Gravitational form factors in holographic QCD

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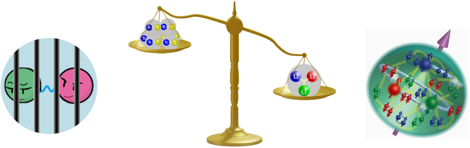
Gravitational form factors

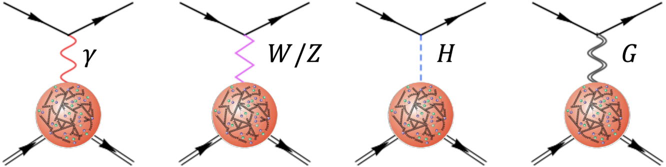
- Big puzzles of the strong force within hadrons
  - Origin of confinement
  - Origin of >99% nucleon mass
  - Origin of the nucleon spin
  - ......



Gross and Klempt (eds.), *50 Years of quantum chromodynamics*

- Hadronic energy-momentum tensor encodes the energy, spin and stress distributions widthin hadrons [Kobzarev:1962wt, Pagels:1966zza]

$$\langle p', s' | T^{\mu\nu}(0) | p, s\rangle = \frac{1}{M}\overline{u}_{s'}(p')\Big[P^\mu P^\nu A(q^2) + \frac{1}{2} i P^{\{\mu}\sigma^{\nu\}\rho} q_\rho J(q^2) + \frac{1}{4}(q^\mu q^\nu - g^{\mu\nu} q^2) D(q^2)\Big] u_s(p)$$



Physical interpretation

[Li:2024vgv, cf. Sachs:1962zzc, Miller:2018ybm]

$$\langle\Psi|T^{\alpha\beta}(x)|\Psi\rangle = \langle \mathcal{E}\mathcal{U}^\alpha\mathcal{U}^\beta - \mathcal{P}\Delta^{\alpha\beta} + \tfrac{1}{2}\partial_\rho(\mathcal{U}^{\{\alpha}\mathcal{S}^{\beta\}\rho}) + \varPi^{\alpha\beta} - g^{\alpha\beta}\Lambda\rangle_\Psi$$

where, $\langle\mathcal{O}(x)\rangle_\Psi = \int \mathrm{d}^3z\,\overline{\Psi}(z)\mathcal{O}(x-z)\Psi(z)$ is the convolution with the hadronic wavepacket $\Psi(x)$.

- Gravitational form factors are F.T. of physical densities

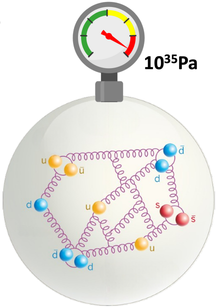
energy density: $\mathcal{E}(x) = M\int\frac{\mathrm{d}^3q}{(2\pi)^3}e^{iq\cdot x}\Big\{\Big(1-\frac{q^2}{4M^2}\Big)A(q^2) + \frac{q^2}{4M^2}\Big[2J(q^2)-D(q^2)\Big]\Big\}$,

spin density: $\mathcal{S}^{\alpha\beta}(x) = \int\frac{\mathrm{d}^3q}{(2\pi)^3}e^{iq\cdot x}\Big\{i\sigma^{\alpha\beta}\sqrt{1-\frac{q^2}{4M^2}} - \frac{U^{[\alpha}q^{\beta]}}{2M}\Big\}J(q^2)$,

pressure: $\mathcal{P}(x) = \frac{1}{6M}\int\frac{\mathrm{d}^3q}{(2\pi)^3}e^{iq\cdot x}q^2D(q^2)$,

shear: $\varPi^{\alpha\beta}(x) = \frac{1}{4M}\int\frac{\mathrm{d}^3q}{(2\pi)^3}e^{iq\cdot x}\Big(q^\alpha q^\beta - \frac{q^2}{3}\Delta^{\alpha\beta}\Big)D(q^2)$,

cosmological constant: $\Lambda = -M^2\int\frac{\mathrm{d}^3q}{(2\pi)^3}e^{iq\cdot x}\bar{c}(q^2)$



- Factorization of the dependence on hadron momentum:
  $\vec{P}=0$ - Sachs distribution, $P_z\to\infty$ - light-front distribution

