavor Model
- Machine Learning Approach
- Results and Comparison with Lattice QCD
- Quark Mass Phase Diagram
- Summary

Intr. & Moti. — QCD phase transition

- Quantum Chromodynamics
  - Governs strong interactions in particle physics
  - Predicts a deconfining transition (hadrons $\to$ QGP) at high T
  - Chiral transition (with chiral symmetry restoration)
- QCD phase transitions: Crucial for understanding:
  - Early universe evolution
  - Neutron star interiors
  - $\cdots$

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- QCD phase transitions: Crucial for understanding:
  - Early universe evolution
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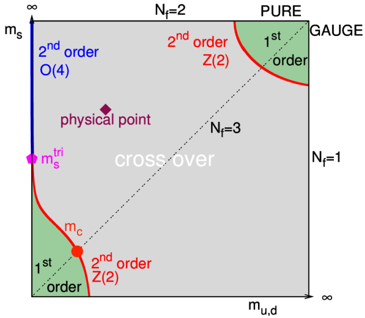
  - $\cdots$
- Key challenge: Non-perturbative nature (Asymptotic freedom)
- Focus: Map phase transitions at zero chemical potential ($\mu = 0$)
  - Deconfinement
  - Chiral transition
  - Quark mass dependence

- Maps QCD phase transition order vs. $(m_{u,d}, m_s)$ at $\mu = 0$
- Key Features (Three regimes):
  - Physical point: Smooth crossover
  - Chiral limit ($m_{u,d} \to 0$): 2nd-order transition (O(4) class)
  - Heavy quarks: 1st-order transition (Z(3) symmetry)



[arXiv:1504.05274]

Holographic QCD Overview

AdS/QCD

- Using AdS/CFT ideas to model QCD non-perturbative aspects holographically. (Links gravity in higher dimensions to QCD)

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Holographic Models

- Top-down: WSS, D3-D7, D4-D6, ...
- Bottom-up: Hard-wall, Soft-wall, Light-front, VQCD, EMD, ...
- Capture hadron spectra, chiral dynamics, thermodynamics, phase transition, ... [arXiv:hep-ph/0501218, 0501128, 0501022, 0306018, 0412141, ..., arXiv:1112.1261, 1501.07272, ...]

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Challenges & Problems

- Previous models struggle to couple gluonic and flavor sectors
  Unify deconfinement and chiral transitions
- (Match lattice QCD? Produce the Columbia plot? μ-T phase diagram? Locate CEP? Hadron spectra? Neutron stars? ...)

Einstein-Dilaton-Flavor (EDF) System

Objective

- Couple gluonic and flavor sectors self-consistently by holography
  Unify deconfinement and chiral transitions (conform to lattice QCD)
- Produce the standard Columbia Plot (as much as possible)

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Metric (Einstein Frame)

$$ds^2 = \frac{L^2 e^{2A_E(z)}}{z^2}\left(-f(z)dt^2 + \frac{dz^2}{f(z)} + dx_i dx^i\right)$$

Einstein-Dilaton-Flavor (EDF) System

$$S = \frac{1}{2\kappa_N^2}\int d^5x\sqrt{-g}\left[R - \frac{4}{3}(\partial\phi)^2 - V_E(\phi) - \beta e^{\phi}\left((\partial\chi_u)^2 + \frac{1}{2}(\partial\chi_s)^2 + V(\chi_u, \chi_s, \phi)\right)\right]$$

- Fields: Metric g_{MN}, Dilaton ϕ, bulk scalar VEVs (χ_u, χ_s)

Einstein-Dilaton-Flavor System

Dilaton Potential

[arXiv:0804.1950 [hep-th]]

$$V_E(\phi) = V_c(\phi_c) = \frac{1}{L^2}\left(-12\cosh\gamma_1\phi_c + b_2\phi_c^2 + b_4\phi_c^4\right)$$

with $\phi_c = \sqrt{8/3}\,\phi$

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Flavor Potential

[arXiv:2312.01346, 1805.05019]

$$V(\chi_u, \chi_s, \phi) = e^{4\phi/3}\left[-\frac{1}{2}(3+\Phi(\phi))(2\chi_u^2+\chi_s^2) + \frac{\gamma}{2\sqrt{2}}\chi_u^2\chi_s + \frac{\lambda}{4}(2\chi_u^4+\chi_s^4)\right]$$

