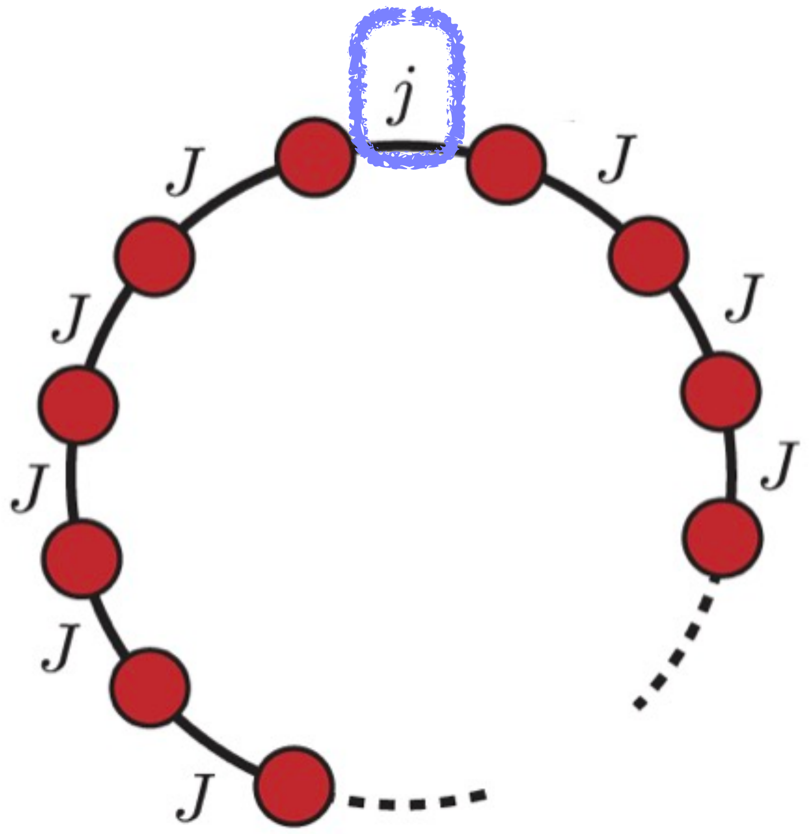
Holographic non-conformal interfaces

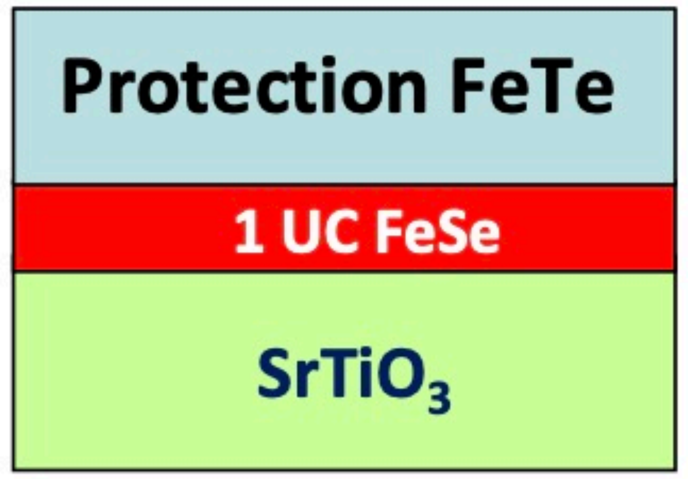
Yan Liu

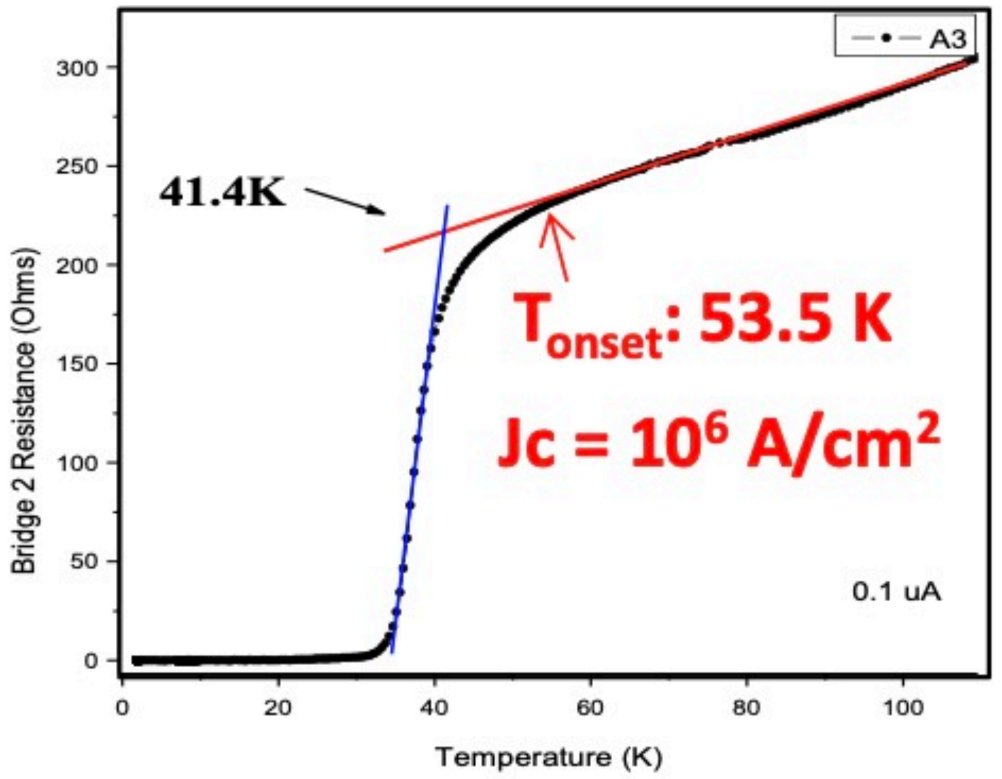
Based on arXiv: 2403.20102，2503.20399， 2506.19553

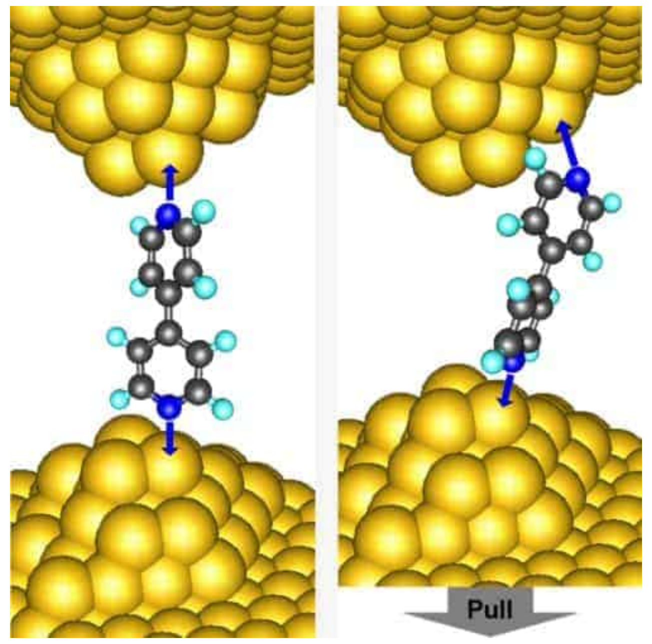
with Hong-Da Lyu (吕宏达), Chuan-Yi Wang (王传艺), You-Jie Zeng (曾佑捷)

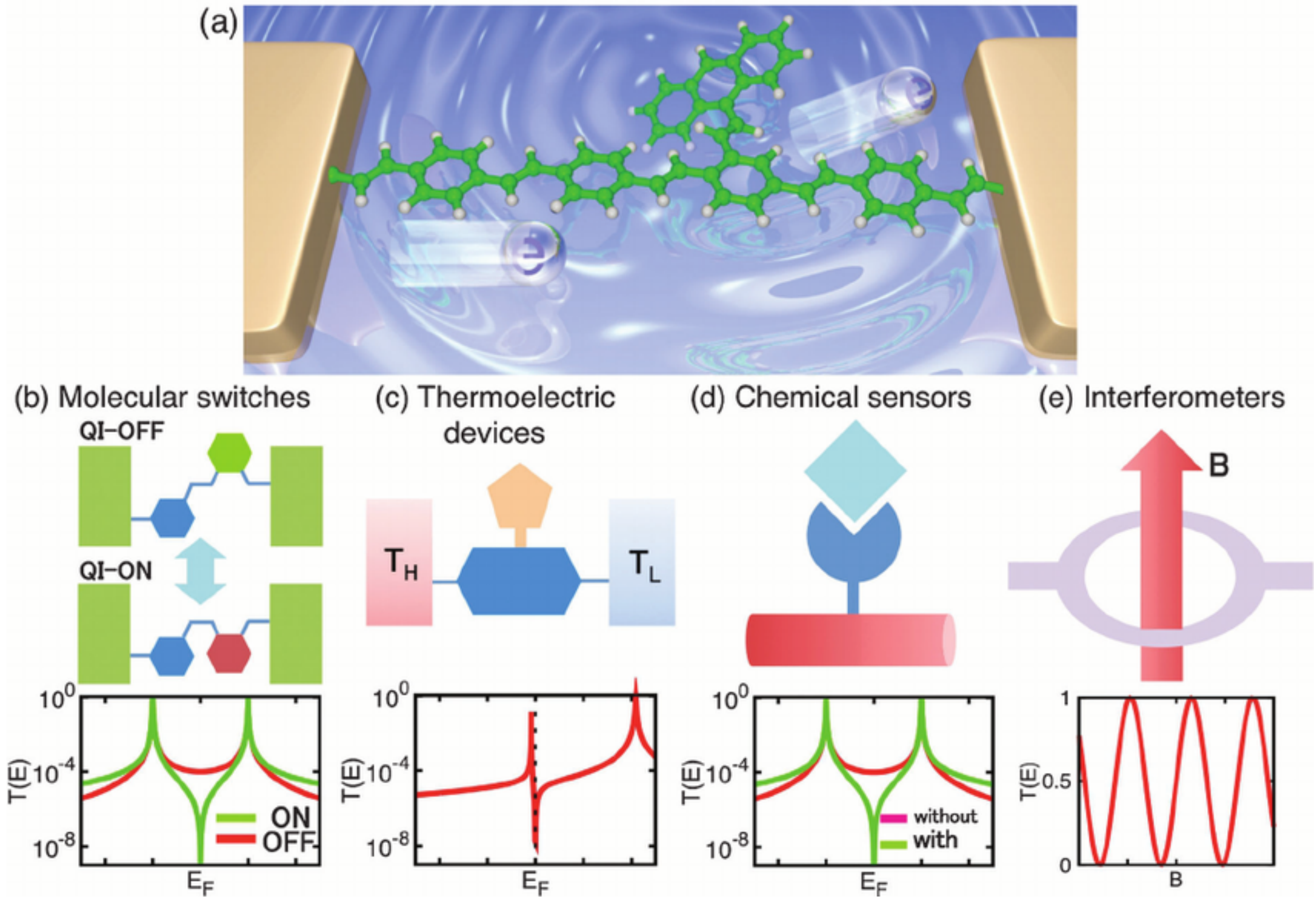
Interface systems are ubiquitous in nature











[Talks by Elias, Hao-Tian, Carlos, Rong-Xin]

Interface field theory

- Interface field theory has rich physics

$$S = S_{\rm I} + S_{\rm II} + S_{\rm interface}$$

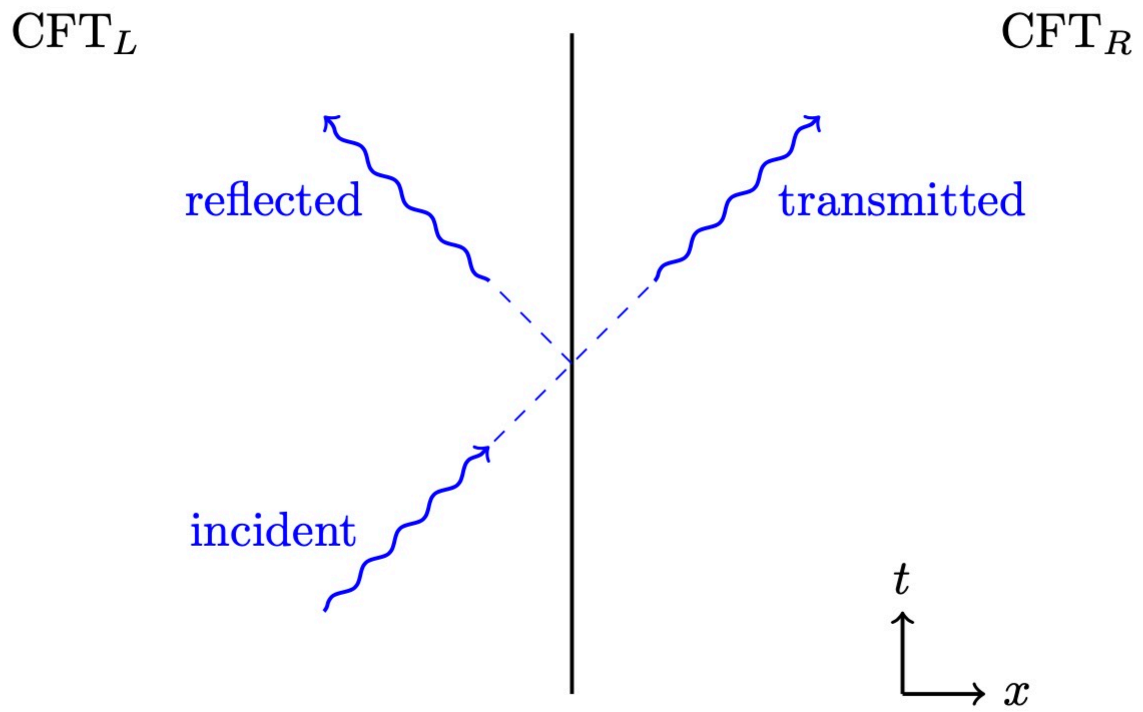
$$S_{\rm interface} \supset \int \lambda \hat{\mathcal{O}}_{\rm I} \hat{\mathcal{O}}_{\rm II} + S[\mathcal{O}]$$

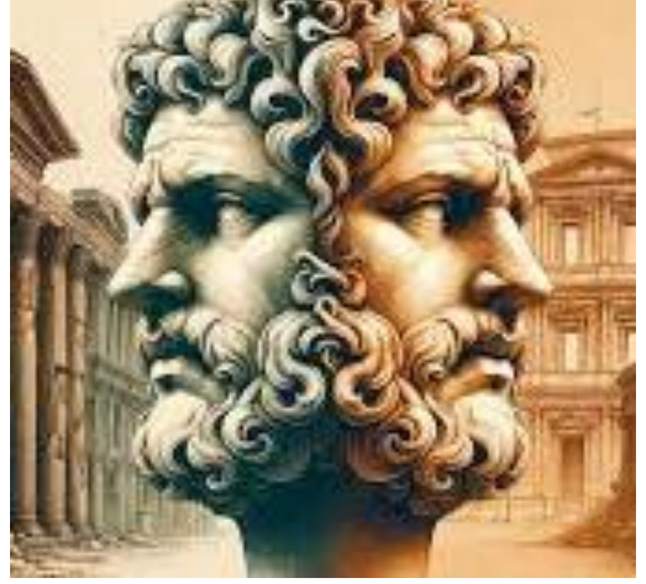
  - Boundary field theory, BCFT
  - Defect field theory, DCFT

- More interesting physical observables

  Entanglement entropy (e.g. “effective central charge” for ICFT)

  Energy transports: transmission/reflection coefficients





Interface in holography (1)

Janus/ICFT

1) top-down, D1-D5 systems, IIB SUGRA on $\mathrm{AdS}_3 \times S^3 \times T^4$

$$ds^2_{\mathrm{IIB}} = e^{\phi/2}(ds^2_{(3)} + d\Omega_3^2) + e^{-\phi/2} ds^2_{T^4}$$

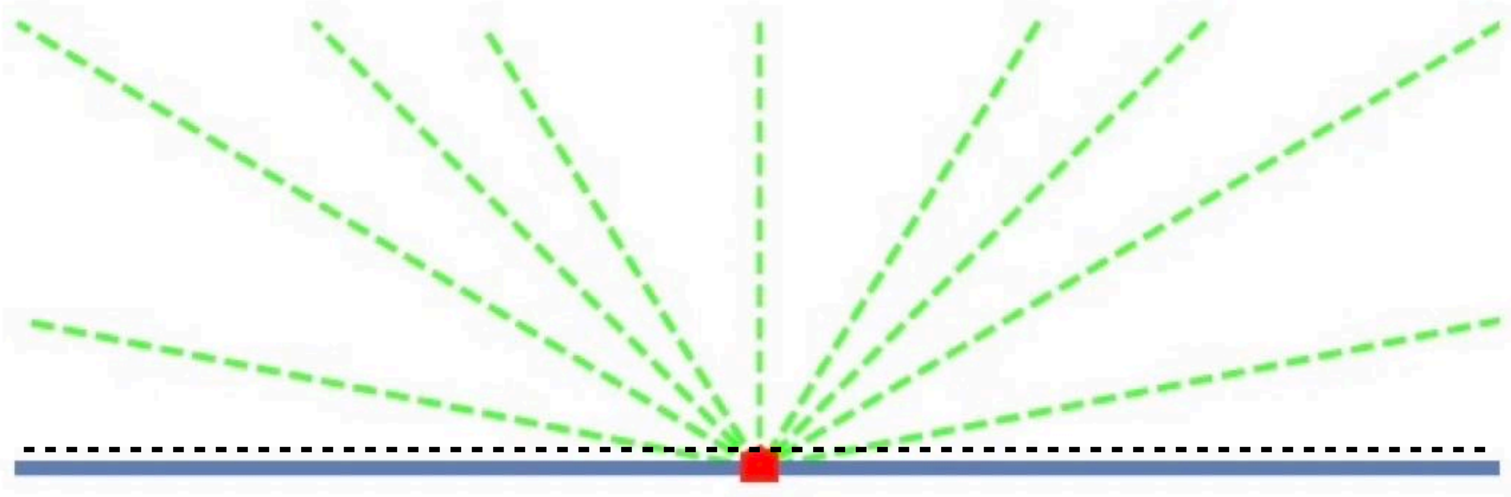
2) bottom-up

3D Einstein-massless scalar theory

$$ds_3^2 = f(y) ds^2_{AdS_2} + dy^2$$

$$f(y) = \frac{1}{2}(1 + \sqrt{1-2\gamma^2}\cosh 2y)\,,$$

$$\phi = \phi_0 + \frac{1}{\sqrt{2}} \ln\left(\frac{1+\sqrt{1-2\gamma^2}+\sqrt{2}\gamma\tanh y}{1+\sqrt{1-2\gamma^2}-\sqrt{2}\gamma\tanh y}\right)$$



[Bachas,Karch, Ooguri, 2001; Skenderis, Bak, Gutperle, Hirano]

Interface in holography (1)

PRL **118,** 201601 (2017) PHYSICAL REVIEW LETTERS

Surface States in Holographic Weyl Semimetals

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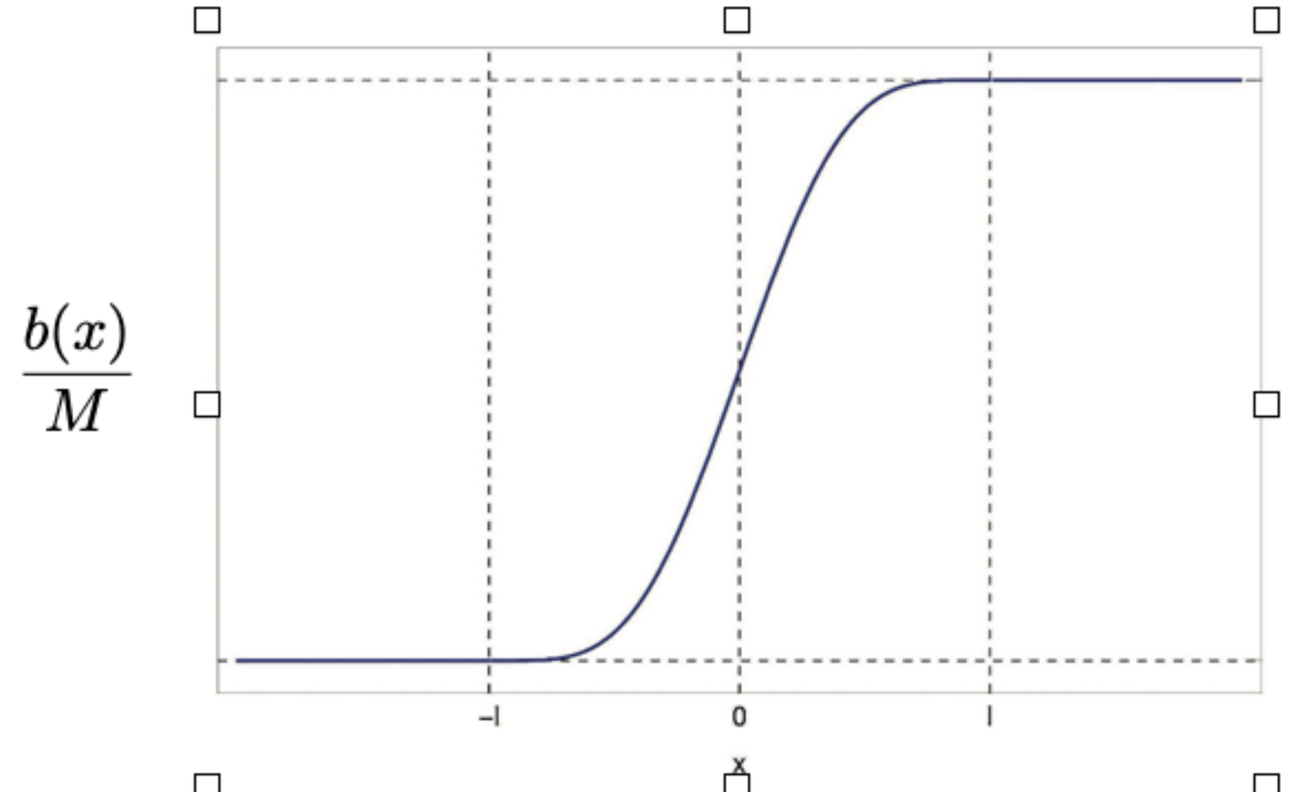
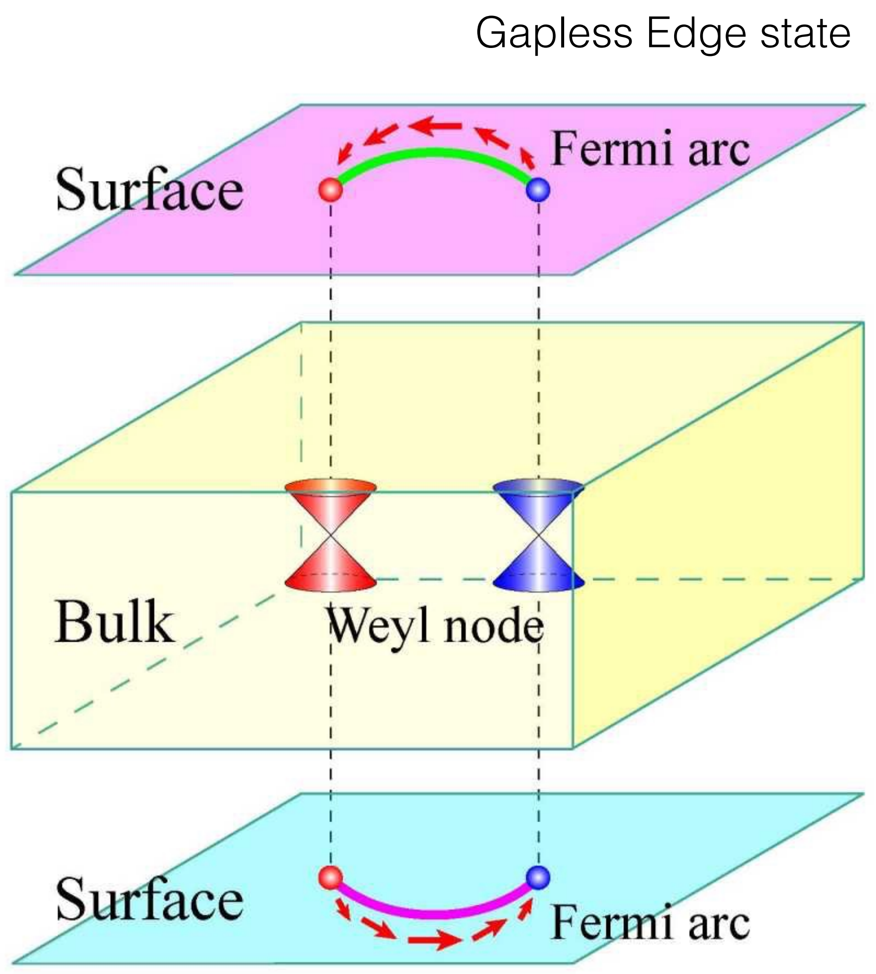
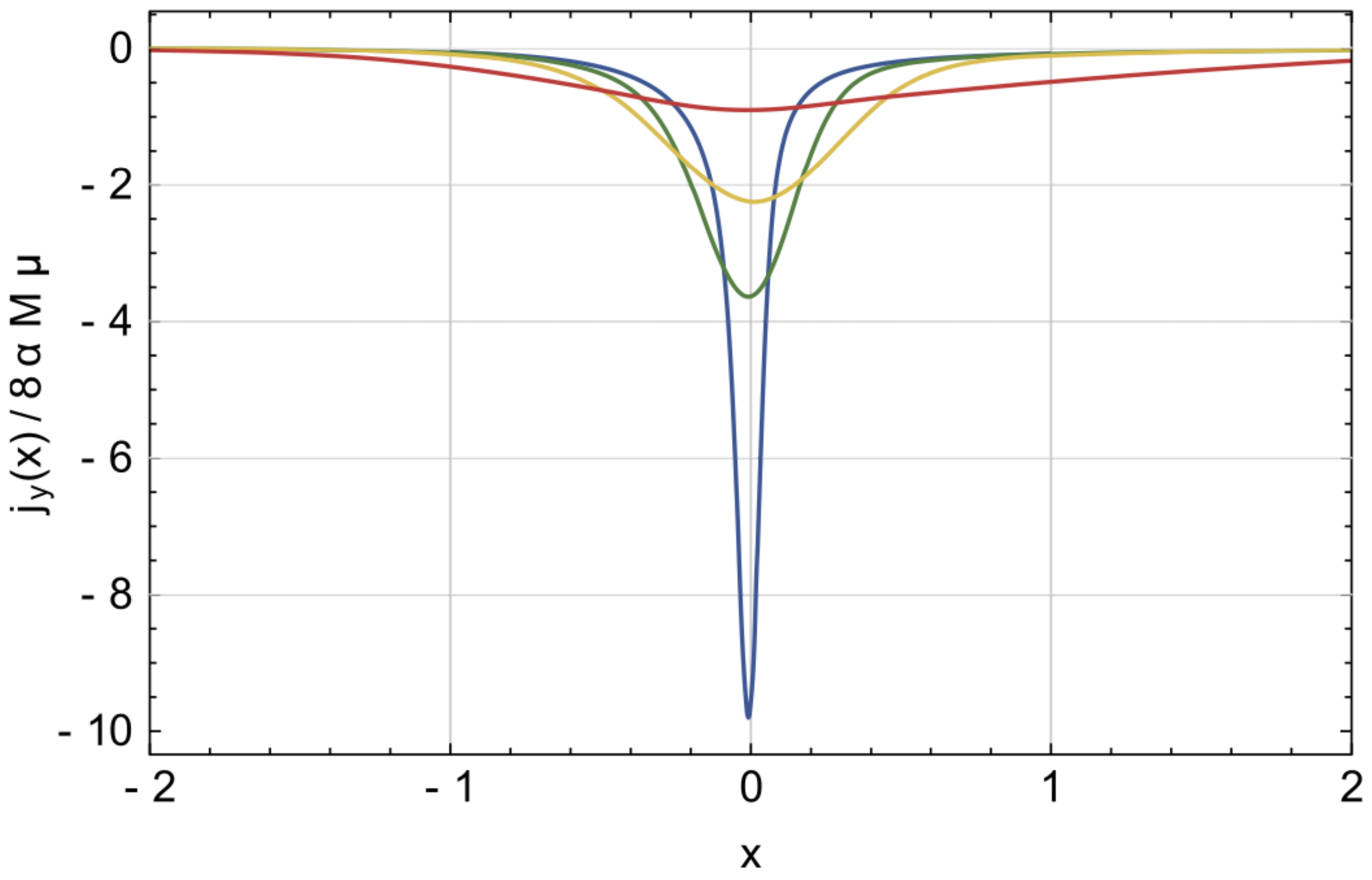
(Received 15 December 2016; published 18 May 2017)

We study the surface states of a strongly coupled Weyl semimetal within holography. By explicit numerical computation of an inhomogeneous holographic Weyl semimetal, we observe the appearance of an electric current restricted to the surface in the presence of an electric chemical potential. The integrated current is universal in the sense that it only depends on the topology of the phases showing that the bulk-boundary correspondence holds even at strong coupling. The implications of this result are subtle and may shed



Trivial state
Interface
Weyl metal





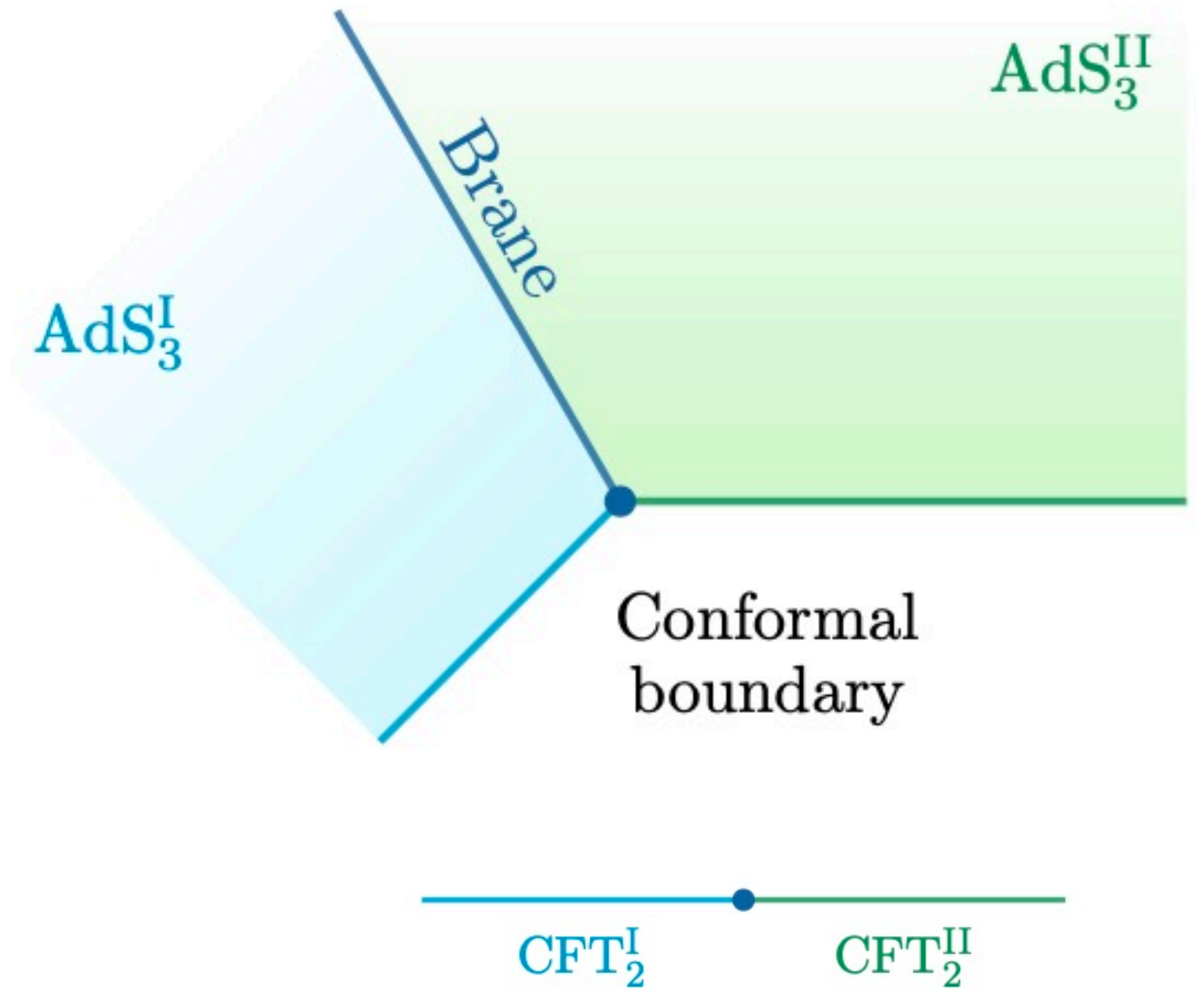
Interface in holography (2)

- AdS/ICFT

  A new ingredient in AdS/CFT: a junction brane

  The left and right bulk can be different

- A junction of two AdS/BCFTs
- Lots of applications: double holography, Janus, …



[Karch, Randall, 2001, Erdmenger…2014, Simidzija, Raamsdonk, 2020 …,Bachas, 2020,…Sonner,…2022]

Interface in holography (2)

