On Carrollian Conformal Field Theory

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Collaborators: Jue Hou, Reiko Liu, Hao-wei Sun and Yu-fan Zheng
Based on 2112.10514, 2301.06011, 2405.04105, 2406.17451 and 2503.22160

Galilei & Carroll

The Galilei group could be produced by considering the $c \to \infty$ (non-relativistic) limit of the Poincaré group.
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Carrollian boosts

$$\vec{x}\,' = \vec{x}, \qquad t' = t - \vec{b} \cdot \vec{x}.$$

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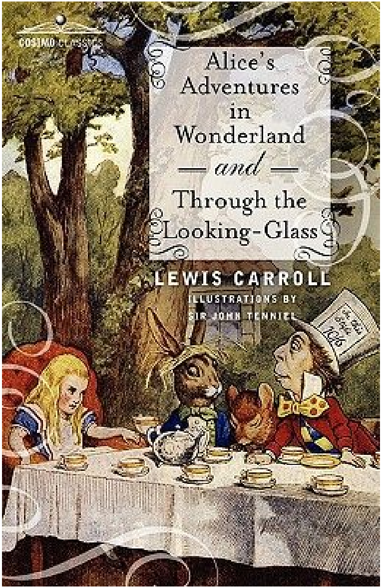
Intuitively, under the Carrollian limit, the lightcones collapse.

"*since absence of causality as well as arbitrarinesses in the length of time intervals is especially clear in Alice's adventures (in particular in the Mad Tea-Party) this did not seem out of place to associate Lewis Carroll's name*" (Lévy-Leblond (1965))

Carroll's world



The Red Queen effect: running without moving, "ultralocal"

... The most curious part of the thing was, that the trees and the other things round them never changed their places at all: however fast they went, they never seemed to pass anything.

A free Carrollian particle is at rest and does not move! C. Duval et.al 1402.0657, E. Bergshoeff et al. 1405.2264

Carrollian physics

The Carrollian boosts appear in the isometry group of plane-gravitational wave. J. M. Souriau (1973);...

The Carrollian limit controls the dynamics of the gravitational field near a spacelike singularity (BKL limit) M. Henneaux (1979);...

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   with P.X. Hao, Z.F. Yu and R. Liu,
   2011.11092, 2203.10490, 2207.01474

1) Multiplet structure
2) Galilean conformal blocks for multiplets
3) Harmonic analysis of GCA: GCPW
4) Shadow formalism ($\xi \neq 0$)
5) Four-point function in GGFT and BMS free scalar in different ways
6) Spectral density by using Hardy-Littlewood tauberian theorem.

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6. Carrollian (conformal) superalgebra: with Y.F. Zheng, 2503.22160

Carrollian conformal algebra (CCA) and H.W. Reps.

BC, R. Liu and Y.F. Zheng, 2112.10514

One can obtain CCA_d by taking the Carrollian limit of the usual d-dim. conformal algebra.

$$\{D, P^\mu, K^\mu, J^{\mu\nu}\} \longrightarrow \{D, P^\mu, K^\mu, B^i, J^{ij}\},$$

where $\mu = 0, 1, \ldots, d-1,\ i, j = 1, \ldots, d-1.$ The Carrollian boost generators B^i come from the rotation generators: $J^{i0} \xrightarrow{c \to 0} B^i$.

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$$\begin{aligned}
&[D, P^\mu] = P^\mu, \quad [D, K^\mu] = -K^\mu, \quad [D, B^i] = [D, J^{ij}] = 0,\\
&[J^{ij}, G^k] = \delta^{ik} G^j - \delta^{jk} G^i, \quad G \in \{P, K, B\}\\
&[J^{ij}, P^0] = [J^{ij}, K^0] = 0,\\
&[J^{ij}, J^{kl}] = \delta^{ik} J^{jl} - \delta^{il} J^{jk} + \delta^{jl} J^{ik} - \delta^{jk} J^{il},\\
&[B^i, P^j] = \delta^{ij} P^0, \quad [B^i, K^j] = \delta^{ij} K^0, \quad [B^i, B^j] = [B^i, P^0] = [B^i, K^0] = 0,\\
&[K^0, P^0] = 0, \quad [K^0, P^i] = -2B^i, \quad [K^i, P^0] = 2B^i, \quad [K^i, P^j] = 2\delta^{ij} D + 2J^{ij}.
\end{aligned}$$

