ISOSPIN ASYMMETRY AND NEUTRON STARS IN V-QCD

Lorenzo Bartolini – Henan University, School of Mathematics and Statistics

Work in collaboration with S. B. Gudnason, M. Järvinen

HOLOGRAPHIC APPLICATIONS: FROM QUANTUM REALMS TO THE BIG BANG, UCAS, Beijing, 17/07/2025

OVERVIEW

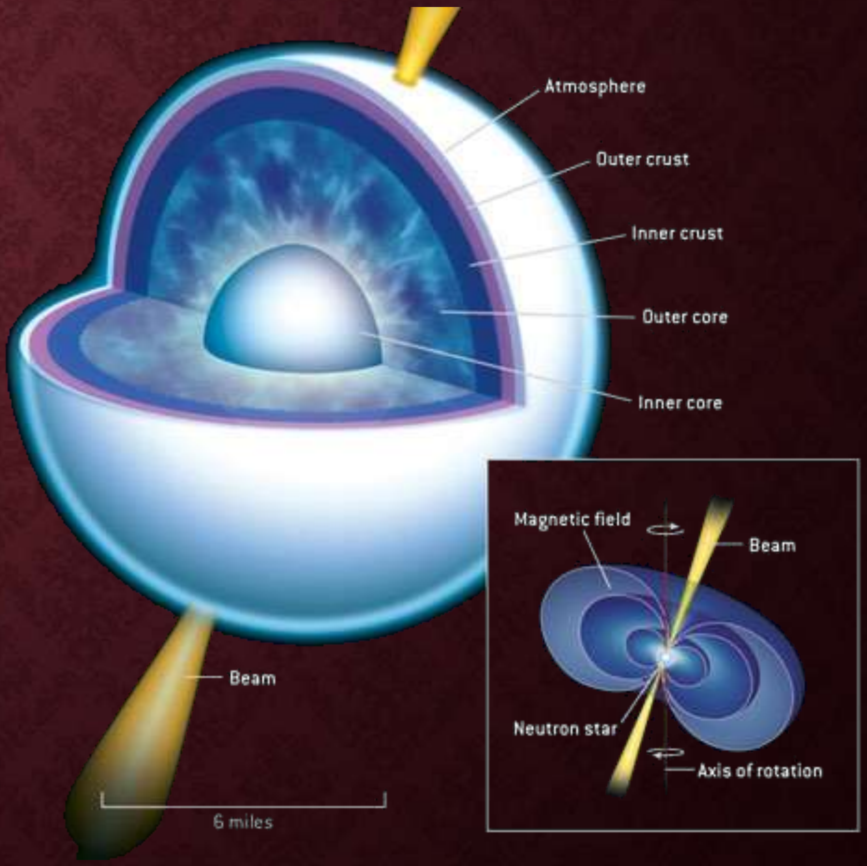
- Introduction: Few Words on Neutron Stars and Symmetry Energy
- VQCD
- Holographic Homogeneous Nuclear Matter
- Hybrid VQCD Equations of State and Neutron Stars
- Conclusions and Future Directions

NEUTRON STARS

- Remnants of supernovae from supergiant stars.
- Most compact astrophysical objects (excluding BHs).
- Tolman-Oppenheimer-Volkov equations:

$$\frac{dP}{dr} = -G(\varepsilon + P)\frac{m + 4\pi r^3 P}{r(r - 2Gm)},$$
$$\frac{dm}{dr} = 4\pi r^2 \varepsilon.$$

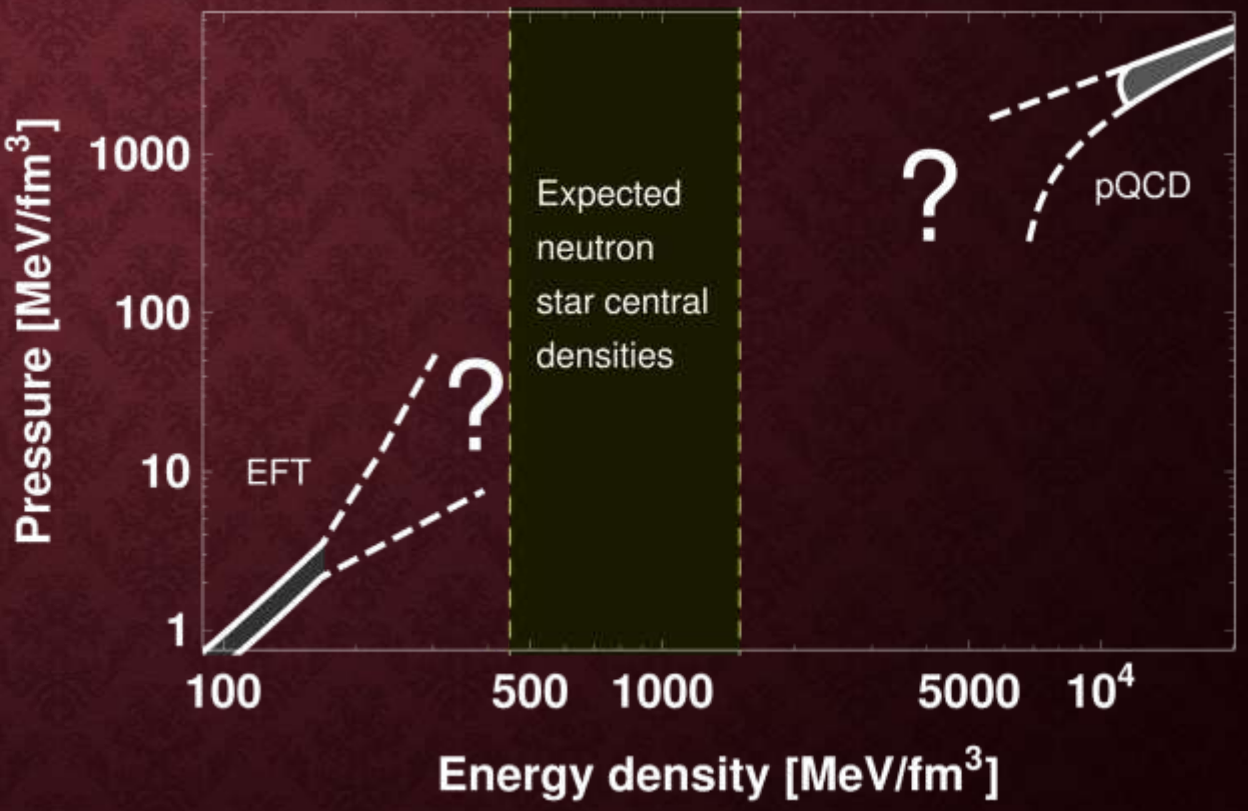
- Neutron rich: proton fraction dictated by Symmetry Energy.
- Problem: equation of state $P(\varepsilon)$ for nuclear matter is not well known at such densities.