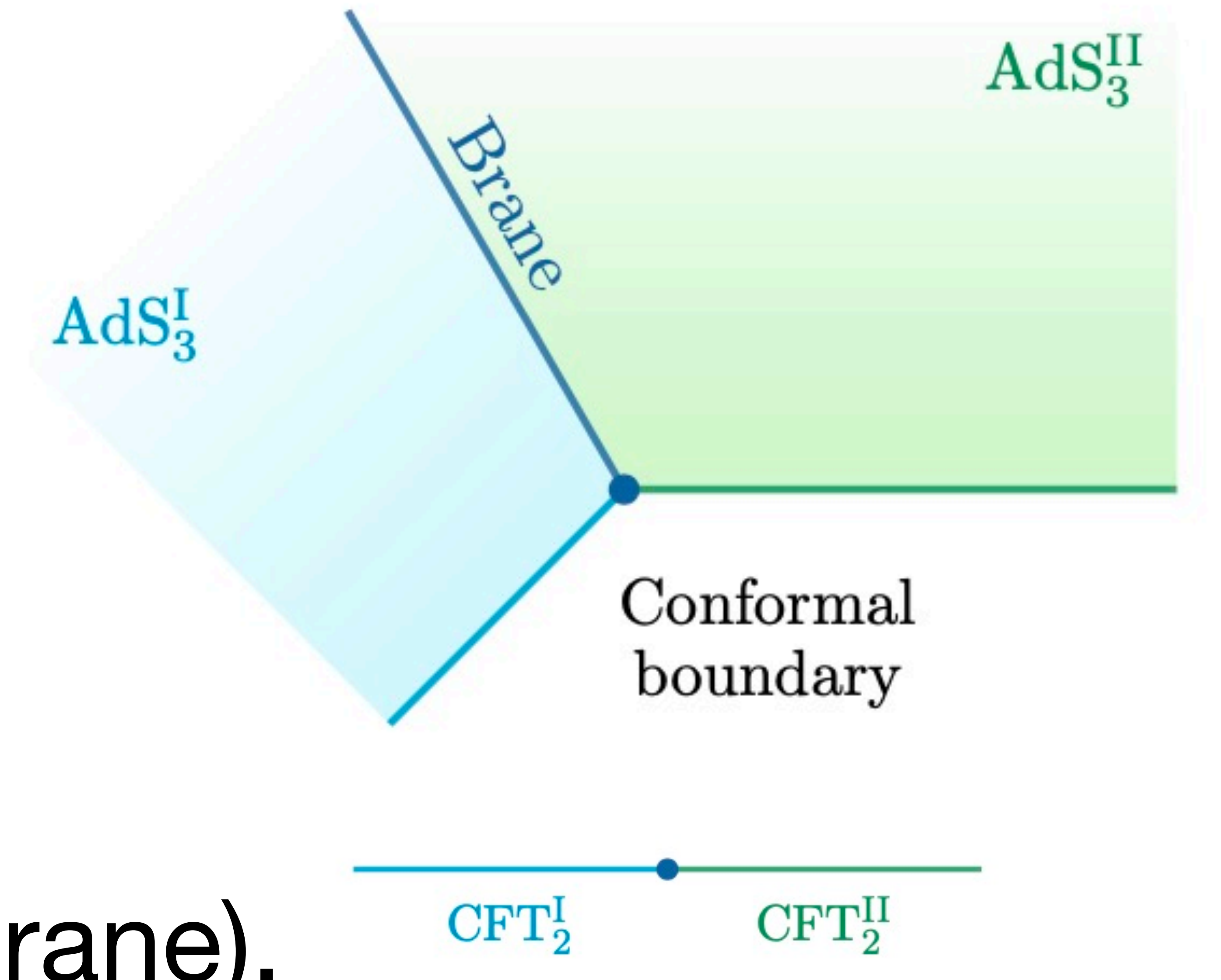
- AdS/ICFT

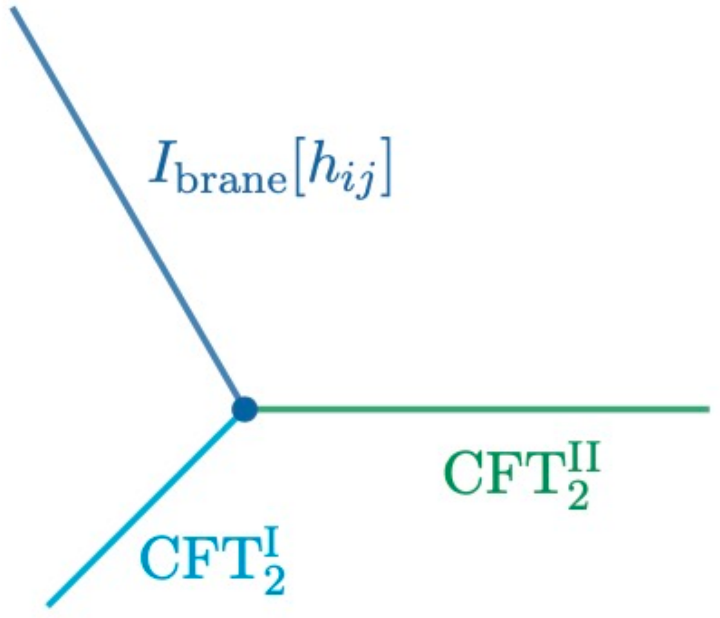
  A new ingredient in AdS/CFT: a junction brane

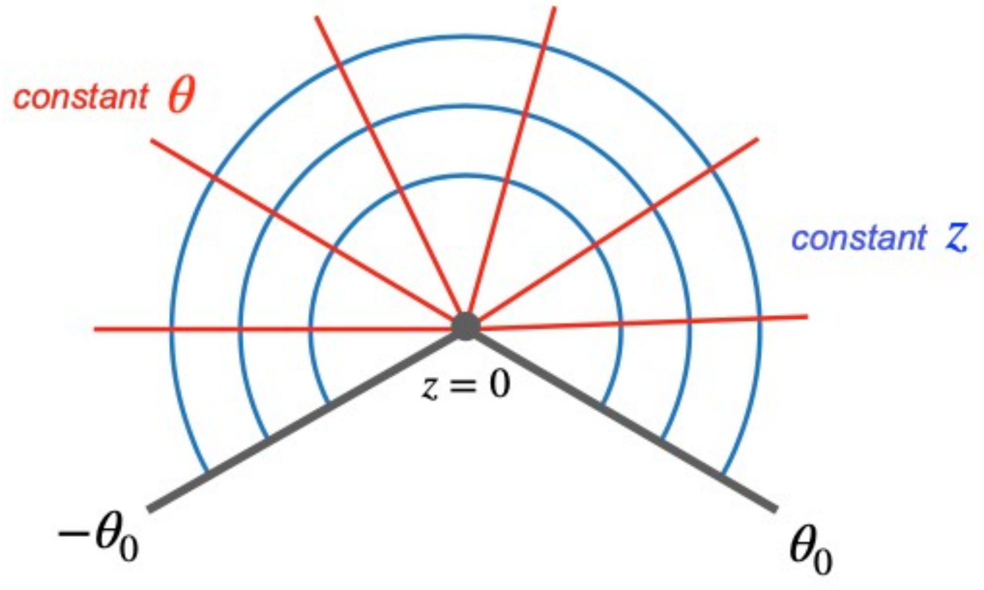
  The left and right bulk can be different

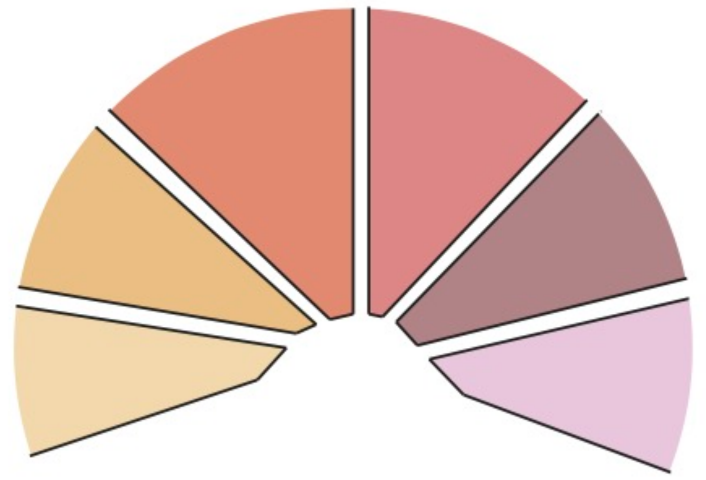
- A junction of two AdS/BCFTs

- Lots of applications: double holography, Janus (thick brane),









[Karch, Randall, 2001; Erdmenger et al. 2014; Simidzija, Raamsdonk, 2020; Bachas et al., 2020; Sonner et al. 2022]

Holographic non-conformal interface

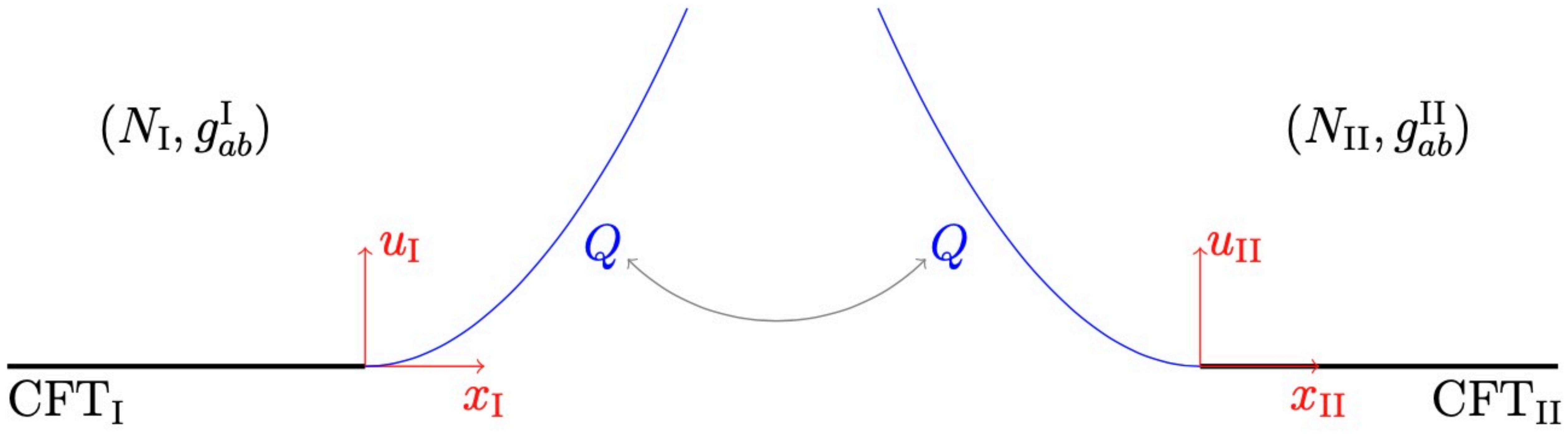
- Beyond conformal interface

$$S = S_{\rm I} + S_{\rm II} + S_{\rm interface}$$

$$S_{\rm interface} \supset \int \lambda \hat{\mathcal{O}}_{\rm I} \hat{\mathcal{O}}_{\rm II} + S[\mathcal{O}]$$

Holographic non-conformal interface

- Beyond conformal interface



$$S_{\rm bulk} = S_{\rm I} + S_{\rm II} + S_Q$$

$$S_{\rm I} = \int_{N_{\rm I}} d^3x\sqrt{-g_{\rm I}}\left[\frac{1}{16\pi G}\left(R_{\rm I} + \frac{2}{L_{\rm I}^2}\right)\right],$$

$$S_{\rm II} = \int_{N_{\rm II}} d^3x\sqrt{-g_{\rm II}}\left[\frac{1}{16\pi G}\left(R_{\rm II} + \frac{2}{L_{\rm II}^2}\right)\right],$$

$$S_Q = \frac{1}{8\pi G}\int_Q d^2y\sqrt{-h}\left[(K_{\rm I} - K_{\rm II}) - (\partial\phi)^2 - V(\phi)\right].$$

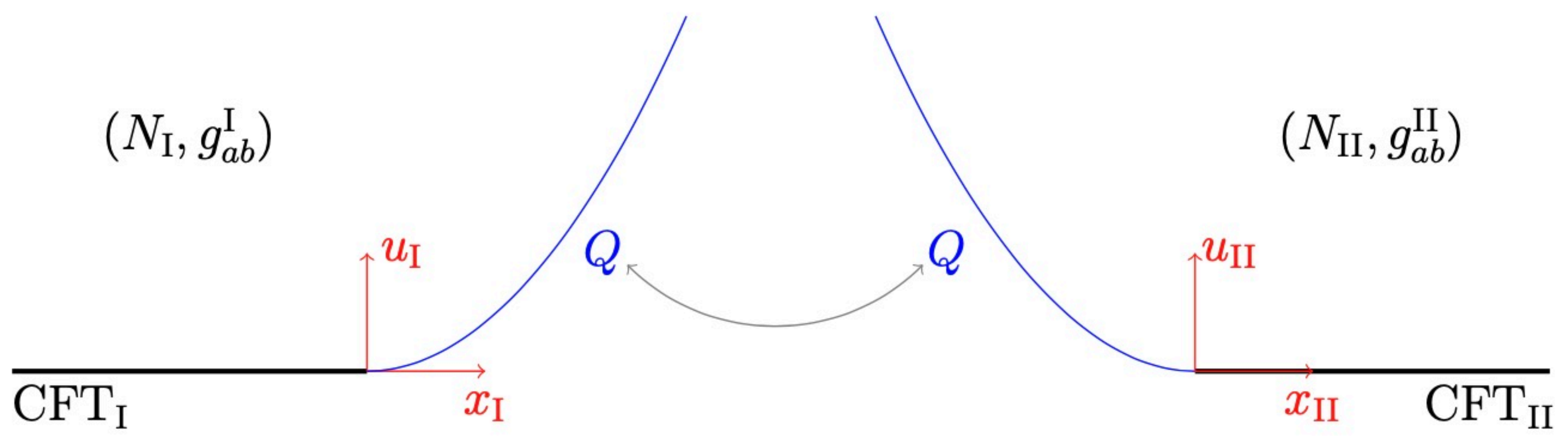
$$h_{\mu\nu} = \frac{\partial x^a_{\rm I}}{\partial y^\mu}\frac{\partial x^b_{\rm I}}{\partial y^\nu}\, g^{\rm I}_{ab} = \frac{\partial x^a_{\rm II}}{\partial y^\mu}\frac{\partial x^b_{\rm II}}{\partial y^\nu}\, g^{\rm II}_{ab}\,.$$

$$\Delta K_{\mu\nu} - h_{\mu\nu}\Delta K + \left[(\partial\phi)^2 + V(\phi)\right] h_{\mu\nu} - 2\partial_\mu\phi\partial_\nu\phi = 0\,,$$

$$2\partial_\mu(\sqrt{-h}h^{\mu\nu}\partial_\nu\phi) - \sqrt{-h}\frac{dV(\phi)}{d\phi} = 0\,,$$

Zero temperature solution

- We focus on static configuration: existence of a global time
  - Left/Right bulk: AdS3 $$ds_A^2 = \frac{L_A^2}{u_A^2}\left[-dt_A^2 + dx_A^2 + du_A^2\right], \qquad A = \mathrm{I},\ \ \mathrm{II}.$$
  - Interface $$\nu \equiv \frac{c_{\mathrm{II}}}{c_{\mathrm{I}}} = \frac{L_{\mathrm{II}}}{L_{\mathrm{I}}} \qquad (0 < \nu \leq 1)$$



Parametrization

$$x_{\mathrm{I}}^a = (t, \psi_{\mathrm{I}}(z), z/\sqrt{\nu}) \qquad x_{\mathrm{II}}^a = (t, \psi_{\mathrm{II}}(z), \sqrt{\nu} z)$$

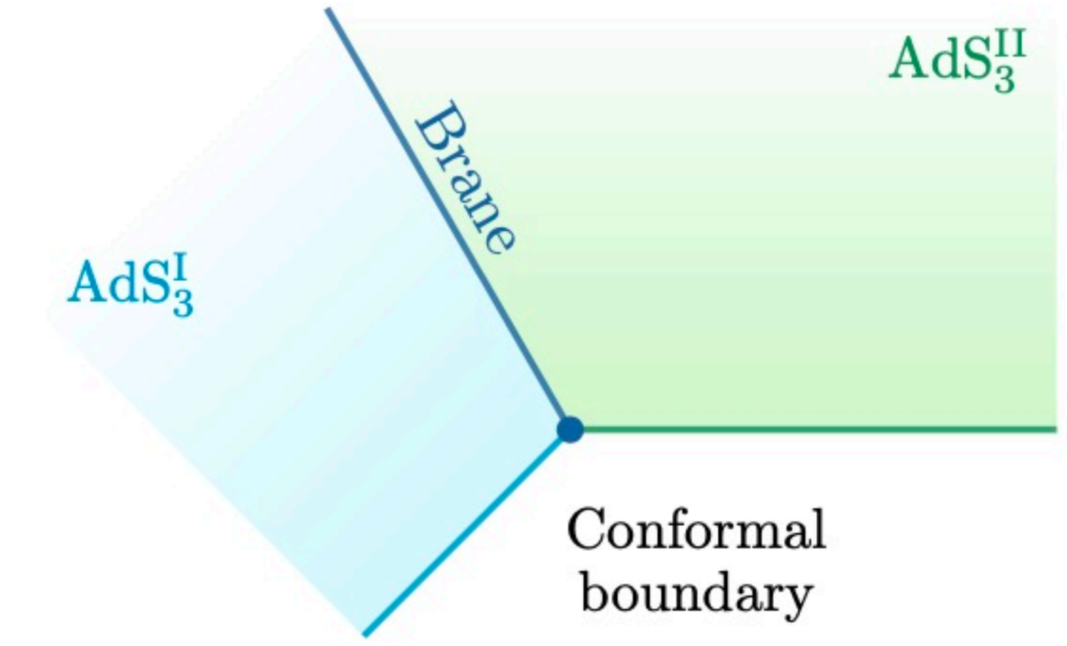
$$\frac{1}{\nu} + \psi_{\mathrm{I}}'^2 = \nu + \psi_{\mathrm{II}}'^2$$

$$\phi'^2 = \frac{L_{\mathrm{I}}}{2z} \frac{-\psi_{\mathrm{I}}'' + \nu \psi_{\mathrm{II}}''}{\sqrt{\nu + \psi_{\mathrm{II}}'^2}},$$

$$V(\phi(z)) = \frac{\sqrt{\nu}\,(2(\psi_{\mathrm{I}}' - \nu\psi_{\mathrm{II}}') + 2(\nu\psi_{\mathrm{I}}'^3 - \psi_{\mathrm{II}}'^3) - z(\psi_{\mathrm{I}}'' - \nu\psi_{\mathrm{II}}''))}{2L_{\mathrm{I}}(1 + \nu\psi_{\mathrm{I}}'^2)^{3/2}},$$

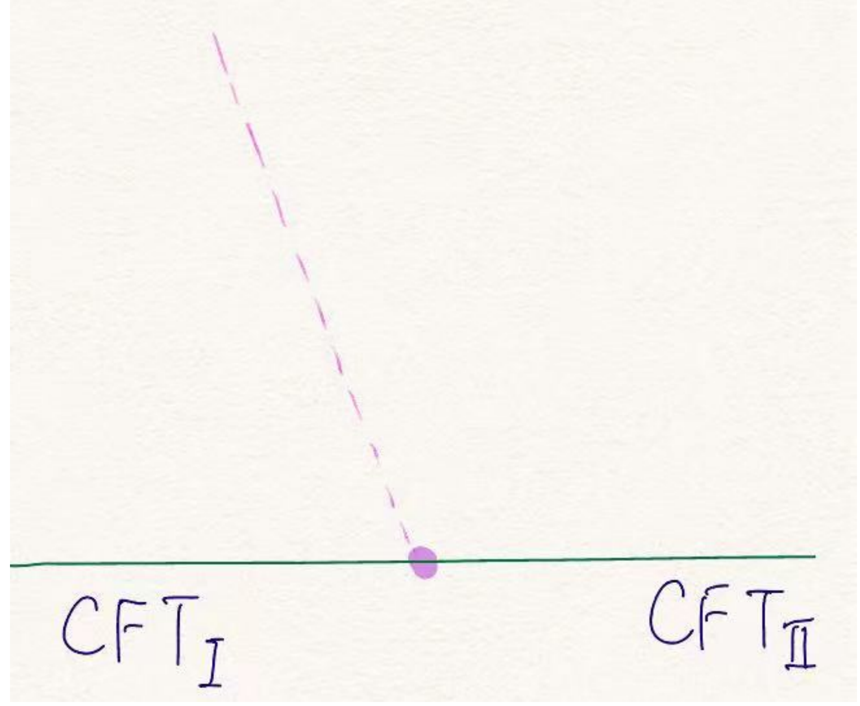
Zero temperature solution

- When the brane is a straight line, or constant potential

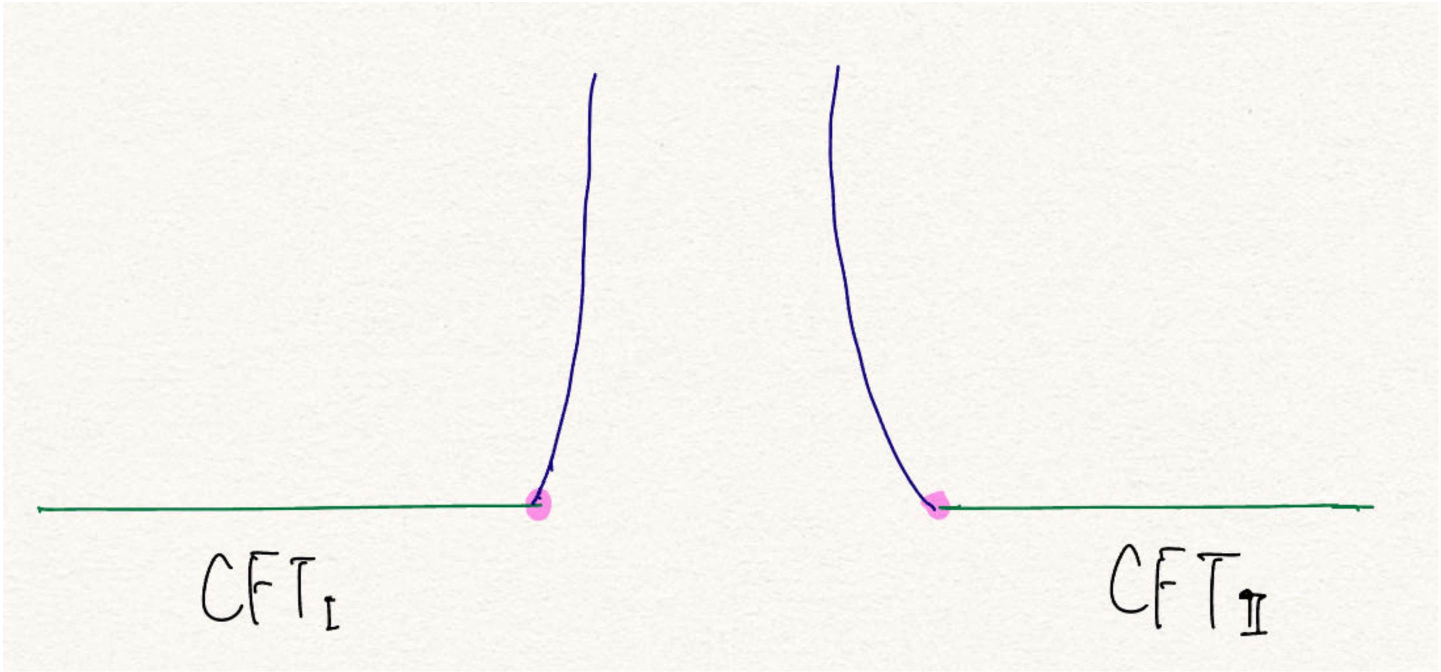


$$T_{\min} < |T| < T_{\max} \qquad T_{\min} = \frac{1-\nu}{L_I \nu}\,, \quad T_{\max} = \frac{1+\nu}{L_I \nu}$$

- When $\nu = \frac{L_{II}}{L_I} = 1$, there are two possible solutions
  - supplementary configuration: $\psi'_I = \psi'_{II}\,, \quad \phi = c_1\,, \quad V = 0$
  - nontrivial configuration: $\psi_I(z) = -\,\psi_{II}(z)$



- BCFT limit [Kanda, Sato, Suzuki, Takayanagi, Wei, 2023]
  - $\nu = \frac{L_{II}}{L_I} = 1$: folding, or Z2 identification
  - limit $\nu = \frac{L_{II}}{L_I} \to 0$

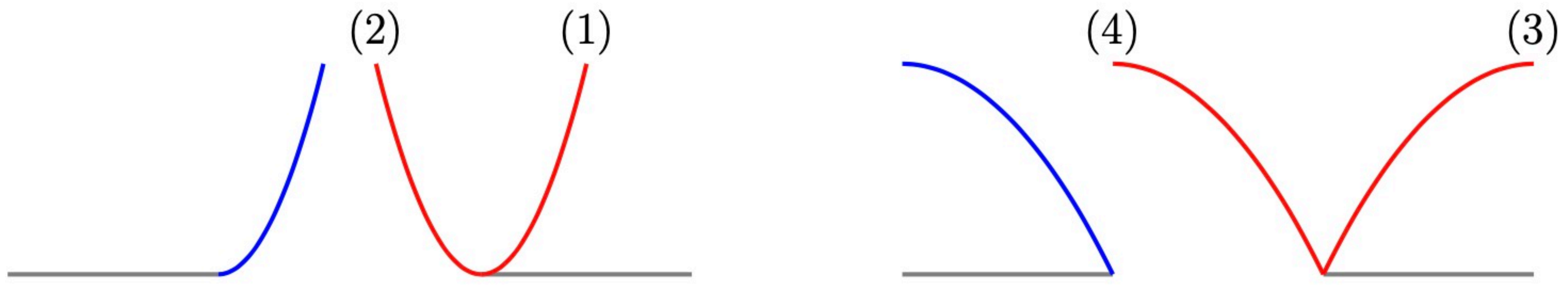


Null energy condition on the brane

- NEC = permissible configuration

$$(\Delta K_{\mu\nu} - h_{\mu\nu}\Delta K)N^{\mu}N^{\nu} = \frac{L_{\mathrm{I}}}{z}\frac{(-\psi_{\mathrm{I}}'' + \nu\psi_{\mathrm{II}}'')}{\sqrt{\frac{1}{\nu} + \psi_{\mathrm{I}}'^2}} \geq 0 \quad \Longleftrightarrow \quad \phi'^2(z) \geq 0$$

  - linear configuration (= constant tension on the brane)
  - Other allowed configurations (presume monotonic)

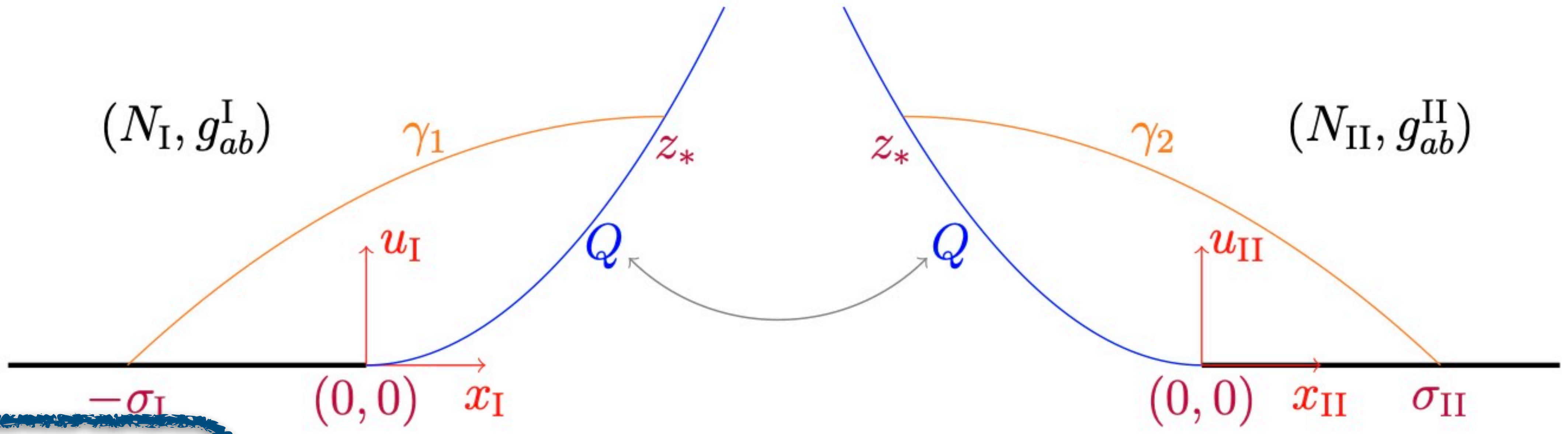


Interface entropy and g-theorem

- Start from HEE of the interval $[-\sigma_{\rm I},0]\cup[0,\sigma_{\rm II}]$, a finite part by setting $\sigma_{\rm I}=\sigma_{\rm II}=\sigma$

$$S_{\rm iE}(\sigma)=S_{\rm E}-\frac{1}{2}S^L_{\rm E}-\frac{1}{2}S^R_{\rm E}$$

$$S^L_{\rm E}=\frac{c_{\rm I}}{3}\log\frac{2\sigma}{\epsilon_{\rm I}}\,,\qquad S^R_{\rm E}=\frac{c_{\rm II}}{3}\log\frac{2\sigma}{\epsilon_{\rm II}}$$



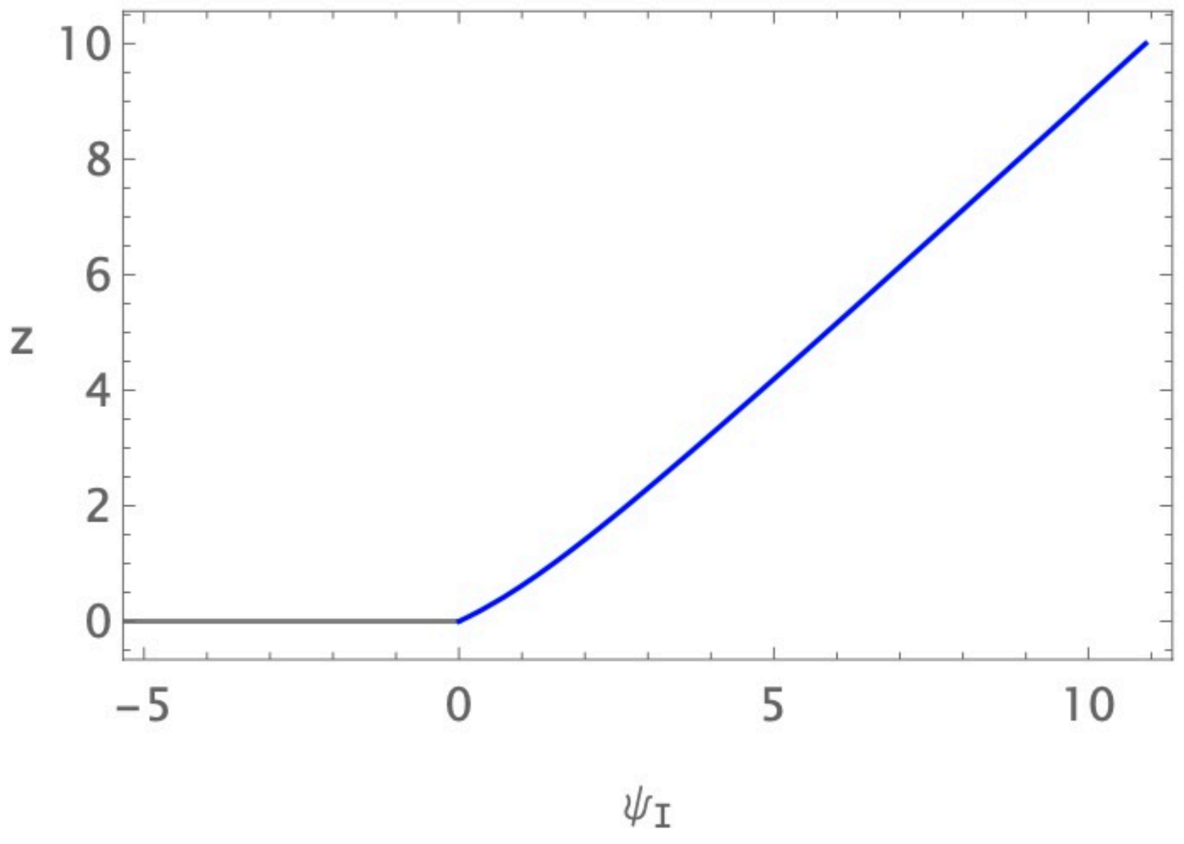
$$\begin{aligned}\log g(\sigma)&=S_{\rm iE}(\sigma)\\&=\frac{c_{\rm I}}{6}\log\frac{\frac{z_*^2}{\sqrt{\nu}}+\sqrt{\nu}(\sigma+\psi_{\rm I}(z_*))^2}{2z_*\sigma}+\frac{c_{\rm II}}{6}\log\frac{z_*^2\nu+(\sigma-\psi_{\rm II}(z_*))^2}{2\sqrt{\nu}z_*\sigma}\end{aligned}$$

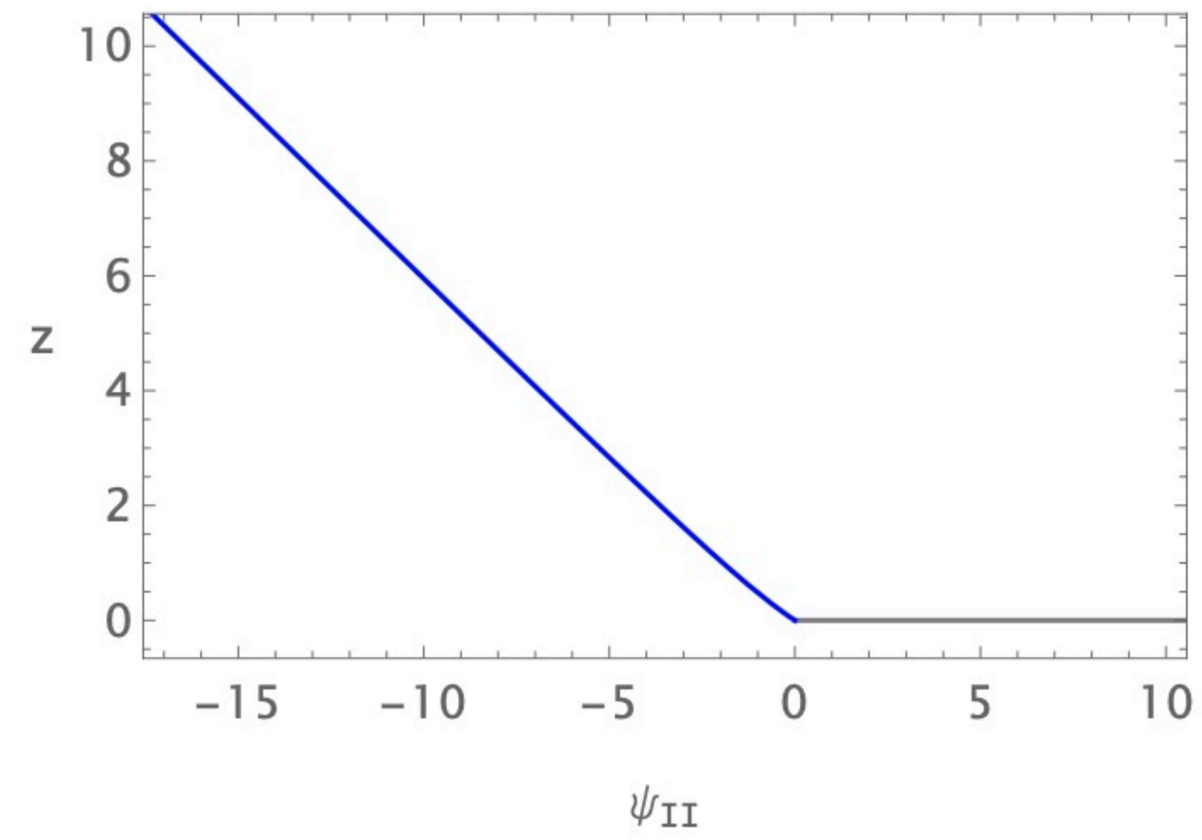
- For $\nu=\frac{L_{\rm II}}{L_{\rm I}}=1$ with supplementary configuration: $S_{\rm iE}=0$
- For ICFT (i.e. with a trivial scalar field), the interface entropy is a constant
- in general, $\log g(\sigma)\in(-\infty,\infty)$
- For a special case (CASE 2), $\log g(\sigma)\geq 0$ and $\frac{d}{d\sigma}S_{\rm iE}(\sigma)\leq 0$

