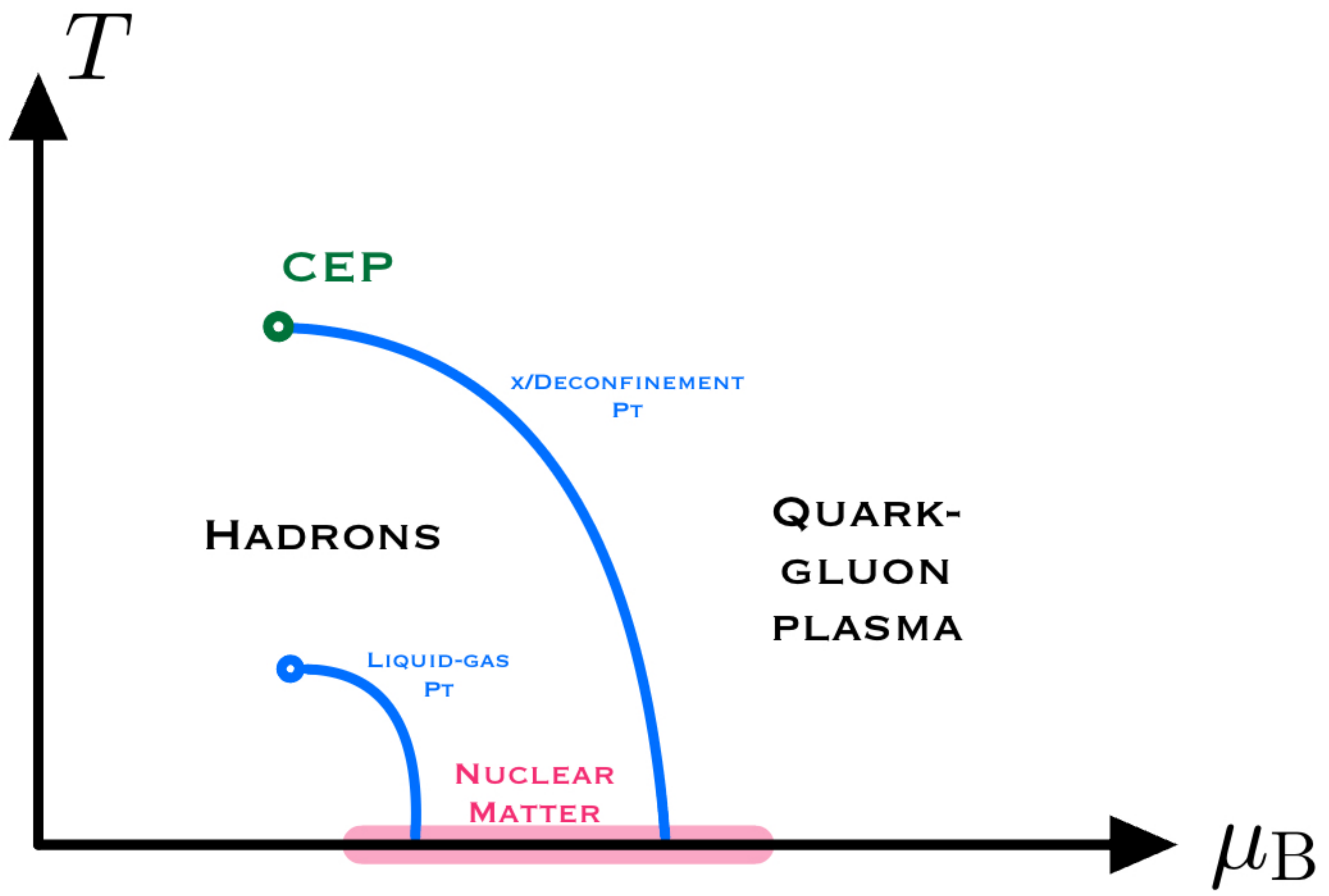
Softening Holographic Nuclear Matter

soon to appear, with Christian Ecker, Nicolas Kovensky & Andreas Schmitt

Orestis Papadopoulos — Holographic Applications — 17/07/2025

QCD Phase Diagram

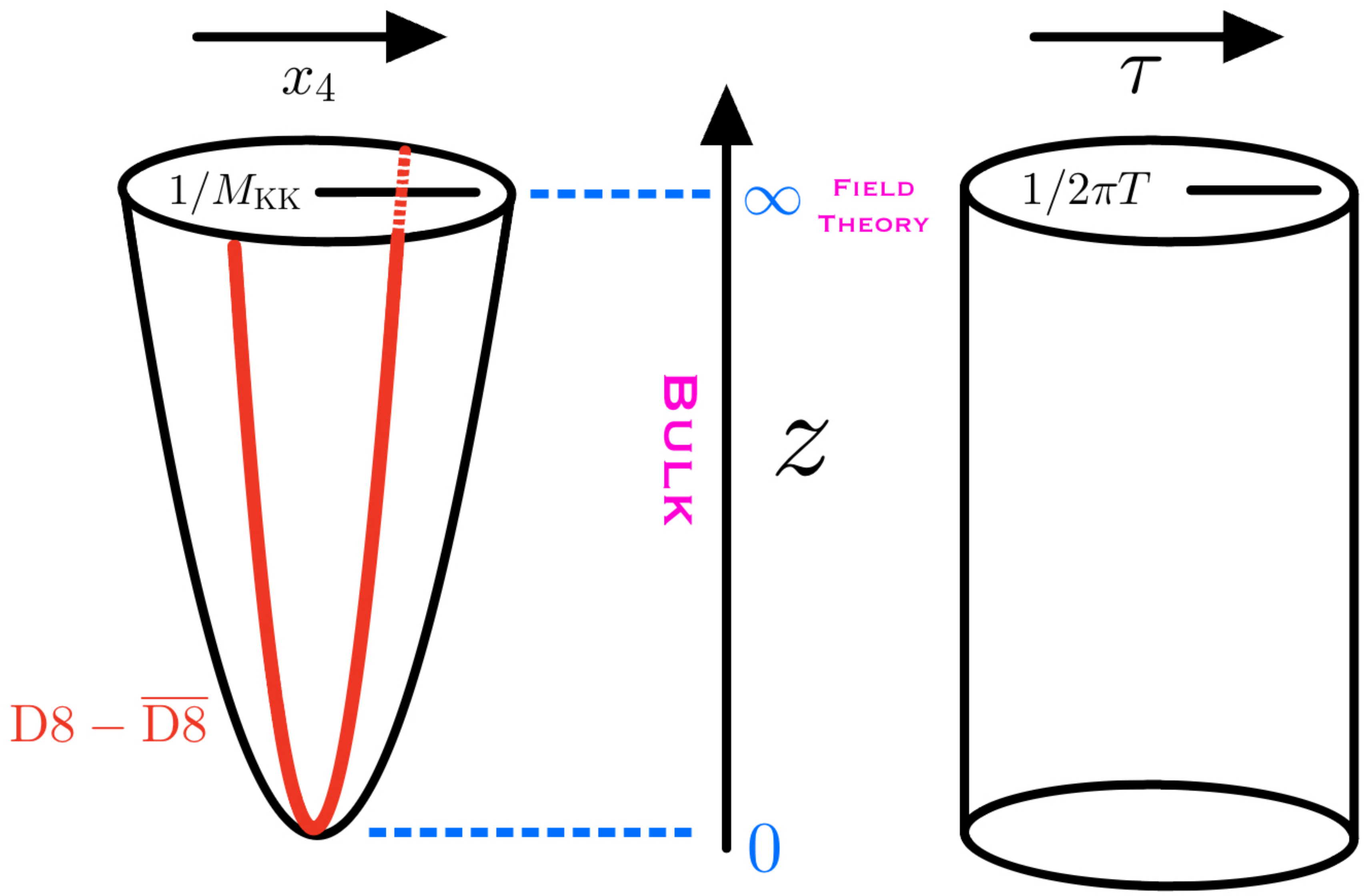
- Interested in finite density, small temperature phases of QCD
- Regime interesting for neutron stars (in equilibrium)
- Strongly coupled regime: use AdS/CFT correspondence
- Would like to reproduce low-density nuclear matter properties in holography



The Witten-Sakai-Sugimoto Model (WSS)

- Top-down model, N_c - D4 branes [Witten '98], N_f - D8 branes [Sakai, Sugimoto '04, '05]
- Two geometries: confined and deconfined
- Antipodal D8 branes
- Chiral symmetry broken:

$$SU(2)_L \times SU(2)_R \to SU(2)_V$$



- Action on the D8 branes: $S_{\mathrm{D8}} = S_{\mathrm{DBI}} + S_{\mathrm{CS}}$

$$S_{\mathrm{DBI}} \propto \int d^4x \int_{-\infty}^{\infty} dz\; e^{-\Phi} \left[\det(g + 2\pi\alpha' \mathscr{F})\right]^{1/2}$$ — use Yang-Mills approximation

$$S_{\mathrm{CS}} \propto \int d^4x \int_{-\infty}^{\infty} dz\; \hat{a}_0 \mathscr{F} \wedge {}^*\mathscr{F}$$

Baryonic Matter in WSS

- Baryons = Wrapped D4 branes
- In WSS, instantons on the D8 branes

[Hashimoto, Sakai, Sugimoto '08]

- Pointlike baryons

[Bergman, Lifschytz, Lippert '07]

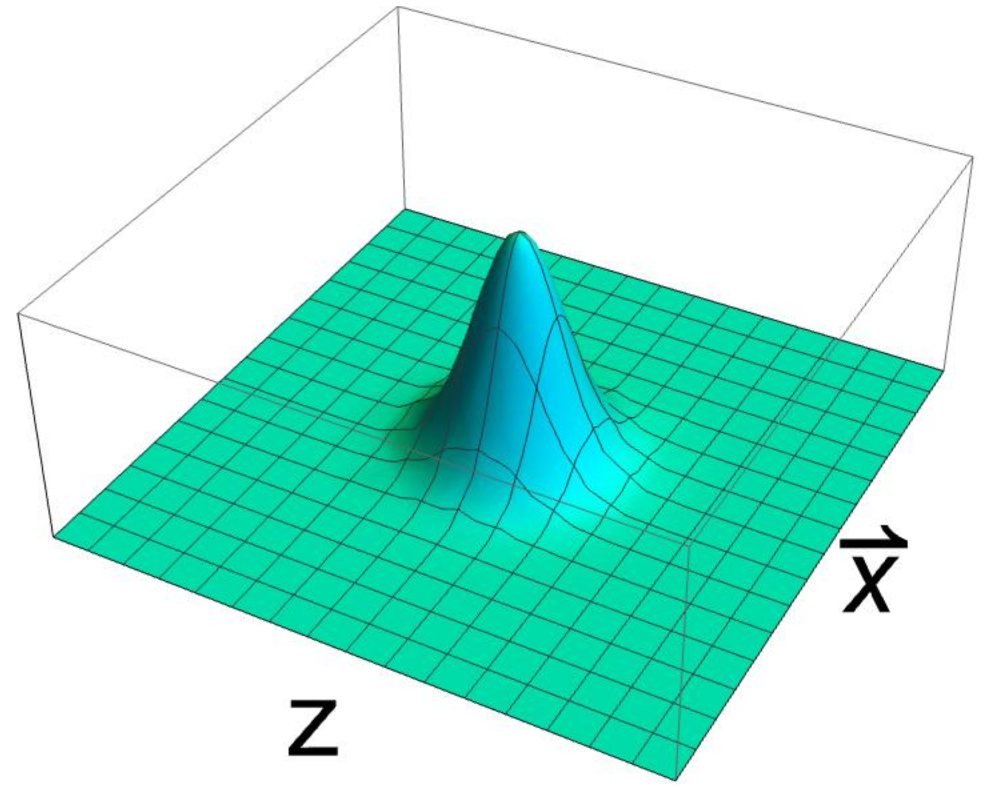
[Kaplunovsky, Melnikov, Sonnenschein '12]

- Homogeneous, finite-width baryons

[Rozali, Shieh, Van Raamsdonk, Wu '07]

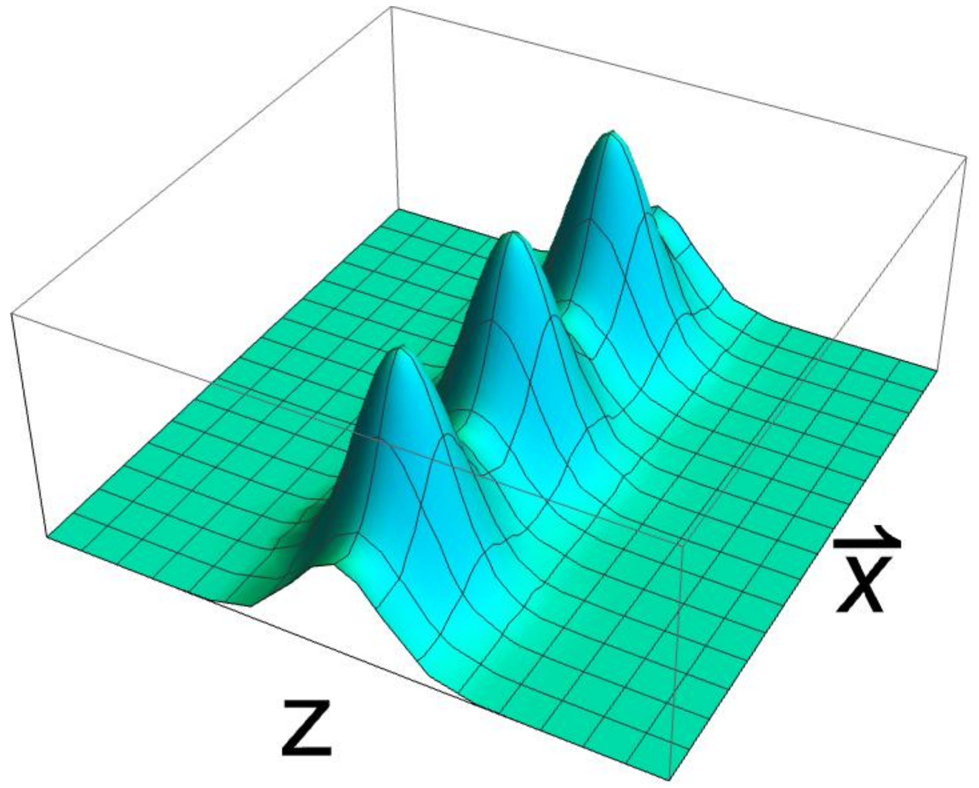
- In VQCD

[Jokela, Järvinen, Nijs, Remes, '20]