Mechanical properties of hadrons

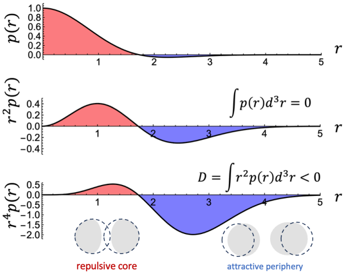
- Mechanical equilibrium → mechanical stability [Polyakov:2018zvc]

$$\text{von Laue condition:} \quad \int d^3r\, \mathcal{P}(r) = 0, \quad \rightarrow \quad D = \int d^3r\, r^2 \mathcal{P}(r) \overset{???}{<} 0$$

- Trace anomaly: $\Theta \equiv T^{\mu}_{\mu} = \frac{\beta(g_s)}{2g_s} G^{\mu\nu a} G^{a}_{\mu\nu} + O(m_q)$

$$\langle p', s' | T^{\mu}_{\mu} | p, s \rangle = \bar{u}_{s'}(p') \Big\{ A(Q^2) + \frac{Q^2}{4M^2} \big[A(Q^2) - 2J(Q^2) + 3D(Q^2) \big] \Big\}$$

- Analogy to superconductor, and dark energy [Teryaev:2016edw, Ji:2021mtz, Liu:2023cse]

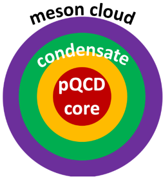


$$r_A^2 < r_\mathcal{E}^2 < r_\theta^2$$

$-6A'(0)$
valence

$-6E'(0) = r_A^2 - \frac{3}{2}\lambda_C^2 D$
condensate

$-6\Theta'(0) = r_A^2 - \frac{9}{2}\lambda_C^2 D$
glueball, meson cloud



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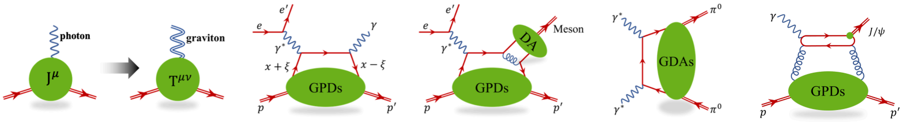
Charmonium's onion-like structure is revealed by new calculations

05 Jul 2024

The last global unknown

[Reviews: Polyakov:2018zvc, Burkert:2023wzr]



- Ji's sum rules: one graviton ≈ two photons → generalized parton distributions (GPDs)

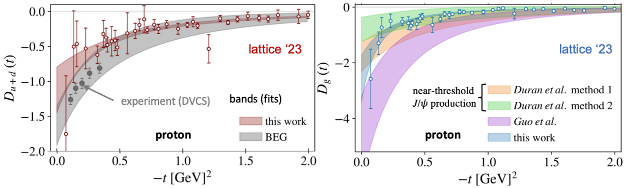
$$\int_{-1}^{1} \mathrm{d}x\, x H^{q,g}(x,\xi,t) = A^{q,g}(t) + \xi^2 D^{q,g}(t)$$

[Ji:1996nm, Polyakov:2002yz]

  - Deeply virtual Compton scattering & deeply vector meson production [Burkert:2018bqq, Burkert:2021ith]
  - Di-photon pair production [Kumano:2017lhr]
- Near threshold vector meson production [Kharzeev:2021qkd, Duran:2022xag]
- Large uncertainties from both the theory and experiments → Electron-Ion Colliders

[Lattice '23: Hackett:2023nkr; DR: Broniowski:2024mpw, Cao:2024zlf, Cao:2025dkv]

electromagnetic	$G_E(0) = Q = 1.602176487(40) \times 10^{-19}\mathrm{C}$ $G_M(0) = \mu = 2.792847356(23)\mu_N$
weak	$G_A(0) = g_A = 1.2694(28)$ $G_P(0) = g_p = 8.06(55)$
gravitational	$mA(0) = m = 938.272013(23)\ \mathrm{MeV}/c^2$ $J(0) = J = \frac{1}{2}$ $D(0) = D = ?$




The Answer to the Great Question Of Life, the Universe and Everything is 42!
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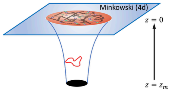
银河系漫游指南
THE HITCHHIKER'S GUIDE TO THE GALAXY

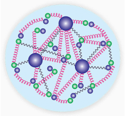
Holographic QCD

[Polchinski:2000uf]

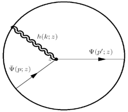


[Maldacena:1997re]

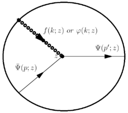
Weak coupling		Strong coupling
?	AdS/QCD	QCD
↑		↑
Semiclassical fields in AdS_5	LF holography	Semiclassical LFQCD
↑		↑
Type IIB string theory in $AdS_5 \times S^5$	AdS/CFT	N=4 SUSY Yang-Mills theory (CFT)