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Parameters

- 6 action params: $\gamma, \gamma_1, b_4, \lambda, d_1, d_2$
- 3 boundary params: p_1, m_u, m_s
- Newton constant G_5 $(\kappa_N = \sqrt{8\pi G_5})$

Formulate the EDF system

Improved 2+1-flavor soft-wall model

[arXiv:2312.01346, 1805.05019]

$$S_M = -\kappa \int d^5x \sqrt{-g_S}\, e^{-\Phi(z)} \left\{ \mathrm{Tr}\left[|DX|^2 + V_X(X,\Phi) + \frac{1}{4g_5^2}(F_L^2 + F_R^2) \right] + \gamma \det|X| \right\}$$

with

$$V_X(X,\Phi) = \left(m_5^2 - \Phi(\phi)\right)|X|^2 + \lambda |X|^4$$

$$\langle X \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} \chi_u(z) & 0 & 0 \\ 0 & \chi_d(z) & 0 \\ 0 & 0 & \chi_s(z) \end{pmatrix}, \qquad \chi_u = \chi_d$$

living on a dynamical gravitational background determined by

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Einstein-Dilaton system

$$S_G = \frac{1}{2\kappa_N^2} \int d^5x\sqrt{-g_S}\, e^{-2\phi} \left[R_S + 4\,\partial_M\phi\partial^M\phi - V(\phi) \right]$$

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with the metric (string frame)

$$ds^2 = \frac{L^2 e^{2A_S(z)}}{z^2}\left(-f(z)dt^2 + \frac{dz^2}{f(z)} + dx_i dx^i\right), \qquad A_S = A_E + 2\phi/3$$

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Full Action

$$S = S_G + S_M$$

Neglecting the vacuum fluctuations leads to the EDF action:

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Notes

- Solve the full EDF system (No probe limit)
- Similar system maybe obtained from other AdS/QCD models (e.g., Veneziano-QCD)

Equations of Motion

- Derived from action for the metric, dilaton ϕ and scalar VEVs (χ_u, χ_s)
- Five ODEs, fields depend on radial coordinate z

$$f'' - \frac{3f'}{z} + 3A_E' f' = 0,$$

$$A_E'' - A_E'^2 + \frac{2A_E'}{z} + \frac{4}{9}\phi'^2 + \frac{1}{3}\beta e^{\phi}\chi_u'^2 + \frac{1}{6}\beta e^{\phi}\chi_s'^2 = 0,$$

$$\phi'' + \left(\frac{f'}{f} + 3A_E' - \frac{3}{z}\right)\phi' - \frac{3e^{2A_E}\partial_\phi V_E(\phi)}{8z^2 f}$$
$$- \frac{3\beta}{8}e^{\phi}\chi_u'^2 - \frac{3\beta}{16}e^{\phi}\chi_s'^2 - \frac{3\beta e^{2A_E}\partial_\phi(e^{\phi}V(\chi_u, \chi_s, \phi))}{8z^2 f} = 0,$$

$$\chi_u'' + \left(\frac{f'}{f} + 3A_E' - \frac{3}{z} + \phi'\right)\chi_u' - \frac{e^{2A_E}\partial_{\chi_u}V(\chi_u, \chi_s, \phi)}{2z^2 f} = 0,$$

$$\chi_s'' + \left(\frac{f'}{f} + 3A_E' - \frac{3}{z} + \phi'\right)\chi_s' - \frac{e^{2A_E}\partial_{\chi_s}V(\chi_u, \chi_s, \phi)}{z^2 f} = 0.$$

Boundary Conditions

- UV asymptotic solutions:

$$
\begin{aligned}
f(z) &= 1 - f_4 z^4 + \dots, \\
A_E(z) &= \frac{1}{108}(-8p_1^2 - 3(2m_u^2 + m_s^2)\beta\zeta^2)z^2 \\
&\quad + a_3 z^3 + a_4 z^4 + a_{4l} z^4 \ln z + \dots, \\
\phi(z) &= p_1 z + \frac{3}{16}\beta\zeta^2(2m_u^2 + m_s^2)(6 + d_1)z^2 + p_3 z^3 \\
&\quad + p_{3l} z^3 \ln z + p_4 z^4 + p_{4l} z^4 \ln z + \dots, \\
\chi_u(z) &= m_u \zeta z - \frac{1}{4} m_u \zeta(\sqrt{2} m_s \gamma\zeta - 4p_1(5 + d_1))z^2 + \frac{\sigma_u}{\zeta} z^3 \\
&\quad + c_{3l} z^3 \ln z + c_4 z^4 + c_{4l} z^4 \ln z + \dots, \\
\chi_s(z) &= m_s \zeta z - \frac{1}{4}\zeta(\sqrt{2} m_u^2 \gamma\zeta - 4m_s p_1(5 + d_1))z^2 + \frac{\sigma_s}{\zeta} z^3 \\
&\quad + \tilde{c}_{3l} z^3 \ln z + \tilde{c}_4 z^4 + \tilde{c}_{4l} z^4 \ln z + \dots
\end{aligned}
$$