EQUATION OF STATE

- HIGH DENSITY: Perturbative QCD
- LOW DENSITY: Nuclear Physics, EFTs.
- PROBLEM: We are missing a description of the EOS in the «sweet spot» regions for neutron stars.

HOLOGRAPHY can come to the rescue?



HOLOGRAPHIC MODELS APPLIED TO NEUTRON STARS

- **D3-D7**

1. [C. Hoyos, D. Rodríguez Fernández, N. Jokela, A. Vuorinen]
2. [K. B. Fadafan, J. C. Rojas, N. Evans]

- **D4-D8**

1. [N. Kovensky, A. Poole, A. Schmitt]
2. [LB, S. B. Gudnason]

- **VQCD** [M. Järvinen, N. Jokela, J. Remes, C. Ecker, L.B, S.B. Gudnason] ⟸ THIS TALK
- **HARD-WALL** [LB, S. B. Gudnason, J. Leutgeb, A. Rebhan]

MERGER EVENTS SIMULATIONS FROM HOLOGRAPHY:

- **VQCD** [C. Ecker, M. Järvinen, G. Nijs, W. van der Schee]
- **HARD-WALL** [LB, S. B. Gudnason, J. Leutgeb, A. Rebhan]

SOME HOLOGRAPHIC RESULTS FOR SYMMETRY ENERGY

- **D4/D6:** 27.7 MeV [Y. Kim, Y. Seo, I.J. Shin, S.J. Sin; 1011.0868]
- **D4/D8:**

1. ~1 GeV ***** [N. Kovensky, A. Schmitt; 2105.03218] ⟸ Via μ_I as UV boundary condition
2. ~0.1 GeV [LB, S. B. Gudnason; 2209.14309] ⟸ Via time dependence in SU(2) moduli

- **VQCD:**
- ~[28.73,39.4] MeV [LB, S. B. Gudnason, M.Jarvinen; 2504.01758]**
- ~[50,60] MeV [LB, S. B. Gudnason, M.Jarvinen; 2504.01758]**

* Factor of N_c^2 from different normalization of Isospin density. The two methods are equivalent.
** Different values due to different matching procedures to obtain the hybrid EOS.

VQCD

GLUE SECTOR: IMPROVED HQCD

$$S_g = M_p^3 N_c^2 \int d^5x \sqrt{-\det g} \left[R - \frac{4}{3} (\partial_M \phi)^2 + V_g(\phi) \right], \qquad ds^2 = e^{2A(r)} \left(\frac{dr^2}{f(r)} - f(r) dt^2 + d\vec{x}^2 \right).$$

$$S_{\rm GH} = M_p^3 N_c^2 \int d^5x \sqrt{-\det h}\, K.$$

FLAVOR SECTOR: TACHYONIC DBI + CHERN-SIMONS ACTION

$$S_{\rm TDBI} = -\frac{1}{2} M_p^3 N_c \,{\rm tr} \int d^5x \left(V_f(\phi, T^\dagger T) \sqrt{-\det {\rm A_L}} + V_f(\phi, T T^\dagger) \sqrt{-\det {\rm A_R}} \right),$$

$$S_{\rm TCS} = \frac{i N_c}{4\pi^2} \int \Omega_5^s,$$

$$\Omega_5^s = \frac{1}{24} F_1(\tau)\, {\rm tr} \left[A \wedge A \wedge A \wedge \left(F^{(L)} + F^{(R)} \right) \right] - \frac{i}{24} F_3(\tau)\, {\rm tr} \left[A \wedge \left(F^{(L)} - F^{(R)} \right) \wedge \left(F^{(L)} - F^{(R)} \right) \right]$$

VQCD

GLUE SECTOR: IMPROVED HQCD

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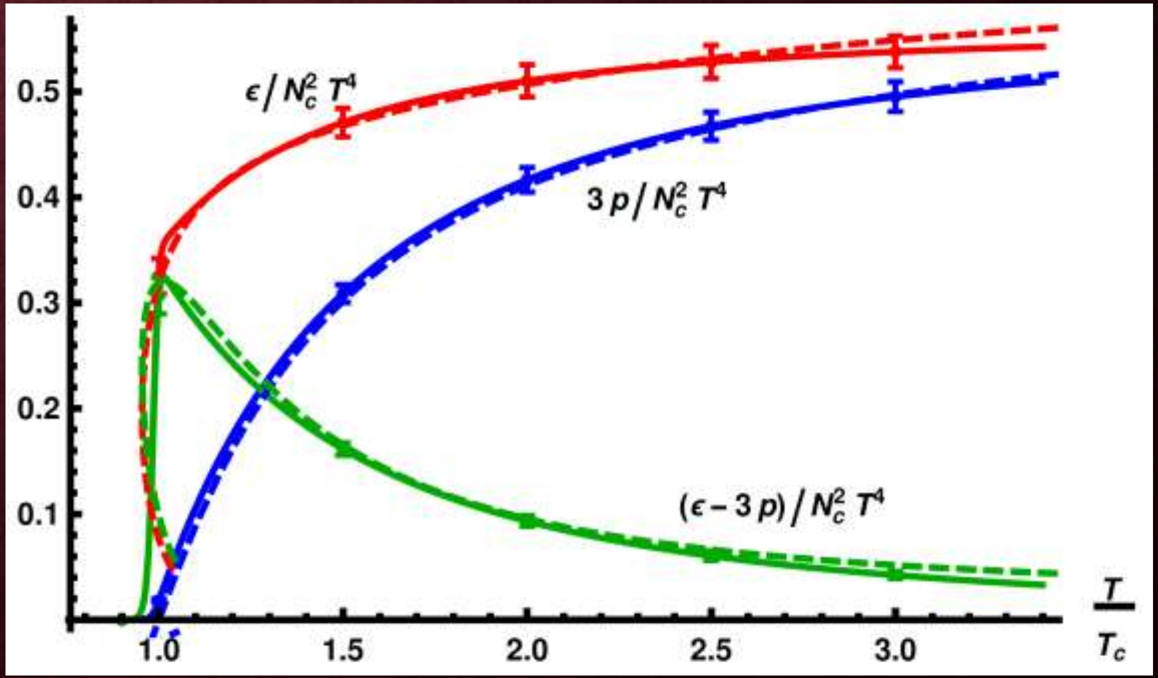
$$S_{\rm GH} = M_p^3 N_c^2 \int d^5x \sqrt{-\det h}\, K.$$