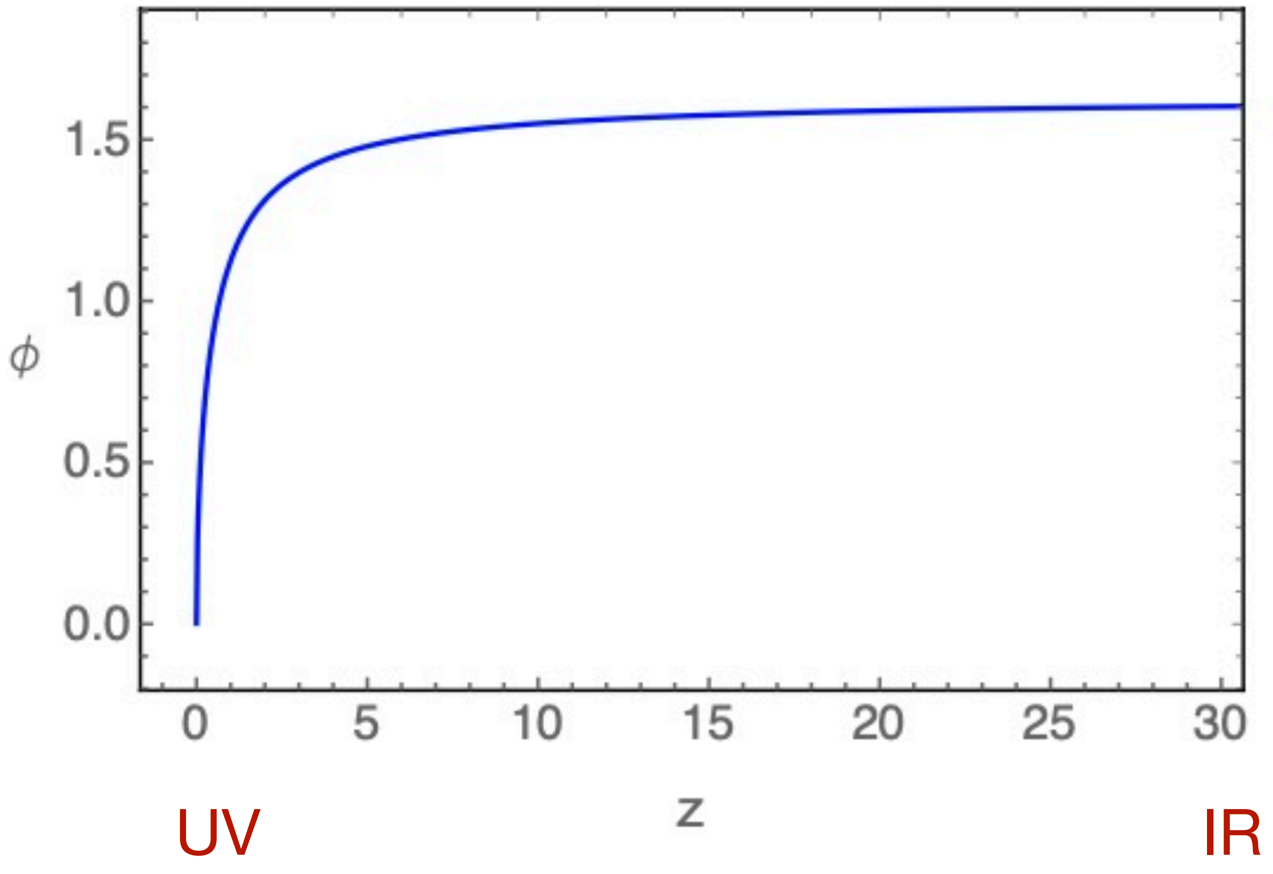
example of solution (1)

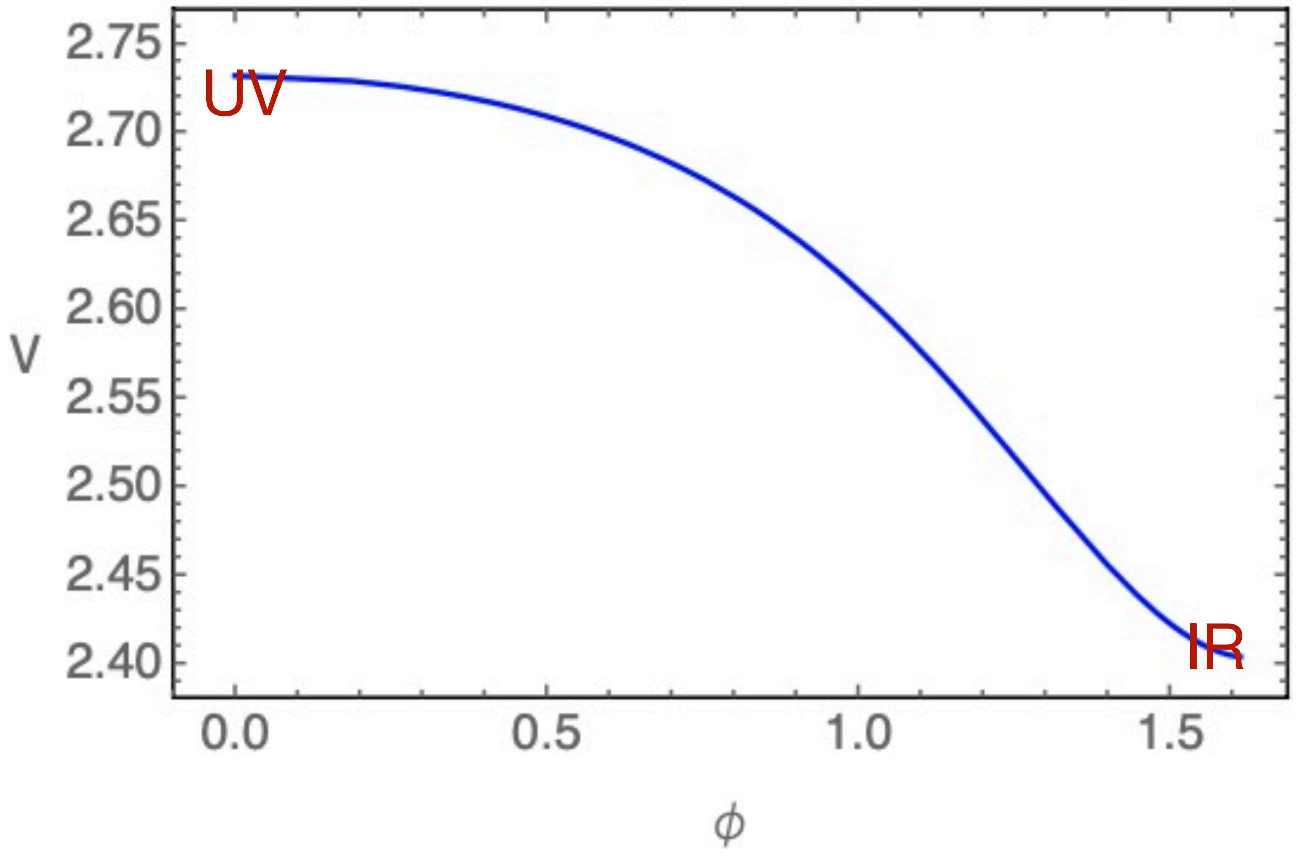
$$\psi_{\rm I}(z) = \frac{az + bz^2}{1+z}$$

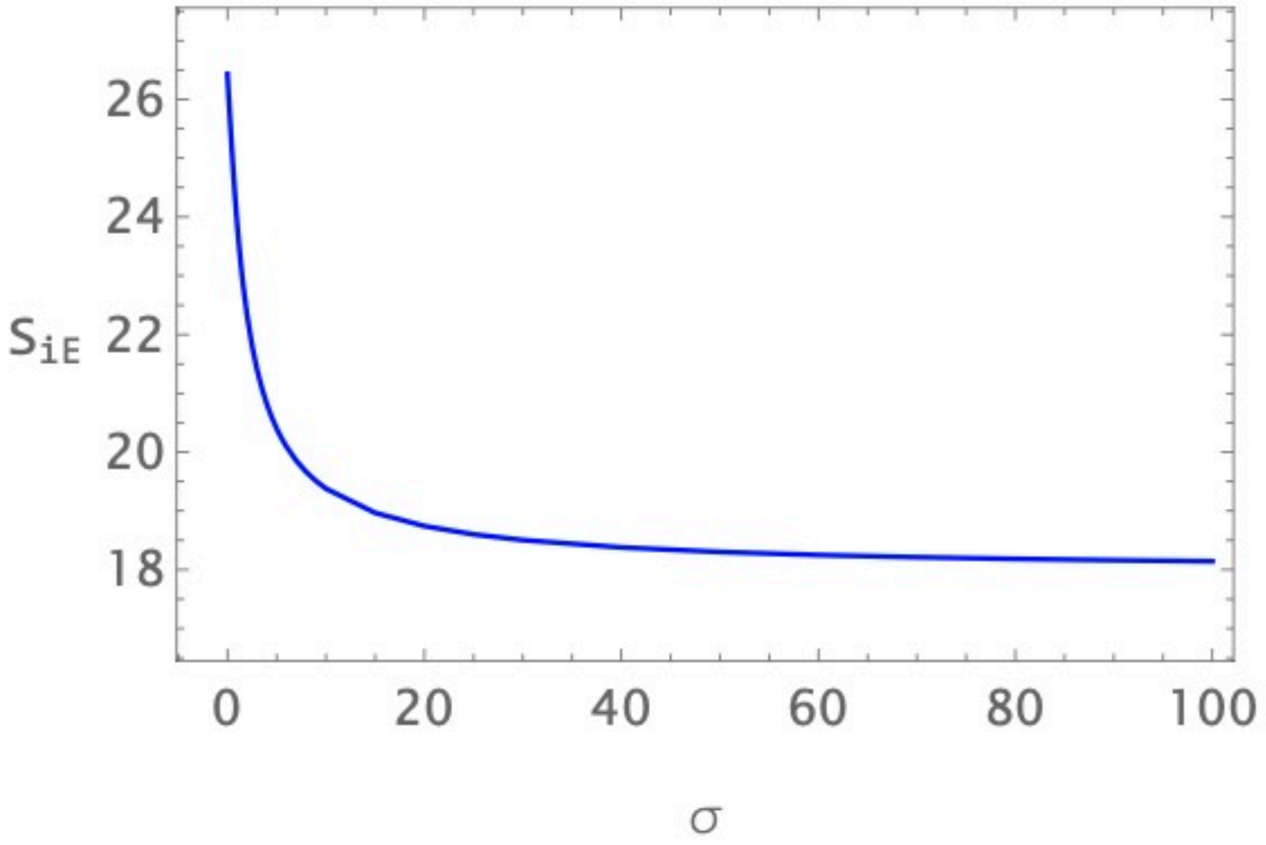
- NEC $\quad a \geq b$
- UV AdS2 ; IR AdS2









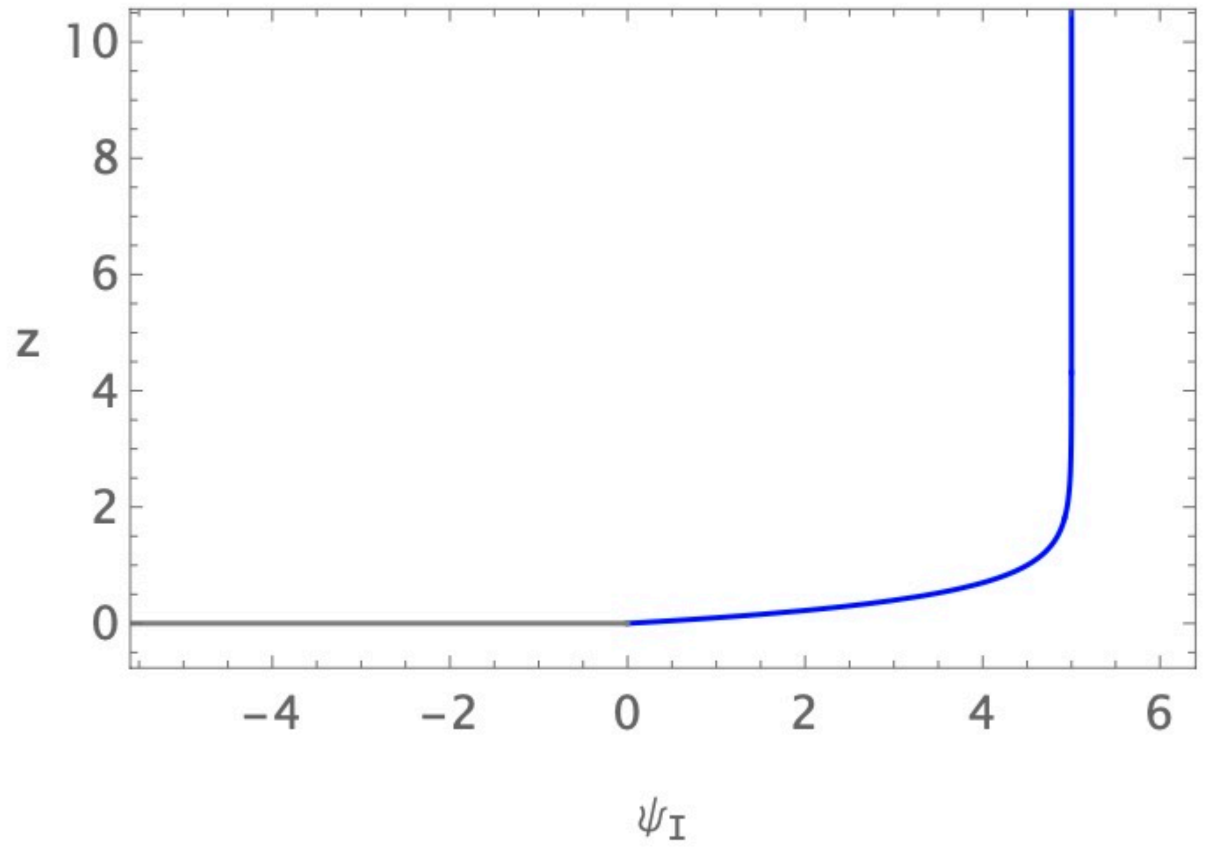


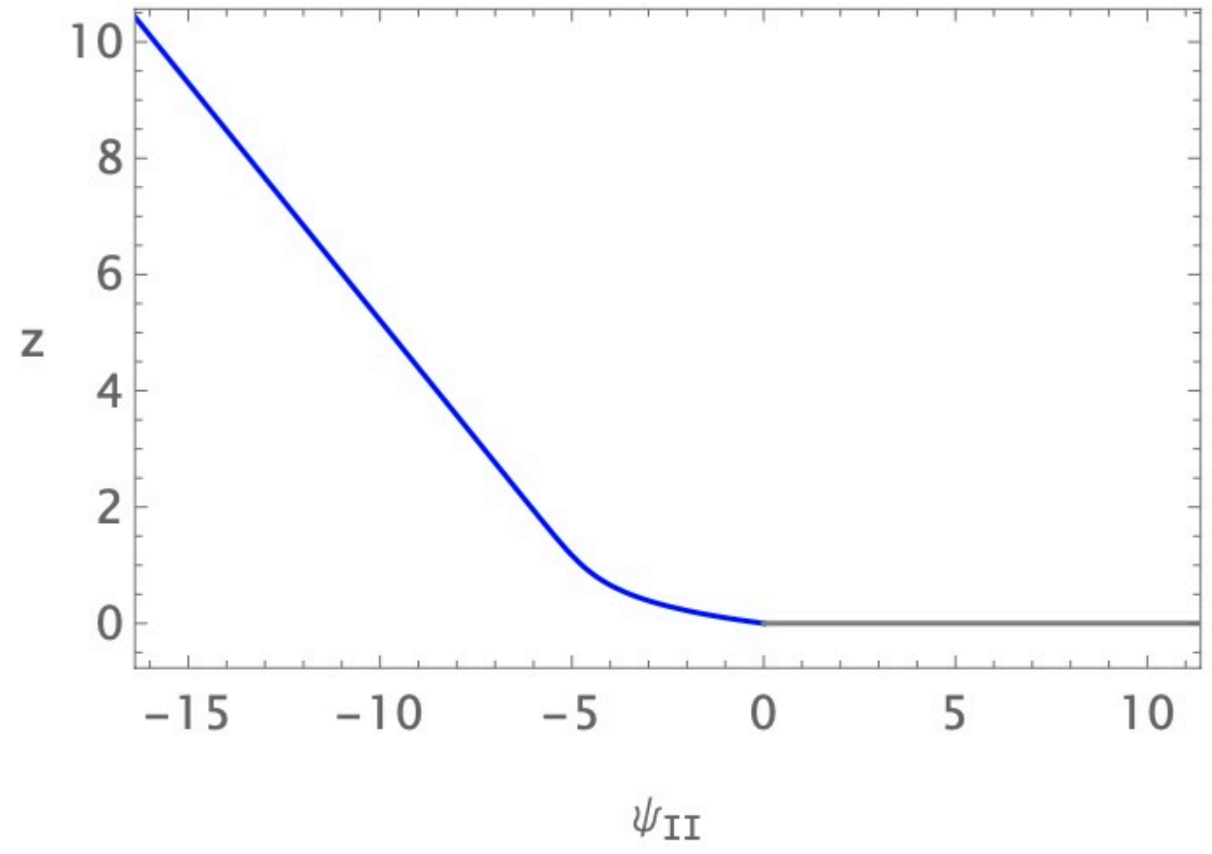
$a = 2\,, b = 1\,, L_{\rm I} = 1\,, \nu = 0.5.$

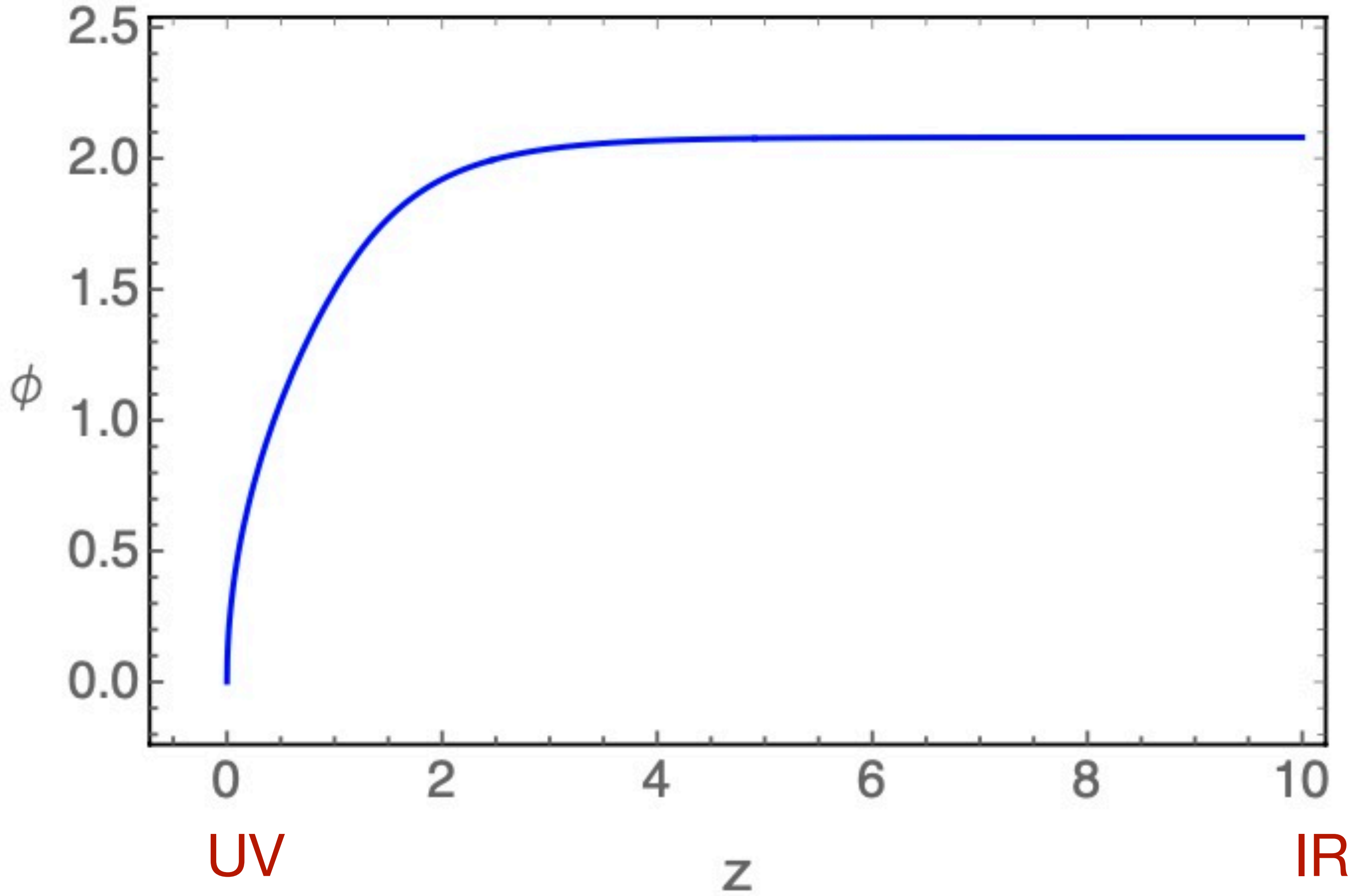
example of solution (2)

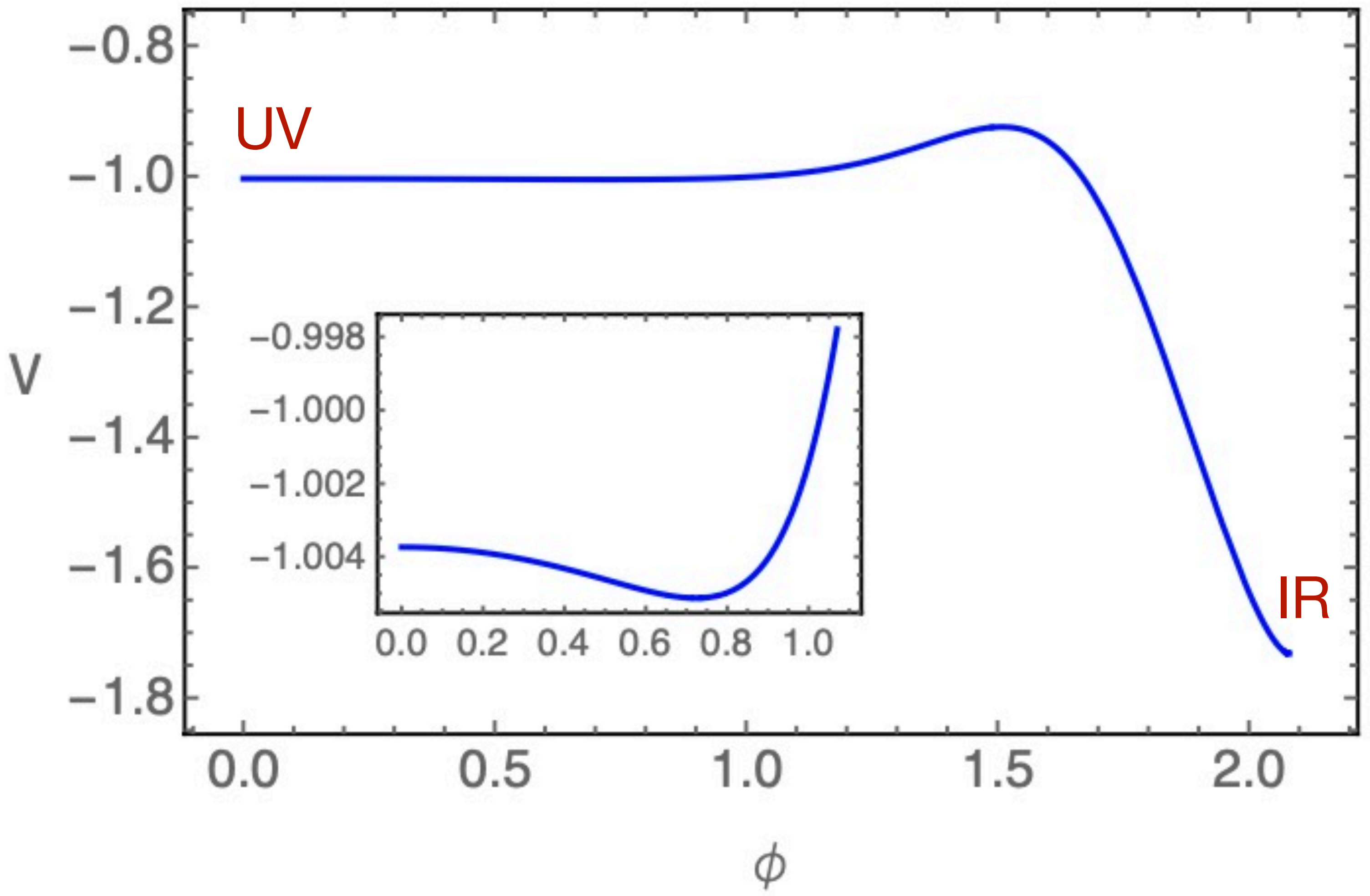
$$\psi_{\mathrm{I}} = \gamma \; a^{-z} - \gamma$$

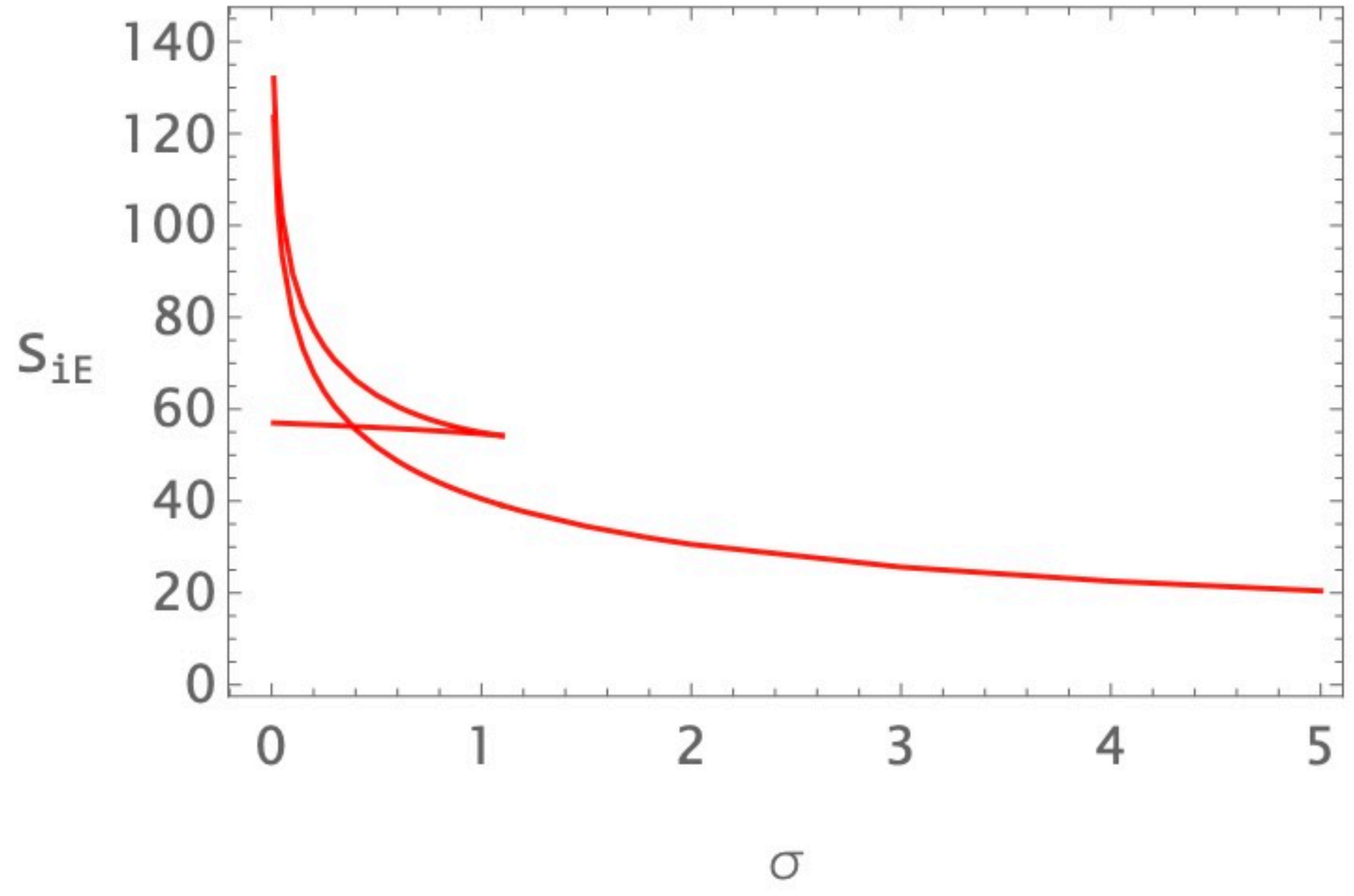
- NEC $\quad \gamma < 0$
- UV AdS2 ; IR AdS2







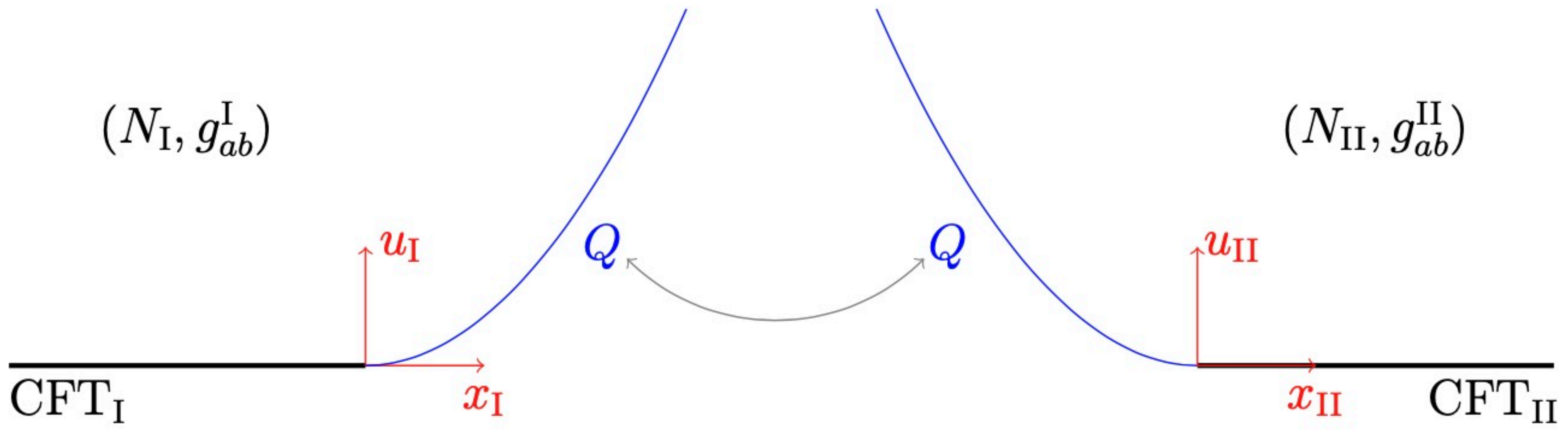




$\gamma = -5, a = 10, L_{\mathrm{I}} = 1$ and $\nu = 0.5$

Finite temperature

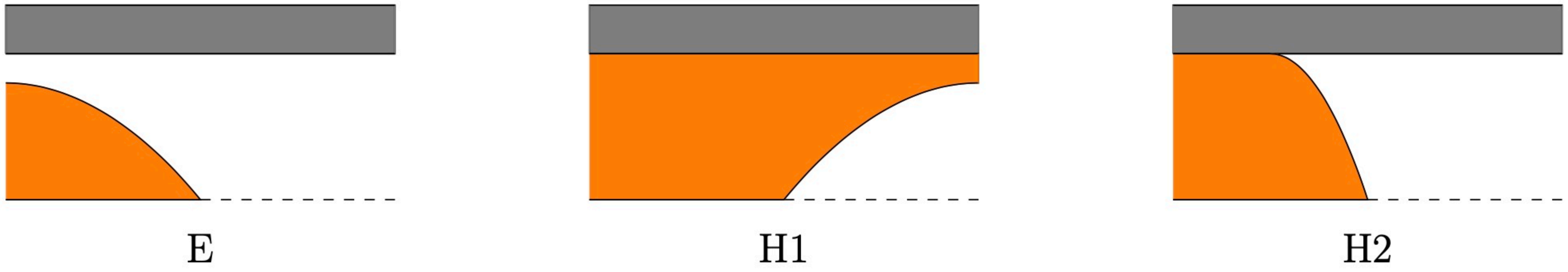
- Two possibilities
  - Gluing two BTZ black holes
  - Gluing a thermal AdS3 with a BTZ black hole



Gluing two BTZ black holes

- BTZ black hole

$$ds^2 = \frac{L_A^2}{u_A^2}\left[-f_A(u_A)dt_A^2 + \frac{du_A^2}{f_A(u_A)} + dx_A^2\right], \quad A = I, II, \qquad f_A(u_A) = 1 - \frac{u_A^2}{(u_A^H)^2}, \quad A = I, II. \qquad \Theta_I = \frac{1}{2\pi u_I^H}, \quad \Theta_{II} = \frac{1}{2\pi u_{II}^H}.$$