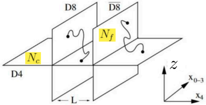
- AdS/QCD: semiclassical field theory in 5D anti-de Sitter space w./ input from QCD
  - Introduce a dilaton field $\Phi(z)$ to break the conformal symmetry in IR: hard-wall, soft-wall models
  - Good agreement with QCD within 10%-15% level [Erlich:2005qh, Karch:2006pv]
- GFFs in AdS/QCD: graviton-hadron scattering in 5D [Abidin:2008hn, Abidin:2008ku, Abidin:2009hr]
  - Problem: GFF $D(Q^2)$ inaccessible, since graviton only couples to the traceless part of $T^{\mu\nu}$
  - Solutions beyond AdS/QCD: scalar graviton, Sakai-Sugimoto model [Mamo:2019mka, Fujita:2022jus]



tensor graviton



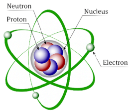
scalar graviton



Sakai-Sugimoto model in 10D

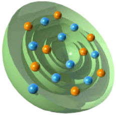
Light front holography

[Review: Brodsky:2014yha]



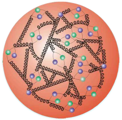
Bohr Model

$$\left(\frac{\vec{p}^2}{2m} - \frac{Z\alpha}{r}\right)\psi = E\psi$$

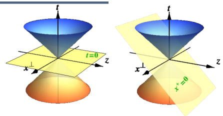


Shell Model

$$\left(\frac{\vec{p}^2}{2m} + \frac{m\omega^2}{2}r^2\right)\psi = E\psi$$



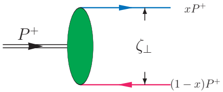
?



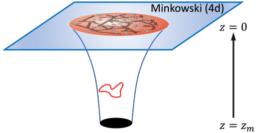
$$x^{\pm} = t \pm z/c$$

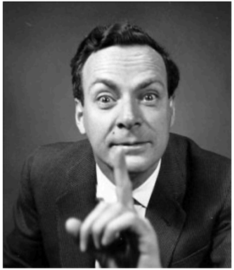
$$H_{\rm QCD}|\psi\rangle = M^2|\psi\rangle \quad \overset{\text{semiclassical appr.}}{\Rightarrow} \quad \left[-\nabla^2_{\zeta_\perp} + V^{\rm eff}_{q\bar{q}}(\vec{\zeta}_\perp)\right]\varphi_h(\vec{\zeta}_\perp) = M_h^2\varphi_h(\vec{\zeta}_\perp)$$

where $\vec{\zeta}_\perp = \sqrt{x(1-x)}\vec{r}_\perp$



semiclassical LFQCD	$\leftrightarrow$	semiclassical field theory in AdS_5
$\zeta_\perp = \sqrt{x(1-x)}r_\perp$	$\leftrightarrow$	fifth coordinate z,
LF amplitude	$\leftrightarrow$	string amplitude
confining potential $V^{\rm eff}_{q\bar{q}}$	$\leftrightarrow$	dilation field Φ
$L^2 - (J-2)^2$	$\leftrightarrow$	$(\mu R)^2$
form factors	$\leftrightarrow$	form factors





If you can't solve a problem in three different ways, you don't really understand it.

— R. P. Feynman

Pion charge form factor F_π in holographic QCD

- Drell-Yan-West formula in LFQCD: [Brodsky:2007hb]

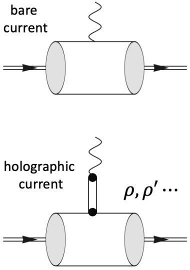
$$F_\pi(Q^2) = \int \zeta_\perp \mathrm{d}\zeta_\perp \big|\varphi_\pi(\zeta_\perp)\big|^2 \zeta_\perp Q K_1(\zeta_\perp Q) + \text{higher Fock sector contributions}$$