FIT TO LATTICE DATA:

Fit V_g to Large-N_c pure Yang-Mills data



VQCD

FLAVOR SECTOR: TACHYONIC DBI + CHERN-SIMONS ACTION

$$S_{\mathrm{TDBI}} = -\frac{1}{2} M_p^3 N_c \mathrm{tr} \int \mathrm{d}^5x \left(V_f(\phi, T^\dagger T) \sqrt{-\det \mathbf{A}_L} + V_f(\phi, TT^\dagger) \sqrt{-\det \mathbf{A}_R} \right),$$

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$$- \frac{i}{24} F_3(\tau)\, \mathrm{tr} \left[A \wedge \left(F^{(L)} - F^{(R)} \right) \wedge \left(F^{(L)} - F^{(R)} \right) \right]$$

TACHYON DEPENDENCE IN CS FROM FLAT SPACE: [R. Casero, E. Kiritsis, A. Paredes; 0702155]

$$F_1(\tau) = e^{-b_1\tau^2}(1 - 2b_1\tau^2), \qquad F_3(\tau) = e^{-b_3\tau^2},$$

b_1, b_3 rescalings to account from curved space?

VQCD

FLAVOR SECTOR: TACHYONIC DBI + CHERN-SIMONS ACTION

$$S_{\rm TDBI} = -\frac{1}{2} M_p^3 N_c {\rm tr} \int d^5x \left(V_f(\phi, T^\dagger T)\sqrt{-\det \mathbf{A}_L} + V_f(\phi, TT^\dagger)\sqrt{-\det \mathbf{A}_R}\right),$$

$$S_{\rm TCS} = \frac{iN_c}{4\pi^2}\int \Omega_5^s,$$

$$\Omega_5^s = \frac{1}{24}F_1(\tau)\,{\rm tr}\left[A\wedge A\wedge A\wedge\left(F^{(L)}+F^{(R)}\right)\right] - $$
$$- \frac{i}{24}F_3(\tau)\,{\rm tr}\left[A\wedge\left(F^{(L)}-F^{(R)}\right)\wedge\left(F^{(L)}-F^{(R)}\right)\right]$$

$$\mathbf{A}_{L,MN} = g_{MN} + w(\phi, T^\dagger T)F^{(L)}_{MN} + \frac{\kappa(\phi, T^\dagger T)}{2}\left[(D_M T)^\dagger (D_N T) + (D_N T)^\dagger (D_M T)\right]$$

$$\mathbf{A}_{R,MN} = g_{MN} + w(\phi, T^\dagger T)F^{(R)}_{MN} + \frac{\kappa(\phi, TT^\dagger)}{2}\left[(D_M T)(D_N T)^\dagger + (D_N T)(D_M T)^\dagger\right]$$

$$T = \tau(r)\mathbb{1}$$

VQCD

FLAVOR SECTOR: TACHYONIC DBI + CHERN-SIMONS ACTION

$$S_{\mathrm{TDBI}} = -\frac{1}{2} M_p^3 N_c \mathrm{tr} \int \mathrm{d}^5x \left(V_f(\phi, T^\dagger T)\sqrt{-\det \mathbf{A}_L} + V_f(\phi, TT^\dagger)\sqrt{-\det \mathbf{A}_R}\right),$$

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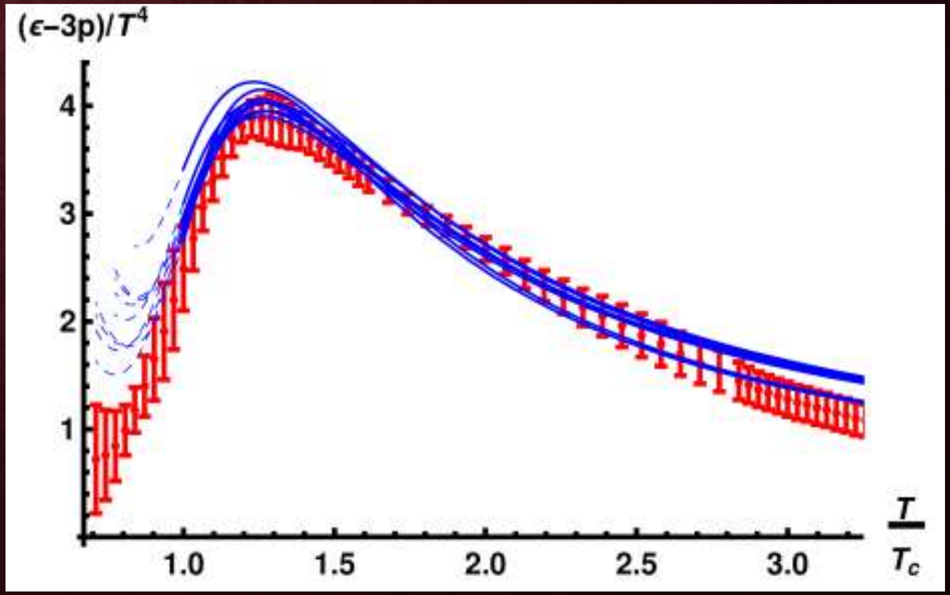
FIT TO LATTICE DATA:

Fit V_f to QCD lattice data for interaction measure:

- Different blue curves correspond to different choices of background potentials.
- We employed two of these choices (“7a, 5b”).



