THE INFLUENCE OF WILSON LINES ON HOLOGRAPHIC HEAVY QUARK POTENTIAL AND EXCITATIONS

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References: MF-He-Sun, Phys. Rev. D102, 126019(2020), MF-He-Sun-Zhang JHEP01(2024)079, Chen-Chen-MF-He-Zhang, arXiv: 2502.16215 [hep-th]

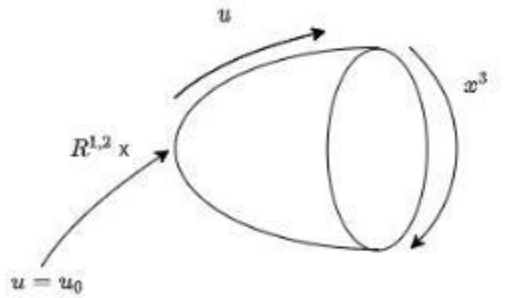
A bottom-up model for color superconductivity

Basu-Nogueira-Rozali-Stang-Raamsdonk ``11

- 6 dimensional gravity dual has an extra scale

- AdS solitons have an IR scale originating from the radius of the compactified direction
  - ⇒ A confined phase with a mass gap
  - ⇒ The emergence of a discrete spectrum of glueball states
  - ⇒ Negative energy and being stable against perturbations

Csaki, Ooguri, Oz, Terning ``98, Horowitz-Myers ``98



- For AdS_7 soliton, dual theory can be seen as the high-temperature limit of 5d SYM theory, essentially representing 4d pure Yang-Mills theory at long distances

Witten ``98

Motivations of this paper

- The mass of operators such as heavy quarkonium of QFT with Wilson lines
  - Wilson lines (background gauge potential) can shift the mass of charged particles

J. Polchinski

- Gauge potential can change Casimir energy (twisted boundary condition)

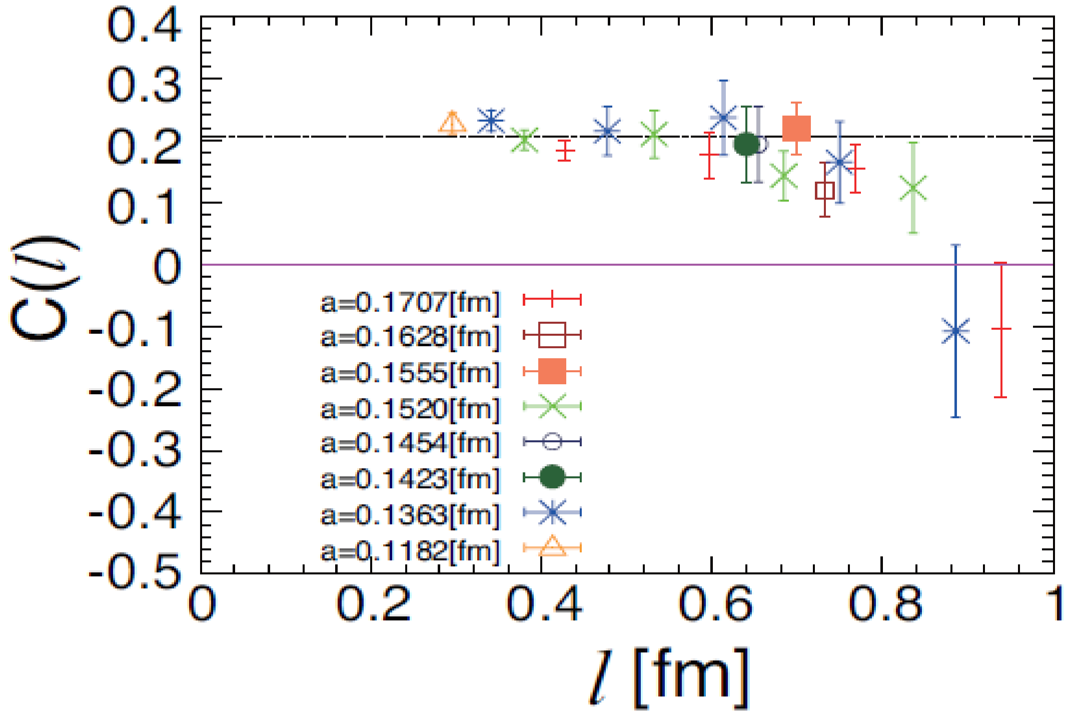
cf. imaginary chemical potential in QCD

  - The twisting parameter changes degrees of freedom

- DOF from EE (e.g. the coefficient of the A-type anomaly for a spherical entangling surface in CFT, *Solodukhin `08*)
  - Renormalized EE with a spherical entangling *Liu-Mezei `12*
  - The entropic C function from the EE with the striped subsystem