Stabilizer algebra and highest weight representations

The stabilizer algebra g_0 is generated by dilation D, generalized rotations $M = \{J, B\}$ and special conformal transformations (SCTs) K

$$[D, M] = 0, \quad [D, K] \subset K, \ \ [M, K] \subset K.$$

The commutativity of the dilatation and the rotations implies that the local operators $\mathcal{O}^a$ can be diagonalized simultaneously into the eigenstates of the dilation and the representations of generalized rotations,

$$[D, \mathcal{O}] = \Delta_{\mathcal{O}}\mathcal{O}, \quad [M, \mathcal{O}^a] = M^a_b\mathcal{O}^b.$$

$\Delta_{\mathcal{O}}$: conformal weight

Highest weight repr.: $[K, \mathcal{O}^a] = 0$.
This is often referred to as the primary condition.

The nontrivial part is the representation of generalized rotation!

Multiplets

For $d \geq 3$ CCFT, the generalized rotation group, CCA rotation group, is the Euclidean group $ISO(d-1)$. It is not semi-simple, and its finite dimensional representations are generally reducible but indecomposable, and can be organized as multiplet representations.

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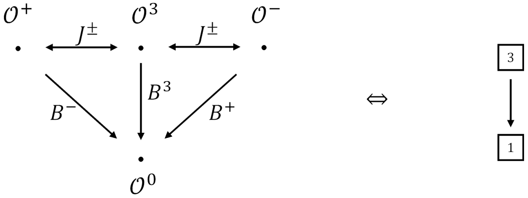
Example: vector representation $\mathcal{O}^\mu$ of CCA$_4$

$$[J^{ij}, \mathcal{O}^k] = \delta^{ik}\mathcal{O}^j - \delta^{jk}\mathcal{O}^i, \quad [B^i, \mathcal{O}^j] = \delta^{ij}\mathcal{O}^0, \quad [J^{ij}, \mathcal{O}^0] = [B^i, \mathcal{O}^0] = 0.$$



Here

$$J = -iJ^{12}, \qquad J^\pm = \frac{1}{\sqrt{2}}(\mp J^{23} + iJ^{31}), \qquad B^\pm = \frac{1}{\sqrt{2}}(iB^1 \pm B^2)$$

Tensor representation

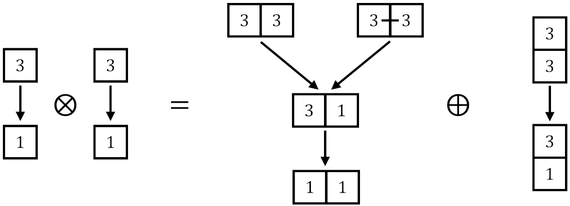


Figure: The rank-2 tensor representation of CCA_4. It is decomposed into a 10-dimensional representation and a 6-dimensional representation

The multiplet representations for $d > 2$ case have much more complicated structures since there is a non-trivial $ISO(d-1)$ part, and lead to net representations rather than just chain-like ones in $\log\mathrm{CFT}_2$ or CCFT_2.

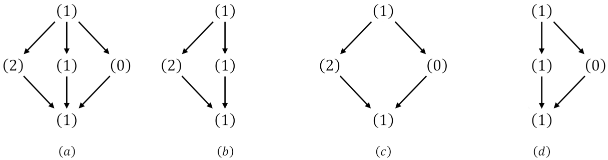


Figure: All the four net representations are legal.

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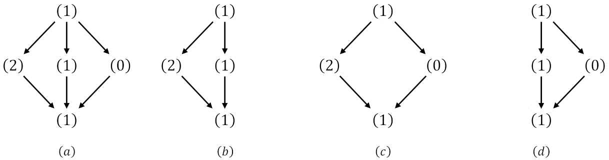

Figure: All the four net representations are legal.

Nevertheless, the finite dimensional representation of the CCA rotations are all multiplet representations with every sub-sector being irreducible representation of $SO(d-1)$, due to a theorem by H. P. Jakobsen (2011).

Notations: the numbers in the bracket indicate the irr. representations w.r.t. $SO(d-1)$, the arrows stand for the actions of the generators B_i.