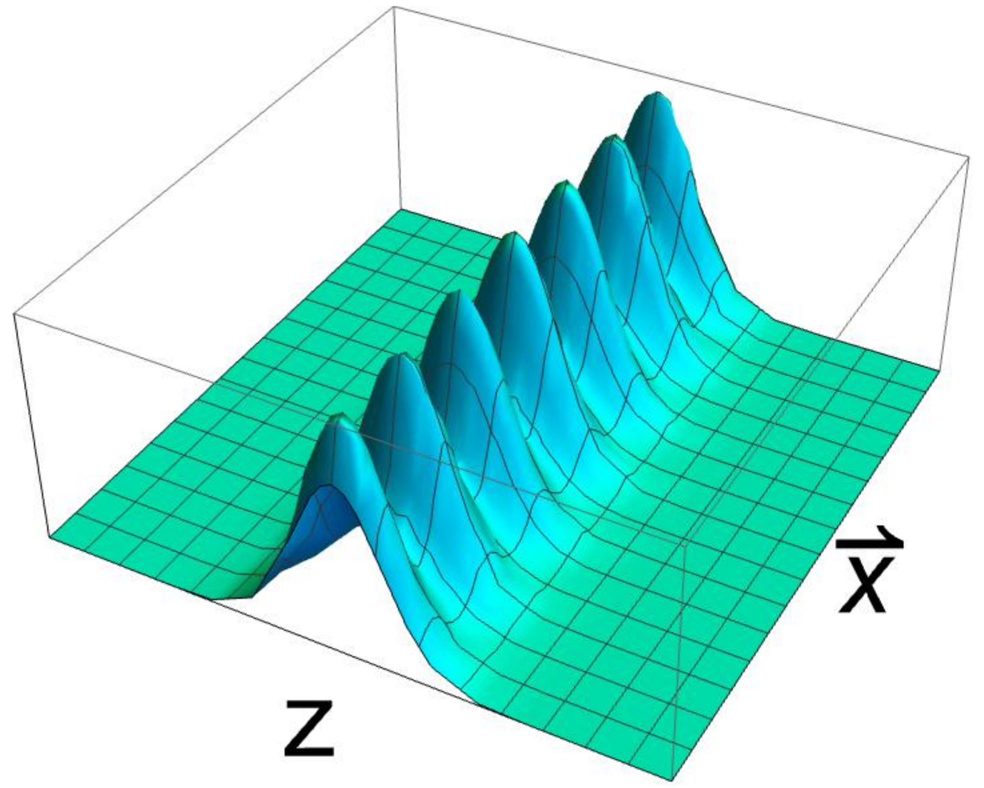
Single baryon

Instanton in space and holographic direction



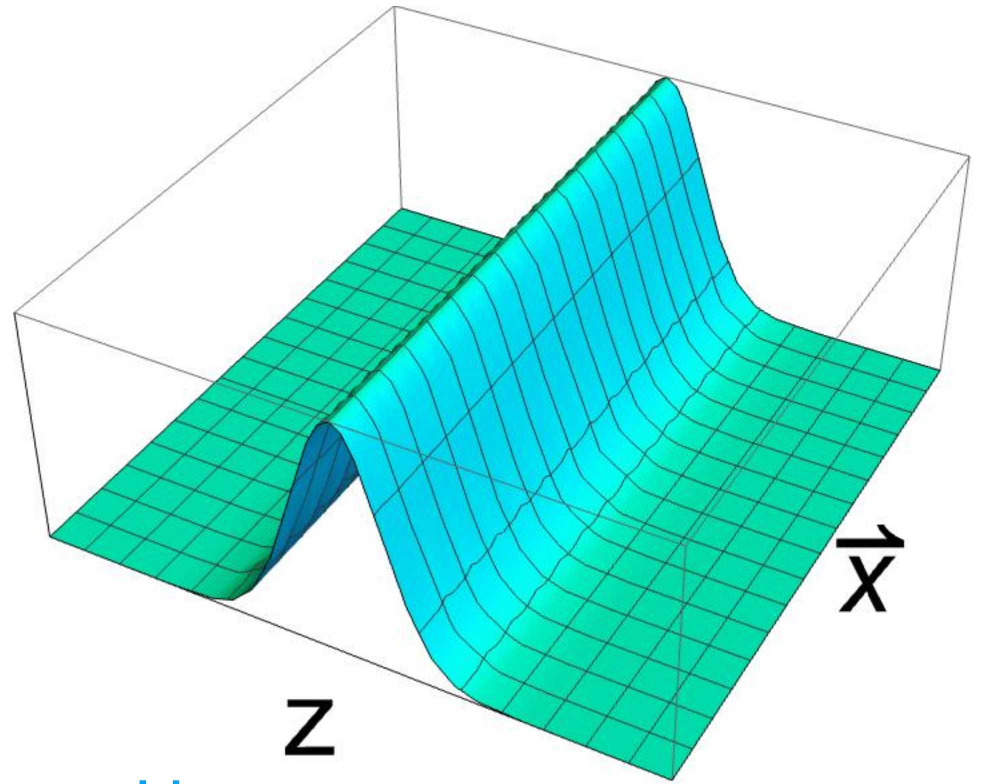
Baryonic matter

Need more than one instanton



Higher densities

Many instantons make computations harder



Homogeneous ansatz

For high densities, homogeneity in spatial directions

→instanton **layer**

Homogeneous Ansatz

Gauge field on D8 branes, from $U(2)$ flavour symmetry on the boundary

Abelian part $\hat{a}_\mu$

Temporal component $\hat{a}_0$ gives baryon chemical potential on the boundary:

$$\hat{a}_0(z = \infty) = \mu_B$$

Non abelian part a_μ

Gives rise to baryon density through Chern-Simons term in action

(Finite width) **Homogeneous ansatz:**

$$a_i(z) = -\frac{\lambda}{8\pi} h(z) \sigma_i$$

Need for Jumps

- Baryon density given by Chern-Simons term

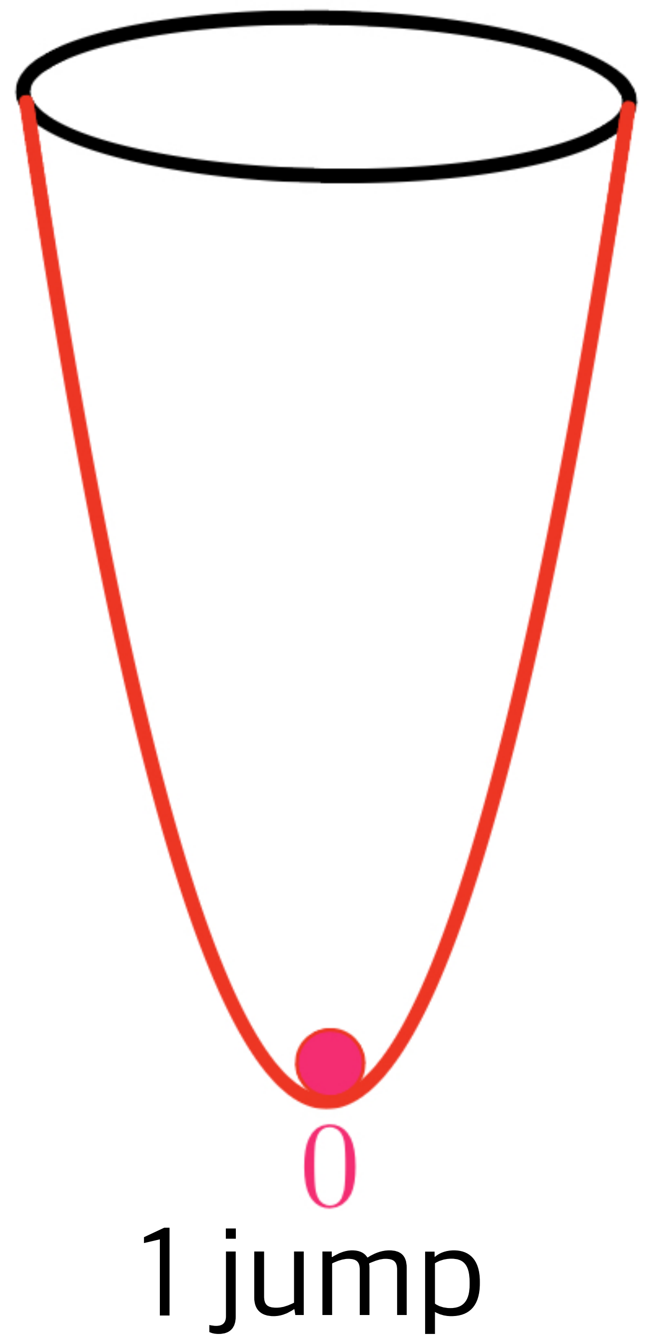
$$n_{\mathrm{B}} = \frac{3\lambda}{32\pi} \int_{-\infty}^{\infty} dz\ \partial_z(h^3)$$

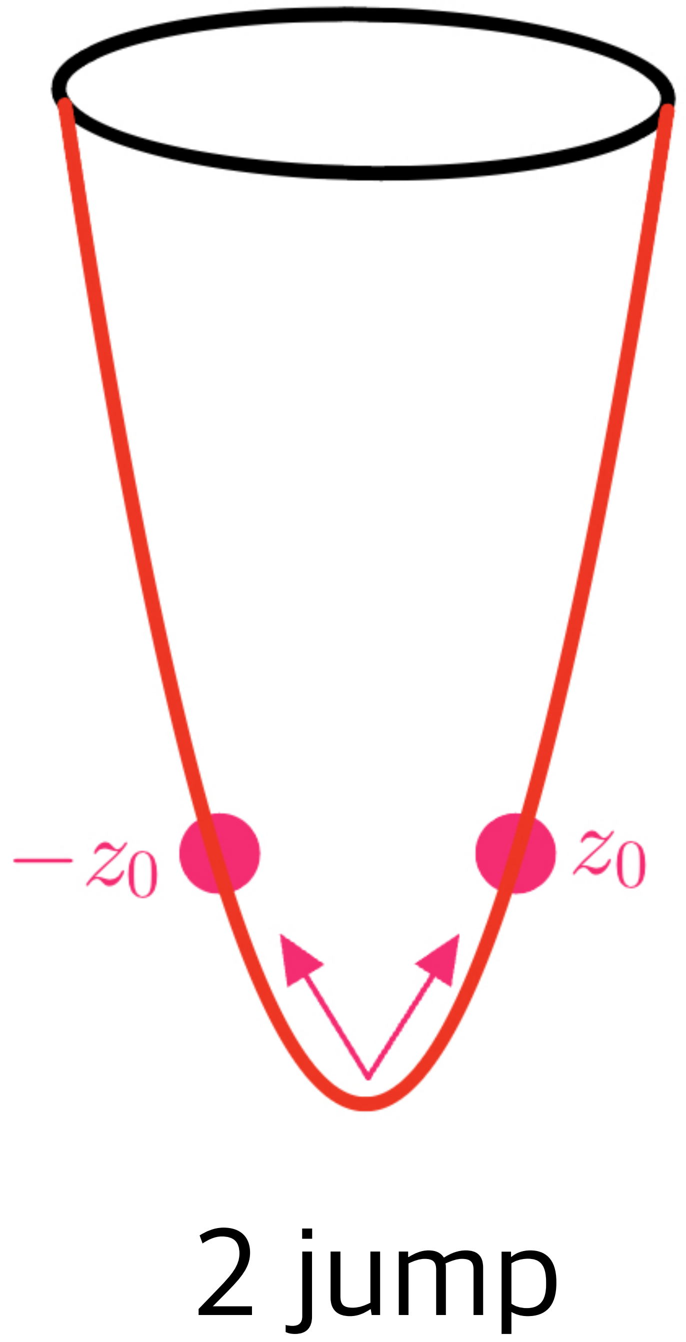
- But also $h(-\infty) = h(\infty) = 0 \quad \Longrightarrow \quad h(z)$ must have a discontinuity (jump)
- 1-jump solution has been studied before [Rozali, Shieh, Van Raamsdonk, Wu '07 / Li, Schmitt, Wang '15]
- One can fix $(\lambda, M_{\mathrm{KK}})$ to reproduce $(\tilde{\mu}_{\mathrm{B}}, \tilde{n}_{\mathrm{B}}) = (922.7\ \mathrm{MeV},\ 0.153\ \mathrm{fm}^{-3})$ at saturation
- Incompressibility K of nuclear matter for this choice $\Longrightarrow$ 8 times too large

What we Studied

- Two goals:

1. To study the multi-jump phase structure within the homogeneous ansatz
2. To improve the description of nuclear matter by reducing the incompressibility at saturation





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Behaviour at the Jump

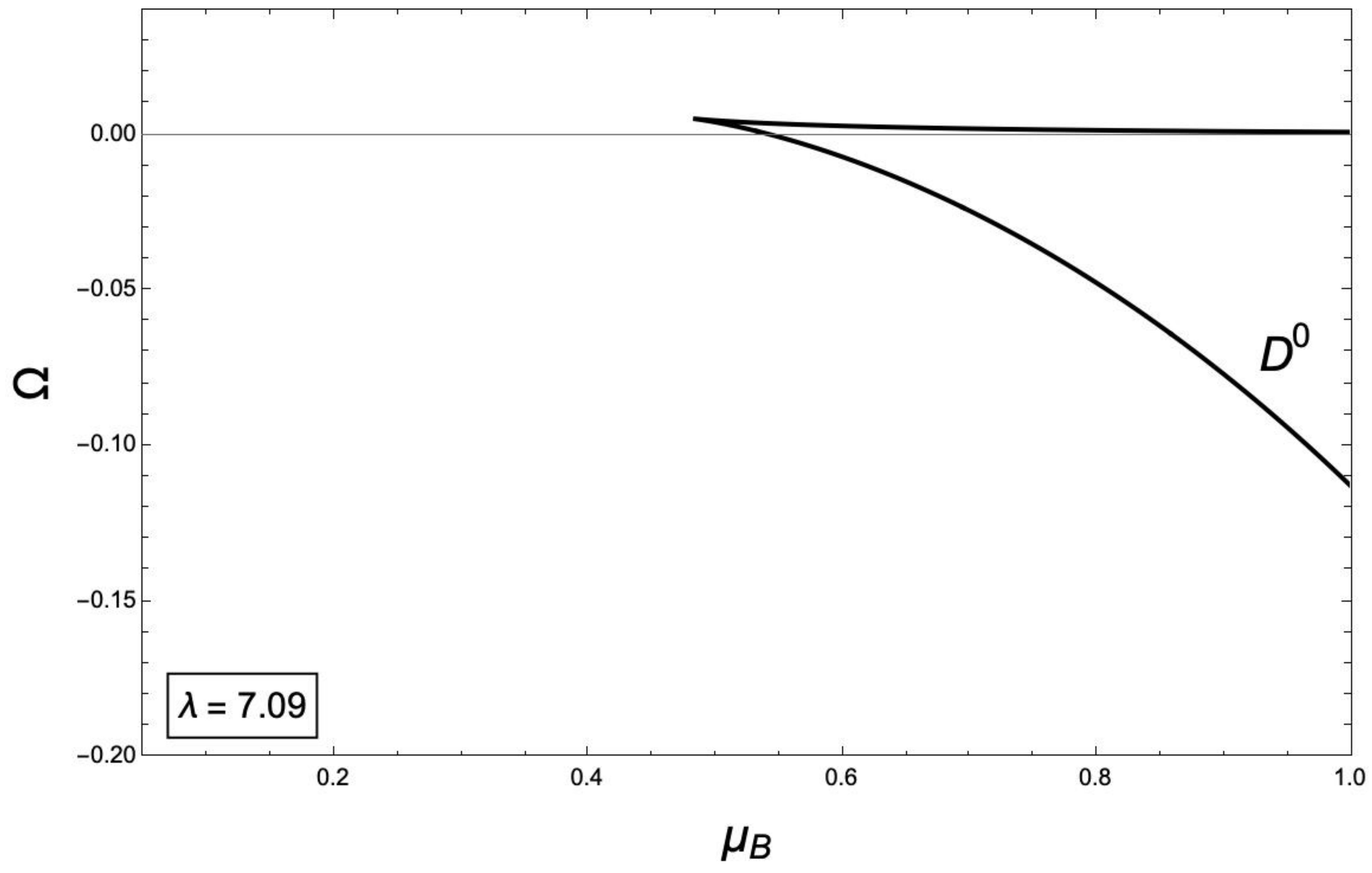
- Location of jump (z_k) and value of h before and after it ($h_\pm$): completely dynamical – extremise free energy w.r.t. them
- Can have 4 different behaviours

