Dynamics of chiral phase transition in the soft-wall AdS/QCD

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Based on: JHEP 07 (2025) 029, Phys.Rev.D 107, 086001

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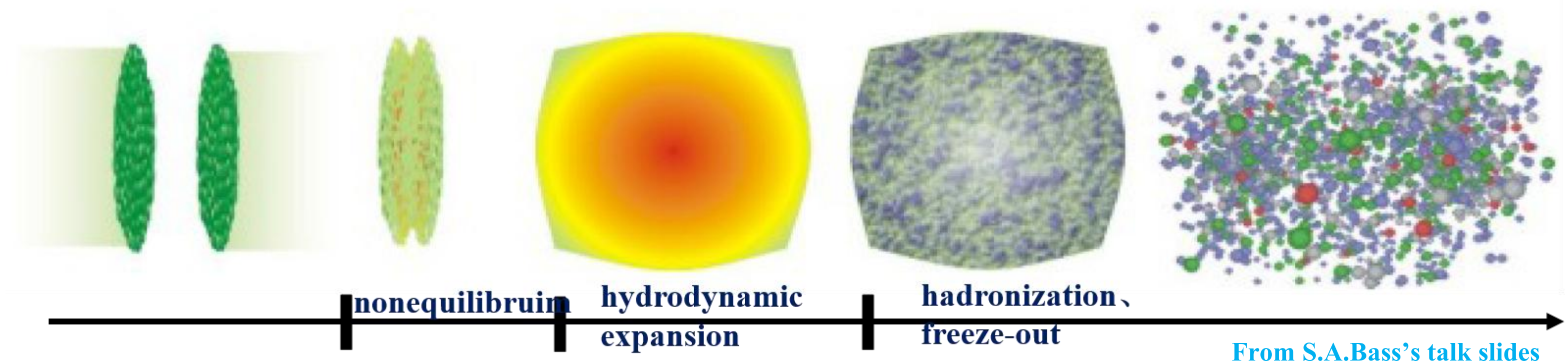
Holographic applications: from Quantum Realms to the Big Bang

Outlines

- Introduction
- Soft-wall AdS/QCD and chiral phase transition
- Real-time dynamics in the soft-wall model
- Summary

Nonequilibrium stage in HIC



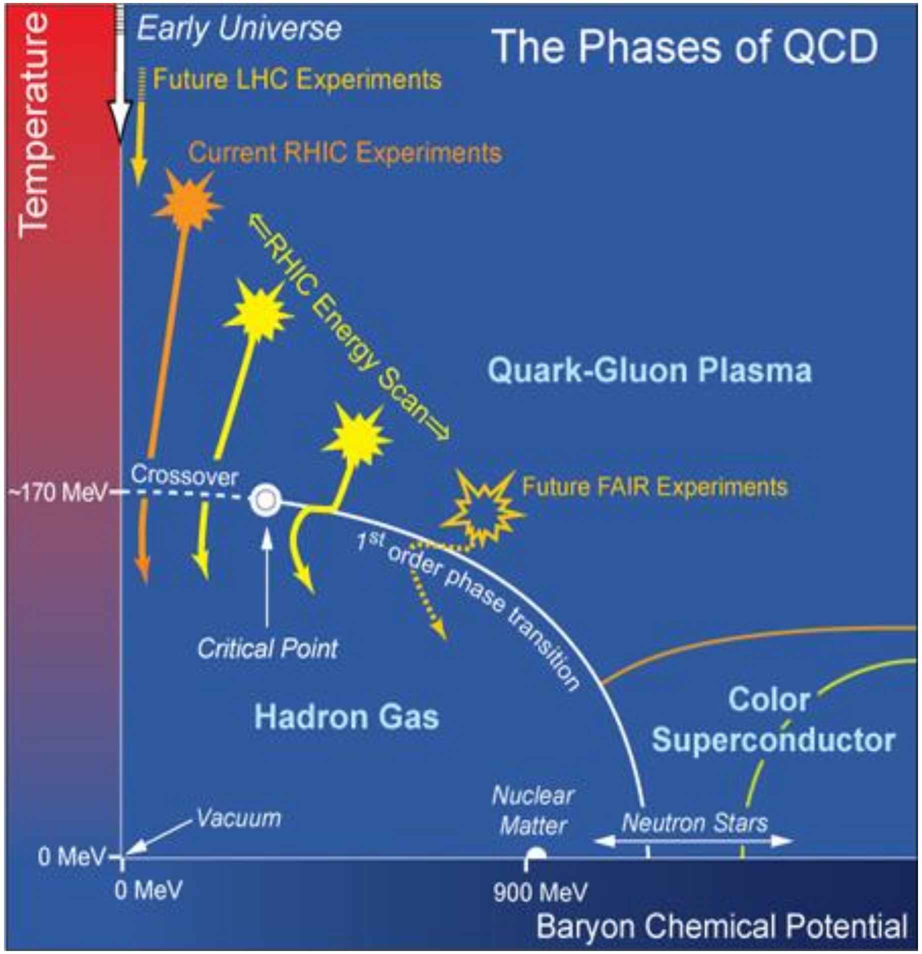
- fluid dynamics: initial conditions, EOS, transport coefficients
  local equilibrium: τ~0.1-1fm/c, fast thermalization?

- transport models: hadronic cascade model, UrQMD models…

- nonequilibrium stage:
  Kinetic theory: $f(x, p)$, relativistic Boltzman equation
  Color glasma: gluon saturation
  weak coupling picture

η/s~0.2, indicating a strong coupled QGP in hydrodynamics.
At least, at the end of the non-equilibrium period.

Phase transitions



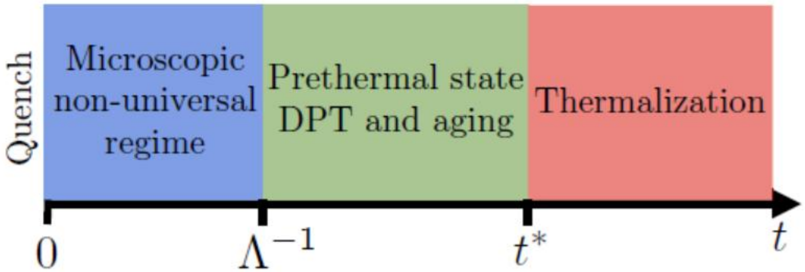
1st order region:
bubble dynamics, inhomogeneous effect (see also Jin's, Chen's and Schee's talks)

Near phase transition:
non-perturbative

Short-time dynamics

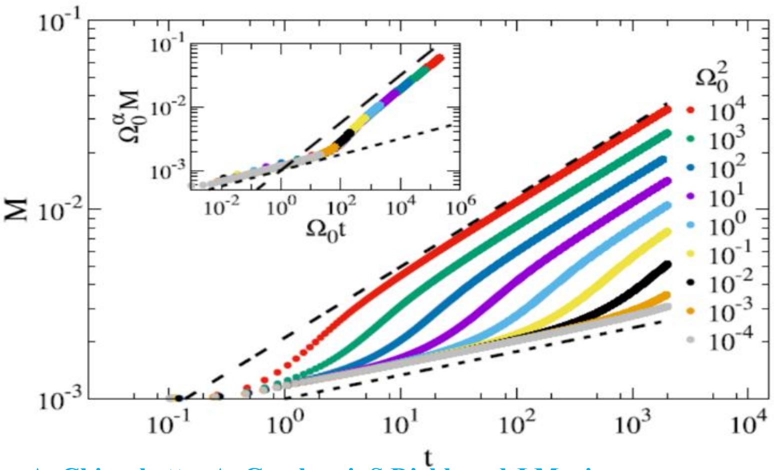
Originally, the term "prethermalization" was introduced by Berges, Borsányi, and Wetterich for matter under extreme conditions in a quasisteady state far from equilibrium.

J. Berges, Sz. Borsanyi, and C.Wetterich, Phys.Rev.Lett. 93 (2004) 142002



A. Chiocchetta, M.Tavora, A.Gambassi and A.Mitra, Phys.Rev.B91, 220302(2015)

Now widely used in many fields:
cold atom (theoretical + experiments)
condensed matter physics,
statistical physics



A. Chiocchetta, A. Gambassi, S.Diehl, and J.Marino, Phys.Rev.Lett. 118 (2017) 13, 135701

Possible explanations:
generalized Gibbs ensembles, Non-thermal fixed point (see Heller's talk)…

Near critical point: strong correlation

Information about nonequilibrium physics from strong coupling point of view is necessary!

Holographic method: strong coupling picture, easier to deal with nonequilibrium physics.