Chain representations

The possible chain representations must take the following patterns:
rank 2

$$\begin{aligned}
&(j) \to (j+1), \\
&(j) \to (j), \quad j \neq 0, \\
&(j) \to (j-1).
\end{aligned}$$

rank 3 or higher

$$\begin{gathered}
(0) \to (1) \to (0), \\
\cdots \to (j) \to (j+1) \to (j+2) \to \cdots , \\
\cdots \to (j) \to (j-1) \to (j-2) \to \cdots ,
\end{gathered}$$

where the patterns works for all possible values of $j \in \{0\} \cup \mathbb{Z}_+/2$.

Correlators of singlets

In principle, the 2-pt and 3-pt functions of the operators in CCFT can be determined by using the Ward identities. However, due to complicated structure in representations, it is hard to discuss the most general case. We discussed the correlators of the operators in chain representations carefully. BC, Reiko Liu and Yu-fan Zheng, 2112.10514

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For a singlet in CCFT$_4$, there is

$$\langle \mathcal{O}_1(t_1,\vec{x}_1)\mathcal{O}_2(t_2,\vec{x}_2)\rangle = c_1\frac{1}{r^{\Delta_1+\Delta_2}} + c_2\delta^{(3)}(\vec{x}_{12})\frac{1}{t^{\Delta_1+\Delta_2-3}},$$

- If $c_1 \neq 0$, $c_2 = 0$, the Ward identities of K^i will force $\Delta_1 = \Delta_2$, and the resulting 2-pt function coincides with the scalar 2-pt function in CFT$_3$.
- If $c_1 = 0$, $c_2 \neq 0$, it can be understood in an concrete model: the Carrollian free scalar with the action

$$S = \int d^3\vec{x}dt\,\phi\partial_t^2\phi.$$

Close relation between 3D Carrollian CFT and celestial holography!

L. Donnay *et al.*. 2202.04702,2212.12553; Bagchi *et al.*. 2202.08438; ...

Correlators of chain representations: trivial one

In the following discussion on correlators, we focus on the ones with only spatial dependence.
Generic structure of 2-pt correlators:

$$\left\langle \mathcal{O}_1^{(m_1,q_1)}(x_1)\mathcal{O}_2^{(m_2,q_2)}(x_2)\right\rangle = f\,^{m_1,m_2}_{q_1,q_2}(x_{12})$$

where q_i is the order of the i-th operator in a multiplet.

For the relatively trivial case that $\mathcal{O}_1, \mathcal{O}_2 \in (1) \to (0)$,

$$\begin{array}{llll} \text{Level 3:} & f\,^{m_1,m_2}_{2,2} = \dfrac{C\, I^{m_1,m_2}_{1,1}}{|\vec{x}_{12}|^{2\Delta}}, & & \\ \text{Level 2:} & f\,^{0,m_2}_{1,2} = 0, \quad f\,^{m_1,0}_{2,1} = 0, & & \\ \text{Level 1:} & f\,^{0,0}_{1,1} = 0, & & \end{array} \qquad \mathcal{O}_1 \in \begin{array}{c}(1)\\ \downarrow\\ (0)\end{array} \quad \mathcal{O}_2 \in \begin{array}{c}(1)\\ \downarrow\\ (0)\end{array}$$

with $\Delta_1 = \Delta_2 = \Delta$.

Correlators of chain representations: the simplest nontrivial case

For the simplest nontrivial case,

$$\mathcal{O}_1 \in (1) \to (0), \qquad \mathcal{O}_2 \in (0) \to (1).$$

Level 3: $$f_{2,2}^{\,m_1,0} = \frac{C\ t_{12}/|\vec{x}_{12}|\ I_{1,0}^{m_1}}{|\vec{x}_{12}|^{2\Delta}},$$

Level 2: $$f_{1,2}^{\,0,0} = \frac{C}{|\vec{x}_{12}|^{2\Delta}}, \qquad f_{2,1}^{\,m_1,m_2} = \frac{C\ I_{1,1}^{m_1,m_2}}{|\vec{x}_{12}|^{2\Delta}},$$

Level 1: $$f_{1,1}^{\,0,m_2} = 0.$$

$$\mathcal{O}_1 \in \begin{matrix}(1)\\ \downarrow\\ (0)\end{matrix} \qquad \mathcal{O}_2 \in \begin{matrix}(0)\\ \downarrow\\ (1)\end{matrix}$$

with $\Delta_1 = \Delta_2 = \Delta$.