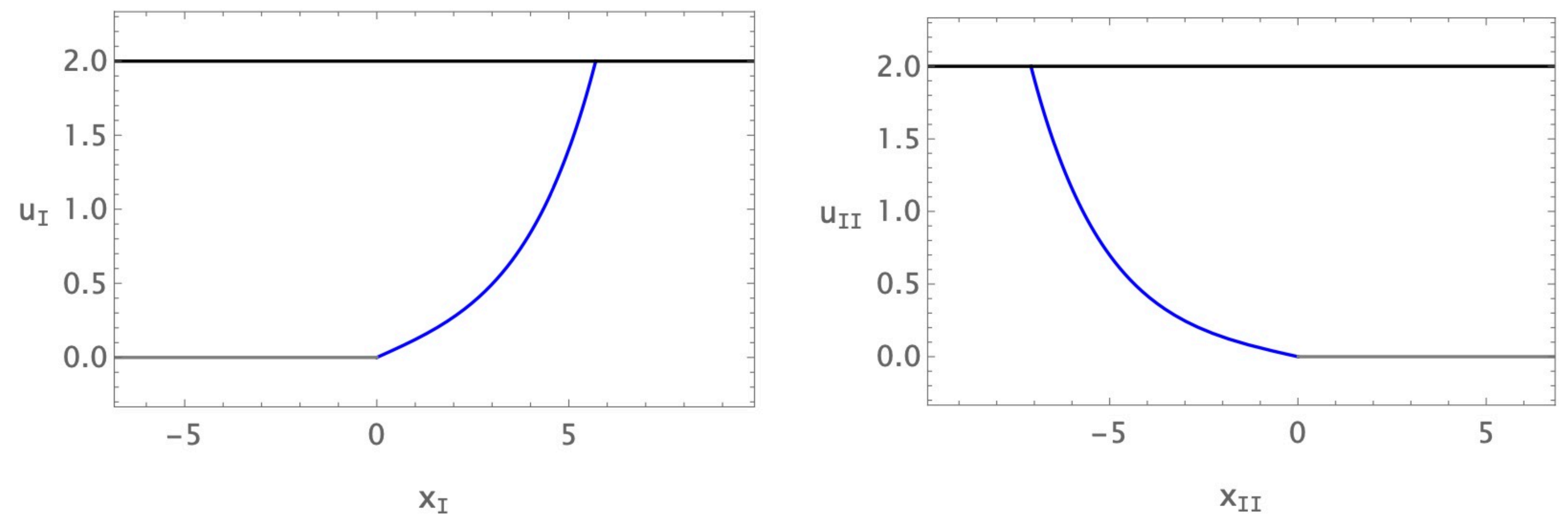
- For configurations H1,H2, the temperature of dual FT is the Hawking temperature, while for E, the temperature is arbitrary

- NEC is consistent with the existence of configurations

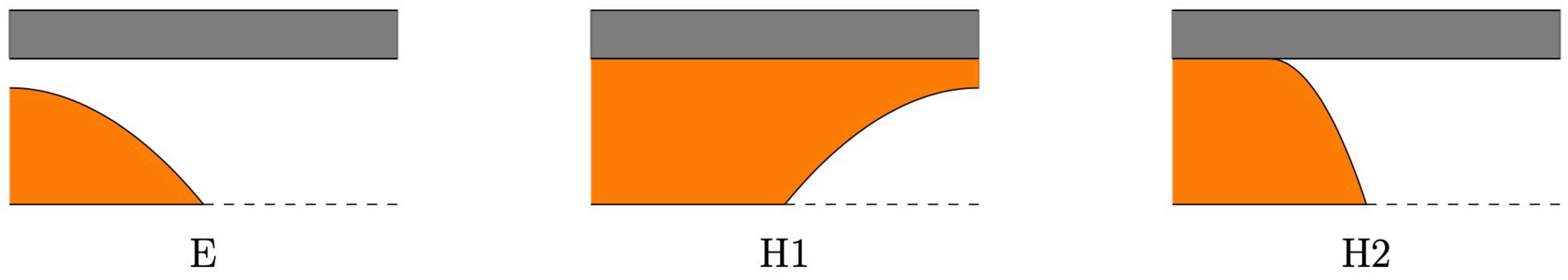
Gluing two BTZ black holes

- With trivial scalar field $\phi = 0, V = T$

[H2, H2]



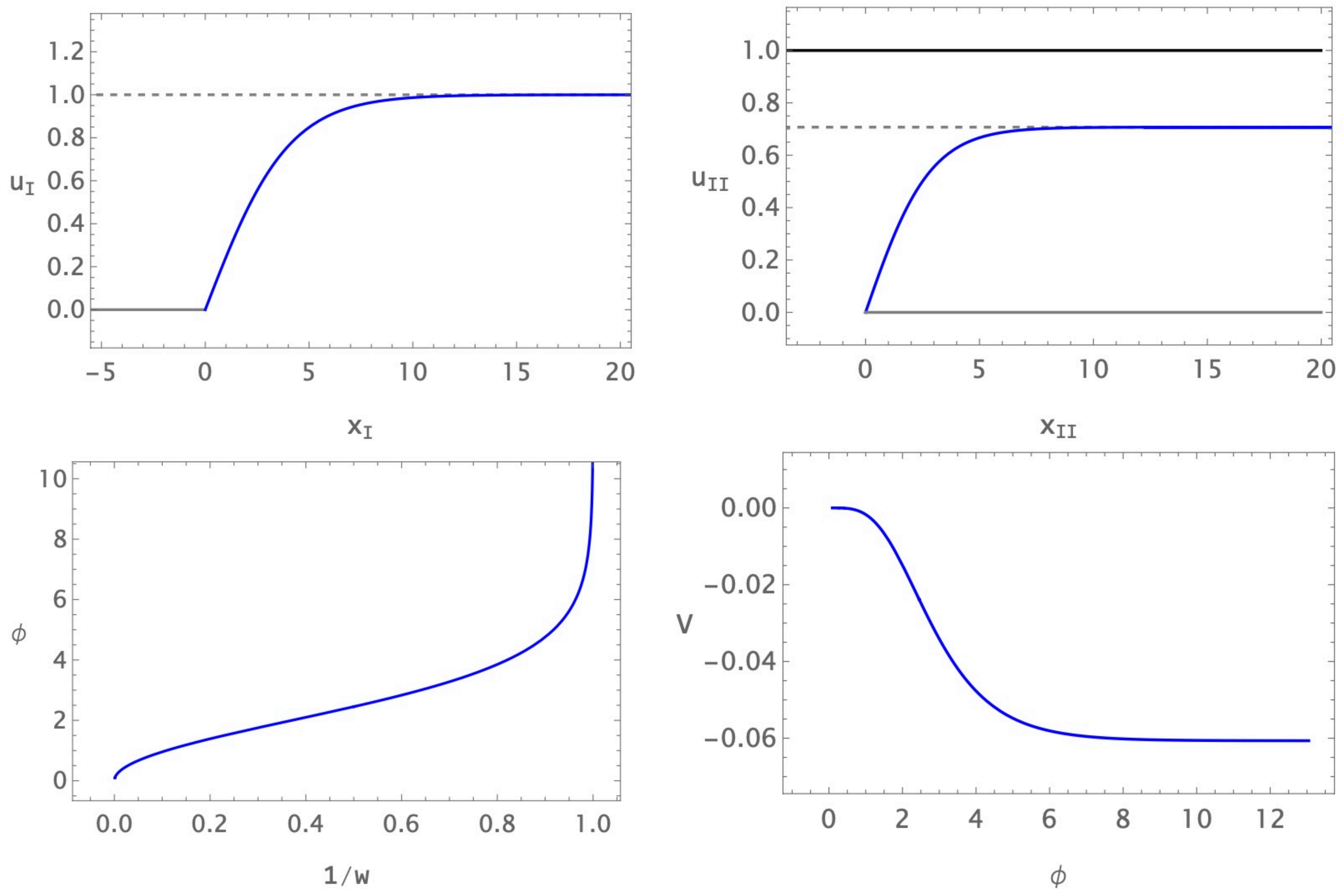
- With nontrivial scalar field,



	E	H1	H2
E	√	√	×
H1	√	×	×
H2	×	×	√

Gluing thermal AdS and BTZ black hole

- No permissible configuration without scalar field
- With a scalar field, the only allowed configuration is $[\mathrm{E_{tAdS}},\, \mathrm{E}]$ and $[\mathrm{E},\, \mathrm{E_{tAdS}}]$



no $[\mathrm{E_{tAdS}},\, \mathrm{H2}]$ and $[\mathrm{H2},\, \mathrm{E_{tAdS}}]$ due to metric compatibility

no $[\mathrm{E_{tAdS}},\, \mathrm{H1}],\, [\mathrm{H1},\, \mathrm{E_{tAdS}}]$ due to NEC

comments on [E, E]

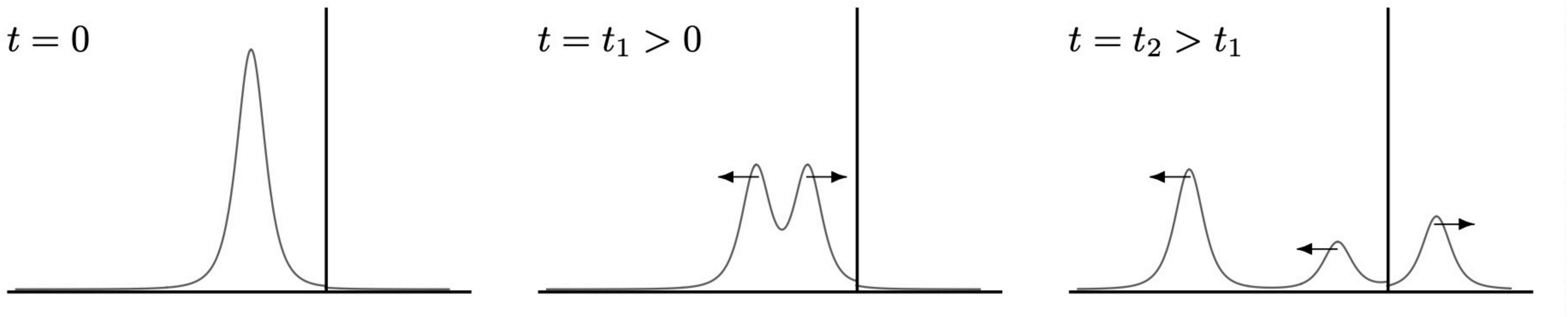
- With a scalar field, we have different types of empty-empty configurations

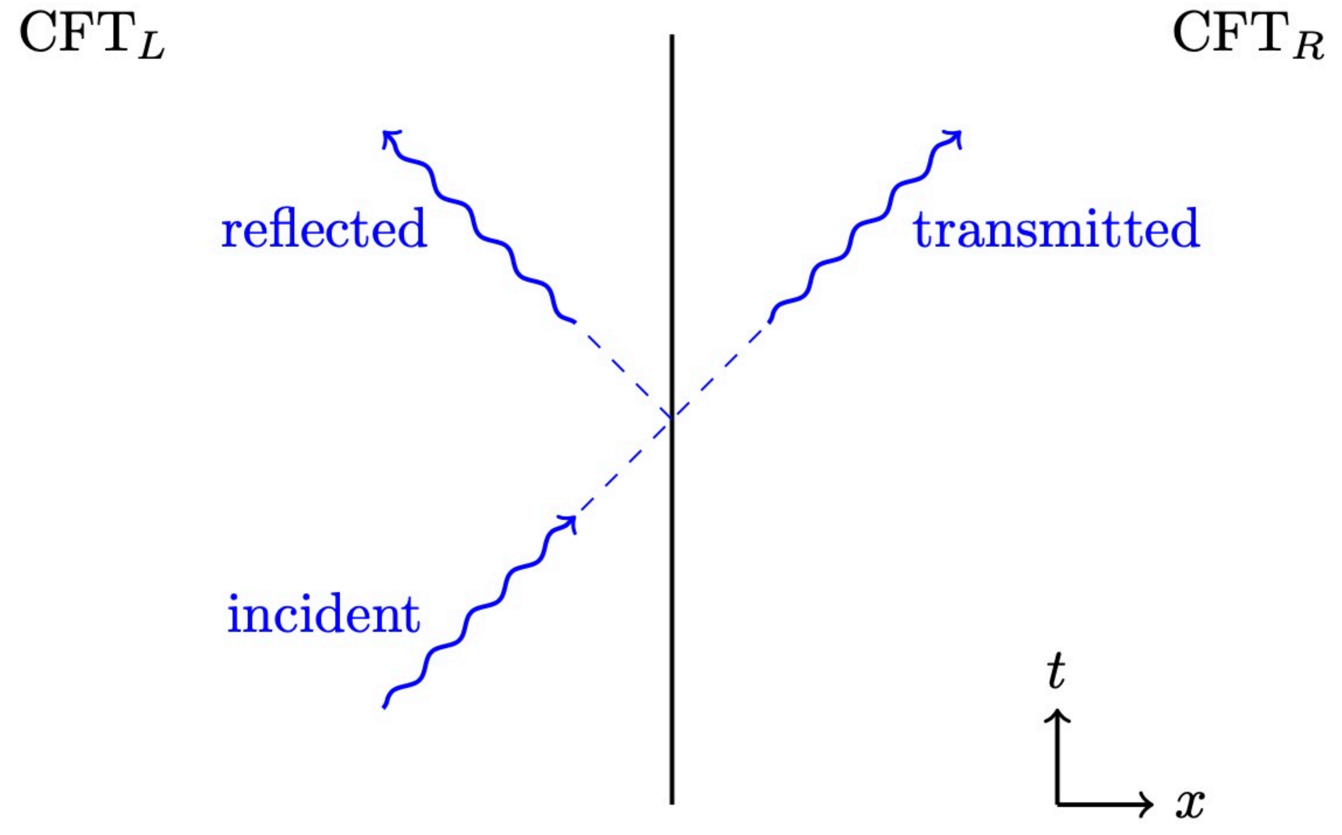
$$[\mathrm{E}_{\mathrm{tAdS}},\mathrm{E}],\ [\mathrm{E},\mathrm{E}_{\mathrm{tAdS}}],\ [\mathrm{E}_{\mathrm{AdS}},\mathrm{E}_{\mathrm{AdS}}]\ \text{and}\ [\mathrm{E},\mathrm{E}]$$

- In principle, all these configurations can be set to T=0
- The dual field theories are different
- stability? dual CFT?

Energy transport

- in field theory:





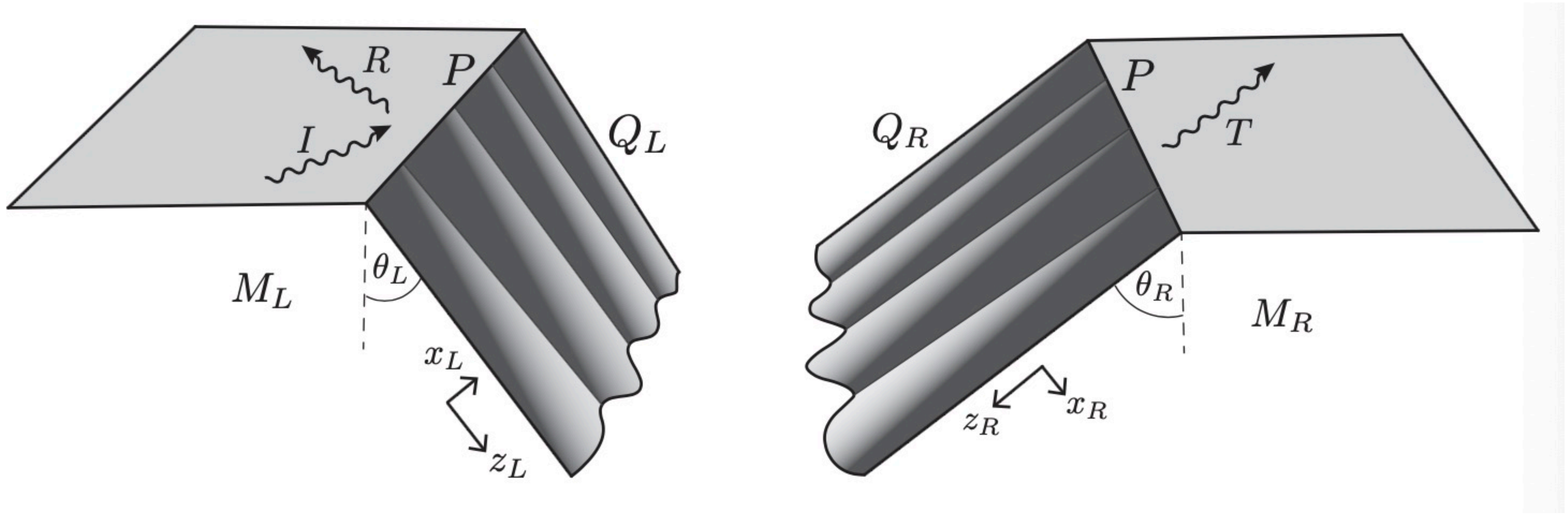
$$\langle T_L(z) T_R(w) \rangle = \frac{c_{LR}}{2(z-w)^4}$$

$$\mathcal{T}_{LR} = \frac{\text{transmitted energy}}{\text{injected energy}} = \frac{c_{LR}}{c_L}$$

[Quella, Runkel, Watts , 2007; Meineri, Penedones, Rousset, 2020]

Energy transport in conformal interface

- In holography



$$ds_L^2 = \frac{\ell_L^2}{y_L^2}[dy_L^2 + du_L^2 - dt_L^2] \quad \text{for } u_L \leq y_L \tan\theta_L,$$

$$ds_R^2 = \frac{\ell_R^2}{y_R^2}[dy_R^2 + du_R^2 - dt_R^2] \quad \text{for } u_R \geq -y_R \tan\theta_R,$$

$$[ds^2]_L^{(2)} = 4G\ell_L\epsilon\Big[e^{i\omega(t_L-u_L)}d(t_L-u_L)^2 + \mathcal{R}_L\, e^{i\omega(t_L+u_L)}d(t_L+u_L)^2\Big] + c.c.$$

$$[ds^2]_R^{(2)} = 4G\ell_R\epsilon\,\mathcal{T}_L e^{i\omega(t_R-u_R)}\,d(t_R-u_R)^2 + c.c.\,,$$

$$\mathcal{T}_{L,R} = \frac{2}{\ell_{L,R}}\left[\frac{1}{\ell_L} + \frac{1}{\ell_R} + 8\pi G\sigma\right]^{-1}$$

$$\mathcal{R} = 1 - \mathcal{T}$$

$$0 \leq \frac{1}{\ell_R} - \frac{1}{\ell_L} \leq 8\pi G\sigma \leq \frac{1}{\ell_R} + \frac{1}{\ell_L}$$

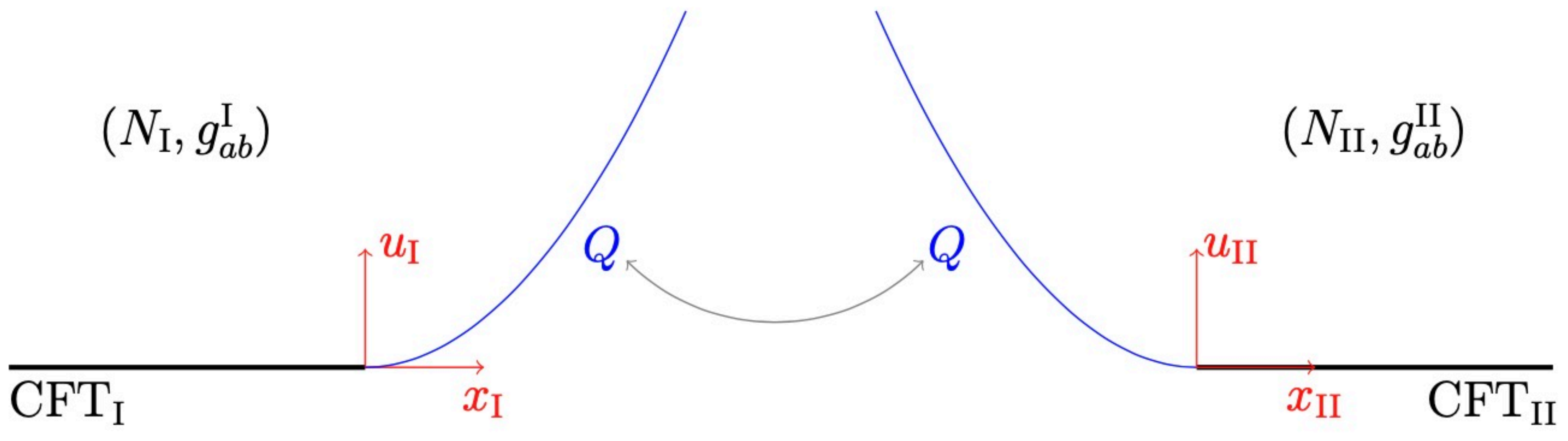
When the central charges are the same,

$$\frac{1}{2} \leq \mathcal{T}_L \leq 1$$

[Bachas, Chapman, Ge, Policastro, 2020]

Energy transport in non-conformal interface

- non-conformal interface (with a nontrivial scalar field)



- Complex energy transmission coefficient

$$T_{\rm in}/\epsilon = e^{i\omega(t_{\rm I}-x_{\rm I})} + c.c.\,, \quad T_{\rm re}/\epsilon = \mathcal{R} e^{i\omega(t_{\rm I}+x_{\rm I})} + c.c.\,, \quad T_{\rm tr}/\epsilon = \mathcal{T} e^{i\omega(t_{\rm II}-x_{\rm II})} + c.c.\,.$$

$$\tilde{\mathcal{R}} = \frac{\text{reflected energy flux}}{\text{injected energy flux}} = \frac{T_{\rm re}}{T_{\rm in}} = \frac{|\mathcal{R}|\cos(\omega t + \phi_r)}{\cos(\omega t)}$$

$$\tilde{\mathcal{T}} = \frac{\text{transmitted energy flux}}{\text{injected energy flux}} = \frac{T_{\rm tr}}{T_{\rm in}} = \frac{|\mathcal{T}|\cos(\omega t + \phi_t)}{\cos(\omega t)}$$

$$\langle \tilde{\mathcal{R}} \rangle = \mathcal{R}'\,, \qquad \langle \tilde{\mathcal{T}} \rangle = \mathcal{T}'$$

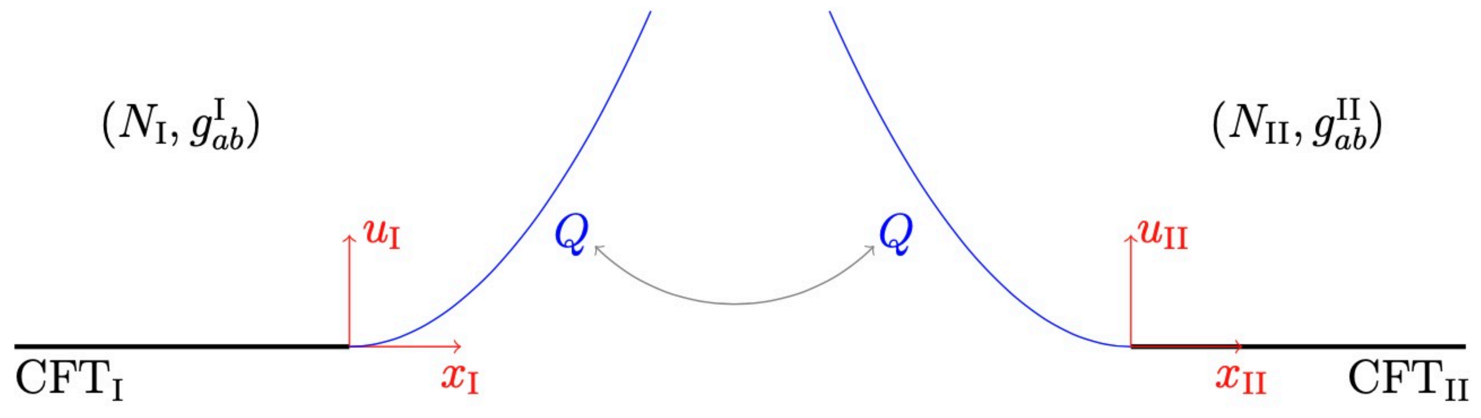
$$\mathcal{T} = |\mathcal{T}|\, e^{i\phi_t} = \mathcal{T}' + i\mathcal{T}''$$

$$\mathcal{R} + \mathcal{T} = 1$$

Energy transport in non-conformal interface

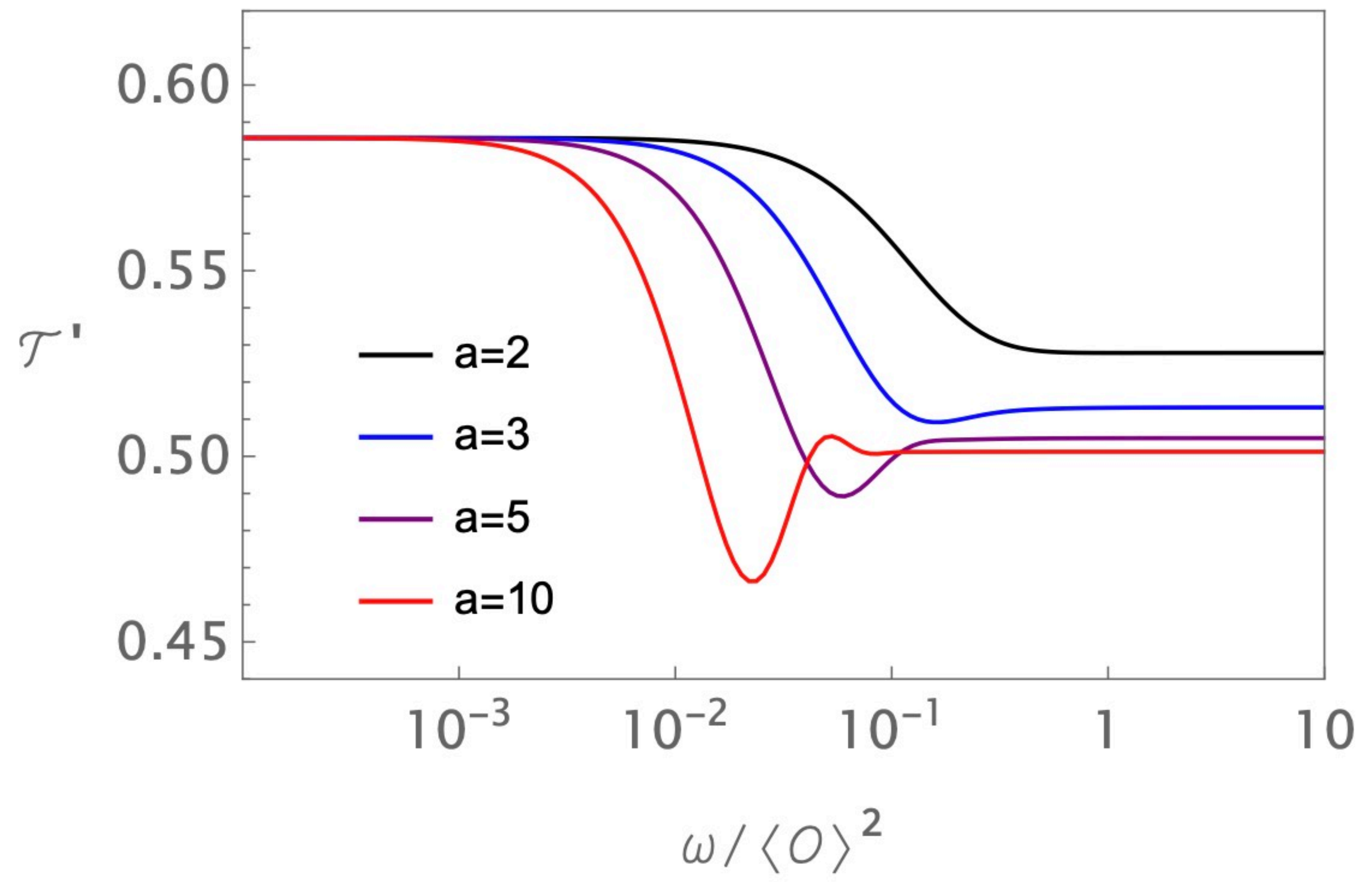
- Profile of non-conformal interface

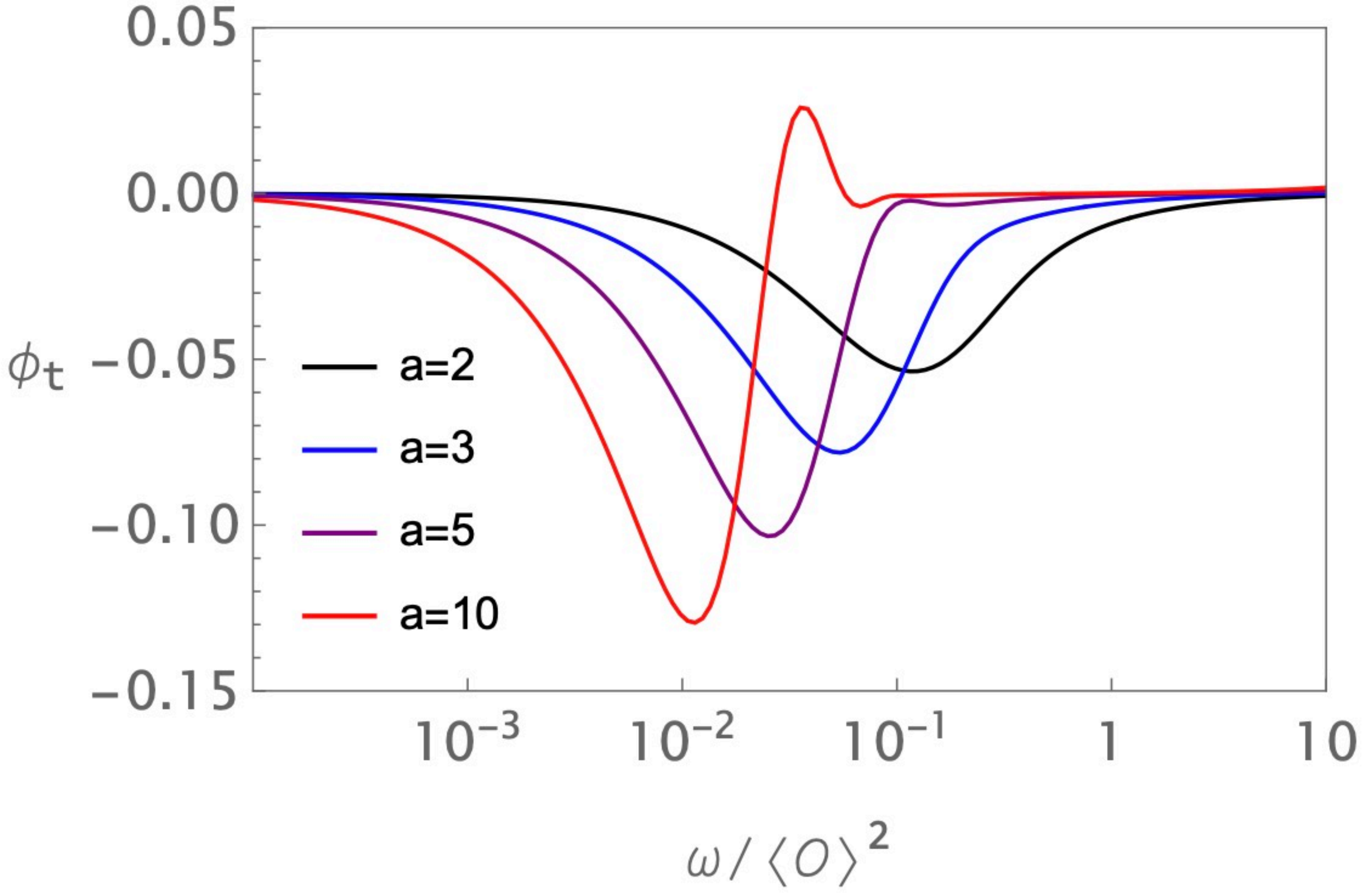
$$\psi(z) = az + \frac{(b-a)z^2}{1+z}$$



$$\psi(z) = az + (b-a)z^2 + \mathcal{O}(z^3), \qquad z \longrightarrow 0 \qquad \text{(UV)}$$

$$\psi(z) = bz - (b-a) + \mathcal{O}\left(\frac{1}{z}\right), \qquad z \longrightarrow \infty \qquad \text{(IR)}$$