Nishioka-Takayanagi `06

Generalized entropic C-function of SU(3) Yang-Mills theory on the lattice *Itou-Nagata-Nakagawa-Nakamura-Zakharov `15*



The size of the subregion

Entropic C function captures DOF at energy $E{\sim}1/l$

$$C(l) = \frac{l^{d-1}}{V}\frac{dS}{dl}$$

(V: the volume S: entanglement entropy)

The black line: C=0.206

Decrease in the middle l=0.88 fm

Agreement with the critical temperature $T_c^{-1} = 0.714\text{fm}$ ($T_c = 280$ MeV) and the Lambda scale Λ_{MS}^{-1} ~0.8 fm

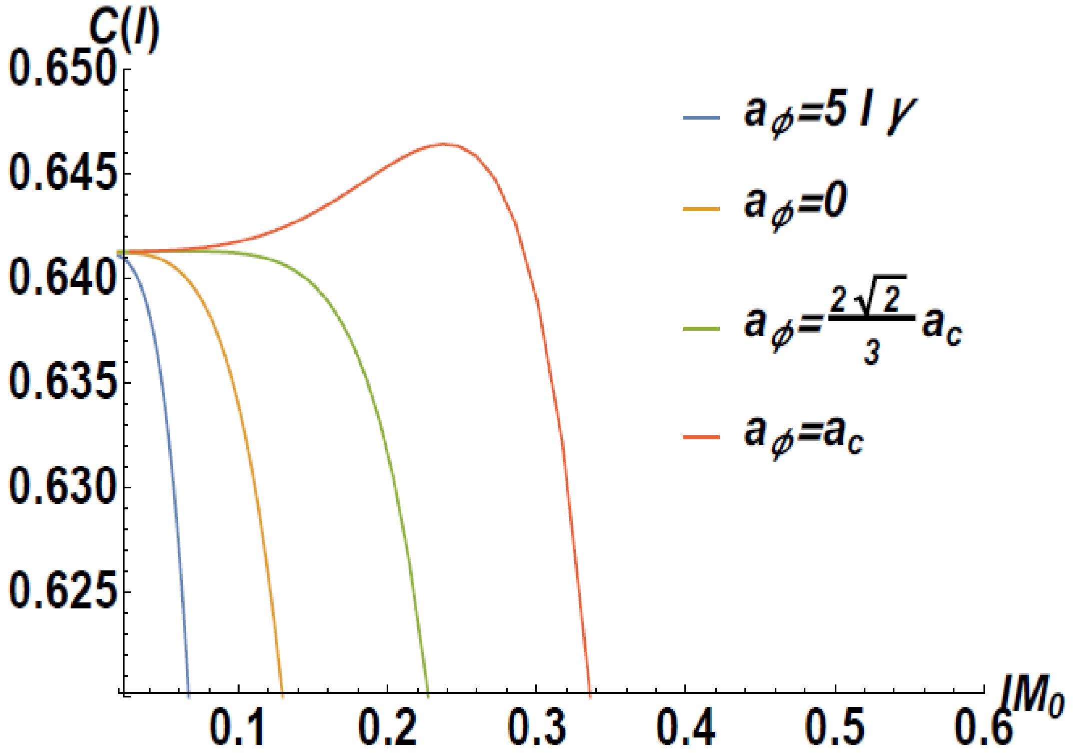
Entropic C function for QFT with Wilson lines

MF-He-Sun `20

- Entropic C function can capture DOF along a circle $\phi \sim \phi + {}^{1}/_{M_0}$ and Wilson lines A_ϕ
- $a_\Phi << a_c$: C decreases
  like *2d* entropic c function
  Nishioka-Takayanagi `06
- $a_\Phi \sim M_0$: C increases
  until the middle ($l \sim 1/M_0$)

