Entanglement and Symmetries in Conformal Field Theories and Holography

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Symmetry-resolved entanglement in AdS_3/CFT_2 coupled to U(1) Chern-Simons theory

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Charged Moments in W_3 Higher Spin Holography

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On the boundary conformal field theory approach to symmetry-resolved entanglement

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Symmetry-resolved entanglement for excited states and two entangling intervals in AdS_3/CFT_2

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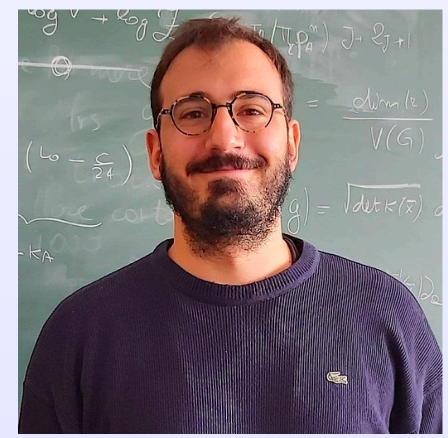

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Quantum Information and AdS/CFT

Building up spacetime with quantum entanglement

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Abstract

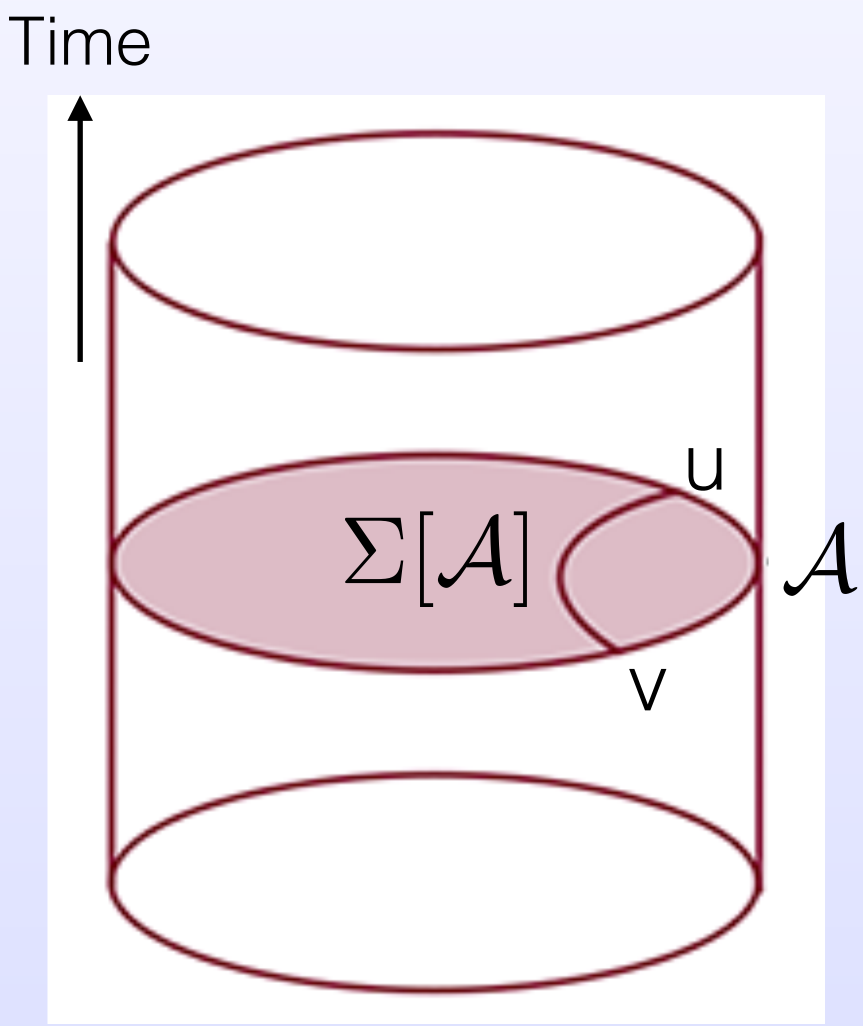
In this essay, we argue that the emergence of classically connected spacetimes is intimately related to the quantum entanglement of degrees of freedom in a non-perturbative description of quantum gravity. Disentangling the degrees of freedom associated with two regions of spacetime results in these regions pulling apart and pinching off from each other in a way that can be quantified by standard measures of entanglement.

arXiv:1005.3035 [hep-th]

Fine-grained information on bulk (quantum) gravity from symmetry resolved entanglement?