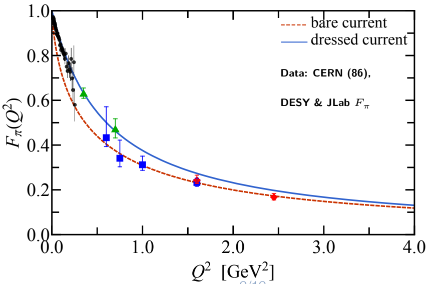
- Electromagnetic coupling in AdS$_5$: [Grigoryan:2007wn, Abidin:2009hr]

$$S_{\text{int}} = e_5 \int \mathrm{d}^5 x \sqrt{g} g^{NM} \Phi^*(x) i \overleftrightarrow{\nabla}_N \Phi(x) A_M(x) \quad \Rightarrow \quad F_\pi(Q^2) = \int z \mathrm{d}z \big|\varphi_\pi(z)\big|^2 V(Q^2, z)$$

where, $V(Q^2, z)$ is the bulk-to-boundary propagator of the 5D EM field $A_N(x)$. In soft-wall model,

$$V(Q^2, z) = \Gamma\big(1 + \tfrac{Q^2}{4\kappa^2}\big) U\big(\tfrac{Q^2}{4\kappa^2}, 0; \kappa^2 z^2\big) \overset{Q^2 \to \infty}{=} zQK_1(zQ)\Big[1 + O(Q^{-1})\Big]$$





Parameters:
soft-wall: $\kappa = \frac{1}{2} m_\rho = 0.388$ GeV;
hard-wall: $\Lambda_{\text{QCD}} = z_m^{-1} = 0.322$ GeV

Soft-wall: r_C = 0.62 fm
Hard-wall: r_C = 0.57 fm
Experiment: r_C = 0.67 fm

Pion gravitational form factor A_π in holographic QCD

- Brodsky-Hwang-Ma-Schmidt formula in LFQCD: [Brodsky:2008pf]

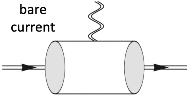
$$A_\pi(Q^2) = \int \zeta_\perp \mathrm{d}\zeta_\perp \big|\varphi_\pi(\zeta_\perp)\big|^2 \frac{1}{2}\zeta_\perp^2 Q^2 K_2(\zeta_\perp Q) + \text{higher Fock sector contributions}$$

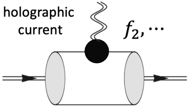
- Gravitational coupling in AdS$_5$: $g_{NM} \to g_{NM} + \delta g_{NM}$ [Abidin:2008hn, Katz:2005ir]

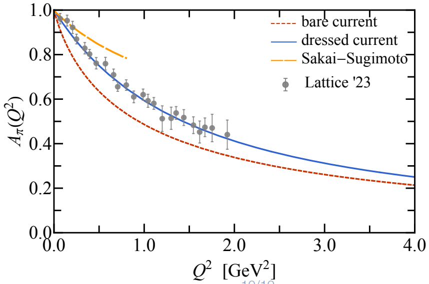
$$A_\pi(Q^2) = \int z\mathrm{d}z \big|\varphi_\pi(z)\big|^2 H(Q^2, z)$$

where, $H(Q^2, z)$ is the bulk-to-boundary propagator of the 5D gravitational field. In soft-wall model,

$$H(Q^2, z) = \Gamma\big(2 + \frac{Q^2}{8\kappa^2}\big) U\big(\frac{Q^2}{8\kappa^2}, -1; 2\kappa^2 z^2\big) \overset{Q^2\to\infty}{=} \frac{1}{2} z^2 Q^2 K_2(zQ)\Big[1 + O(Q^{-1})\Big]$$







Parameters:
soft-wall: $\kappa = \frac{1}{2} m_\rho = 0.388$ GeV;
hard-wall: $\Lambda_{\text{QCD}} = z_m^{-1} = 0.322$ GeV

Soft-wall: r_A = 0.39 fm
Hard-wall: r_A = 0.32 fm
Lattice '23: r_A = 0.41(1) fm
[Hackett:2023nkr]

GFF D in LFQCD

International Journal of Modern Physics A | Vol. 33, No. 26, 1830025 (2018)

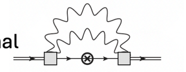
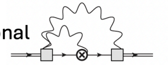
| Reviews

Forces inside hadrons: Pressure, surface tension, mechanical radius, and all that

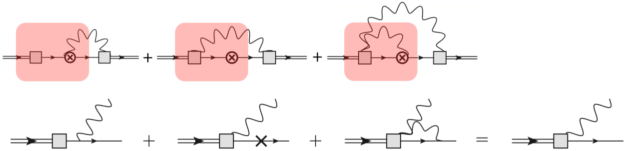
Maxim V. Polyakov and Peter Schweitzer

https://doi.org/10.1142/S0217751X18300259 | **Cited by:** 212 **(Source: Crossref)**

$\hat{T}_{++}$ of the EMT. Being related to the stress tensor $\hat{T}_{ij}$ the form factor $D(t)$ naturally "mixes" good and bad light-front components and is described in terms of transitions between different Fock state components in overlap representation. As a quantity intrinsically nondiagonal in a Fock space, it is difficult to study the D-term in approaches based on light-front wave functions. This is due to the rela-

diagonal  non-diagonal 

- In an explicit model calculation (scalar model in 3+1D), we showed that all non-diagonal contributions add up to a diagonal contribution [Cao:2023ohj]



- How could this miracle happen?