⇒ It implies that Wilson lines make particles light

Massive modes decouple others soon



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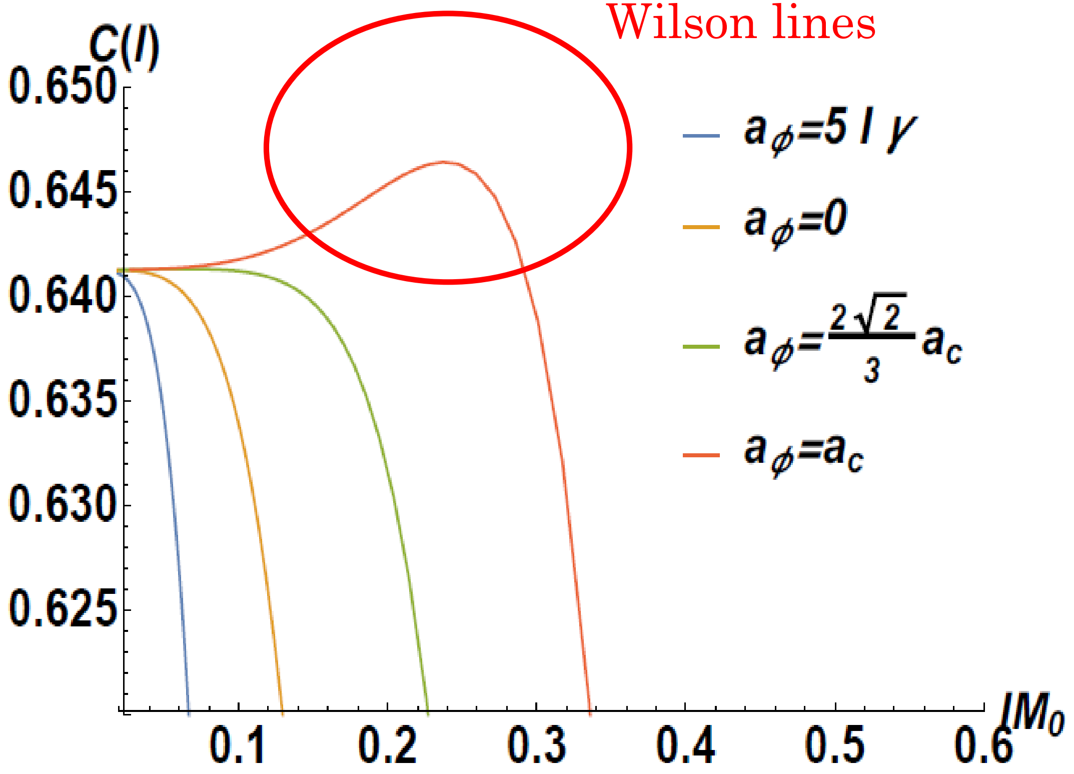
Particles with Wilson lines



GRAVITY DUAL: SPACETIME WITH A BACKGROUND GAUGE FIELD

The AdS soliton with a background gauge field

- The double Wick rotation of the AdS Reissner Nordstrom black hole with imaginary chemical potential

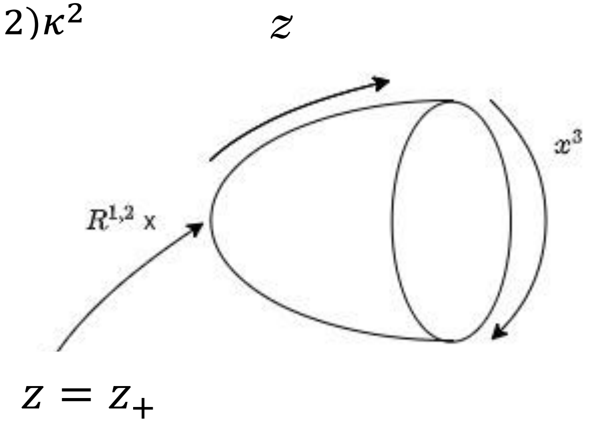
- The metric of the AdS soliton with a background gauge field