HOMOGENEOUS NUCLEAR MATTER

HOMOGENEOUS ANSATZ:

$$A_i^L = -A_i^R = -\frac{H(r)}{2}\sigma^i, \quad \text{with} \quad i = 1, 2$$

$$A_3^L = -A_3^R = -\frac{H_3(r)}{2}\sigma^3 - \frac{L_3(r)}{2}\mathbb{1},$$

$$A_t^L = A_t^R = a_0(r)\sigma^3 + \frac{\hat{a}_0}{2}\mathbb{1}.$$

FITS AND MATCHING TO LOW DENSITY

- Fit potentials to reproduce lattice data.

Now only two parameters left:

- c_b (action rescaling)
- b_1 (TCS rescaling)

Two different matching procedures:

a) Choose a transition density and impose continuity of $n_B, P, \mathcal{E}$.
b) Choose $c_b = 1, b_1$ to fit saturation density.

UV BOUNDARY CONDITIONS:

$$H(r=0) = H_3(r=0) = L_3(r=0) = 0,$$

$$\hat{a}_0(r=0) = 2\mu,$$

$$a_0(r=0) = \frac{\mu_I}{2}.$$

BARYON DENSITY (IR BC):

$$H_3(r_c)H(r_c)^2 = -\frac{4\pi^2 n_B}{F_1(r_c)}$$

SYMMETRY ENERGY

SMALL μ_I EXPANSION:

$$a_0(r) = \tilde{a}_0(r)\mu_I + \mathcal{O}(\mu_I^2),$$

$$L_3(r) = \tilde{L}_3(r)\mu_I + \mathcal{O}(\mu_I^2),$$

$$H_3(r) = H(r) + \mathcal{O}(\mu_I^2),$$

$$S_N(n_B) = \frac{n_B}{8\Lambda(n_B)}$$

FREE ENERGY ⟹

$$\Omega = -\mathcal{E}_0(n_B) - \frac{1}{2}\Lambda(n_B)\mu_I^2, \qquad n_I = -\frac{\partial\Omega}{\partial\mu_I} = \Lambda(n_B)\mu_I.$$

MOMENT OF INERTIA
(As bulk integral)

$$\Lambda(n_B) \equiv 2M_p^3N_c\int_0^{r_c}\mathrm{d}r\,\frac{V_f(\phi,\tau)}{4\sqrt{e^{2A(r)}+\kappa(\phi)\tau'^2}}\Big[e^{2A(r)}w(\phi)^2(4\tilde{a}_0'^2+\tilde{L}_3'^2) + 4(e^{2A(r)}+\kappa(\phi)\tau'^2)(2w(\phi)^2H^2\tilde{a}_0^2 + e^{2A(r)}\kappa(\phi)\tau^2\tilde{L}_3^2)\Big].$$

MOMENT OF INERTIA
(As boundary VEV)

$$\Lambda(n_B) = -M_p^3N_c\left(\frac{e^{2A(r)}V_f(\phi,\tau)w(\phi)^2}{\sqrt{e^{2A(r)}+\kappa(\phi)\tau'^2}}\tilde{a}_0'\right)\Bigg|_{r\to0}$$