Holographic method

Closed strings in AdS background

$$\frac{R^4}{l_s^4} = 4\pi g_s N_c >> 1$$

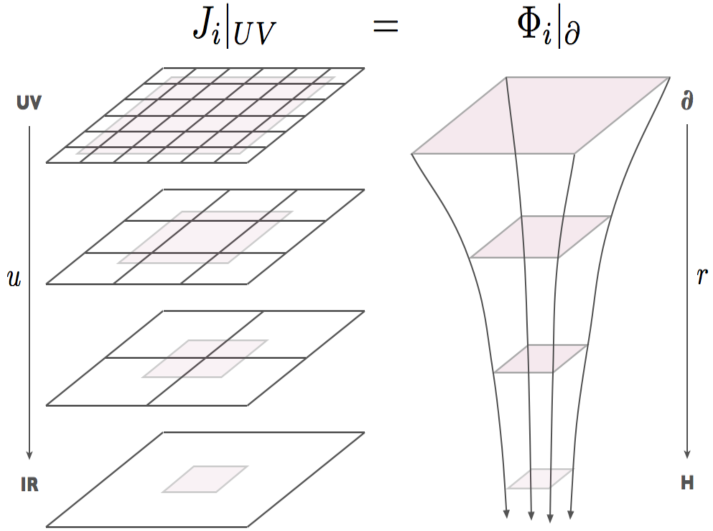
J.Maldacena, Adv.Theor.Math.Phys.2:231-252,1998

SYM in Minkovski Space Time

$$\lambda = g_{YM}^2 N = 4\pi g_s N >> 1$$

application to QCD: try to break the conformal symmetry
top-down (SS, Dp-Dq…)
bottom-up (EMD, IHQCD, VQCD, light-front HQCD, soft-wall model,…)

Holography: 5th dimension(r) plays as energy scale



Mapped the flow equation of FRG to the wave equation of HQCD:

F.Gao, M.Yamada, Phys.Rev.D 106.126003(2022)

$$\left[\partial_z^2 - \frac{(d-1-2J)}{z}\partial_z - \frac{(\mu R)^2}{z^2} - U_J(z) + \mathcal{M}^2\right]\Phi_J(z) = 0.$$

$$k = 1/z$$

$$k\partial_k\{k^{-\delta}k\partial_k\Gamma_k^{(n)}\} = k\partial_k(k^{-\delta}\beta_\Gamma^{(n)})$$

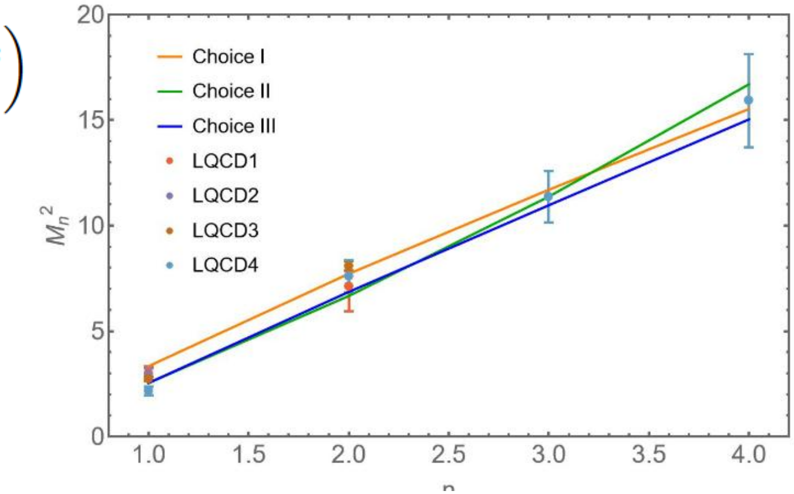
Gluedynamics

$$S_b^s = \frac{1}{16\pi G_5}\int d^5x\sqrt{-g^s}e^{-2\phi_s}\left[R_s + 4\partial_M\Phi_s\partial^M\Phi_s - V_s\left(\Phi_s\right)\right]$$

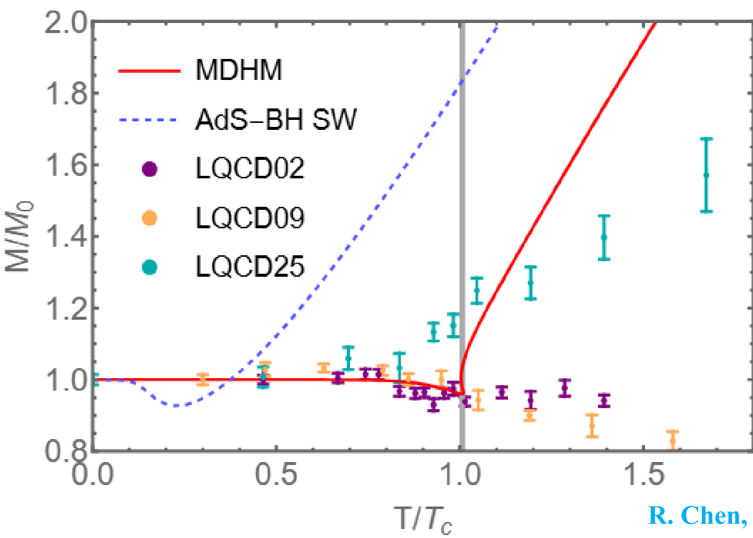
$$S_g^s = -c_g\int d^5x\sqrt{-g_s}h(\Phi(z))\left(\frac{1}{2}g_s^{MN}\partial_M S\partial_N S + \frac{1}{2}M_5^2 S^2\right)$$

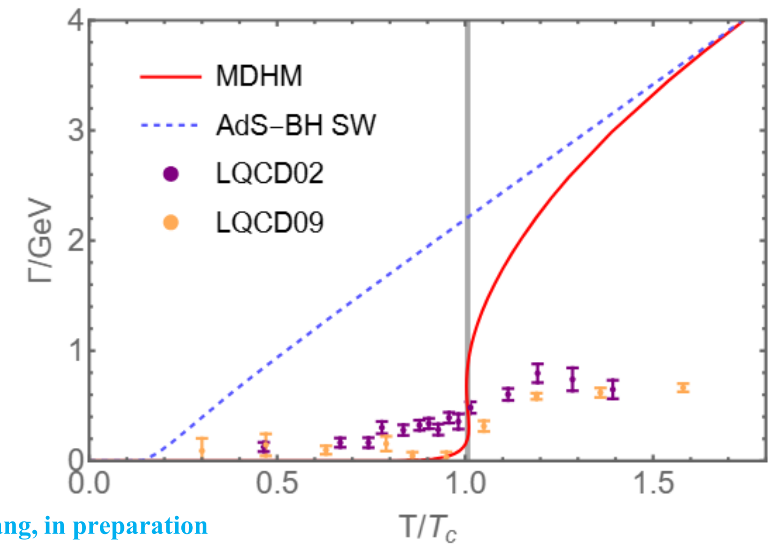
$$ds^2 = \frac{L^2 e^{2A(z)}}{z^2}\left[-f(z)dt^2 + \frac{dz^2}{f(z)} + d\vec{x}^2\right]$$

$$A(z) = d\ln(az^2+1) + d\ln(bz^4+1)$$



Thermodynamical quantities, see
[X. Chen et.al, Phys.Rev.D 109, L051902]
(See also Matti's lecture, Song's talk…)





R. Chen, DL, M.Huang, in preparation

Holographic thermalization

Hard to deal with in traditional field tools, but easy(relatively) in HQCD:

$T_{ab} \leftarrow\rightarrow g_{ab}$ (appear in its expanding coefficients)

Time evolution: Einstein equations

$T_{ab}(t) \rightarrow g_{ab}(t)$

- **Early studies: linearize Einstein equations**

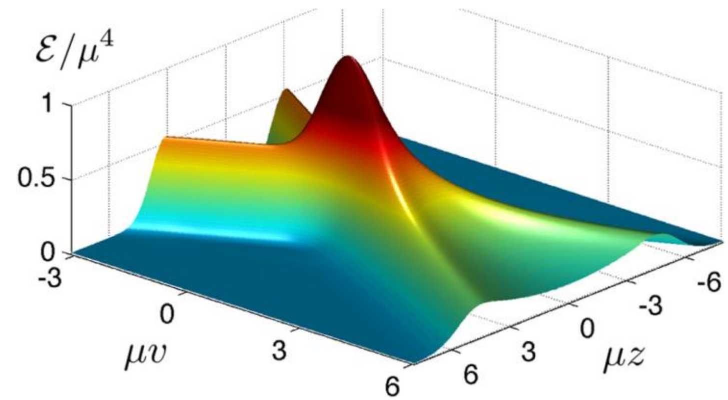
G.T.Horowitz et al., Phys.Rev.D62,024027(2000)
K.Murata et al. JHEP 0808,027(2008)
R.A.Janik et al., Phys.Rev.D73,045013(2006)

- **Direct mimicking: full dynamics in SYM(R+Λ)**

P.M.Chesler et al., Phys.Rev.Lett.106,021601(2011)
M.P.Heller et al., Phys.Rev.Lett.108,201602(2012)
M.P.Heller et al., Phys.Rev.D85,126002(2012)

- **Describing the experimental data:**

W.V.Schee et al., Phys.Rev.Lett.111,222302(2013)
W.V.Schee et al., Phys.Rev.C92,064907(2015)
K.Rajagopal et al., Phys.Rev.Lett.116,211603(2016)



P.M.Chesler et al., Phys.Rev.Lett.106,021601(2011)

Chiral dynamics ? Order parameter?