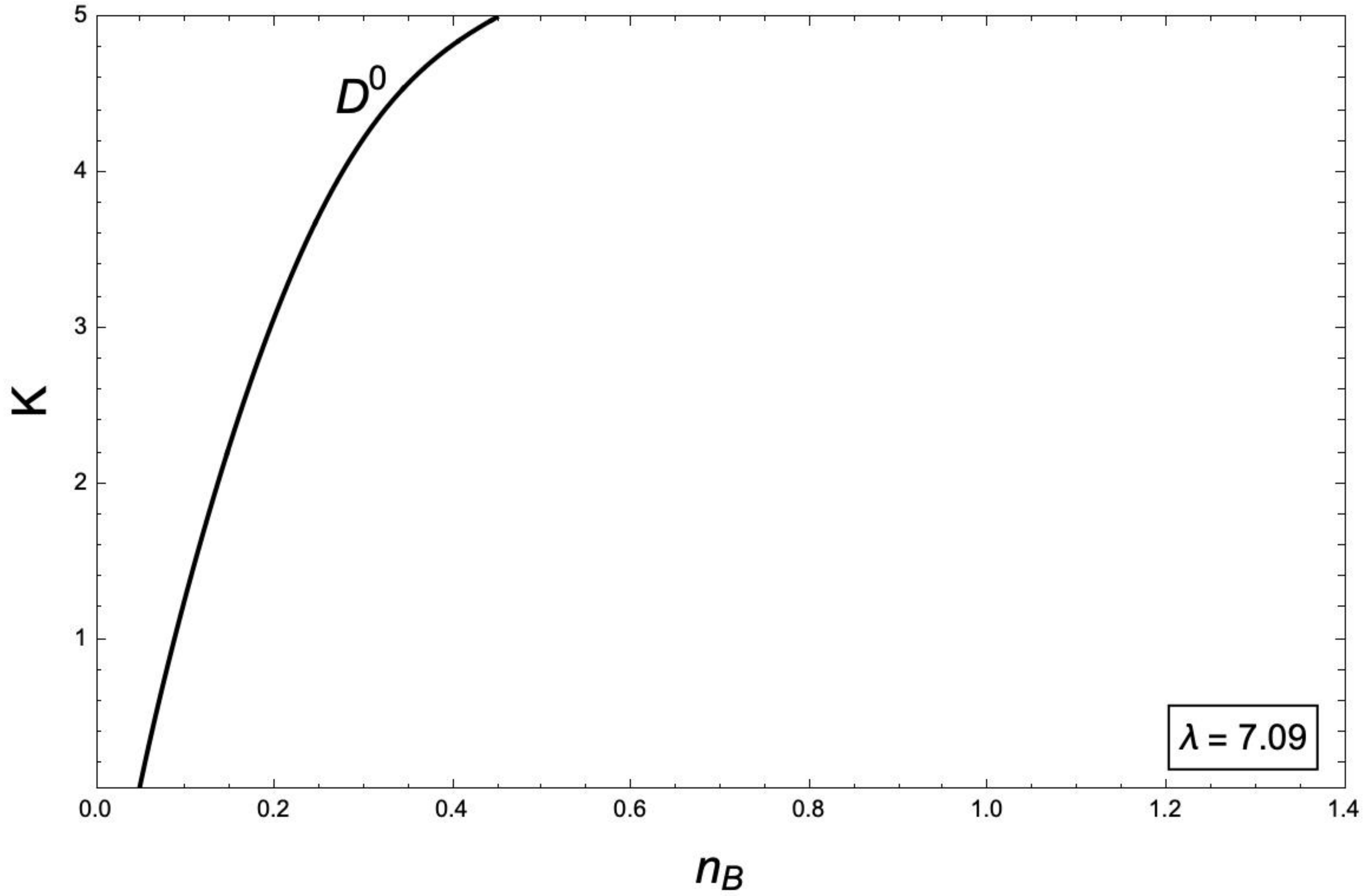
Left [Rozali, Shieh, Van Raamsdonk, Wu '07] | Right [Cruz Rojas, Demircik, Järvinen '23] | Symmetric | Asymmetric

- We solve for $\hat{a}_0, h$ numerically
- Construct phases with up to 4 jumps, and compare free energies (from on shell action)