$$ds^2_{d+1} = \frac{L^2}{z^2}\left(\frac{dz^2}{f_d(z)} + f_d(z)d\phi^2 - dt^2 + dR^2 + R^2 d\Omega_{d-3}\right) \qquad f_d(z) = 1 - \left(1 + \frac{\epsilon_1 z_+^2 a_\phi^2}{\gamma^2}\right)\left(\frac{z}{z_+}\right)^d + \frac{\epsilon_1 z_+^2 a_\phi^2}{\gamma^2}\left(\frac{z}{z_+}\right)^{2(d-1)},$$

a_ϕ a constant gauge field , $A_\phi = a_\phi\left(1 - \left(\frac{z}{z_+}\right)^{d-2}\right)$, and $\gamma^2 = \frac{(d-1)g_e^2 L^2}{(d-2)\kappa^2}$

- The Kaluza-Klein mass $M_0 = \frac{1}{4\pi z_+}\left(d - \frac{\epsilon_1 (d-2) z_+^2 a_\phi^2}{\gamma^2}\right) > 0$

- Dual 4d gauge theory corresponds to the high-temperature limit of a 5d gauge theory at an imaginary chemical potential and at long distances dual to the Reissner-Nordstr¨om AdS black hole



Total energy of spacetime

- $M = \langle T_{00} \rangle V_{d-2}/M_0$

$$M = \frac{1}{\kappa^2}\int d^{d-1}x N\sqrt{\sigma}(K - K_0) = -\frac{V_{d-2}}{M_0}\frac{R^{d-1}}{2\kappa^2 z_+^d}\left(1 - z_+^2 a_\phi^2\right)$$

- The boundary energy changes the sign when we change Wilson lines a_φ

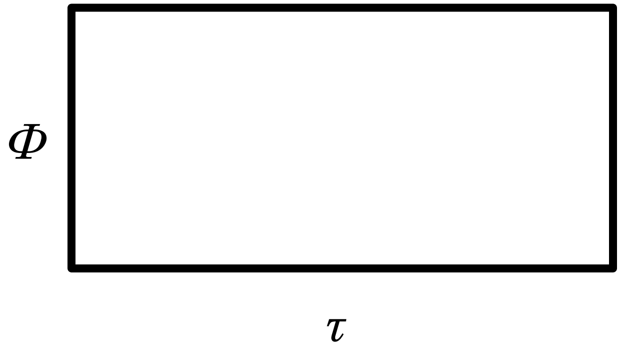
$$\begin{cases} M < 0 & a_\phi < \dfrac{2\pi M_0}{d-1} \\ M > 0 & a_\phi > \dfrac{2\pi M_0}{d-1}. \end{cases}$$

For $a_\varphi = 0$, it realizes Casimir energy of 4d SYM theory.
Casimir energy is different among periodic and anti-periodic b.c.

Heavy quark potential (x=const)

- Namb-Goto action with the static gauge

$$E = \frac{S}{\tau}, S = S_{NG} = \frac{1}{2\pi\alpha'}\int d\xi^0 d\xi^2 \sqrt{-\det g_{\alpha\beta}}$$