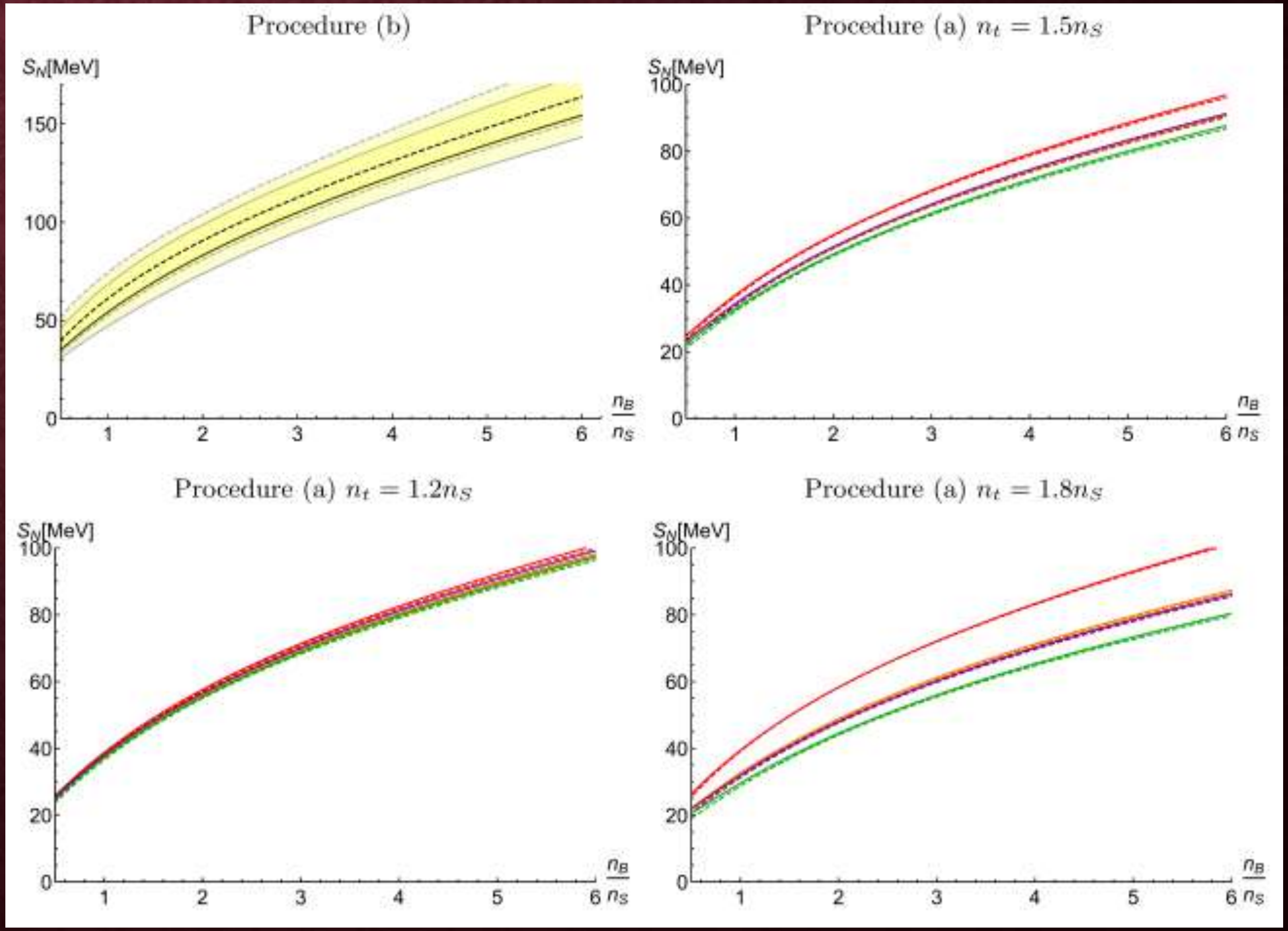
SYMMETRY ENERGY & PARTICLE FRACTIONS

$$S_N(n_B) = S_0 + \frac{1}{3}L\frac{n_B - n_S}{n_S} + \frac{1}{18}K_{\text{sym}}\left(\frac{n_B - n_S}{n_S}\right)^2 + \cdots$$

- VQCD (a) gives reasonable S_0:
  - STIFF $[36.4,39.4]$ MeV
  - MED $[31.9,37.6]$ MeV
  - SOFT $[28.7,37.3]$ MeV
  - Sly $[31.3,48.0]$ MeV