Light-front wave function representation of D-term

In light-front dynamics, operators can be exactly localized in the transverse direction and then form factors are expressed as F.T. of one-body densities,

- Localizing the charge: Drell-Yan-West formula [Drell:1969km, West:1970av]

$$Q = \int \mathrm{d}^3x\, J^+ \quad \to \quad J^+_{\mathrm{LF}}(r_\perp) = \sum_i e_i \delta^2(r_\perp - r_{i\perp}) \quad \to \quad \mathcal{F}(r_\perp) = \Big\langle \sum_i e_i \delta^2(r_\perp - r_{i\perp}) \Big\rangle$$

- Localizing the momentum: Brodsky-Hwang-Ma-Schmidt formula [Brodsky:2008pf]

$$P^{+,\perp} = \int \mathrm{d}^3x\, T^{++,\perp} \quad \to \quad \mathcal{A}(r_\perp) = \Big\langle \sum_i x_i \delta^2(r_\perp - r_{i\perp}) \Big\rangle$$

- Localizing the light-front Hamiltonian: Cao-Li-Vary formula [Cao:2023ohj, Xu:2024hfx]

$$P^- = \int \mathrm{d}^3x\, T^{+-} \quad \to \quad \mathcal{T}^{+-}(r_\perp) = \Big\langle \sum_i \Big\{ \frac{-\frac{1}{4}\overleftrightarrow{\nabla}^2_{i\perp} + m_i^2 + \frac{1}{4}\nabla^2_\perp}{x_i} + V_i \Big\} \delta^2(r_\perp - r_{i\perp}) \Big\rangle$$

where, $V = M^2 - \sum_i \frac{-\nabla^2_{i\perp} + m_i^2}{x_i}$ is the potential energy, $\langle O \rangle \equiv \sum_n \int [\mathrm{d}x_i \mathrm{d}^2 r_{i\perp}]_n \widetilde{\psi}^*_n(\{x_i, \vec{r}_{i\perp}\}) O_n \widetilde{\psi}_n(\{x_i, \vec{r}_{i\perp}\})$.

Pion gravitational form factor D_π

[Li:2023izn]

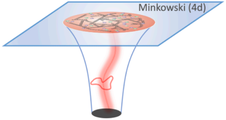
$$T^{+-}(Q^2) = (M_\pi^2 + \frac{1}{4}Q^2)A_\pi(Q^2) + \frac{1}{2}Q^2 D_\pi(Q^2)$$

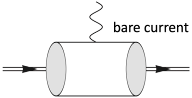
$$\Rightarrow D_\pi(Q^2) = \int z\mathrm{d}z|\varphi_\pi(z)|^2\Big\{-\frac{1}{4}z^2Q^2K_2(zQ) - 2K_0(zQ) - \frac{2U(z)}{Q^2}\Big[zQK_1(zQ) - \frac{z^2Q^2}{2}K_2(zQ)\Big]\Big\}$$

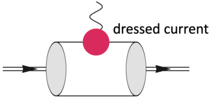
$$\Downarrow$$

$$D_\pi^{\text{dress}}(Q^2) = \int z\mathrm{d}z|\varphi_\pi(z)|^2\Big\{-\frac{1}{2}H(Q^2,z) - \frac{1}{2}S(Q^2,z) - \frac{2U(z)}{Q^2}\Big[V(Q^2,z) - H(Q^2,z)\Big]\Big\}$$

- Large Q^2 scaling consistent with pQCD prediction $D_\pi(Q^2) \sim 1/Q^2$ [Tong:2021ctu, Tong:2022zax]
- The dressed scalar current $S(Q^2,z) = \Gamma(2+\frac{Q^2}{4\kappa^2})U(\frac{Q^2}{4\kappa^2}, -1, \kappa^2z^2)$ is the bulk-to-boundary propagator of a scalar field $\mathrm{tr}G^2$ ($\Delta = 4$), i.e. scalar glueball [Colangelo:2007pt, Forkel:2007ru, Chen:2015zhh]