- Boundary conditions:

$$
f(0) = 1, \quad f(z_h) = 0, \quad \phi'(0) = p_1, \quad \chi_u'(0) = m_u\zeta, \quad \chi_s'(0) = m_s\zeta
$$

EoS and normalized chiral condensate

Temperature T and entropy density s

$$T = \frac{|f'(z_h)|}{4\pi}, \qquad s = \frac{2\pi e^{3A_E(z_h)}}{\kappa_5^2 z_h^3}$$

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Pressure p and energy density ε (by thermodynamic relations)

$$dp = sdT, \qquad \varepsilon = -p + sT$$

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Chiral condensates (via holographic renormalization)

$$\langle\bar{\psi}\psi\rangle_u^T = \frac{\delta S_r}{\delta m_u} = k_1\sigma_u + b_1, \qquad \langle\bar{\psi}\psi\rangle_s^T = \frac{\delta S_r}{\delta m_s} = k_2\sigma_s + b_2$$

Normalized chiral condensates:

$$\frac{\langle\bar{u}u\rangle_T}{\langle 0|\bar{u}u|0\rangle} = \frac{\langle\bar{\psi}\psi\rangle_u^T}{\langle\bar{\psi}\psi\rangle_u^0}, \qquad \frac{\langle\bar{s}s\rangle_T}{\langle 0|\bar{s}s|0\rangle} = \frac{\langle\bar{\psi}\psi\rangle_s^T}{\langle\bar{\psi}\psi\rangle_s^0}$$

Machine Learning Integration

Solution

[arXiv:2303.15136, 2005.02636, 2401.06417, 2406.12772, ...]

- Use machine learning for efficient parameter optimization

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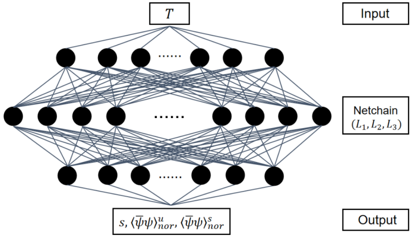
Neural Network Architecture

- Input: Temperature T
- 3 Hidden layers (128, 256, 128 neurons)
- Output: $s(T)$, $\langle\bar{\psi}\psi\rangle_{nor}(T)$
- Loss function: Mean square error
- Optimizer: Adam



Machine Learning Integration

Parameter Optimization

- Gradient Descent method: Gradient $= \frac{L(x+\epsilon)-L(x-\epsilon)}{2\epsilon}$
- Physical constraints:
  - 2 MeV $\leq m_u \leq$ 5 MeV
  - 90 MeV $\leq m_s \leq$ 120 MeV
- Training: 10,000 epochs
- Output: Smooth interpolations for observables

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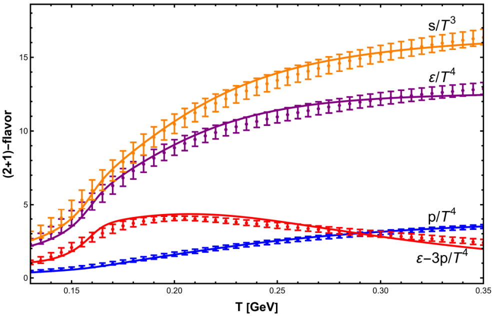
Optimized Model Parameters

- Quark masses: $m_u = 3\,\text{MeV}$, $m_s = 120\,\text{MeV}$

Case	γ	γ_1	b_4	λ	G_5	p_1	d_1	d_2
2+1-flavor	1.99	0.737	0.10	3	0.459	0.349	-0.89	-0.0385
two-flavor	/	0.75	0.02	10	0.582	0.473	-1.00	/

Thermodynamic Quantities

Energy density ε, entropy density s, pressure p, trace anomaly $\varepsilon - 3p$