Entanglement Entropy in AdS3/CFT2

Minimal length curve (geodesic)
anchored at the ends of the entangling interval



$$S(\mathcal{A}) = \frac{\mathrm{Length}(\Sigma[\mathcal{A}])}{4G_3}$$

$$c = \frac{3L}{2G_3} \gg 1$$

L… Curvature Radius of AdS3 space-time

For ground state of 2D CFTs:

$$S(\mathcal{A}) = \frac{c}{3} \log \frac{|v-u|}{\epsilon}$$

ϵ … Short Distance Cutoff

Ryu, Takayanagi 2006
Hubeny-Rangamani-Takayanagi 2007

Holzhey-Larsen-Wilczek 1994,
Cardy-Calabrese 2004

Symmetry Resolved Entanglement

Entanglement entropy in each charge sector, e.g. U(1)

charge operator Q $\qquad Q = Q_{\mathcal{A}} \oplus Q_{\mathcal{B}}.$

eigenstate of Q, $\quad [\rho, Q] = 0. \quad \Rightarrow \quad [\rho_{\mathcal{A}}, Q_{\mathcal{A}}] = 0.$

Block decomposition: $\qquad \rho_{\mathcal{A}} = \oplus_q \rho_{\mathcal{A}}(q)$

Symmetry Resolved Renyi and Entanglement Entropy:

$$S_n(q) = \frac{1}{1-n} \log \mathrm{Tr} \left(\frac{\rho_{\mathcal{A}}(q)}{P_{\mathcal{A}}(q)} \right)^n \qquad P_{\mathcal{A}}(q) = \frac{\mathrm{Tr}\rho_{\mathcal{A}}(q)}{\mathrm{Tr}\rho_{\mathcal{A}}} = \mathrm{Tr}\rho_{\mathcal{A}}(q)$$

$$S_1(q) = \lim_{n \to 1} S_n(q) = -\mathrm{Tr} \left(\frac{\rho_{\mathcal{A}}(q)}{P_{\mathcal{A}}(q)} \log \frac{\rho_{\mathcal{A}}(q)}{P_{\mathcal{A}}(q)} \right)$$