Here $I_{j_1,j_2}^{\,m_1,m_2}$ is the 2-point tensor structure.

Remarks on correlators

- Due to the multiplet structure of the representations, the correlators present multi-level structures. At each level, there are more than one 2-pt coefficients. Even if considering the basis change and renormalization of the operators, not all 2-pt coefficients can be fixed by the Ward identities;

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- As the representations are reducible, there is short of selection rules on the representations. This means that the 2-pt correlators of the operators in different representations could be nonvanishing.
- We explored the 2-pt correlators of net representations and the 3-pt correlators of chain representations. It turns out that the constraints from the Ward identities are quite loose, and we had to compute them case by case.

Constructions of CCFT BC, H.W. Sun and Y.F. Zheng 2301.06011

The explicit examples are vital to understand various properties of CCFT.

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$$\text{2D Carrollian conformal group} \simeq \text{BMS}_3$$

In 2D, a few field theories with BMS symmetry have been constructed and studied:

BMS free scalar theory P.x. Hao *et al.*. 2111.04701,

BMS free fermion theory Z.f. Yu & BC. 2211.06926; P.x. Hao *et al.* 2211.06927; A. Banerjee *et al.* 2211.11639

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In higher dim., the study of Carrollian field theories got revived in the past few years. There are two existing ways to construct theories

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We proposed a novel off-shell way to construct Carrollian (conformal) field theories, starting from the Bargmann field theories. We have successfully reproduced all Carrollian field theories in the literatures.

Bargmann symmetry

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$$\mathcal{B} = \mathbb{R} \times \mathbb{R}^{d-1} \times \mathbb{R}, \qquad G = 2dudv + \delta_{ij}dx^i dx^j, \qquad \xi = \partial_u,$$

where u, v are the lightcone coordinates.

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where u, v are the lightcone coordinates. The Bargmann group is the isometries of the flat Bargmann structure, which is a subgroup of Poincaré group

$$Barg(d,1) = ISO(d,1)/\{J^{\ 0}{}_{d+1}, 1/\sqrt{2}\left(J^{\ i}{}_{0} - J^{\ i}{}_{d+1}\right)\}$$

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that keep the null vector ξ invariant. The Bargmann generators are $\{P_\alpha, J^i{}_j, B_i^{\mathcal{B}}\}$, where $B_i^{\mathcal{B}}$ is the Bargmann boost. The actions on point $(u, \vec{x}, v)$ in the manifold are shown in the following Table

generator	vector field	finite transformation
p_α	∂_α	$x^\alpha + x_0^\alpha$
$m^i{}_j$	$x^i\partial_j - x_j\partial^i$	$(u, \mathbf{M}\vec{x}, v)$
$b_i^{\mathcal{B}}$	$v\partial_i - x_i\partial_u$	$\left(u - \vec{\nu}\cdot\vec{x} - \frac{1}{2}\vec{\nu}^2 v, \vec{x} + \vec{\nu}v, v\right)$

Carrollian symmetry from Bargmann symmetry

Restricting the Bargmann group on the null hyper-surface $v = 0$, we can immediately see the Bargmann structure reduce to Carrollian structure $(\mathcal{C}, g, \xi)$ with the coordinates $(t = u, \vec{x})$, the degenerated metric

$$g_{\mu\nu} = G_{\mu\nu} = \delta^i_\mu \delta^j_\nu \delta_{ij}$$

while ξ^μ being the timelike vector, and the Carroll group is subgroup of Bargmann group $Carr(d) = Barg(d, 1)/\{P_v\}$.

Carrollian symmetry from Bargmann symmetry

Restricting the Bargmann group on the null hyper-surface $v = 0$, we can immediately see the Bargmann structure reduce to Carrollian structure $(\mathcal{C}, g, \xi)$ with the coordinates $(t = u, \vec{x})$, the degenerated metric

$$g_{\mu\nu} = G_{\mu\nu} = \delta^i_\mu \delta^j_\nu \delta_{ij}$$

while ξ^μ being the timelike vector, and the Carroll group is subgroup of Bargmann group $Carr(d) = Barg(d, 1)/\{P_v\}$.

This motivates us to construct Carrollian field theories by restricting Bargmann field theories on the null hyper-surface.