Solutions

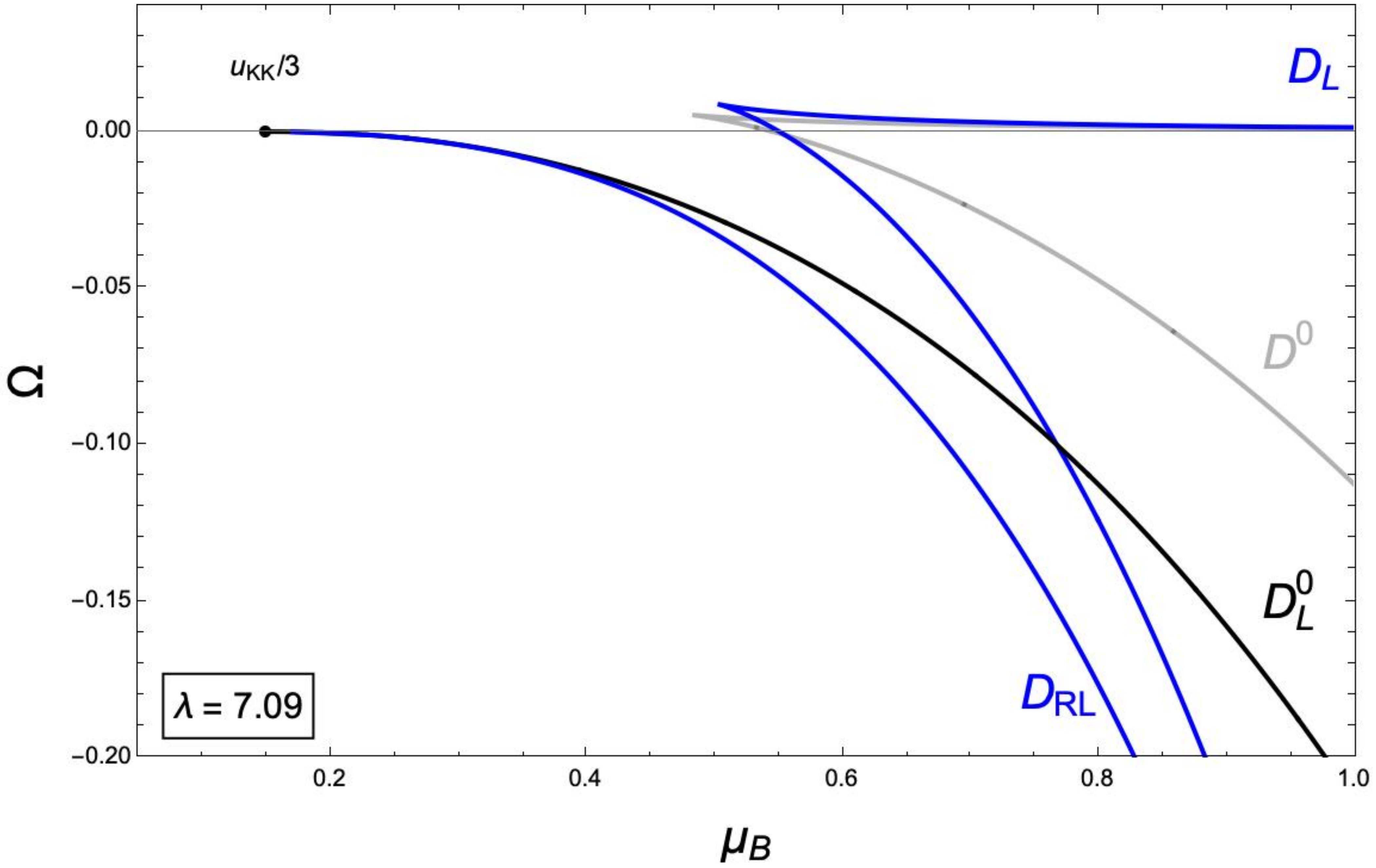
Solutions

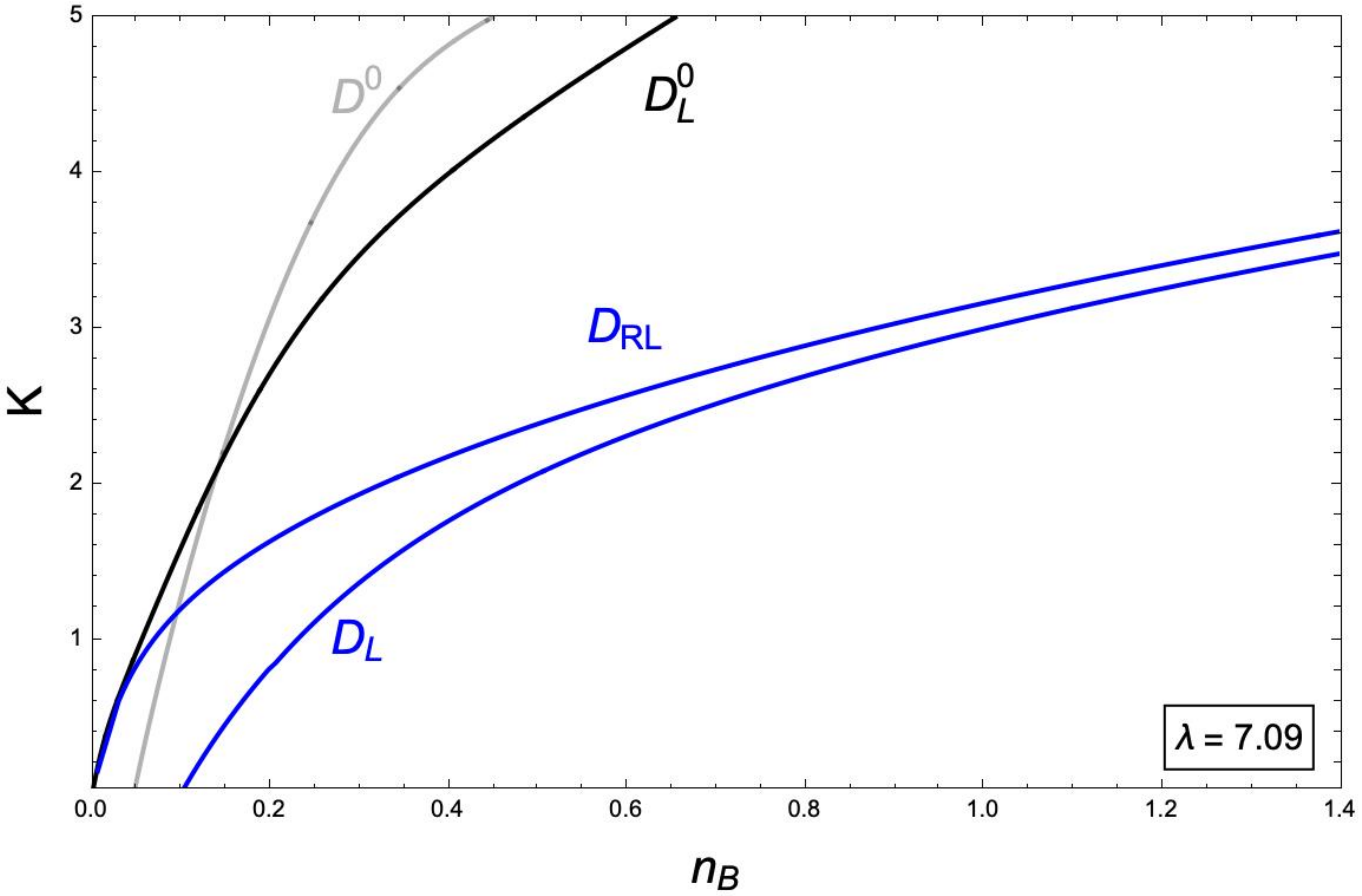




Original 1-jump solution D^0

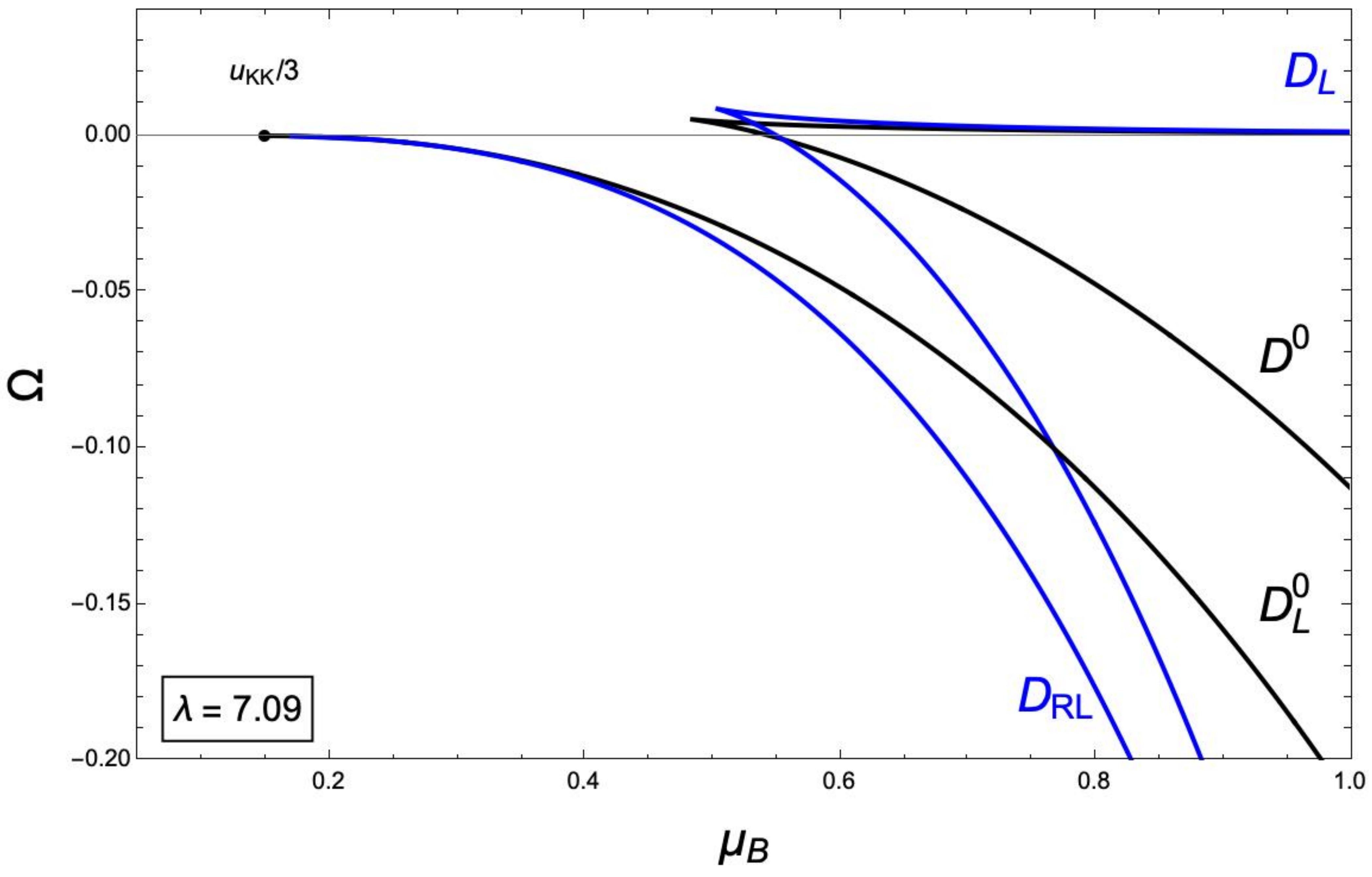
Solutions

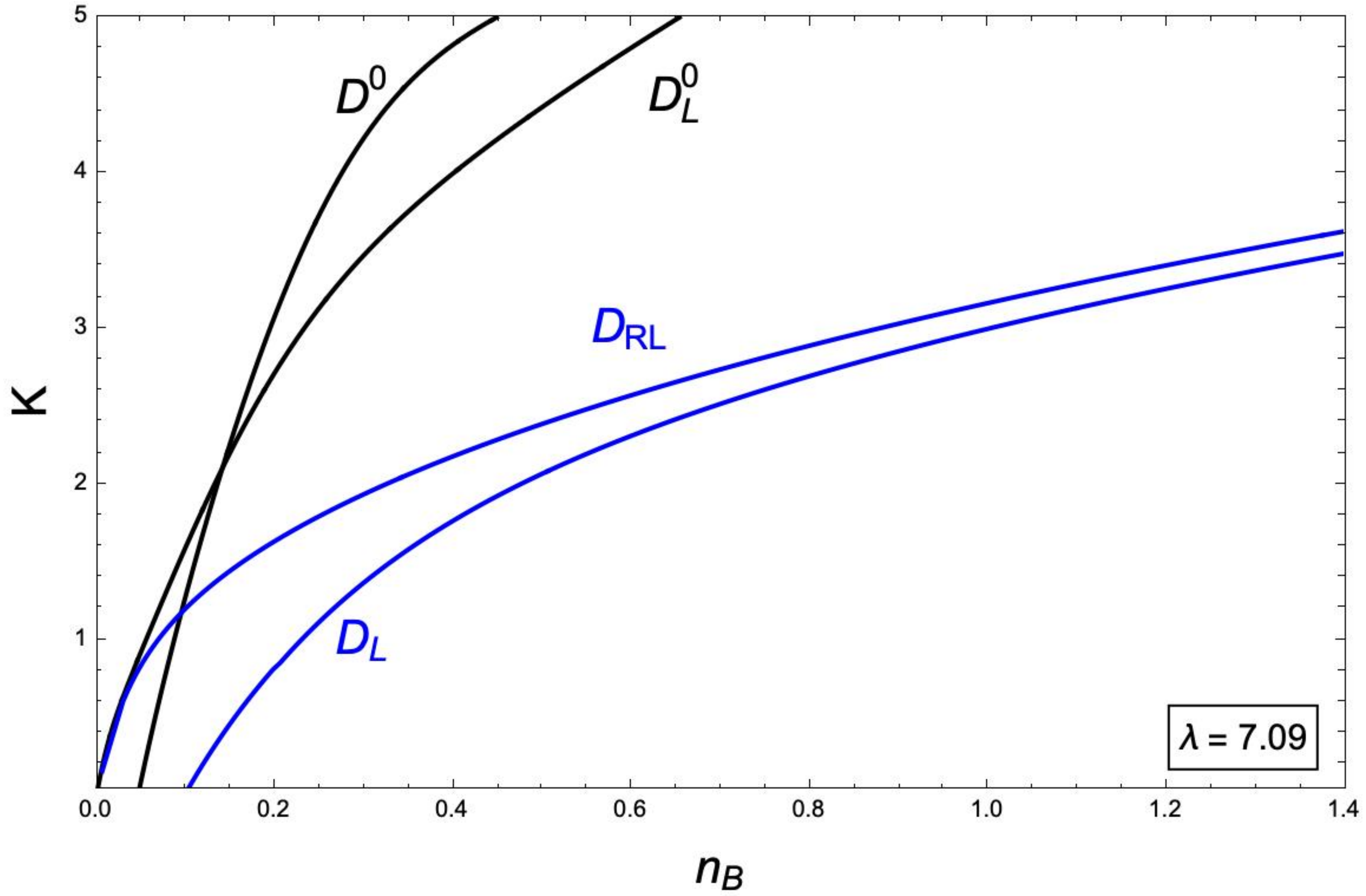




- Most important multi-jump solutions
- Why blue and black? Jump $\neq$ Layer

Solutions





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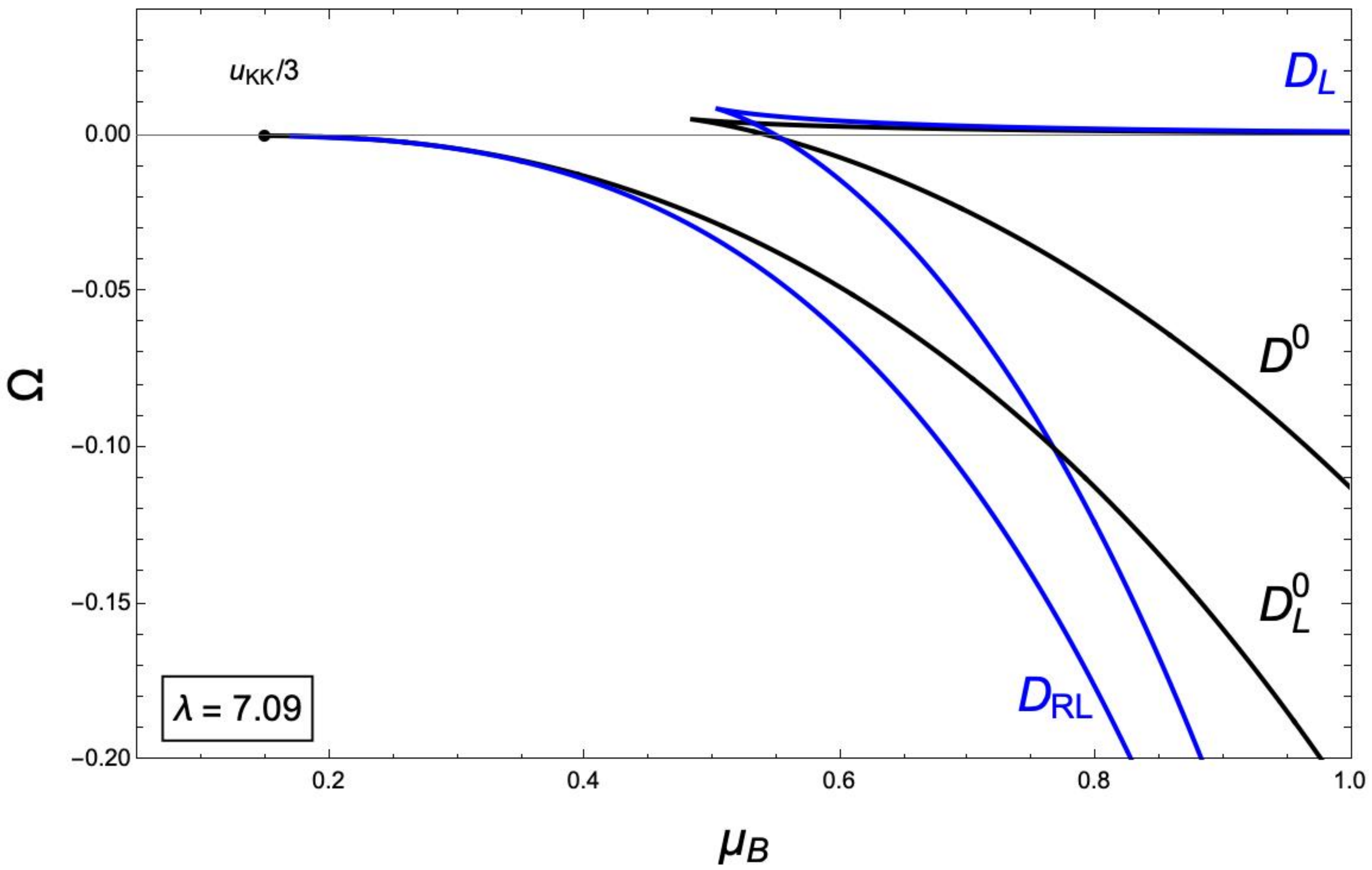