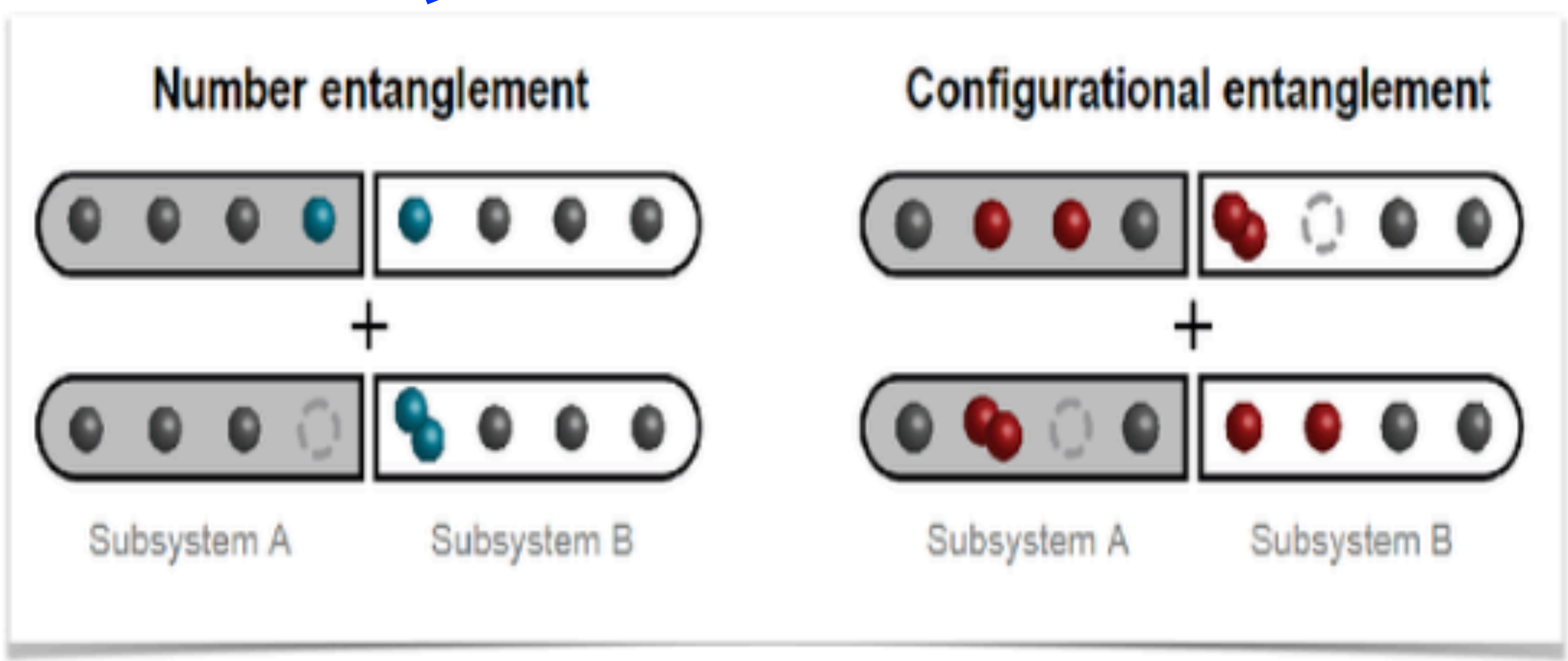
Sela, Goldstein PRL 2018

Symmetry Resolved Entanglement

Entanglement entropy in each charge sector

$$S_1 = \sum_q P_{\mathcal{A}}(q) S_1(q) - \sum_q P_{\mathcal{A}}(q) \log P_{\mathcal{A}}(q)$$

$$P_{\mathcal{A}}(q) = \frac{\mathrm{Tr}\rho_{\mathcal{A}}(q)}{\mathrm{Tr}\rho_{\mathcal{A}}} = \mathrm{Tr}\rho_{\mathcal{A}}(q)$$



A. Lukin, M. Rispoli, R. Schittko, M. E. Tai, A. M. Kaufman,S. Choi, V. Khemani, J. Leonard, and M. Greirer, Probing entanglement in a many-body localized system, Science 364, 6437 (2019).

Example: 2-Qubit system

$$|EPR\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle_A \times |1\rangle_B + |1\rangle_A \times |0\rangle_B\right)$$

$$\rho_A = \frac{1}{2}\left(|0\rangle_A\langle 0|_A) + |1\rangle_A\langle 1|_A\right)$$

Conserved Charge: Occupation number (0 or 1)

$$\rho_A(0) = |0\rangle_A\langle 0|_A \qquad \rho_A(1) = |1\rangle_A\langle 1|_A$$

$$P_A(0) = P_A(1) = \frac{1}{2}$$
$$S_1(0) = S_1(1) = 0$$

Only number entanglement (classical charge correlations)
No configurational entanglement (quantum correlations)

U(1) Kac-Moody CFTs

Holographic CFT with U(1) conserved current: $c = \frac{3L}{2G_3} \gg 1$

$$J(z) = \sum_{n=-\infty}^{\infty} \frac{J_n}{z^{n+1}}$$

Conserved current

$$T(z) = \sum_{n=-\infty}^{\infty} \frac{L_n}{z^{n+2}}$$

Energy-Momentum-Tensor

U(1)$_k$ Kac-Moody algebra at level k

$$[J_n, J_m] = \frac{1}{2} n k \delta_{m+n}$$
$$[L_n, J_m] = -m J_{n+m}$$
$$[L_n, L_m] = (n-m) L_{n+m} + \frac{c}{12}\left(n^3 - n\right) \delta_{n+m,0}$$

S. Zhao, C. Northe, RM, JHEP 2021 (arXiv: 2012.11274)
K. Weisenberger, S. Zhao, C. Northe, RM, JHEP 2021 (arXiv: 2108.09210)