Soft-wall model

A.Karch, E.Katz, D.T.Son, M.A.Stephonov, Phys.Rev.D74:015005,2006

Promote 4D global chiral symmetry to 5D:

$$I = \int d^5x\, e^{-\Phi(z)} \sqrt{g} \left\{ -|DX|^2 + 3|X|^2 - \frac{1}{4g_5^2}(F_L^2 + F_R^2) \right\} \qquad SU_L(N_f) \times SU_R(N_f)$$

Field and Operator Correspondence:

4D: $\mathcal{O}(x)$	5D: $\phi(x,z)$	p	Δ	$(m_5)^2$
$\bar{q}_L \gamma^\mu t^a q_L$	$A^a_{L\mu}$	1	3	0
$\bar{q}_R \gamma^\mu t^a q_R$	$A^a_{R\mu}$	1	3	0
$\bar{q}^\alpha_R q^\beta_L$	$(2/z)X^{\alpha\beta}$	0	3	-3

$$A^a_{L,\mu}(z) = j^a_{L,\mu}(x) + J^a_{L,\mu}(x) z^2 + \ldots$$

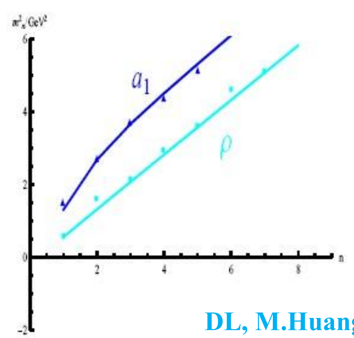
$$A^a_{R,\mu}(z) = j^a_{R,\mu}(x) + J^a_{R,\mu}(x) z^2 + \ldots$$

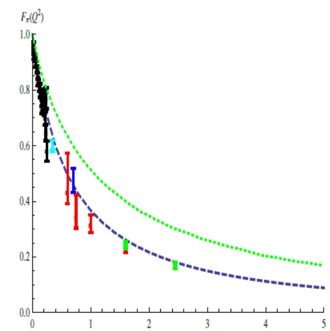
$$X(z) = m_q z + \langle \bar{q} q \rangle z^3 + \cdots$$

$$Z_{QCD} \sim Z_{Gravity} \sim e^{-S_{Gravity}} \sim e^{-\int c(z) j_i J^i + d(z) m\bar{q}q}$$

Extensions:

T. Gherghetta, J. I. Kapusta and T. M. Kelley,Phys. Rev. D 79 (2009) 076003
T. M. Kelley, S. P. Bartz and J. I. Kapusta, Phys. Rev. D 83 (2011) 016002
Y. -Q. Sui, Y. -L. Wu, Z. -F. Xie and Y. -B. Yang, Phys. Rev. D 81(2010) 014024
Y.Q.Sui, Y.L.Wu, Y.B.Yang, Phys.Rev. D83 (2011), 065030
DL, M.Huang, Q.S.Yan, Eur.Phys.J. C73 (2013) 2615
S.He, S.Y.Wu, Y.Yang and P.H.Yuan,JHEP 1304 (2013) 093
Y.Chen, M. Huang, Phys.Rev. D105 (2022), 026021
………..





DL, M.Huang, JHEP11(2013)088

AdS$_5$ metric+quadratic dilaton (κz^2, linear confinement)

A 5D chiral perturbation theory?

P. Colangelo et.al, JHEP 11 (2012) 012

$$S = S_{\rm YM} + S_{\rm CS}$$
$$S_{\rm YM} = -\int d^5x \,{\rm tr}\left[-f^2(z)\mathcal{F}_{z\mu}^2 + \frac{1}{2g^2(z)}\mathcal{F}_{\mu\nu}^2\right],$$
$$S_{\rm CS} = -\kappa \int {\rm tr}\left[\mathcal{A}\mathcal{F}^2 + \frac{i}{2}\mathcal{A}^3\mathcal{F} - \frac{1}{10}\mathcal{A}^5\right].$$

$$\xi_R(x) = \xi_L(x)^\dagger \equiv u(\pi) = \exp\{i\pi^a t^a/f_\pi\}$$
$$u_\mu(x) \equiv {\rm i}\left\{\xi_R^\dagger(x)(\partial_\mu - {\rm i}r_\mu)\xi_R(x) - \xi_L^\dagger(x)(\partial_\mu - {\rm i}\ell_\mu)\xi_L(x)\right\}$$
$$\Gamma_\mu(x) \equiv \frac{1}{2}\left\{\xi_R^\dagger(x)(\partial_\mu - {\rm i}r_\mu)\xi_R(x) + \xi_L^\dagger(x)(\partial_\mu - {\rm i}\ell_\mu)\xi_L(x)\right\}$$

$$\mathcal{A}_\mu(x,z) = i\Gamma_\mu(x) + \frac{u_\mu(x)}{2}\psi_0(z) + \sum_{n=1}^{\infty} v_\mu^n(x)\psi_{2n-1}(z) + \sum_{n=1}^{\infty} a_\mu^n(x)\psi_{2n}(z)$$

$$\begin{aligned} S_2[\pi] + S_4[\pi] = \int d^4x \Big[& \frac{f_\pi^2}{4} < u_\mu u^\mu > \\ & + L_1 < u_\mu u^\mu >^2 + L_2 < u_\mu u_\nu >< u^\mu u^\nu > + L_3 < u_\mu u^\mu u_\nu u_\nu > \\ & - iL_9 < f_{+\mu\nu} u^\mu u^\nu > + \frac{L_{10}}{4} < f_{+\mu\nu} f_+^{\mu\nu} - f_{-\mu\nu} f_-^{\mu\nu} > \\ & + \frac{H_1}{2} < f_{+\mu\nu} f_+^{\mu\nu} + f_{-\mu\nu} f_-^{\mu\nu} > \Big], \end{aligned}$$

$$f_\pi^2 = 4\left(\int_{-z_0}^{z_0} \frac{{\rm d}z}{f^2(z)}\right)^{-1},$$
$$L_1 = \frac{1}{2}L_2 = -\frac{1}{6}L_3 = \frac{1}{32}\int_{-z_0}^{z_0} \frac{(1-\psi_0^2)^2}{g^2(z)}{\rm d}z,$$
$$L_9 = -L_{10} = \frac{1}{4}\int_{-z_0}^{z_0} \frac{1-\psi_0^2}{g^2(z)}{\rm d}z,$$
$$H_1 = -\frac{1}{8}\int_{-z_0}^{z_0} \frac{1+\psi_0^2}{g^2(z)}{\rm d}z.$$

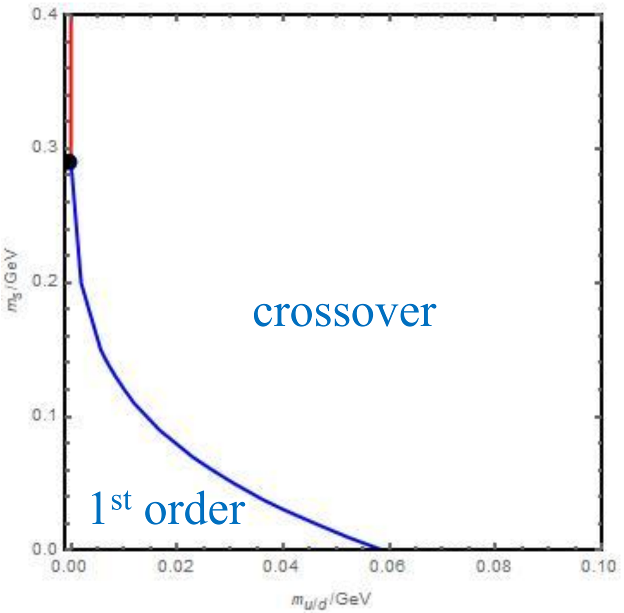
Map the soft-wall model to the 4D chiral perturbation theory

Chiral phase transition

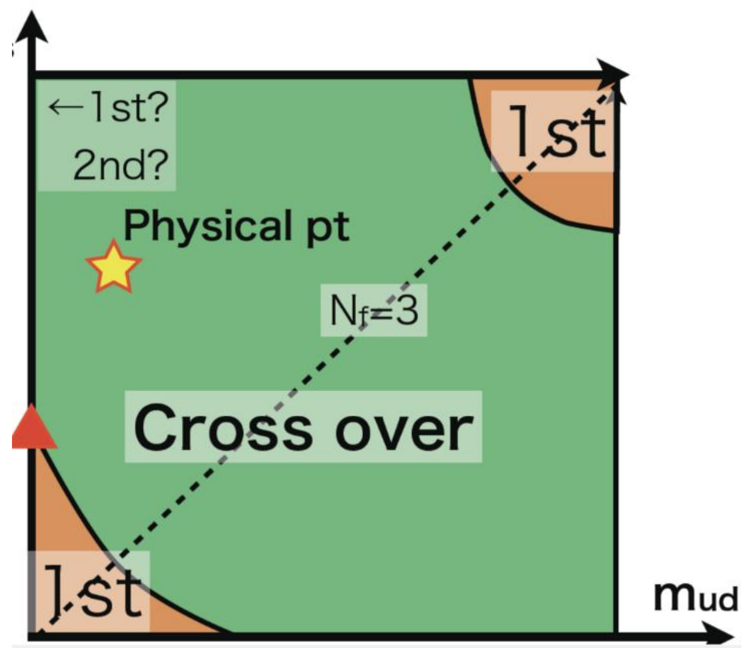
A simple extension to finite temperature:

AdS-Schwarzchild black holes + an additional scale + higher order scalar potential :
1. modify the dilaton configuration or the overall coupling (e.g. JHEP04(2016)046)
2. Modify the 5D mass of the scalar (e.g. Phys.Lett.B 762 (2016) 86-95)

The qualitative behaviors are the same



J.Chen, S.He, M.Huang, DL, JHEP01(2019)165

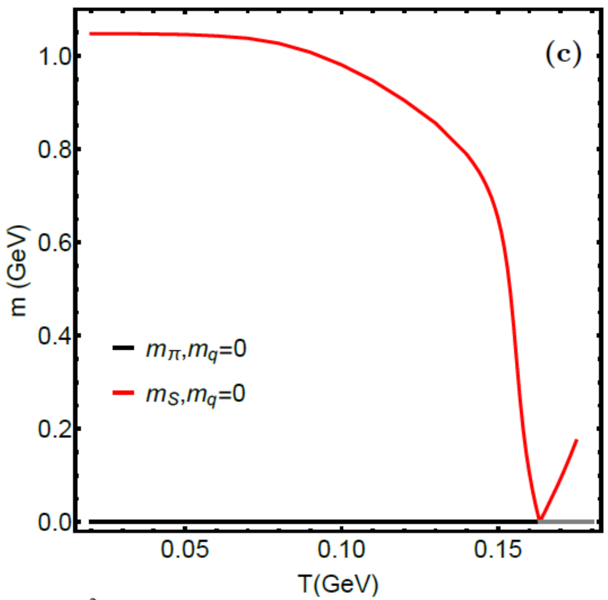


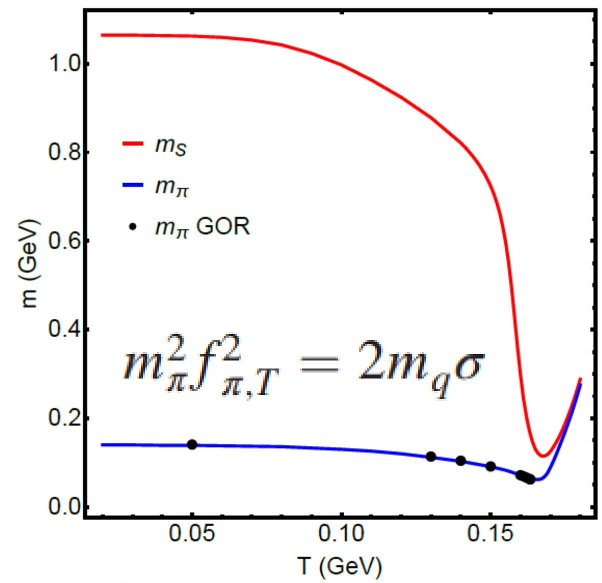
A.Tomiya et.al, EPJ Web of Conferences 175, 07041 (2018)