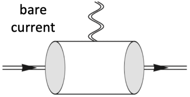


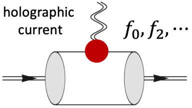
$D_\pi(Q^2)$ with holographic currents

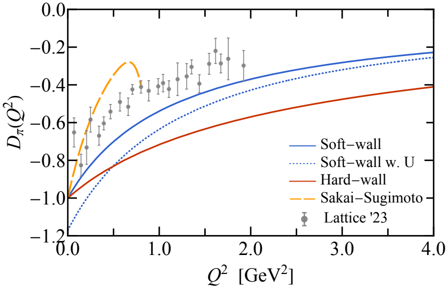
[Li:2023izn]

$$D_\pi(Q^2) = \int z\mathrm{d}z|\varphi_\pi(z)|^2\Big\{-\frac{1}{2}H(Q^2,z) - \frac{1}{2}S(Q^2,z) - \cancel{\frac{2U(z)}{Q^2}\big[V(Q^2,z) - H(Q^2,z)\big]}\Big\}$$

- In hard-wall model, the U-term vanishes and $D_\pi^{\mathrm{HW}}(0) = -1$, consistent with χPT
- In soft-wall model, the U-term leads to a small deviation from -1: $D_\pi^{\mathrm{SW}}(0) = -1.17$ due to the imperfect χSB, and should be neglected
- Glueball dominance: scalar glueball $D_\pi^S(0) = -0.5$, tensor meson $D_\pi^T(0) = -0.5$







Parameters:
soft-wall: $\kappa = \frac{1}{2}m_\rho = 0.388$ GeV;
hard-wall: $\Lambda_{\mathrm{QCD}} = z_m^{-1} = 0.322$ GeV

Soft-wall: r_D = 0.43 fm
Hard-wall: r_D = 0.32 fm
Lattice '23: r_D = 0.61(7) fm
[Hackett:2023nkr, ($M_\pi = 170\,\mathrm{MeV}$)]

Nucleon gravitational form factor A_N

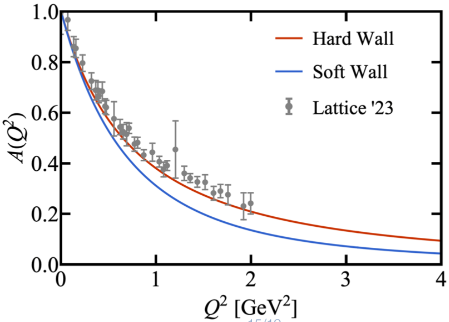
- Brodsky-Hwang-Ma-Schmidt formula: [Brodsky:2008pf]

$$A_N(Q^2) = \int \zeta_\perp \mathrm{d}\zeta_\perp \frac{1}{2}\Big\{\big|\varphi_+(\zeta_\perp)\big|^2 + \big|\varphi_-(\zeta_\perp)\big|^2\Big\} \frac{1}{2}\zeta_\perp^2 Q^2 K_2(\zeta_\perp Q) + \text{higher Fock sector contributions}$$

- Gravitational coupling in AdS_5: $g_{NM} \to g_{NM} + \delta g_{NM}$ [Abidin:2008hn]

$$A_N(Q^2) = \int z\mathrm{d}z \frac{1}{2}\Big\{\big|\varphi_+(z)\big|^2 + \big|\varphi_-(z)\big|^2\Big\} H(Q^2, z)$$

here, recall $H(Q^2, z)$ is the bulk-to-boundary propagator of the 5D gravitational field.



Parameters (simultaneously fixed to the proton mass M_p = 0.98 GeV and rho mass m_ρ = 0.70 GeV):
soft-wall: κ = 0.35 GeV;
hard-wall: $\Lambda_{\text{QCD}} = z_m^{-1}$ = 0.245 GeV;

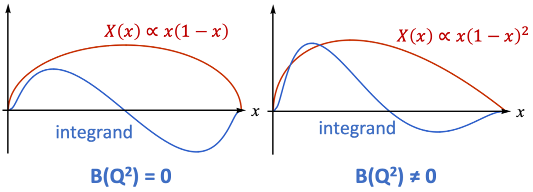
soft-wall: r_A = 0.562 fm; hard-wall: r_A = 0.562 fm
Lattice: r_A = 0.51(1) fm
[Broniowski:2025ctl, Cao:2025dkv]