- Solid lines: Our model
- Points: Lattice QCD data

  [arXiv:1407.6387 [hep-lat]]

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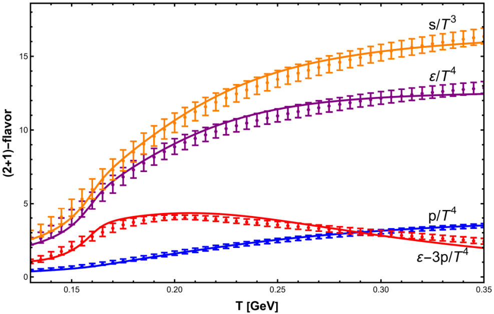

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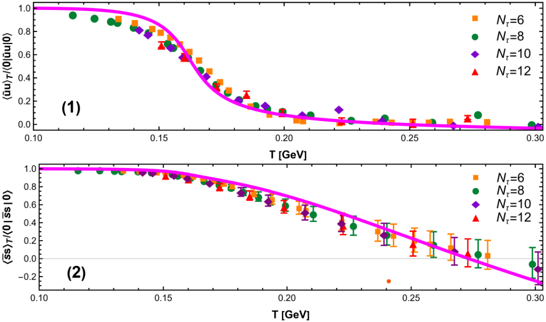
[arXiv:1407.6387 [hep-lat]]

- Excellent agreement with lattice data.

Chiral Condensates

Normalized chiral condensates (light, strange quarks)



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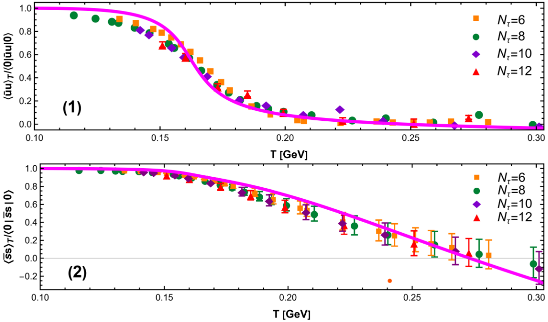

- Matches lattice QCD data with error bars [arXiv:1407.6387, 1812.00385]
- Light quarks: Sharp crossover vs. Strange quark: Gradual decrease

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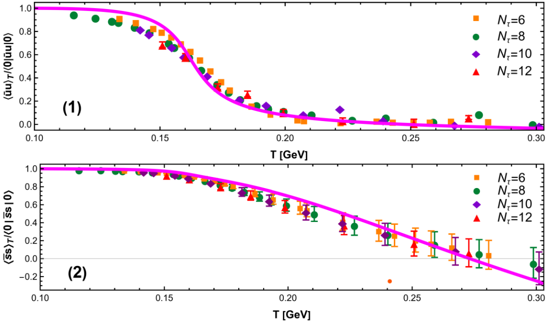


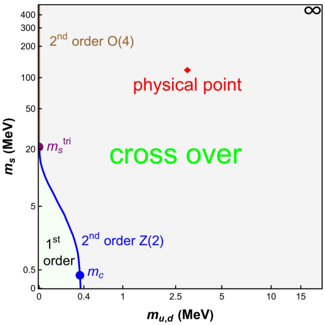
- Matches lattice QCD data with error bars [arXiv:1407.6387, 1812.00385]
- Light quarks: Sharp crossover vs. Strange quark: Gradual decrease

Lesson

- A coupled EDF system can model the EoS and chiral transitions simultaneously and consistently

Quark Mass Phase Diagram (Columbia Plot)

$(m_{u,d}, m_s)$ Phase Diagram from EDF system



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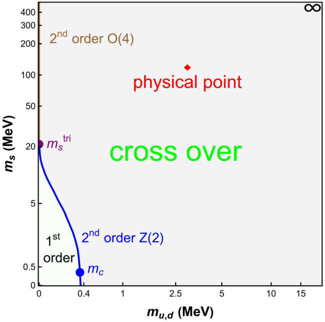