Mean field exponents and scaling functions (See also Zhen's talk)

Thermal excitations (σ, π)

X. Cao, H. Liu, DL, Phys.Rev.D 102 (2020) 126014





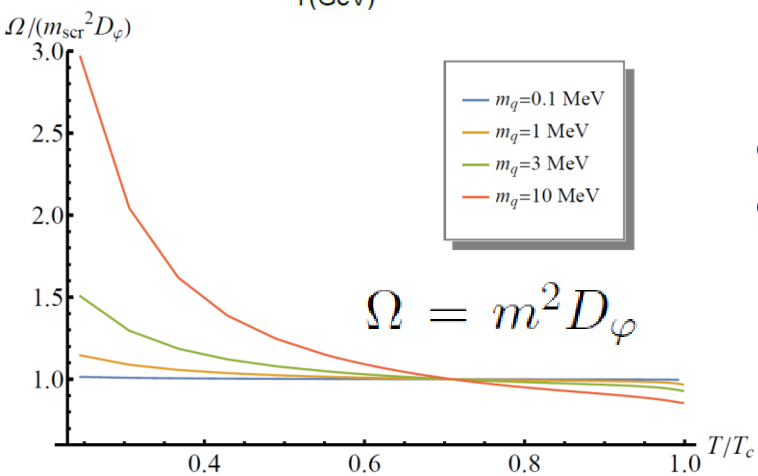
consistent with lattice simulations in Phys.Rev.D 92, 094510

consistent with scaling law near the critical point:

$$\langle\bar{\psi}\psi\rangle \sim t^{\beta} \qquad f \sim t^{\nu/2}$$

$$m_p^2 \equiv u^2 m^2 = -\frac{m_q\langle\bar{\psi}\psi\rangle}{\chi_{I5}} \sim m_q t^{\beta}$$

D.T.Son and M.A.Stephanov, Phys.Rev.Lett. 88 (2002) 202302



X. Cao, M.Baggioli, H. Liu, DL, JHEP 12 (2022) 113

The soft-wall model can describe hadronic data and chiral phase transition well !

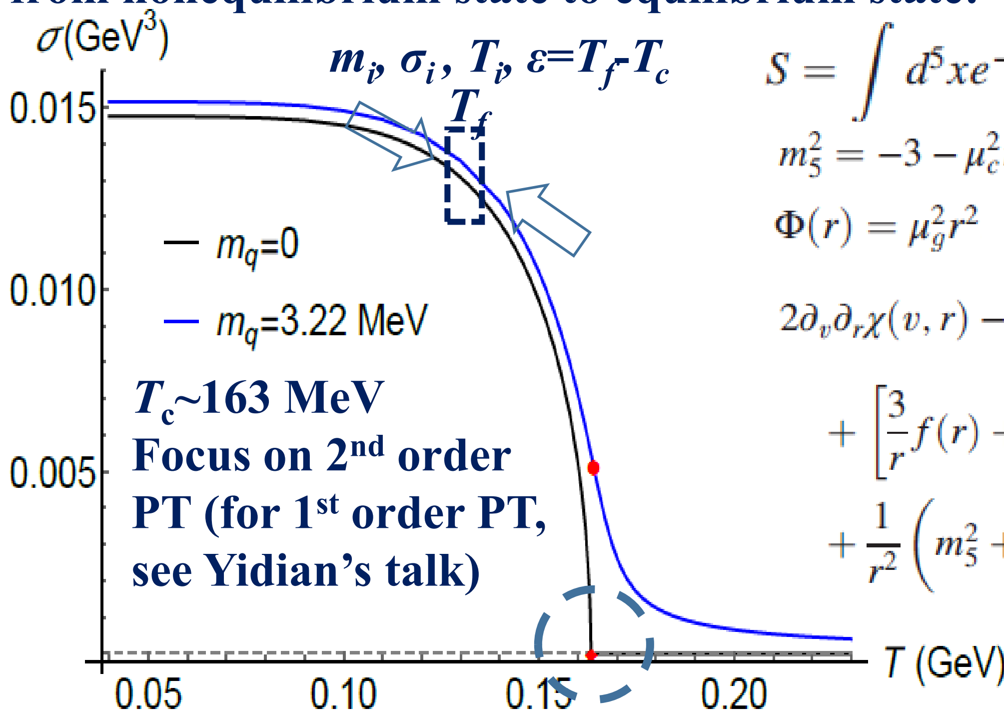
Quite similar (qualitatively) to the finite temperature chiral perturbative theory !

Real-time dynamics in SW

Extend to nonequilibrium case: $\chi(z) \Rightarrow \chi(t,z)$

Time dependent chiral condensate: $\chi(z\to 0) = m_q \gamma z + \ldots + \frac{\sigma(t)}{\gamma} z^3$

Evolve from nonequilibrium state to equilibrium state:



$$S = \int d^5x e^{-\Phi(r)} \sqrt{g} \mathrm{Tr}\{|DX|^2 - (m_5^2|X|^2 + \lambda|X|^4)\}$$

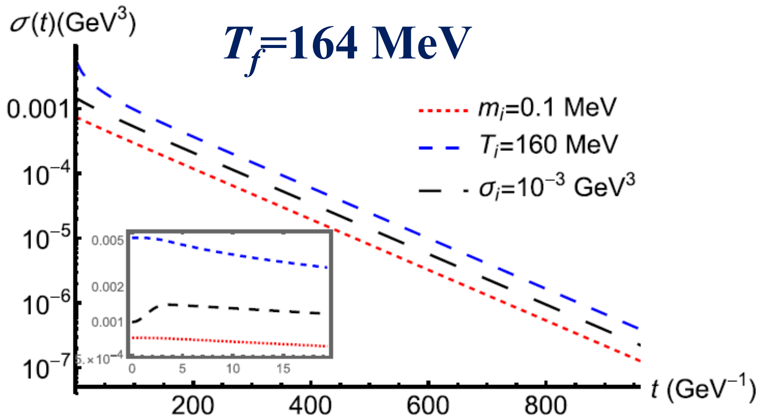
$m_5^2 = -3 - \mu_c^2 r^2$ $\quad \mu_c$=1.45GeV,

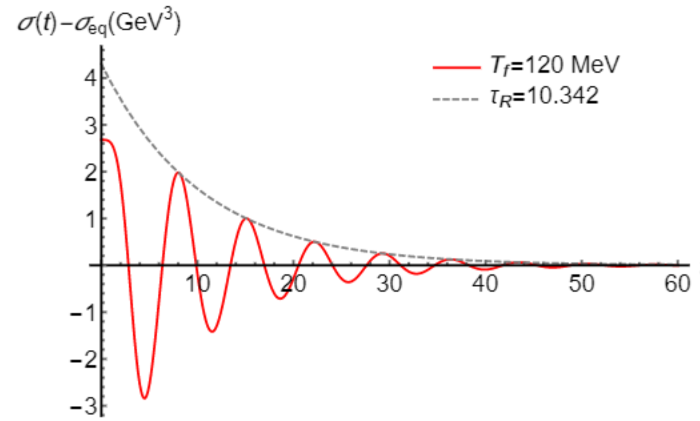
$\Phi(r) = \mu_g^2 r^2$ $\quad \mu_g$=0.44GeV, λ=80

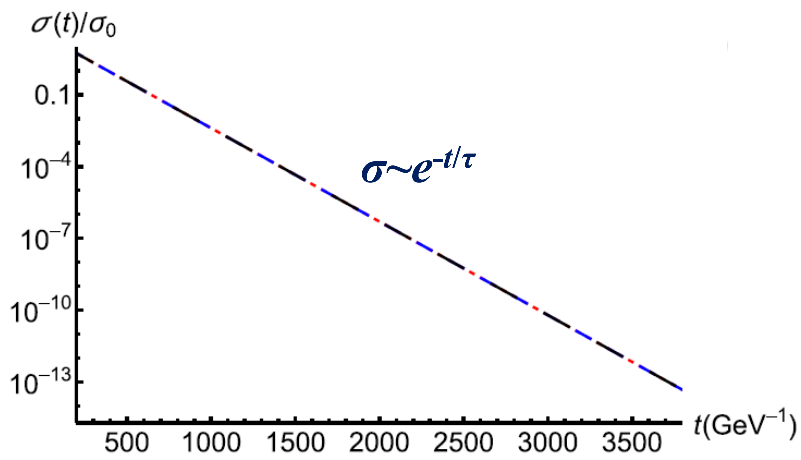
$$2\partial_v\partial_r\chi(v,r) - \left[\frac{3}{r} + \Phi'(r)\right]\partial_v\chi(v,r) - f(r)\partial_r^2\chi(v,r)$$
$$+\left[\frac{3}{r}f(r) + \Phi'(r)f(r) - f'(r)\right]\partial_r\chi(v,r)$$
$$+\frac{1}{r^2}\left(m_5^2 + \frac{\lambda}{2}\chi(v,r)^2\right)\chi(v,r) = 0.$$