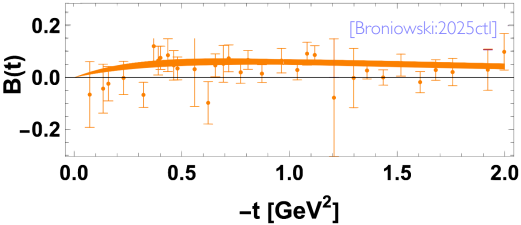
Nucleon gravitational form factor B_N

- $B(0) = 0$ regardless of the choice of $\varphi_\pm$, consistent with the Equivalence Principle [Teryaev:1999su]
- In AdS$_5$, $B(Q^2) = 0$ for all Q^2 due to the absence of the tensor coupling to a bulk Dirac fermion
- In LFQCD, Brodsky-Hwang-Ma-Schmidt formula: [Brodsky:2008pf]

$$\frac{q^1 + iq^2}{2M_N} B(Q^2) = \int \mathrm{d}x \int \mathrm{d}^2\zeta_\perp \varphi_-^*(\zeta_\perp)\varphi_+(\zeta_\perp)\Big\{x e^{-i\sqrt{\frac{1-x}{x}}\vec{q}_\perp\cdot\vec{\zeta}_\perp} + (1-x)e^{+i\sqrt{\frac{x}{1-x}}\vec{q}_\perp\cdot\vec{\zeta}_\perp}\Big\} + \cdots$$

  - $B(Q^2) = 0$ also for all $Q^2 \neq 0$ due to a symmetry of $\varphi_\pm(\zeta_\perp)$ under swapping $x \leftrightarrow 1-x$
  - $\Rightarrow$ For a general longitudinal profile $\psi(x, r_\perp) = X(x)\varphi(\zeta_\perp)$, $B(Q^2)$ is non-vanishing, but $B(Q^2)$ is still suppressed for longitudinal S-waves

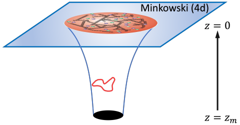


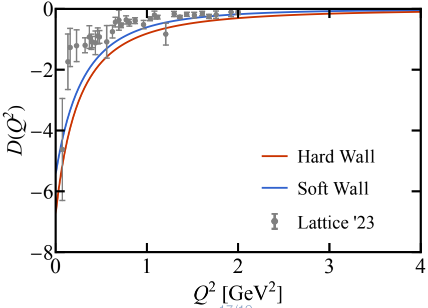


Nucleon gravitational form factor D_N

$$D_N(Q^2) = \int z\mathrm{d}z\Big[|\varphi_+(z)|^2 + |\varphi_-(z)|^2\Big]\Big\{-\frac{1}{4}S(Q^2,z) + \frac{4M^2}{Q^2}[V(Q^2,z) - H(Q^2,z)]\Big\}$$
$$-\int \mathrm{d}^2z\Big[U_+(z)|\varphi_+(z)|^2 + U_-(z)|\varphi_-(z)|^2\Big]\frac{2}{Q^2}\Big\{V(Q^2,z) - H(Q^2,z)\Big\}$$

- Large Q^2 scaling $D_N(Q^2) \sim 1/Q^4$ in contrast to the pQCD prediction
- Predictions to the D-term: $D_N^{\mathrm{SW}}(0) = -5.4$, $D_N^{\mathrm{HW}}(0) = -6.7$, compared to the MIT Lattice result $D = -3.87(97)$





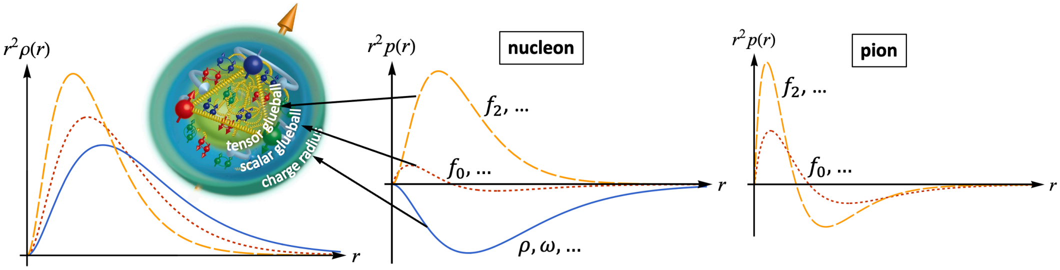
Parameters (simultaneously fixed to the proton mass M_p = 0.98 GeV and rho mass m_ρ = 0.70 GeV):
soft-wall: κ = 0.35 GeV;
hard-wall: $\Lambda_{\mathrm{QCD}} = z_m^{-1}$ = 0.245 GeV;