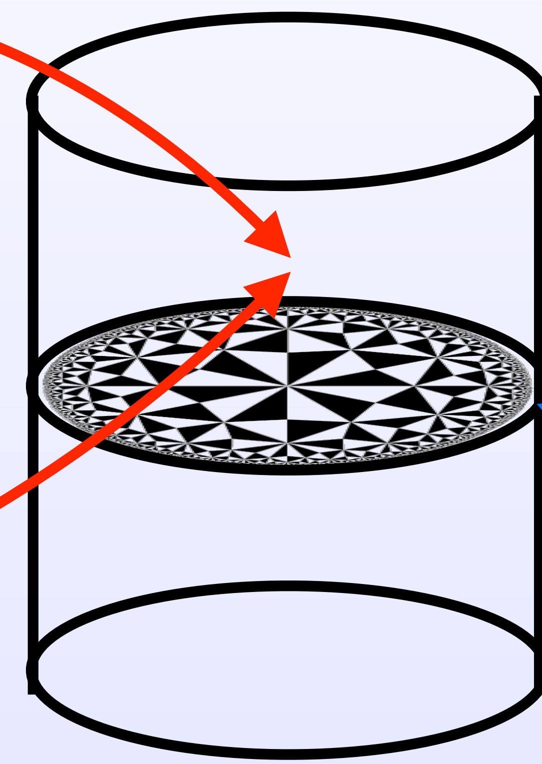
AdS$_3$ dual to U(1)$_k$ Kac-Moody CFT

3D Einstein-Hilbert gravity

$$S_g = \frac{1}{16\pi G_3}\int d^3x\sqrt{g}\left(R+\frac{2}{L^2}\right)$$

L… Curvature Radius of AdS3 space-time

To suppress quantum gravity effects: $c = \frac{3L}{2G_3} \gg 1$



U(1)$_k$ Chern-Simons theory

$$S_{CS} = \frac{ik}{8\pi}\int A\wedge dA$$

k… Chern-Simons level

Asymptotic symmetry analysis:

Boundary theory has U(1)$_k$ Kac-Moody symmetry

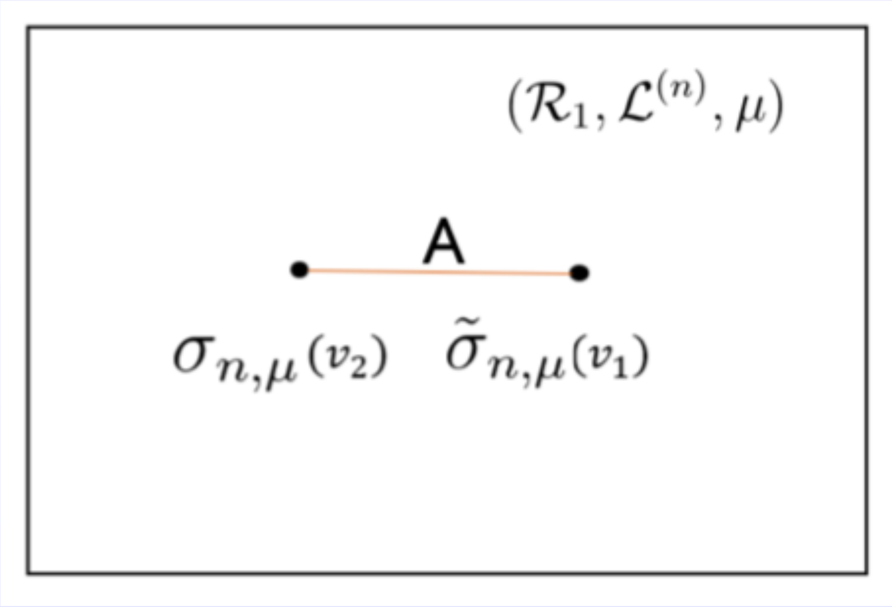
P. Kraus arXiv:hep-th/0609074

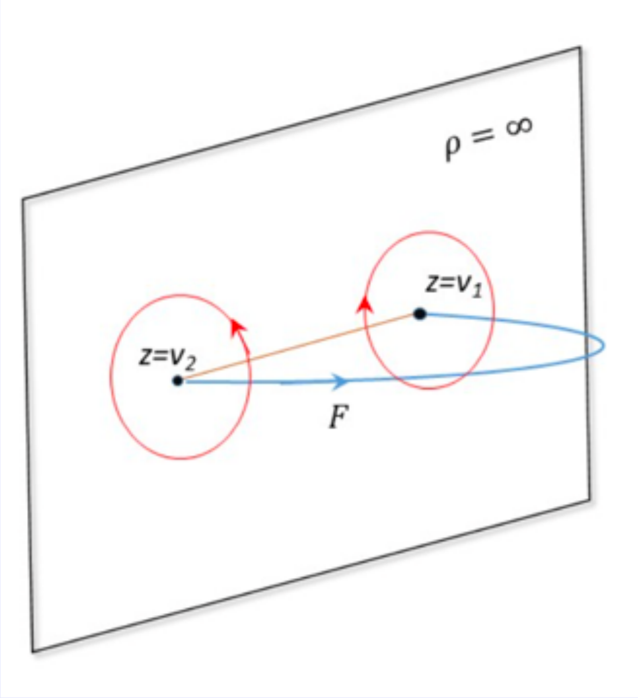
Single Interval SREE in AdS_3/CFT_2

Charged twist operator induces flux:

Sela, Goldstein PRL 2018

Wilson line following the Ryu-Takayanagi geodesic





CFT_2 result matches AdS_3 result for c>>1

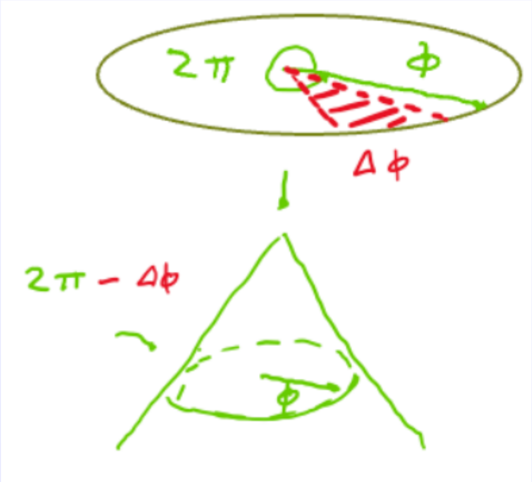
$$S_1(q) = \frac{c}{6}\ell - \frac{1}{2}\log\left(\frac{k\ell}{2\pi}\right) \quad \text{with} \quad \ell = 2\log\frac{|v_1 - v_2|}{\epsilon}$$

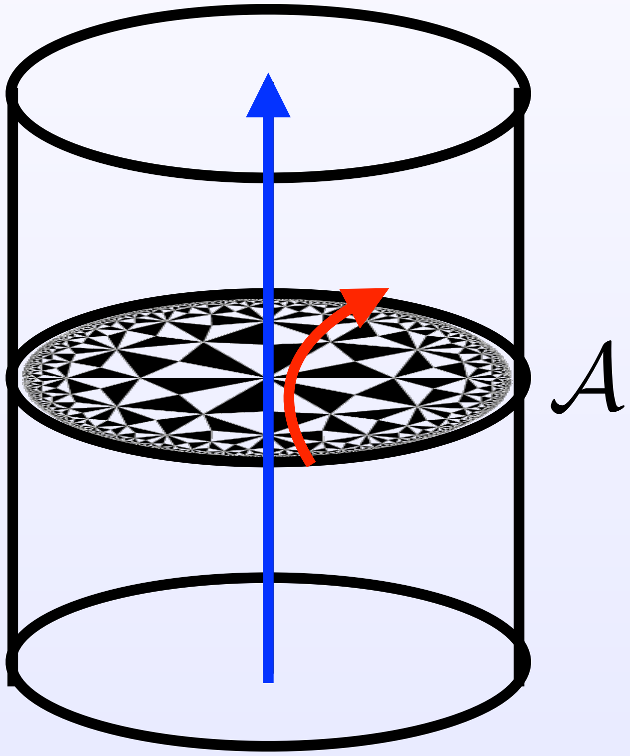
Equipartiton of entanglement!