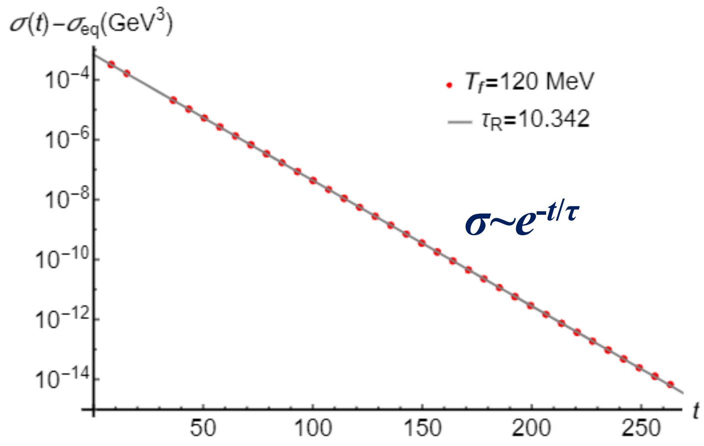
$$\sigma(\sigma_i, \epsilon, m_q, t) = \sigma_i^{1/x} f_{\sigma_i}(\epsilon\sigma_i^{-1/x\beta}, m_q\sigma_i^{-\delta/x}, t\sigma_i^{\nu z/x\beta})$$

Thermalization of σ

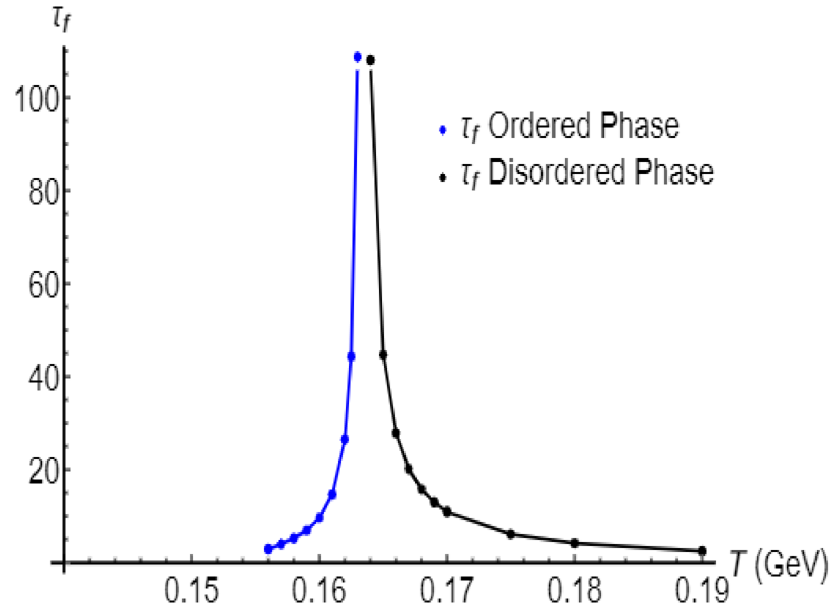


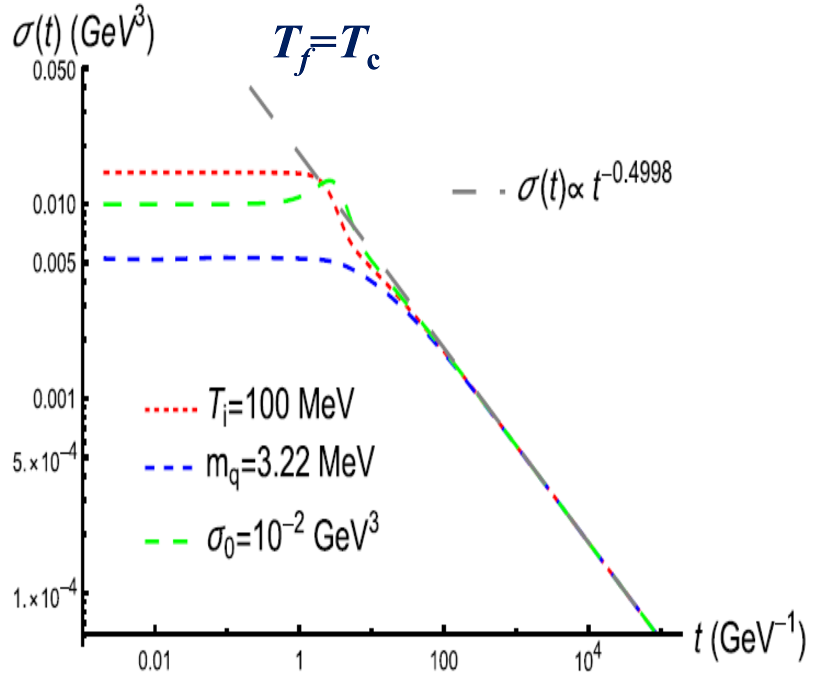






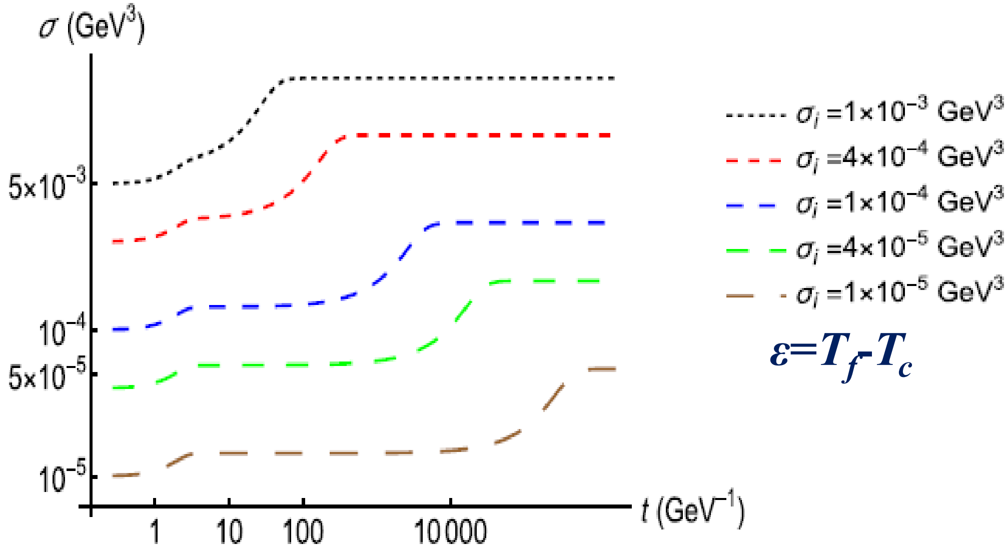
Critical slowing down





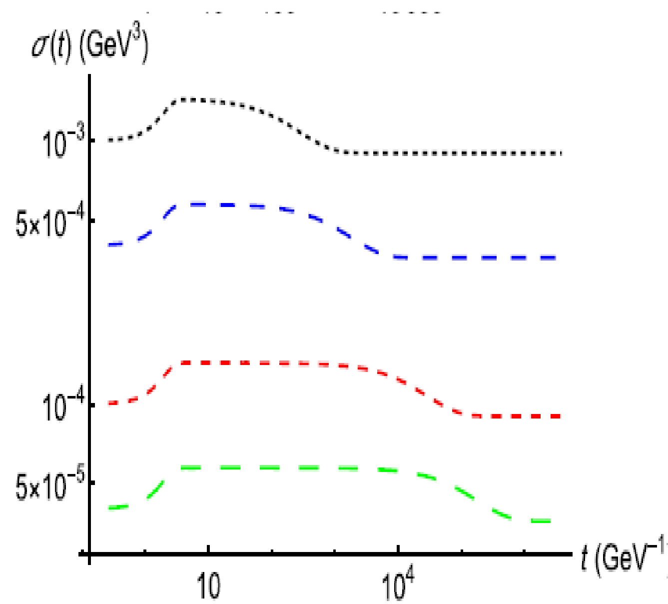
The short time dynamics

Short time:
compared to the thermalization time (intermediate)