soft-wall: r_D = 0.882 fm; hard-wall: r_D = 0.953 fm

Physical contributions at low Q^2

- Vector current, e.g. ρ, ω, is attractive and long-ranged
- Tensor current, e.g. f_2, is repulsive and short-ranged
- Scalar current, e.g. f_0, is in mechanical equilibrium, and mid-ranged
- For the pion, both the scalar glueball and tensor glueball are self-balanced
- Large-Q^2 scaling is related to the parton distributions through Brodsky-Farrar parton counting rule

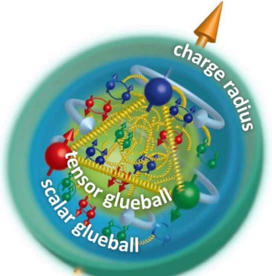
[Liu:2019vsn, deTeramond:2021lxc]



Summary

- The gravitational form factors emerge as a vital observable to unravel the internal structure of hadrons
- Holographic QCD provides a unique non-perturbative access to these observables with convincing physical pictures
- By exploiting the remarkable correspondence between light-front QCD and AdS/QCD, we computed the GFFs of the pion and the nucleon, and compared the results with recent Lattice simulations

Thank you!



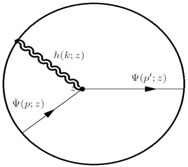

I think we’re gonna need
Backup

GFFs in AdS/QCD in 5D

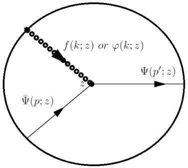
- Access to the tensor GFF $A(Q^2)$: graviton-hadron scattering in AdS$_5$ [Abidin:2008hn, Abidin:2008ku, Abidin:2009hr]
- NO access to GFF $D(Q^2)$, since graviton only couples to the traceless part of $T^{\mu\nu}$
- Adding the scalar gravitaton allows the access to GFF $D(q^2)$; however, GFF $D(q^2)$ vanishes at large N_c due to the degeneracy between the scalar and tensor glueballs [Mamo:2019mka, Mamo:2021krl, Mamo:2022eui]

$$D_N(q^2) = \frac{4M_N^2}{3q^2}\left[A(q^2) - \Theta(q^2)\right]$$

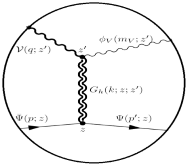
  - Fit scalar and tensor glueballs separately and obtained $A_N(0) = 0.43$ and $D_N(0) = -1.275$
  - GFFs interpreted as the gluon contributions, more relevant for the near-threshold VM production



tensor graviton coupling



scalar graviton coupling

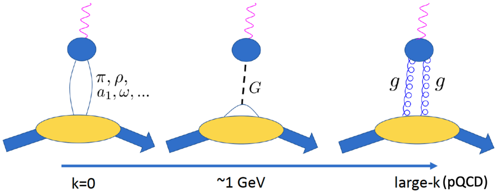


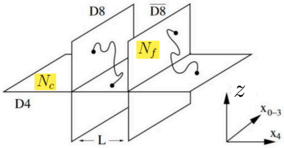
Near threshold VM photo-production

GFFs in top-down holographic QCD

Sakai-Sugimoto (SS) model in 10D [Fujita:2022jus, Fujii:2024rqd]

- Key observation: the 4D part of the traceless $T^{\mu\nu}$ becomes traceful
- Obtained $D_\pi(0) = -1$ consistent with χEFT
- Large q^2 scaling different from pQCD prediction
- $D_N(0) = -0.140(22)$ resulting from cancellation between $U(1)$ and $SU(2)$ fields
- New extraction using non-perturbative soliton solution gives $D_N(0) = -2.05$ [Sugimoto:2025btn]





Holographic light-front QCD

- The D-term involves non-minimal coupling terms in gravitational EFT, [Donoghue:1994dn]

$$S_D = -\frac{D}{4}\int \mathrm{d}^d x\,\sqrt{-g}R\phi^2$$

  - Need constraints from both the QCD side and the gravity side
- Light-front holography: correspondence between semi-classical LFQCD and AdS/QCD in 5D
  - HLFQCD allows us to impose constraints from both the QCD and the gravity sides
  - Further insights: super-conformal algebra, Veneziano amplitudes, parton counting rules and GPD sum rules [Review: Brodsky:2014yha]