However, trivially restricting Bargmann fields with configuration $\Phi(u, \vec{x}, v) = \Phi(u, \vec{x})\delta(v)$ on $v = 0$ causes many difficulty from the Dirac delta function. The trick is to introduce an uniformly distributed function over small interval of v.

Bargmann scalar field theories

The building blocks of Bargmann field theories are geometric invariants $G^{\alpha\beta}$ and ξ^{α}. For a free scalar Φ, the Bargman invariant action could be

$$S_E^{\mathcal{B}} = \frac{1}{2}\int d^{d+1}x\, \xi^{\alpha}\xi^{\beta}\partial_{\alpha}\Phi\partial_{\beta}\Phi, \qquad S_M^{\mathcal{B}} = -\frac{1}{2}\int d^{d+1}x\, G^{\alpha\beta}\partial_{\alpha}\Phi\partial_{\beta}\Phi.$$

The subscript M is for magnetic sector and E for electric sector, corresponding to magnetic/electric Carrollian field theories. M. Henneaux and P. Salgado-Rebolledo, 2109.06708

Electric sector

$$S_E^{\mathcal{B}} = \frac{1}{2}\int d^{d+1}x\, \xi^\alpha \xi^\beta \partial_\alpha \Phi \partial_\beta \Phi.$$

Expanding Φ near $v = 0$, we have

$$\Phi(u, \vec{x}, v) = \phi(u, \vec{x}) + v\pi(u, \vec{x}) + \mathcal{O}(v^2).$$

Inserting this in the action, and noticing $\xi^\alpha = (1, \vec{0}, 0)$, we have

$$S_E^{\mathcal{B}} = -\frac{1}{2}\int d^{d+1}x\, \partial_u \Phi \partial_u \Phi = -\frac{1}{2}\int d^{d+1}x\, \partial_u \phi \partial_u \phi + 2v\partial_u \pi \partial_u \phi + \mathcal{O}(v^2),$$

and thus we have the Carrollian action

$$S_E^{\mathcal{C}} = \lim_{\epsilon \to 0} S_{E,\epsilon}^{\mathcal{B}} = -\frac{1}{2}\int d^d x\, \partial_0 \phi \partial_0 \phi.$$

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$$S_E^{\mathcal{C}} = \lim_{\epsilon \to 0} S_{E,\epsilon}^{\mathcal{B}} = -\frac{1}{2}\int d^d x\, \partial_0 \phi \partial_0 \phi.$$

Actually, it is not only Carrollian invariant, but even Carrollian conformal invariant. From

$$\langle \phi(x)\phi(y)\rangle = \frac{i|t|}{2}\delta^{(d-1)}(\vec{x}),$$

we see that ϕ is a primary operator when $d > 2$.

Magnetic sector

$$S_M^{\mathcal{B}} = -\frac{1}{2}\int d^{d+1}x\; G^{\alpha\beta}\partial_\alpha\Phi\partial_\beta\Phi.$$

Inserting the expansion of Φ, we get:

$$S_M^{\mathcal{B}} = -\frac{1}{2}\int d^{d+1}x\, 2\partial_u\Phi\partial_v\Phi + \partial_i\Phi\partial_i\Phi = -\frac{1}{2}\int d^{d+1}x\, 2\pi\partial_u\phi + \partial_i\phi\partial_i\phi + \mathcal{O}(v).$$

Thus we reproduce the action of magnetic Carrollian scalar theory M. Henneaux and P. Salgado-Rebolledo, 2109.06708.

$$S_M^{\mathcal{C}} = -\frac{1}{2}\int d^dx\; 2\pi\partial_0\phi + \partial_i\phi\partial_i\phi$$

The fundamental fields in this theory are ϕ and π.