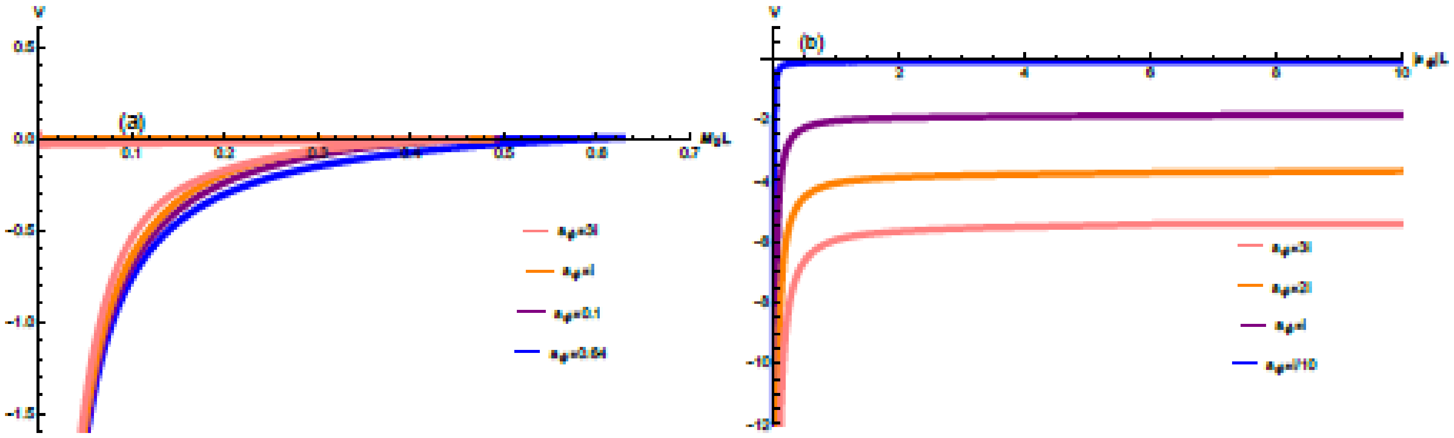
- The boundary condition

$$z(\pm\frac{L}{2}) = 0, \quad z(0) = z_0, \quad (\partial_\phi z)^2\Big|_{z=z_0} = 0.$$

- The regularized energy (the subtraction up to the soliton's tip)

$$E = \frac{R^2}{\pi\alpha'}\int_0^{z_0}\left(\frac{z_0^2}{z^2}\sqrt{\frac{1}{z_0^4 f(z) - z^4 f(z_0)}} - \frac{1}{z^2\sqrt{f(z)}}\right)dz - \frac{R^2}{\pi\alpha'}\int_{z_0}^{z_+}\frac{dz}{z^2\sqrt{f(z)}}.$$

Quark anti-quark potential



- Left: physics analogous to the dissociation
  - For large a_Φ , the potential deepens.
- Right: $M_0=0$.
  - Kaluza-Klein modes become massless and no dissociation

Solving the Schrodinger equation

- 5d quark anti-quark potential probing extra dimensions

$$\bar{V}(L) = -\gamma L^{-a} + \kappa L^{b} + v_0, \quad a, b > 0. \quad \bar{V}'(L) > 0, \quad \bar{V}''(L) \leq 0$$

- Binding energy in the leading order $E_{n,l} \sim \bar{V}(L_a) + \frac{1}{2} L_a \bar{V}'(L_a)$ (L_a: constant)

- The mass of bound state $m_{\bar{Q}Q} = 2m_Q + 2E_{n,l}$

- Fitting with a bottom-quark system
  ⇒ A bound state called bottomonium is formed

Ikhdair-Sever, `09, Kim-Lee-Park-Sin, `08

Mass of Bottomonium

- Bottomonium for $d=5$ and $M_0=0.145$ in units of GeV

a_ϕ	L_a	Meson mass	Binding energy
$3i$	0.77	9.1	-0.27
i	0.68	9.3	-0.15
0.1	0.66	9.0	-0.30
0.23	0.66	8.9	-0.36

- The mass of bottom quark is 4.80 GeV

  cf. $\Upsilon(1S)$ 9.46 GeV and $\eta_b(1S)$ 9.40 GeV

Spectrum of spin 0^{++} glueball-like operators for QCD_4

- The mass is in units of GeV

States	Lattice QCD	$a_\phi = 0,\ M_0 = 0.145$	$a_\phi = \pi M_0/2,\ M_0 = 0.218$
0^{++}	1.48-1.73	1.48	1.48
0^{++*}	2.67-2.84	2.43	2.46
0^{++**}	3.37	3.36	3.41
0^{++***}	3.99	4.27	4.35

- The fourth column: Kaluza-Klein mass is almost equal to the QCD critical temperature T_c=0.28 GeV and Lambda scale: Λ_{MS}=0.25 GeV

Discussion

- Holographic heavy quark potential is analyzed from holographic Wilson loops in the AdS soliton with gauge potential

- Case 1: physics analogous to the dissociation occurs
  - The mass of heavy quarkonium decreases with increase of the gauge potential

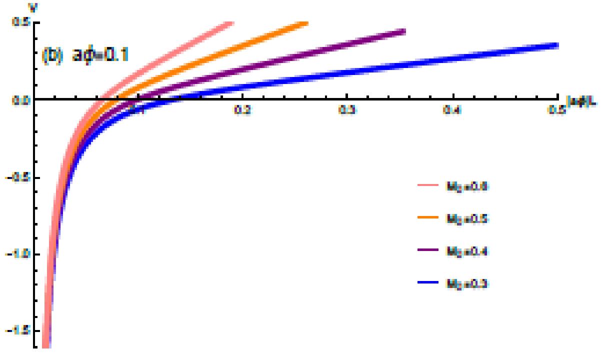
- Case 2: heavy quark potential shows the area law behavior
  - The mass of the excitation of QCD strings decreases with increase of the gauge potential