Suting Zhao, Christian Northe, RM, JHEP 2021 (arXiv: 2012.11274), Belin et.al. 2013, Xavier et.al.. 2019

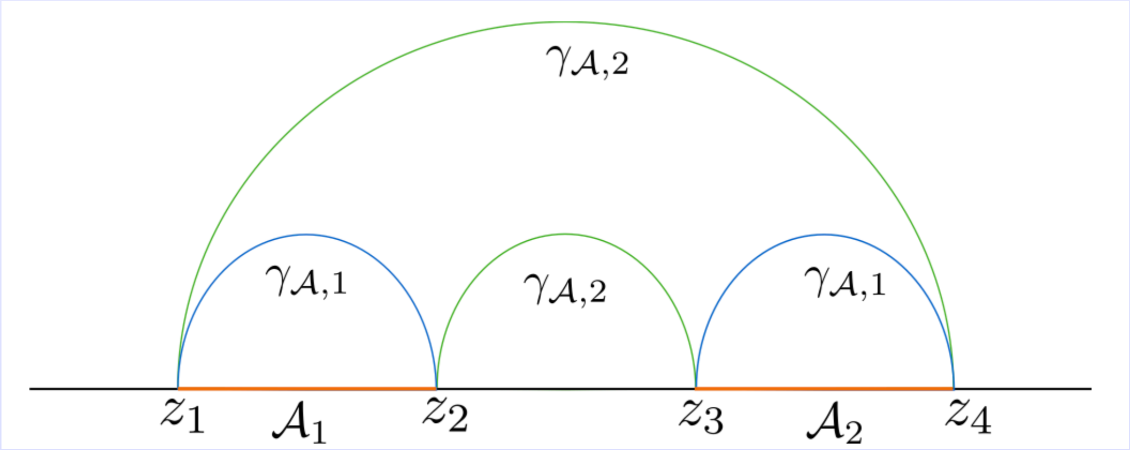
Further Checks

- Single interval with uncharged/charged heavy primary insertions





- Two intervals in the ground state



S. Zhao, C. Northe, RM, JHEP 2021 (arXiv: 2012.11274)
K. Weisenberger, S. Zhao, C. Northe, RM, JHEP 2021 (arXiv: 2108.09210)

Breakdown of Equipartition

SL(3,R) Higher Spin Gravity in 3D: W_3 symmetric CFT
Energy-momentum tensor plus Spin 3 current

$$T(z)W(w) = \frac{3W(w)}{(z-w)^2} + \frac{\partial W(w)}{z-w} + \cdots$$

Charged moments for a single interval:

Topological black hole grand canonical partition function

Perturbative result to quartic order in μ

$$\log \mathrm{Tr}\left(e^{-2\pi n \mathcal{H} + 2\pi i \mu Q_{\mathcal{A}}}\right) = \frac{c\ell}{6n}\left(-\frac{1}{3}\frac{\mu^2}{n^4} + \frac{10}{27}\frac{\mu^4}{n^8} + \cdots\right)$$

Fourier transformation and taking the replica limit yields breakdown of equipartition in SREE at large c.

Suting Zhao, Christian Northe, Konstantin Weisenberger, RM, JHEP (2022), 2202.11111

Boundary Conformal Field Theory

Subleading corrections to (SR)EE: Boundary Conditions

- Definition of entanglement entropy requires bipartiton

(Ohmori, Tachikawa, 2015)

$$\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$$

Quantum fields are distributions

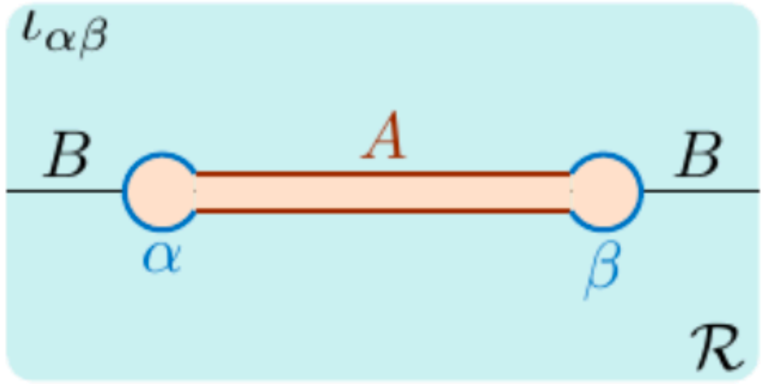
Must be smeared against test functions

Naive bipartition is insufficient

Ohmori & Tachikawa (2015)

Boundary Conformal Field Theory

Solution: Boundary Conditions



Choose boundary conditions which preserve conformal symmetry and $U(1)$

$$T = \bar{T}|_{bdy} \qquad J = \pm\bar{J}|_{bdy}$$

Provide local set of conserved charges: Chiral algebra

$$K_A = \int_A \frac{(2A)^2 - x^2}{4A}(T(x) + \bar{T}(x)) \qquad Q_A = \int_A (J(x) \mp \bar{J}(x))$$

Their eigenstates furnish the Hilbert space $\mathcal{H}_{A,\alpha\beta}$, i.e. the entanglement spectrum associated with the region A.