The above action is Carrollian conformal invariant as well. The scalar ϕ is still a primary fields, and the field π appears as part of staggered module of ϕ's conformal family.

$$\begin{tikzcd} & \phi \arrow[d, shift right, "P_i"'] \arrow[dr, "P_0"] & \\ \pi \arrow[ur, "K_0"] \arrow[r, "B_i"'] & \partial_i\phi \arrow[u, shift right, "K_i"'] \arrow[r, "B_i"'] & \partial_0\phi \end{tikzcd}$$

Figure: The staggered structure of fields ϕ, $\partial_\mu \phi$ and π.

$$\begin{aligned} \langle \phi(\vec{x}_1, t_1)\phi(\vec{x}_2, t_2)\rangle &= 0 \\ \langle \phi(\vec{x}_1, t_1)\pi(\vec{x}_2, t_2)\rangle &= -\frac{i\ sign(t)}{2(1-\alpha^2)}\delta^{(d-1)}(\vec{x}) \\ \langle \pi(\vec{x}_1, t_1)\pi(\vec{x}_2, t_2)\rangle &= \frac{i|t|}{2(1-\alpha^2)}\vec{\partial}^2\delta^{(d-1)}(\vec{x}) \end{aligned}$$

where $t = t_1 - t_2$ and $\vec{x} = \vec{x}_1 - \vec{x}_2$, and α is a parameter determined by quantization. They indeed satisfy the Ward identities of CCA.

Quantum aspects of CCFT BC, W.H. Sun and Y.F. Zheng 2406.17451

Some essential questions: quantizations, vacuum, state-operator correspondence, ...

Quantization: Path-integral BC *et al.* 2301.06011, 2302.05975; Canonical quantization on massive scalar J. de Boer *et al.* 2307.06827; J. Cotler *et al.* 2407.11971

We studied the quantization of Carrollian conformal scalar theories in 2D and 3D BC *et al.* 2406.17451

Q1: induced vacuum or highest-weight (h.w.) vacuum?

h.w. vacuum: no unitary Hilbert space

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- h.w. vacuum: no unitary Hilbert space
- induced vacuum: can define unitary Hilbert space

Q2: state-operator correspondence: No!

ModMax theory

ModMax: Modified Maxwell (ModMax)

$$\mathcal{L}_\gamma(\mathcal{S},\mathcal{P}) = -\frac{1}{2}\cosh\gamma\mathcal{S} + \frac{1}{2}\sinh\gamma\sqrt{\mathcal{S}^2+\mathcal{P}^2}, \quad \gamma \in \mathbf{R},$$

with

$$\mathcal{S} \equiv \frac{1}{2}F_{\mu\nu}F^{\mu\nu}, \qquad \mathcal{P} \equiv \frac{1}{2}F_{\mu\nu}(*F)^{\mu\nu}.$$

1. It is maximally symmetric nonlinear extensions of Maxwell: conformal invariant and EM invariant $(\mathbf{E}+i\mathbf{B}) \to e^{-i\theta}(\mathbf{E}+i\mathbf{B})$. I. Bandos *et al.* 2007.09092; B. Kosyakov 2007.13878

2. ModMax can be generated from the Maxwell theory by the $\sqrt{T\bar{T}}$ flow perturbatively in $d=4$ in the sense that H. Babaei-Aghbolagh *et al.* 2202.11156; C. Ferko *et al.* 2203.01085

$$\frac{\partial\mathcal{L}_\gamma^{\mathrm{ModMax}}}{\partial\gamma} = O^\gamma_{\sqrt{T^2}}, \qquad \mathcal{L}_\gamma^{\mathrm{ModMax}} = \mathcal{L}^{\mathrm{Maxwell}} + \int O^\gamma_{\sqrt{T^2}}d\gamma,$$

where

$$O^\gamma_{\sqrt{T^2}} = \sqrt{\frac{1}{d}\left(T^\mu{}_\nu T^\nu{}_\mu - rT^\mu{}_\mu T^\nu{}_\nu\right)}.$$

Carrollian ModMax BC, J. Hou and H.W. Sun, 2405.04105

$$\mathcal{L}_{\gamma}^{\mathsf{CMM}}(\mathcal{S},\mathcal{P}) = \frac{1}{4}\left(e^{\gamma}\mathcal{S} \mp e^{-\gamma}\frac{\mathcal{P}^2}{\mathcal{S}}\right),$$

with

$$\mathcal{S} \equiv \frac{1}{2}F_{\mu\nu}F^{\mu\nu} = m^{\mu\rho}\gamma^{\nu\sigma}F_{\mu\nu}F_{\rho\sigma}, \quad \mathcal{P} \equiv \frac{1}{2}F_{\mu\nu}\widetilde{F}^{\mu\nu}.$$

Note: The theory is defined on Carrollian geometry, and the Hodge dual should be defined carefully.