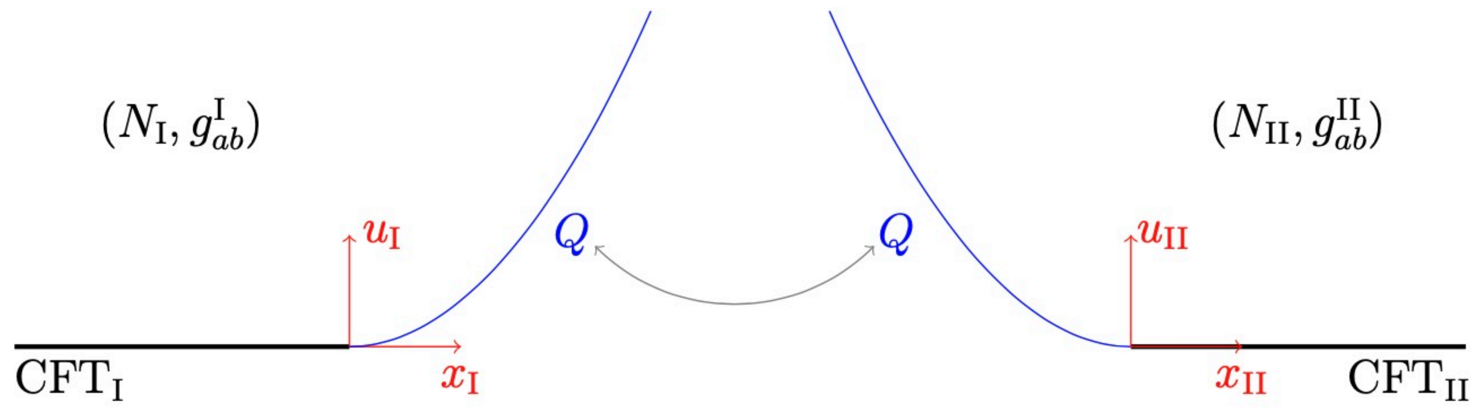


complex energy transmission coefficients

Energy transport in non-conformal interface

- Profile of non-conformal interface

$$\psi(z) = az + \frac{(b-a)z^2}{1+z}$$



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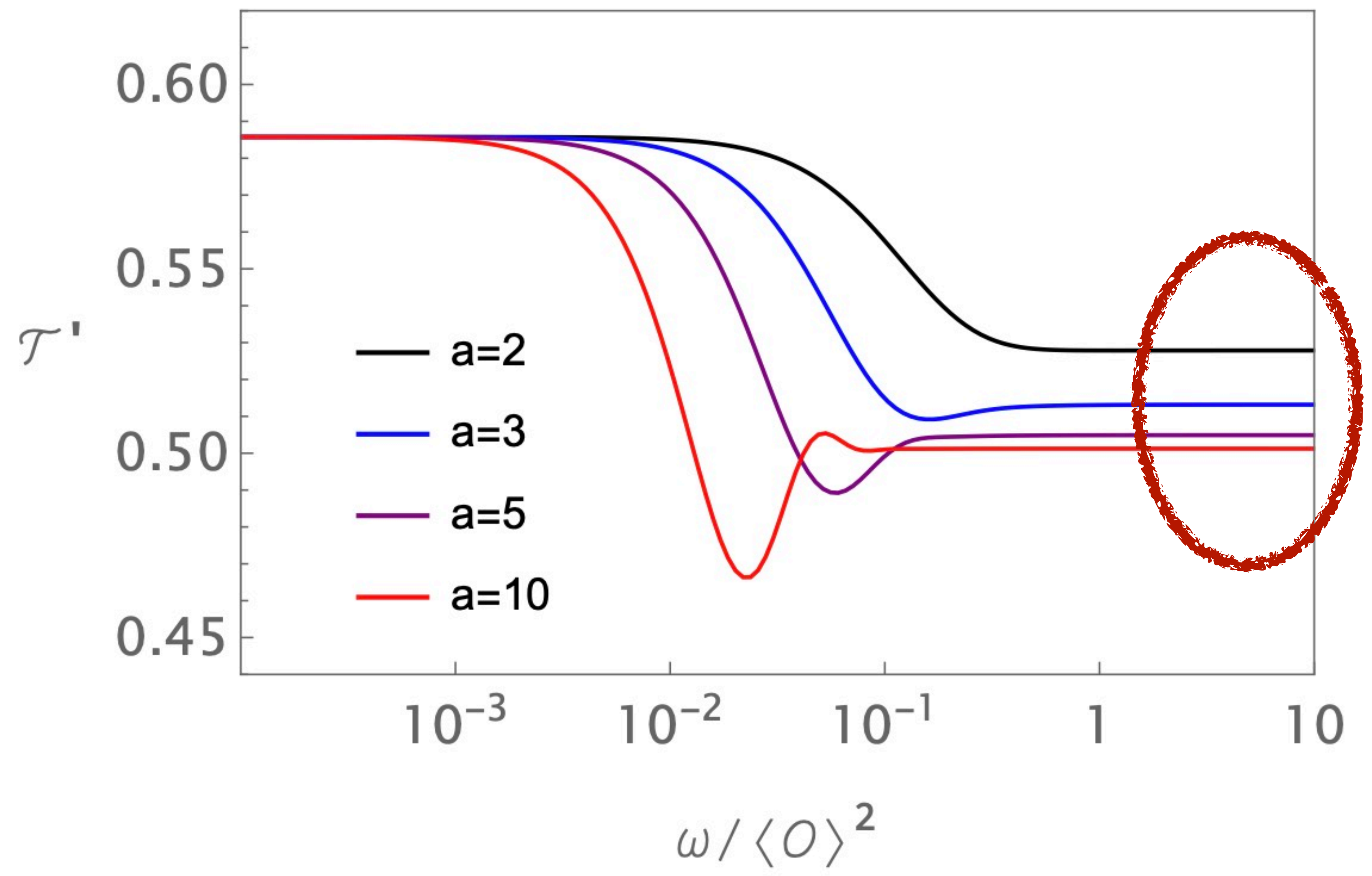
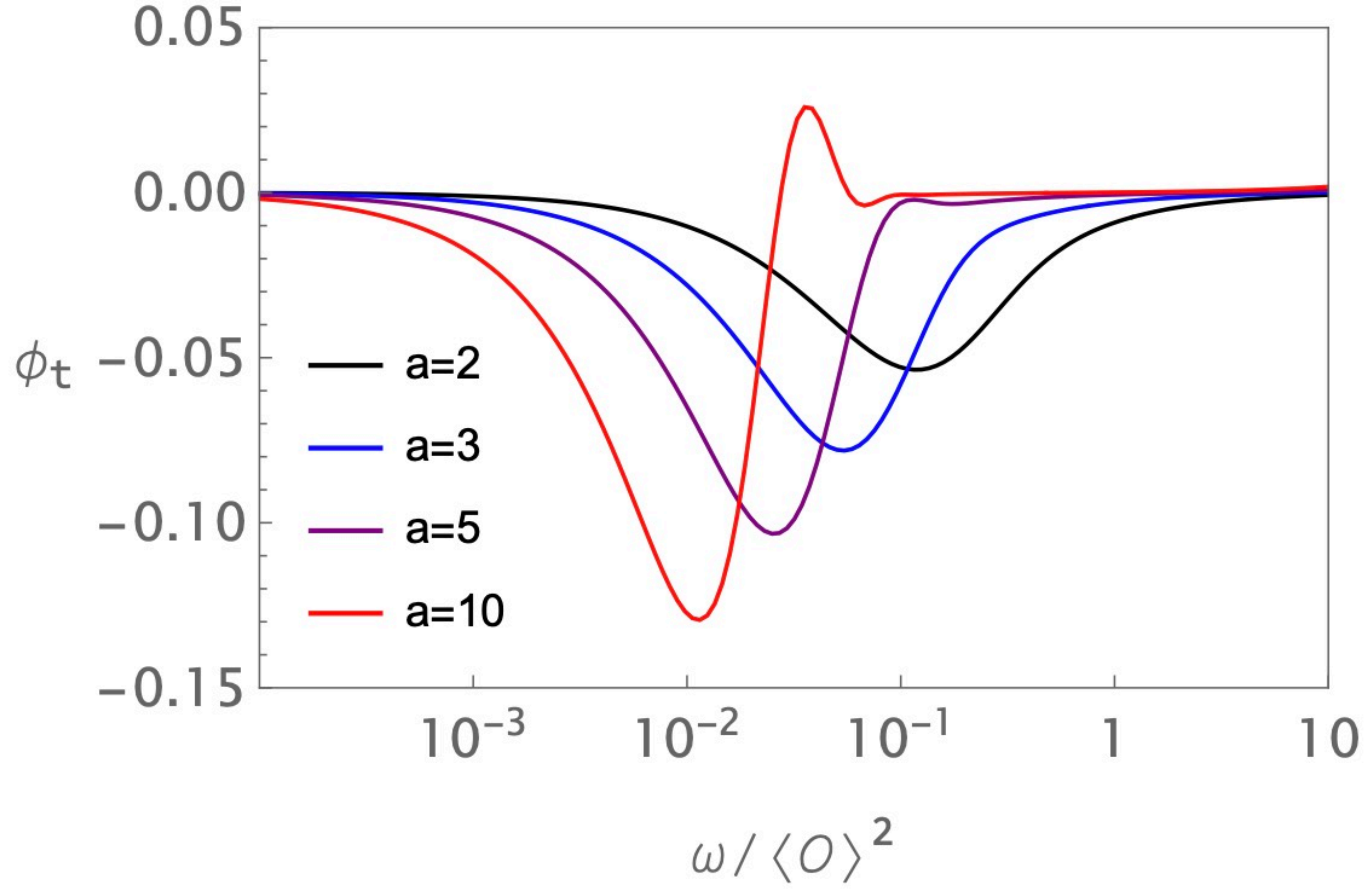


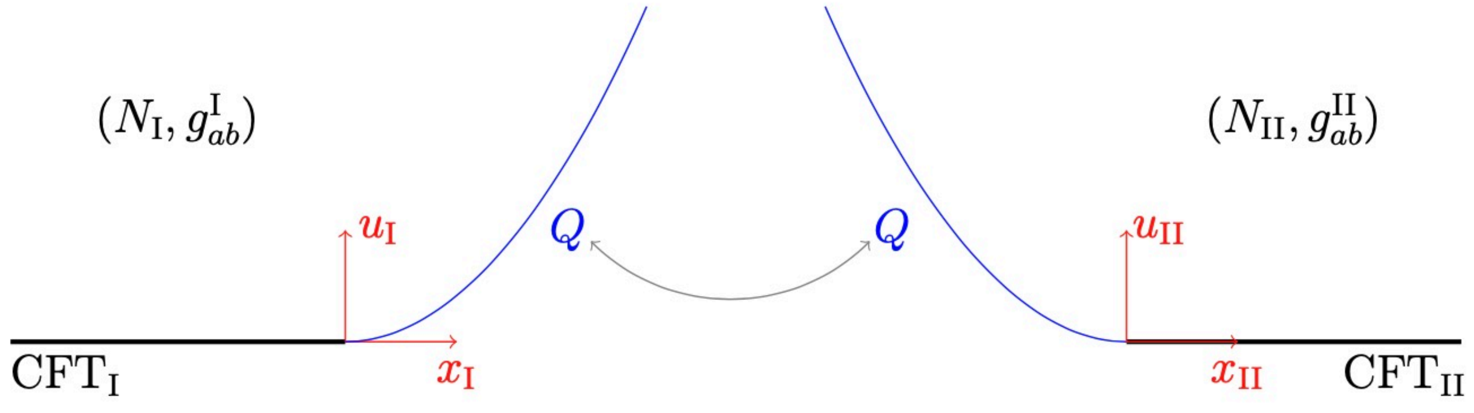


high frequency: real, the transmission coefficient approaches values of UV ICFT

Energy transport in non-conformal interface

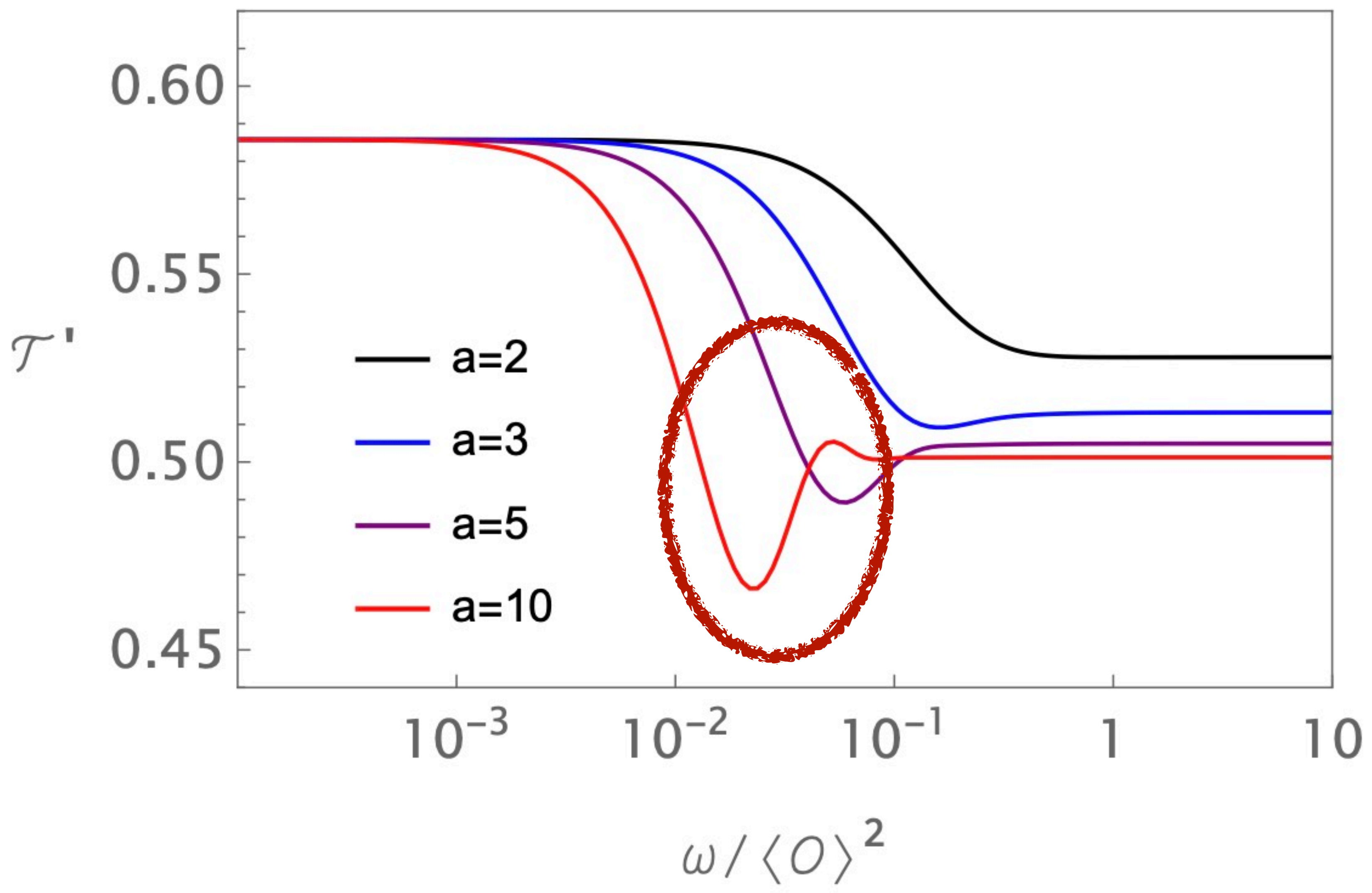
- Profile of non-conformal interface

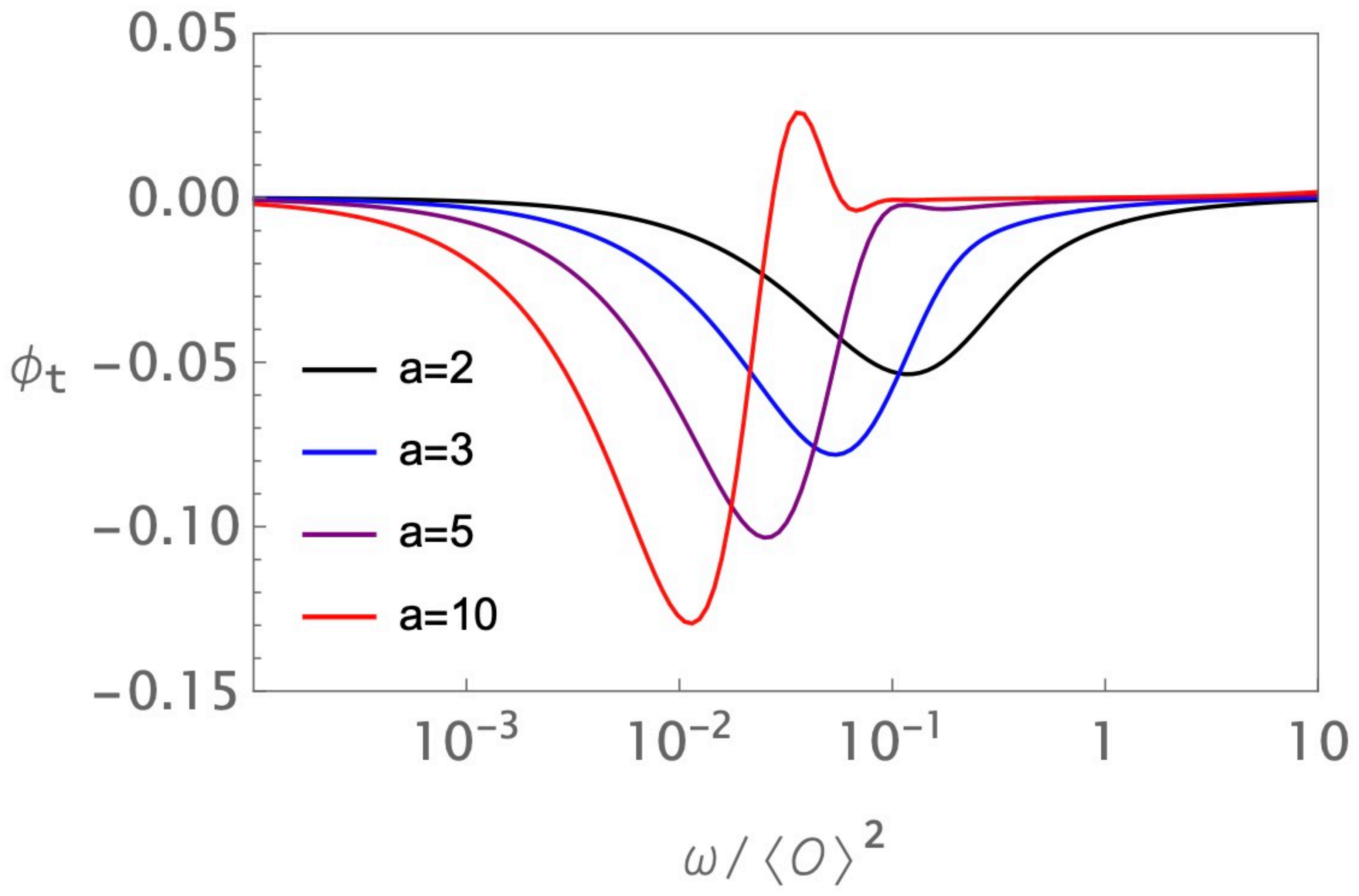
$$\psi(z) = az + \frac{(b-a)z^2}{1+z}$$



$$\psi(z) = az + (b-a)z^2 + \mathcal{O}(z^3), \qquad z \longrightarrow 0 \qquad \text{(UV)}$$

$$\psi(z) = bz - (b-a) + \mathcal{O}\left(\frac{1}{z}\right), \qquad z \longrightarrow \infty \qquad \text{(IR)}$$



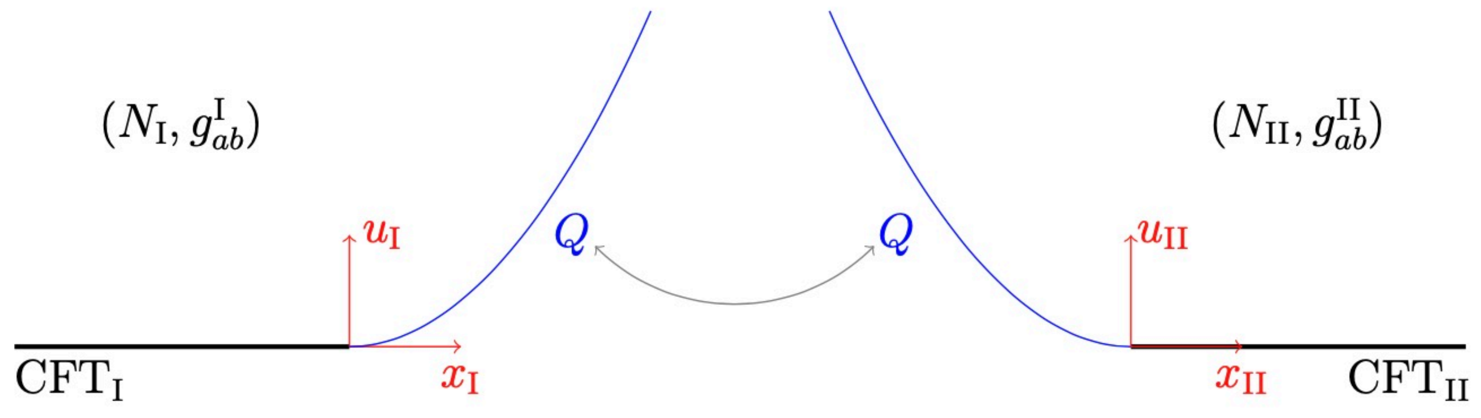


high frequency: real, the transmission coefficient approaches values of UV ICFT

oscillations in the intermediate energy scale, (break the bound of ICFT)

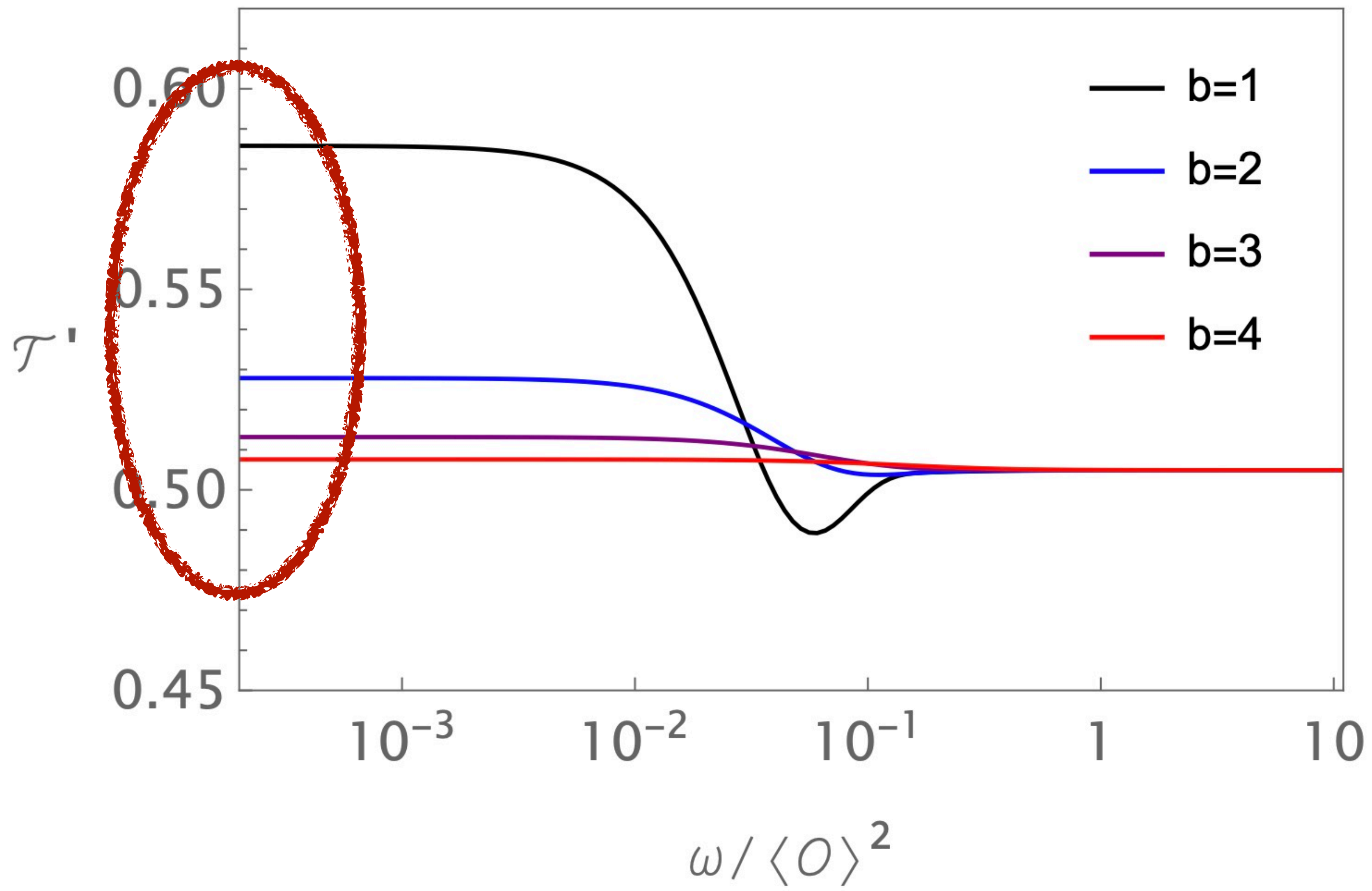
$\frac{1}{2}$ **X** $\mathcal{T}_L \leq 1$

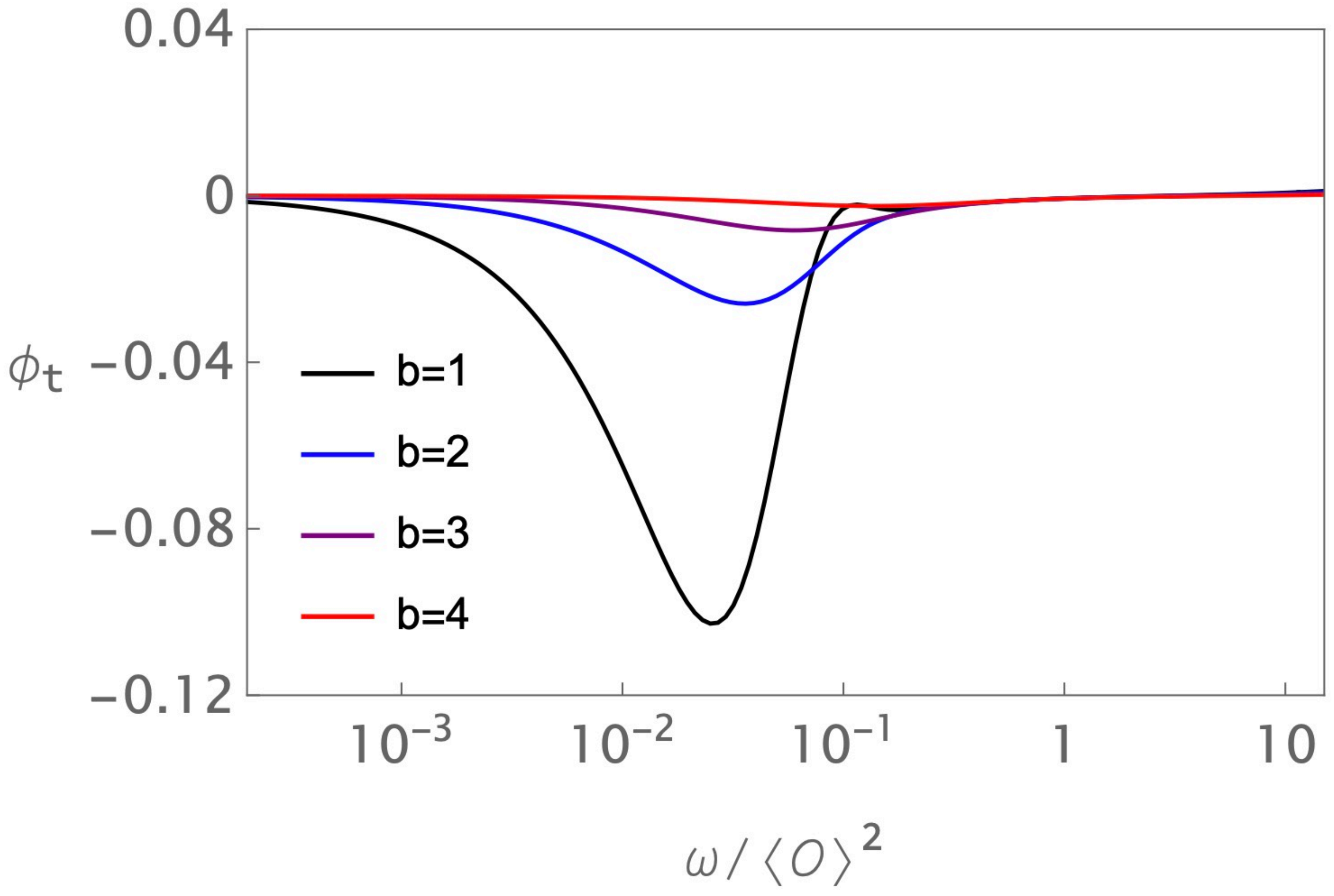
Energy transport in non-conformal interface



$$\psi(z) = az + (b-a)z^2 + \mathcal{O}(z^3)\,, \qquad z \longrightarrow 0 \qquad \text{(UV)}$$

$$\psi(z) = bz - (b-a) + \mathcal{O}\left(\frac{1}{z}\right), \qquad z \longrightarrow \infty \qquad \text{(IR)}$$

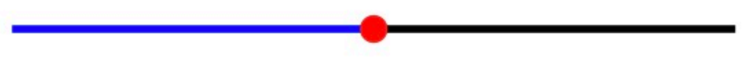


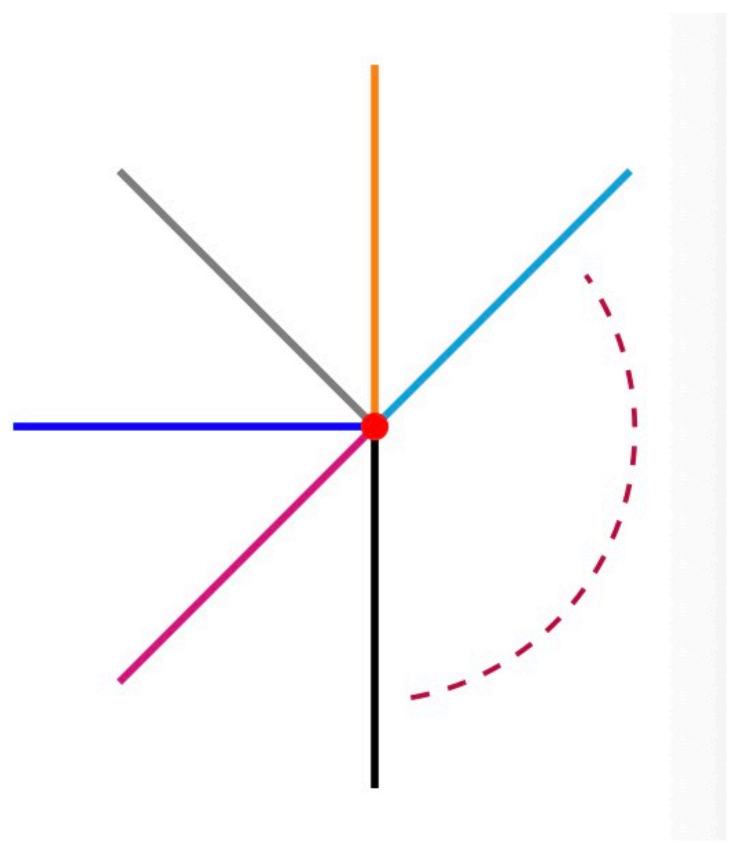


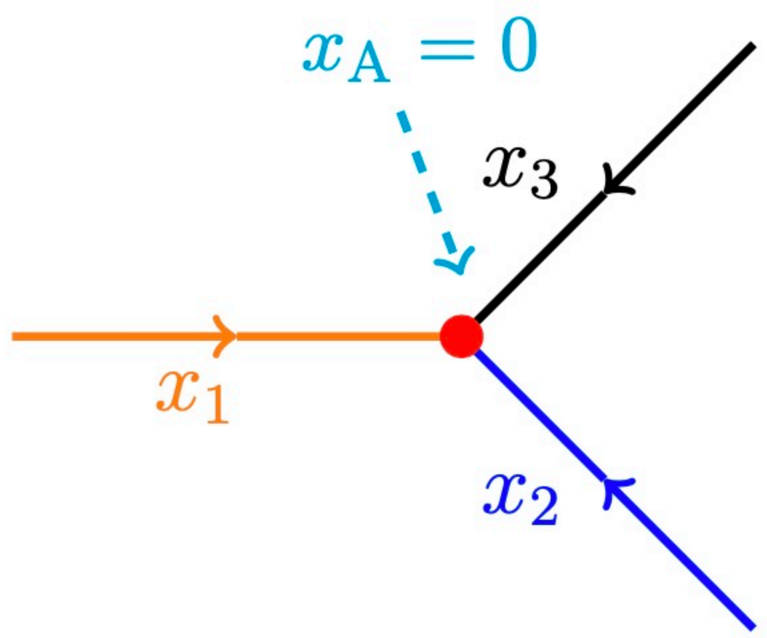
small frequency: the transmission coefficient approaches the value of IR ICFT