$\varepsilon = T_f - T_c$

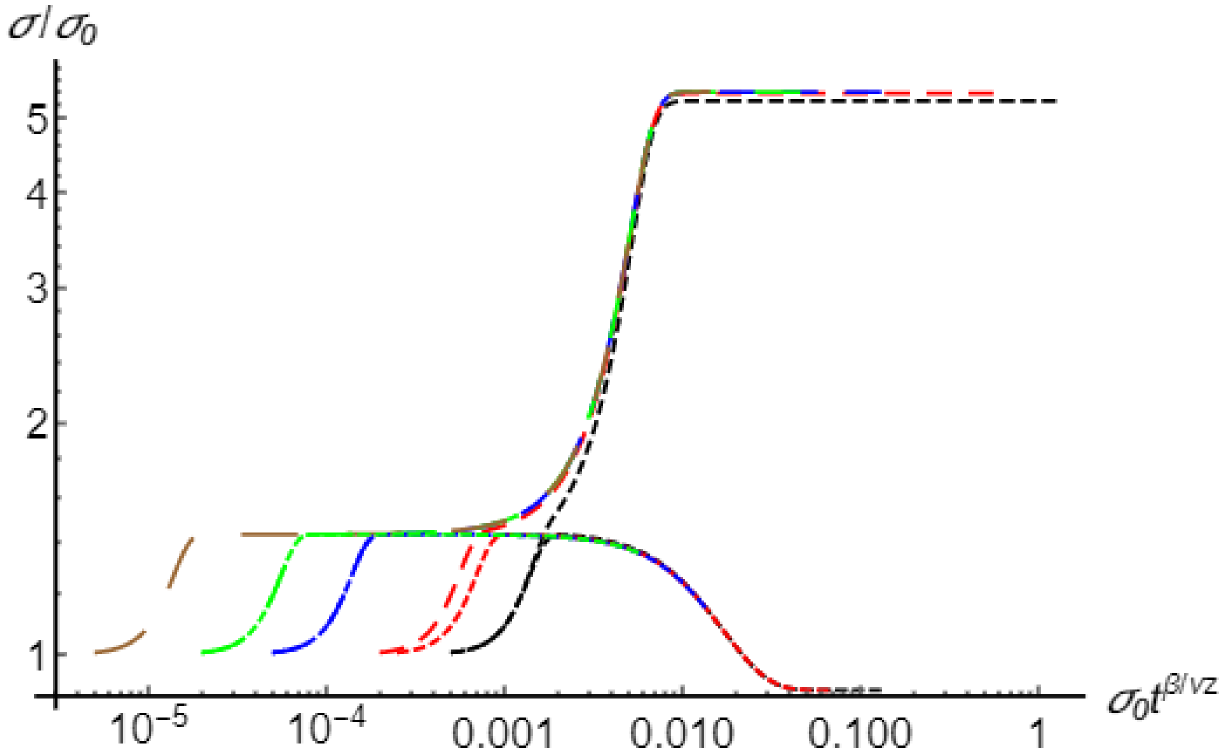
$$-\epsilon\sigma_i^{-1/\beta} = 3323$$



$$-\epsilon\sigma_i^{-1/\beta} = 92$$

The short time dynamics

Short time:
compared to the thermalization time (intermediate)



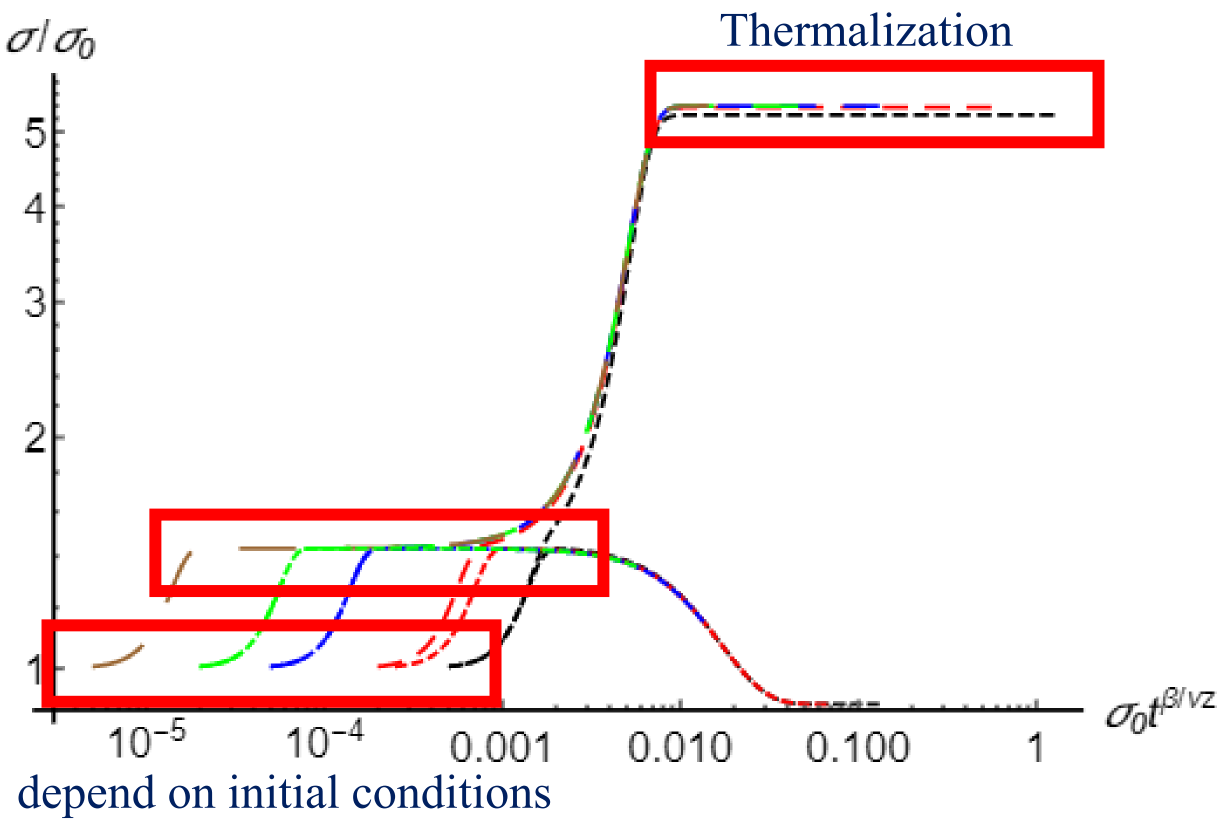
$$\sigma(\sigma_i, \epsilon, m_q, t) = \sigma_i^{1/x} f_{\sigma_i}(\epsilon\sigma_i^{-1/x\beta}, m_q\sigma_i^{-\delta/x}, t\sigma_i^{\nu z/x\beta})$$

***β*=1/2, *δ*=3,**
***ν*=1/2, *z*=2**

$$\xi \sim m_q^{-\nu/\beta\delta}, \qquad \sigma \sim m_q^{1/\delta}, \qquad \tau_R \sim m_q^{-\nu z/\beta\delta},$$

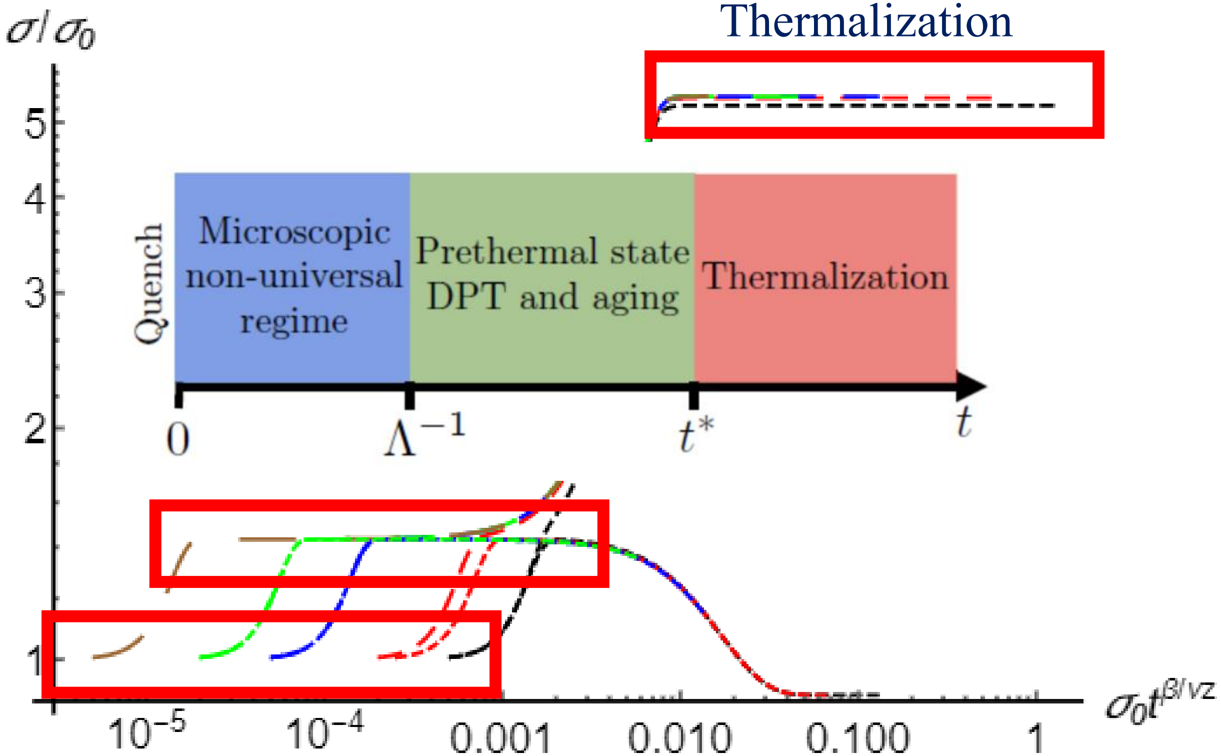
$$\xi \sim \epsilon^{-\nu}, \qquad \sigma \sim \epsilon^{\beta}, \qquad \tau_R \sim \epsilon^{-\nu z}.$$