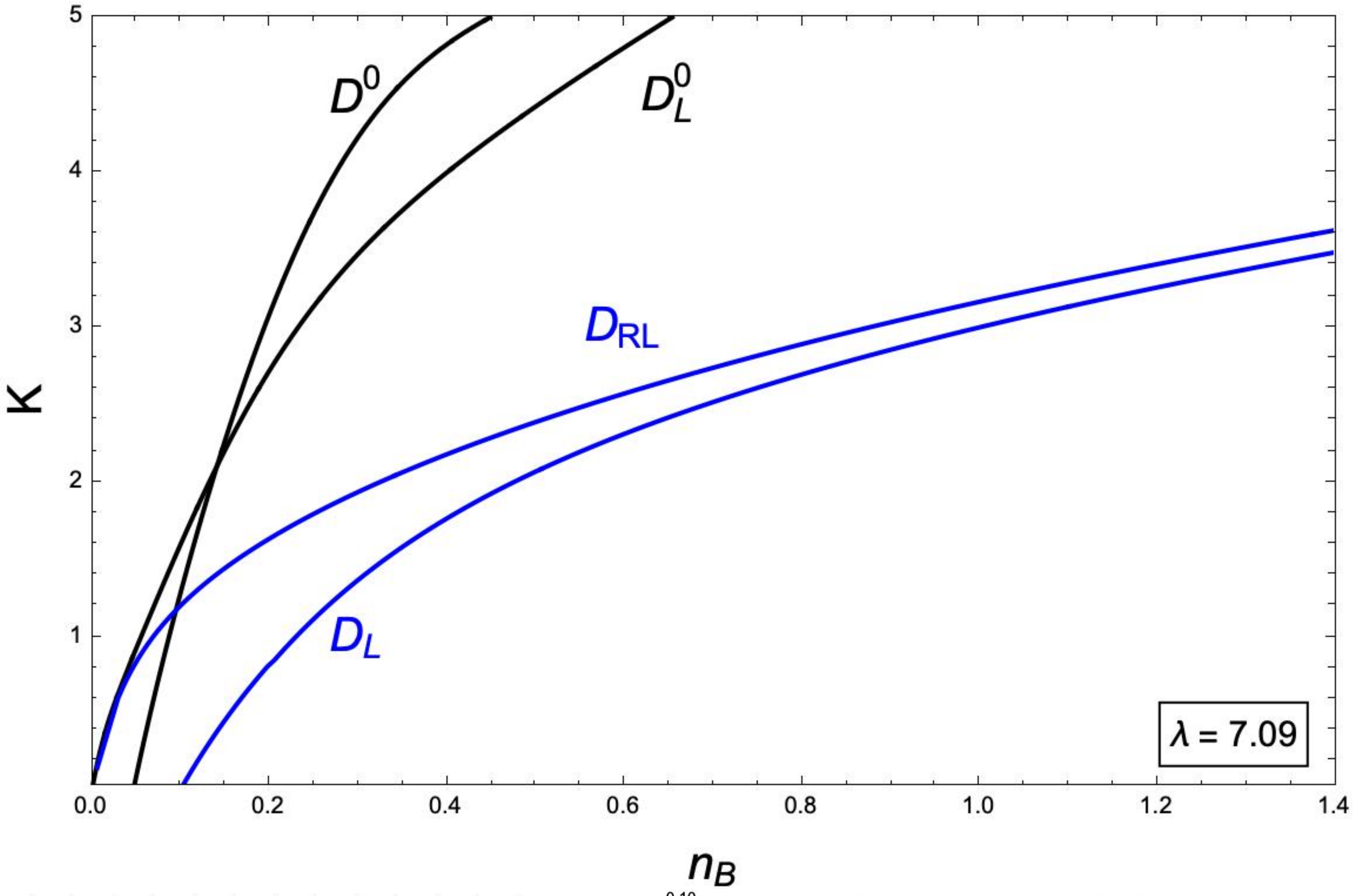

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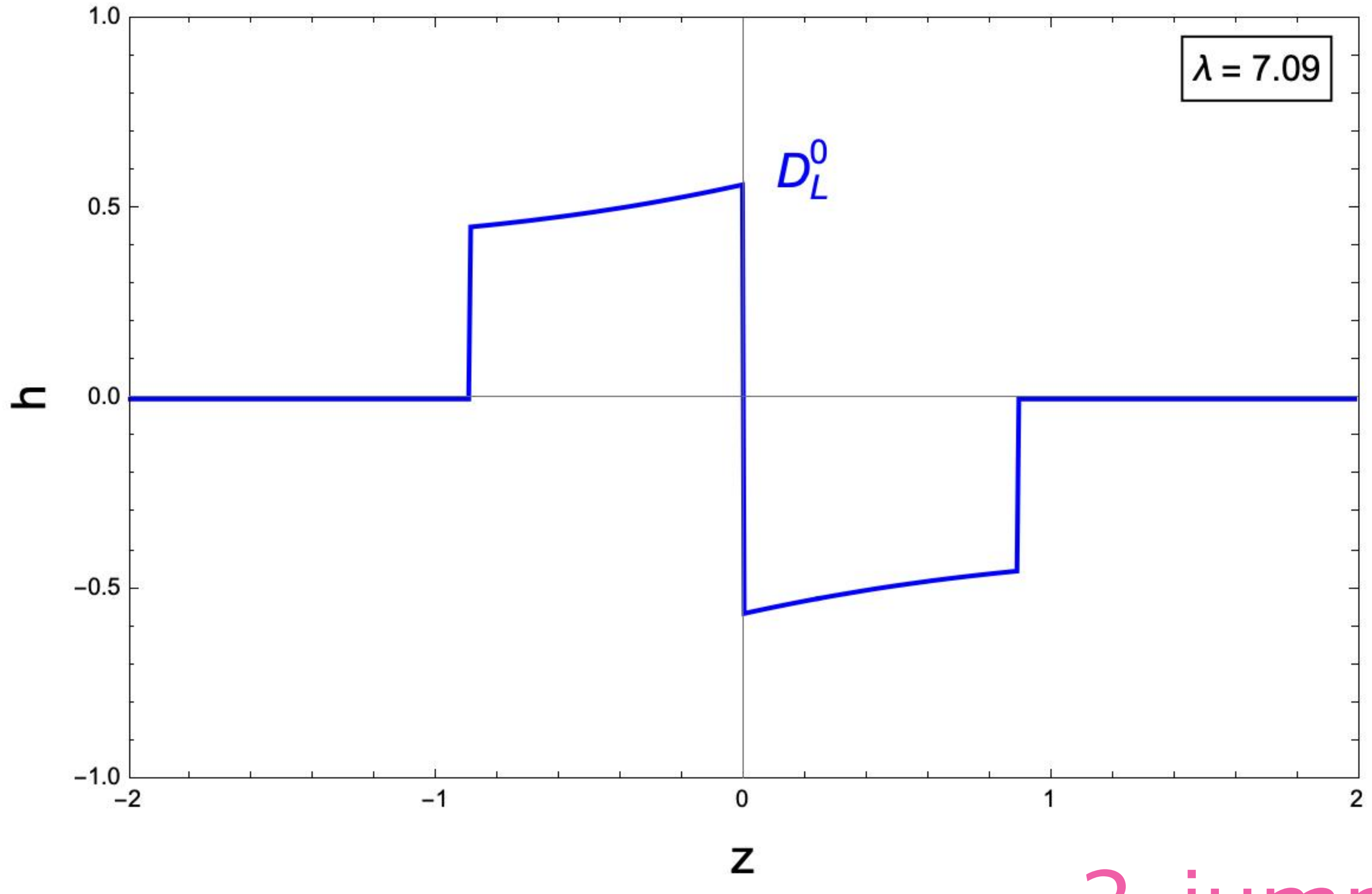
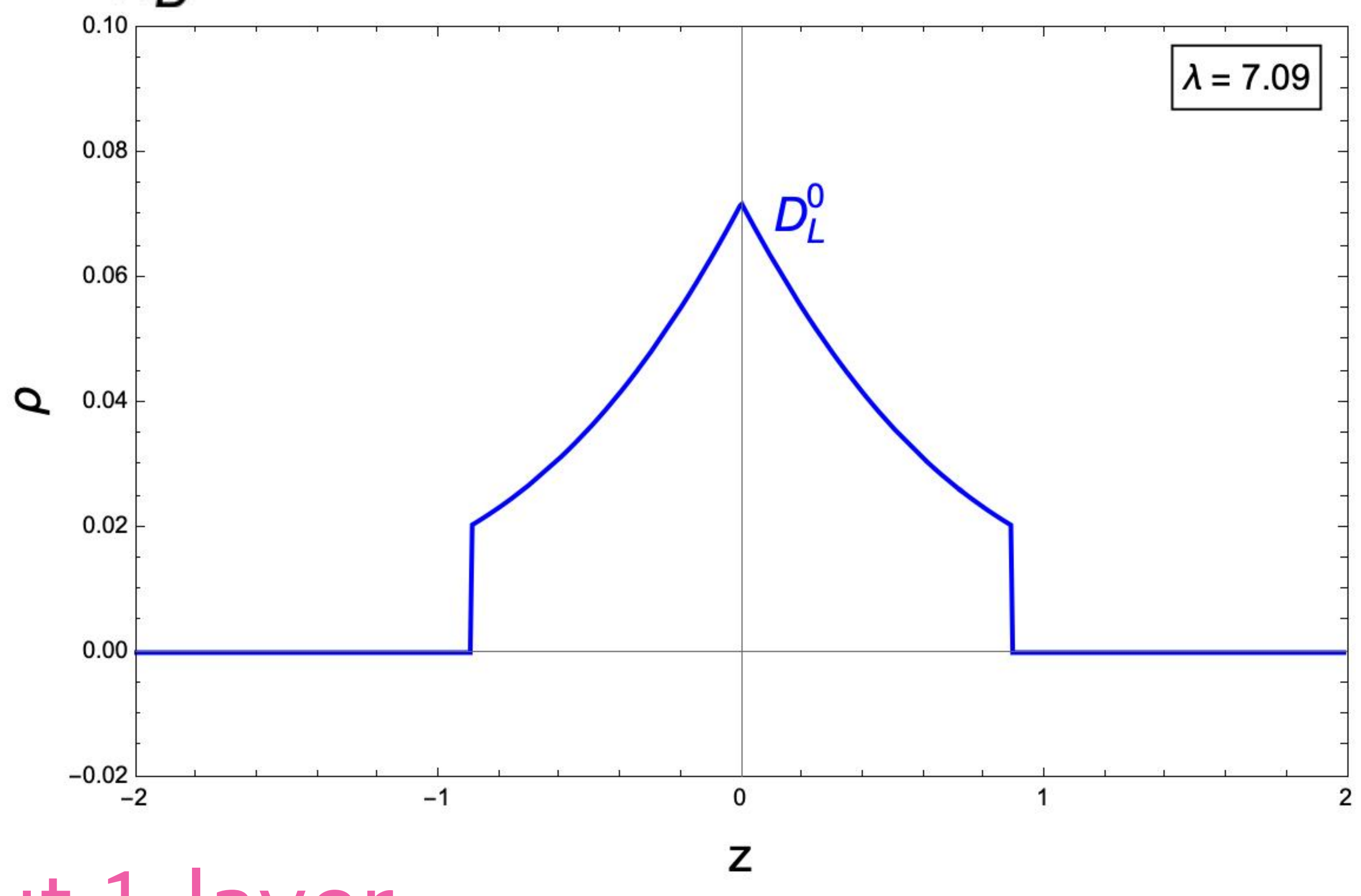
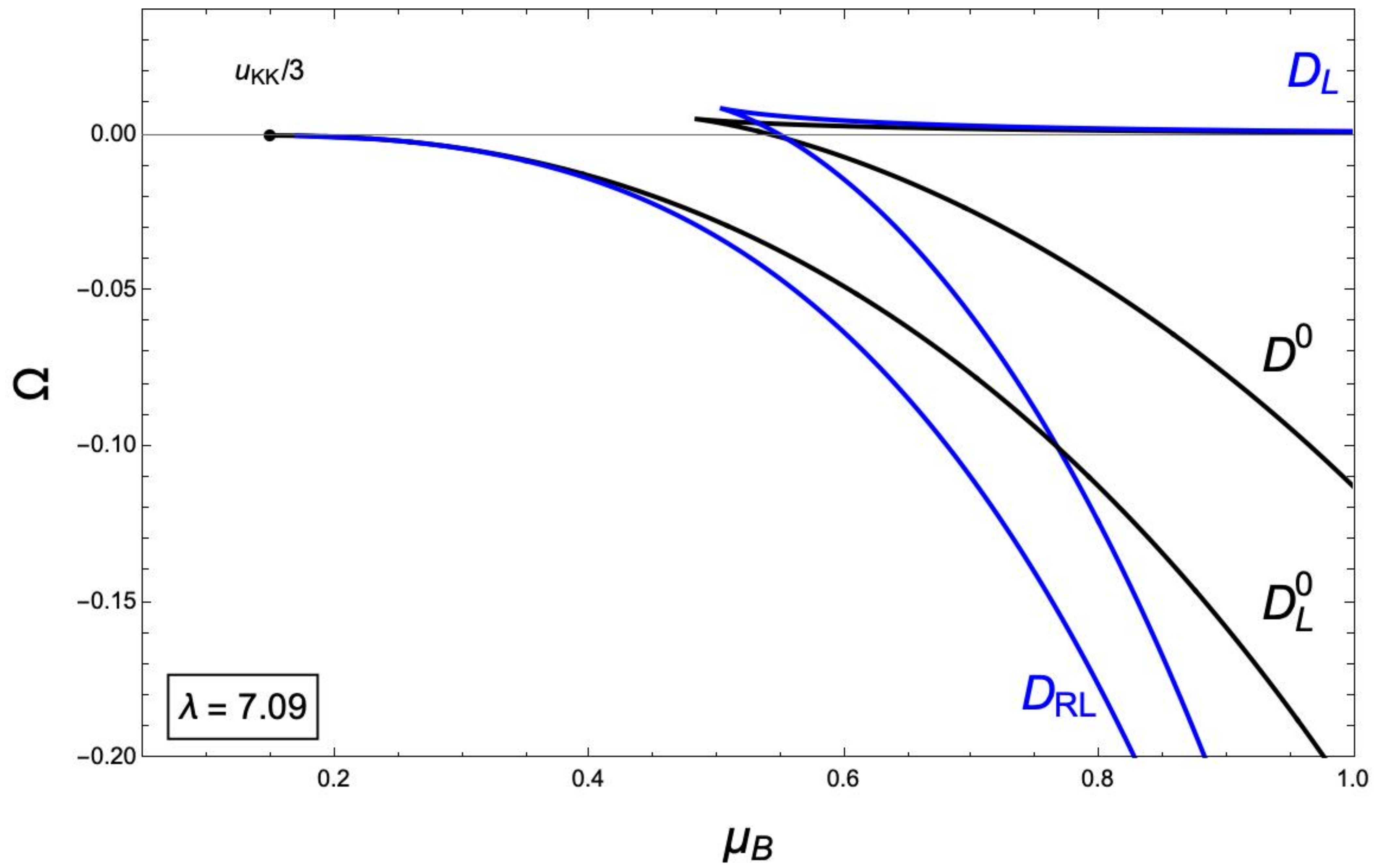