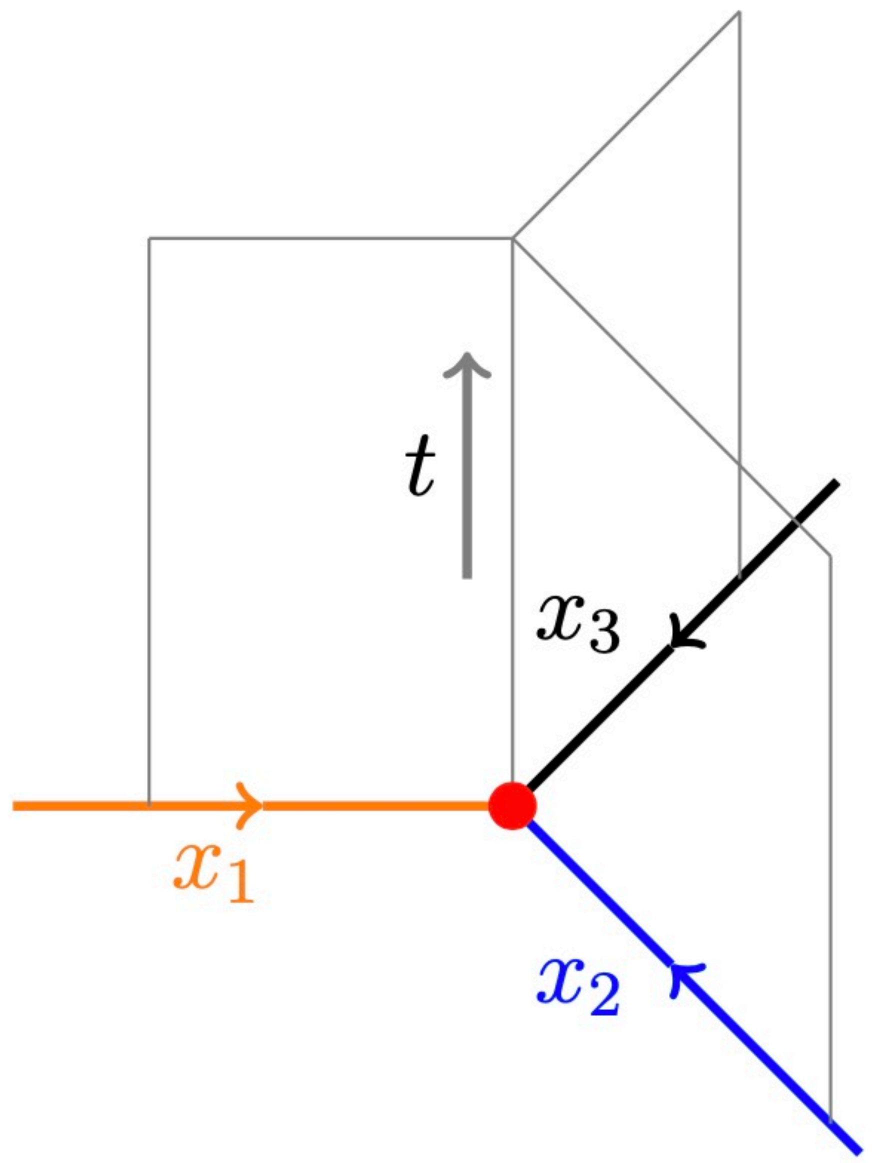
From ICFT$_2$ to ICFT$_3$

- at a constant time slice



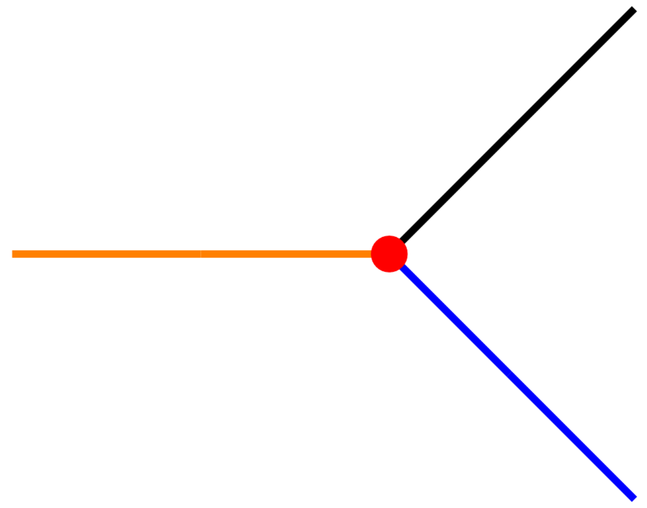




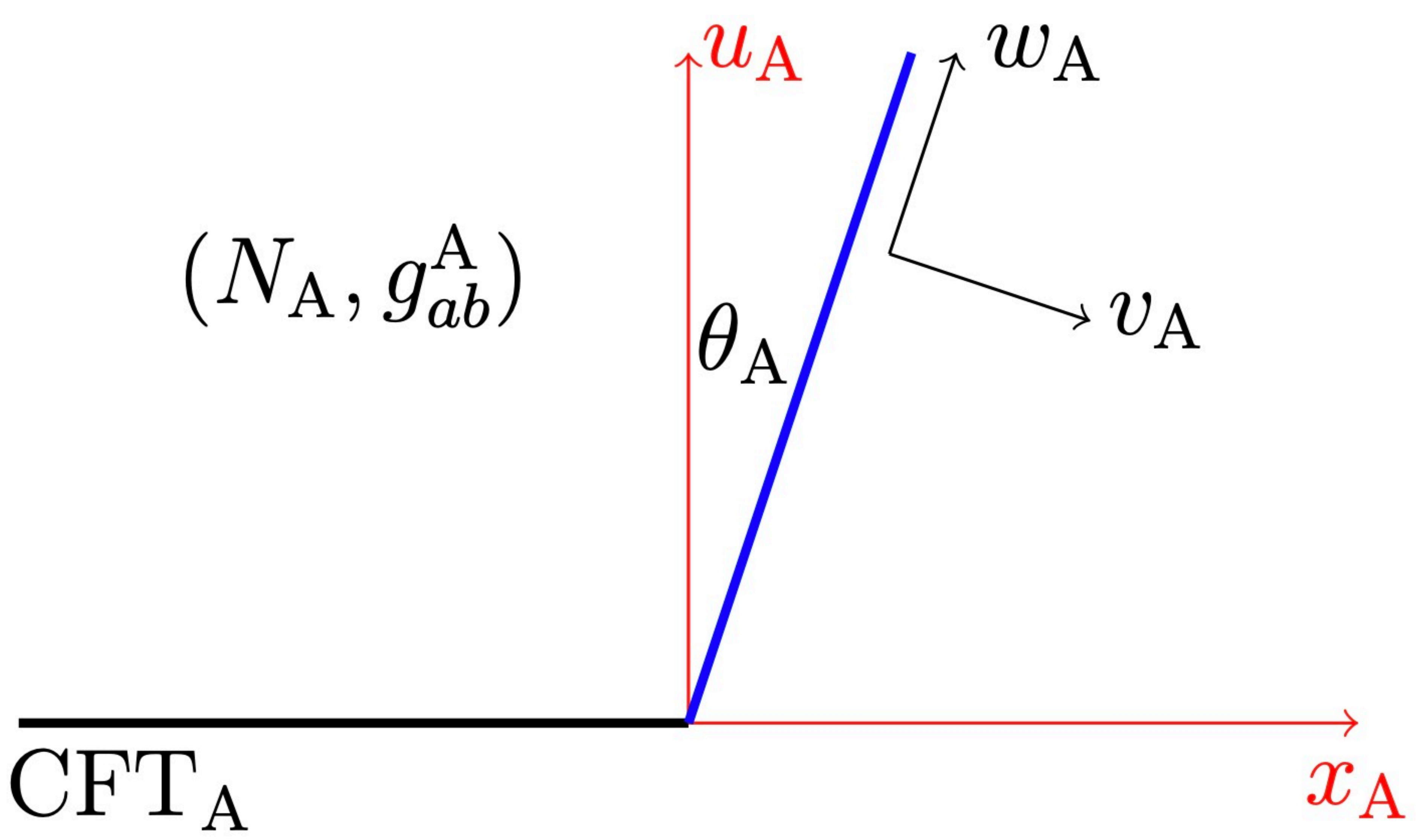


[Talk by Rong-Xin]

AdS/ICFT₃



- Gluing three AdS/BCFT along a common junction



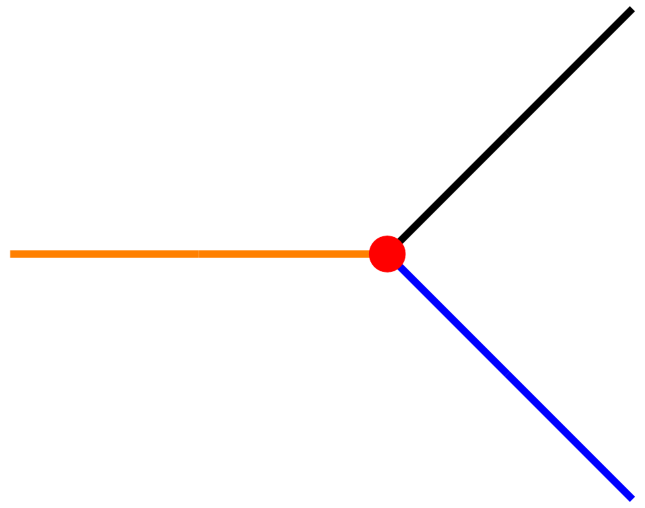
Junction condition:

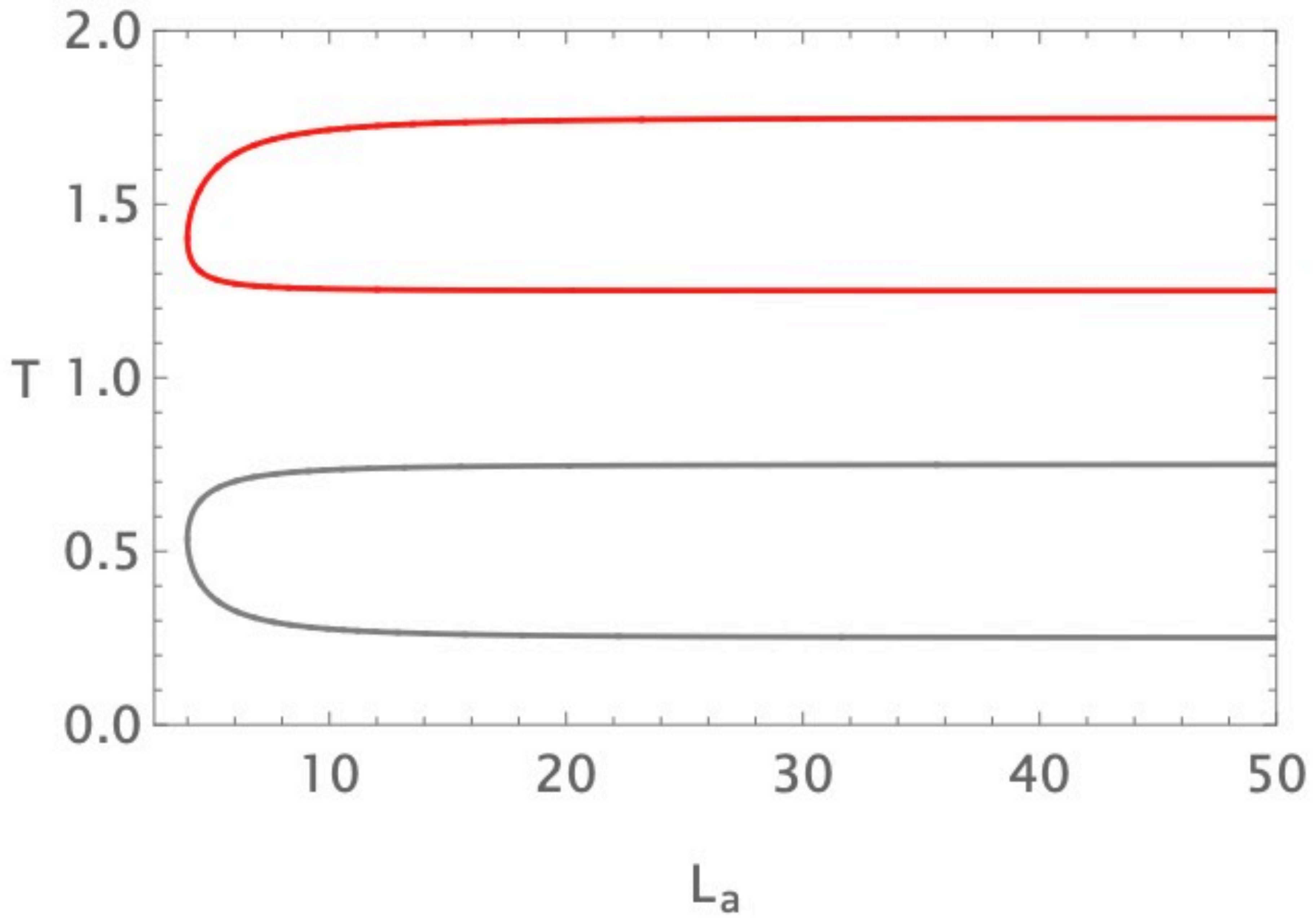
$$h_{\mu\nu} = g^{A}_{ab} \frac{\partial x^a_A}{\partial y^\mu} \frac{\partial x^b_A}{\partial y^\nu}\bigg|_Q$$

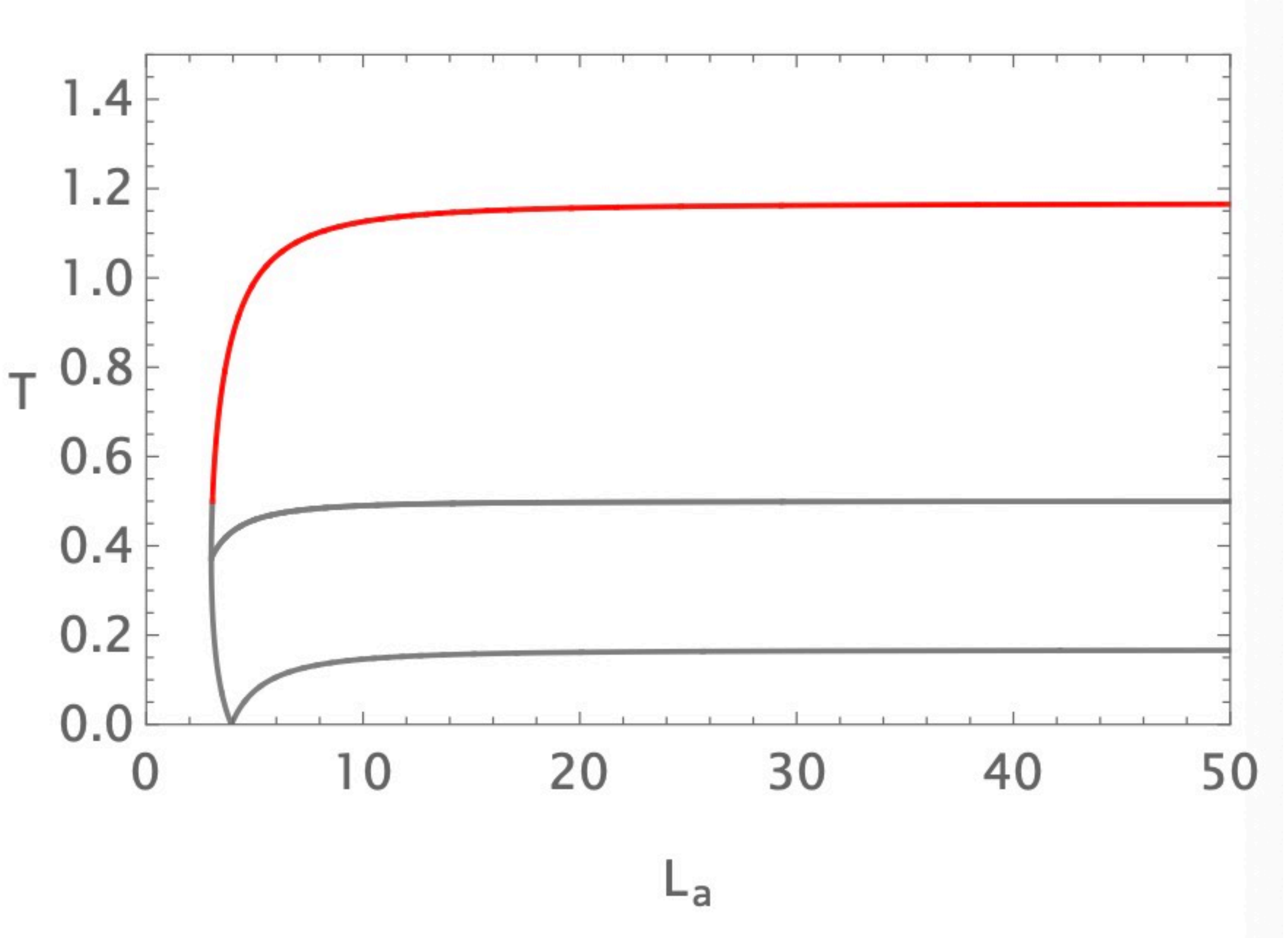
$$\sum_{A=1}^{3} K^A_{\mu\nu} - \left(\sum_{A=1}^{3} K^A - T\right) h_{\mu\nu} = 0$$

[Shen, Peng, Li, 2024]

Unique bulk solution







$L_1 \leq L_2 < L_3$ $\qquad L_1 + L_2 < L_3$

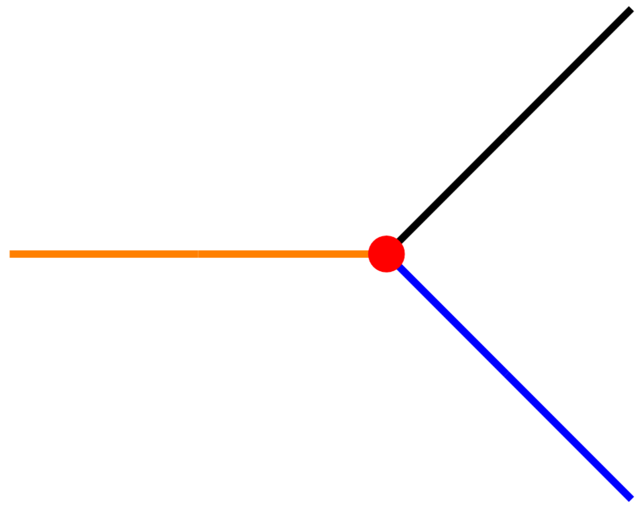
$$T \in \left(\frac{1}{L_1} + \frac{1}{L_2} - \frac{1}{L_3},\ \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}\right)$$

$$\log g \in (-\infty, +\infty)$$

$L_1 \leq L_2 = L_3$

$$T \in \left[\frac{1}{L_1},\ \frac{1}{L_1} + \frac{2}{L_2}\right)$$

Energy transport (I)



- Energy conservation

$$\mathcal{T}_{12} + \mathcal{T}_{13} + \mathcal{R}_1 = 1$$

- Total energy transmission

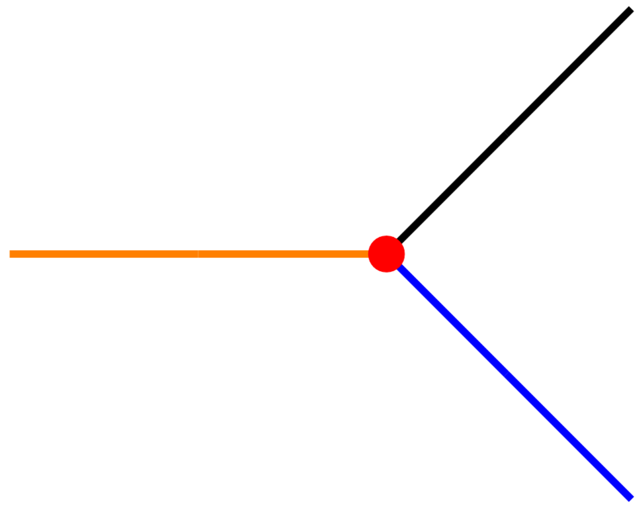
$$\mathcal{T}_A = \sum_{B \neq A} \mathcal{T}_{AB} = 1 - \mathcal{R}_A$$

- effective central charge

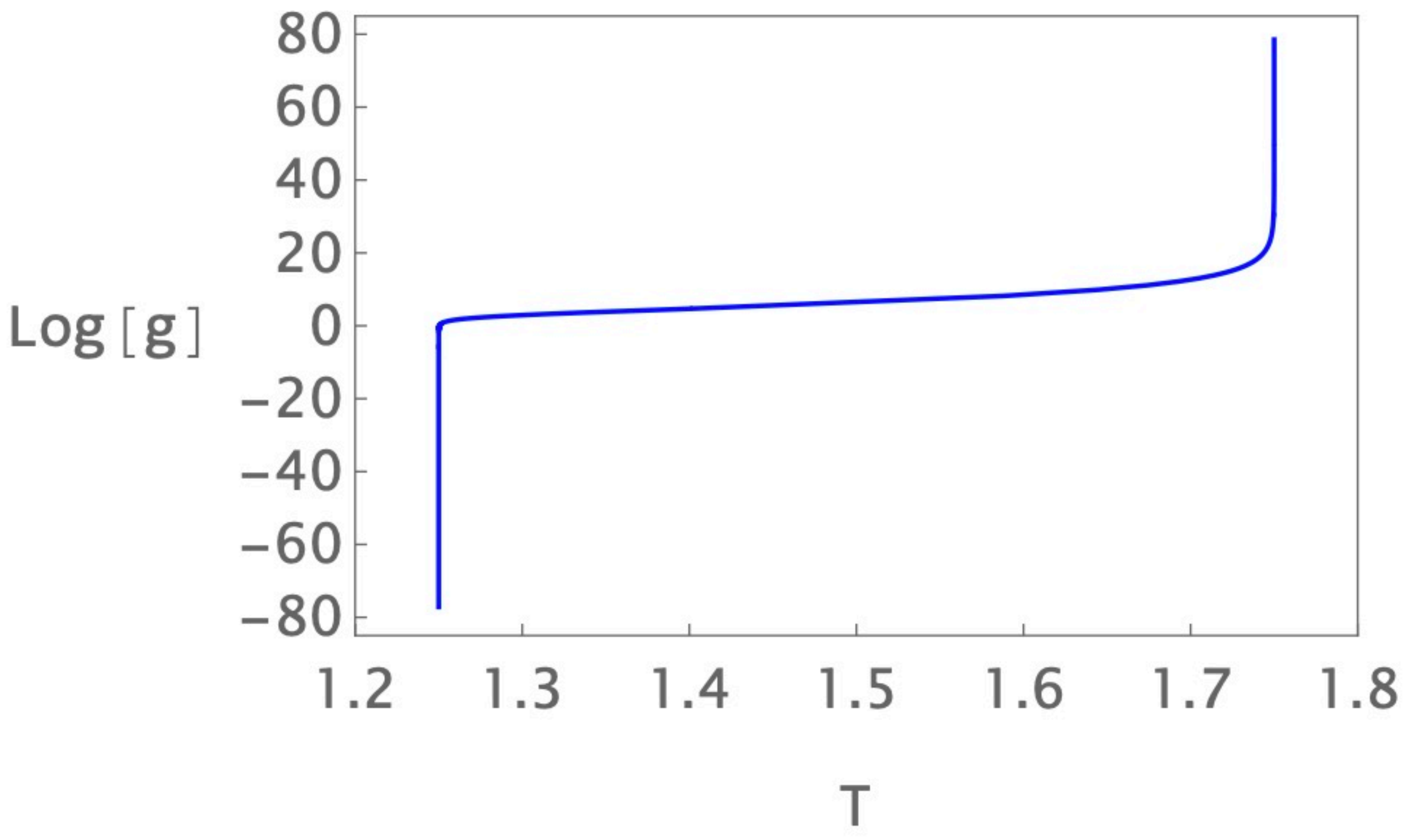
$$c_{\text{eff}}^{\text{A}} = \min\left(c_A, \sum_{\text{B}\neq\text{A}} c_{\text{B}}\right)$$

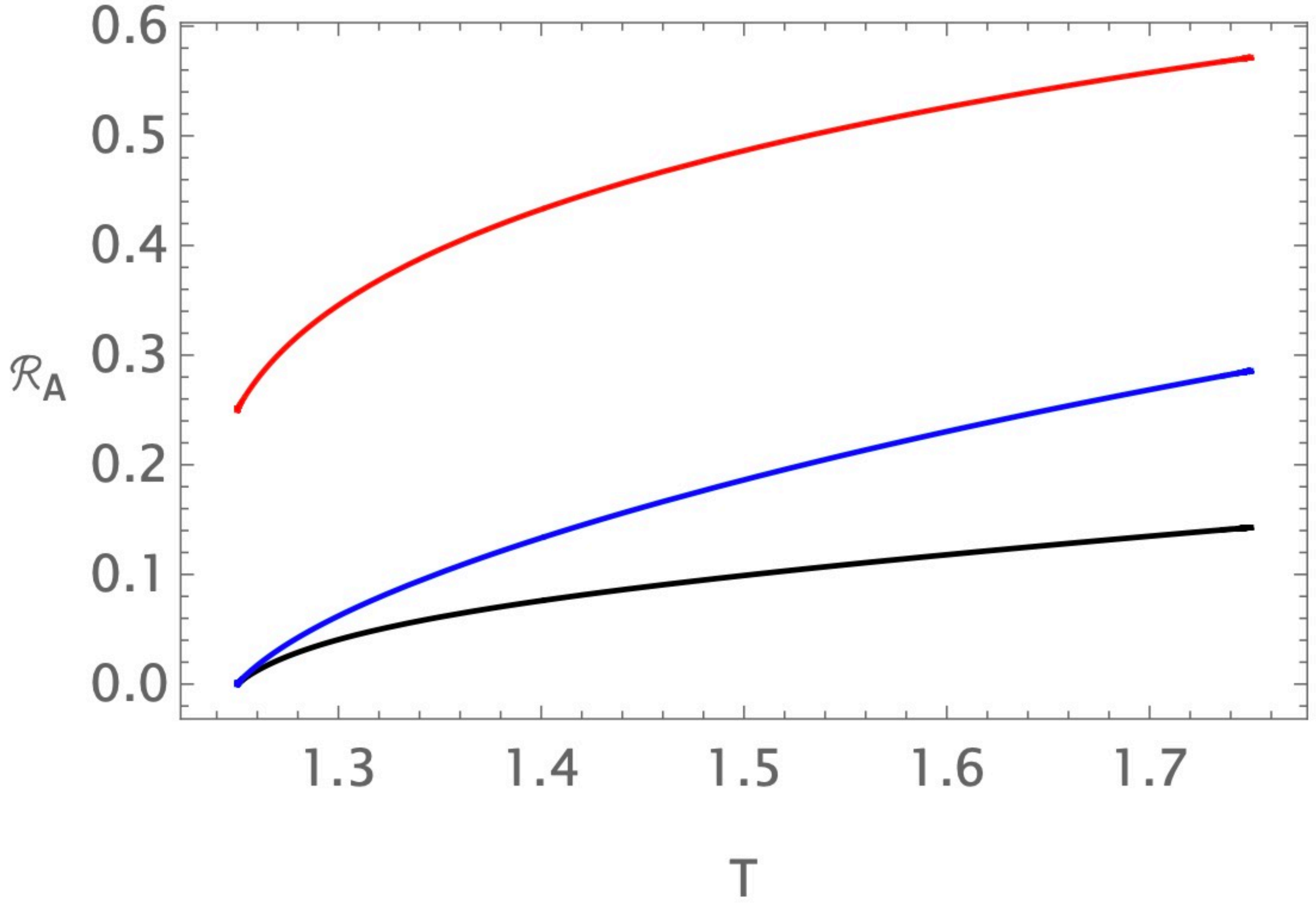
-

Energy transport (2)



- $L_1 \leq L_2 < L_3 \qquad L_1 + L_2 < L_3$





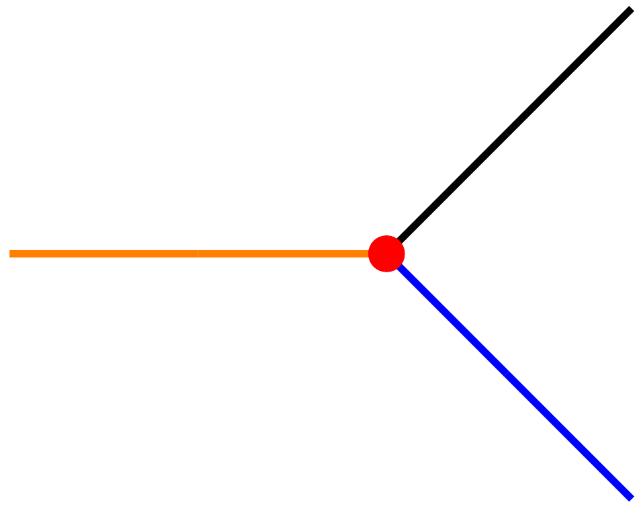
When T increases, the transmission coefficients decrease.

$$\mathcal{T}_1 \in \left(\frac{L_2 + L_3}{L_1 + L_2 + L_3}, 1\right), \quad \mathcal{T}_2 \in \left(\frac{L_1 + L_2}{L_1 + L_2 + L_3}, 1\right), \quad \mathcal{T}_3 \in \left(0, \frac{L_1 + L_2}{L_3}\right)$$

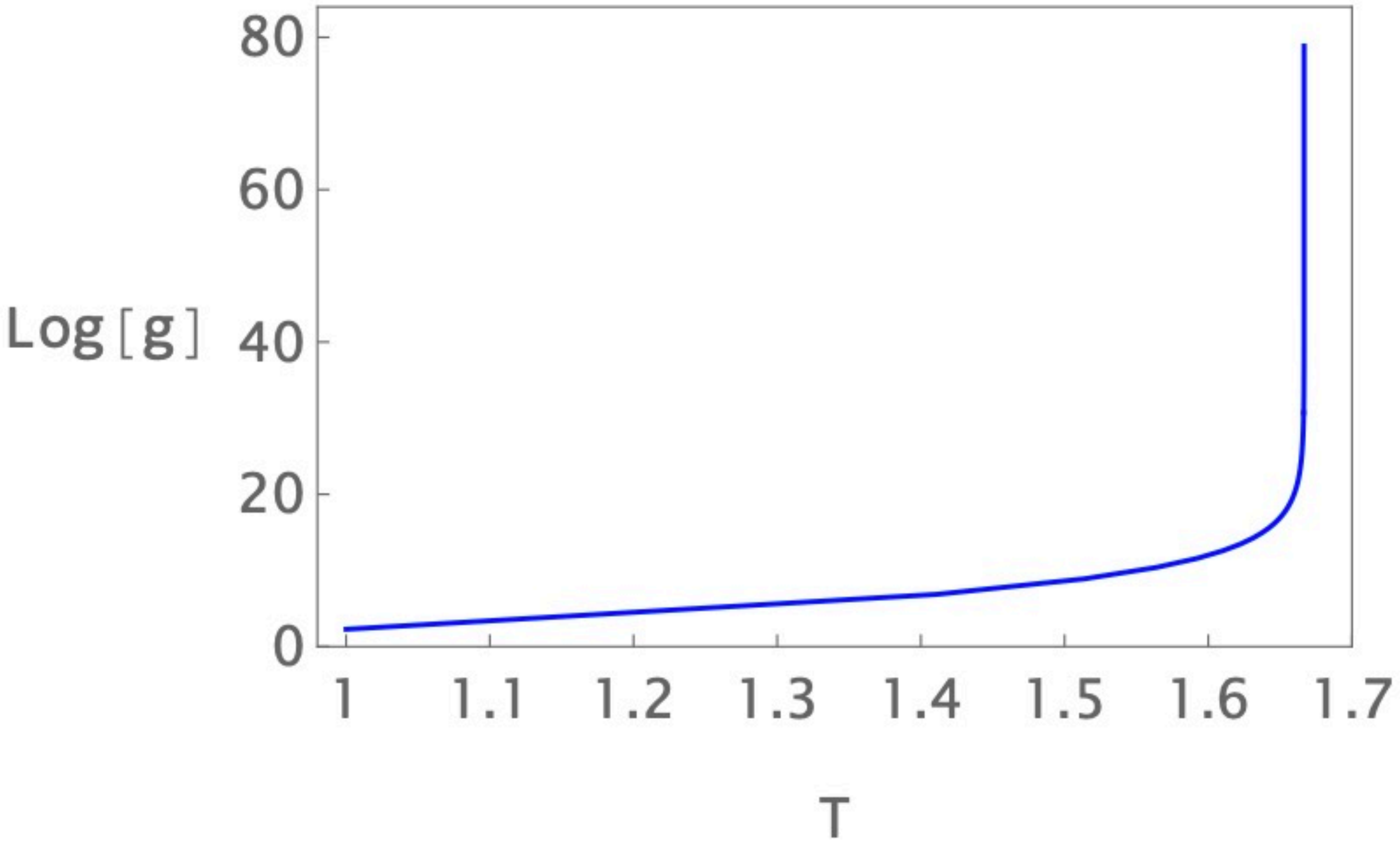
$$c_1 \mathcal{T}_1 < c_{\text{eff}}^1 , \; c_2 \mathcal{T}_2 \leq c_{\text{eff}}^2 , \; c_3 \mathcal{T}_3 \leq c_{\text{eff}}^3$$

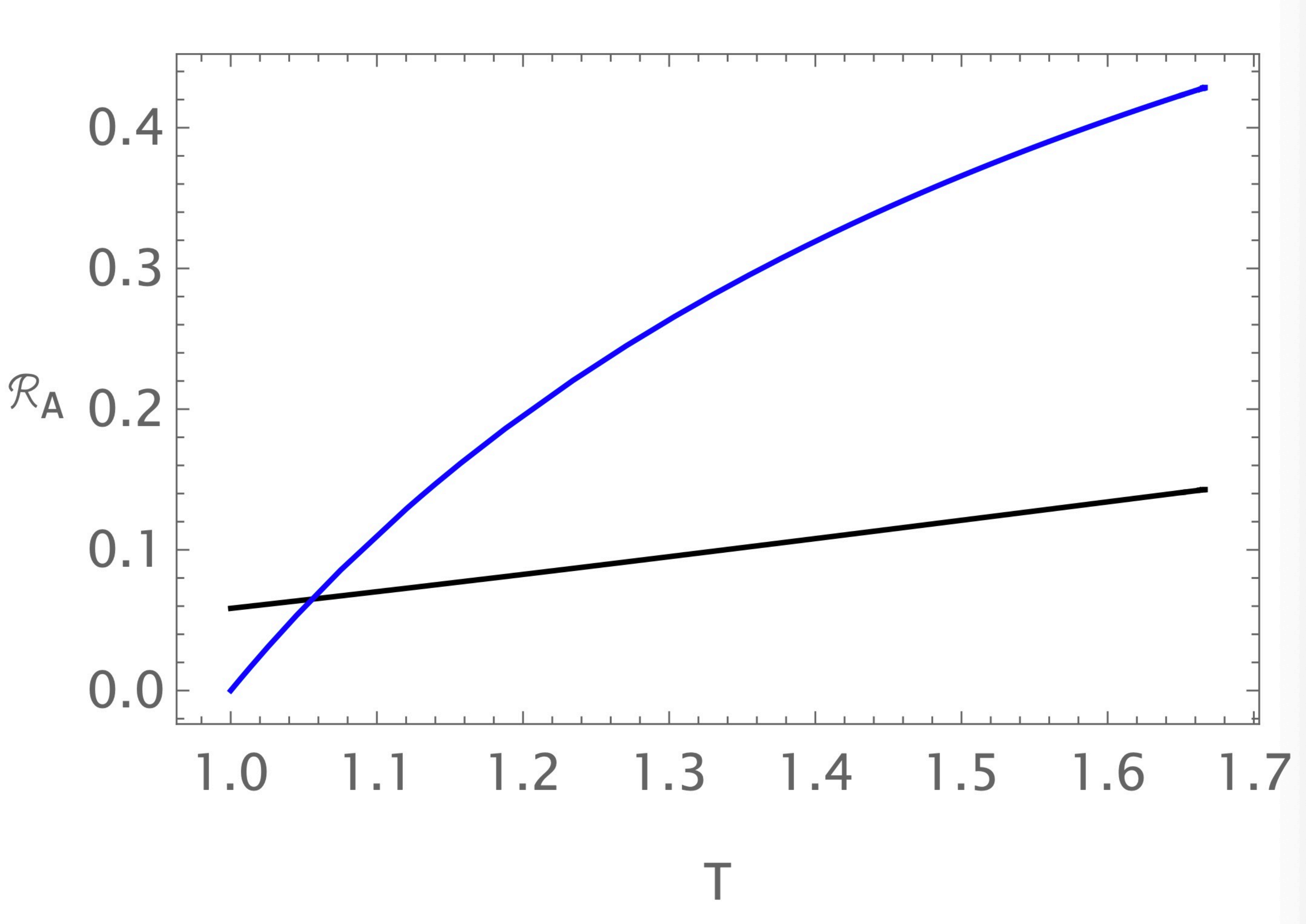
$$0 \leq \sum_{B \neq A} c_{AB} \leq c_{\text{eff}}^A \leq \min\left(c_A, \sum_{B \neq A} c_B\right)$$

Energy transport (3)



- $L_1 \leq L_2 = L_3$



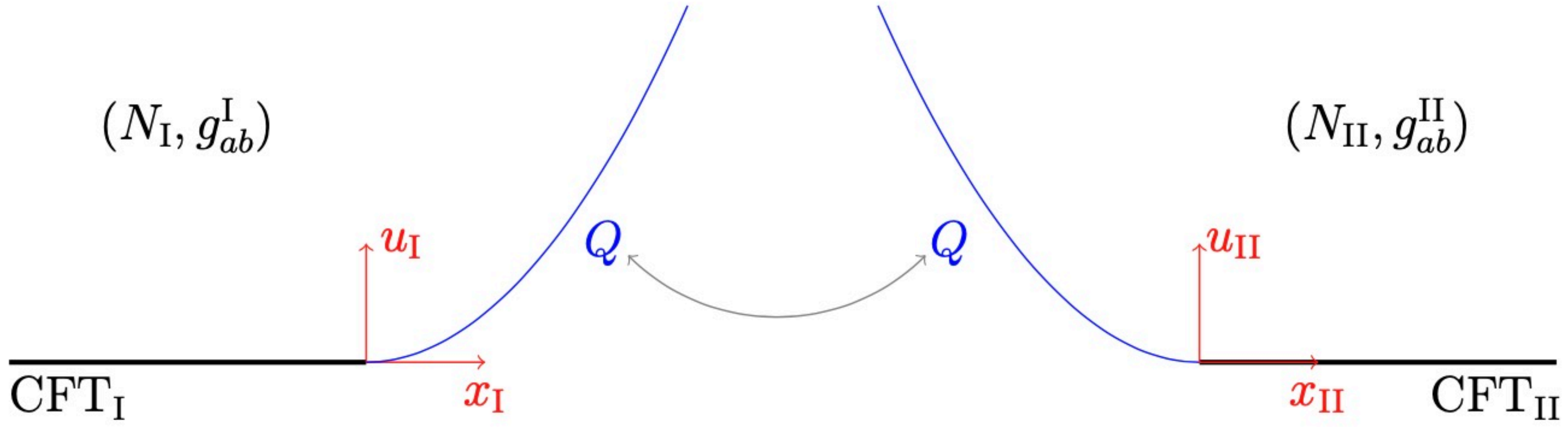


$$\mathcal{T}_1 \in \left(\frac{L_0 - L_1}{2L_0 + L_1}, \; 2\mathcal{T}_{12}^{\max}\right], \quad \mathcal{T}_2 = \mathcal{T}_3 \in \left(\frac{L_0 - L_1}{2L_0 + L_1}, \; 1\right]$$

$$c_1\mathcal{T}_1 < c_{\text{eff}}^1, \; c_2\mathcal{T}_2 \leq c_{\text{eff}}^2, \; c_3\mathcal{T}_3 \leq c_{\text{eff}}^3$$

$$0 \leq \sum_{B \neq A} c_{AB} \leq c_{\text{eff}}^A \leq \min\left(c_A, \sum_{B \neq A} c_B\right)$$

Conclusion



- AdS3/ICFT2 with a brane-localized scalar field
- At zero temperature
  - NEC is consistent with the existence of configuration
  - g-theorem is satisfied, the properties of interface entropy is linked to the potential
- At finite temperature: more allowed configurations

	E	H1	H2
E	√	√	×
H1	√	×	×
H2	×	×	√

- Energy transport at zero temperature
  - for non-conformal interface: complex, oscillations
  - for junctions: $0 \leq \sum_{B\neq A} c_{AB} \leq c^A_{\rm eff} \leq \min\Big(c_A, \sum_{B\neq A} c_B\Big)$

Open questions

- Adding other matter fields on the brane
- Surface/interface states of topological matter
- Phase transitions
- AdS/ICFT$_3$ at finite temperature
- Other transports
- …

Thank you !