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Note: The theory is defined on Carrollian geometry, and the Hodge dual should be defined carefully.

1. It is Carrollian SO(2) EM duality invariant as well as conformal invariant.
2. The Carrollian ModMax theories in the family deform among themselves under $\sqrt{T\bar{\bar{T}}}$ flow, except two end-points $\gamma \to \pm\infty$, where the flow is non-invertible.

$$\mathcal{L} \sim -\frac{\mathcal{P}^2}{2\mathcal{S}} \qquad\qquad \mathcal{L} \sim \frac{\mathcal{S}}{2}$$

$$-\infty \; \xleftrightarrow[\sqrt{T\bar{\bar{T}}}-\gamma \text{ flow}]{\text{Carrollian ModMax}} \; +\infty$$

Structure of Carrollian (conformal) superalgebra

Y.F Zheng and BC, 2503.22160

Aim: try to classify all possible structure of Carrollian (conformal) superalgebra in $d = 4$ and $d = 3$.

Basic ansatz: the supercharges transforming in chain representations and $\{Q, Q\} \sim P, [P, Q] = 0$.
The Jacobi identities constrain the possible structures.

In 4D, we find nontrivial super-Carrollian algebra with certain pattern for the Q representations, which can not be derived by taking ultra-relativistic limit of usual super-Poincaré algebra.

In 3D, it is significantly challenging to systematically discuss the nonconformal superalgebra since the restriction from the Carrollian rotation is not enough. We therefore focus on the Carrollian superconformal algebra

For the Carrollian superconformal case, there are less allowed structures. We demonstrate that the nontrivial Carrollian superconformal algebras for $d = 4$ and $d = 3$ are isomorphic to super-Poincaré algebra of $d = 5$ and the one of $d = 4$ respectively.

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3D Carrollian conformal algebra can be infinitely extended to the BMS_4 algebra. How about supersymmetric version?
Singlet super-BMS_4 algebra is analogous to a homogeneous superalgebra in 2D. Bagchi, et al., 1606.09628, 1710.03482,1801.03245;I. Lodato, et al., 1610.07506;BC, et al., 2302.05975

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Moreover, we discover two multiplet chiral super-BMS_4 algebras, which parallel to the inhomogeneous superalgebra. Remarkably, they can not be found from Carrollian superconformal algebra by extension.

Summary

1. We tried to study the higher dimensional ($d \geq 3$) Carrollian conformal invariant theories in a systematic way. As the conformal algebra is not semi-simple, the finite dimensional h.w.r. present some novel features: multiplet structure, staggered module, chain-like and even net-like representations.

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- We discussed the 2-pt and 3-pt correlators of operators in chain-like representations.

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Thanks for your attention!

Carroll group as kinematical group

Both the Galilei group and Carroll group are kinematical groups.H. Bacry and J. Lévy-Leblond (1968).

Possible relativity groups in 4D: possible invariance groups of a 4D physical theory that contains the generators of relativity, i.e. time translations, space translations, spatial rotations and boosts.

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Poincaré group, Galilei group, AdS/dS isometry group,
Newton-Hooke group, Carroll group

Different relativity groups are related by Inönü-Wigner contractions. Actually all these groups can be obtained by certain contraction of AdS and dS groups.

Backup: Carrollian particle

To study the motion of a free Carrollian particle, we may start from the massive particle moving in AdS/dS spacetime and then take the Carrollian limit. In the end, we find the action

$$S_C = \int d\tau \left(-E\dot{t} + \dot{\vec{x}} \cdot \vec{p} - \frac{e}{2}(E^2 - M^2)\right)$$

which is invariant under the Carrollian transformation

$$\begin{aligned} t' &= t - \vec{b} \cdot R\vec{x} + a_t, & \vec{x}\,' &= R\vec{x} + \vec{a}, \\ \vec{p}\,' &= R\vec{p} - \vec{b}E, & E' &= E. \end{aligned}$$

The free Carrollian particle is at rest and does not move! C. Duval et al 1402.0657, E. Bergshoeff *et al.* 1405.2264

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Symmetries: E. Bergshoeff *et al.* 1405.2264

1. The free Carrollian particle has infinite dimensional symmetry.
2. For the massless one, the symmetries get enhanced to Carrollian conformal symmetry.

However, for two-particle system, there is non-trivial dynamics!