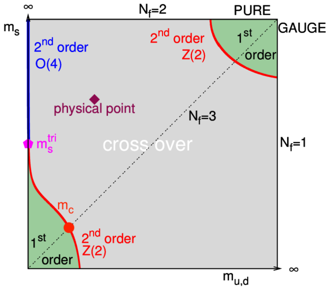

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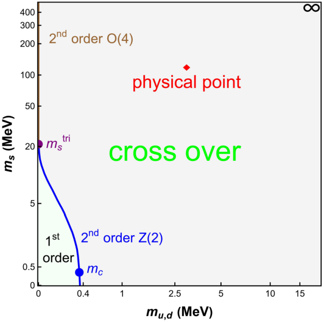
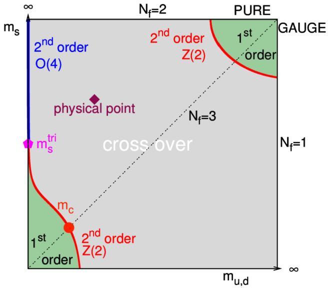
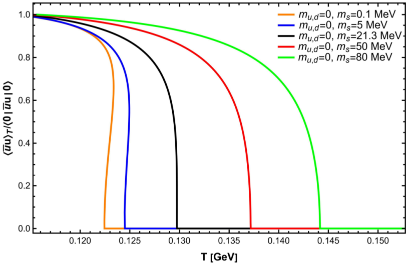


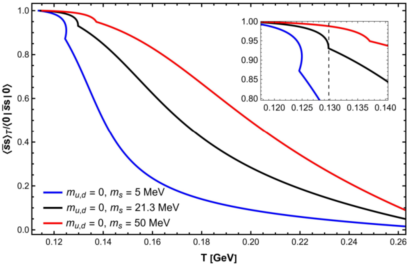


- Physical point: $(3, 120)$ MeV (crossover)
- Tri-critical point: $(0, 21.3)$ MeV $\quad (m_s^{tri} < m_s^{phy})$
- Flavor-symmetric point: $(0.355, 0.355)$ MeV (Close to lattice results)

Determine the Tri-critical Point

Light and strange quark chiral transitions at $m_{u,d} = 0$





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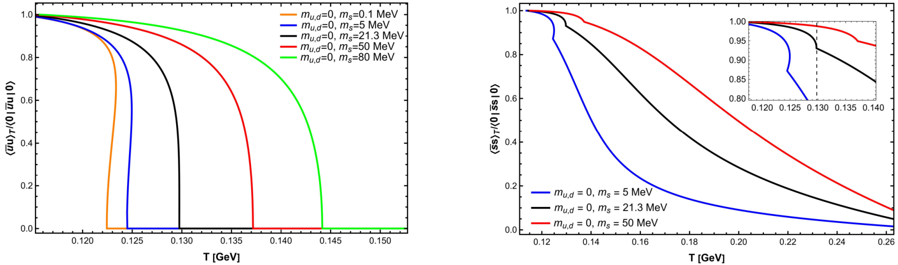
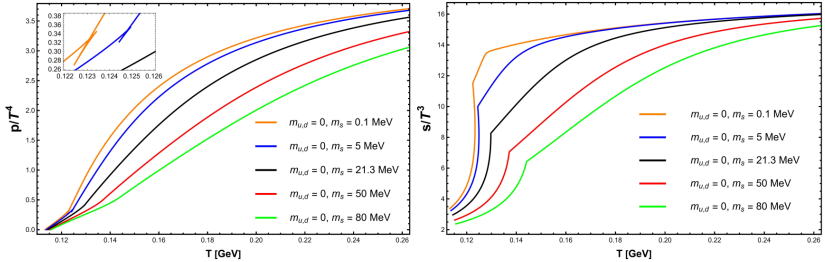


- $m_{u,d} = 0$, vary m_s
- Transition from 1st-order to 2nd-order
- Left panel: Sharp decrease near critical temperature
  Indicates chiral symmetry restoration
- Right panel: Gradual decrease compared to light quarks
  Indicates different restoration temperatures for flavors