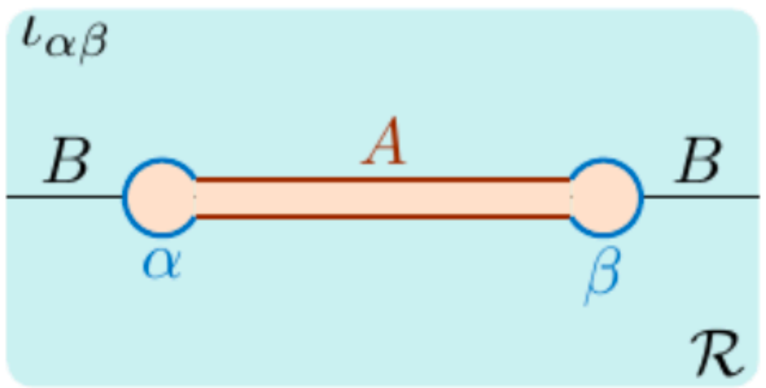
Mathematically, this is encoded in a factorization map

$$\iota_{\alpha\beta} : \mathcal{H} \to \mathcal{H}_{A,\alpha\beta} \otimes \mathcal{H}_{B,\alpha\beta}$$

Ohmori & Tachikawa (2015), Cardi & Tonni (2016)

Example: Compact Free Boson

Two conserved currents: $J^\mu = \partial^\mu \varphi$ $\tilde{J}^\mu = \epsilon^{\mu\nu}\partial_\nu \varphi$



Choose boundary conditions which preserve conformal symmetry and $U(1)$

$$T = \bar{T}|_{bdy} \qquad J = \pm \bar{J}|_{bdy}$$

Dirichlet

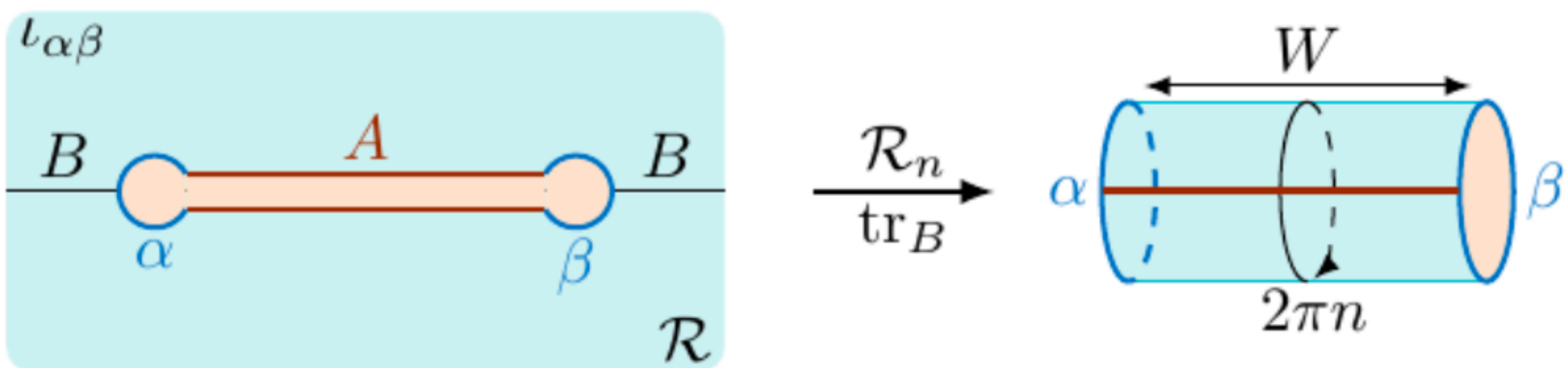
Neumann

- After cutting: NN and DD boundary conditions preserve conservation law for either w or m
- ND and DN boundary conditions break both conservation laws, remaining $\mathbb{Z}_2$ symmetry