Thank you !

Gluing two BTZ black holes

- Without scalar field: $\phi = 0, V = T$

$$u_{\rm I}(w) = \frac{u_{\rm I}^H L_{\rm I}}{\sqrt{L_{\rm I}^2 + (u_{\rm I}^H)^2 w}}\,, \qquad u_{\rm II}(w) = \frac{u_{\rm II}^H L_{\rm II}}{\sqrt{L_{\rm II}^2 + (u_{\rm II}^H)^2 w}}\,,$$

$$x_{\rm I}(w) = -\int_w^{+\infty} \frac{u_{\rm I}^H \eta_{\rm I}(\sigma)}{\sqrt{L_{\rm I}^2 + (u_{\rm I}^H)^2\sigma}} d\sigma\,, \quad x_{\rm II}(w) = -\int_w^{+\infty} \frac{u_{\rm II}^H \eta_{\rm II}(\sigma)}{\sqrt{L_{\rm II}^2 + (u_{\rm II}^H)^2\sigma}} d\sigma\,.$$

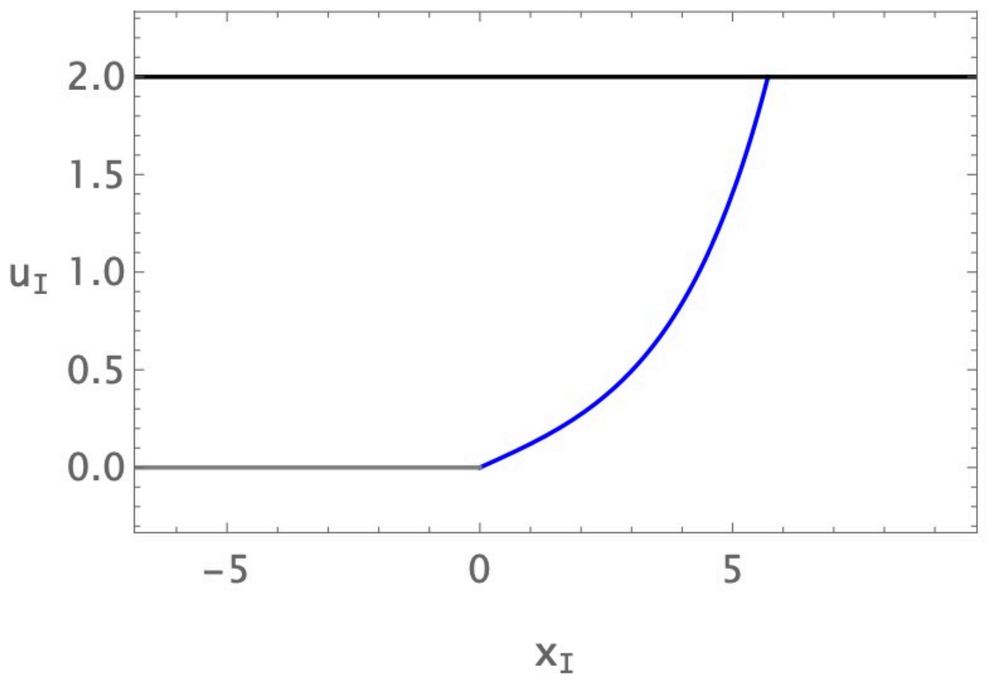
  - If $u_{\rm I}^H \neq u_{\rm II}^H$, e.g. [E, E], we have

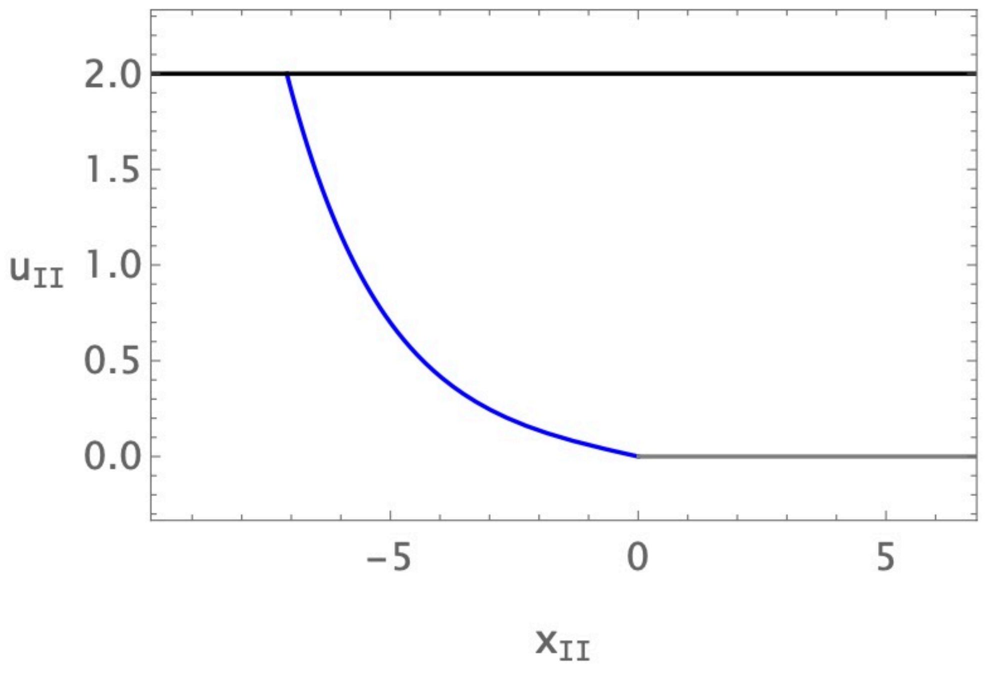
    $\eta_{\rm I} \propto \frac{1}{\sqrt{w - w_0}}$, thus $w \in (w_0, \infty)$, $x_{\rm I}$ terminates at a finite value

  - Therefore, $u_{\rm I}^H = u_{\rm II}^H$ [H2, H2]

$$x_{\rm I} = u_{\rm I}^H \,\mathrm{arctanh}\left(\frac{u_{\rm I}(-1+(1+L_{\rm I}^2T^2)\nu^2)}{\sqrt{u_{\rm I}^2(-1+(1+L_{\rm I}^2T^2)\nu^2)^2 + (u_{\rm I}^H)^2(-1+(1+L_{\rm I}^2T^2)\nu^2 - (-1+L_{\rm I}^2T^2)^2\nu^4)}}\right)$$

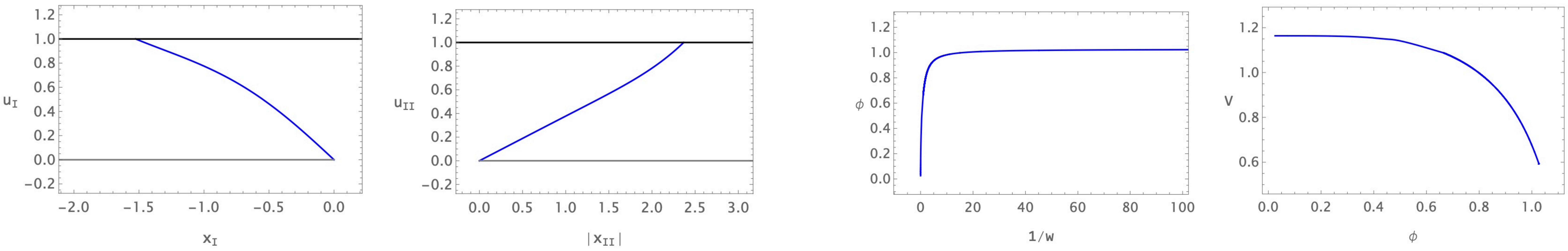
$$x_{\rm II} = u_{\rm I}^H \,\mathrm{arctanh}\left(\frac{u_{\rm II}(-1+(1-L_{\rm I}^2T^2)\nu^2)}{\sqrt{u_{\rm II}^2(1+(-1+L_{\rm I}^2T^2)\nu^2)^2 + (u_{\rm I}^H)^2(-1+(1+L_{\rm I}^2T^2)\nu^2 - (-1+L_{\rm I}^2T^2)^2\nu^4)}}\right)$$



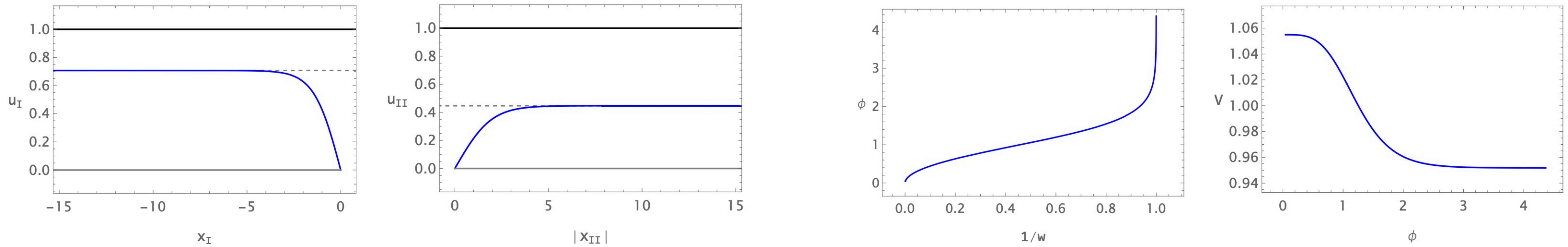


Gluing two BTZ black holes

- [H2, H2] $\eta_{\mathrm{I}} = \frac{1}{2L_{\mathrm{I}}}\frac{1}{\sqrt{a+bw^2}}$

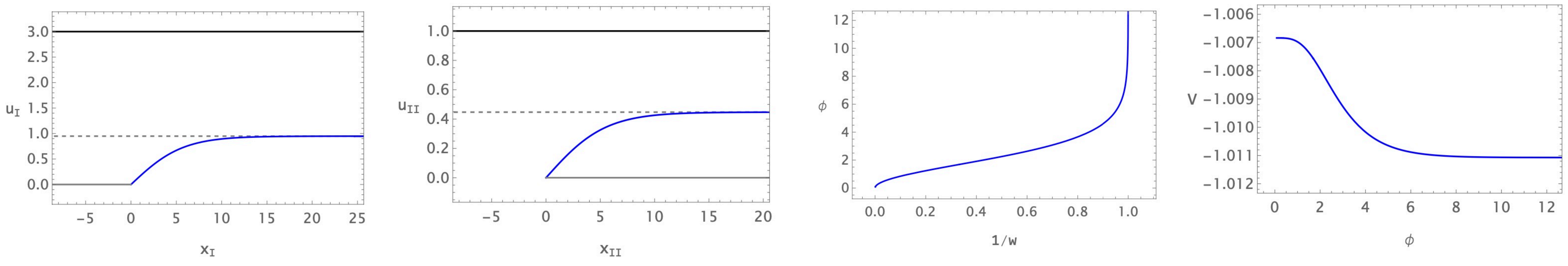


- [E, E] or [E H1] $\eta_{\mathrm{I}} = \frac{a}{w - bL_{\mathrm{I}}^2}$



Gluing two BTZ black holes

- [H1, E] $\eta_{\rm I} = -\dfrac{a}{w - bL_{\rm I}^2}$

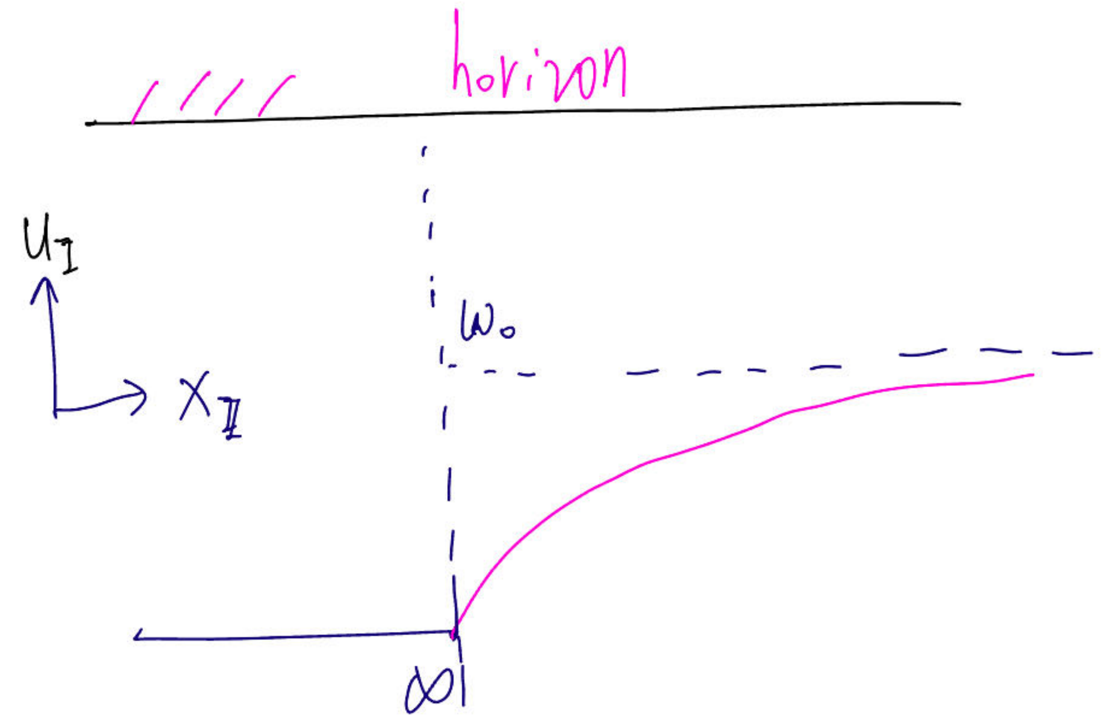


- no [E, H2], [H1, H2], [H2, E], [H2, H1] (the continuous condition of metric)
- no [H1, H1], forbidden by NEC $u_{\rm I}^H = u_{\rm II}^H$

$$\left(2\eta_{\rm I}^2 - \frac{(u_{\rm I}^H)^4}{(u_{\rm I}^H)^2 w + L_{\rm I}^2}\right)\eta_{\rm I} - (u_{\rm I}^H)^2\,\eta_{\rm I}' \geq 0$$

$$x_{\rm I}'(u_{\rm I}) \sim -\eta_{\rm I}(w) \to \infty \ \text{ when } w \to w_{0+}$$

$$\eta_{\rm I}' \to -\infty$$



g-theorem (1)

- When the induced metric on the interface brane is asymptotically AdS in both UV and IR, we have $S_{\rm iE}^{\rm UV} \geq S_{\rm iE}^{\rm IR}$

UV AdS2

$$\psi_{\rm I}(z) \simeq \gamma z^n + \cdots, \qquad \psi_{\rm II}(z) \simeq \pm\sqrt{\frac{1}{\nu} - \nu + \gamma^2 n^2 \delta_{1n}}\, z + \cdots,$$

$$S_{\rm iE}^{\rm UV} = \lim_{\sigma\to 0} S_{\rm iE}(\sigma) = \frac{c_{\rm I}}{6}\log\left(\gamma\sqrt{\nu}\delta_{1n} + \sqrt{1+\gamma^2\nu\delta_{1n}}\right) + \frac{c_{\rm II}}{6}\log\left(\frac{\sqrt{1+\gamma^2\nu\delta_{1n}} \mp \sqrt{1-\nu^2+\gamma^2\nu\delta_{1n}}}{\nu}\right)$$

IR AdS2

$$\psi_{\rm I}(z) \simeq \psi_{\rm I}'(+\infty) z + \ldots, \qquad \psi_{\rm II}(z) \simeq \pm\sqrt{\psi_{\rm I}'^2(\infty) + \frac{1}{\nu} - \nu}\, z + \ldots,$$

$$S_{\rm iE}^{\rm IR} = \lim_{\sigma\to +\infty} S_{\rm iE}(\sigma) = \frac{c_{\rm I}}{6}\log\left(\psi_{\rm I}'(+\infty)\sqrt{\nu} + \sqrt{1+\psi_{\rm I}'(+\infty)^2\nu}\right) + \frac{c_{\rm II}}{6}\log\frac{\sqrt{1+\psi_{\rm I}'(+\infty)^2\nu} \mp \sqrt{1+\psi_{\rm I}'(+\infty)^2\nu}}{\nu}$$

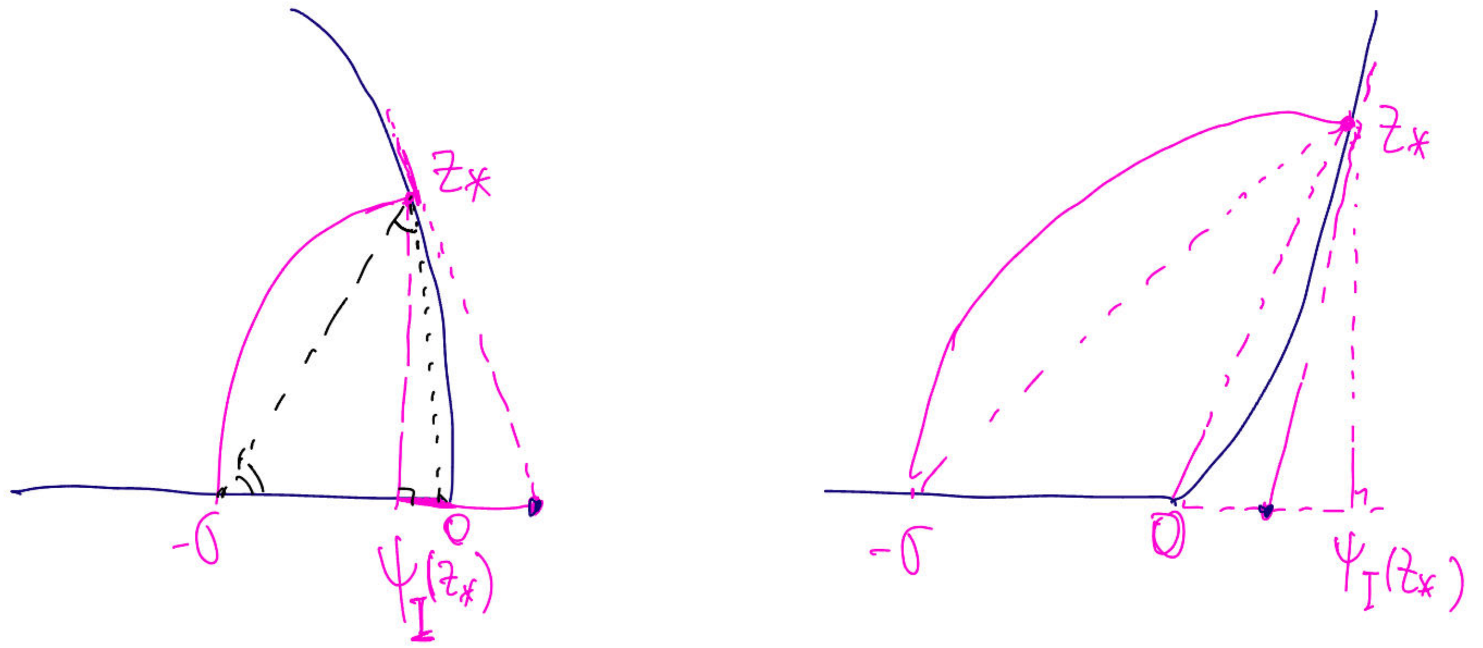
NEC

$$\psi_{\rm I}'(0) \geq \psi_{\rm I}'(+\infty).$$

- Along the RG flow, g-theorem is $\frac{d}{d\sigma} S_{\rm iE}(\sigma) \leq 0$

g-theorem (2)

- Along the RG flow, g-theorem is $\frac{d}{d\sigma} S_{\rm iE}(\sigma) \leq 0$
    - When $\nu = \frac{L_{\rm II}}{L_{\rm I}} = 1$, $\psi_{\rm I}(z) = -\psi_{\rm II}(z)$ unfolding the BCFT result, [SSA by takayanagi et al.], NEC,



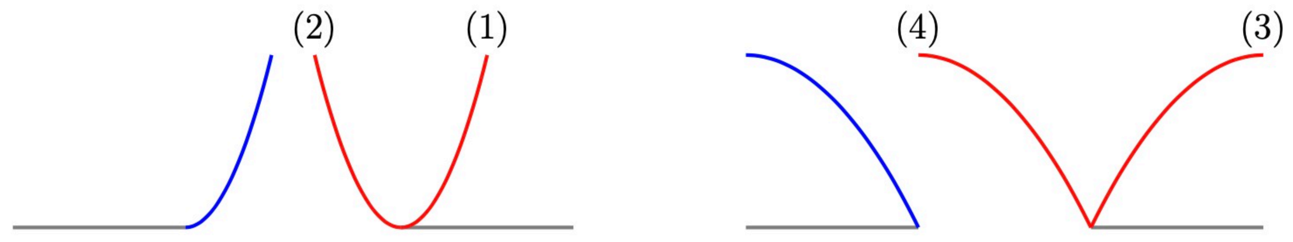
$$\frac{d}{d\sigma} S_{\rm iE}(\sigma) = -\frac{c_{\rm I}(z_*^2 - \sigma^2 + \psi_{\rm I}^2(z_*))}{3\sigma(z_*^2 + (\sigma + \psi_{\rm I}(z_*))^2)}$$

- When $0 < \nu < 1$ (for profiles of case 2)

$$\frac{d}{d\sigma} S_{\rm iE}(\sigma) = \frac{c_{\rm I}}{6\sigma}\left[-\frac{2\nu(\sigma + \psi_{\rm I}(z_*))(\psi_{\rm I}(z_*) - z\psi_{\rm I}'(z_*))}{z_*^2 + \nu(\sigma + \psi_{\rm I}(z_*))^2} + \frac{2\nu(\sigma - \psi_{\rm II}(z_*))(\psi_{\rm II}(z_*) - z\psi_{\rm II}'(z_*))}{z_*^2\nu + \nu(\sigma - \psi_{\rm II}(z_*))^2}\right]$$

(from NEC)

- For other cases, we numerically construct concrete examples.



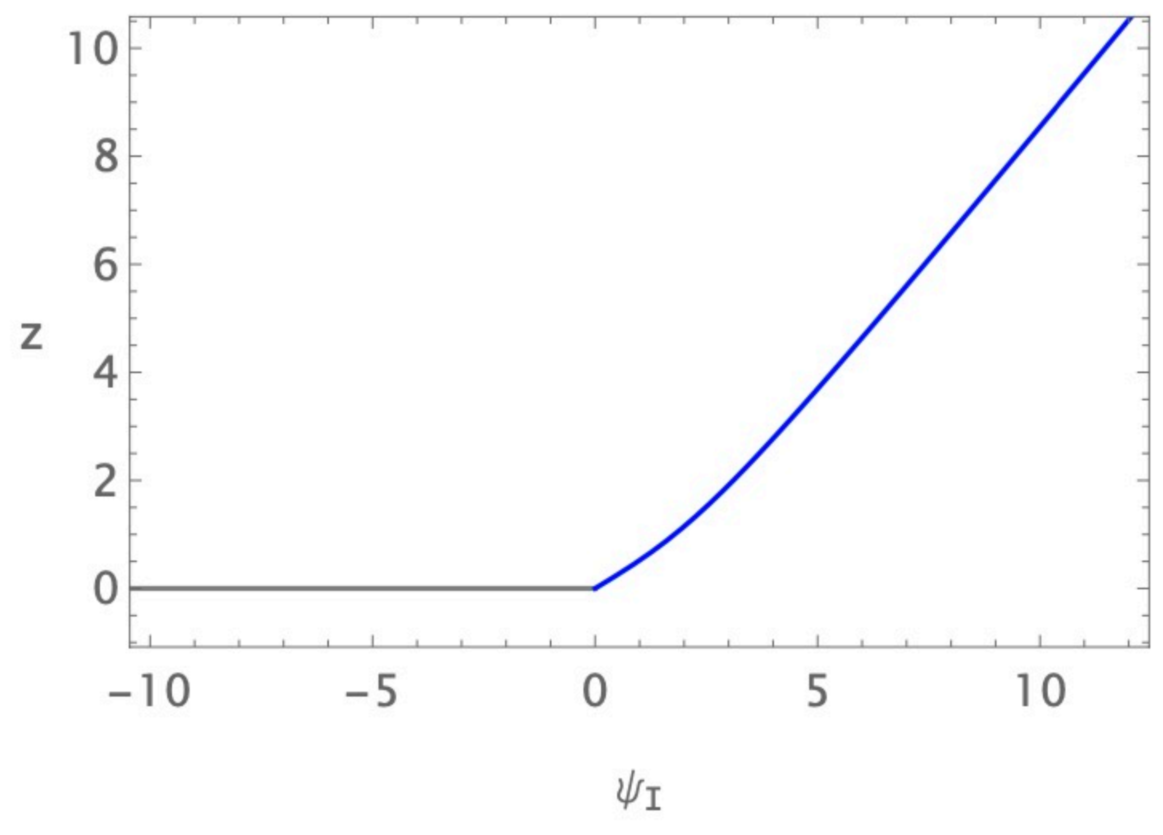
Features from case studies

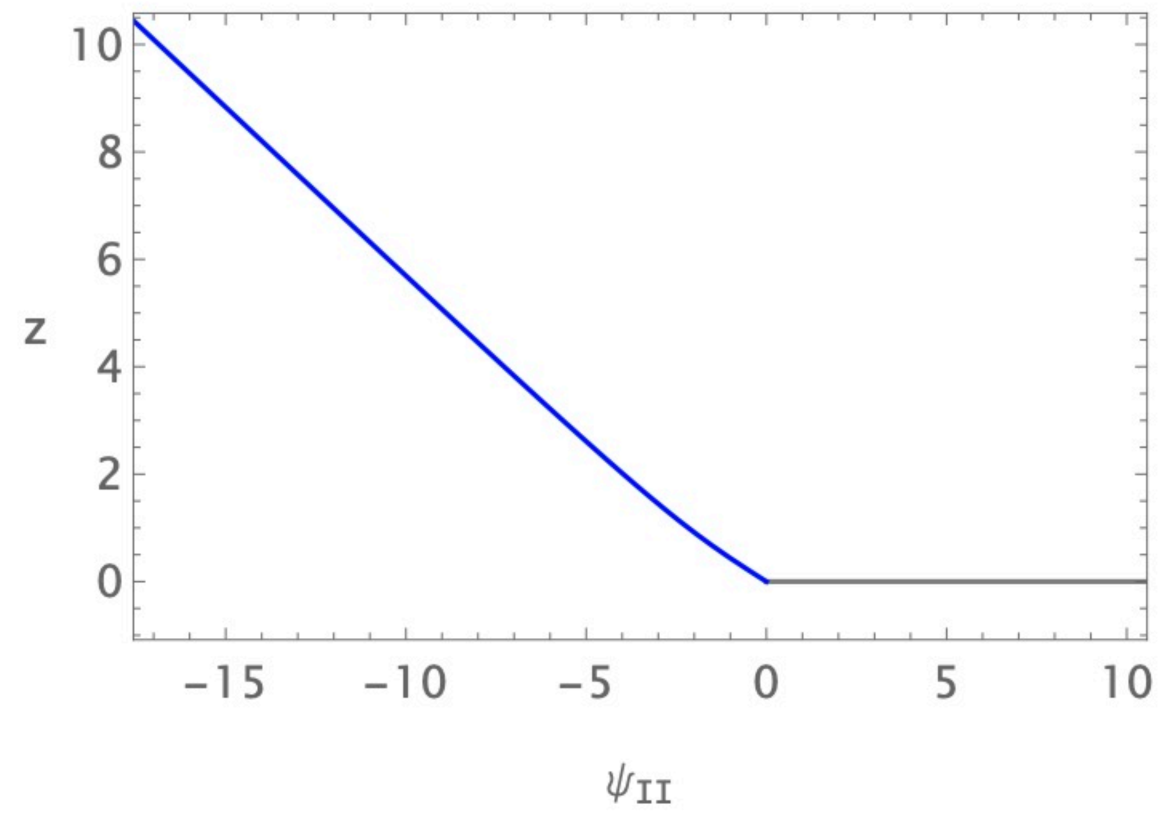
- Relation $\partial_\phi V(\phi)|_{\phi_{\rm UV}} = \partial_\phi V(\phi)|_{\phi_{\rm IR}} = 0$
- UV AdS_2 to IR AdS_2, potential evolves from locally minimal to globally maximum
- UV AdS_2 to IR flat, potential evolves from a locally maximal (VI) or a locally minimal (IV) in UV to a globally minimal in IR
- When the scalar potential is non-monotonic, (Solutions IV, V), there exits multiple extremal surfaces. A first-order phase transition occurs for the interface entropy
- When the induced metric is asymptotically flat in IR, $S_{\rm iE}(\sigma \to \infty)$ goes to $-\infty$
- The g-theorem is always consistently satisfied