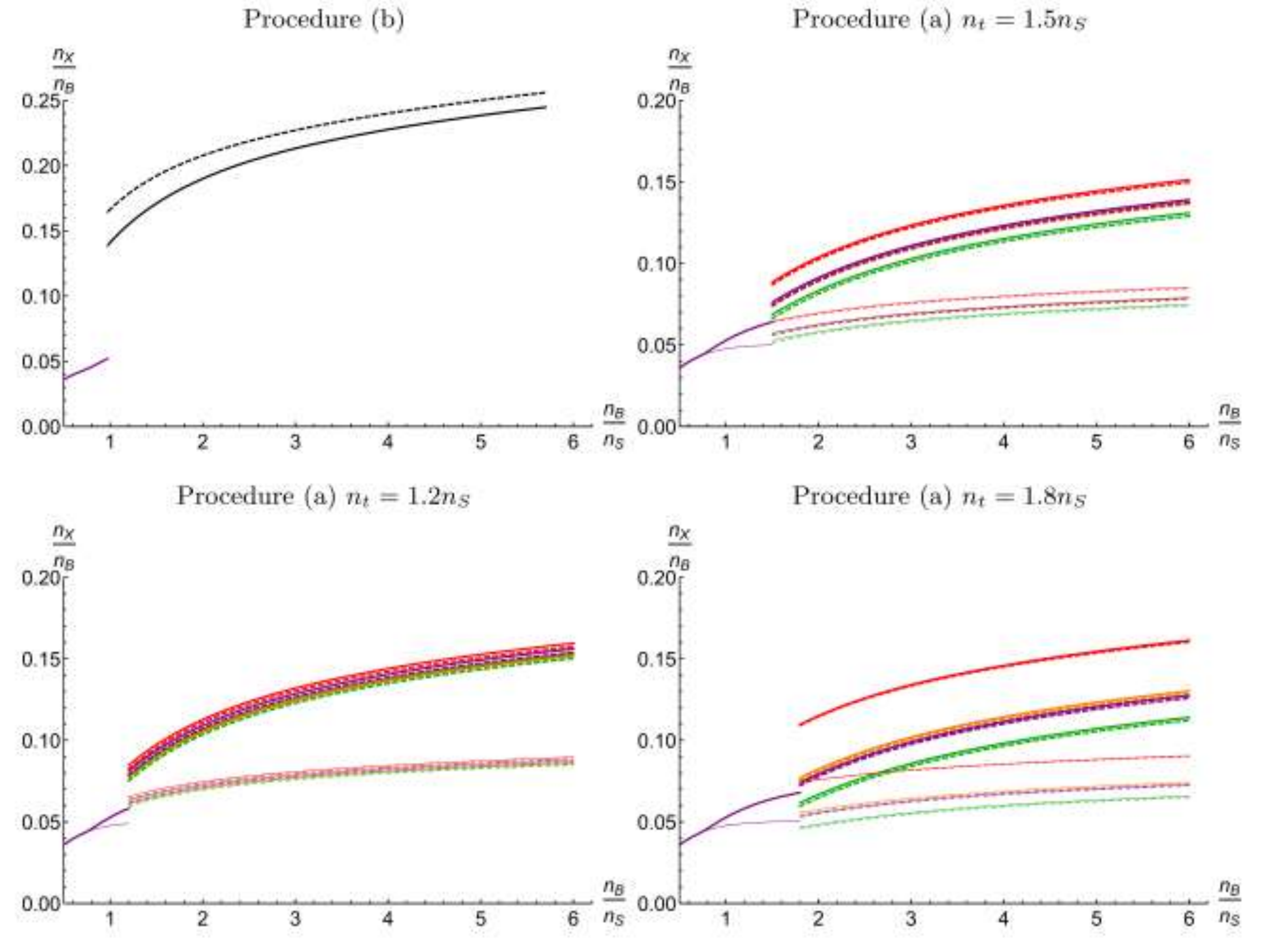
- VQCD (b) Overestimates S_0

- VQCD (a) predicts:
  - $L \in [51.6,71.0]$ MeV
  - $K \in [-106.1,-42.9]$ MeV



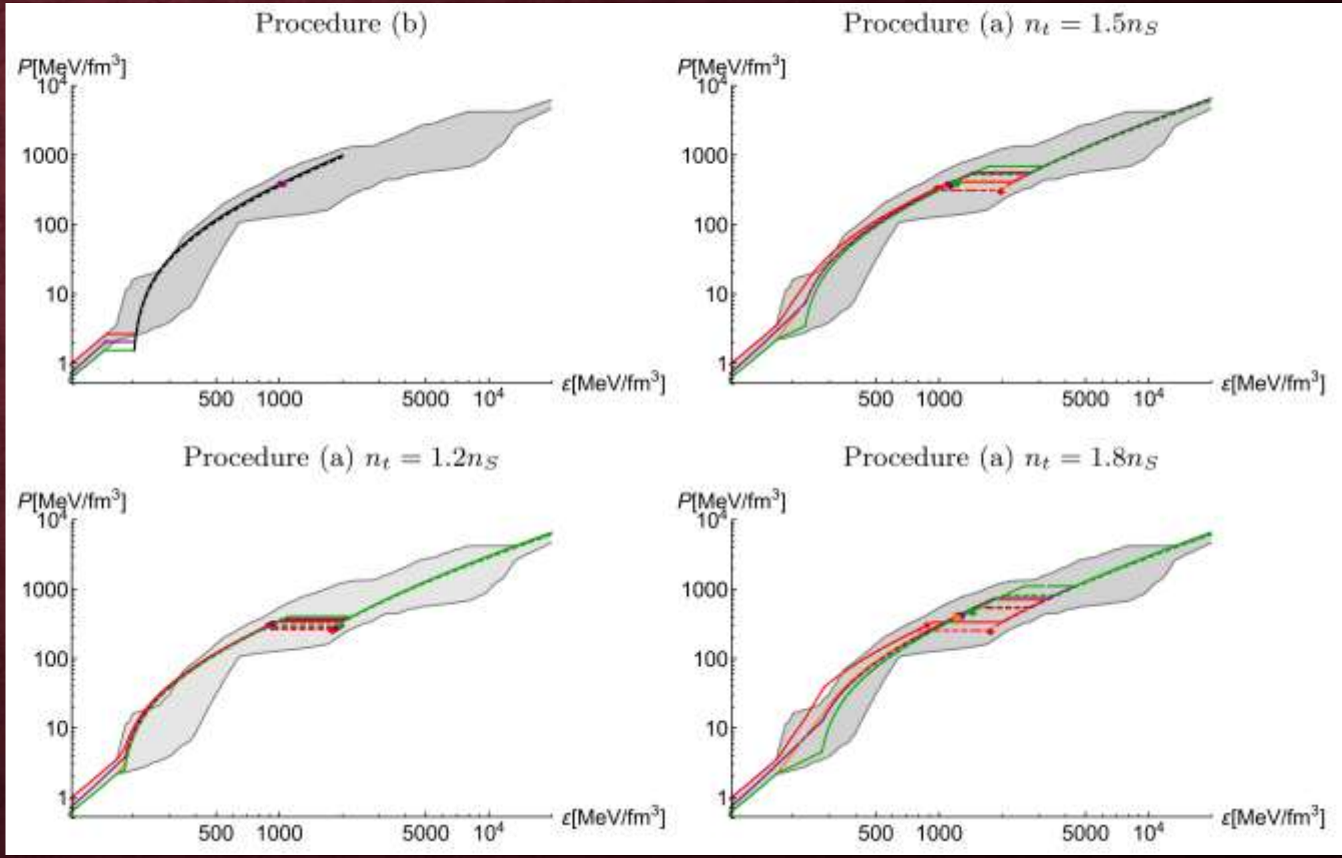
SYMMETRY ENERGY & PARTICLE FRACTIONS

- At low density we show the proton fractions for SLy4.
- VQCD (a) gives reasonable results because it can reproduce S_0.
- VQCD (b) gives high proton fractions because of high Symmetry Energy: not enough free parameters? Break down of Homogeneous Ansatz?