Prethermalization



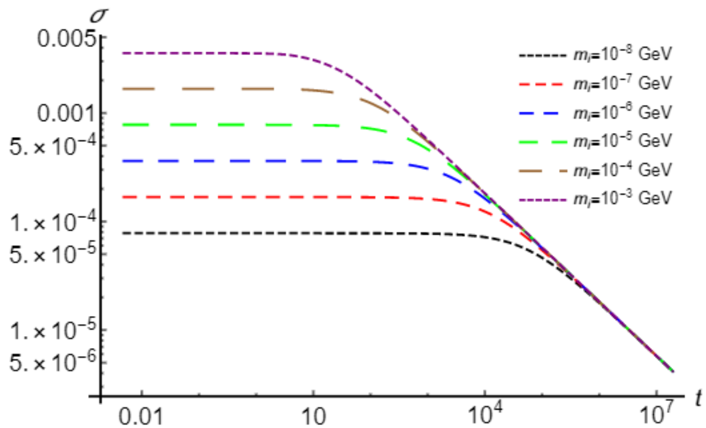
Prethermalization

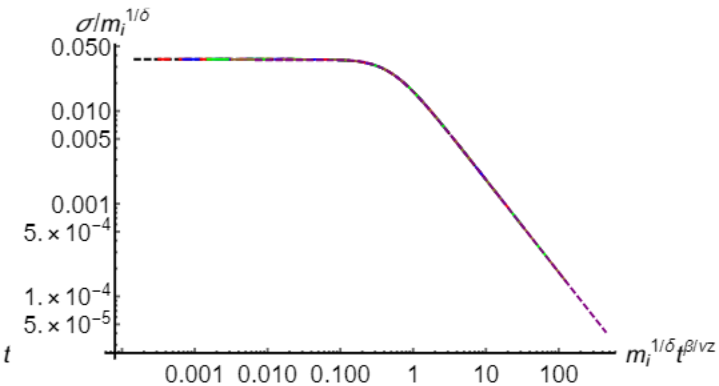
Thermalization



depend on initial conditions

Dynamic critical scaling

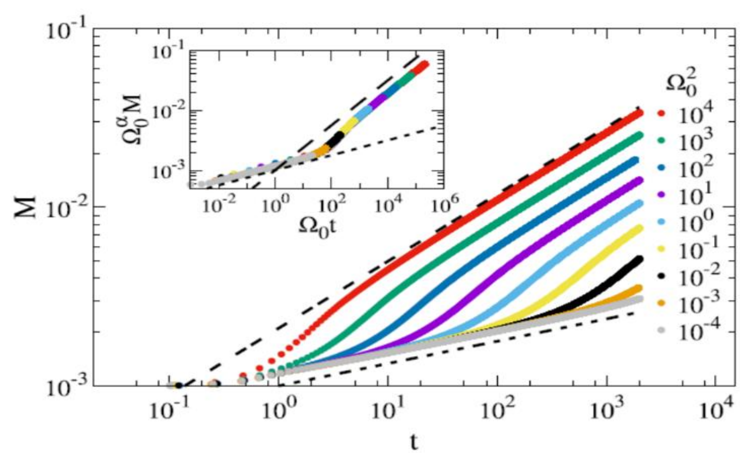




However, the initial-slip exponent θ=0, again mean field!

$$\sigma(\sigma_i, t) \propto \sigma_i t^{(x-1)\beta/\nu z}$$

$$\theta \equiv (x-1)\beta/\nu z$$



But qualitatively they have similar patterns

Including the Goldstone

Linear representation: $X = \frac{1}{2}(\chi \mathbf{I}_{2\times 2} + i\pi^a\tau^a)$ $\qquad \tilde{\boldsymbol{\sigma}} = \sqrt{\langle\boldsymbol{\sigma}\rangle^2 + \langle\boldsymbol{\pi}^a\rangle^2}$

The action:

$$S_{\text{eff}} = -\int d^5x\sqrt{-g}e^{-\Phi}\left\{\frac{1}{2}\partial_M\chi\partial^M\chi + \frac{1}{2}\partial_M\vec{\pi}\cdot\partial^M\vec{\pi} + (\partial_M\chi)\vec{A}^M\cdot\vec{\pi} - \frac{1}{2}\chi(\partial_M\vec{\pi})\cdot\vec{A}^M\right.$$
$$+\frac{1}{8}\chi^2\vec{A}_M\cdot\vec{A}^M + \frac{1}{2}(\vec{A}_M\cdot\vec{\pi})(\vec{A}^M\cdot\vec{\pi}) + \frac{1}{2}m_5^2\left(\chi^2 + \vec{\pi}\cdot\vec{\pi}\right) + \frac{1}{8}\lambda\left(\chi^2 + \vec{\pi}\cdot\vec{\pi}\right)^2$$
$$\left.+\frac{1}{4g_5^2}\left[(\vec{A}_M\cdot\vec{A}^M)(\vec{A}_N\cdot\vec{A}^N) - (\vec{A}_M\cdot\vec{A}_N)(\vec{A}^M\cdot\vec{A}^N) + F^{MN,a}F^a_{MN}\right]\right\},$$

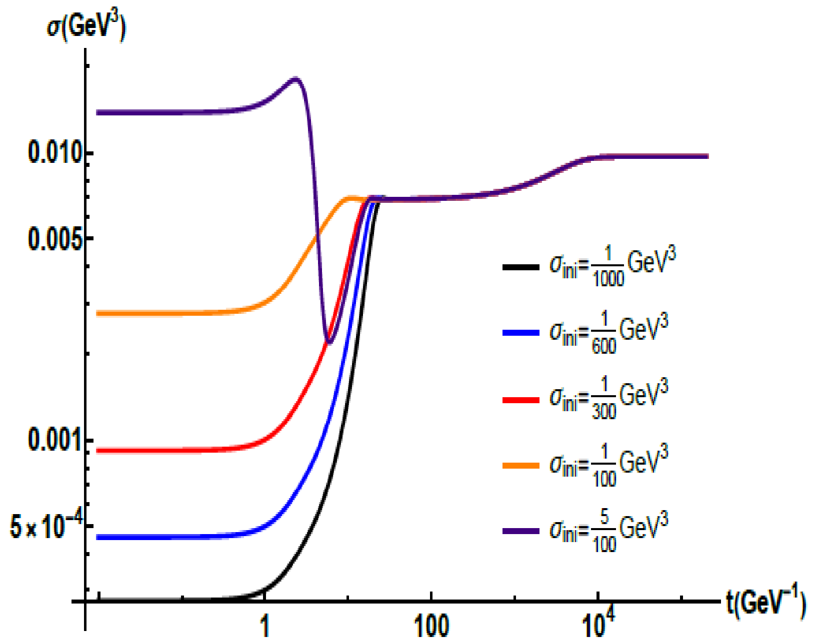
The EOM:

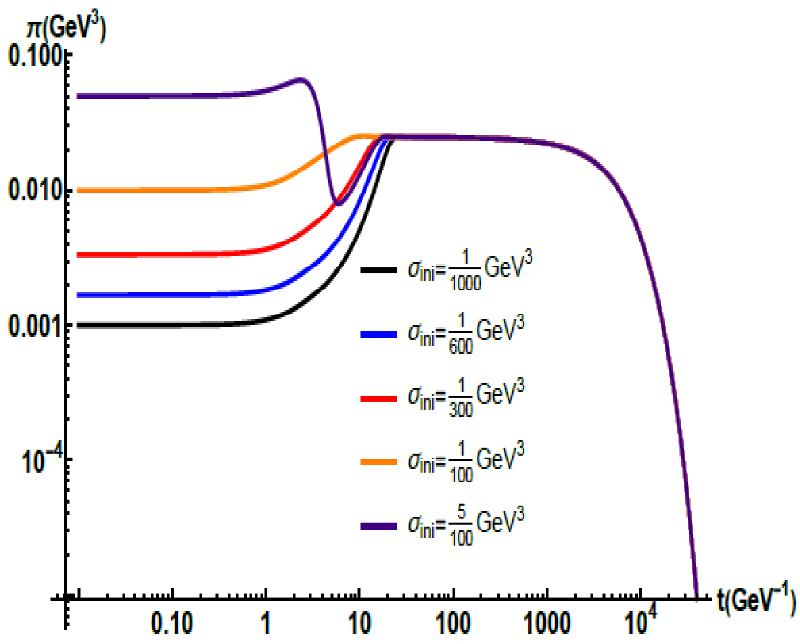
$$z^2f(z)(\partial_z^2\chi) + \left[z^2f'(z) - f(z)(3z + z^2\Phi'(z))\right](\partial_z\chi) - \left[2z^2\partial_z - 3z - z^2\Phi'(z)\right]\partial_t\chi$$
$$= \frac{1}{4}z^2A^2\chi + m_5^2\chi + \frac{\lambda}{2}\left(\chi^2 + \pi^2\right)\chi,$$

$$z^2f(z)(\partial_z^2\pi) + \left[z^2f'(z) - f(z)(3z + z^2\Phi'(z))\right](\partial_z\pi) - \left[2z^2\partial_z - 3z - z^2\Phi'(z)\right]\partial_t\pi$$
$$= \frac{1}{3}z^2A^2\pi + m_5^2\pi + z^2p^2\pi + \frac{\lambda}{2}\left(\chi^2 + \pi^2\right)\pi,$$

$$z^2f(z)(\partial_z^2A) + \left[z^2f'(z) - f(z)(z + z^2\Phi'(z))\right](\partial_zA) - \left[2z^2\partial_z - z - z^2\Phi'(z)\right]\partial_tA$$
$$= g_5^2\left(\frac{1}{4}\chi^2 + \frac{1}{3}\pi^2\right)A + \frac{2}{3}z^2A^3,$$