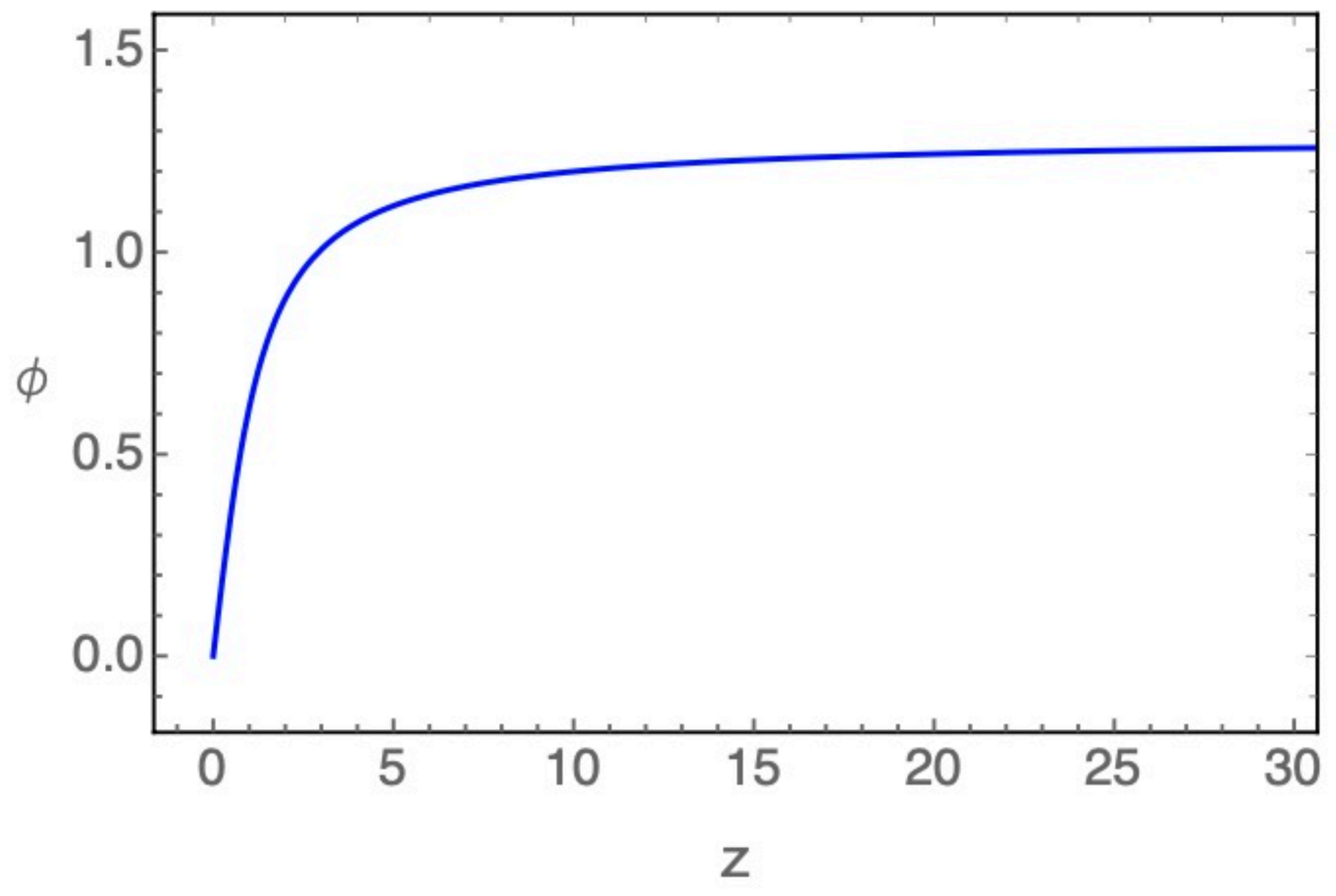
Solution III

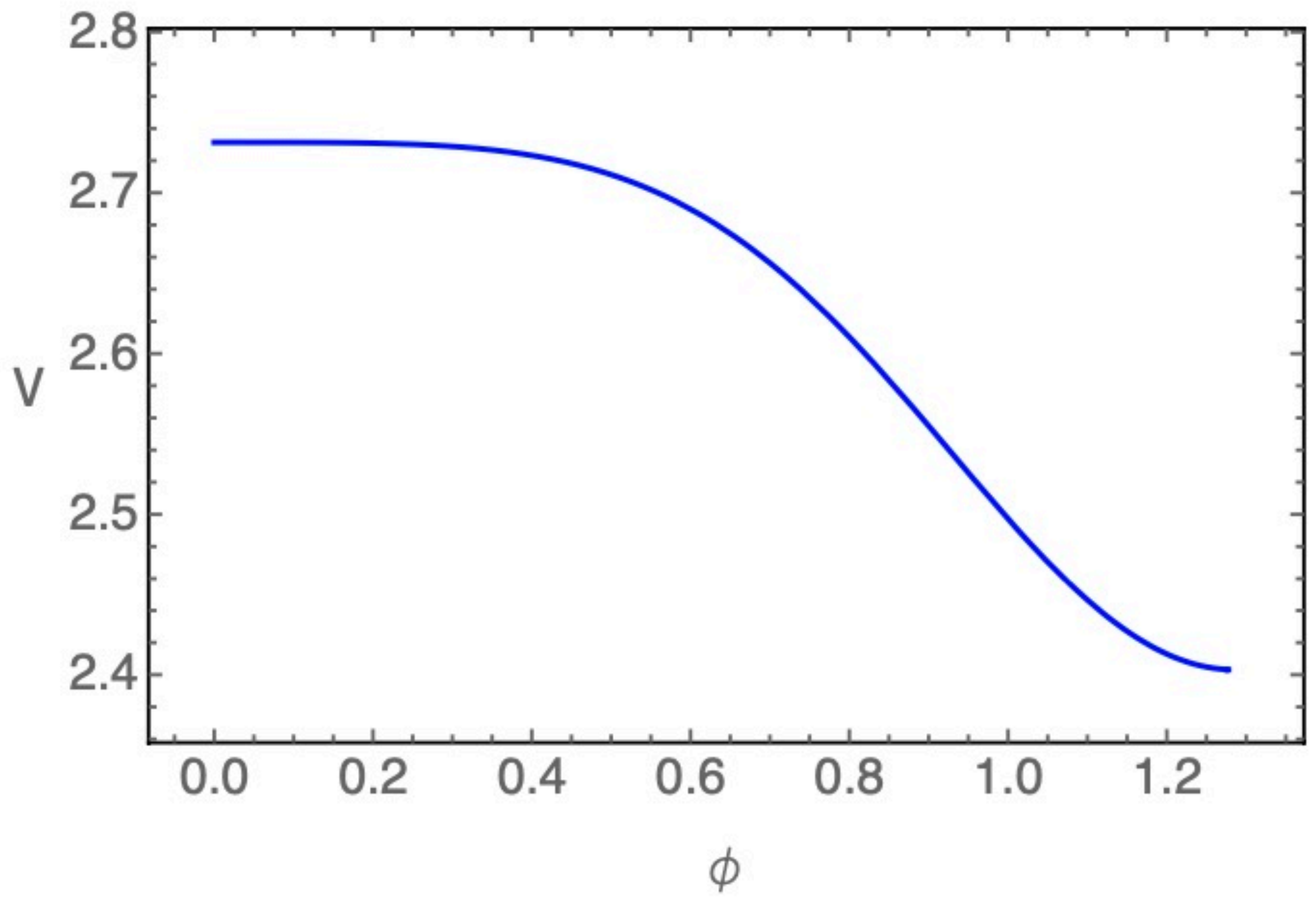
$$\psi_{\rm I}(z) = a\arctan(bz) + cz$$

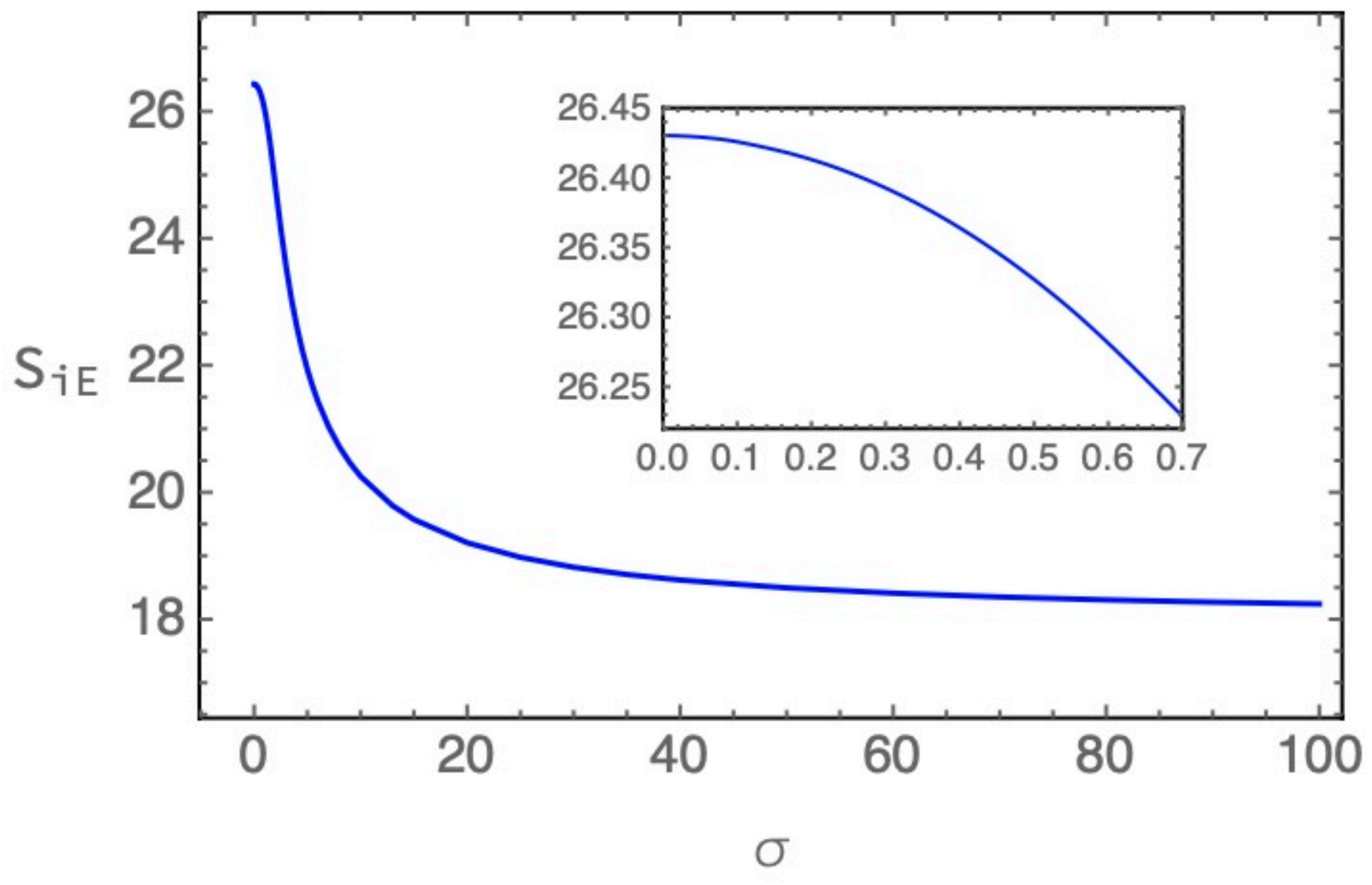
- NEC $\quad ab > 0$
- UV AdS2 ; IR AdS2









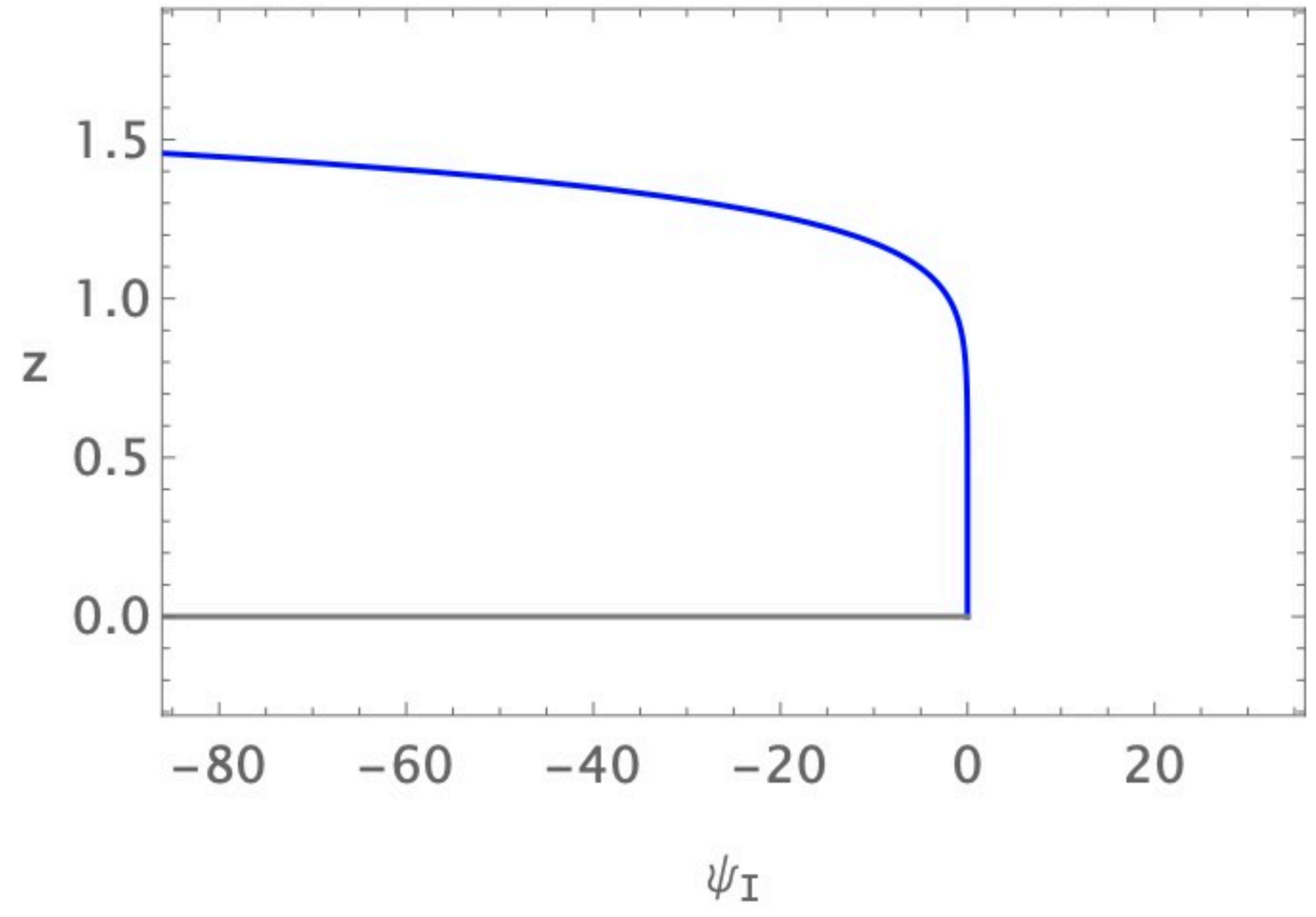


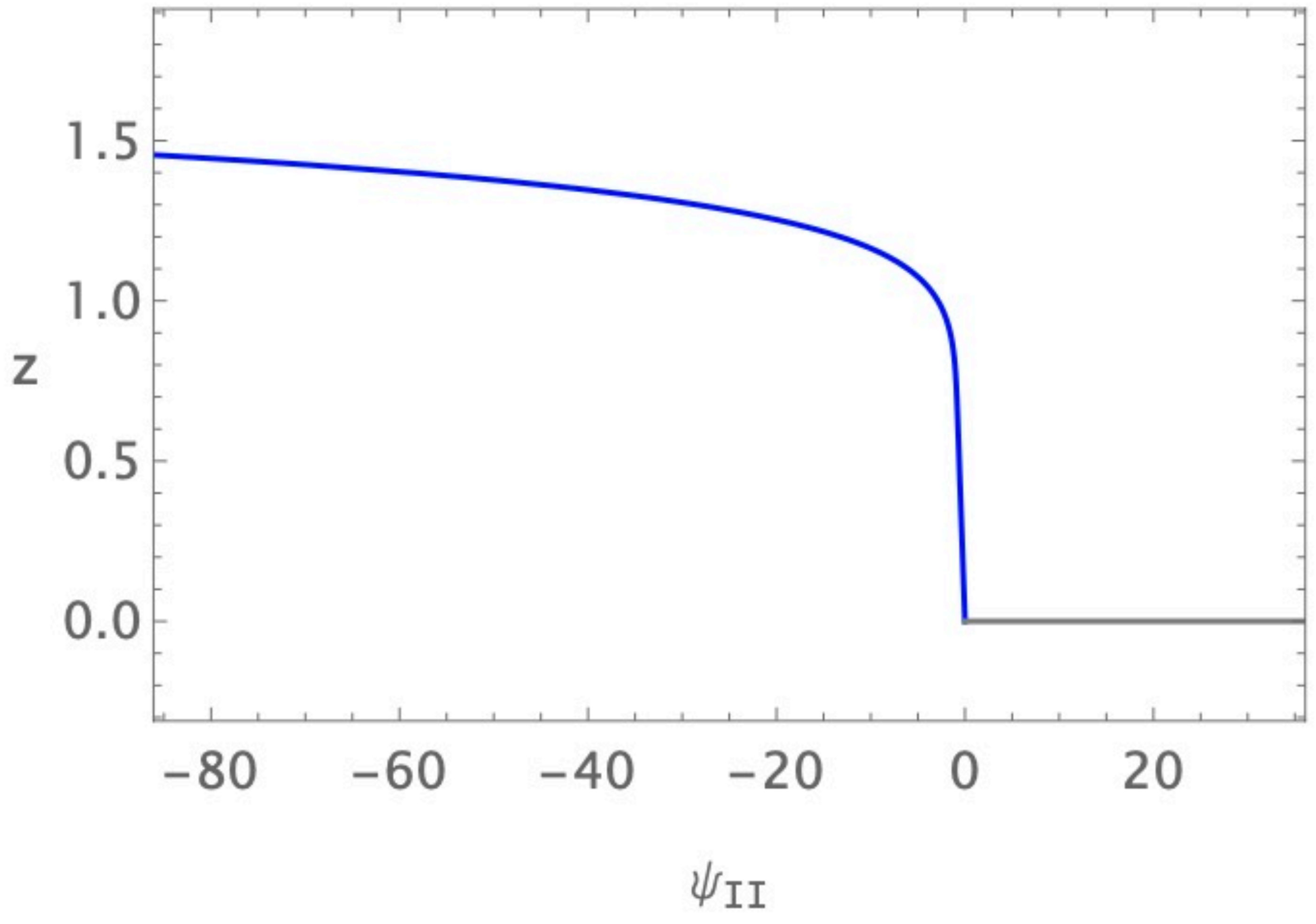
$a = b = c = 1\,,\nu = 0.5\,,L_{\rm I} = 1$

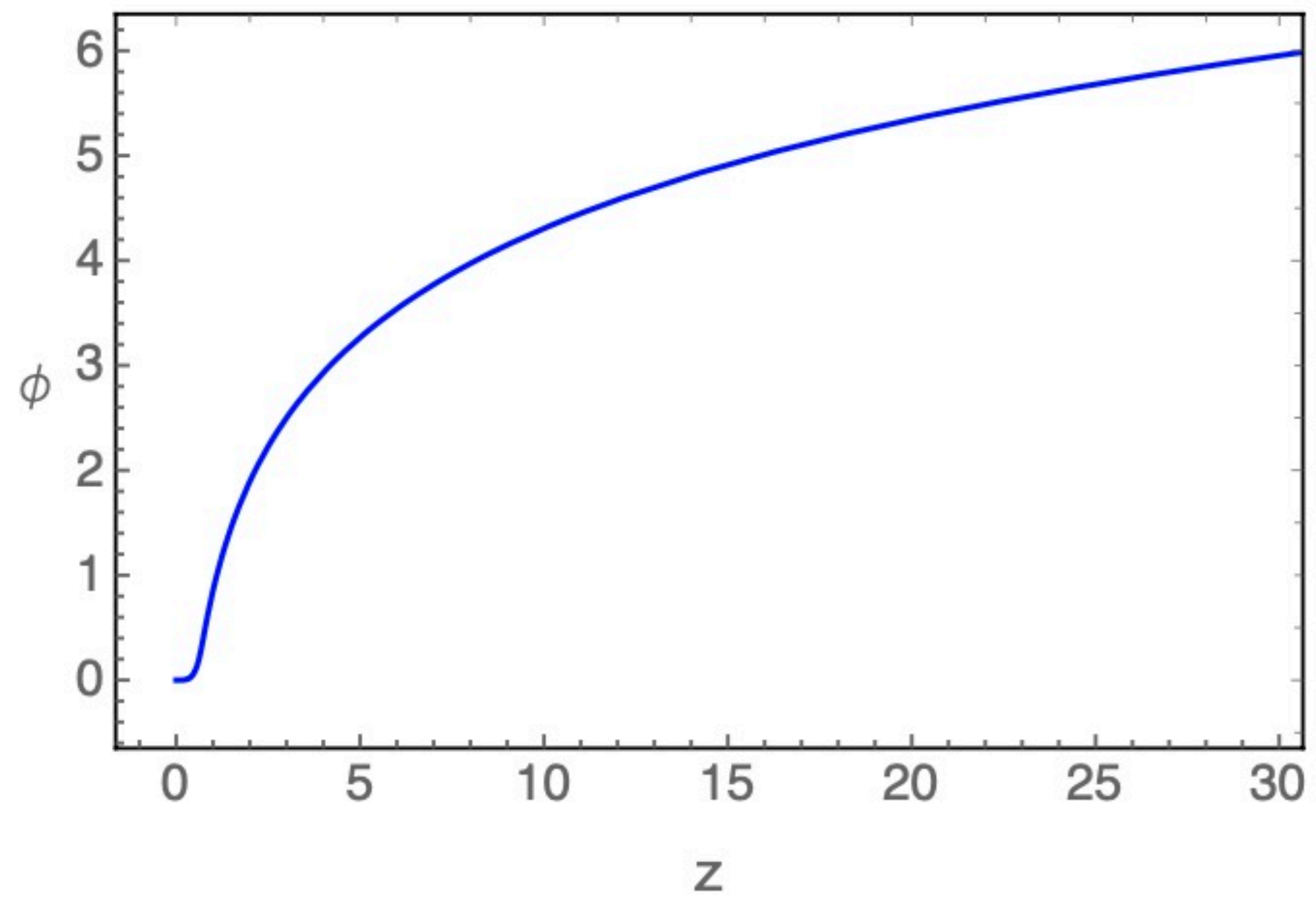
Solution VI

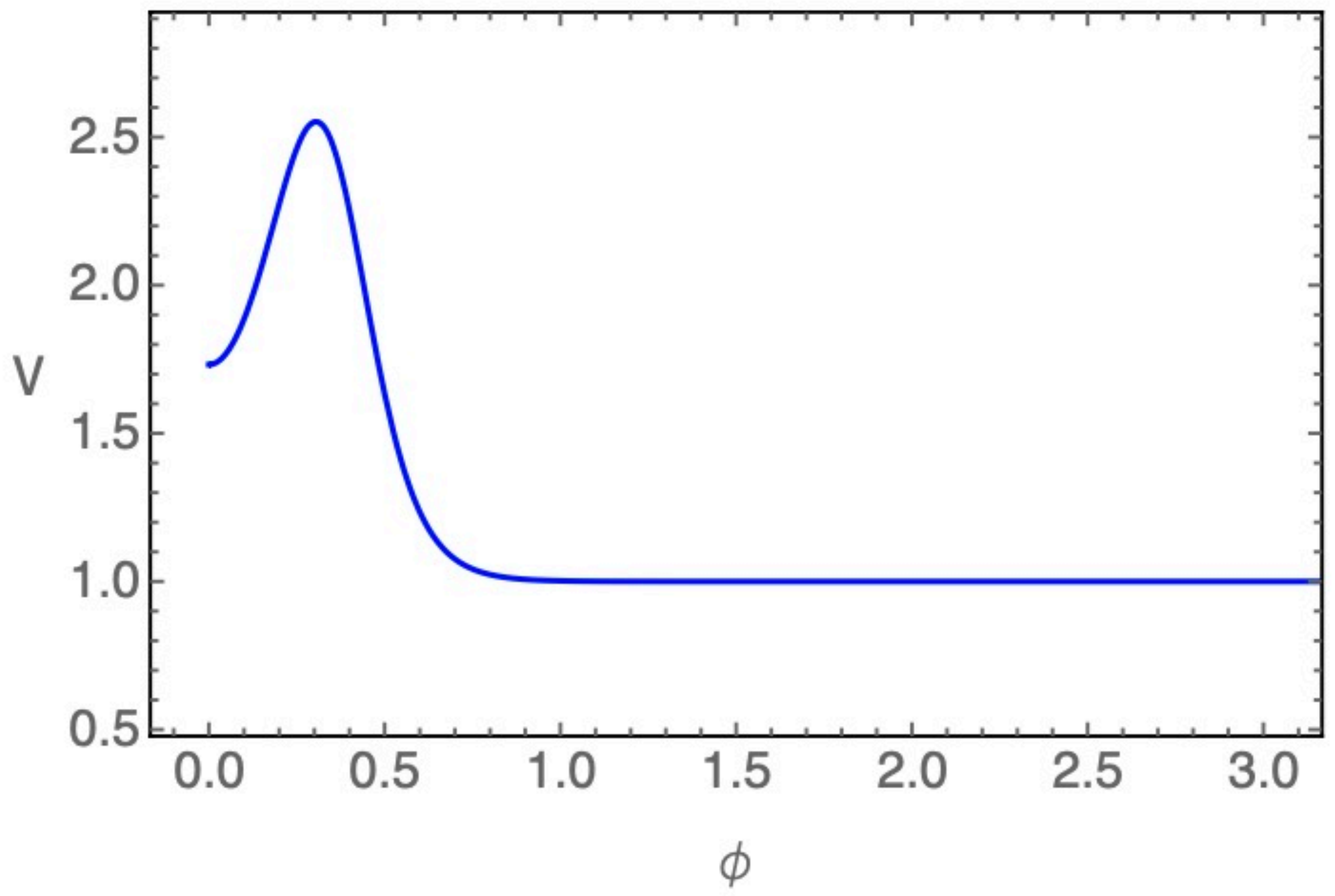
$$\psi_{\rm I} = \gamma \; z^n$$

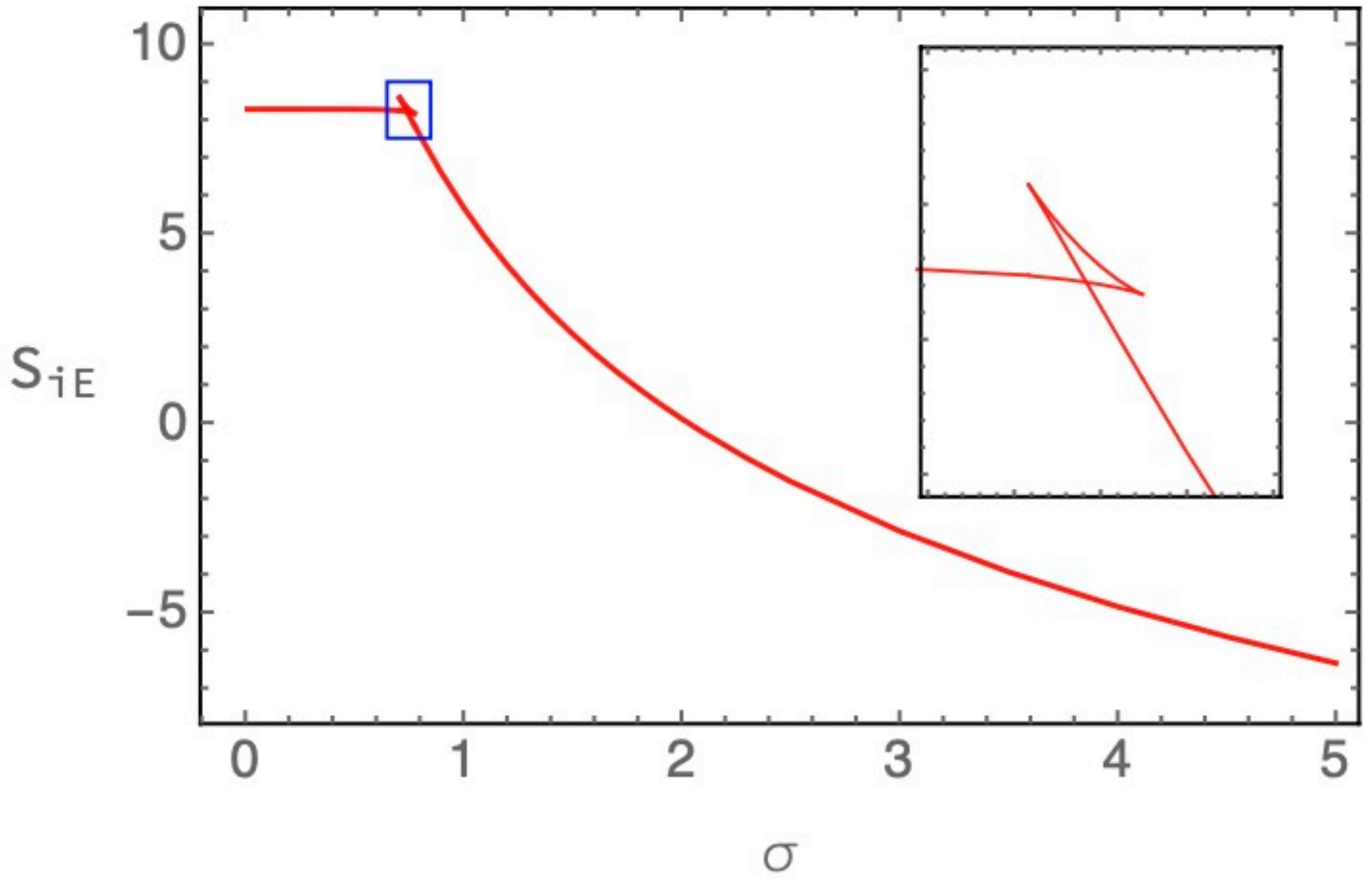
- NEC $\gamma < 0$ when $n > 1$
- UV AdS2 ; IR flat









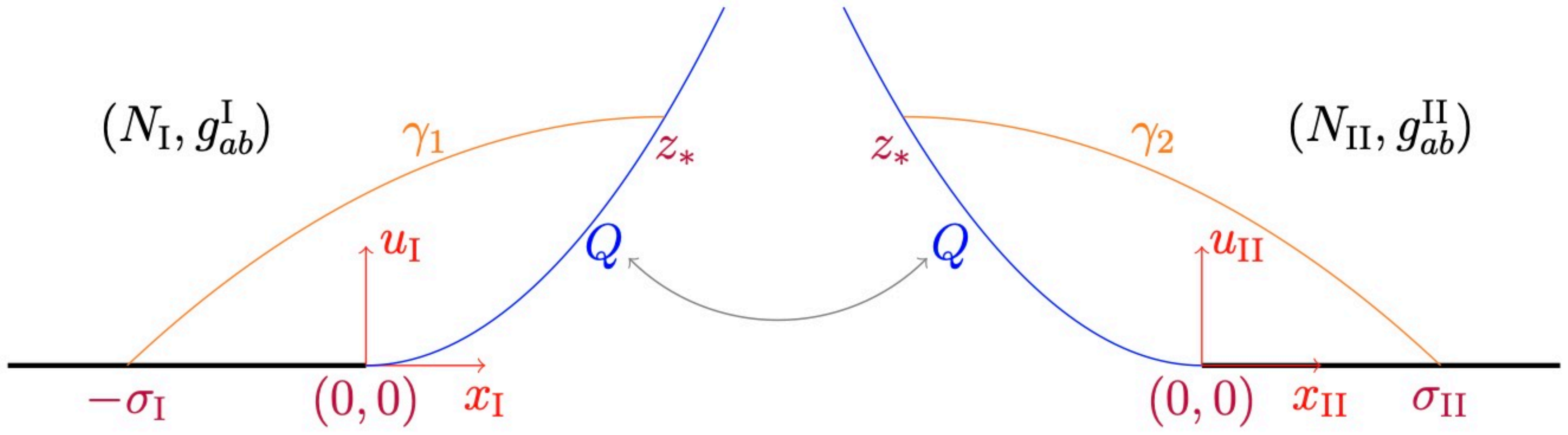


$\gamma = -2$, $n = 10$, $L_{\rm I} = 1$ and $\nu = 0.5$.

Holographic entanglement entropy

- Consider the interval $[-\sigma_{\mathrm{I}}, 0] \cup [0, \sigma_{\mathrm{II}}]$
- RT-formulae

    Boundary condition



Holographic entanglement entropy

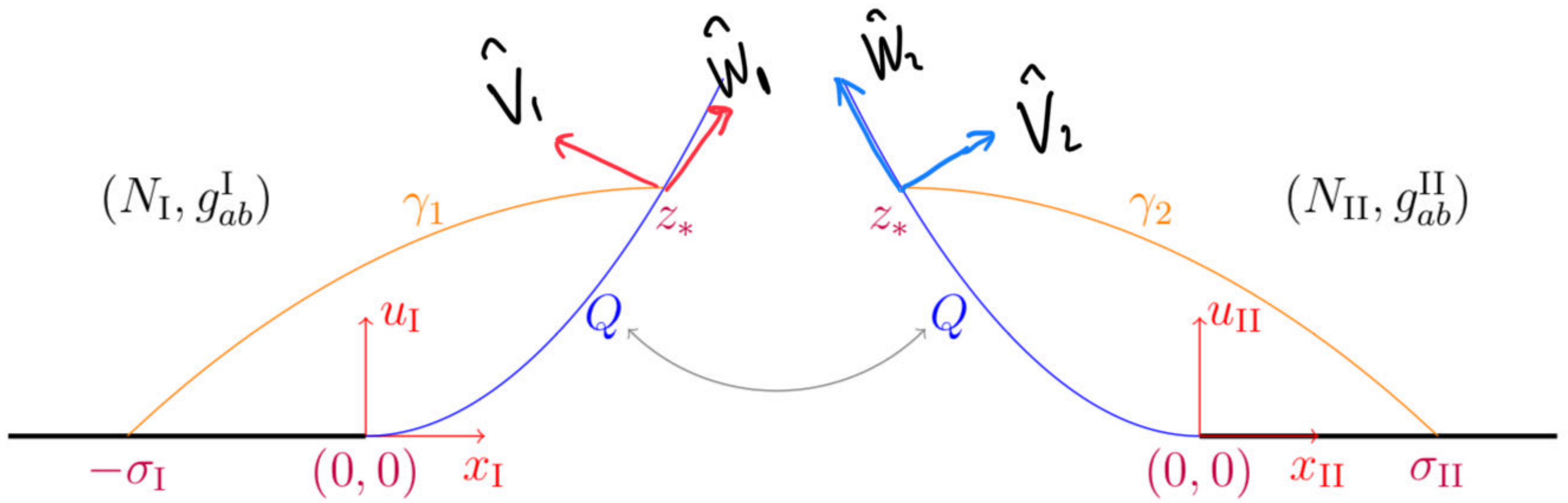
- Consider the interval $[-\sigma_{\mathrm{I}}, 0] \cup [0, \sigma_{\mathrm{II}}]$
- RT-formula

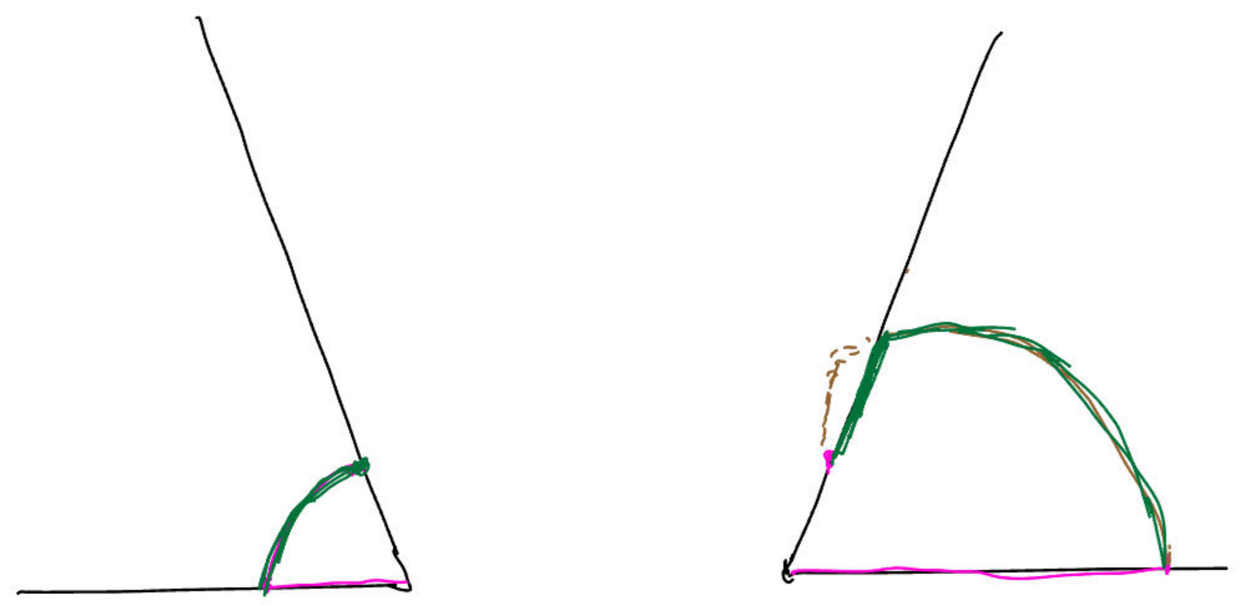
  Boundary condition

  $$\hat{V}_1 \cdot \hat{W}_1 + \hat{V}_2 \cdot \hat{W}_2 = 0$$

  Squashed geodesics

- BCFT limits
- Effective central charge from holographic entanglement entropy





[Anous, Meineri, Pelliconi, Sonner; Czech, Nguyen, Swaminathan]

Interface in holography

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Holographic Josephson Junctions

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We construct a gravitational dual of a Josephson junction. Calculations on the gravity side reproduce the standard relation between the current across the junction and the phase difference of the condensate. We also study the dependence of the maximum current on the temperature and size of the junction and reproduce familiar results.