- The mass of 0++ glueball-like operator decreases with increase of the gauge potential

Phase transitions

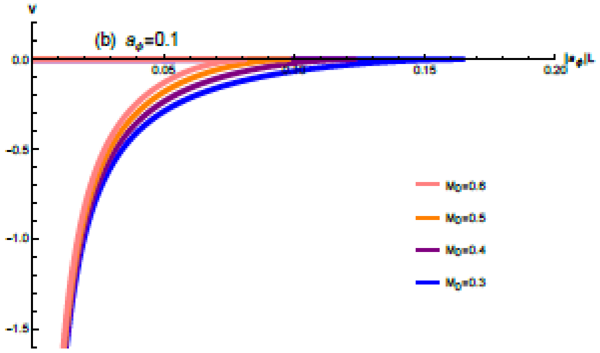
- AdS soliton with a gauge potential for Small $a_{\Phi} < a_{0}$:

Dissociation of quarks or an area law for 5d quarks



- AdS black hole for large $a_{\Phi} > a_{0}$:

Quark potential disappears and dissociation of quarks occurs



Thank you!

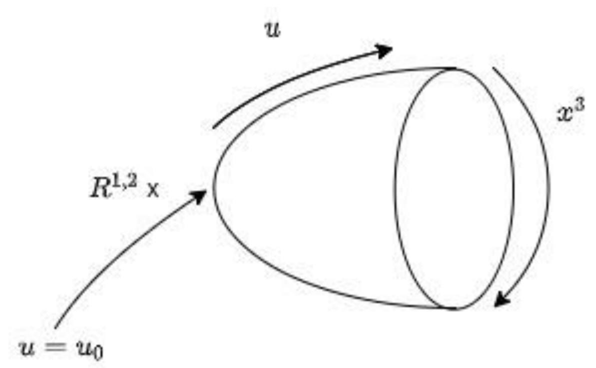
The AdS soliton

- The double Wick rotation of the AdS black hole

- It corresponds to the ground state of QFT with the anti-periodic boundary condition on fermions

$$ds^2 = \frac{L^2}{u^2}\left(-dt^2 + \frac{du^2}{f(u)} + f(u)d\varphi^2 + \sum dx^i dx^i\right),$$

where $f(u) = 1 - \left(\frac{u}{u_0}\right)^4$ and $x^3 = \varphi$



- The mass of the AdS soliton = negative energy

Spectrum of spin 0^{++} glueball-like operators for QCD

- Decrease with increase of energy M (also in other dimensions)

- $M_0 = \frac{1}{\pi}$

$$m^2 = 11.6, \quad 34.5, \quad 69.0, \quad 115, \ldots \qquad \text{for } a_\phi = 0,$$
$$m^2 = 3.60, \quad 11.1, \quad 22.4, \quad 37.3, \ldots \qquad \text{for } a_\phi = \frac{1}{\sqrt{2}},$$
$$m^2 = 19.8, \quad 58, \quad 115, \quad 192, \ldots \qquad \text{for } a_\phi = i.$$

- $M_0 = 0$ $\qquad m^2 = 0$

(the extremal case)

Glueball-like operators are massless

The relation to QCD with imaginary chemical potential

Ghoroku-Kashiwa-Nakano-Tachibana-Toyoda `20

- A remnant of Z_N center symmetry remains in *4d* QCD with imaginary chemical potential.
-
- Quarks in *4d* QCD satisfy the twisted boundary condition along the temporal circle.
  - The twist parameter=imaginary chemical potential μ_I

$$\phi(\vec{x},\beta) \sim e^{i\mu_I\beta}\phi(\vec{x},0)$$

- The periodicity of the partition function $Z[\mu_I] = \mathrm{Tr}\left(e^{-\beta H + i\beta\mu_I N_q}\right)$
  - First order Roberge-Weiss phase transition