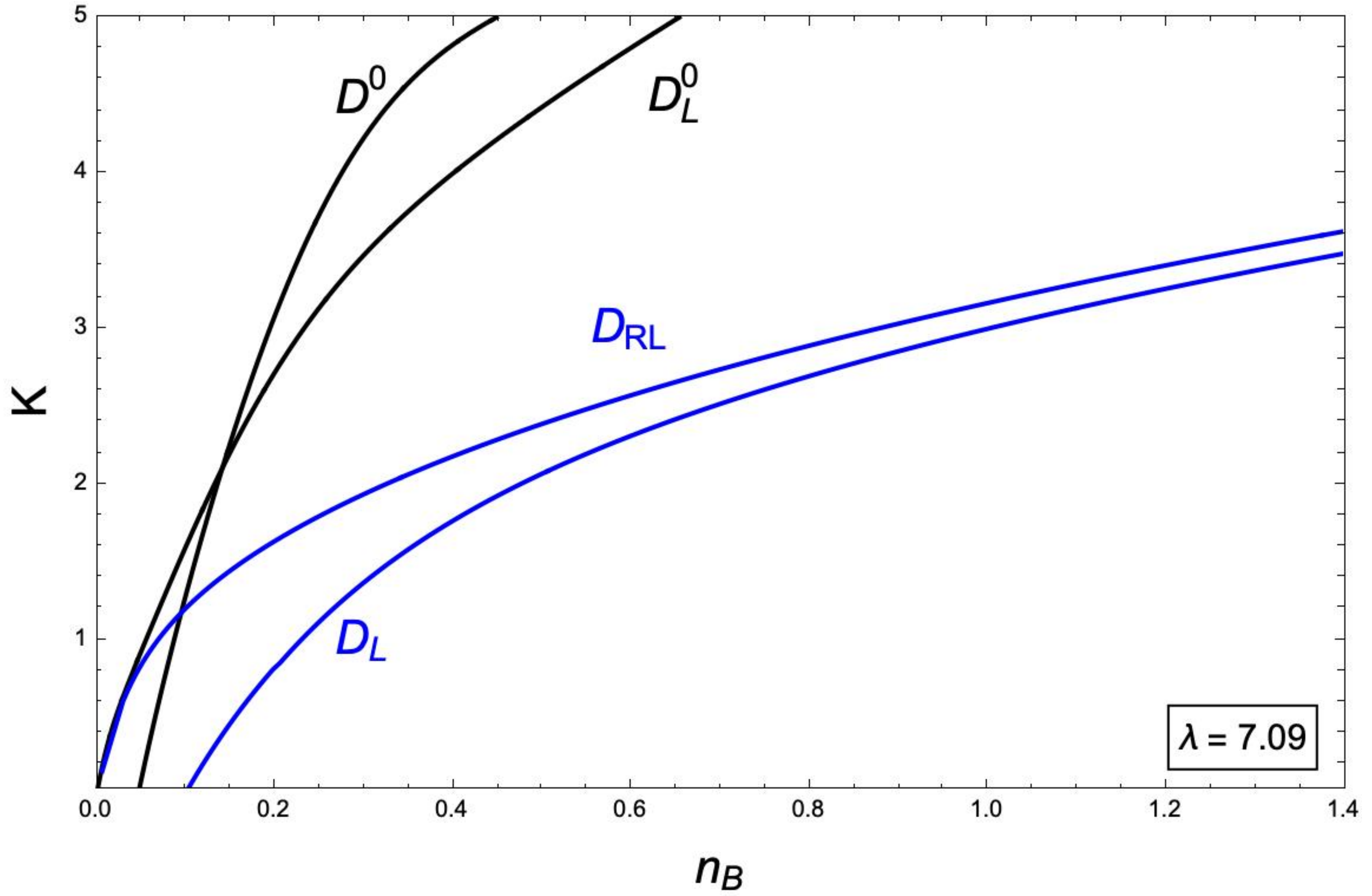


3-jump but 1-layer

Solutions

Black: 1 Layer — Blue: 2 Layers

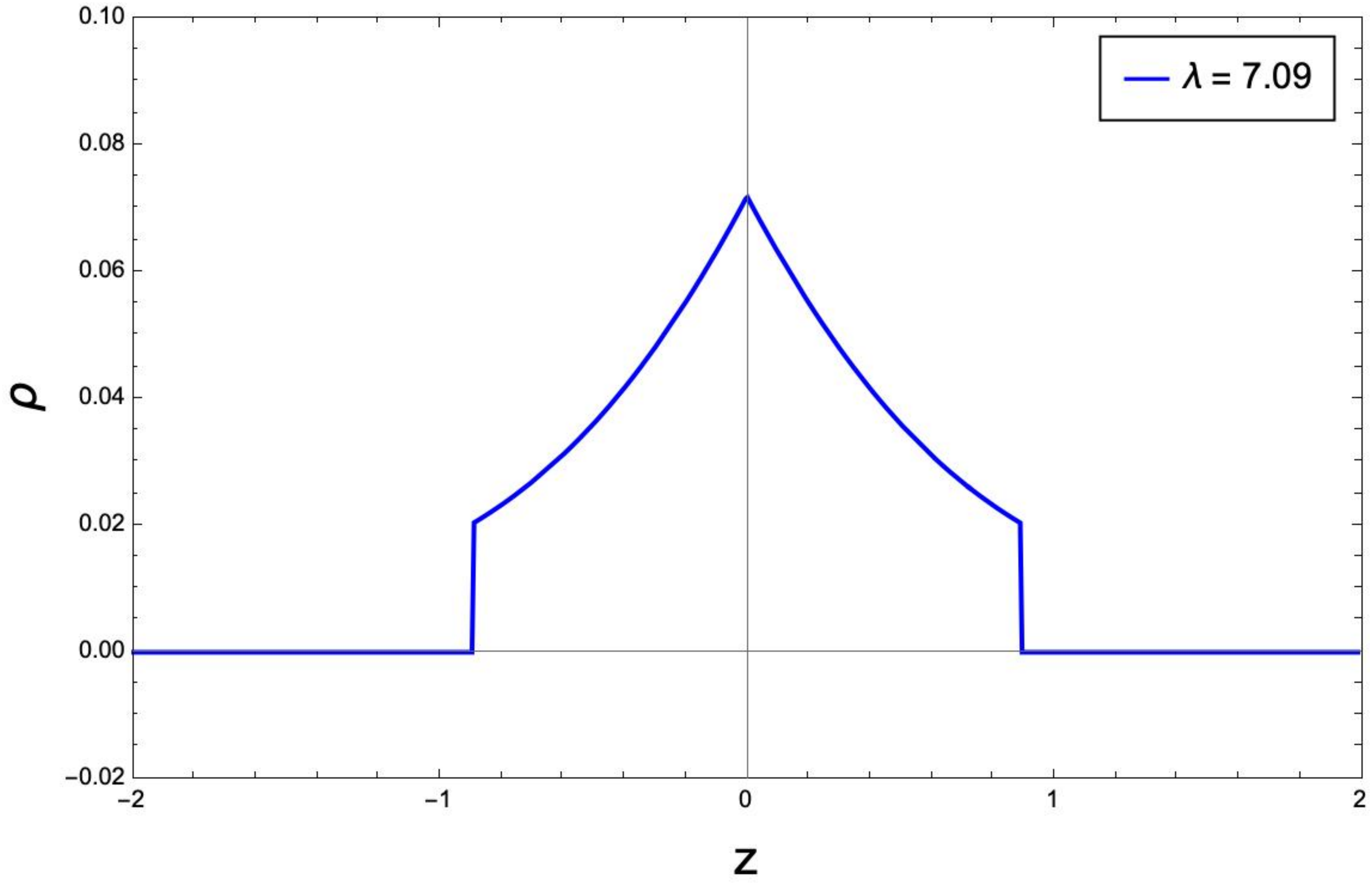




- Most important multi-jump solutions
- Preferred phase D_{RL}
- 2-layer phases decrease the incompressibility throughout the density range
- Two phases with 2nd order phase transition to vacuum, at $\mu_{\mathrm{B}} = u_{\mathrm{KK}}/3$, like in pointlike ansatz

Towards Pointlike

D_L^0 phase for $\mu_{\rm B} = 0.25$



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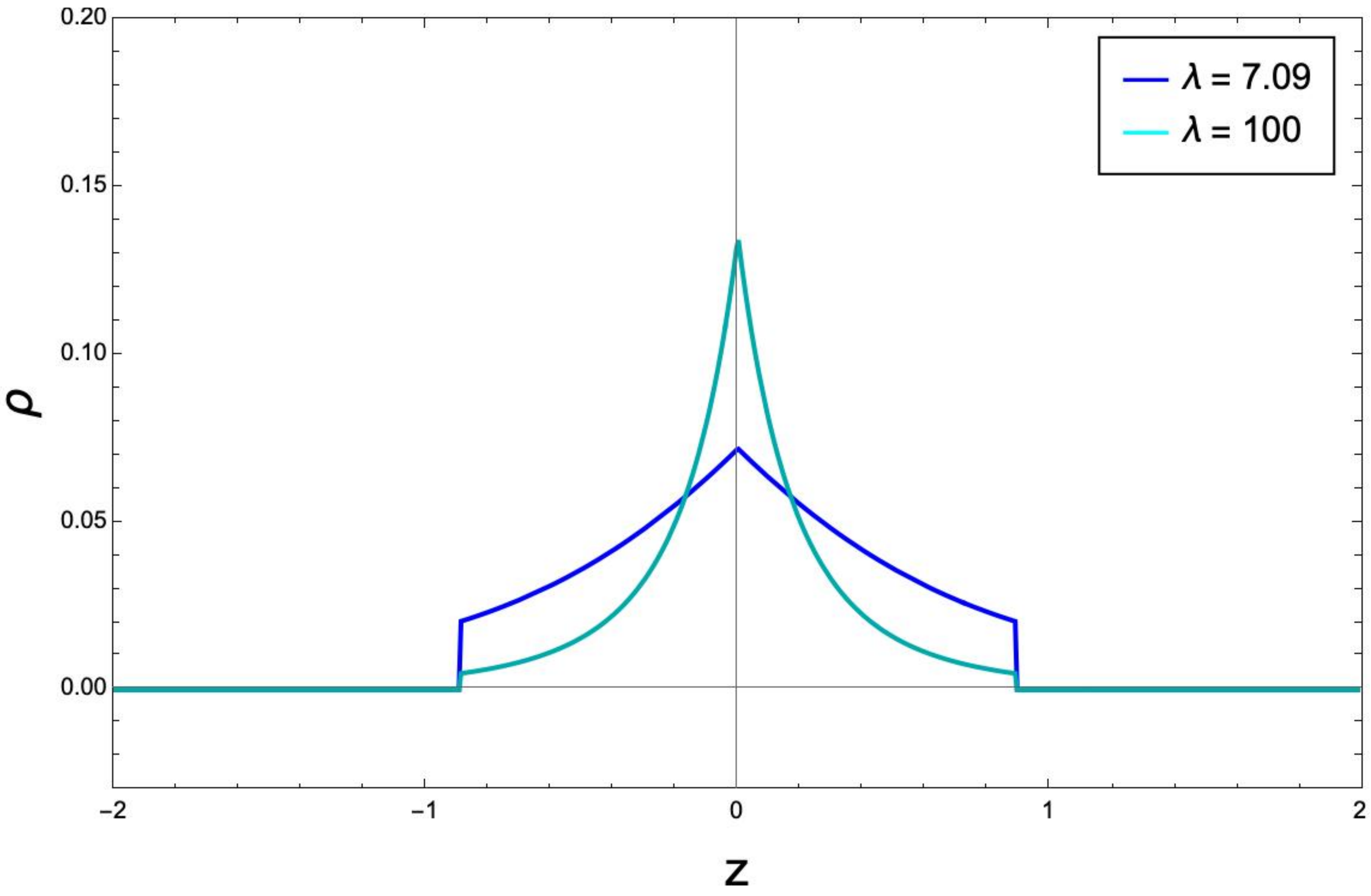

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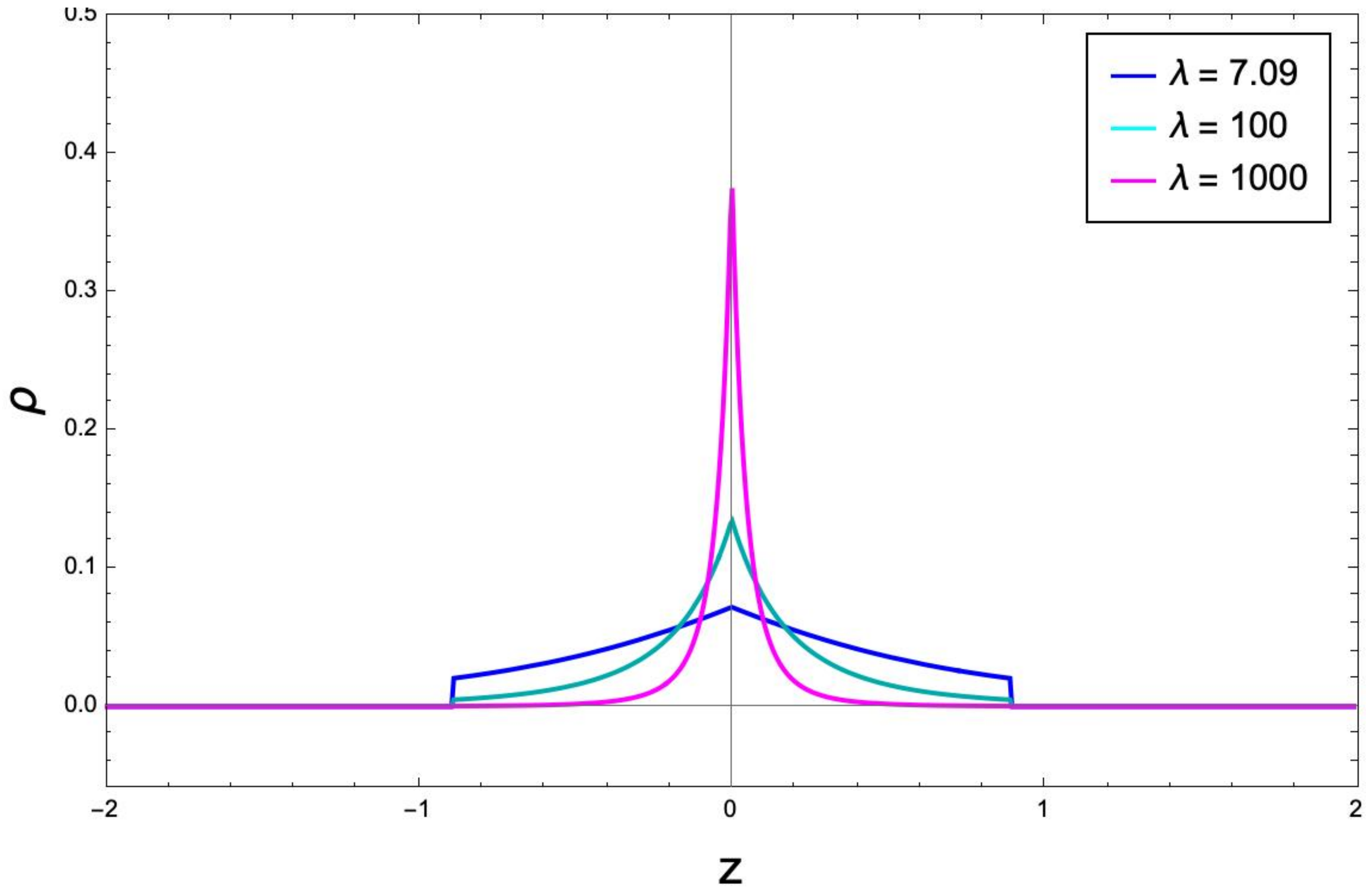

Charge distribution more localised in z as coupling increases → pointlike baryons

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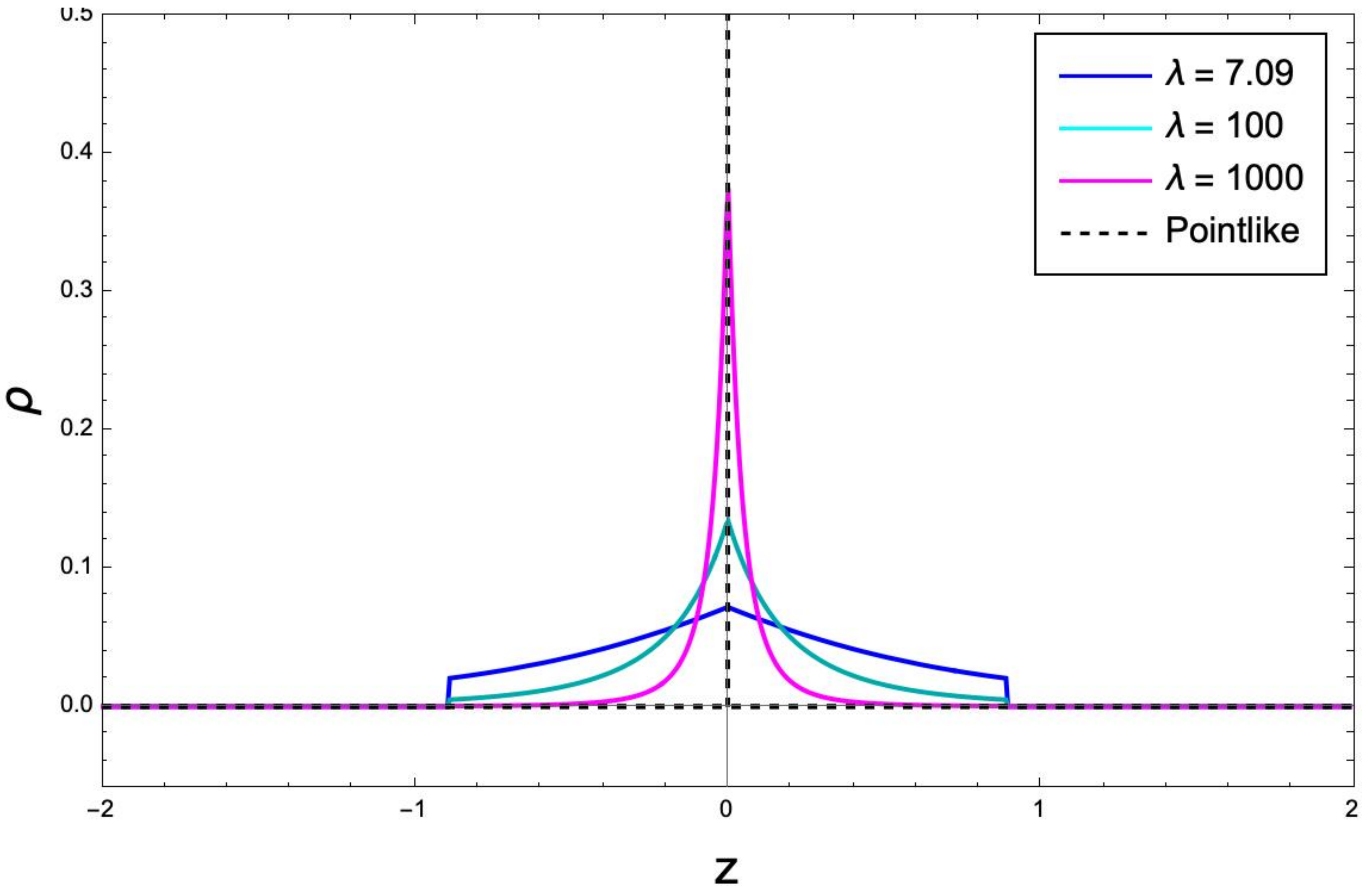