Pressure p and entropy density s at $m_{u,d} = 0$



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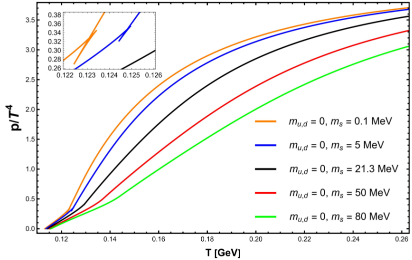
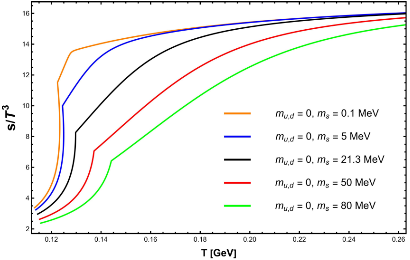
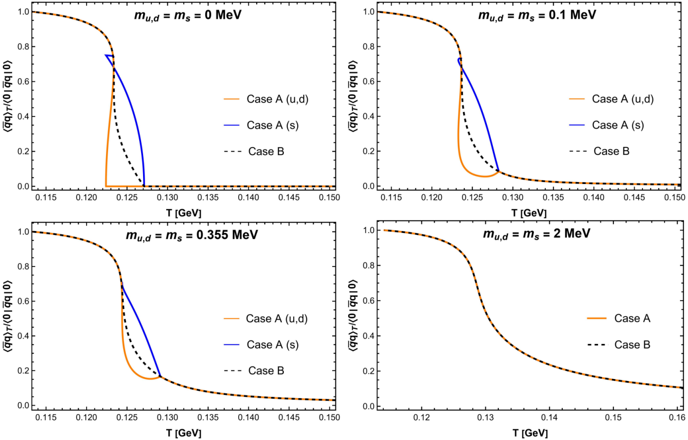




- Transition from 1st-order to 2nd-order
- Dovetail in $p(T)$ confirms 1st-order
- Tri-critical point at $m_s^{tri} = 21.3$ MeV
- Convergence at high temperatures (QGP phase)

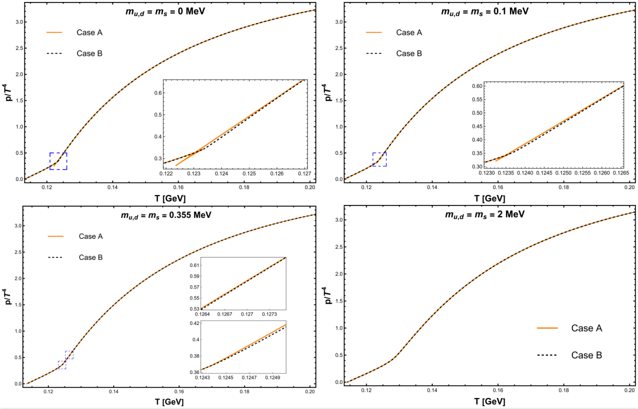
Chiral transitions for $m_{u,d} = m_s$



- Two solutions: Case A: $\chi_u \neq \chi_s$ (stable); Case B: $\chi_u = \chi_s$

Flavor-Symmetric Case ($m_{u,d} = m_s$)

Pressure Analysis



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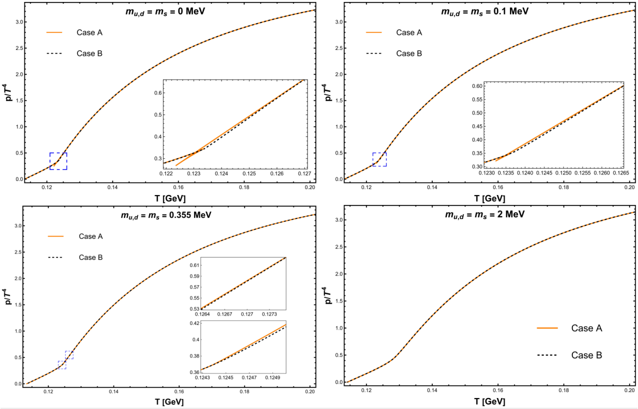
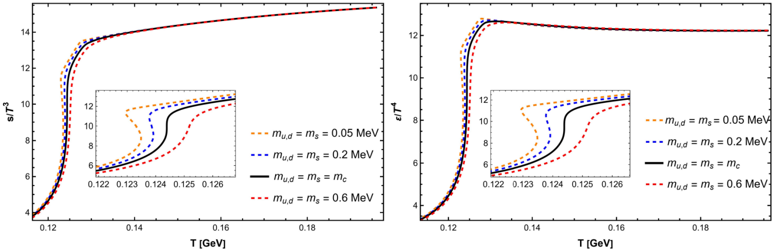
Pressure Analysis



- Pressure for Case A ($\chi_u \neq \chi_s$) is larger, and so stable
- Shows first-order (dovetail) and crossover transitions
- Critical mass: $m_c = 0.355$ MeV (Consistent with lattice)

Entropy and Energy Density



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- Future works and Applications:
  - Extending to finite chemical potential?
  - Applying ML to other QCD observables?
  - Early universe: QCD transitions post-Big Bang
  - Neutron stars: Strongly interacting matter

Thanks for your attention!