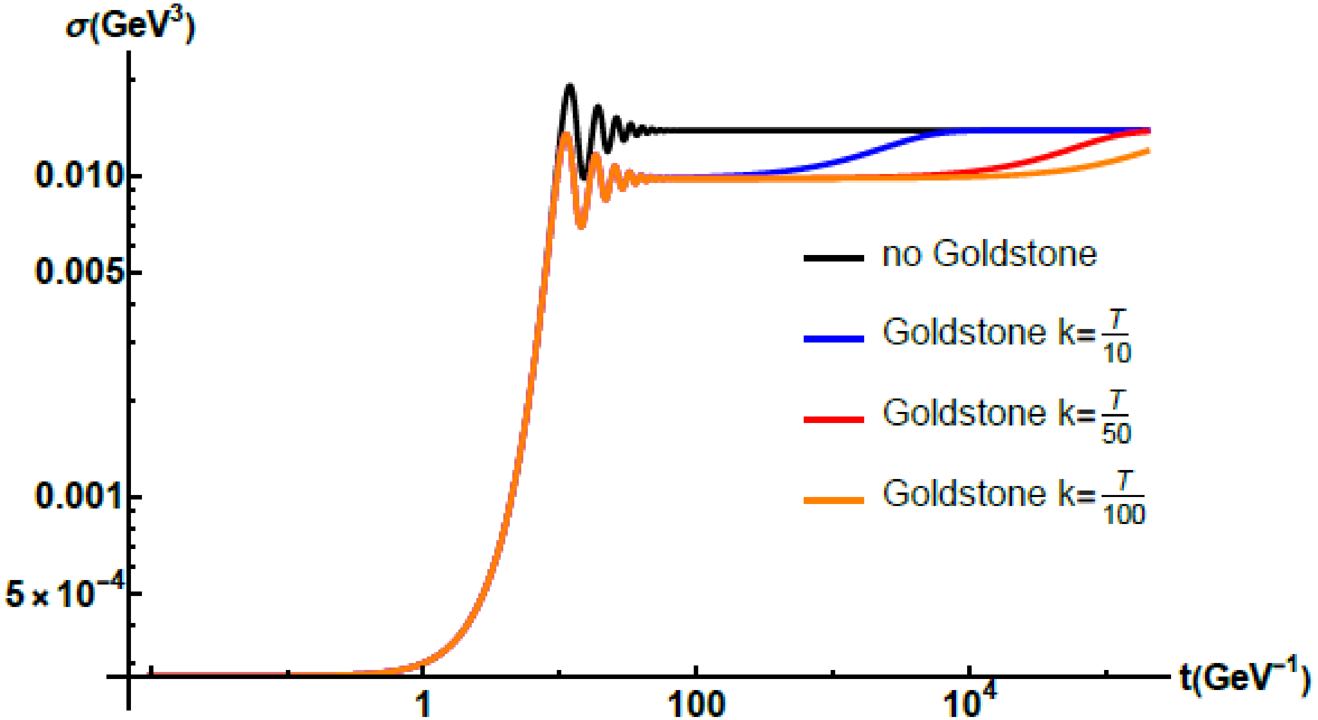
Dynamics of pion





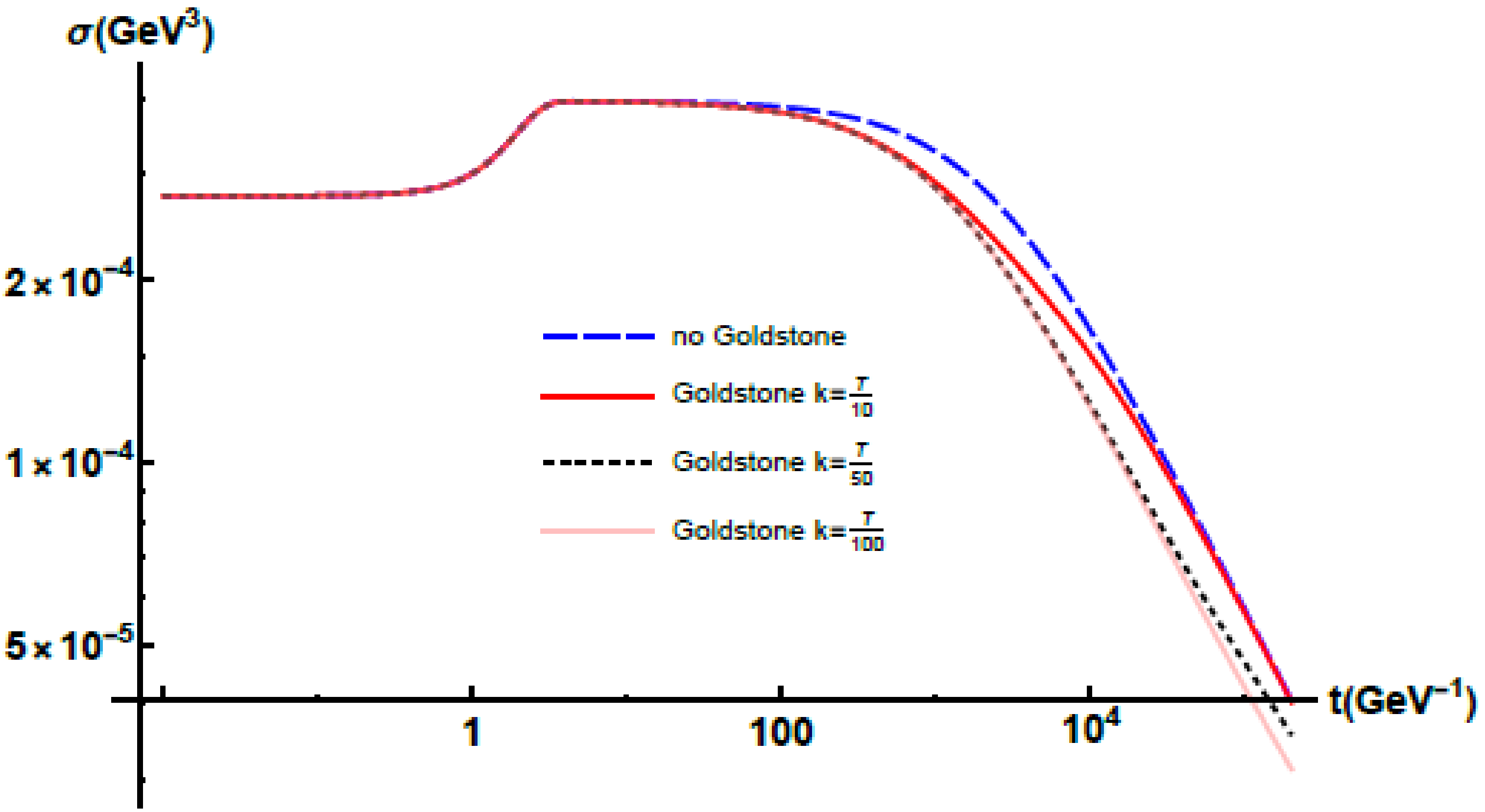
P.Zheng, Y. Chen, DL, M.Huang, Y. Liu, JHEP 07 (2025) 029

Non-critical point



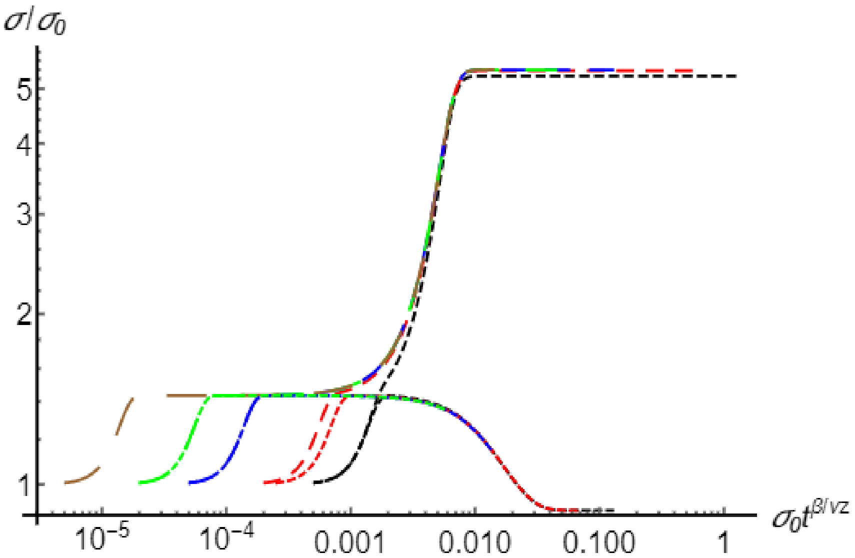
P.Zheng, Y. Chen, DL, M.Huang, Y. Liu, JHEP 07 (2025) 029

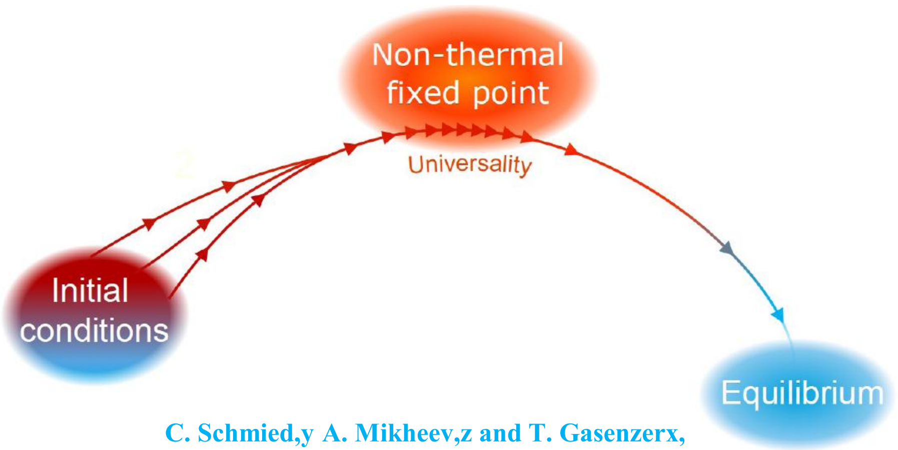
Near critical point



P.Zheng, Y. Chen, DL, M.Huang, Y. Liu, JHEP 07 (2025) 029

NTFP?





C. Schmied,y A. Mikheev,z and T. Gasenzerx,
Int.J.Mod.Phys.A 34 (2019) 29, 1941006

**Linearize and study
the QNMs?
(future study)**

Summary

- **Soft-wall AdS/QCD model provides a good start point to consider chiral phase transition in holographic framework. The phase diagram is in agreement with the Columbia plot. It is like a 5D finite temperature CPT.**

- **The real-time dynamics of chiral phase transition show non-trivial behavior in the intermediate time. This might be related with the so called "prethermalization" phenomena.**

- **In the future: beyond the probe limit, close to the CEP in *T-μ* plane, effective particle distributions……**

Thanks for your attention!