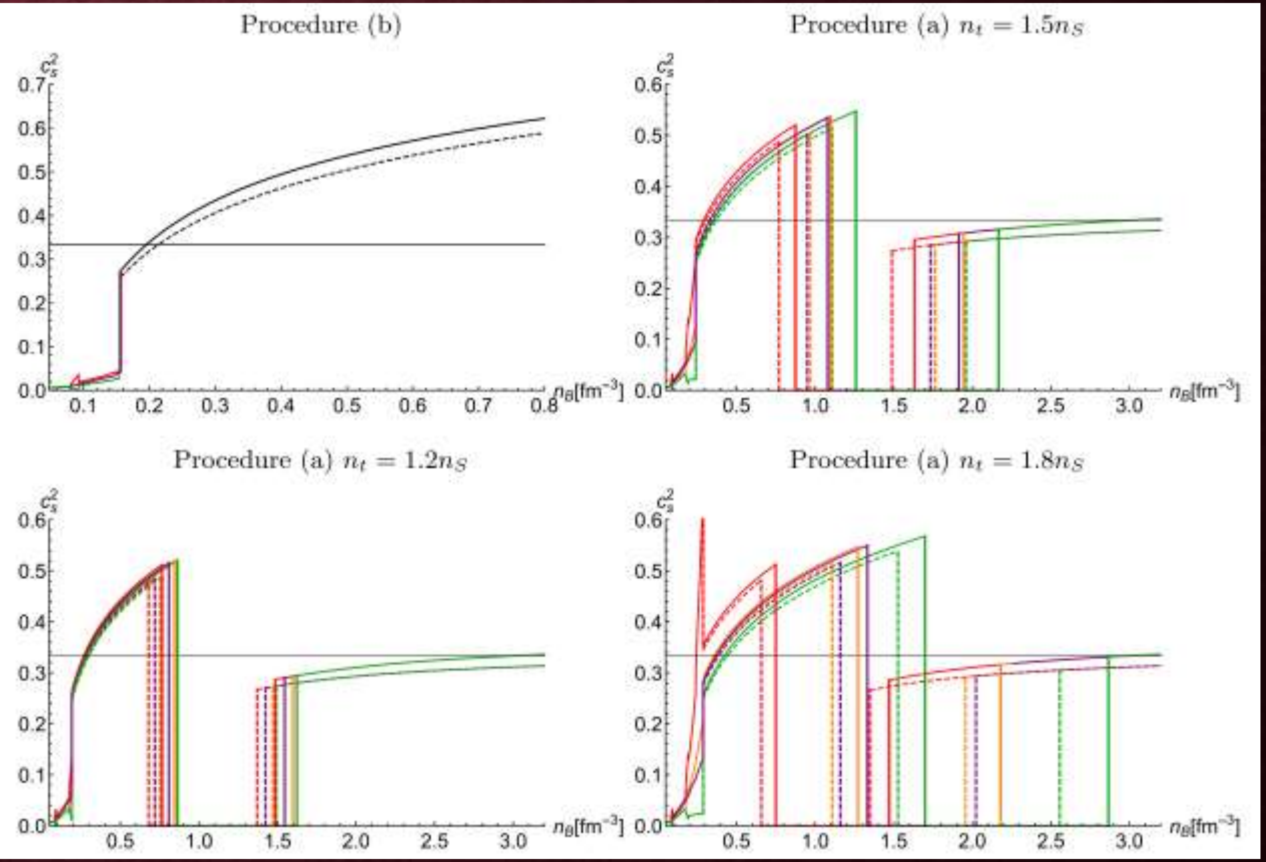
HYBRID EQUATIONS OF STATE

- Holographic EOS are naturally stiff at high density.
- Models (b) with one free parameter fail to connect with phenomenology around saturation density.
- Models (a) provide sets of EOS, some of which fall within the allowed band.
- Models (a) accomodates for the introduction of a quark phase too!
- Models (b): wrong scale for chemical potentials, PT to quark phase cannot be introduced.



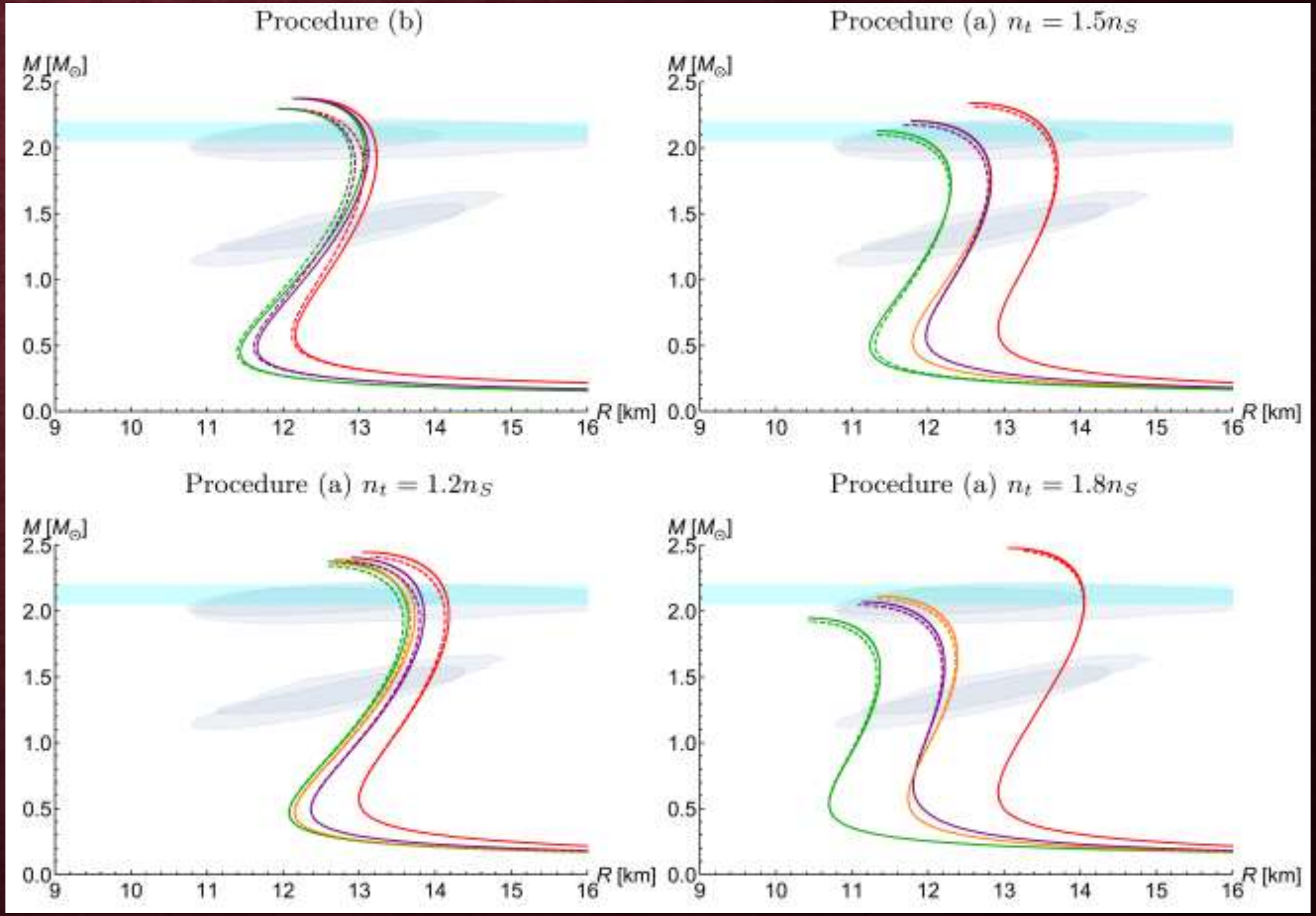
SPEED OF SOUND

- LOW DENSITY PHASE: for EOS compatible with data, remains below conformal barrier.
- HOMOGENEOUS PHASE: discontinuity in speed of sound, then rapidly grows above conformal bound: reaches values typical of the stiffest polytropic interpolations.
- QUARK PHASE: drop in speed of sound, approaches again the conformal bound from below.
- Density gap between the phases! (1st order PT).