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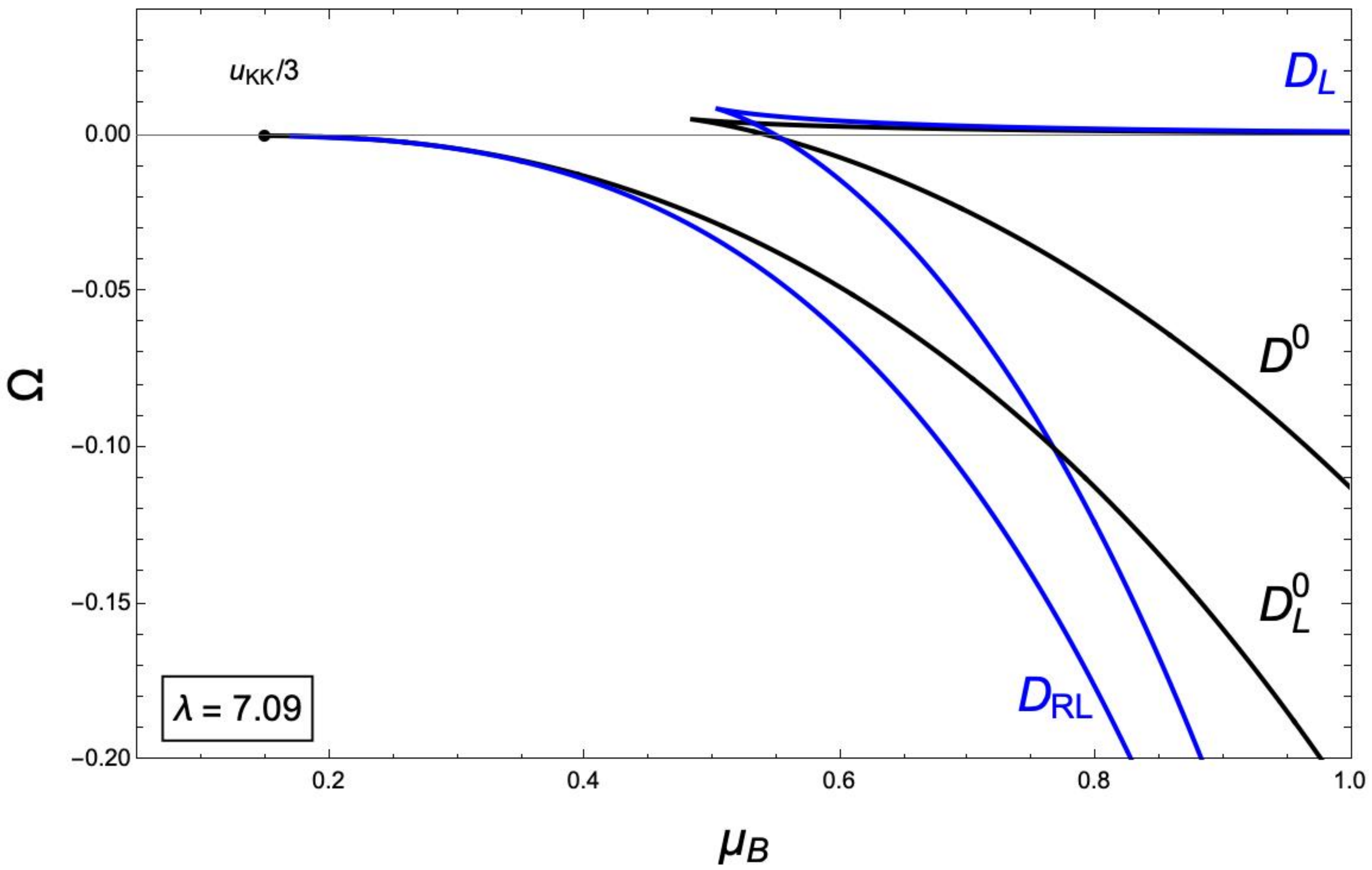
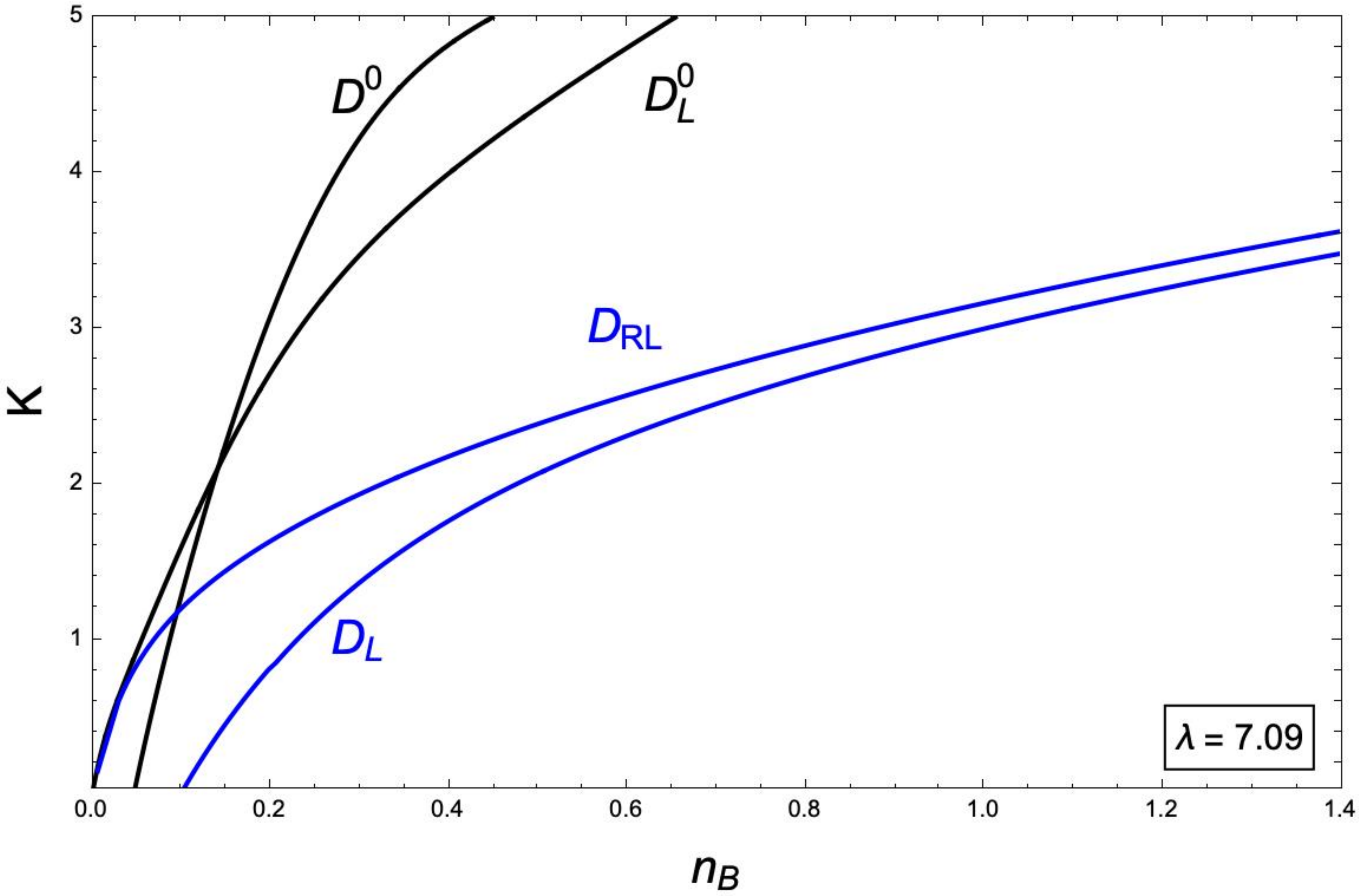
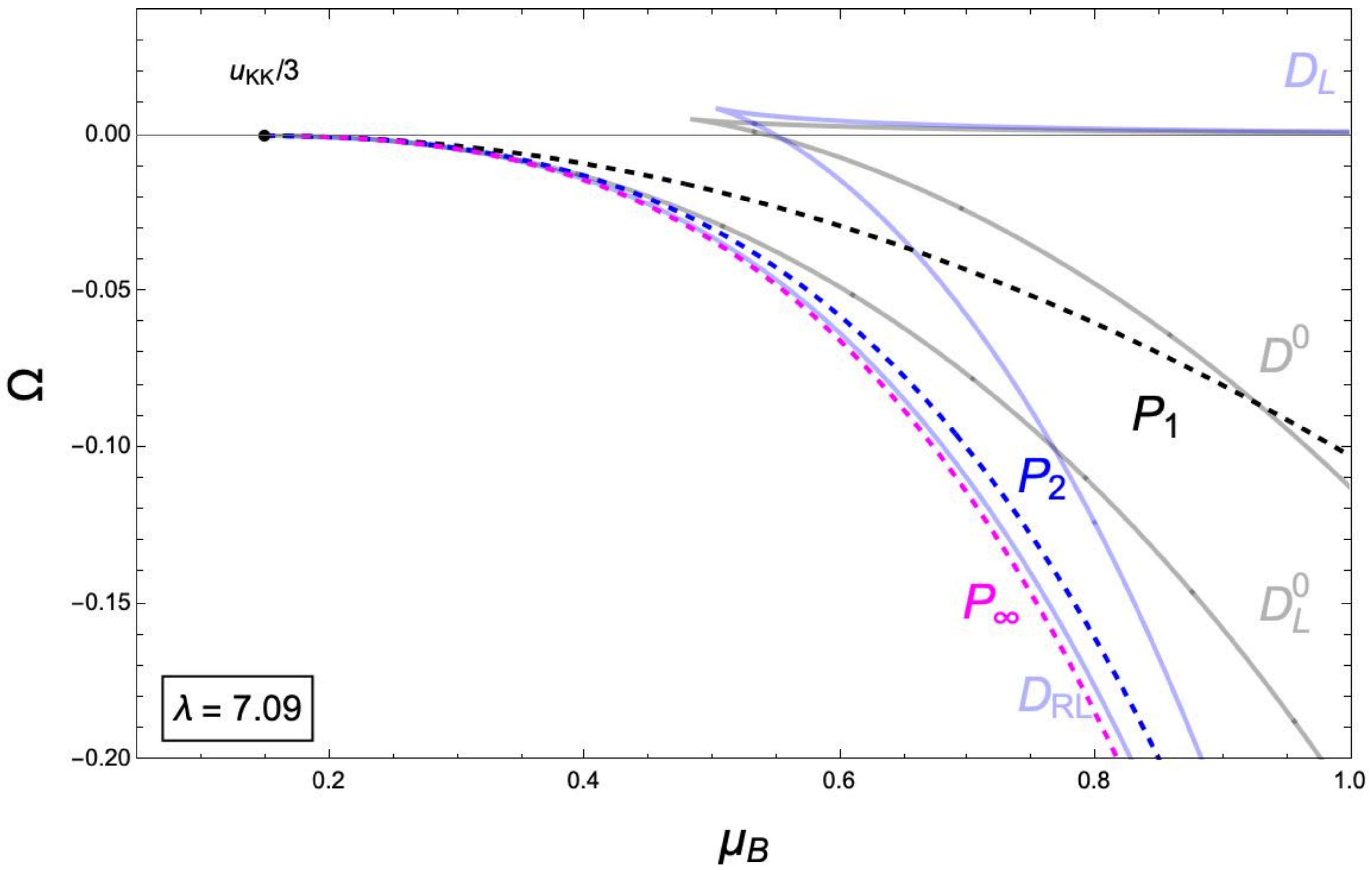
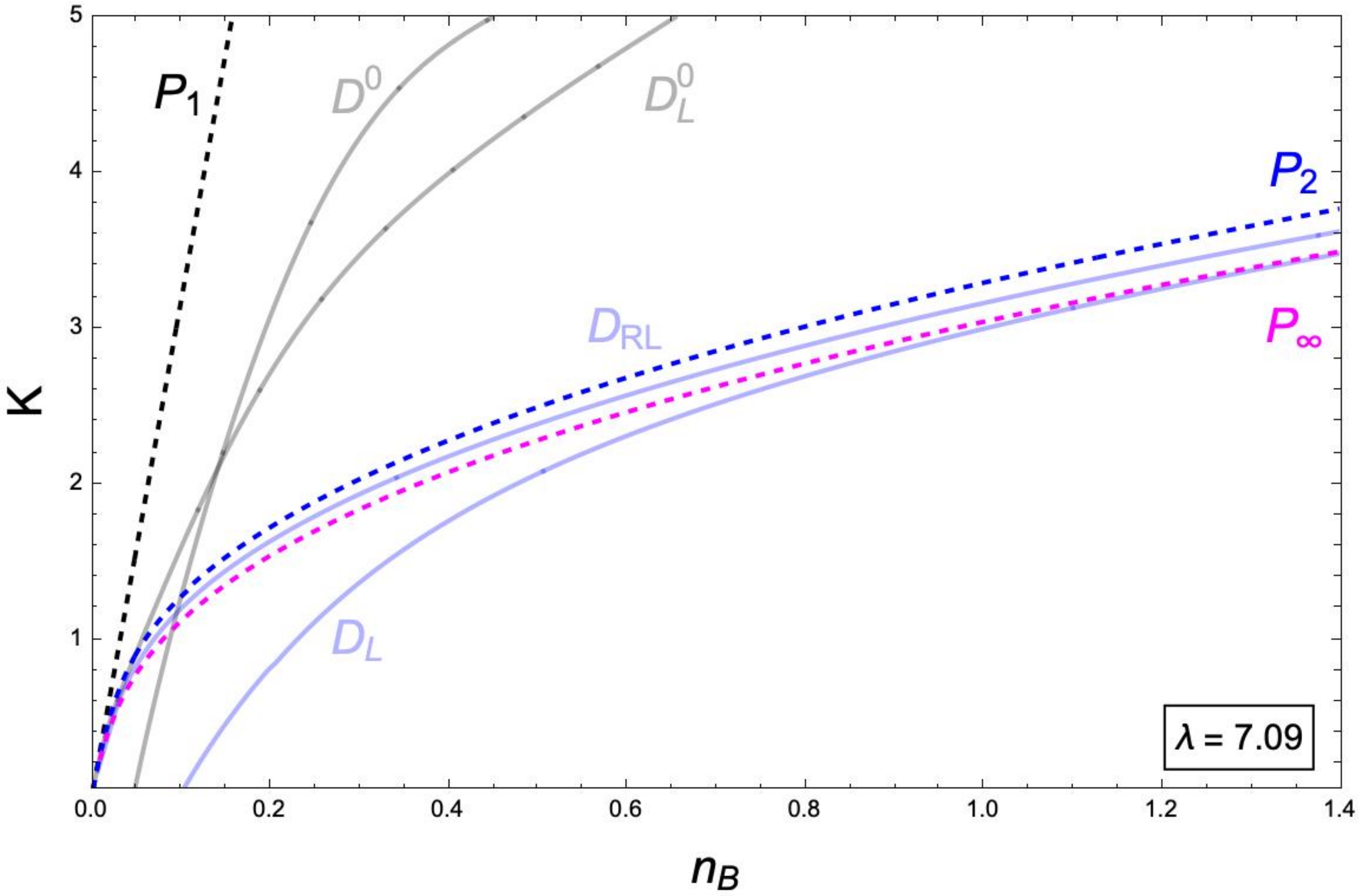