Boundary Conformal Field Theory

Solution: Boundary Conditions

- Replicate geometry n times $\to \mathcal{R}_n$
- trace over B
- map onto annulus of width $W = 2\log A/\epsilon$



Focus on vacuum of CFT, $\rho = |0\rangle\langle 0|$. Reduced density matrix

$$\rho_A = \frac{q^{L_0 - c/24}}{Z_{\alpha\beta}(q)},$$

where $Z_{\alpha\beta}(q) = \mathrm{tr}\,_A q^{L_0 - c/24}$ and $q = e^{-2\pi^2/W}$

Ohmori & Tachikawa (2015), Cardi & Tonni (2016)

U(1) Resolution and BCFT

Direct calculation of charged partition functions

Boundary conditions are $J = \pm\bar{J}|_{bdy}$ and $Z_\pm(q) = \sum_{Q\in\sigma_\pm} \chi_Q(q)$

$$\chi_Q(q) = \frac{q^{Q^2}}{\eta(q)} \qquad \eta(q) = q^{1/24}\prod_{k=1}^{\infty}(1-q^k)$$

These correspond to Fock spaces of $U(1)$ charge Q

$$a_{-n_l}\dots a_{-n_2}a_{-n_1}|Q\rangle$$

Charge-projected partition functions select one representation in $Z_\pm$

$$\mathcal{Z}_n(Q) = \mathrm{tr}_A[\Pi_Q\rho_A^n] = \frac{\chi_Q(q^n)}{(Z_\pm(q))^n} \qquad \mathrm{p}_Q = \frac{\chi_Q(q)}{Z_\pm(q)}$$

It vanishes when Q does not appear in $Z_\pm$

Di Giulio, RM, Northe, Scheppach, Zhao, SciPost Physics Core, arXiv: 2212.09767

U(1) Resolution and BCFT

Equipartition to all orders in the UV cutoff expansion

Charge dependence cancels out to all orders in symmetry-resolved entropies

$$S_n(Q) = \frac{1}{1-n} \log \left[\frac{q^{nQ^2}}{\left(q^{Q^2}\right)^n} \frac{\eta^n(q)}{\eta(q^n)} \right]$$

All higher orders are easily found $(W = 2 \log A/\epsilon)$

$$S_n(Q) = \underbrace{\frac{W}{12} \frac{n+1}{n} - \frac{1}{2} \log \left[\frac{W}{\pi} \right] + \frac{1}{2} \frac{\log n}{1-n}}_{\text{Known terms}} + \frac{1}{1-n} \sum_{k=1}^{\infty} \log \left[\frac{\left(1 - e^{-2Wk}\right)^n}{1 - e^{-2Wk/n}} \right]$$

These expressions are reasonable only when Q appears in $Z_\pm$, i.e. in $\mathcal{H}_A \rightsquigarrow$
Determined by boundary conditions

Di Giulio, RM, Northe, Scheppach, Zhao, SciPost Physics Core, arXiv: 2212.09767

Conclusions & Outlook

- New insight into AdS/CFT from quantum information, e.g. Entanglement entropy, Symmetry resolved entanglement
- New tests of AdS_3/CFT_2 plus Chern-Simons
- First example of breakdown of equipartition at large c
- BCFT approach yields exact results due to symmetries, U(1) equipartition to all orders for free bosons

- SREE Analysis of further instances of AdS/CFT **WIP**
- Symmetry resolving other quantum information measures **WIP**
- Higher dimensions? **P. Bueno, P.A. Cano, A. Murcia, A.R. Sanchez, PRL 2022**
- Other symmetry groups? **Y. Kusuki et. al., arXiv:2309.03287**
- Implications on bulk entanglement? **WIP w. T. Kögel, J. Xu, W. Weng, G. Di Giulio**
- Timelike (Pseudo) SREE **WIP w. T. Kögel, C. Englert**
- Entanglement in non-hermitean systems **WIP w. Z. Chen, Z.-Y. Xian**
- Field theory and CondMat Applications? **C. Northe, Phys. Rev. Lett., arXiv:2303.07724**