NEUTRON STARS

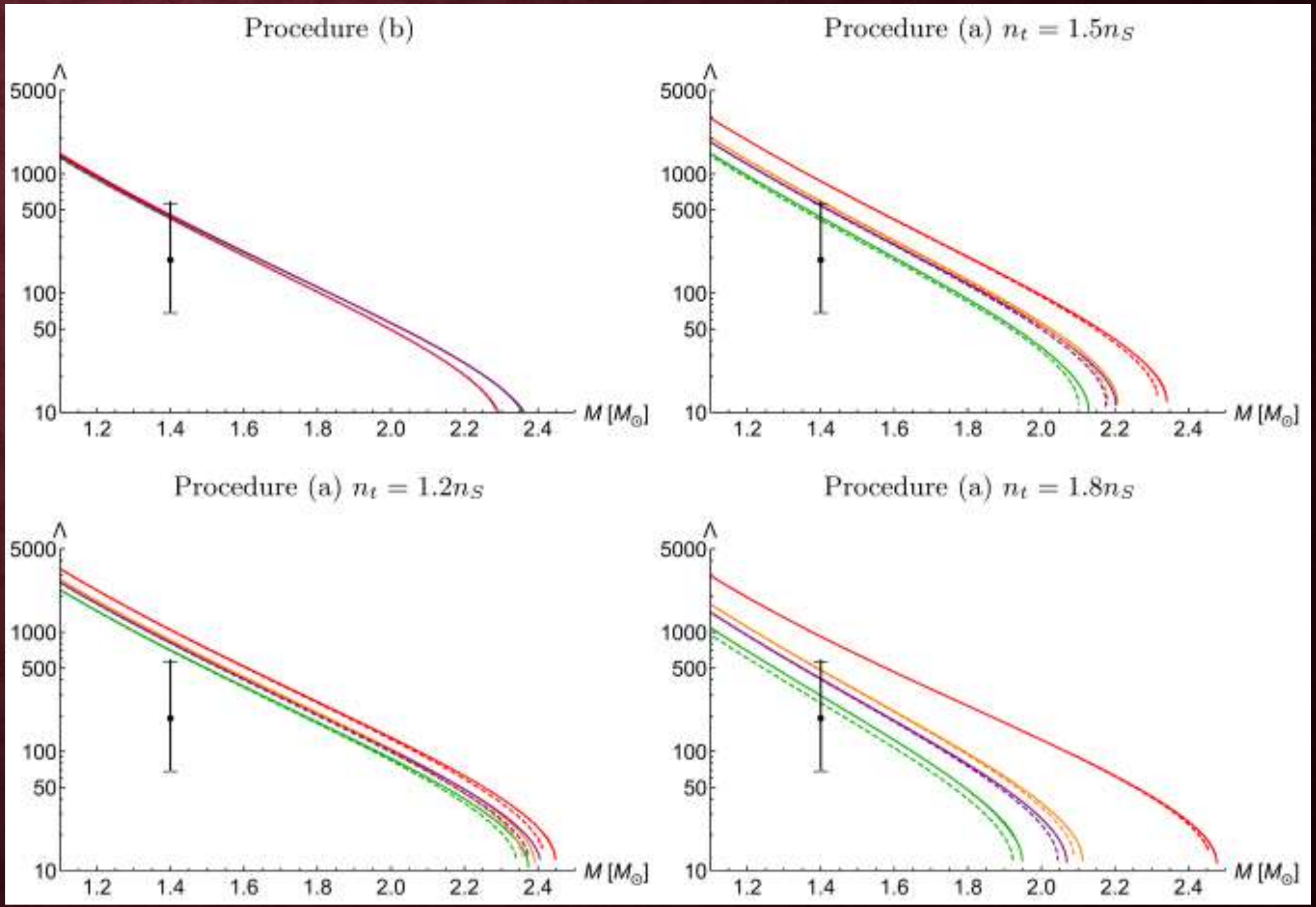
- All the approaches result in curves that succesfully pass through the NICER bands.
- While VQCD (a) allows for a quark phase, no quark matter is present unless for the heaviest star generated from the stiffest EOS: it then triggers instability.
- Moreover: the only stars that would have traces of quark matter, are generated by EOS that are disfavored. There is no quark matter in our most realistic stars.



NEUTRON STARS

- All three approaches result in at least some curves that succesfully pass through the LIGO/Virgo band.

- The stiffest construction from VQCD (a) seems to be excluded. Consistently with the MR results. Intermediate and SLy4 constructions are at some tension with the data.



CONCLUSIONS AND FUTURE DIRECTIONS

- Holographic models can be a powerful tool to obtain EOS in regimes that are difficult for other approaches.
- Holographic EOS for homogeneous matter are found (many times) to be quite stiff.
- Neutron stars phenomenology can be recovered quite successfully, including proton fractions, but only when introducing the additional parameter c_b.
- Shortcomings: What can substitute c_b, b_1? What is the real TCS? Homogeneous ansatz can be improved? Backreaction of baryonic matter?
- For the future: phase diagram at finite μ_I, possibly including quark masses and meson condensation? What about hyperons?

THANK YOU FOR YOUR ATTENTION!