Solutions

Black: 1 Layer — Blue: 2 Layers — Pink: Inf. Points



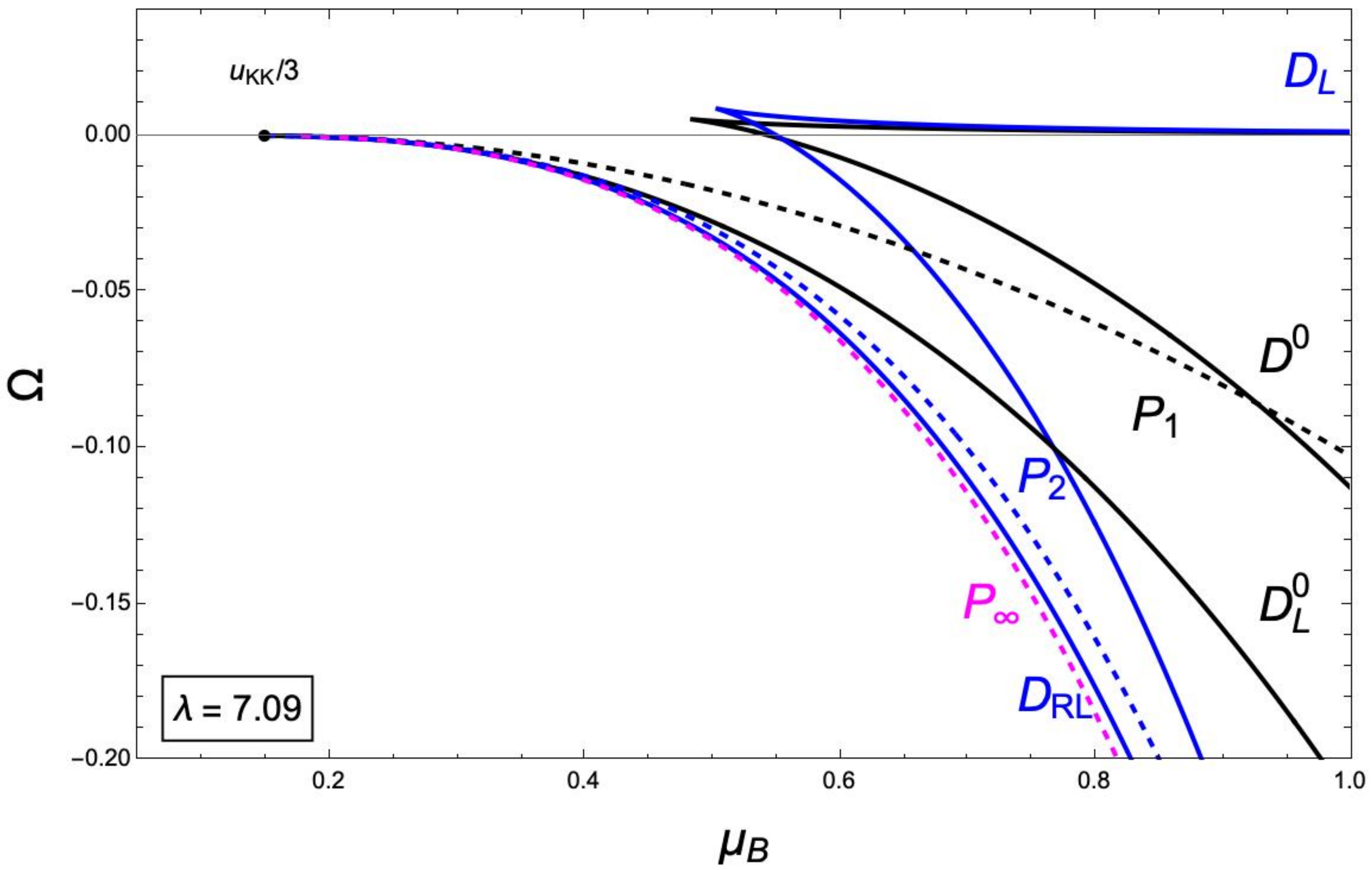


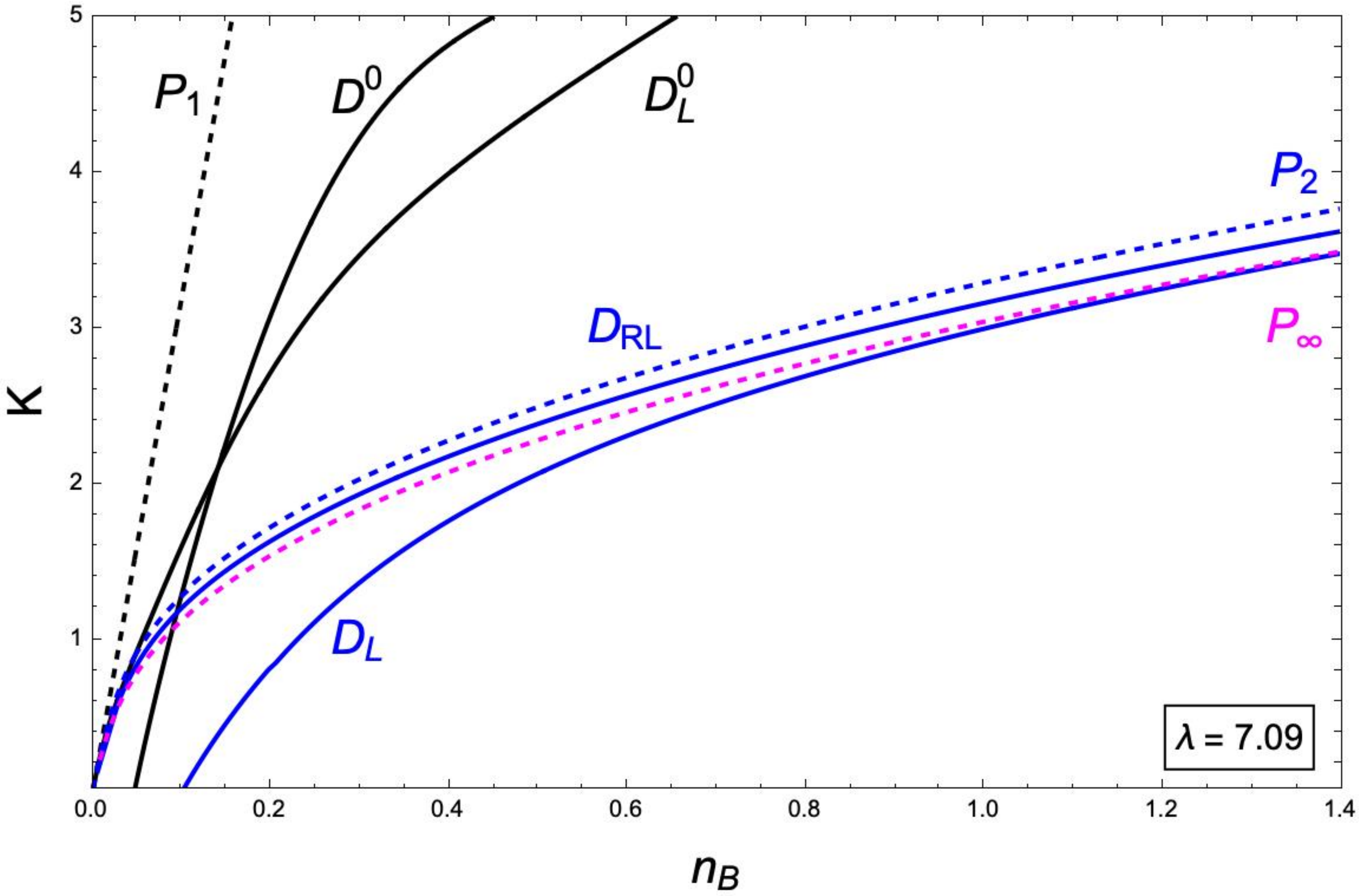
Multi-layer pointlike phases converge to finite limit, with no. of points $\to \infty$

Pointlike phase P_∞ is preferred, at least for small μ_B

Solutions

Black: 1 Layer — Blue: 2 Layers — Pink: Inf. Layers





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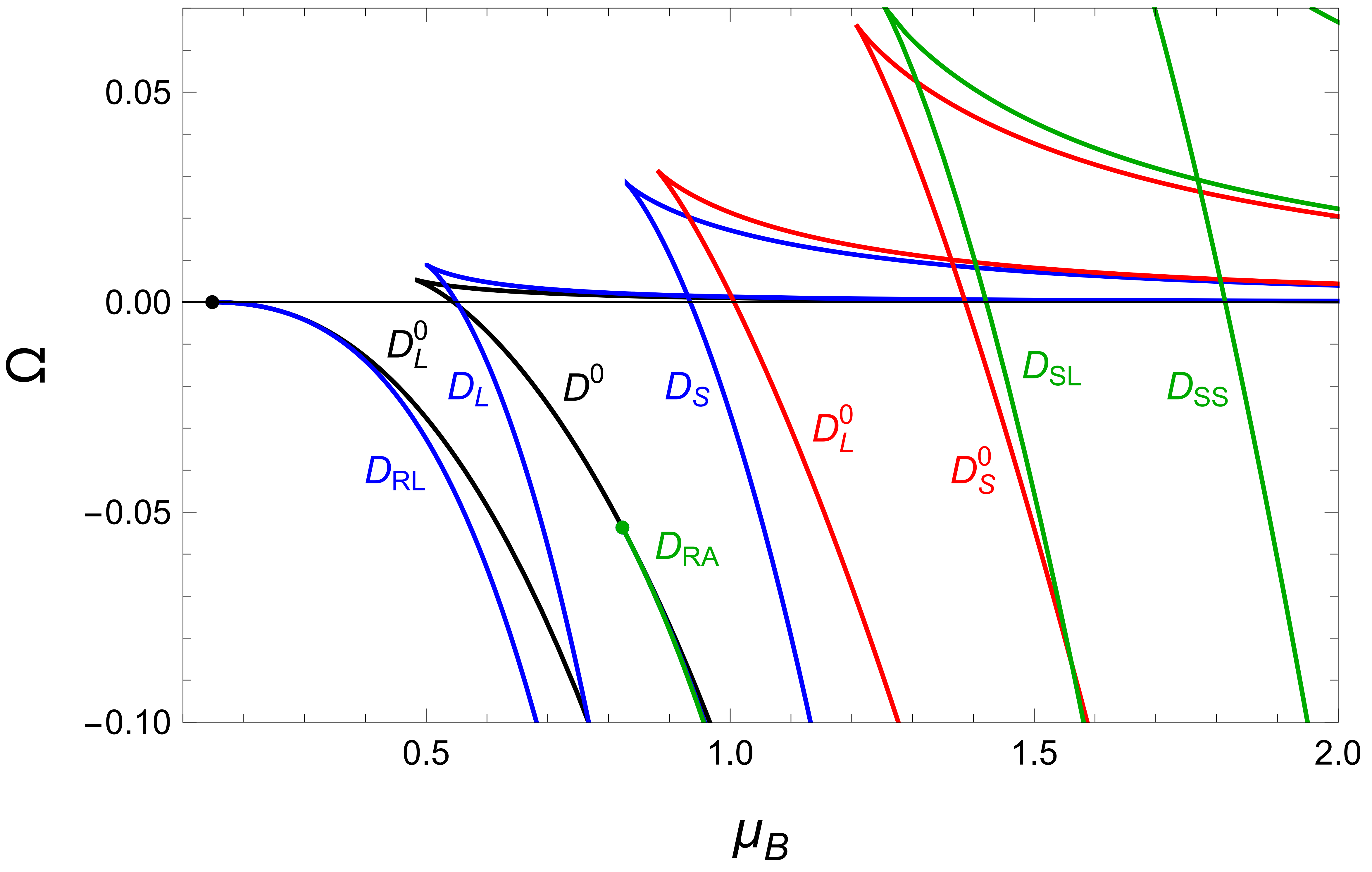
More Layers



Higher number of layers are less preferred with the homogeneous ansatz

Black: 1 Layer — Blue: 2 Layers — Red: 3 Layers — Green: 4 Layers

Summary

- Studied phases of homogeneous ansatz with up to 4 jumps systematically
- New preferred phase (D_{RL}) within this ansatz
- Can decrease incompressibility with new phases
- Also constructed pointlike baryon solution with infinite points, preferred phase overall

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Outlook

- Add isospin asymmetry: pure-neutron matter
- Deconfined geometry: finite temperature
- EoS for finite $\mu_{\mathrm{B}}, T \to$ neutron stars
- Study transport properties

Thank you for your attention!

谢谢你!